| 7.1 | 7.2 | 7.3 | $\Sigma$ |
|---|---|---|---|
| 4 | 8 | 8 | ~~22~~ 21 |

Foundations of Audio Signal Processing — Exercise sheet 7


**Group Members:**
Muhammad Ahmed (3304158)
Cüneyt Erem (3277992)
Ali Mohammadi (3289515)
Rozhin Bayati (3314202)


## Exercise 7.1:

Given: $\langle \mathrm{sinc}(\cdot - k) | \mathrm{sinc}(\cdot - l) \rangle = \delta_{kl}$

Using Plancherel's Theorem

$$\langle \mathrm{sinc}(\cdot - k) | \mathrm{sinc}(\cdot - l) \rangle = \langle \widehat{\mathrm{sinc}(\cdot - k)} | \widehat{\mathrm{sinc}(\cdot - l)} \rangle$$

$$= \langle \widehat{\mathrm{sinc}_k} | \widehat{\mathrm{sinc}_l} \rangle$$

used property $\left\{ \hat{f}_{t_0}(\omega) = e^{-2\pi i \omega t_0} \hat{f}(\omega) \right\}$ $\quad = \langle e^{-2\pi i k t} \widehat{\mathrm{sinc}} | e^{-2\pi i l t} \widehat{\mathrm{sinc}} \rangle$

We also know that the sinc function is the Fourier transform of the normalized box-function

$$\boxed{f := \chi_{[-1/2, 1/2]}}$$

then,

$$= \langle e^{-2\pi i k t} \chi_{[-1/2,1/2]} | e^{-2\pi i l k t} \chi_{[-1/2,1/2]} \rangle$$

$$= \int_{-1/2}^{1/2} e^{-2\pi i (l-k) t} \, dt$$

conjugation missing

Now consider the case $k = l$, then

$$\int_{-1/2}^{1/2} e^{-2\pi i (l-l) t} \, dt = \int_{-1/2}^{1/2} 1 \, dt = [t]_{-1/2}^{1/2} = 1$$

and for $k \neq l$:

$$\int_{-1/2}^{1/2} e^{-2\pi i (l-k) t} \, dt = \left[ \frac{e^{-2\pi i (l-k) t}}{-2\pi i (l-k)} \right]_{-1/2}^{1/2}$$

$$= \frac{e^{-\pi i (l-k)} - e^{\pi i (l-k)}}{-2\pi i (l-k)}$$

$$= \frac{(\cos(-\pi(l-k)) + i \sin(-\pi(l-k)) - (\cos(\pi(l-k)) + i \sin(\pi(l-k))}{-2\pi i (l-k)}$$

$$= \frac{\cos(\pi(l-k)) - i \sin(\pi(l-k)) - \cos(\pi(l-k)) - i \sin(\pi(l-k))}{-2\pi i (l-k)}$$

$$= \frac{-2i \sin(\pi(l-k))}{-2\pi i (l-k)} = \frac{\sin(\pi(l-k))}{\pi(l-k)} = \mathrm{sinc}(\pi(l-k))$$

$$= 0 \qquad \left\{ \begin{array}{l} \text{For multiples of } \pi \\ \sin \text{ is zero} \end{array} \right.$$

So only when $k \neq l$, both functions don't intersect, so their inner product is zero, and are orthogonal.

4/5

## 7.2

a) $\hat{g}$ of the $g \in L^2(\mathbb{R})$, T-sampled version of f, result?

$$f(kT) = \int_{-5\Omega}^{+5\Omega} \hat{f}(w) e^{2\pi i w kT} dw$$

$$= \int_{-5\Omega}^{-3\Omega} \hat{f}(w) e^{2\pi i w kT} dw + \int_{-3\Omega}^{-\Omega} \hat{f}(w) e^{2\pi i w kT} dw + \int_{-\Omega}^{\Omega} \hat{f}(w) e^{2\pi i w kT} dw + \int_{\Omega}^{+3\Omega} \hat{f}(w) e^{2\pi i w kT} dw + \int_{3\Omega}^{5\Omega} \hat{f}(w) e^{2\pi i w kT} dw$$

$$= \int_{-\Omega}^{\Omega} \hat{f}(w-4\Omega) e^{2\pi i (w-4\Omega)kT} dw + \int_{-\Omega}^{\Omega} \hat{f}(w-2\Omega) e^{2\pi i (w-2\Omega)kT} dw + \int_{-\Omega}^{\Omega} \hat{f}(w) e^{2\pi i w kT} dw + \int_{-\Omega}^{\Omega} \hat{f}(w) e^{2\pi i (w+2\Omega)kT} dw + \int_{-\Omega}^{\Omega} \hat{f}(w) e^{2\pi i (w+4\Omega)kT} dw$$

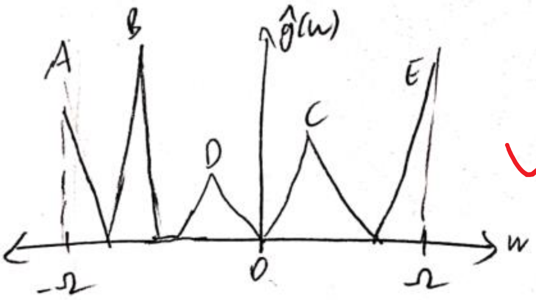
we know that $T = \frac{1}{2\Omega}$. $e^{2\pi i (\mp 2\Omega)k}$ periodic so that we can substitute $e^{2\pi i (w \mp 2\Omega)kT} = e^{2\pi i w kT}$

$$= \int_{-\Omega}^{\Omega} \hat{f}(w-4\Omega) e^{2\pi i w kT} dw + \int_{-\Omega}^{\Omega} \hat{f}(w-2\Omega) e^{2\pi i w kT} dw + \text{----}$$

$$= \int_{-\Omega}^{\Omega} \underbrace{[\hat{f}(w-4\Omega) + \hat{f}(w-2\Omega) + \hat{f}(w) + \hat{f}(w+2\Omega) + \hat{f}(w+4\Omega)]}_{\hat{g}(w) \text{ where } |w| \leq \Omega,\ 0 \text{ otherwise}} e^{2\pi i w kT} dw$$

✓ 4/4

b)




✓ 4/4

## Exercise 7.3:

**Script to call the function to get the following plot:**

```
Sheet7Exercise3('random_signal.mat')
```