Foundations of Audio Signal Processing

Exercise Sheet 2

2.1	2.2	Σ
4	6	10/12

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2.1. a) $1+i\sqrt{3} = r(\cos\theta + i\sin\theta) = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right) = 2\left(1+\frac{\sqrt{3}}{2}i\right) = \boxed{2e^{i\frac{\pi}{3}}}$

$r = \sqrt{1+3} = 2$

$\tan^{-1}\theta = \tan^{-1}\sqrt{3} = \frac{\pi}{3}$ ✓

b) $\frac{e^{i\frac{\pi}{3}}-1}{1+i\sqrt{3}} = \frac{(1+i\sqrt{3})-1}{1+i\sqrt{3}} = \frac{i\sqrt{3}}{1+i\sqrt{3}} \times \frac{1-i\sqrt{3}}{1-i\sqrt{3}} = \frac{i\sqrt{3}+3}{1+3} = \frac{3}{4} + \frac{i\sqrt{3}}{4} = \frac{2\sqrt{3}}{4}\left(\overset{\frac{\sqrt{3}}{2}}{\cos\frac{\pi}{6}} + i\overset{\frac{1}{2}}{\sin\frac{\pi}{6}}\right) = \boxed{\frac{2\sqrt{3}}{4}e^{i\frac{\pi}{6}}}$

$r = \sqrt{\frac{9}{16}+\frac{3}{16}} = \frac{2\sqrt{3}}{4}$

$\tan^{-1}\theta = \tan^{-1}\frac{\sqrt{3}}{3} = \frac{\pi}{6}$

c) $(1-i\sqrt{3}) = 2\left(\overset{\frac{1}{2}}{\cos\frac{-\pi}{3}} + i\overset{-\frac{\sqrt{3}}{2}}{\sin\frac{-\pi}{3}}\right) = 1 - i\frac{\sqrt{3}}{2} = 2e^{-i\frac{\pi}{3}} \Rightarrow (1-i\sqrt{3})^3 = \left(2e^{-i\frac{\pi}{3}}\right)^3 = \boxed{8e^{-i\pi}}$

$r = \sqrt{1+3} = 2$

$\tan^{-1}\theta = \tan^{-1}(-\sqrt{3}) = -\frac{\pi}{3}$ ✓

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Exercise 2.2: $z_\alpha := e^{2\pi i \alpha}$

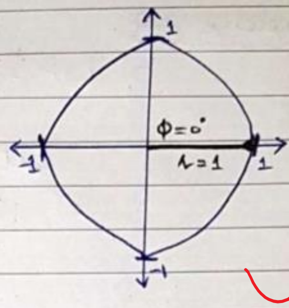
a) For $\alpha = 0$,

$$z_0 = e^{i2\pi(0)}$$

and using

$$z = r.e^{i\phi},$$

we have $r = 1$ and $\phi = 0$

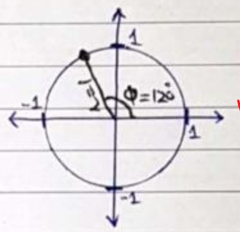


[redacted] For $\alpha = 1/3$,

$$z_{1/3} = e^{i2\pi/3}$$

and using $z = r.e^{i\phi}$, we have

$r = 1$, $\phi = 2\pi/3$

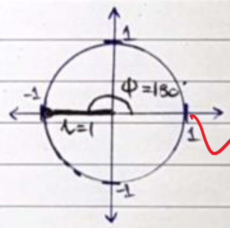


For $\alpha = 1/2$,

$$z_{1/2} = e^{i\pi}$$

and using $z = r.e^{i\phi}$, we have

$r = 1$ and $\phi = \pi$

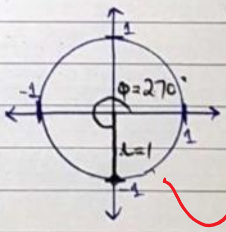


For $\alpha = 3/4$,

$$z_{3/4} = e^{i2\pi(3/4)} = e^{i3\pi/2}$$

and using $z = r.e^{i\phi}$, we have

$r = 1$ and $\phi = 3\pi/2$



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Exercise 2-2

b) L.H.S

$z\bar{z}$

Let $z = r.e^{i\phi}$ then $\bar{z} = r.e^{-i\phi}$,

$\Rightarrow z\bar{z} = (r.e^{i\phi})(r.e^{-i\phi})$

$= r^2 e^{i\phi - i\phi}$

$= r^2 e^0$

$= r^2$

$= |z|^2$

$=$ R.H.S , Hence proved.

2.2

c) Using euler $e^{i\alpha} = \cos\alpha + i.\sin\alpha$. and $e^{z_1 + z_2} = e^{z_1 \cdot z_2}$

prove $\sin\alpha = \frac{1}{2i} \cdot (e^{i\alpha} - e^{-i.\alpha})$?

answer:

$$\sin\alpha \overset{?}{=} \frac{1}{2i} \cdot (e^{i\alpha} - e^{-i\alpha})$$

$$= \frac{1}{2i} \cdot (\cos\alpha + i.\sin\alpha - (\underbrace{\cos(-\alpha)}_{\cos\alpha} + \underbrace{i.\sin(-\alpha)}_{-i.\sin\alpha}))$$

$$= \frac{1}{2i} \cdot (\cos\alpha + i.\sin\alpha - \cos\alpha + i.\sin\alpha)$$

$$\sin\alpha = \frac{1}{2i} \cdot (2i.\sin\alpha) = \sin\alpha$$

so it is proved.