the actual speed of the cars (electrons): (Figure 14.4)

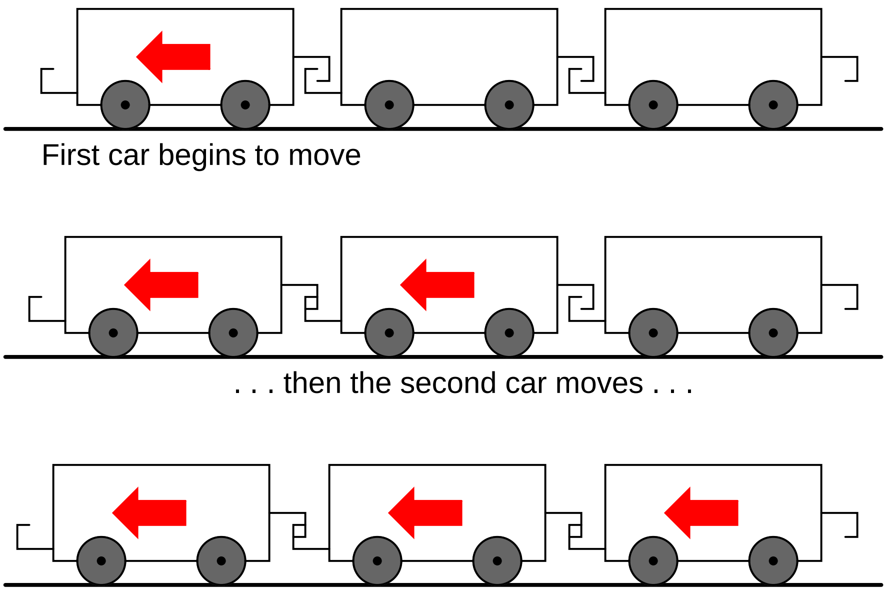


Figure 14.4: *Motion is transmitted sucessively from one car to next.*

Another analogy, perhaps more fitting for the subject of transmission lines, is that of waves in water. Suppose a flat, wall-shaped object is suddenly moved horizontally along the surface of water, so as to produce a wave ahead of it. The wave will travel as water molecules bump into each other, transferring wave motion along the water's surface far faster than the water molecules themselves are actually traveling: (Figure 14.5)

Likewise, electron motion "coupling" travels approximately at the speed of light, although the electrons themselves don't move that quickly. In a very long circuit, this "coupling" speed would become noticeable to a human observer in the form of a short time delay between switch action and lamp action.

- **REVIEW:**

- In an electric circuit, the effects of electron motion travel approximately at the speed of light, although electrons within the conductors do not travel anywhere near that velocity.

14.3 Characteristic impedance

Suppose, though, that we had a set of parallel wires of *infinite* length, with no lamp at the end. What would happen when we close the switch? Being that there is no longer a load at the end of the wires, this circuit is open. Would there be no current at all? (Figure 14.6)

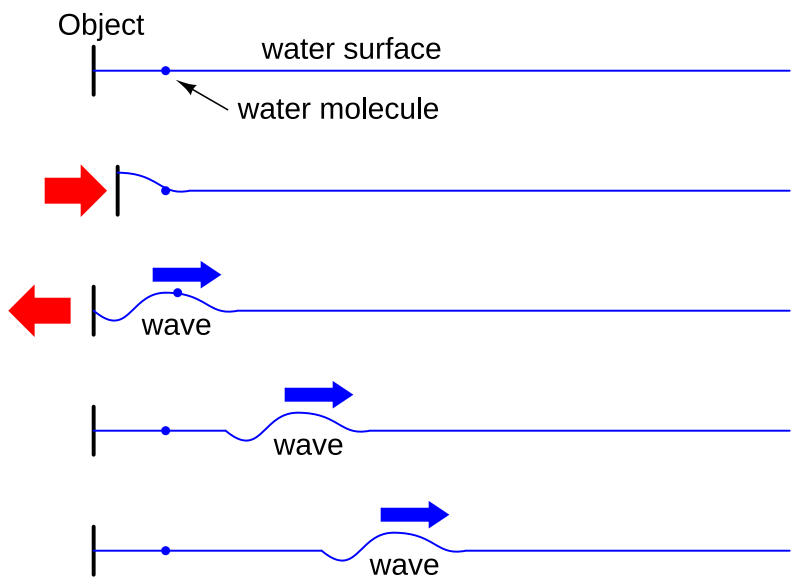


Figure 14.5: *Wave motion in water.*

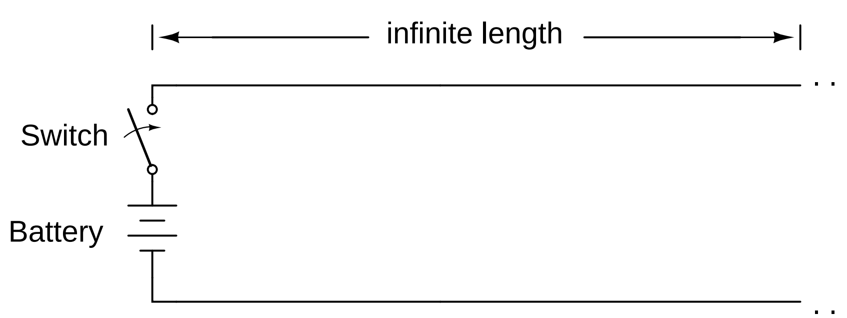


Figure 14.6: *Driving an infinite transmission line.*

Despite being able to avoid wire resistance through the use of superconductors in this “thought experiment,” we cannot eliminate capacitance along the wires’ lengths. *Any* pair of conductors separated by an insulating medium creates capacitance between those conductors: (Figure 14.7)

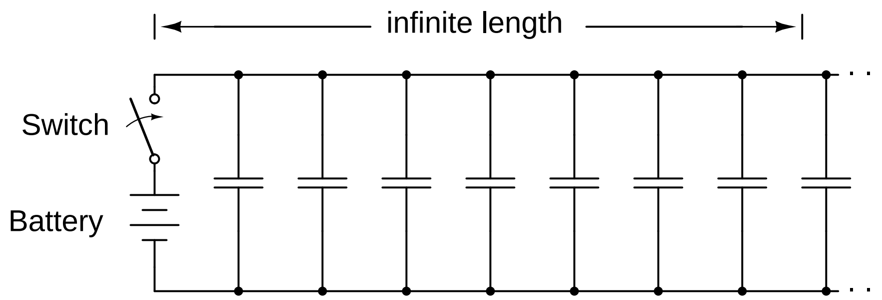


Figure 14.7: *Equivalent circuit showing stray capacitance between conductors.*

Voltage applied between two conductors creates an electric field between those conductors. Energy is stored in this electric field, and this storage of energy results in an opposition to change in voltage. The reaction of a capacitance against changes in voltage is described by the equation i = C(de/dt), which tells us that current will be drawn proportional to the voltage’s rate of change over time. Thus, when the switch is closed, the capacitance between conductors will react against the sudden voltage increase by charging up and drawing current from the source. According to the equation, an instant rise in applied voltage (as produced by perfect switch closure) gives rise to an infinite charging current.

However, the current drawn by a pair of parallel wires will not be infinite, because there exists series impedance along the wires due to inductance. (Figure 14.8) Remember that current through *any* conductor develops a magnetic field of proportional magnitude. Energy is stored in this magnetic field, (Figure 14.9) and this storage of energy results in an opposition to change in current. Each wire develops a magnetic field as it carries charging current for the capacitance between the wires, and in so doing drops voltage according to the inductance equation e = L(di/dt). This voltage drop limits the voltage rate-of-change across the distributed capacitance, preventing the current from ever reaching an infinite magnitude:

Because the electrons in the two wires transfer motion to and from each other at nearly the speed of light, the “wave front” of voltage and current change will propagate down the length of the wires at that same velocity, resulting in the distributed capacitance and inductance progressively charging to full voltage and current, respectively, like this: (Figures 14.10, 14.11, 14.12, 14.13)

The end result of these interactions is a constant current of limited magnitude through the battery source. Since the wires are infinitely long, their distributed capacitance will never fully charge to the source voltage, and their distributed inductance will never allow unlimited charging current. In other words, this pair of wires will draw current from the source so long as the switch is closed, behaving as a constant load. No longer are the wires merely conductors of electrical current and carriers of voltage, but now constitute a circuit component in themselves,

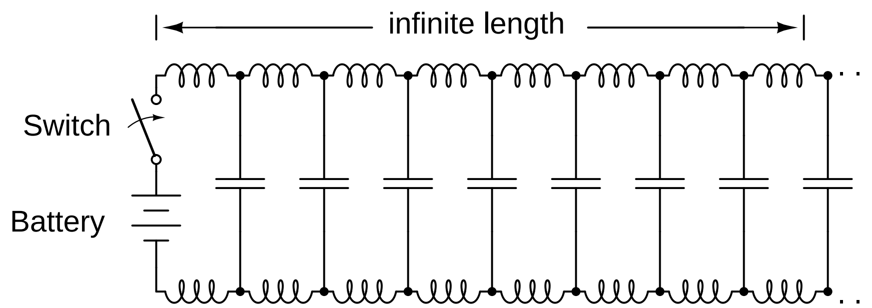


Figure 14.8: *Equivalent circuit showing stray capacitance and inductance.*

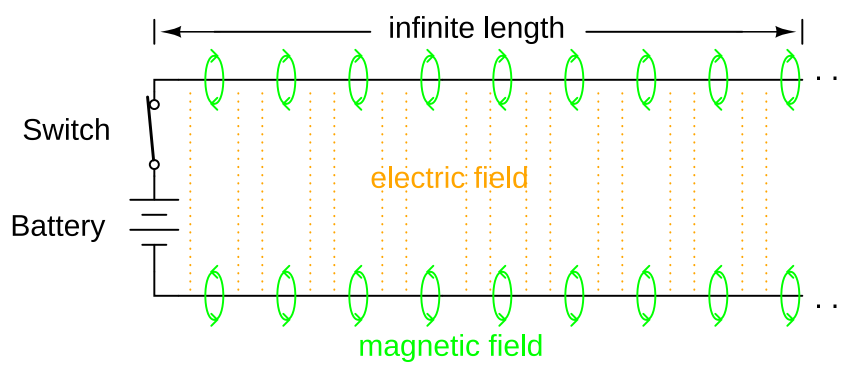


Figure 14.9: *Voltage charges capacitance, current charges inductance.*

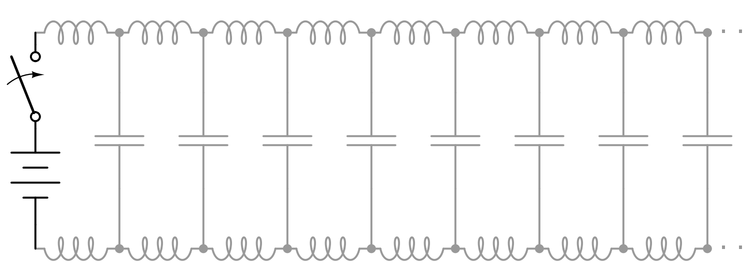


Figure 14.10: *Uncharged transmission line.*

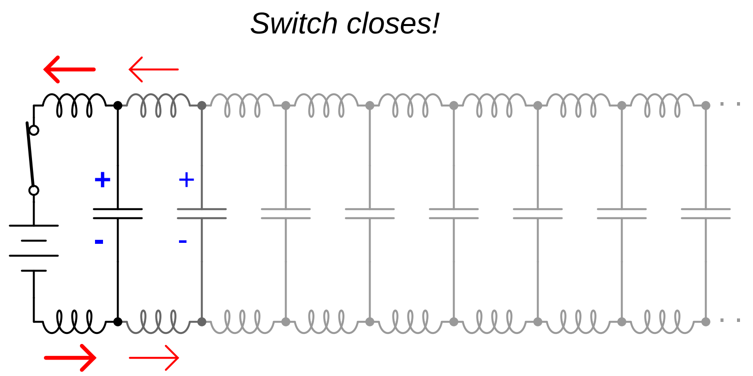


Figure 14.11: *Begin wave propagation.*

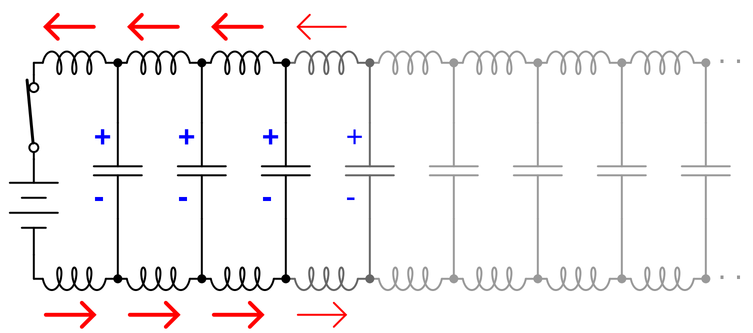


Figure 14.12: *Continue wave propagation.*

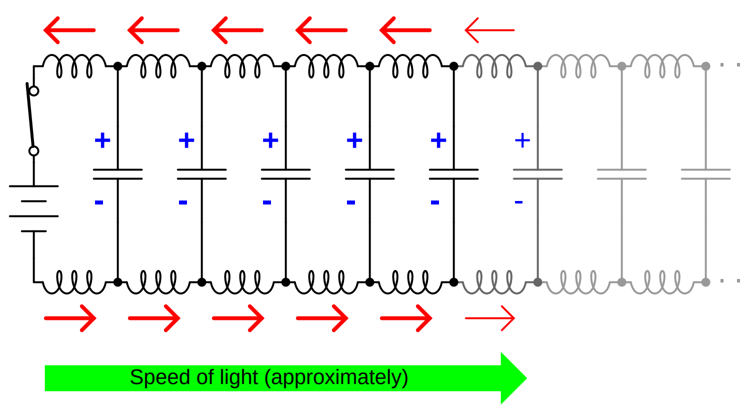
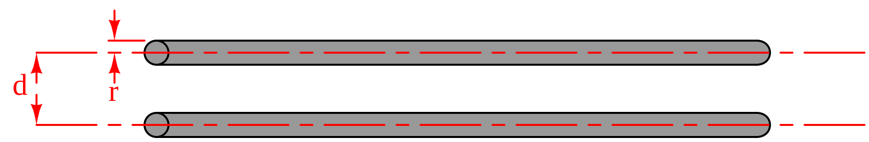


Figure 14.13: *Propagate at speed of light.*

with unique characteristics. No longer are the two wires merely *a pair of conductors*, but rather a *transmission line*.

As a constant load, the transmission line's response to applied voltage is resistive rather than reactive, despite being comprised purely of inductance and capacitance (assuming super-conducting wires with zero resistance). We can say this because there is no difference from the battery's perspective between a resistor eternally dissipating energy and an infinite trans-mission line eternally absorbing energy. The impedance (resistance) of this line in ohms is called the *characteristic impedance*, and it is fixed by the geometry of the two conductors. For a parallel-wire line with air insulation, the characteristic impedance may be calculated as such:



$$\boxed{Z_0 = \frac{276}{\sqrt{k}} \log \frac{d}{r}}$$

Where,

Z_0 = Characteristic impedance of line
d = Distance between conductor centers
r = Conductor radius
k = Relative permittivity of insulation between conductors

If the transmission line is coaxial in construction, the characteristic impedance follows a different equation:

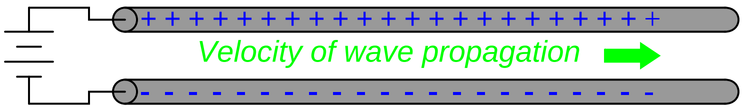
$$\boxed{Z_0 = \frac{138}{\sqrt{k}} \log \frac{d_1}{d_2}}$$

Where,

Z_0 = Characteristic impedance of line
d_1 = Inside diameter of outer conductor
d_2 = Outside diameter of inner conductor
k = Relative permittivity of insulation between conductors

In both equations, identical units of measurement must be used in both terms of the frac-tion. If the insulating material is other than air (or a vacuum), both the characteristic impedance and the propagation velocity will be affected. The ratio of a transmission line's true propagation velocity and the speed of light in a vacuum is called the *velocity factor* of that line.

Velocity factor is purely a factor of the insulating material's relative permittivity (otherwise known as its *dielectric constant*), defined as the ratio of a material's electric field permittivity to that of a pure vacuum. The velocity factor of any cable type – coaxial or otherwise – may be calculated quite simply by the following formula:



$$\text{Velocity factor} = \frac{v}{c} = \frac{1}{\sqrt{k}}$$

Where,

v = Velocity of wave propagation

c = Velocity of light in a vacuum

k = Relative permittivity of insulation between conductors

Characteristic impedance is also known as *natural impedance*, and it refers to the equivalent resistance of a transmission line if it were infinitely long, owing to distributed capacitance and inductance as the voltage and current "waves" propagate along its length at a propagation velocity equal to some large fraction of light speed.

It can be seen in either of the first two equations that a transmission line's characteristic impedance (Z_0) increases as the conductor spacing increases. If the conductors are moved away from each other, the distributed capacitance will decrease (greater spacing between capacitor "plates"), and the distributed inductance will increase (less cancellation of the two opposing magnetic fields). Less parallel capacitance and more series inductance results in a smaller current drawn by the line for any given amount of applied voltage, which by definition is a greater impedance. Conversely, bringing the two conductors closer together increases the parallel capacitance and decreases the series inductance. Both changes result in a larger current drawn for a given applied voltage, equating to a lesser impedance.

Barring any dissipative effects such as dielectric "leakage" and conductor resistance, the characteristic impedance of a transmission line is equal to the square root of the ratio of the line's inductance per unit length divided by the line's capacitance per unit length:

$$Z_0 = \sqrt{\frac{L}{C}}$$

Where,

Z_0 = Characteristic impedance of line

L = Inductance per unit length of line

C = Capacitance per unit length of line

- **REVIEW:**
- A *transmission line* is a pair of parallel conductors exhibiting certain characteristics due to distributed capacitance and inductance along its length.
- When a voltage is suddenly applied to one end of a transmission line, both a voltage "wave" and a current "wave" propagate along the line at nearly light speed.
- If a DC voltage is applied to one end of an infinitely long transmission line, the line will draw current from the DC source as though it were a constant resistance.
- The *characteristic impedance* (Z_0) of a transmission line is the resistance it would exhibit if it were infinite in length. This is entirely different from leakage resistance of the dielectric separating the two conductors, and the metallic resistance of the wires themselves. Characteristic impedance is purely a function of the capacitance and inductance distributed along the line's length, and would exist even if the dielectric were perfect (infinite parallel resistance) and the wires superconducting (zero series resistance).
- *Velocity factor* is a fractional value relating a transmission line's propagation speed to the speed of light in a vacuum. Values range between 0.66 and 0.80 for typical two-wire lines and coaxial cables. For any cable type, it is equal to the reciprocal (1/x) of the square root of the relative permittivity of the cable's insulation.

14.4 Finite-length transmission lines

A transmission line of infinite length is an interesting abstraction, but physically impossible. All transmission lines have some finite length, and as such do not behave precisely the same as an infinite line. If that piece of 50 Ω "RG-58/U" cable I measured with an ohmmeter years ago had been infinitely long, I actually would have been able to measure 50 Ω worth of resistance between the inner and outer conductors. But it was not infinite in length, and so it measured as "open" (infinite resistance).

Nonetheless, the characteristic impedance rating of a transmission line is important even when dealing with limited lengths. An older term for characteristic impedance, which I like for its descriptive value, is *surge impedance*. If a transient voltage (a "surge") is applied to the end of a transmission line, the line will draw a current proportional to the surge voltage magnitude divided by the line's surge impedance (I=E/Z). This simple, Ohm's Law relationship between current and voltage will hold true for a limited period of time, but not indefinitely.

If the end of a transmission line is open-circuited – that is, left unconnected – the current "wave" propagating down the line's length will have to stop at the end, since electrons cannot flow where there is no continuing path. This abrupt cessation of current at the line's end causes a "pile-up" to occur along the length of the transmission line, as the electrons successively find no place to go. Imagine a train traveling down the track with slack between the rail car couplings: if the lead car suddenly crashes into an immovable barricade, it will come to a stop, causing the one behind it to come to a stop as soon as the first coupling slack is taken up, which causes the next rail car to stop as soon as the next coupling's slack is taken up, and so on until the last rail car stops. The train does not come to a halt together, but rather in sequence from first car to last: (Figure 14.14)

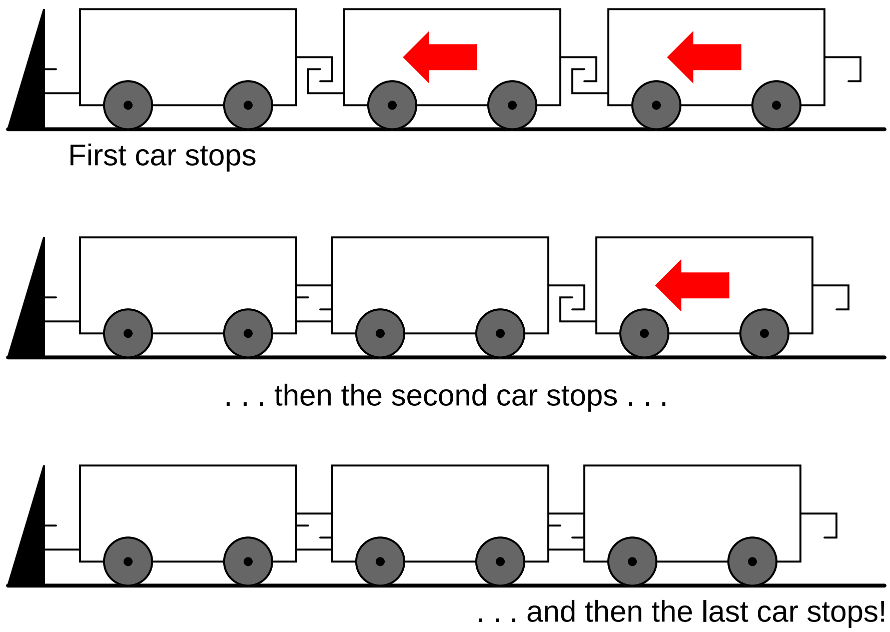


Figure 14.14: *Reflected wave.*

A signal propagating from the source-end of a transmission line to the load-end is called an *incident wave*. The propagation of a signal from load-end to source-end (such as what happened in this example with current encountering the end of an open-circuited transmission line) is called a *reflected wave*.

When this electron "pile-up" propagates back to the battery, current at the battery ceases, and the line acts as a simple open circuit. All this happens very quickly for transmission lines of reasonable length, and so an ohmmeter measurement of the line never reveals the brief time period where the line actually behaves as a resistor. For a mile-long cable with a velocity factor of 0.66 (signal propagation velocity is 66% of light speed, or 122,760 miles per second), it takes only 1/122,760 of a second (8.146 microseconds) for a signal to travel from one end to the other. For the current signal to reach the line's end and "reflect" back to the source, the round-trip time is twice this figure, or 16.292 μs.

High-speed measurement instruments are able to detect this transit time from source to line-end and back to source again, and may be used for the purpose of determining a cable's length. This technique may also be used for determining the presence *and* location of a break in one or both of the cable's conductors, since a current will "reflect" off the wire break just as it will off the end of an open-circuited cable. Instruments designed for such purposes are called *time-domain reflectometers* (TDRs). The basic principle is identical to that of sonar range-finding: generating a sound pulse and measuring the time it takes for the echo to return.

A similar phenomenon takes place if the end of a transmission line is short-circuited: when the voltage wave-front reaches the end of the line, it is reflected back to the source, because voltage cannot exist between two electrically common points. When this reflected wave reaches the source, the source sees the entire transmission line as a short-circuit. Again, this happens as quickly as the signal can propagate round-trip down and up the transmission line at whatever velocity allowed by the dielectric material between the line's conductors.

A simple experiment illustrates the phenomenon of wave reflection in transmission lines. Take a length of rope by one end and "whip" it with a rapid up-and-down motion of the wrist. A wave may be seen traveling down the rope's length until it dissipates entirely due to friction: (Figure 14.15)

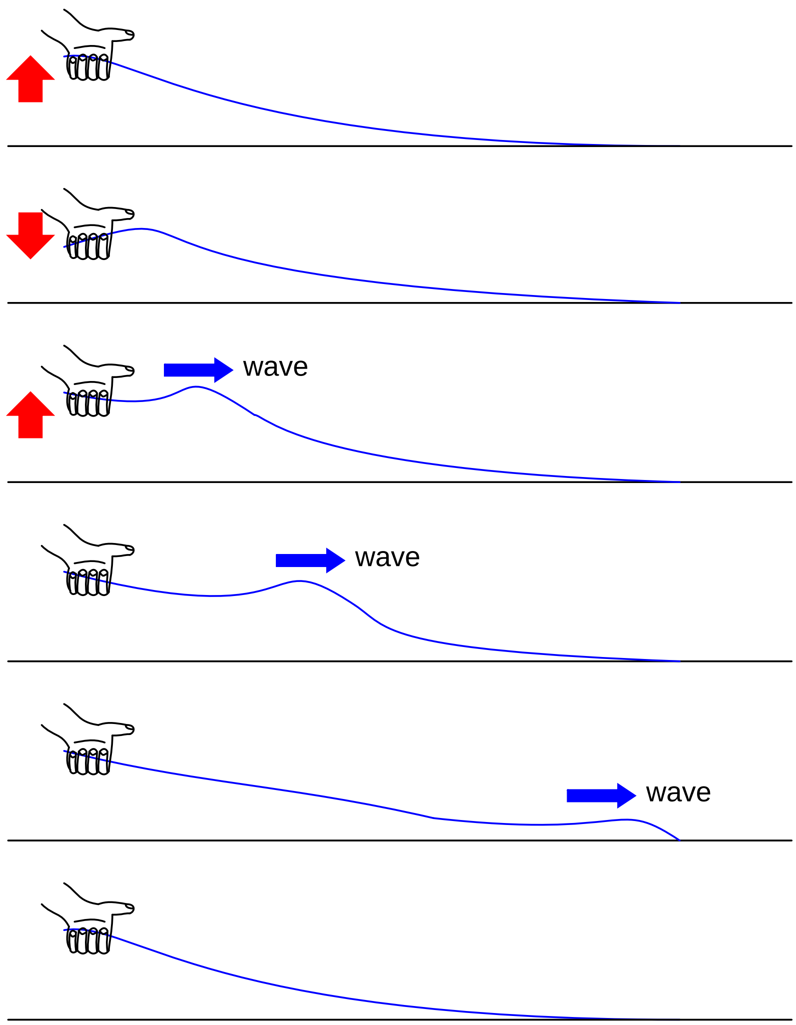


Figure 14.15: *Lossy transmission line.*

This is analogous to a long transmission line with internal loss: the signal steadily grows weaker as it propagates down the line's length, never reflecting back to the source. However, if the far end of the rope is secured to a solid object at a point prior to the incident wave's total dissipation, a second wave will be reflected back to your hand: (Figure 14.16)

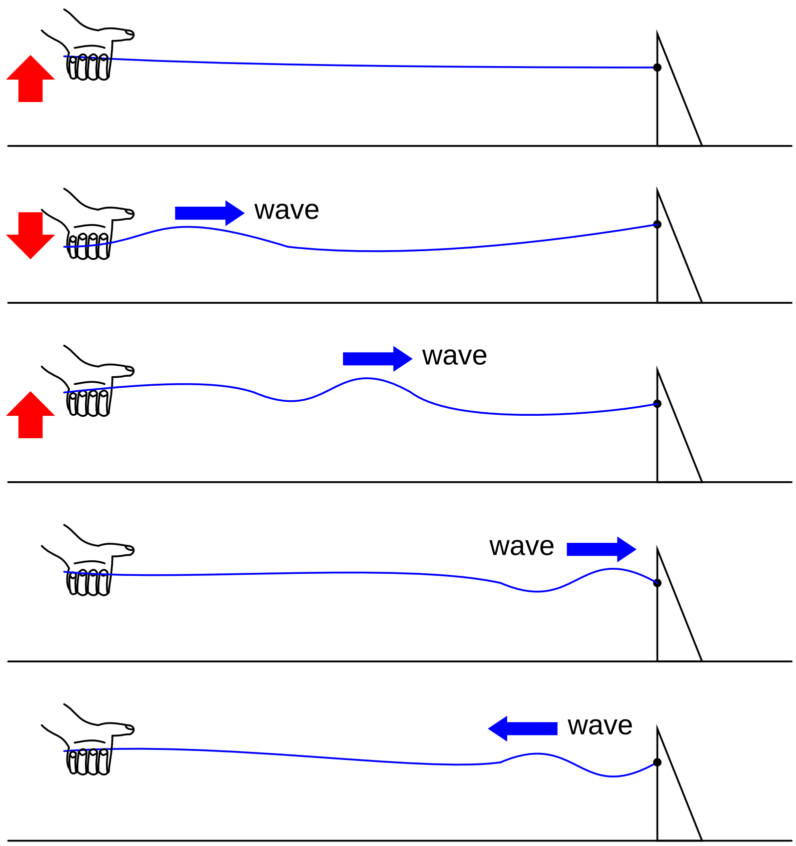


Figure 14.16: *Reflected wave.*

Usually, the purpose of a transmission line is to convey electrical energy from one point to another. Even if the signals are intended for information only, and not to power some significant load device, the ideal situation would be for all of the original signal energy to travel from the source to the load, and then be completely absorbed or dissipated by the load for maximum signal-to-noise ratio. Thus, "loss" along the length of a transmission line is undesirable, as are reflected waves, since reflected energy is energy not delivered to the end device.

Reflections may be eliminated from the transmission line if the load's impedance exactly equals the characteristic ("surge") impedance of the line. For example, a 50 Ω coaxial cable that is either open-circuited or short-circuited will reflect all of the incident energy back to the source. However, if a 50 Ω resistor is connected at the end of the cable, there will be no reflected energy, all signal energy being dissipated by the resistor.

This makes perfect sense if we return to our hypothetical, infinite-length transmission line example. A transmission line of 50 Ω characteristic impedance and infinite length behaves exactly like a 50 Ω resistance as measured from one end. (Figure 14.17) If we cut this line to some finite length, it will behave as a 50 Ω resistor to a constant source of DC voltage for a brief time, but then behave like an open- or a short-circuit, depending on what condition we leave the cut end of the line: open (Figure 14.18) or shorted. (Figure 14.19) However, if we *terminate* the line with a 50 Ω resistor, the line will once again behave as a 50 Ω resistor, indefinitely: the same as if it were of infinite length again: (Figure 14.20)

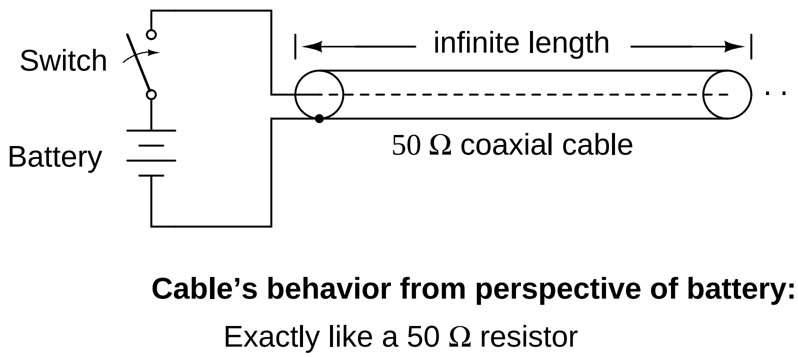


Figure 14.17: *Infinite transmission line looks like resistor.*

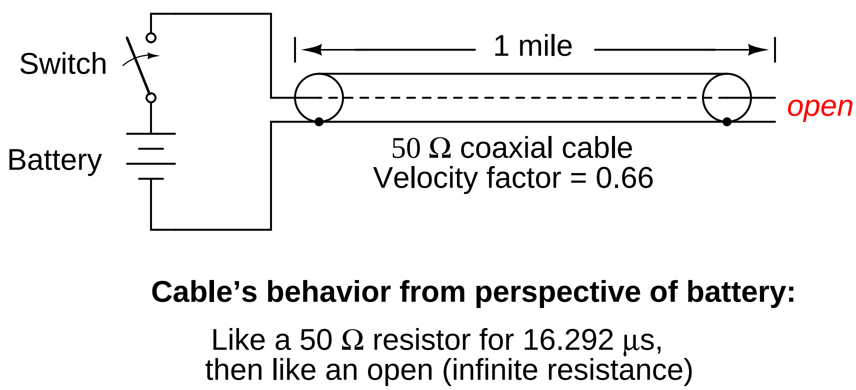


Figure 14.18: *One mile transmission.*

In essence, a terminating resistor matching the natural impedance of the transmission line makes the line "appear" infinitely long from the perspective of the source, because a resistor has the ability to eternally dissipate energy in the same way a transmission line of infinite length is able to eternally absorb energy.

Reflected waves will also manifest if the terminating resistance isn't precisely equal to the characteristic impedance of the transmission line, not just if the line is left unconnected (open) or jumpered (shorted). Though the energy reflection will not be total with a terminating impedance of slight mismatch, it will be partial. This happens whether or not the terminating resistance is *greater* or *less* than the line's characteristic impedance.

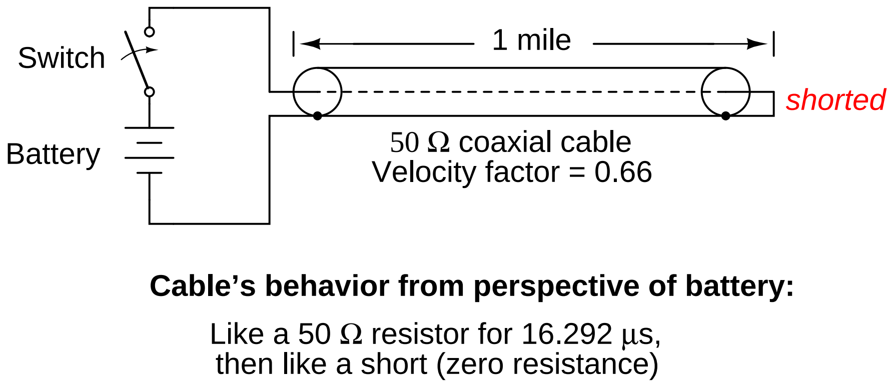


Figure 14.19: *Shorted transmission line.*

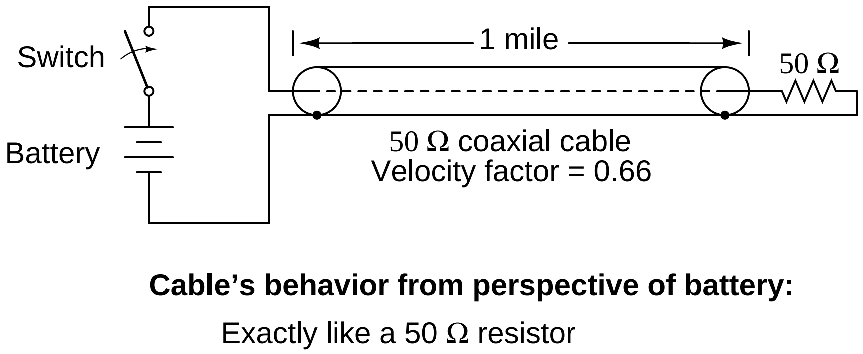


Figure 14.20: *Line terminated in characteristic impedance.*

Re-reflections of a reflected wave may also occur at the *source end* of a transmission line, if the source's internal impedance (Thevenin equivalent impedance) is not exactly equal to the line's characteristic impedance. A reflected wave returning back to the source will be dissipated entirely if the source impedance matches the line's, but will be reflected back toward the line end like another incident wave, at least partially, if the source impedance does not match the line. This type of reflection may be particularly troublesome, as it makes it appear that the source has transmitted another pulse.

- **REVIEW:**
- Characteristic impedance is also known as *surge impedance*, due to the temporarily resistive behavior of any length transmission line.
- A finite-length transmission line will appear to a DC voltage source as a constant resistance for some short time, then as whatever impedance the line is terminated with. Therefore, an open-ended cable simply reads "open" when measured with an ohmmeter, and "shorted" when its end is short-circuited.
- A transient ("surge") signal applied to one end of an open-ended or short-circuited transmission line will "reflect" off the far end of the line as a secondary wave. A signal traveling on a transmission line from source to load is called an *incident wave*; a signal "bounced" off the end of a transmission line, traveling from load to source, is called a *reflected wave*.
- Reflected waves will also appear in transmission lines terminated by resistors not precisely matching the characteristic impedance.
- A finite-length transmission line may be made to appear infinite in length if terminated by a resistor of equal value to the line's characteristic impedance. This eliminates all signal reflections.
- A reflected wave may become re-reflected off the source-end of a transmission line if the source's internal impedance does not match the line's characteristic impedance. This re-reflected wave will appear, of course, like another pulse signal transmitted from the source.

14.5 "Long" and "short" transmission lines

In DC and low-frequency AC circuits, the characteristic impedance of parallel wires is usually ignored. This includes the use of coaxial cables in instrument circuits, often employed to protect weak voltage signals from being corrupted by induced "noise" caused by stray electric and magnetic fields. This is due to the relatively short timespans in which reflections take place in the line, as compared to the period of the waveforms or pulses of the significant signals in the circuit. As we saw in the last section, if a transmission line is connected to a DC voltage source, it will behave as a resistor equal in value to the line's characteristic impedance only for as long as it takes the incident pulse to reach the end of the line and return as a reflected pulse, back to the source. After that time (a brief 16.292 μs for the mile-long coaxial cable of the last example), the source "sees" only the terminating impedance, whatever that may be.