Foundations of Audio Signal Processing

Exercise Sheet 8

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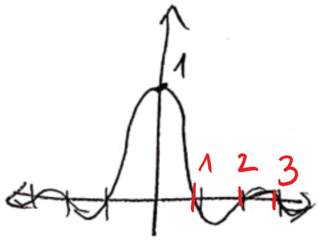
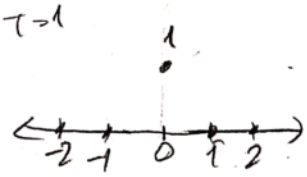
8.1	8.2	8.3	Σ
5,5	3,5	8	16/18

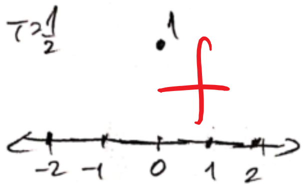
8.1

a) — $f(t) = \mathrm{sinc}(t) \Rightarrow$ signal f sampled;

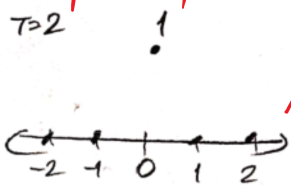


— T-sampled for $n \to f(n \cdot T)$ for $T \in \{1, \frac{1}{2}, 2\}$





f

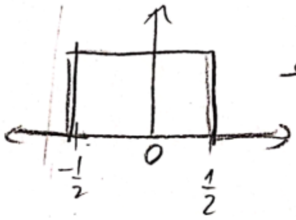


1,5/2

sampling rate is $Hz = \frac{1}{T} \Rightarrow$ $T=1$, $1\,Hz$, $T=\frac{1}{2}$, $2\,Hz$, $T=2$, $0{,}5\,Hz$ ✓

b) smallest Ω for f is bandlimited;

Ω for f is bandlimited for $f(t) = \mathrm{sinc}(t) \Rightarrow \hat{f}(\omega)$ is a box function.



$\Omega = \frac{1}{2}$ ✓

c) Sampling theorem, reconstructible f for $T \in \{1, \frac{1}{2}, 2\}$;

f is reconstructible for $T \leq \frac{1}{2\Omega}$ where $\Omega = \frac{1}{2} \Rightarrow T \leq 1$
$\hookrightarrow 1/2$

$\Rightarrow$ $T = 1 \Rightarrow 1 \leq 1$ it is reconstructible, ✓

$T = \frac{1}{2} \Rightarrow \frac{1}{2} \leq 1$ it is reconstructible with oversampling,

$T = 2 \Rightarrow 2 \not\leq 1$ it is <u>not</u> reconstructible with undersampling

Exercise 8.2

For a signal $x \in \ell^2(\mathbb{Z})$ the upsampling operator is defined as:

$$(\uparrow M)[x](n) = \begin{cases} x(n/M), & \text{if } M|n, \\ 0, & \text{otherwise} \end{cases}$$

a) Linear

* $$(\uparrow M)[x+y](n) = \begin{cases} (x+y)(n/M), & \text{if } M|n, \\ 0, & \text{otherwise} \end{cases}$$
$$= \begin{cases} x(n/M) + y(n/M), & \text{if } M|n \\ 0, & \text{otherwise} \end{cases}$$
$$= (\uparrow M)[x](n) + (\uparrow M)[y](n)$$

* $$(\uparrow M)[\lambda x](n) = \begin{cases} (\lambda x)(n/M), & \text{if } M|n \\ 0, & \text{otherwise} \end{cases}$$
$$= \begin{cases} \lambda[x(n/M)], & \text{if } M|n \\ 0, & \text{otherwise} \end{cases}$$
$$= \lambda (\uparrow M)[x](n)$$

2/2

Hence upsampling operator is linear.

b) Time-invariant

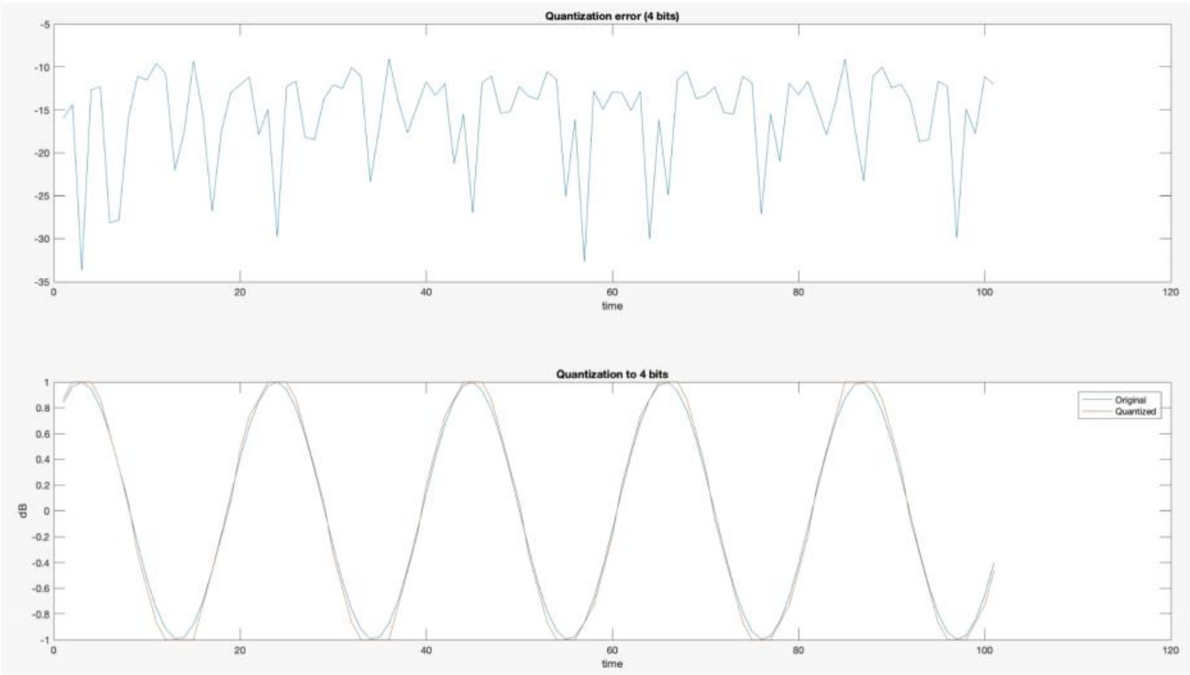
* $$(\uparrow M \circ T^k)[x](n) = (\uparrow M[T^k[x]])(n)$$
$$= (\uparrow M[y])(n)$$
$$= \begin{cases} y(n/M), & \text{if } M|n \\ 0, & \text{otherwise} \end{cases}$$
$$= \begin{cases} x(n/M - k), & \text{if } M|n \\ 0, & \text{otherwise} \end{cases}$$

* $$(T^k \circ \uparrow M)[x](n) = (T^k[(\uparrow M)[x]])(n)$$
$$= (T^k[y])(n)$$
$$= y(n-k)$$
$$= \begin{cases} x\left(\frac{n-k}{M}\right), & \text{if } M|(n-k) \\ 0, & \text{otherwise} \end{cases}$$

1,5/2

and since they are not equal, upsampling operator is not time-invariant.

Exercise 8.3:



Quantization error (4 bits)
-5
-10
-15
-20
-25
-30
-35
0
20
40
60
80
100
120
time
Quantization to 4 bits
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
dB
Original
Quantized
0
20
40
60
80
100
120
time