Hortonian Overland Flow

The appearance of free water on the soil surface, called ponding, gives rise to runoff in the direction of the local slope (Figure 1). Rainfall can produce ponding by two mechanisms, as outlined in the infiltration section. The first mechanism involves a rate of rainfall which exceeds the infiltrability of the soil at the surface. The second mechanism is soil filling, when a soil layer deeper in the soil restricts downward flow and the surface layer fills its available porosity. In the first mechanism, the surface soil water pressure head is not more than the depth of water, and decreases with depth, while in the second mechanism, soil water pressure head increases with depth until the restrictive layer is reached.

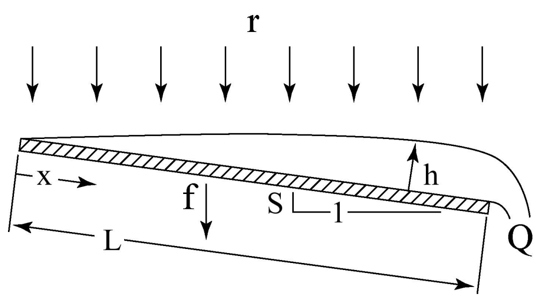


Figure 1. Definition sketch for overland flow.

Viewed at a very small scale, overland flow is an extremely complex three-dimensional process. At a larger scale, however, it can be viewed as a one-dimensional flow process in which flux is related to the unit area storage by a simple power relation:

$$Q = \alpha h^m \tag{1}$$

where Q is discharge per unit width and h is the storage of water per unit area. Parameters α and m are related to slope, surface roughness, and flow regime. Note that we take a larger scale view of this equation and do not assume literal sheet flow . High slopes or supercritical flow are not required. Figure 2 illustrates some of the possible configurations that the flow may assume in relation to local cross-slope microtopography.

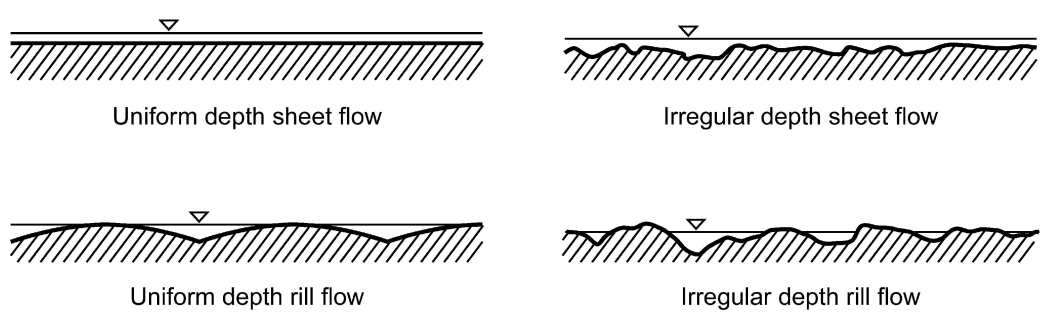


Figure 2. Examples of several types of overland flow (after Wilgoose and Kuczera, 1995)

Equation (1) is used in conjunction with the equation of continuity:

$$\frac{\partial h}{\partial t} + \frac{\partial Q}{\partial x} = q(x,t) \tag{2}$$

where t is time, x is the distance along the slope direction, and $q()$ is the lateral inflow rate.

For overland flow, equation (1) may be substituted into equation (2) to obtain

$$\frac{\partial h}{\partial t} + \alpha m h^{m-1} \frac{\partial h}{\partial x} = q(x,t) \tag{3}$$

The kinematic wave equations are simplifications of the de Saint Venant equations, and do not preserve all of the properties of the more complex equations, such as backwater and diffusive wave attenuation. Attenuation does occur in kinematic routing from shocks or from spatially variable infiltration. The kinematic routing method is an excellent approximation for most overland flow conditions (Woolhiser and Liggett, 1967; Morris and Woolhiser, 1980).

Boundary Conditions The depth or unit storage at the upstream boundary must be specified to solve equation (3). If the upstream boundary is a flow divide, the boundary condition is

$$h(0,t) = 0 \tag{4}$$

If another surface is contributing flow at the upper boundary, the boundary condition is

$$h(0,t) = \left[\frac{\alpha_u h_u (L,t)^{m_u} W_u}{\alpha W} \right]^{\frac{1}{m}} \tag{5}$$

where subscript u refers to the upstream surface, W is width and L is the length of the upstream element. This merely states an equivalence of discharge between the upstream and downstream elements.

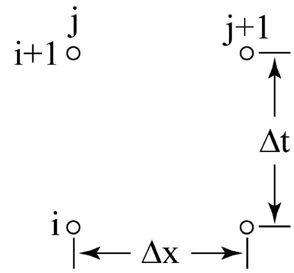
Recession and Microtopography Microtopographic relief can play an important role in determining hydrograph shape (Woolhiser et al., 1997). The effect is most pronounced during recession, when the extent of soil covered by the flowing water determines the opportunity for water loss by infiltration. KINEROS2 provides for treatment of this relief by assuming the relief geometry has a maximum elevation, and that the area covered by surface water (see Figure 2, above) varies linearly with elevation up to this maximum. The geometry of microtopography is completed by specifying a relief scale, which geometrically represents the mean spacing between relief elements.

Numerical Solution KINEROS2 solves the kinematic wave equations using a four-point implicit finite difference method. The finite difference form for equation (3) is

$$
\begin{aligned}
&h_{j+1}^{i+1} - h_{j+1}^{i} + h_{j}^{i+1} - h_{j}^{i} + \\
&\frac{2\Delta t}{\Delta x}\left\{\theta_w\left[\alpha_{j+1}^{i+1}\left(h_{j+1}^{i+1}\right)^m - \alpha_j^{i+1}\left(h_j^{i+1}\right)^m\right] + \left(1-\theta_w\right)\left[\alpha_{j+1}^{i}\left(h_{j+1}^{i}\right)^m - \alpha_j^{i}\left(h_j^{i}\right)^m\right]\right\} - \\
&\Delta t\left(\bar{q}_{j+1} + \bar{q}_j\right) = 0
\end{aligned}
\tag{6}
$$

where θ_w is a weighting parameter (usually 0.6 to 0.8) for the x derivatives at the advanced time step. The notation for this method is shown in Figure 3.

Figure 3. Notation for space and time dimensions of the finite difference grid



A solution is obtained by Newton's method (sometimes referred to as the *Newton-Raphson* technique). While the solution is unconditionally stable in a linear sense, the accuracy is highly dependent on the size of Δx and Δt values used. The difference scheme is nominally of first order accuracy.

Roughness Relationships Two options for α and m in equation (3) are provided in KINEROS:

1. The Manning hydraulic resistance law may be used. In this option

$$\alpha = 149\frac{S^{\frac{1}{2}}}{n} \qquad and \qquad m = \frac{5}{3} \tag{7}$$

where S is the slope, n is a Manning's roughness coefficient for overland flow, and English units are used. A table values of n for use on surfaces are suggested in the User Manual.

2. The Chezy law may be used. In this option,

$$\alpha = CS^{\frac{1}{2}} \qquad and \qquad m = \frac{3}{2} \tag{8}$$

where C is the Chezy friction coefficient.

Overland flow response characteristics are controlled by the slope, slope length, and hydraulic resistance parameters as well as the rainfall intensity and infiltration characteristics. For example, if the Manning resistance law is used, the time to equilibrium (t_e)[seconds] of a surface of length (L) and slope (S) with a constant rate of lateral inflow (q)[L/s], is

$$t_e = \left[\frac{nL}{149 S^{\frac{1}{2}} q^{\frac{2}{3}}} \right]^{\frac{3}{5}} \tag{9}$$

where n is the Manning resistance coefficient. Therefore, to maintain the time response characteristics of the catchment we must retain the slope, slope length, and hydraulic resistance in our simplified version of a complex catchment. The length, width, and slope of the overland flow elements can be determined from topographic maps. The hydraulic resistance parameter is more difficult to determine. Ranges of Manning's n and Chezy C obtained from experiments reported in the literature are presented in the KINEROS User Manual (Woolhiser et al., 1990).

References

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