Deep Random Splines for Point Process Intensity Estimation of Neural Population Data

Gabriel Loaiza-Ganem[1], Sean M. Perkins[2], Karen E. Schroeder[3], Mark M. Churchland[3] and John P. Cunningham[1]

Department of Statistics[1], Department of Biomedical Engineering[2] and Department of Neuroscience[3], Columbia University

Columbia University in the City of New York

Introduction

- We use random splines to model intensity functions of Poisson Processes.
- We use neural networks to endow splines with rich distributions that avoid some pitfalls of Gaussian Processes (GPs).
- Unlike popular alternatives, we treat time as truly continuous by not discretizing events into bins.
- This allows to recover low-dimensional representations whose dimension is unrelated to the number of time bins (there are no time bins).

Contributions of the paper

- Introducing Deep Random Splines (DRS) as a tool to model random functions.
- Ensuring splines are nonnegative via the method of alternating projections.
- Proposing a Variational AutoEncoder (VAE) with a novel encoder architecture, which recovers low-dimensional latent structure.
- Outperforming alternatives on simulated and both old and new real microelectrode array data.

Acknowledgments and References

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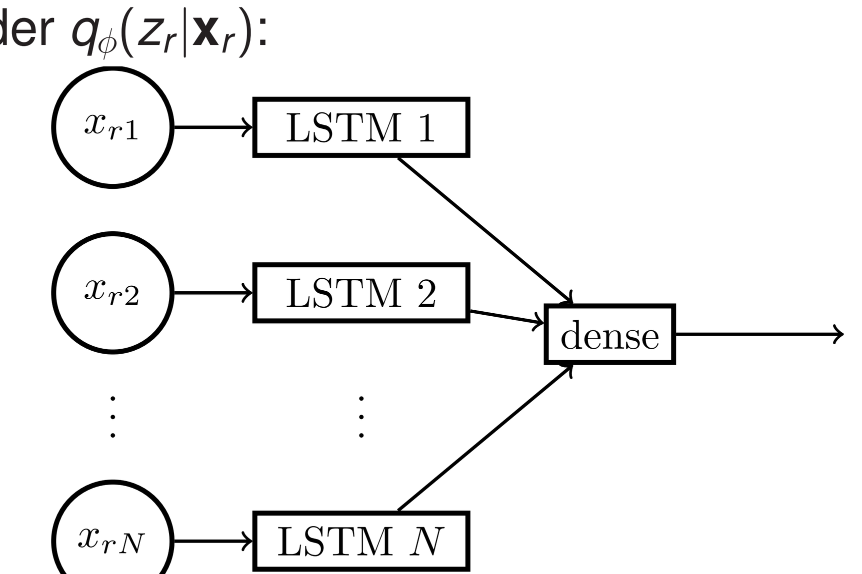
[KW13] Diederik P Kingma and Max Welling, *Auto-encoding variational bayes*, arXiv preprint arXiv:1312.6114 (2013).

Deep Random Splines

- Splines are smooth piecewise polynomial functions of degree d.
- The fixed knot locations $T_1 = t_0 < \cdots < t_I = T_2$ define the pieces and Ψ is the set of parameters of the polynomials in each of those pieces.
- g_ψ denotes a spline on $[T_1, T_2)$ parametrized by $\psi \in \Psi$.
- Simple distributions over Ψ result in oversimplified distributions on functions.
- DRS are a distribution over splines.
- Sampling from a DRS is done by transforming Gaussian noise $Z \in \mathbb{R}^m$ through a neural network $f_\theta : \mathbb{R}^m \to \Psi$, the result being $g_{f_\theta(Z)}$.
- To enforce nonnegativity, we use the method of alternating projections and parameterize splines of degree $2k+1$ with $(Q_1^{(i)}, Q_2^{(i)})_i$ as:
$$p^{(i)}(t) = (t_i - t)[t]^\top Q_1^{(i)}[t] + (t - t_{i-1})[t]^\top Q_2^{(i)}[t]$$
where $[t] = (1, t, \ldots, t^k)^\top$ and $Q_1^{(i)}, Q_2^{(i)} \succeq 0$, for $i = 1, \ldots, I$.

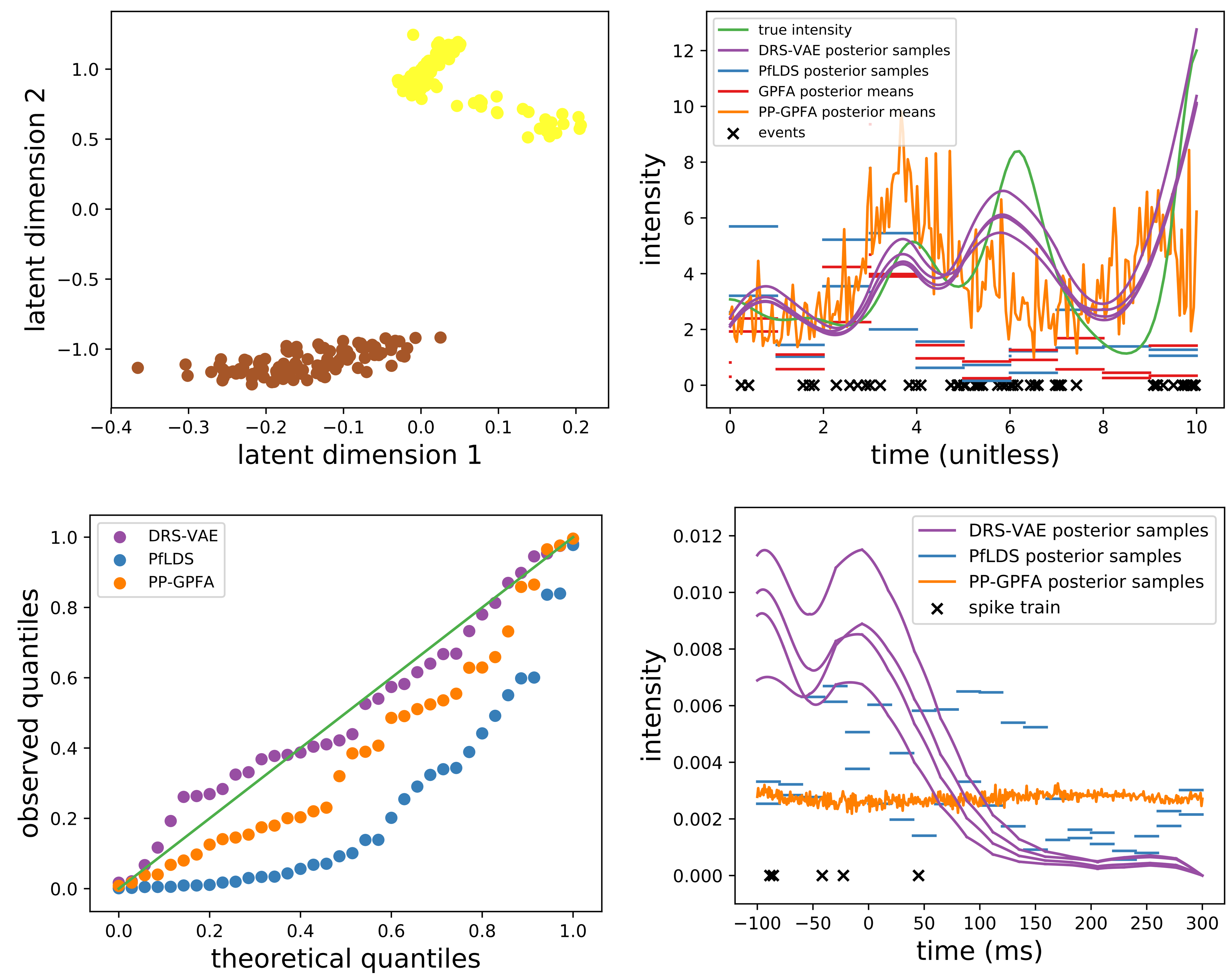
Our Model

- We model each of the R repetitions (trials) of N simultaneous point processes $X_{r,n}$ as:
$$\begin{cases} Z_r \sim \mathcal{N}(0, I_m) \text{ for } r = 1, \ldots, R \\ \psi_{r,n} = f_\theta^{(n)}(Z_r) \text{ for } n = 1, \ldots, N \\ X_{r,n}|\psi_{r,n} \sim \mathcal{PP}_{[T_1,T_2)}(g_{\psi_{r,n}}) \end{cases}$$
- The latent variable is shared across point processes.
- Truly continuous time: latent dimension m does not depend on temporal resolution I.
- We perform inference with a VAE [KW13] , maximizing the ELBO:
$$\sum_{r=1}^{R} E_{q_\phi}[p_\theta(\mathbf{x}_r|z_r)] - KL(q_\phi(z_r|\mathbf{x}_r)||p(z_r))$$
- Computing $p_\theta(\mathbf{x}_r|z_r)$ is tractable due to our spline choice, and we use the following architecture for the encoder $q_\phi(z_r|\mathbf{x}_r)$:



- The input for the encoder are N point processes.
- Each point process is fed to its LSTM and outputs are concatenated at the end.
- Novel encoder allows us to process actual event times (not binned counts).

Experiments



METHOD	SIMULATED DATA ELBO	SIMULATED DATA L^2	SIMULATED DATA p-VALUE	REACHING DATA ELBO	REACHING DATA 15-NN	REACHING DATA SSG/SST	CYCLING DATA ELBO	CYCLING DATA 15-NN	CYCLING DATA SSG/SST
DRS-VAE	**57.1**	$\mathbf{0.11 \pm 0.09}$	—	**−500.8**	**23.7%**	**73.9%**	6372	**55.9%**	**70.0%**
PfLDS	52.3	0.21 ± 0.10	$< 10^{-44}$	−505.7	3.1%	6.2%	**6532**	11.7%	3.2%
GPFA	—	0.21 ± 0.10	$< 10^{-45}$	—	—	—	—	—	—
PP-GPFA	29.0	0.38 ± 0.24	$< 10^{-70}$	−523.2	14.1%	30.5%	6079	51.1%	14.6%

Conclusions

- DRS: rich distribution class over functions and tractable constraints.
- Truly continuous time: better dimensionality reduction.
- We outperform commonly used alternatives.

Email: gl2480@columbia.edu