Damped harmonic oscillator DSN hyperparameter search

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1: Damped harmonic oscillator

The classical damped harmonic oscillator is a good system to show proof of principle with DSNs. A DHO models a system often described as a swinging mass acted on by forces of gravity, a spring, and friction. The dynamics are described by the simple equation:

$$m\frac{d^2x}{dt^2} = kx + c\frac{dx}{dt}$$

with parameters corresponding to mass m, fricition coefficient c and spring constant k. It is straightforward to see that there is a degeneracy in the parameterization by dividing each side of the equation by m.

$$m\frac{d^2x}{dt^2} - kx - c\frac{dx}{dt} = 0$$

If we ask a DSN to learn the parameters $\phi = \{m, c, k\}$, that result in a give length-T trajectory given some initial conditions: $x(0) = x_0$ and $\frac{dx}{dt}(0) = \dot{x}_0$, there should be a uniform distribution on a linear subspace in three dimensions of solutions found by the algorithm described by: $r(m, c, k) = [m_0 t, c_0 t, k_0 t] \, \forall t \in \left[\frac{a}{min(m_0,c_0,k_0)}, \frac{b}{max(m_0,c_0,k_0)}\right]$ assuming the final layer of the deep generative model maps onto an interval $[a, b]$, and $a, b \geq 0$.

We have already validated that the DSN code yeilds a uniform distribution on a plane and line with one and two, respectively, linear constraints imposed on a three dimensional parameter space.

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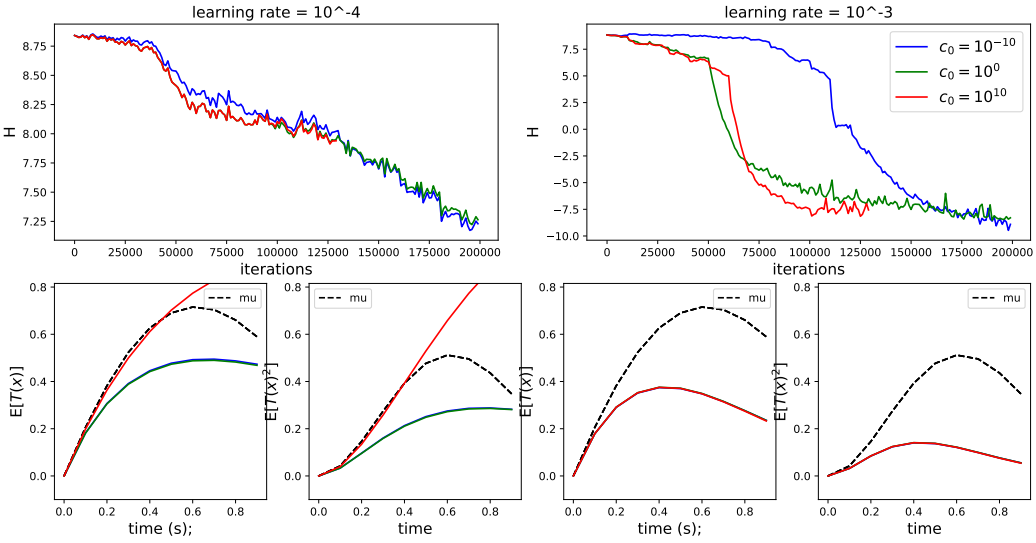
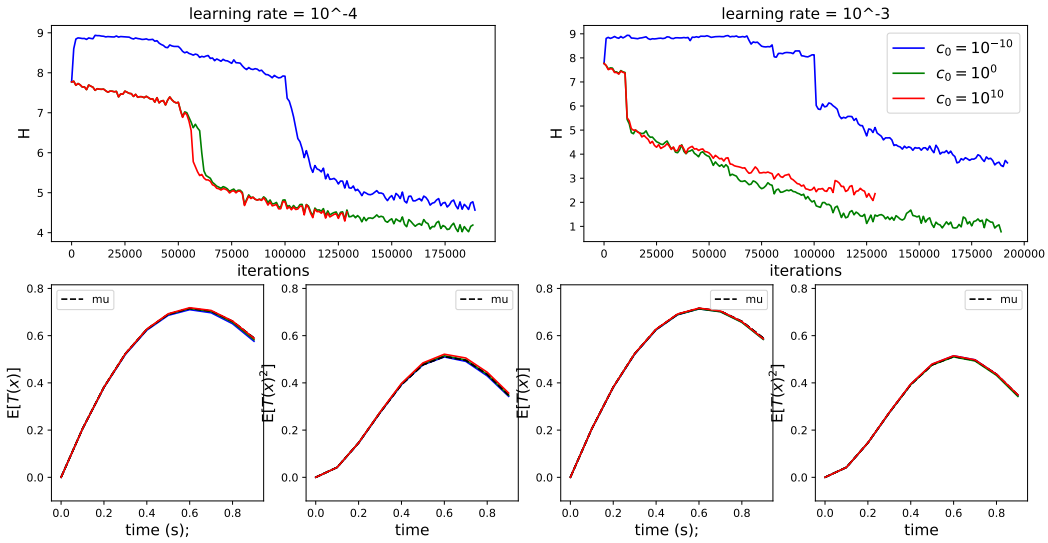
2: Hyperparameter search

DHO with T = 10, 5 planar flows



DHO with T = 10, 10 planar flows



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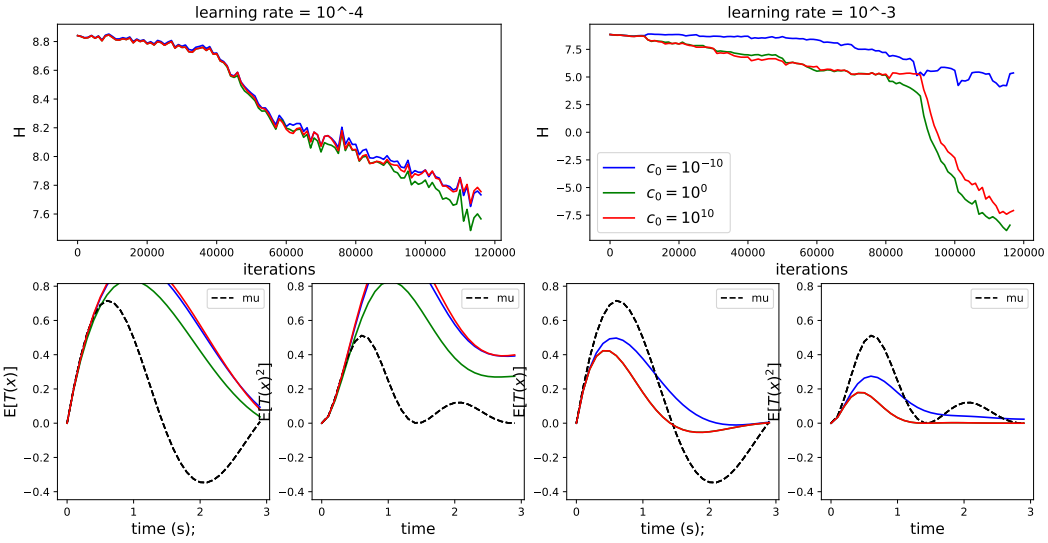
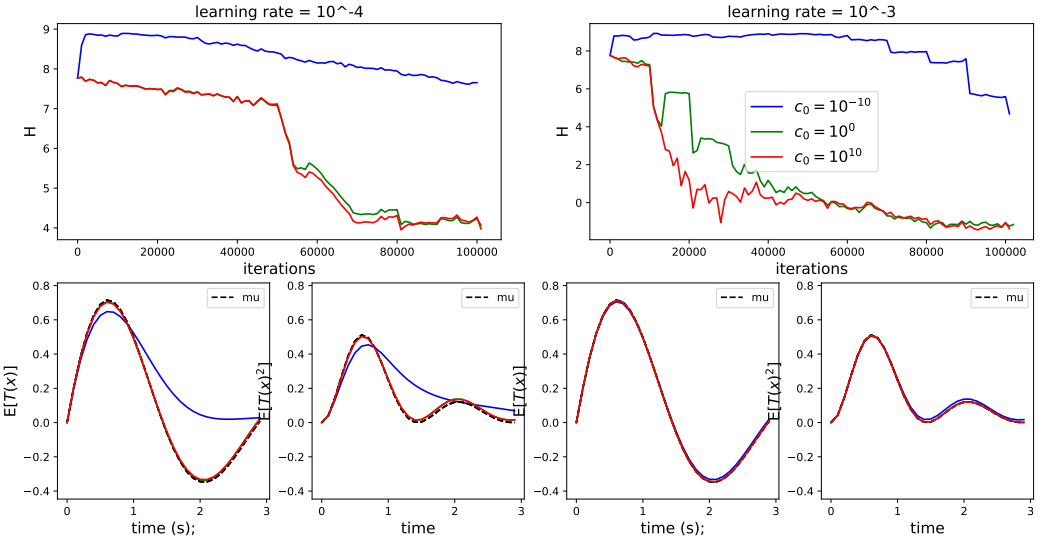
DHO with T = 30, 5 planar flows



DHO with T = 30, 10 planar flows



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DHO with 10 planar flows $T = 30$, $lr = 10^{-3}$, $c_0 = 0$