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2)

a) $a \times (b+c) = a \times b + a \times c$?

Let $a = (a_1, a_2, a_3)^T$
$b = (b_1, b_2, b_3)^T$
$c = (c_1, c_2, c_3)^T$

(we normally use collumn vectors)

$$a \times (b+c) = (a_1, a_2, a_3)^T \times (b_1+c_1, b_2+c_2, b_3+c_3)^T \quad (1)$$

$$= \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1+c_1 & b_2+c_2 & b_3+c_3 \end{vmatrix}$$

Here you are using what you are supposed to proove in (c)

$$= i(a_2(b_3+c_3) - a_3(b_2+c_2))$$
$$- j(a_1(b_3+c_3) - a_3(b_1+c_1))$$
$$+ k(a_1(b_2+c_2) - a_2(b_1+c_1))$$

$$a \times b = (a_1, a_2, a_3) \times (b_1, b_2, b_3)$$

$$= \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

$$= i(a_2 b_3 - a_3 b_2) - j(a_1 b_3 - a_3 b_1) + k(a_1 b_2 - a_2 b_1) \quad (2)$$

$$a \times c = (a_1, a_2, a_3) \times (c_1, c_2, c_3)$$

$$= \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$= i(a_2 c_3 - a_3 c_2) - j(a_1 c_3 - a_3 c_1) + k(a_1 c_2 - a_2 c_1) \quad (3)$$

by (2) + (3) $= i(a_2(b_3+c_3) - a_3(b_2+c_2))$
$- j(a_1(b_3+c_3) - a_3(b_1+c_1))$
$+ k(a_1(b_2+c_2) - a_2(b_1+c_1))$

so (1) = (2) + (3) then $a \times (b+c) = a \times b + a \times c$

Just use the definition 0/1

2c) $a = a_1 e_x + a_2 e_y + a_3 e_z$ | $b = b_1 e_x + b_2 e_y + b_3 e_z$

$$a \times b = (a_1 e_x + a_2 e_y + a_3 e_z) \times (b_1 e_x + b_2 e_y + b_3 e_z)$$
$$= a_1 b_1 (e_x \times e_x) + a_1 b_2 (e_x \times e_y) + a_1 b_3 (e_x \times e_z)$$
$$+ a_2 b_1 (e_y \times e_x) + a_2 b_2 (e_y \times e_y) + a_2 b_3 (e_y \times e_z)$$
$$+ a_3 b_1 (e_z \times e_x) + a_3 b_2 (e_z \times e_y) + a_3 b_3 (e_z \times e_z)$$

Here,

$$e_x \times e_x = e_y \times e_y = e_z \times e_z = 0 \; [\because \sin 0° = 0]$$
$$e_x \times e_y = e_y \times e_z = e_z \times e_x = 1 \; [\because \sin 90° = 1]$$
$$e_y \times e_x = e_z \times e_y = e_x \times e_z = -1 \; [\because \sin(-90°) = -1]$$

why? you don't need this

This is pretty close to the definition from the lecture

$$a \times b = e_x(a_2 b_3 - a_3 b_2) - e_y(a_1 b_3 - a_3 b_1) + e_z(a_1 b_2 - a_2 b_1)$$

$$= \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} \cdot e_x - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} \cdot e_y + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} e_z \quad ✓ \quad = \begin{vmatrix} e_x & e_y & e_z \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

$$\therefore |a \times b| = \left| \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} e_x - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} \cdot e_y + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} e_z \right| \quad ✓$$

$$= \left\| \begin{matrix} e_x & e_y & e_z \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{matrix} \right\|$$

This is not asked in the exercise

1.5/2

b) $a \times (a \times b) = 0$?

let $a = (a_1, a_2, a_3)$
$b = (b_1, b_2, b_3)$

Why?

$a \times b = (a_1, a_2, a_3) \times (b_1, b_2, b_3)$

$$= \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

$$= i\cdot(a_2b_3 - a_3b_2) - j(a_1b_3 - a_3b_1) + k(a_1b_2 - a_2b_1)$$

$$a\cdot(a\times b) = a_1\cdot(a_2b_3 - a_3b_2) - a_2\cdot(a_1b_3 - a_3b_1) + a_3(a_1b_2 - a_2b_1)$$

$$= \cancel{a_1a_2b_3} - \cancel{a_1a_3b_2} - \cancel{a_1a_2b_3} + \cancel{a_2a_3b_1} + \cancel{a_1a_3b_2} - \cancel{a_2a_3b_1}$$

$$= 0$$

(*)

(*) is close to what is asked of you 0.5/1

d) $a\cdot(b\times c) = b\cdot(c\times a) = c\cdot(a\times b)$?

$$a\cdot(b\times c) = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$= a_1(b_2c_3 - b_3c_2) - a_2(b_1c_3 - b_3c_1) + a_3(b_1c_2 - b_2c_1) \quad ①$$

$$b\cdot(c\times a) = \begin{vmatrix} b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \\ a_1 & a_2 & a_3 \end{vmatrix}$$

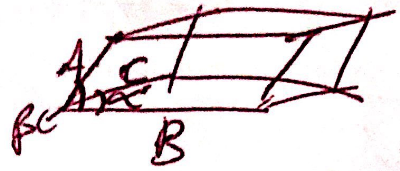
$$= b_1(c_2a_3 - c_3a_2) - b_2(c_1a_3 - c_3a_1) + b_3(c_1a_2 - c_2a_1) \quad ②$$

$$① = ② = a_1b_2c_3 - a_1b_3c_2 - a_2b_1c_3 + a_2b_3c_1 + a_3b_1c_2 - a_3b_2c_1$$

so they are same.

OK. 1/1

e) $a \cdot (b \times c) \simeq$ V of parallelepiped ?



Introduce new variables!

volume = height × surface area

~~surface area [illegible] sin α~~

surface area is $|b| \cdot |c| \cdot \sin\alpha = |(b \times c)|$ because it's a parallel

for height: $|a| \cdot \cos\beta$

$\Rightarrow$ volume $= a \cdot (b \times c) = |a| \cdot \cos\beta \cdot (b \times c)$

What you mean is Vol. $= |b \times c| |a| \cos\beta$

$= a \cdot (b \times c)$ 0.5/1

Please read the exercises carefully and use the definitions from the lecture!

Practical Part

Assignment 1) Transformations
a) 1/1
b) 1/1
c) 1/2 (correct idea, but wrong result due to implementation)

3/4

Theoretical Part

Assignment 2) Cross and Dot Product
a) 0/1
b) 0.5/1
c) 1.5/2
d) 1/1
e) 0.5/1

3.5/6