Cüneyt Erem
Sheikh Mastura Farzana
Md Emtazul Haque

1.5/2

2a) $t_0 = 0$, $x(0) = [x_0 \ \ y_0]^T$, $a = [\pi \cos t, \ \pi \sin t]^T$

$v(t_0) = [0 \ \ 0]^T$

You are using "t" for two different things. Not OK!

$$v(t) = v(t_0) + \int_{t_0}^{t} a(t)\, dt$$

$$= \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \int_{t_0}^{t} \begin{bmatrix} \pi \cos t \\ \pi \sin t \end{bmatrix} dt$$

$$= \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \pi(\sin t - \sin t_0) \\ \pi(-\cos t + \cos t_0) \end{bmatrix}$$

$$= \begin{bmatrix} \pi \sin t \\ \pi(1 - \cos t) \end{bmatrix} \checkmark$$

$$\therefore v(t_1) = \begin{bmatrix} \pi \sin t_1 \\ \pi(1 - \cos t_1) \end{bmatrix}$$

$$x(t) = x(t_0) + \int_{t_0}^{t} v(t)\, dt$$

$$= \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} + \int_{t_0}^{t} \begin{bmatrix} \pi \sin t \\ \pi(1 - \cos t) \end{bmatrix} dt$$

$$= \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} + \begin{bmatrix} \pi(-\cos t + \cos t_0) \\ \pi((t - t_0) - (\sin t - \sin t_0)) \end{bmatrix}$$

$$= \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} + \begin{bmatrix} \pi(1 - \cos t) \\ \pi(t - \sin t) \end{bmatrix}$$

$$x(t) = \begin{bmatrix} x_0 + \pi(1 - \cos t) \\ y_0 + \pi(t - \sin t) \end{bmatrix}$$

$$\therefore x(t_1) = \begin{bmatrix} x_0 + \pi(1 - \cos t_1) \\ y_0 + \pi(t_1 - \sin t_1) \end{bmatrix} \checkmark$$

0.5/1

2b) Forward Euler method is a first order numerial procedure to solve ODE with a given initial value.
Given that we know the initial value of position $\vec{x}(t_0)$, we can calculate the position at time (t_0+h),

$$\vec{x}(t_0+h) = \vec{x}(t_0) + h \cdot \vec{x}'(t_0) + O(h^2)$$

~~which can be written as~~

$$\cancel{\vec{x}(t_0+h) = \vec{x}(t_0) + h \cdot \vec{v}(t_0)}$$

Here, $O(h^2)$ is the error of single step.
Total error $= O(h)$
For particle position and velocity we can write as following,

$$\vec{v}_{n+1} = \vec{v}_n + \vec{a}_n \cdot h$$

$$\vec{x}_{n+1} = \vec{x}_n + \vec{v}_{n+1} \cdot h$$

$$= \vec{x}_n + (\vec{v}_n + \vec{a}_n \cdot h)\, h$$

$$= \vec{x}_n + h \cdot \vec{v}_n + h^2 \cdot \vec{a}_n$$

write clearly.

Practical Part

Assignment 1) Lorenz Attractor
a) 2/2
b) 2/2
c) 3/3
d) 1/1

8/8

Theoretical Part

Assignment 3) Particle Systems
a) 1.5/2
b) 0.5/1
c) 0.5/1
d) 1/1

3.5/5

0.5/1

2c) $x(t_0) = [-1, 0]^T$, $h = \pi/2$, $r = 1$, $t_0 = 0$

$a = [r \cos t, \; r \sin t]^T$, $v(t_0) = [0 \; 0]^T$

$$v(t_0+h) = v(t_0) + h \cdot a(t_0)$$

$$= \begin{bmatrix} 0 \\ 0 \end{bmatrix} + h \cdot \begin{bmatrix} r \cos(t_0 + \underline{h}) \\ r \sin(t_0 + \underline{h}) \end{bmatrix}$$

$$= \pi/2 \begin{bmatrix} \cos \pi/2 \\ \sin \pi/2 \end{bmatrix}$$

cos 0

sin 0

$$= \begin{bmatrix} 0 \\ \pi/2 \end{bmatrix}$$

$$\begin{bmatrix} \pi/2 \\ 0 \end{bmatrix}$$

$$x(t_0+h) = x(t_0) + v(t_0+h) \cdot h$$

$$= \begin{bmatrix} -1 \\ 0 \end{bmatrix} + \pi/2 \begin{bmatrix} 0 \\ \pi/2 \end{bmatrix}$$

$$= \begin{bmatrix} -1 \\ \pi^2/4 \end{bmatrix}$$

⋮

1/1

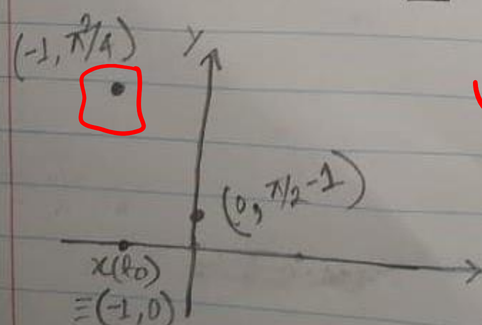
2d) $v(t_0+h) = \begin{bmatrix} \pi \sin(t_0+h) \\ \pi(1-\cos(t_0+h)) \end{bmatrix}$

$$= \begin{bmatrix} \sin \pi/2 \\ 1-\cos \pi/2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$x(t_0+h) = \begin{bmatrix} x_0 + \pi(1-\cos(t_0+h)) \\ y_0 + \pi(t_0+h - \sin(t_0+h)) \end{bmatrix}$

$$= \begin{bmatrix} -1 + (1-\cos \pi/2) \\ 0 + (\pi/2 - \sin \pi/2) \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ \pi/2 - 1 \end{bmatrix}$$ (✓)



wrong, as above but OK.

1.5 + 0.5 + 0.5 + 1