Solution

From the official solution

For each $1 \leq i \leq N^2$, we compute the minimum number of viewers that will hate viewer i forever (the answer is the sum of these values).

This number coincides with the minimum cost of a path from the seat of viewer i to the sides of the square, considering that going through an empty seat has cost 0 and going through an occupied seat has cost 1

Let $H_k(i)$ be the minimum cost (as defined above) of a path from the seat of viewer i to the outside after the first k viewers have left the cinema.

Our strategy is to keep all values $H_k(i)$ updated at all times. When the viewer P_k goes away, we perform a DFS starting from the seats of P_k and updating the values. During the k^{th} DFS, we will visit only the seats i such that $H_k(i) < H_{k-1}(i)$, hence the total number of seats visited in the N^2 steps (for $1 \leq k \leq N^2$) is $O(N^3)$ (see key observation)

Key observation

The values $H_k(i)$ are decreasing (for all a k). At the beginning we have:

$$H_0(1) + H_0(2) + \cdots + H_0(N^2) \approx \frac{N^3}{6}$$

Because we visit seats will strictly decreasing value of H, and since the sum of all H are $O(N^3)$, we visit at most $O(N^3)$ seats for updating