Modelling interdependent infrastructures using interacting dynamical models

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Abstract: We investigate the consequence of failures, occurring on the electrical grid, on a telecommunication network. We have focused on the Italian electrical transmission network and the backbone of the internet network for research (GARR). Electrical network has been simulated using the DC power flow method; data traffic on GARR by a model of the TCP/IP basic features. The status of GARR nodes has been related to the power level of the (geographically) neighbouring electrical nodes (if the power level of a node is lower than a threshold, all communication nodes depending on it are switched off). The electrical network has been perturbed by lines removal: the consequent re-dispatching reduces the power level in all nodes. This reduces the number of active GARR nodes and, thus, its Quality of Service (QoS). Averaging over many configurations of perturbed electrical network, we have correlated the degradation of the electrical network with that of the communication network. Results point to a sizeable amplification of the effects of faults on the electrical network on the communication network, also in the case of a moderate coupling between the two networks.

Keywords: interdependent networks; complex systems; topological analysis.

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1 Introduction

Modelling Critical Infrastructures (CIs hereafter) is a key problem in basic research of complex systems (of which CIs represent an excellent metaphor) and for the technological activities aimed at analysing and controlling large infrastructures whose fate and functioning have a large impact on nations and citizens (Dunn and Wigert, 2004).

CIs, as complex systems, show a number of structural and 'behavioural' features, which have been widely investigated in recent years (Albert *et al.*, 2004; Crucitti *et al.*, 2004; Pastor-Satorras *et al.*, 2001; Issacharoff *et al.*, 2006; Tiriticco *et al.*, 2006). All these studies have dealt, usually, with a single infrastructure, analysed on both the structural and, in some cases, the functional viewpoints. The intrinsic complexity of these tasks has prevented scientific efforts aimed at understanding the effects related to the *interaction* among CIs. Far from being 'stand-alone' systems, in fact, CIs are mutually, and sometimes strongly, dependent and interdependent; the incorrect functioning of a CI might have dramatic repercussions on other CIs, which are functionally related (*e.g.*, the effects of an electrical outage, which rapidly spread over railways, communication networks and many others, through the *domino* effect).

The infrastructure's complexity resides, in general, on two main factors: an 'intrinsic' complexity of the single infrastructure, originating from its topological structure and the specific dynamical process running on it, and an 'extrinsic' one, induced by the presence of dependency elements 'logically' and/or 'physically' outside the system's domain.

The relevance of these extrinsic relations has been emphasised by recent episodes (Bologna and Setola, 2005). The electric blackout that affected Italy in September 2003, for instance, induced large degradation in railway network, in healthcare systems, in financial services and also in different types of communication networks. On the other side, the partial failure of communication systems affected the capability of the SCADA network (*i.e.*, the system used to manage the electric grid) to perform its function, thus producing a negative feedback on the restore phase (Italian Government Working Group on Critical Information Infrastructure Protection, 2004).

In this episode, electrical and communication networks showed a bi-directional functional dependency. This phenomenon is generally indicated as 'interdependency' and it should be carefully considered in risk assessment strategies because it might induce large amplification of negative consequences (positive feedback).

A great deal of effort has been recently devoted to the study and the modelling of CIs, seen as complex systems from the standpoint of theoretical analysis (Albert and Barabasi, 2002; Boccaletti *et al.*, 2006). The promise of these efforts is to unveil relevant insights on growth mechanisms, causes of vulnerability, dynamic behaviour under perturbation, onset of emerging phenomena, *etc*. The ambitious goal of understanding the combined behaviour, *i.e.*, the dynamic behaviour of clusters of interdependent complex systems, will open the way to the modelling of groups of CIs to analyse their behaviour and to predict the occurrence of critical events triggered by their interdependency.

On the basis of recent developments in the field of complex systems, there are two aspects which, if properly analysed, may allow gaining of relevant insights on CIs:

1 the study of the topology of the graph representing their structure

2 the study of their 'behaviour', as it can be deduced from the analysis of some functional model able to reproduce the dynamic process (mainly transport of some entity, such as electricity, data and vehicles) taking place in them.

Topology analysis of graphs describing large and complex CIs has received a renewed impulsion from the works of Strogartz and Barabasi (Watts and Strogartz, 1998; Jeong *et al*., 2001), who emphasised the presence of a 'selective pressure' able to determine a common growth mechanism in a number of diverse complex systems (from those representing technological systems to those related to biological and social networks). The growth mechanisms (known as 'Preferential Attachment') determine the development of these structures towards a class of mathematical graphs known as 'scale-free'. Topological analysis, made on the basis of the classical graph theory, allows to unveil relevant properties of the network, to highlight the role played by several components (nodes and arcs), to make preliminary vulnerability assessments based on the simulation of faults (mainly represented by the removal of nodes and arcs) and the subsequent reevaluation of the major topological properties. It has been shown that a 'scale-free' structure provides a network of a considerable robustness against *random* faults (and a large vulnerability, in turn, to deliberate attacks) with respect to *random* networks. But, it has also been proven that, quite often, large technological infrastructures do not show a clear 'scale-free' structure, mainly related to technological constraints, which limit the arbitrary growth of the node's degrees (Amaral *et al*., 2000).

The functional analysis of CIs is a complex task, mainly because of the lack of accurate and complete information on the infrastructures. Functional models of CIs require, in fact, the knowledge of a much larger amount of data; network's graphs must be complemented by a number of information consisting of the technical characteristics of lines and nodes, load requirements, *etc*. These data are often unavailable as they are treated as *confidential* information from the stakeholders. To overcome this limitation, 'simple' functional models have been developed, able to capture the basic features of the networks, disregarding more complex effects related to the technological complexity of CIs elements. These models have shown the way complex networks react to faults or attacks; their behaviour often results in a dramatic *cascade* phenomenon (Motter and Lai, 2002; Motter, 2004).

Studies of CIs' interdependency are even rarer. Recent works have reported on layered networks (Kurant and Thiran, 2006) where several *homogenous* networks (*i.e.*, of similar nature) interact by exchanging loads. To the best of our knowledge, the only attempt done in the direction of studying *heterogeneous* interdependent networks (*i.e.*, formed by infrastructures of different nature) was that of Newman *et al*. (2005), where the authors studied a system composed of two connected networks (L and M). They assumed that, in the presence of a failure in one component of the system (say L), *e.g.*, an overload condition, this has the effect of producing a redistribution of the load on the components of system M, and increasing the load in the other components of model L itself. The authors showed that this interdependent load increase induces a shift in the critical point or, in other terms, the coupling makes the system more susceptible to large failure.

Other authors (Jiang and Haimes, 2004; Reed *et al*., 2006) have proposed a phenomenological approach based on the Leontief formalism which, given an *interdependence* matrix grouping the sensitivity of the operability of each CI with respect to those of the others, allows to evaluate the repercussions of the decrease of operability of one CI on the others. This analysis, however, relies on the availability of the above-mentioned *interdependence* matrix. The main goal of this study is to define a methological workflow which, starting from the description of functional models of CI and their interdependency, might allow to estimate the sensitivity values that will fill the elements of that matrix.

In this paper, we focus on the analysis of the interdependencies between an electrical grid (the Italian high-voltage electrical transmission network) and a telecommunication network (the Italian high-bandwidth backbone for public research, modelled at the level of large AS-routers).

Results have shown that there is a consistent amplification of the effects of the fault produced by one network into the other. An outage on the electrical network is able to produce a larger effect on the communication network, even under the hypothesis of a moderate coupling. This qualitative result confirms the relevance of the problem, providing several indications for further research.

The paper is organised as follows. Section 2 is devoted to a short description of the overall scheme of the work. Sections 3, 4 and 5 provide the description of the dynamic models used to simulate the high-voltage electrical network, the internet network and the coupling assumptions between them, respectively. Section 6 shows the results obtained in the interdependent network's simulation, while Section 7 is devoted to summarising the results, to drawing preliminary conclusions and to suggesting further developments.

2 The proposed approach

The plan of the present work is as follows: we first introduce the two prototype CI models (the Italian high-voltage electrical transmission network, HVIET hereafter, and the high-bandwidth backbone of the Italian internet network, GARR hereafter). Each of these networks will be analysed under the topological point of view. Then, we will define the dynamical model developed to simulate the electrical power flow in the first and the data traffic in the second network. We will then describe the 'stand-alone' behaviour of each network during their normal functioning and report recent results on their 'stand-alone' behaviour. We will then formulate a geographically based interaction assumption between the two networks and evaluate their 'interconnected' vulnerabilities. Specifically, we will evaluate how a perturbation affecting the electrical transmission network (resulting in its partial operability loss) propagates on the internet network, by producing a service degradation.

This work is a part of a more general project aimed at defining the elements of an 'interdependence' matrix R_{ij} which, for each couple i,j of CIs, defines the mutual variation of the corresponding *operability* functions x_i:

$$R_{ij} = \frac{dx_i}{dx_j}. \tag{1}$$

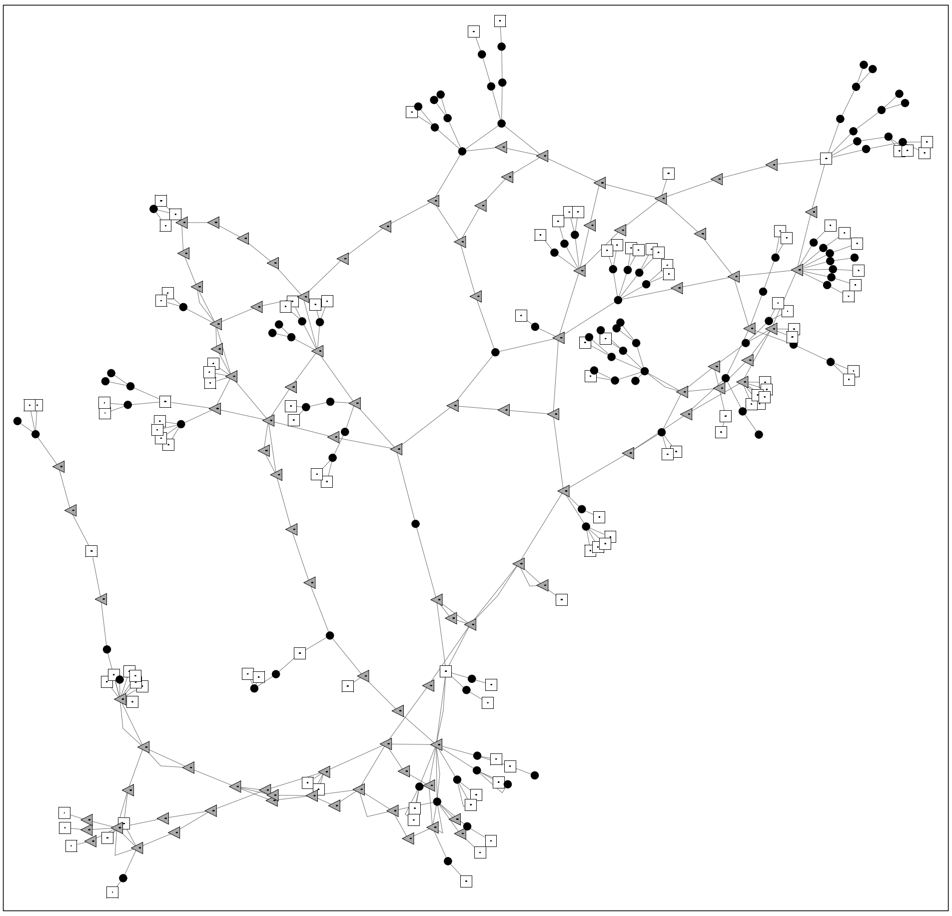
The construction of the matrix R_{ij} where i and j run over all the relevant CIs will open the way to the use of simulations of the behaviour of interdependent infrastructures based on the use of phenomenological master equations (Issacharoff *et al.*, 2006; Jiang and Haimes, 2004).

3 The high-voltage electrical transmission network

We have analysed data relative to the Italian high-voltage (380 kV) electrical transmission network (HVIET). This study follows previous works performed to extract, from topological analysis, a number of features of the HVIET (Crucitti *et al.*, 2004; Tiriticco *et al.*, 2006; Rosato *et al.*, 2006a). The network's data have been inferred from the analysis of the public documentation.

HVIET can be represented by an undirected graph of N nodes and E arcs (also referred to as 'lines'). Available data allow attributing to each node the quality of being a *source* node S (where part of the power is inserted in the network), or a *load* node L (where part of the power is extracted from the network) or a *junction* node J (which is neither a S nor a L node). The topology of the HVIET is reported in Figure 1 where S nodes are squares, L nodes are triangles and J nodes are black circles. HVIET consists of $N = 310$ nodes and $E = 361$ arcs. There are $S = 97$ source nodes, $L = 113$ load nodes and $J = 100$ junction nodes. All lines are *single* lines; 14 are *double* lines (*i.e.*, two point-to-point connections). Points of cut of the network (which is connected to other European networks) have been substituted with ‘fictitious’ source nodes where the same amount of electrical power received by foreigner countries is pumped into the network.

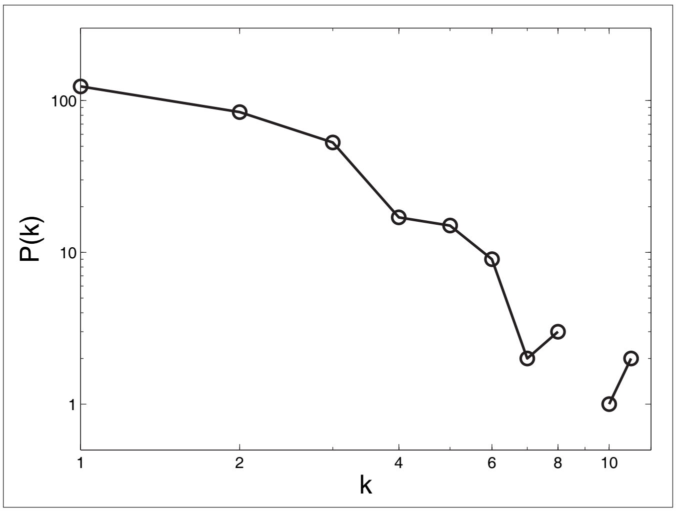
Figure 1 The graph corresponding to the Italian high-voltage (380 kV) transmission grid resulting from the available data.



Notes: Source *S* nodes are square, Load *L* nodes triangles and Junction *J* nodes are black circles

Several topological properties have been analysed on the HVIET network. Among them, the distribution $P(k)$ of the node’s *degree* k (the *degree* is the number of links connecting each node to its nearest neighbours), which allows to ‘classify’ its topology. The HVIET $P(k)$ is reported in Figure 2. The network has a limited number of hubs, whose maximum degree is $k_{max} = 11$. $P(k)$ and the cumulative degree distribution $P(k > K)$ are both likely to be fit by an exponential (single-scale network (Albert *et al*., 2004; Crucitti *et al*., 2004; Amaral *et al*., 2000)). The latter distribution can be fitted $P(k > K) \sim e^{-0.55K}$ in agreement with previous findings for the North-American power grid (Albert *et al*., 2004). A further property measured on the HVIET network is the average *clustering* coefficient C (Crucitti *et al*., 2004), which turns out to be as small as $C = 2.06 \cdot 10^{-2}$.

Figure 2 The distribution of HVIET node's degree (log–log scale)



A complete study on the topological properties of HVIET is reported in Rosato *et al.* (2006a) where a structural vulnerability study has allowed identifying specific sites of the networks as critical vulnerability points.

A further goal of Rosato *et al.* (2006a) has been to highlight the relevance of the structural vulnerability sites on the function of the network, in order to see if structural and functional vulnerability are somehow related to the same sites (*i.e.*, nodes and lines) or, instead, if functional vulnerability (*i.e.*, the sites whose fault produces the highest damage in terms of reduction of operability) is associated to a different set of sites. Following the approach proposed in Rosato *et al.* (2006a), in this work we will use a simplified transport model as we wish to evaluate the effect of the network topology on the steady-state power flow rather than on transitory regimes. We will firstly evaluate the power flow distribution on the unperturbed network, resulting from a specific input–output condition, chosen to be representative of a typical power requirement that HVIET must daily sustain. Then we will perturb the network by removing lines, and analyse the behaviour of the grip under these 'failure' configurations.

3.1 DC power flow model

In the present study, the transport of the electrical flow in the network has been determined by using a DC power flow model. The DC power flow equations (Wood and Wollenberg, 1984) provide a linear relationship between the active power flowing through the lines and the power input into the nodes. They can be formulated as follows:

$$F_{km} = \frac{\theta_k - \theta_m}{x_{km}} \tag{2}$$

where x_{km} is the reactance of the line connecting nodes k and m, F_{km} is the active power flow on this line and θ_k, θ_m are the voltage phases of the k-th and m-th node. Summing up all branches connected to node i, the power flow of that node P_i is:

$$P_i = \sum_j F_{ij} = \theta_i \sum_j x_{ij}^{-1} - \sum_j \frac{\theta_j}{x_{ij}}. \tag{3}$$

The power flow on the network can be written in a matrix form as:

$$P = B\theta \tag{4}$$

where θ and P, respectively, are the vectors composed by voltage phase and the electrical power at each of the N nodes of the HVIET, and B is a $N \times N$ matrix $\left(B_{km} = -1/x_{km}\right.$ and $\left.B_{kk} = \sum_l 1/x_{kl}\right)$. The rank of B is $N - 1$ since the network must comply with the conservation condition $\sum_{i=1}^{N} P_i = 0$ (notice that source nodes are characterised by positive P_i values, junction by vanishing values, loads by negative values).

To solve the system, an equation is removed and the associated link is chosen in a way to introduce a reference node whose phase angle is arbitrarily set to $\theta = 0$. For a given input vector $P^{(0)}$, the linear system in Equation (4) is solved to find θ_i and F_{ij}. Two constraints must be imposed to ensure the physical correctness of the solution. This comes from the fact that the DC power flow method results from the elimination of the imaginary part of the current equations, under the hypothesis that power phase angles differences are small. For this reason, it should result, $\forall(k, m)$, that:

1. $\theta_{km} < 30$ degrees
2. $|F_{km}| < F_{km}^{max}$ (where F_{km}^{max} is some specified limiting power flux on the link between nodes k and m).

If Constraint (1) is not fulfilled, the inductive part of the electrical flux cannot be disregarded and Equation (4) does not hold. Constraint (2) is a technological limit, relating to the specific line's impedance. A too large flux produces an unendurable heat, normally prevented by *ad hoc* elements that disconnect the line.

Then, after having selected a $P^{(0)}$, *i.e.*, a typical snapshot of the vector (injected power-extracted power) experienced daily in the HVIET, one can use Equation (4) and Equation (2) to evaluate the resulting power flux F_{ij} along the lines and the phase angles θ_i.

The solution of the DC power flow system will constitute the 'normal' response of the network to the input conditions $P^{(0)}$.

3.2 Perturbation analysis

The cut of one or more links in the HVIET induces a modification into flow allocation on the network that might be still evaluated using Equations (2) and (4). For some cuts, however, a solution is inhibited by the physical constraints. In other terms, because of overload conditions or unbalancing situations, the electric grid is not any more able to supply energy from the source to the destination nodes. This would imply the presence of more or less large blackouts if any corrective action is taken. In accordance with typical policies adopted by electrical operators to prevent this condition, we reduce the energy extraction from the network, and consequently the energy injected in it, by a suitable modulation of the loads. This procedure is named redispatching.

To this end, we have added to the model a procedure that is able to evaluate the new input conditions P in order to satisfy Equations (2) and (4) with the Constraints (1) and (2) above, by further optimising the distance with respect to the 'normal' load conditions $P^{(0)}$. In other words, P is searched such that ΔP, defined as:

$$\Delta P = \sum_{i \in loads} \left[P_i - P_i^0 \right] \tag{5}$$

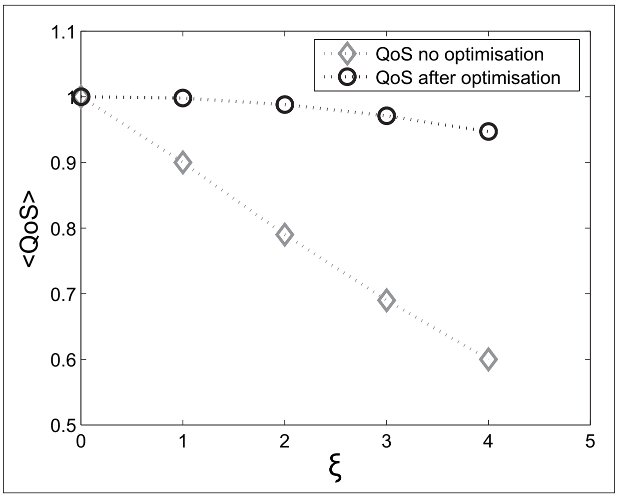
results to be minimum. This strategy allows to find a new solution, although degraded, of the network flow. The vector P_i will thus be the new solution to the dispatching problem (Rosato *et al.*, 2006a).

In Rosato *et al.* (2006a), it is assumed that the 'distance' of this new solution with respect to the normal distribution is a measure of the quality of service of the HVIET in the perturbed condition. Specifically, a suitable 'Quality of Service' (*QoS*) of the network has been defined as a function of the perturbation strength ξ (measured in terms of broken lines) as follows:

$$QoS = 1 - \frac{\Delta P}{\sum_{i \in loads} P_i^0}. \tag{6}$$

Figure 3 reports the *QoS* as a function of ξ either in the case in which 'redispatching' is performed (circles) and when it is not performed (diamonds). In the latter case, *QoS* is set to zero when the system cannot be solved or set to one if the system has a solution within the physical constraints. Data reported in Figure 3 refers to average over a large number of different choices of removed lines. Figure 3 also shows that an (ideally) optimised redispatching strategy of the input and the extracted power could reduce the impact of faults: the decrease of *QoS*, in fact, could be minimised (in principle) to a few percent also in case of a severe network perturbation (*e.g.*, the simultaneous removal of a few lines).

Figure 3 *QoS* values as a function of the perturbation ξ (number of simultaneously removed lines)

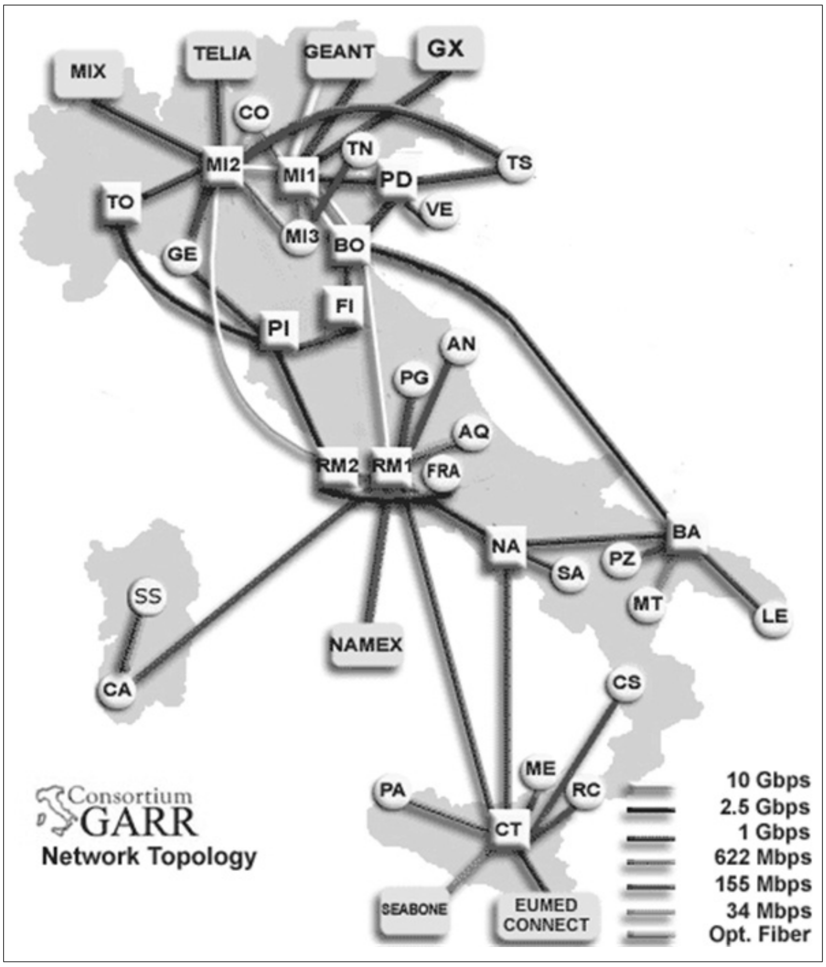


Notes: Circles represent the *QoS* value when redispatching is performed; diamonds when no optimisation is performed.

4 The internet

We have chosen to investigate the traffic dynamics on the Italian high-bandwidth backbone of the internet network dedicated to linking universities and research institutions (GARR) depicted in Figure 4.

Figure 4 The high-bandwidth backbone of the internet network dedicated to linking Italian universities and research institutions (GARR)



The undirected GARR graph G(N,E), inferred from public documentation, consists in the adjacency matrix of $N = 39$ nodes and $E = 58$ arcs. The functional model of the network is the same proposed in Rosato *et al.* (2006b) consisting in a simplified model of traffic of data on a network. Data packets are generated by an Origin Node (ON) and are directed towards a Destination Node (DN). The size of data packets is supposed to be infinitesimal and no hypothesis on the arcs' capacity (in terms of bandwidth) is done. The traffic behaviour is thus regulated only by the network's topology and the model's characteristics. No effects can thus originate from the limited bandwidth of the lines.

The dynamical model issued for describing the functioning of the network assumes that each node represents an AS-level router. At each time-step, each node might perform two basic actions:

1 send a packet of data to a nearest neighbour node (*i.e.*, a node directly connected to it) but cannot send a packet to itself

2 receive one or more packets of data from its nearest neighbour nodes.

These hypotheses stem from the fact that all nodes are supposed to have equal technological properties *independently* on their degree. This can be reasonably supposed as the GARR backbone is quite small and all nodes have very similar degrees. Each data packet contains the value of two different quantities: the time of emission and the DN. The latter information is used to direct the packet throughout its journey until the DN. In order to define the route that the packets follow from the ON to the DN, each node contains two basic elements: (a) a buffer (unlimited in size) allowing the received packets to form a queue and (b) a Routing Table (RT) which associates, for each DN k, two different nodes j_1 and j_2, both belonging to its nearest neighbours and each of them being part of a different minimum-path for reaching the DN. The transit packet will be directed towards one of these two nodes: the choice between the direction j_1 or the direction j_2 is made according to the probabilistic rule (Echenique *et al.*, 2004):

$$P(j_1) = \frac{e^{-\beta X_{j_1}}}{e^{-\beta X_{j_1}} + e^{-\beta X_{j_2}}} \tag{7}$$

$$P(j_2) = \frac{e^{-\beta X_{j_2}}}{e^{-\beta X_{j_1}} + e^{-\beta X_{j_2}}} \tag{8}$$

where X_k, $k = j_1, j_2$ is the number of data packets previously sent to node k through the corresponding paths. β is a suitable parameter that allows constraining the routing strategy to be fully random (if $\beta = 0$, the two directions are chosen with an equal probability), deterministic (if $\beta = \infty$, only direction to j_1 is chosen) or probabilistic (if $\beta = 1$, the two directions are chosen according to the traffic along those directions). Packet dispatching takes place using a FIFO strategy: packets queued in the buffer are treated with *First-in*, *First-out* policy. Notice that the RT is evaluated on the current network's topology. In the presence of a failure (arc or node removal), the RT is reevaluated to eliminate the removed element from the different paths.

The amount of traffic present in the network is measured by the variable λ, which measures the frequency with which nodes emit a packet of data ($0 \le \lambda \le 1$). According to this definition, $\lambda = 0.1$ represents a level of traffic where, at each time-step, 10% of the N nodes of the network generate a packet of data directed towards a randomly chosen set of DNs.

When a traffic simulation on the network starts, all buffers are empty. At a given starting time, each node emits, with probability λ, a data packet directed to a randomly chosen DN. Each node directs the packet towards one of its neighbouring nodes according to its RT and the probability defined in Equations (7)–(8). Data packets are immediately received by that node and placed into the buffer. At the next time-step, the packet creation is repeated. If a node does not create a new packet, it can, during this time-step, forward the first packet residing in its buffer (*i.e.*, that which arrived at first among the others) towards one of its neighbouring nodes according to the data stored in the RT and the routing rules defined by Equations (7)–(8). If, in turn, it gives origin to a further event of data creation, new data packet are processed first and routed according to the RT and the buffer's packets are kept on stand-by; they will be processed as soon as the node will not give origin to new data packets.

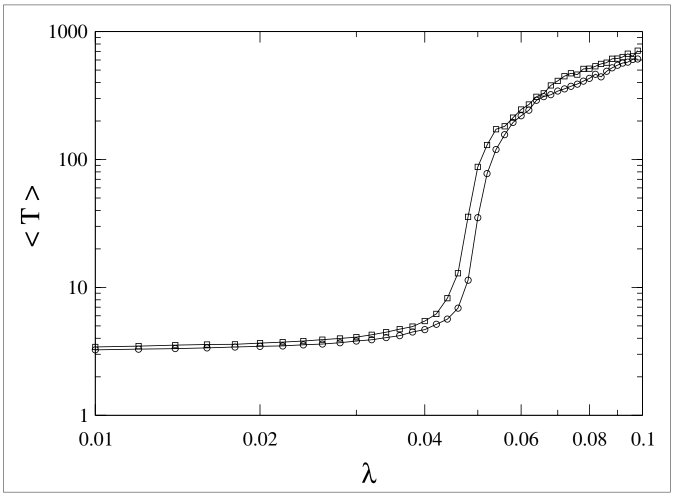
This dynamic is iterated for a large number of time-steps. Packets are received by their DN after a certain *delivery* time τ, which varies according to the distance ON–DN, the network topology and, particularly, the traffic level λ. If λ is quite high, one expects that nodes frequently originate new packets and, thus, cannot promptly deliver previously received data packets, which thus start filling the node's buffer. An average indicator of the efficiency of the network is represented by the value of the 'average delivery time' $< T >$; if the network produces M packets, m out of which are correctly delivered within the simulation time Γ, then:

$$< T > = \frac{1}{m}\sum_{i=1}^{m}\tau_i \tag{9}$$

where τ_i is the delivery time of the packet i.

The behaviour of the network, produced by the action of the basic rules of the model, produces a packet's dynamics which, as a function of the traffic level λ, can be ascribed to two different phases: a *normal* phase, at $\lambda < \lambda_c$, where the $< T >$ behaviour is a (slowly) linearly increasing function of λ. When $\lambda > \lambda_c$, a *congested* phase takes place, producing a rapid, non-linear increase of $< T >$. A typical behaviour of the quantity $< T >$ as a function of the traffic λ for our model of the GARR network is reported in Figure 5. The *congested* phase originates by the presence of buffers which, for large enough traffic values (depending on network size and topology) start filling at a rate larger than their discharge rate.

Figure 5 Log–log scale representation of the behaviour of the average delivery time $< T >$ as a function of the traffic level λ in the unperturbed GARR network (circles) and in the perturbed (four lines simultaneously removed, squares) as predicted by the dynamic model



Also in this case, the behaviour of the system under the presence of structural faults could be measured. Removing one (or more) links, one can evaluate the behaviour of the network in terms of the new form of $< T >$ as a function of λ (Rosato *et al.*, 2006b). We have realised perturbed conditions by removing from the GARR network from one to three links (simultaneously). For each perturbed condition, a traffic simulation has been performed for different traffic levels λ. Figure 5 also reports the behaviour of the perturbed network, which can be directly compared with that of the unperturbed one. A double effect is visible: the average delivery time for the normal phase is slightly increased and the critical traffic value λ_c is shifted to lower values. The effect is quite pronounced as the GARR network is quite small and the removal of four different links corresponds to a severe perturbation.

5 The coupling between the two networks

The interdependency of the two networks has been modelled by making the hypothesis that nodes 'geographically' close are functionally related. Each node of the two networks has been located in the Italian territory according to its geographical position. A GARR node has been assumed to be functionally connected to the closest HVIET load node present in the region. Moreover, we have assumed that a GARR node is connected to one and only one HVIET load node, whereas a HVIET load node can be linked to more than one GARR node. This assumption is correct as HVIET network nodes are those which supply electrical power to large regions; as such, there is a direct correlation between dispatched load and availability of electrical power to supply communication routers. Routers installations are often provided with autonomous power generators for electrical outages. However, they can provide electrical power only for a small amount of time (in the order of hours).

A further assumption taken in the present study is the following. For each load node of HVIET, we determine the unperturbed value of the extracted power, on the basis of the solution of the dispatching problem arising from the normal input condition $P^{(0)}$ and assume that, in this condition, all GARR nodes are correctly supplied (*i.e.*, all GARR nodes are in the on state). If, in turn, some fault affects the HVIET network that triggers the activation of the redispatching procedure, we evaluate the new dispatched power P_i to the load nodes and compare it to the expected power $P_i^{(0)}$. We assume that a GARR node k, supplied by the HVIET node i, is in its on state if the actual dispatched power on load node i is such that:

$$\text{GARR node } k = \begin{cases} \text{on} & \text{if } P_i \geq \alpha P_i^{(0)} \\ \text{off} & \text{otherwise} \end{cases} \tag{10}$$

where $0 \leq \alpha \leq 1$ is a suitable parameter, which determines the strength of the 'coupling' between the two networks. In other words, if the electrical node is not able to dispatch a sufficient power (*i.e.*, at least the α fraction of the normal power) then the communication node is switched off thus affecting the functionality of the GARR network. This is a realistic assumption: in fact, in case of a dispatching problem from the electrical side, it is reasonable to suppose that several users connected to the node on the low-voltage distribution network supplied by that node will be put in blackout. The parameter α introduced here allows to vary the amount of *QoS* that the router is able to accommodate: if α is close to one, a slight reduction of the dispatched power to put the router in its off state will be sufficient. Conversely, if α is small, the router can accommodate a large decrease of the dispatched power. In the absence of an unavailable realistic figure, we have set the α value to an intermediate value of $\alpha = 0.75$, which should represent a reasonable trade-off between a strong interdependence coupling (α close to one) and a substantial absence of interdependence (with α close to zero). We have planned to perform a parametric study to evaluate the interdependent behaviour of the two networks as a function of the coupling parameter α.

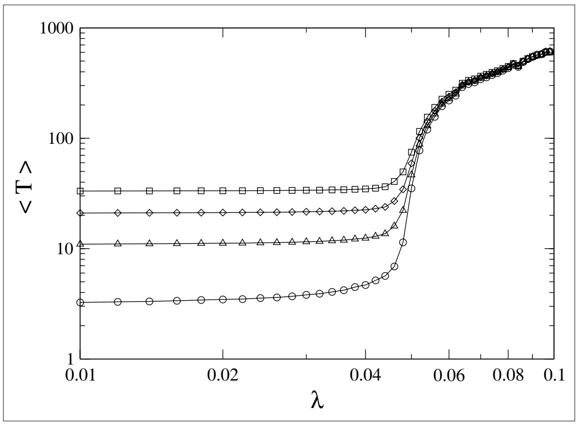
6 Simulations and results

We have performed the following simulations: we have perturbed the HVIET network by removing randomly a line ($\xi = 1$) and for each instance, we have evaluated if Equations (2) and (4) were fulfilled for the normal load condition $P^{(0)}$. If it was not the case, we used the redispatching procedure to evaluate the new load power dispatched to the load nodes. For each final configuration of electric loads, we evaluate the corresponding configuration of the GARR network, putting in the off state those communication nodes that are not correctly supplied according to the rule defined in Equation (10). Such perturbed configuration of the GARR network was then used to host a traffic simulation to evaluate the $< T > = f(\lambda)$ function. A severe perturbation of the electrical network can induce an even stronger perturbation of the communication network. There are cases, in fact, in which the perturbation of the electrical network results in the disconnection of the GARR network (one or more nodes cannot be reached anymore). In those cases, the value of $< T >$was arbitrarily set to the maximum value assumed by this quantity at large values of λ (*i.e.*, the value of $< T > = f(\lambda = 0.1)$ in Figure 5). We then performed the same analysis after the removal of two ($\xi = 2$) and three ($\xi = 3$) randomly selected lines from the HIVET network.

For each value of the perturbation $\xi = \{1,2,3\}$, we evaluated the response of the GARR network as the average over the different configurations issued upon the HVIET faults. In Figure 6 we report the result for different perturbation strengths ($\xi = 1,2,3$) and for the value of $\alpha = 0.75$ (this means that a GARR node is put in the off state only if the power received by the load node, to which it is connected, was lower than 75% of the normal power).

Figure 6 shows that the effect of the perturbation on the GARR network consists in the increase of the average delivery time in the 'normal' regime, while the critical traffic level λ_c, where the congested phase onsets, remains essentially unchanged. This marked effect in the region of 'normal' regime depends on the fact that, owing to the small size of the GARR network, even with an intermediate coupling parameter α, even a small degradation of the electrical load might result in the disconnection of the GARR network. In that case, according to what was previously stated, the whole function $< T > = f(\lambda)$ is set to a large value of $< T >$; when averaged with the results of all the simulations performed for a given perturbation strength ξ of the electrical network, this condition dramatically increases the average delivery time in the 'normal' regime.

Figure 6 Log–log scale representation of the behaviour of the average delivery time $< T >$ as a function of the traffic level λ in the unperturbed GARR network (circles), and in the presence of a perturbation on the HVIET of strength $\xi = 1$ (triangles), $\xi = 2$ (diamonds), $\xi = 3$ (squares). The value of the parameter α of Equation (10) has been set to $\alpha = 0.75$.

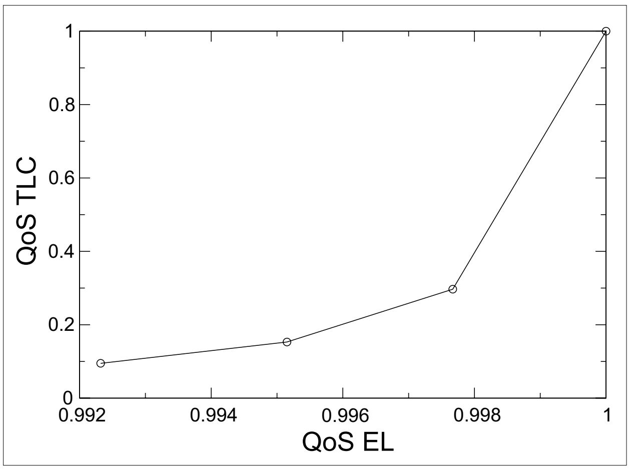


The relative impact of one network on the other could be directly measured by resorting to the definition of CI's *QoS*. If we define *QoS* of the electrical infrastructure as in Equation (6) (Rosato *et al.*, 2006a) and *QoS* of the communication infrastructure is defined as:

$$QoS\ TLC = \frac{\frac{m}{M}}{\frac{< T >}{< T_0 >}} \tag{11}$$

where m and M are the number of generated and dispatched packets, respectively, $< T >$ and $< T_0 >$ are the average delivery times in the normal phase for the perturbed and the unperturbed network, respectively, we can rationalise the interdependency between the two networks by showing *QoS TLC* as a function of *QoS EL* (Figure 7).

Figure 7 Variation of the *QoS* value for the communication infrastructure (*QoS TLC*) with respect to the variation of the *QoS* of the electrical network (*QoS EL*)



7 Conclusion

This work represents a first attempt to define a methodological workflow enabling the characterisation of coupled networks and the emerging effects related to their level of interdependency. We focused our attention on the Italian high-voltage grid (HVIET) and on its functional dependency with the Italian internet high-bandwidth backbone (GARR). Both networks have been modelled on the basis of their topological graphs, giving a suitable model for the electrical flow and data packets, respectively. The networks have been assumed to be interconnected on a geographical basis, taking into account a coupling coefficient α. The strength of the parameter α can be suitably adapted on the basis of information concerning the power supply mechanisms adopted in the real cases. If α is close to 1, the coupling is strong (very low power loss of the electrical load node is sufficient to put the communication nodes in the *off* state). The opposite case (α close to zero) is, in turn, the sign of a weak coupling, where only a severe power loss of the electrical node produces the *off* state of the related communication nodes.

Figure 7 shows that, under the hypotheses done in the present simulation, there is a significant amplification of the effect of a fault, occurring in the electrical network, on the communication network, in the case of an intermediate coupling ($\alpha = 0.75$). In fact, although under the fault, the HVIET network is able (through the redispatching procedure) to partially recover its operability level, the same does not hold for communication which, despite the adoption of an alternative routing strategy (after the introduction of the faults in the GARR network, new RTs are evaluated for eliminating the `off` state nodes from the data paths) is not able to fully recover its operability and to grant an acceptable *QoS*.

Far from being a *quantitative* assessment of such a relevant problem (the interdependency between electrical and communication networks), this work aims to propose a methodology that could open the way to a number of more realistic and detailed assessment of the effects of the coupling between CIs. Moreover, in a number of ongoing works, this methodological framework is applied by the authors to other cases (motorways, railways, *etc*.) in order to establish macroscopic-scale relations among different couples of infrastructures. This will constitute a basis to set a phenomenological analysis of the global framework shown in Section 2.

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