Erosion and Sediment Transport

As an optional feature, KINEROS can simulate the movement of eroded soil along with the movement of surface water. KINEROS accounts separately for erosion caused by raindrop energy and erosion caused by flowing water, and continues the simulation through channel and pond elements. One necessary limitation is that, because a single mean particle size is used to characterize the eroded material, the effective soil particle size needs to be similar for all the eroding elements.

Upland Erosion. The general equation used in KINEROS to describe the sediment dynamics at any point along a surface flow path is a mass balance equation similar to that for kinematic water flow (Bennett, 1974):

$$\frac{\partial(AC_S)}{\partial t} + \frac{\partial(QC_S)}{\partial x} - e(x,t) = q(x,t) \tag{1}$$

in which C_s = sediment concentration [L^3/L^3],

Q = water discharge rate [L^3/T],

A = cross sectional area of flow [L^2],

e = rate of erosion of the soil bed [L^2/T],

q_s = rate of lateral sediment inflow for channels [$L^3/T/L$].

For upland surfaces, e is assumed to be composed of two major components - production of eroded soil by splash of rainfall on bare soil, and hydraulic erosion (or deposition) due to the interplay between the shearing force of water on the loose soil bed and the tendency of soil particles to settle under the force of gravity. Thus e may be positive (increasing concentration in the water) or negative (deposition). Net erosion is a sum of splash erosion rate as e_s and hydraulic erosion rate as e_h,

$$e = e_s + e_h \tag{2}$$

Splash Erosion Based on limited experimental evidence, the splash erosion rate can be approximated as a function of the square of the rainfall rate (Meyer and Wischmeier, 1969). This relationship in KINEROS estimates the splash erosion rate as follows:

$$\begin{aligned} e_s &= c_f k(h) r^2 \quad ; \quad q > 0 \\ &= 0 \quad\quad\quad\quad ; \quad q < 0 \end{aligned} \tag{3}$$

in which c_f is a constant related to soil and surface properties, and $k(h)$ is a reduction factor representing the reduction in splash erosion caused by increasing depth of water. The function

$k(h)$ is 1.0 prior to runoff and its minimum is 0 for very deep flow; it is given by the empirical expression

$$k(h) = exp(-c_h h) \tag{4}$$

The parameter c_h represents the damping effectiveness of surface water, and does not vary widely. Both c_f and $k(h)$ are always positive, so e_s is always positive when there is rainfall and a positive rainfall excess (q).

<u>Hydraulic Erosion</u> The hydraulic erosion rate e_h represents the rate of exchange of sediment between the flowing water and the soil over which it flows, and may be either positive or negative. KINEROS assumes that for any given surface water flow condition (velocity, depth, slope, etc.), there is an equilibrium concentration of sediment that can be carried if that flow continues steadily. Hydraulic erosion rate (e_h) is estimated as being linearly dependent on the difference between the equilibrium concentration and the current sediment concentration. In other words, hydraulic erosion/deposition is modeled as a kinetic transfer process;

$$e_h = c_g (C_m - C_s) A \tag{5}$$

in which C_{mx} is the concentration at equilibrium transport capacity, $C_s = C_s(x,t)$ is the current local sediment concentration, and c_g is a transfer rate coefficient [T^{-1}]. Clearly, the transport capacity is important in determining hydraulic erosion, as is the selection of transfer rate coefficient. Conceptually, when deposition is occurring, c_g is theoretically equal to the particle settling velocity divided by the hydraulic depth, h. For erosion conditions on cohesive soils, the value of c_g must be reduced, and v_s / h is used as an upper limit for c_g.

<u>Transport Capacity</u> Many transport capacity relations have been proposed in the literature, but most have been developed and tested for relatively deep, mildly sloping flow conditions, such as streams and flumes. Experimental work by Govers (1990) and others using shallow flows over soil have demonstrated relations that are similar to the transport capacity relation of Engelund and Hansen (1967):

$$C_m = \frac{0.5 u u_*^3}{g^2 d h (\gamma_s - 1)^2} \tag{6}$$

in which

u is velocity [L/T],

u_* is shear velocity, defined as $\sqrt{ghS}$,

d is particle diameter [L],

γ_s is suspended specific gravity of the particles, γ_s - 1,

h is water depth.[L]

To apply this relation with the results of Govers research, we modify equation (6) to include the unit stream power threshold Ω_c of 0.004 m/s found to apply to shallow flow transport capacity. Unit stream power as used here, Ω, is simply $u \cdot S$. In terms of this variable and the threshold, equation (6), may be modified to:

$$C_{mx} = \frac{0.05}{d(\gamma_s - 1)^2}\sqrt{\frac{Sh}{g}}(\Omega - \Omega_c) \tag{7}$$

This relation would have transportability beginning abruptly after Ω = .004. Actually, KINEROS2 employs a transitional relation as shown in (no figure).

Particle settling velocity is calculated from particle size and density, assuming the particles have drag characteristics and terminal fall velocities similar to those of spheres (Fair and Geyer, 1954). This relation is

$$v_s^2 = \frac{4}{3}\frac{g(\rho_s - 1)d}{C_D} \tag{8}$$

in which C_D is the particle drag coefficient.

The drag coefficient is a function of particle Reynolds number,

$$C_D = \frac{24}{R_n} + \frac{3}{\sqrt{R_n}} + 0.34 \tag{9}$$

in which R_n is the particle Reynolds number, defined as

$$R_n = \frac{v_s d}{\nu} \tag{10}$$

where < is the kinematic viscosity of water [L^2/T]. Settling velocity of a particle is found by solving equations (8), (9), and (10) for v_s.

Treating a Range of Particle Sizes. The above series of erosion relations are applied to each of up to five particle size classes which are used to describe a soil with a range of particle sizes. Our experimental and theoretical understanding of the dynamics of erosion for a mix of particle sizes is incomplete. It is not clear, for example, exactly what results when the distribution of relative particle sizes is contradictory to the distribution of their relative transport capacities. In larger particles on stream bottoms, armoring will ultimately occur when smaller more transportable particles are selectively removed, leaving behind an armor of large particles. In the smaller particle sizes at shallower flows and rapidly changing flow conditions of overland flow, however, there is considerably less understanding of the relations. Sufficient knowledge exists to use the following assumptions in our model formulation, however:

1. If the largest particle size in a soil of mixed sizes is below it s erosion threshold, the erosion of smaller sizes will be limited, since otherwise armoring will soon stop the erosion process.

2. When erosive conditions exist for all particle sizes, particle erosion rates will be proportional to the relative occurrence of the particle sizes in the surface soil. The same is true of erosion by rain splash.

3. Particle settling velocities, when concentrations exceed transportability, are independent of the concentration of other particle sizes.

Treatment of a mix of sizes is most critical for cases where the sediment characterizing the bed of the channels is significantly different than that of the upland slopes, and where impoundments exist in which there is significant opportunity for selective settling.

Numerical Method for Sediment Transport Equations (1- 7) are solved numerically at each time step used by the surface water flow equations, and for each particle size class. A four-point finite-difference scheme is used; however, iteration is not required since, given current and immediate past values for A and Q and previous values for C_s, the finite difference form of this equation is explicit, *i.e.*:

$$C_{s\,j+1}^{\,i+1} = f\left(C_{s\,j}^{\,i}, C_{s\,j+1}^{\,i}, C_{s\,j}^{\,i+1}\right) \tag{11}$$

The value of C_{mx} is found from Eq. (7) using current hydraulic conditions.

Initial Conditions for Erosion When runoff commences during a period when rainfall is creating splash erosion, the initial condition on the vector C_s should not be taken as zero. The initial sediment concentration at ponding, $C_s(t=t_p)$, can be found by simplifying equation (1) for conditions at that time. Variation with respect to x vanishes, and hydraulic erosion is zero. Then,

$$\frac{\partial\left(AC_s\right)}{\partial t} = e(x,t) = c_f rq - C_s \tag{12}$$

where $k(h)$ is assumed to be 1.0 since depth is zero. Since A is zero at time of ponding, and dA/dt is the rainfall excess rate (q), expanding the left-hand side of equation (12) results in

$$C_s\left(t = t_p\right) = \frac{c_f rq}{q + v_s} \tag{13}$$

The sediment concentration at the upper boundary of a single overland flow element, $C_s(0,t)$, is given by an expression identical to equation (13), and a similar expression is used at the upper boundary of a channel.

Channel Erosion and Sediment Transport. The general approach to sediment transport simulation for channels is nearly the same as that for upland areas. The major difference in the equations is that splash erosion (e_s) is neglected in channel flow, and the term q_s becomes important in representing lateral inflows. Equations (1) and (5) are equally applicable to either channel or distributed surface flow, but the choice of transport capacity relation may be different for the two flow conditions. For upland areas, q_s will be zero, whereas for channels it will be the important addition that comes with lateral inflow from surface elements. The close similarity of the treatment of the two types of elements allows the program to use the same algorithms for both types of elements.

The erosion computational scheme for any element uses the same time and space steps employed by the numerical solution of the surface water flow equations. In that context, equations (1) and (5) are solved for $C_s(x,t)$, starting at the first node below the upstream boundary, and from the upstream conditions for channel elements. If there is no inflow at the upper end of the channel, the transport capacity at the upper node is zero and any lateral input of sediment will be subject there to deposition. The upper boundary condition is then

$$C_s\left(0,t\right) = \frac{q_s}{q_c + v_s W_B} \tag{14}$$

where W_B is the channel bottom width. $A(x,t)$ and $Q(x,t)$ are assumed known from the surface water solution.

References

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