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Foundations of Audio Signal Processing

Exercise Sheet 5

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5.1. a.

$$\langle f|A_k\rangle = \sqrt{2}\left(\int_0^{1/2} \cos 2\pi k t \, dt + \int_{1/2}^{1} -\cos 2\pi k t \, dt\right) = \sqrt{2}\left(\frac{\sin \pi k}{2\pi k} + \frac{\sin \pi k - \sin 2\pi k}{2\pi k}\right)$$

//

too fast

$$\langle f|B_k\rangle = \sqrt{2}\left(\int_0^{1/2} \sin 2\pi k t \, dt + \int_{1/2}^{1} -\sin 2\pi k t \, dt\right) = \sqrt{2}\left(\frac{\sin^2\left(\frac{\pi k}{2}\right)}{\pi k} + \frac{\cos 2\pi k - \cos \pi k}{2\pi k}\right)$$

f

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$$\langle f|1\rangle = \int_0^{1/2} 1 \, dt + \int_{1/2}^{1} -1 \, dt = 0$$

b) ✓ 6/6

5.1. c.

5.1. c.

as k increases the expansion of Fourier series of the function $f(t)$ with the first k fourier Coeficients approaches to the real function with more accuracy.

(except: in discontinuities) in which we observe overshoot (gibs phenomenon) no matte how much k increases ($t=0^+$, $t=\frac{1}{2}^-$, $t=\frac{1}{2}^+$, $t=1^-$)

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Exercise 5.2: If $f \in L^2(\mathbb{R})$ then $\hat{f}(\omega) = \int_R f(t) e^{-2\pi i \omega t} dt$

a) If f is imaginary, then $Re(\hat{f})$ is odd and $Im(\hat{f})$ is even.

$$\hat{f}(\omega) = \int_R f(t) e^{-2\pi i \omega t} dt$$

$$= \int_R f(t)\left(\cos(-2\pi\omega t) + i\sin(-2\pi\omega t)\right) dt \quad \Big\} \text{Euler's Formula}$$

$$= \int_R f(t)\cos(-2\pi\omega t)\, dt + i\int_R f(t)\sin(-2\pi\omega t)\, dt$$

Since f is imaginary,

$$Re(\hat{f}(\omega)) = i\int_R f(t)\sin(-2\pi\omega t)\, dt \quad \text{—— } ①$$

To prove $Re(\hat{f}(\omega))$ odd, we need to show $Re(\hat{f}(\omega)) = Re(-\hat{f}(-\omega))$, then consider

$$Re(\hat{f}(\omega)) = i\int_R f(t)\left(-\sin(-2\pi(-\omega)t)\right) dt$$

$$= -i\int_R f(t)\sin(-2\pi(-\omega)t)\, dt$$

$$= Re(-\hat{f}(-\omega)), \text{ from } ①. \text{ Hence proved.}$$

$$Im(\hat{f}(\omega)) = \int_R f(t)\cos(-2\pi\omega t)\, dt \quad \text{—— } ②$$

To prove $Im(\hat{f}(\omega))$ even, we need to show $Im(\hat{f}(\omega)) = Im(\hat{f}(-\omega))$, then consider

$$Im(\hat{f}(\omega)) = \int_R f(t)\cos(-(-2\pi(-\omega)t))dt$$

Date:

$$= \int_{\mathbb{R}} f(t) \cos(-2\pi(-\omega)t)\,dt \qquad \left\{ \cos(x) = \cos(-x) \right.$$

$= \operatorname{Im}(\hat{f}(-\omega))$ from ②, Hence proved

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b) If f is imaginary and even, $\hat{f}$ is imaginary and even

We know that $\hat{f}(\omega) = \int_{\mathbb{R}} f(t) \cdot e^{-2\pi i \omega t} dt = \underbrace{\int_{\mathbb{R}} f(t) \cdot \cos(-2\pi\omega t) dt}_{\text{imaginary}} + \underbrace{i \int_{\mathbb{R}} f(t) \cdot \sin(-2\pi \omega t) dt}_{\text{real parts}}$

because f is imaginary $\Rightarrow$

if f is imaginary and even, then real part should be 0, $Re(\hat{f})(\omega) = 0$

$$Re(\hat{f})(\omega) = i \cdot \int_{-\infty}^{0} f(t) \sin(-2\pi\omega t) dt + i \cdot \int_{0}^{\infty} f(t) \cdot \sin(-2\pi\omega t) dt$$

$f(t) = f(-t)$
then $t \Rightarrow -t$

they are 2 different things

$$= -i \int_{-\infty}^{0} f(-t) \cdot \sin(-2\pi\omega(-t))(-dt) + i \cdot \int_{0}^{\infty} f(t) \sin(-2\pi\omega t) dt$$

$$\overset{f \text{ is even}}{=} i \cdot \int_{-\infty}^{0} f(t) \sin(-2\pi\omega(-t)) dt + i \int_{0}^{\infty} f(t) \sin(-2\pi\omega t) dt$$

$1\frac{5}{3}$

$$\overset{\sin \text{ is odd}}{=} i \cdot \int_{-\infty}^{0} f(t) \cdot -\sin(-2\pi\omega t) dt + i \cdot \int_{0}^{\infty} f(t) \sin(-2\pi\omega t) dt$$

$$= -i \int_{-\infty}^{0} f(t) \sin(-2\pi\omega t) dt + i \cdot \int_{0}^{\infty} f(t) \sin(-2\pi\omega t) dt = 0$$

$\hookrightarrow$ even

are not the same!

$\int_{-\infty}^{0} \dots + \int_{-\infty}^{0} \dots = 0$

c) f is imaginary and odd, $\hat{f}$ is real and odd?

$Im(\hat{f})(\omega) = 0$

$$= \int_{-\infty}^{0} f(t) \cos(-2\pi\omega t) dt + \int_{0}^{\infty} f(t) \cos(-2\pi\omega t) dt$$

where does it come from?

you should substitute $-t$ also in the second part

$t \Rightarrow -t$

$$= \int_{-\infty}^{0} f(-t) \cos(-2\pi\omega(-t))(-dt) + \int_{0}^{\infty} f(t) \cdot \cos(-2\pi\omega t) dt$$

$$\overset{f \text{ is odd}, \cos \text{ is even}}{=} \int_{-\infty}^{0} -f(t) \cdot + \cos(-2\pi\omega t) dt + \int_{0}^{\infty} f(t) \cos(-2\pi\omega t) dt$$

$$= -\int_{-\infty}^{0} f(t) \cdot \cos(-2\pi\omega t) dt + \int_{0}^{\infty} f(t) \cos(-2\pi\omega t) dt = 0$$

$\hookrightarrow$ odd

same as above

$1\frac{5}{3}$