6.1	6.2	6.3	Σ
6	10	8	24

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6.1.

a) $f'(t) = \frac{d}{dt} f(t) = \frac{d}{dt} \mathcal{F}^{-1}\{\hat{f}(\omega)\} = \frac{d}{dt} \int_{-\infty}^{+\infty} \hat{f}(\omega) e^{2\pi i t\omega} d\omega$

$= \int_{-\infty}^{+\infty} \hat{f}(\omega) \frac{d}{dt} e^{2\pi i t \omega} d\omega = \int_{-\infty}^{+\infty} \hat{f}(\omega) (2\pi i \omega) e^{2\pi i \omega t} d\omega$

$= \mathcal{F}^{-1}\{\hat{f}(\omega)(2\pi i \omega)\} \Rightarrow \mathcal{F}\{f'(t)\} = \hat{f}'(\omega) = 2\pi i \omega \hat{f}(\omega)$ ✓

b) $\frac{d}{d(\omega)} \hat{f}(\omega) = \frac{d}{d\omega} \mathcal{F}\{f(t)\} = \frac{d}{d\omega} \int_{-\infty}^{+\infty} f(t) e^{-2\pi i t \omega} dt$

$= \int_{-\infty}^{+\infty} f(t) \frac{d}{d\omega} e^{-2\pi i t\omega} dt = \int_{-\infty}^{+\infty} f(t) (-2\pi i t) e^{-2\pi i t \omega} dt$

$= -2\pi i \int_{-\infty}^{+\infty} f(t)\, t\, e^{-2\pi i t \omega} dt = -2\pi i\, \mathcal{F}\{t f(t)\} = -2\pi i\, \mathcal{F}\{g(t)\}$

$= -2\pi i\, \hat{g}(\omega)$ ✓

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6.2 $x,y \in \ell^2(\mathbb{Z}),\ \lambda \in \mathbb{C},\ k \in \mathbb{Z},\ w_0 \in [0,1)$, Prove;

a) $\widehat{x+y} = \hat{x} + \hat{y}$ and $\widehat{\lambda x} = \lambda . \hat{x}$?

$\xrightarrow{1} \widehat{x+y} = \sum_{n\in\mathbb{Z}} (x+y)(n) e^{-2\pi i w n} = \sum_{n\in\mathbb{Z}} (x(n)+y(n)).e^{-2\pi i w n} = \sum_{n\in\mathbb{Z}} x(n).e^{-2\pi i w n} + \sum_{n\in\mathbb{Z}} y(n).e^{-2\pi i w n}$

$\xrightarrow{2} \widehat{\lambda x} = \sum_{n\in\mathbb{Z}} (\lambda x)(n).e^{-2\pi i w n} = \lambda . \sum_{n\in\mathbb{Z}} x(n).e^{-2\pi i w n} = \lambda \hat{x}$ $= \hat{x} + \hat{y}$

✓ 2/2

b) $\hat{x}_k(w) = e^{-2\pi i w k}.\hat{x}(w)$, $x_k(n) = x(n-k)$?

$\hat{x}_k(w) = \sum_{n\in\mathbb{Z}} x(n-k).e^{-2\pi i w n} \overset{n-k=m}{=\!=} \sum_{m\in\mathbb{Z}} x(m).e^{-2\pi i w(m+k)} = e^{-2\pi i w k}.\sum_{m\in\mathbb{Z}} x(m).e^{-2\pi i w m}$

$= e^{-2\pi i w k}.\hat{x}(w)$ ✓ 2/2

c) $\widehat{x^{w_0}}(w) = \hat{x}(w+w_0)$, $x^{w_0}(n) = e^{-2\pi i w_0 n}.x(n)$?

$\widehat{x^{w_0}}(w) = \sum_{n\in\mathbb{Z}} x^{w_0}(n).e^{-2\pi i w n} = \sum_{n\in\mathbb{Z}} e^{-2\pi i w_0 n}.x(n).e^{-2\pi i w n} = \sum x(n).e^{-2\pi i (w+w_0)n}$

$= \hat{x}(w+w_0)$ ✓ 2/2

d) $y = \bar{x} \Rightarrow \hat{y}(w) = \overline{\hat{x}(-w)}$?

$\hat{y}(w) = \sum_{n\in\mathbb{Z}} \overline{x(n)}.e^{-2\pi i w n} = \overline{\sum_{n\in\mathbb{Z}} x(n).e^{-2\pi i(-w).n}} = \overline{\hat{x}(-w)}$ ✓ 2/2

e) $y(n) = x(-n) \Rightarrow \hat{y}(w) = \hat{x}(-w)$?

$\hat{y}(n) = \sum_{n\in\mathbb{Z}} x(-n).e^{-2\pi i w n} \overset{n=-m}{=\!=} \sum_{m\in\mathbb{Z}} x(m).e^{-2\pi i(-w)m} = \hat{x}(-w)$ ✓ 2/2