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2.5/3

1a) $f(x) = \text{sgn}(\cos(x)) = \begin{cases} 1; & \text{if } \cos(x) > 0 \\ -1, & \text{if } \cos(x) < 0 \end{cases}$

$= \begin{cases} 1, & \text{if } 0 < x \leq \pi/2;\ 3\pi/2 < x \leq 2\pi \\ -1, & \text{if } \pi/2 < x \leq 3\pi/2 \end{cases}$

when, $\cos(x) = 0$,

$\Rightarrow$ sgn(0) can be 1 or -1 to fit the cases of $g(x)$

Now,

$$f^*(x) \cdot g(x) = \underbrace{f(x) \cdot g(x)}_{\text{because } f \text{ is real}} = \begin{cases} \frac{2}{\pi}x; & \text{if } 0 < x \leq \pi/2 \\ \frac{2}{\pi}x - 2; & \text{if } \pi/2 < x \leq 3\pi/2 \\ \frac{2}{\pi}x - 4; & \text{if } 3\pi/2 < x \leq 2\pi \end{cases}$$

So, $\int_0^{2\pi} f(x) \cdot g(x)\, dx$

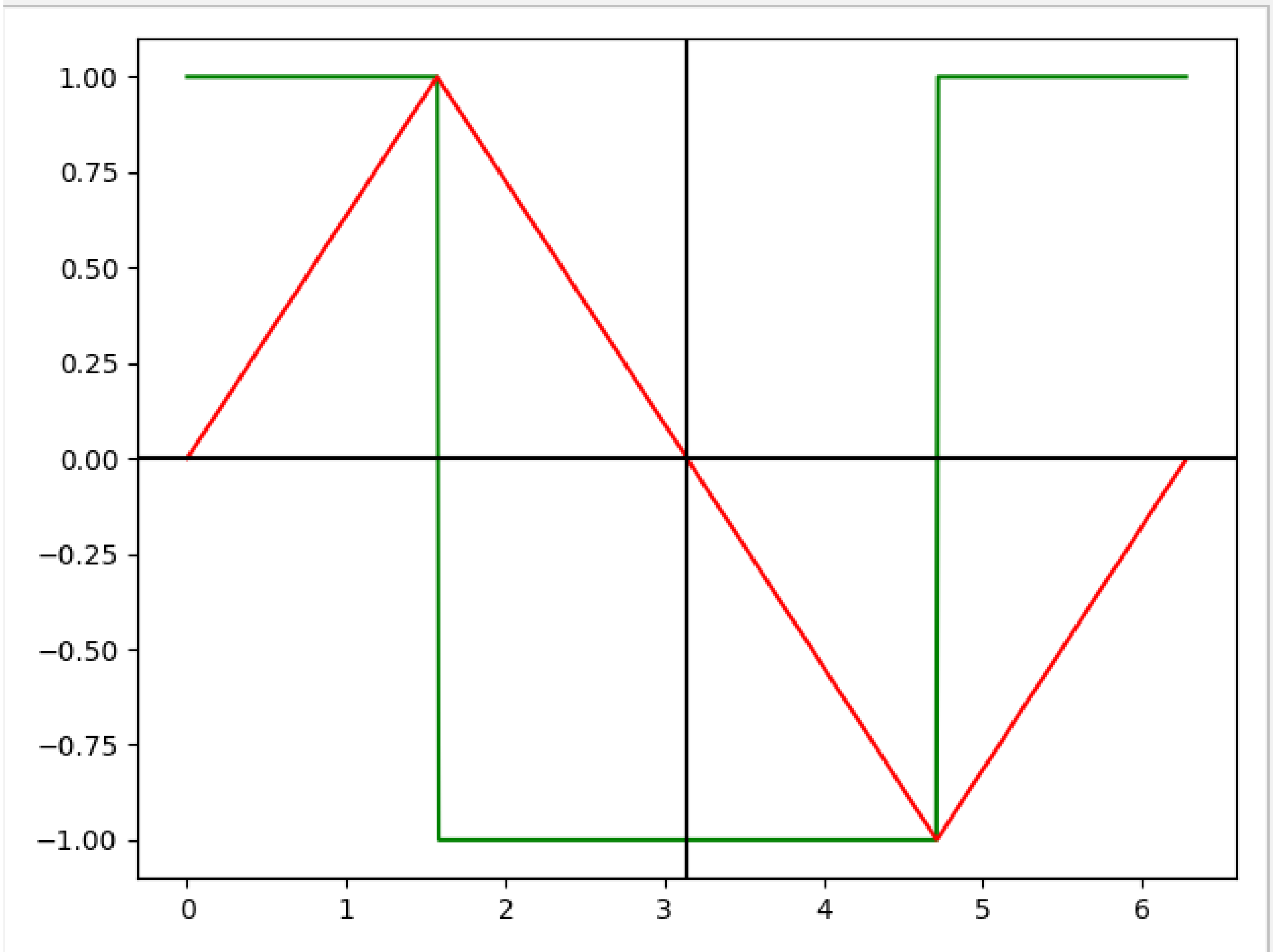
$$= \int_0^{\pi/2} \frac{2}{\pi}x\, dx + \int_{\pi/2}^{3\pi/2} \left(\frac{2}{\pi}x - 2\right) dx + \int_{3\pi/2}^{2\pi} \left(\frac{2}{\pi}x - 4\right) dx$$

$$= \frac{1}{\pi}\left[x^2\right]_0^{\pi/2} + \frac{1}{\pi}\left[x^2\right]_{\pi/2}^{3\pi/2} - 2\left[x\right]_{\pi/2}^{3\pi/2} + \frac{1}{\pi}\left[x^2\right]_{3\pi/2}^{2\pi} - 4\left[x\right]_{3\pi/2}^{2\pi}$$

$$= \frac{1}{\pi}\left(\frac{\pi^2}{4} - 0\right) + \frac{1}{\pi}\left(\frac{9\pi^2}{4} - \frac{\pi^2}{4}\right) - 2\left(\frac{3\pi}{2} - \frac{\pi}{2}\right) + \frac{1}{\pi}\left(4\pi^2 - \frac{9\pi^2}{4}\right) - 4\left(2\pi - \frac{3\pi}{2}\right)$$

$$= \pi/4 + 2\pi - 2\pi + \frac{7}{4}\pi - 2\pi$$

$$= 0 \checkmark$$



0.5/1

1b) f(x) : Green , g(x) : Red

Horizontal axis is at ~~x~~ y = 0 and the vertical axis is at x = π.

f(x) creates vertical and horizontal lines. g(x) creates 2 same sized triangles. One above the horizontal axis and one below the horizontal axis. This 2 triangles are divided equally by the vertical lines created by f(x).

Both functions covers same amount of area above and below the horizonal axis. So after multiplying them and integration they will cancel each other out. So the result will be 0.

Your explanation is not very clear.

... so the result will be 0.

2b) $$f(t) = \sum_{n=-\infty}^{\infty} \hat{f}(n) . e^{2\pi i n t}$$

$$= \sum_{n=-\infty}^{\infty} \langle f, e_n \rangle . e_n$$

Given, $\int_0^1 |f(t)|^2 dt = ||f||^2 = \langle f, f \rangle$

Given what?

$$= \left\langle \sum_{n=-\infty}^{\infty} \langle f, e_n \rangle . e_n \, , \, \sum_{k=-\infty}^{\infty} \langle f, e_k \rangle . e_k \right\rangle$$

$$= \sum_{n,k=-\infty}^{\infty} \overline{\langle f, e_n \rangle} . \langle f, e_k \rangle . \langle e_n, e_k \rangle$$

$$= \sum_{n,k=-\infty}^{\infty} \overline{\langle f, e_n \rangle} \langle f, e_k \rangle \, \delta_{k,n}$$ where $\delta_{k,n} = \begin{cases} 1, & k=n \\ 0, & O/W \end{cases}$

$$= \sum_{n=-\infty}^{\infty} \overline{\langle f, e_n \rangle} \langle f, e_n \rangle$$ [As $e_n(t) = e^{2\pi i n t}$ are orthonormal w.r.t inner product]

$$= \sum_{n=-\infty}^{\infty} |\langle f, e_n \rangle|^2 = \sum_{n=-\infty}^{\infty} |\hat{f}(n)|^2$$

Theory Points: 2.5 + 0.5 + 4 /8

total current points: 23/31 (74%)

Practical Points: 3(large difference) + 4 /8

Total current points: 39.5/46 (86%)