Foundations of Audio Signal Processing

Date: 01.11.2018

Group Members:
Muhammad Ahmed (3304158)
Cüneyt Erem (3277992)
Ali Mohammadi (3289515)
Rozhin Bayati (3314202)

Exercise 3

3.1	3.2	3.3	Σ
6	3	6	15/18

3.1 a) As

$$\Omega_n^k = e^{i\pi 2k/n} \quad \text{where } k \in [1:n-1]$$

then for $n=5$, $k=[0,1,2,3,4]$

so,

$$\Omega_5^0 = e^0 = 1 \Rightarrow \phi = 0$$

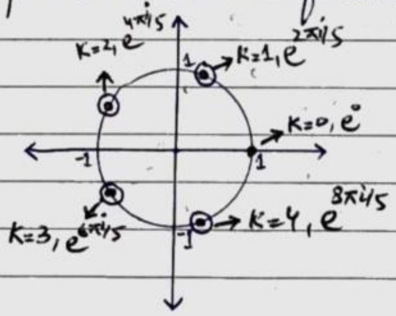
$$\Omega_5^1 = e^{2\pi i/5} \Rightarrow \phi = 2\pi/5$$

$$\Omega_5^2 = e^{4\pi i/5} \Rightarrow \phi = 4\pi/5$$

$$\Omega_5^3 = e^{6\pi i/5} \Rightarrow \phi = 6\pi/5$$

$$\Omega_5^4 = e^{8\pi i/5} \Rightarrow \phi = 8\pi/5$$

Since all values of k less than 5 are coprimes of 5, so all of the above are primitive roots of unity (encircled):



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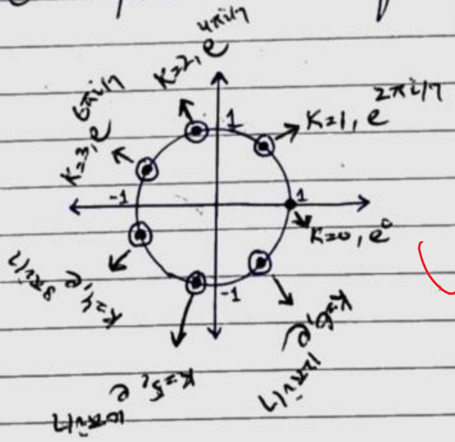
and for $n=7$, $k=[0,1,2,3,4,5,6]$

$$\Omega_7^0 = e^0 = 1 \Rightarrow \phi = 0$$
$$\Omega_7^1 = e^{2\pi i/7} \Rightarrow \phi = 2\pi/7$$
$$\Omega_7^2 = e^{4\pi i/7} \Rightarrow \phi = 4\pi/7$$
$$\Omega_7^3 = e^{6\pi i/7} \Rightarrow \phi = 6\pi/7$$
$$\Omega_7^4 = e^{8\pi i/7} \Rightarrow \phi = 8\pi/7$$
$$\Omega_7^5 = e^{10\pi i/7} \Rightarrow \phi = 10\pi/7$$
$$\Omega_7^6 = e^{12\pi i/7} \Rightarrow \phi = 12\pi/7$$

Since all values k less than 7 are coprimes of 7, so all of the above are primitive roots of unity (encircled below):



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3.1 b) As n^{th} root of unity is given by

$$\Omega_n^k = e^{2\pi i k/n}$$

then sum of all n^{th} roots of unity will be

$$\sum_{k=0}^{n-1} \Omega_n^k = \sum_{k=0}^{n-1} e^{2\pi i k/n}$$

$$= \sum_{k=0}^{n-1} \left(e^{2\pi i/n}\right)^k$$

$$= \frac{1-\left(e^{2\pi i/n}\right)^n}{1-e^{2\pi i/n}} \qquad \left\{ \sum_{n=0}^{N-1} r^n = \frac{1-r^N}{1-r} \right\}$$

$$= \frac{1-e^{2\pi i}}{1-e^{2\pi i/n}}$$

$$= \frac{1-(\cos(2\pi)+i\sin(2\pi))}{1-e^{2\pi i/n}}$$

$$= \frac{1-(1-i(0))}{1-e^{2\pi i/n}}$$

$$= \frac{0}{1-e^{2\pi i/n}}$$

$$= 0$$

Hence sum of all n^{th} roots of unity is equal to zero.

3.2) $f(x+p)=f(x)$, $w\in[0,1)$, $f_w(x):=e^{2\pi i w x}$

a) figures of $f_{1/2}, f_{1/3}, f_{1/4}, f_{1/8}$?

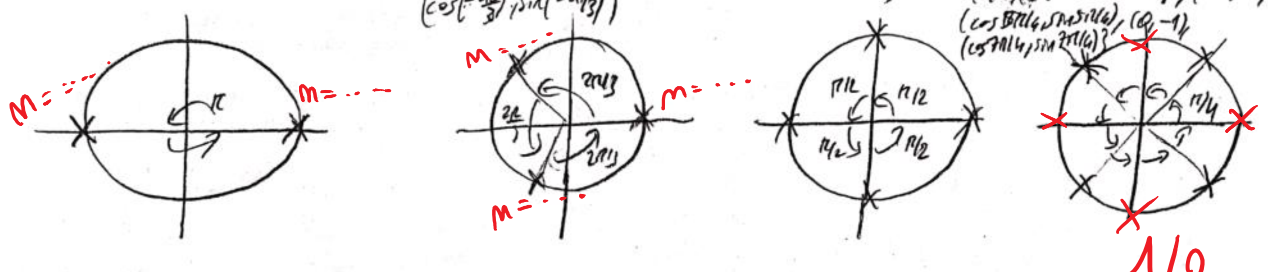
We know that $\exp(x)=\exp(x+2\pi i n)$ for $n\in\mathbb{Z}$

$f_{1/2}=\{(1,0),(-1,0)\}$

$f_{1/3}=\{(1,0),(\cos\frac{2\pi}{3},\sin\frac{2\pi}{3}),(\cos(-\frac{2\pi}{3}),\sin(-\frac{2\pi}{3}))\}$

$f_{1/4}=\{(1,0),(0,1),(-1,0),(0,-1)\}$

$f_{1/8}=\{(1,0),(\cos\pi/4,\sin\pi/4),(0,1),(\cos 3\pi/4,\sin 3\pi/4),(-1,0),(\cos 5\pi/4,\sin 5\pi/4),(0,-1),(\cos 7\pi/4,\sin 7\pi/4)\}$



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b) prove f_w periodic $\Leftrightarrow w\in\mathbb{Q}$?

We know that $\exp(x)$ is periodic of $2\pi i n$, and $\exp(x)=\exp(x+2\pi i n)$

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for $\forall n\in\mathbb{Z}, \exists p\in\mathbb{N};\ f_w(n)=f_w(n+p)$

$\Rightarrow \forall n\in\mathbb{Z}, \exists p\in\mathbb{N};\ \exp(2\pi i w n)=\exp(2\pi i w(n+p))$
$=\exp(2\pi i w n+\underline{2\pi i w p})$) ?

$\Rightarrow \forall n\in\mathbb{Z}, \exists p\in\mathbb{N};\ 2\pi i w p=2\pi i n$ why?

$\Rightarrow \exists n,p\in\mathbb{Z},\ w\cdot p=n \Rightarrow w=\frac{n}{p}$

f_w is periodic $\Leftrightarrow w\in\mathbb{Q}$

?