9.1	9.2	9.3	Σ
4	6	7	17

/24

Foundations of Audio Signal Processing

Exercise Sheet 9

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9.1

a)

$x = (x(0), x(1), x(2)) = (1, 2, 3)$

$y = (y(0), y(1)) = (4, 5)$, convolution x and y?

$$(x * y)(n) = \sum_{k \in \mathbb{Z}} x(k) \cdot y(n-k)$$

$$(x*y)(0) = \underbrace{x(0)}_{1} \cdot \underbrace{y(0-0)}_{4} + \underbrace{x(1)}_{2} \cdot \underbrace{y(0-1)}_{0} + \underbrace{x(2)}_{3} \cdot \underbrace{y(0-2)}_{0} = 4$$

$$(x*y)(1) = \underbrace{x(0)}_{1} \cdot \underbrace{y(1-0)}_{5} + \underbrace{x(1)}_{2} \cdot \underbrace{y(1-1)}_{4} + \underbrace{x(2)}_{3} \cdot \underbrace{y(1-2)}_{0} = 13$$

$$(x*y)(2) = \underbrace{x(0)}_{1} \cdot \underbrace{y(2-0)}_{0} + \underbrace{x(1)}_{2} \cdot \underbrace{y(2-1)}_{5} + \underbrace{x(2)}_{3} \cdot \underbrace{y(2-2)}_{4} = 22$$

$$(x*y)(3) = \underbrace{x(0)}_{1} \cdot \underbrace{y(3-0)}_{0} + \underbrace{x(1)}_{2} \cdot \underbrace{y(3-1)}_{0} + \underbrace{x(2)}_{3} \cdot \underbrace{y(3-2)}_{5} = 15$$

✓

$$(x*y)(4) = 1 \cdot 0 + 2 \cdot 0 + 3 \cdot 0 = 0$$

$$\Rightarrow (x*y)(n) = \begin{cases} (4, 13, 22, 15) & \text{if } 0 \le n \le 3 \\ 0 & \text{otherwise} \end{cases}$$

4/4

b, c) missing

9.2)

a) $x, y \in \ell^2(\mathbb{Z})$ with $(x * y) \in L^2(\mathbb{Z})$

convolution is commutative ?

$k, n \in \mathbb{Z}$ and k is iterative along the all $\mathbb{Z}$ values,

$$(x*y)(n) = \sum_{\cancel{k=0}\; k\in\mathbb{Z}}^{\cancel{N-1}} x(k) \cdot y(n-k)$$

if we substitute $\ell = n-k$

then $$= \sum_{\ell=n}^{n-(N-1)} x(n-\ell) \cdot y(\ell)$$

$$= \sum_{\ell=0}^{N-1} y(\ell) \cdot x(n-\ell) \quad \text{where } \ell \in \mathbb{Z}$$

1.5/2

$$= (y * x)(n) \quad //$$

b) $$\widehat{x*y}(w) = \sum_{k\in\mathbb{Z}} (x*y)(k) \cdot e^{-2\pi i w k}$$

$$= \sum_{k\in\mathbb{Z}} \Big(\sum_{j\in\mathbb{Z}} x(j) \cdot y(k-j) \Big) \cdot e^{-2\pi i w k}$$

$$= \sum_{j\in\mathbb{Z}} x(j) \cdot \sum_{k\in\mathbb{Z}} y(k-j) \cdot e^{-2\pi i w k}$$

substitute $k-j$ with ℓ $$= \sum_{j\in\mathbb{Z}} x(j) \cdot \sum_{\ell\in\mathbb{Z}} y(\ell) \cdot e^{-2\pi i w(\ell+j)}$$

$$= \sum_{j\in\mathbb{Z}} x(j) \cdot e^{-2\pi i w j} \cdot \sum_{\ell\in\mathbb{Z}} y(\ell) \cdot e^{-2\pi i w \ell}$$

$$\Rightarrow \quad (\widehat{x*y})(w) = \hat{x}(w) \cdot \hat{y}(w) \quad //$$

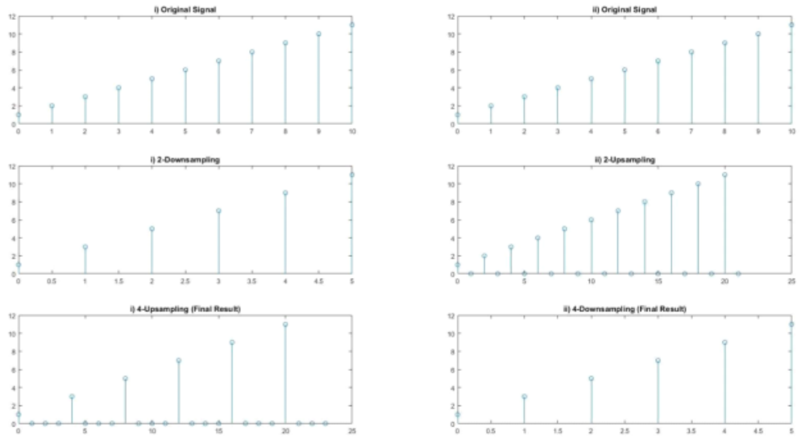
9.3 a)

Function : ResampledSignal = Resampling(Signal, isDownsampling, MorN)

Parameters:

- Signal: The original Signal to be resampled
- isDownsampling: ‘true’ for downsamplling and ‘false’ for upsampling
- MorN: M for M-Downsampling and N for N-Upsampling

9.3 b)



9.3 c)

Even though the result has the same sampling rate and sample numbers as the original signal, but the quality has significantly reduced.

By changing the order of resamplings (performing upsampling at first and downsampling in the next) the result would be as good as the original one.

interpolate instead of inserting zeros