542

cumason

October 2019

1. The following are equivalent WFF's because the x used in both functions are equivalent

$$P(x) \cup Q(x)$$
$$\forall_x.(P(x) \cup Q(x))$$

Unlike the above, these WFF's are equivalent because the variables passed into P and Q aren't necessarily the same

$$P(x) \cup Q(y)$$
$$\forall_x.P(x) \cup \forall_x.Q(x)$$
$$\forall_x.P(x) \cup \forall_y.Q(y)$$
$$\forall_x.\forall_y.(P(x) \cup Q(y))$$

2. the WFF's:

$$\forall_x.\exists_y.P(x, y)$$
$$\exists_x.\forall_y.P(x, y)$$

are not the same. A simple proof by contradiction, let's say the function

$$P(a, b) := b$$

We may observe that

$$\forall_x.\exists_y.P(x, y)$$

is true because y could always be true while

$$\exists_x.\forall_y.P(x, y)$$

is false because the function will always be false if y is false.

3. The following WFF's are not equivalent:

$$\forall_x . P(x) \rightarrow \exists_x . Q(x)$$
$$\forall_x . (P(x) \rightarrow Q(x))$$

   For example, let's define

$$P(x) := True$$

   and

$$Q(x) := x mod 2 == 0$$

   We may now observe that

$$\forall_x . P(x) \rightarrow \exists_x . Q(x)$$

   is true because there will always exist an even number whereas

$$\forall_x . (P(x) \rightarrow Q(x))$$

   will be false because P will evaluate to true and q will be false whenever x is an odd number.

   The first formula can not imply the second as shown with above example, but the second may imply the first. I know this because if the second is true, then the x chosen in the must exist section of the first formula will simply be the same value of x as passed into P

4. x != y