Linear 2D system oscillation DSN

Aug 19, 2018

1: 2D linear system with oscillations

We're considering the 2D linear system

$$\tau \dot{x} = -x + Wx$$

$$W = \begin{bmatrix} w_1 & w_2 \\ w_3 & w_4 \end{bmatrix}$$

Where the linear dynamics can characterized by an eigendecomposition of the linear dynamics matrix

$$\dot{x} = Ax, A = \frac{1}{\tau}(W - I) = \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix}$$

If we want to learn a gaussian distribution of pure oscillations, the eigenvalues of A (and W) should have zero real part, and the imaginary components should be complex conjugate pairs with magnitude corresponding to the desired frequency. The behavior I imposed on this 2D linear system DSN was:

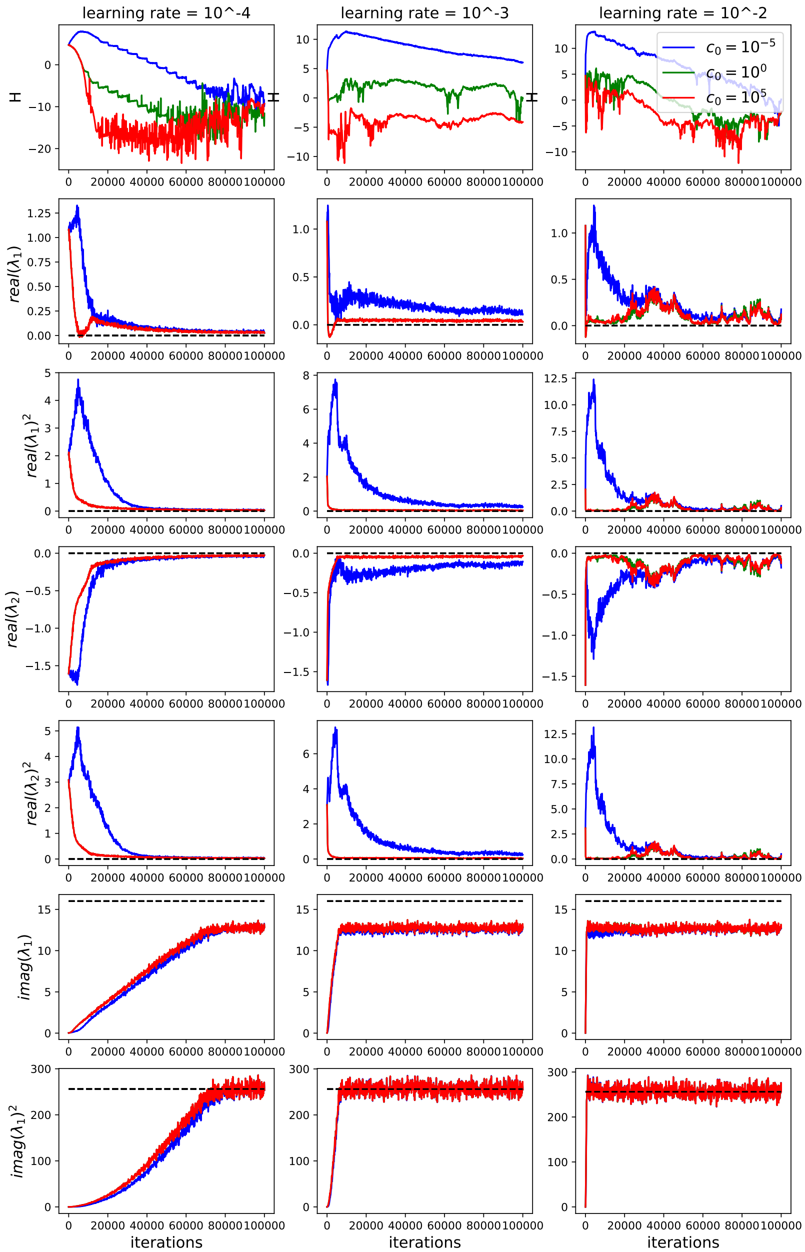
$$E[T(g(\phi))] = \begin{bmatrix} real(\lambda_1) \\ real(\lambda_1)^2 \\ real(\lambda_2) \\ real(\lambda_2)^2 \\ imag(\lambda_1) \\ imag(\lambda_1)^2 \end{bmatrix} = \begin{bmatrix} 0.0 \\ 0.001 \\ 0.0 \\ 0.001 \\ 16.0 \\ 0.1 \end{bmatrix}$$

I trained DSN's to learn the space of real entries of both A (section 2) and W (section3, $\tau = 100ms$).
I tested 5 architectures:
1A - one affine transformation
2P - two planar flows (two nonlinear normalizing flows)
4P, 8P, ...
10P - ten planar flows

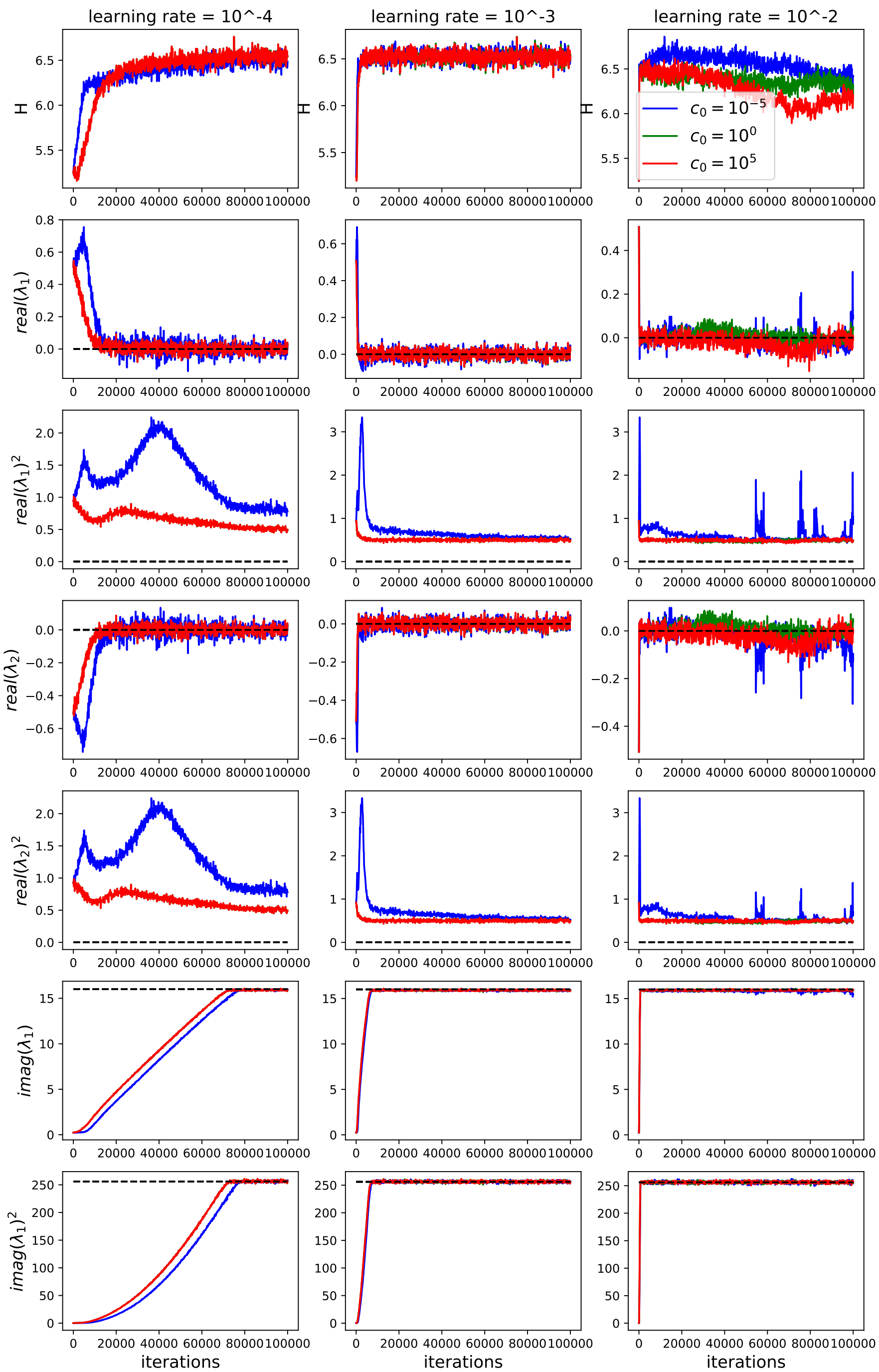
For each architecture I tested three different initializations of the augmented lagrangian optimization paramter $c \in \left[10^{-5}, 1, 10^{5}\right]$, and three separate learning rates $lr \in \left[10^{-4}, 10^{-3}, 10^{-2}\right]$

2: Learning A

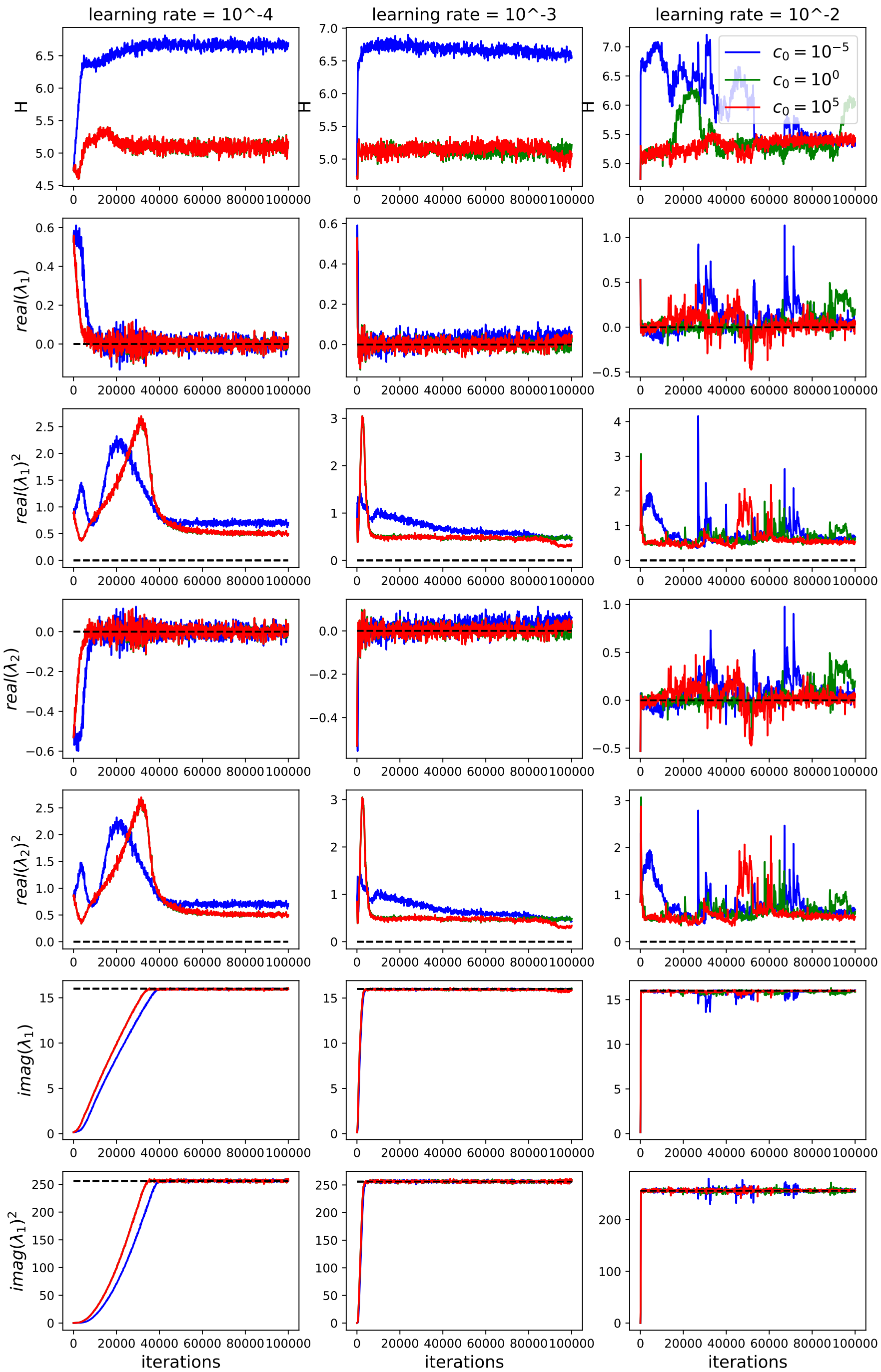
1 affine layer



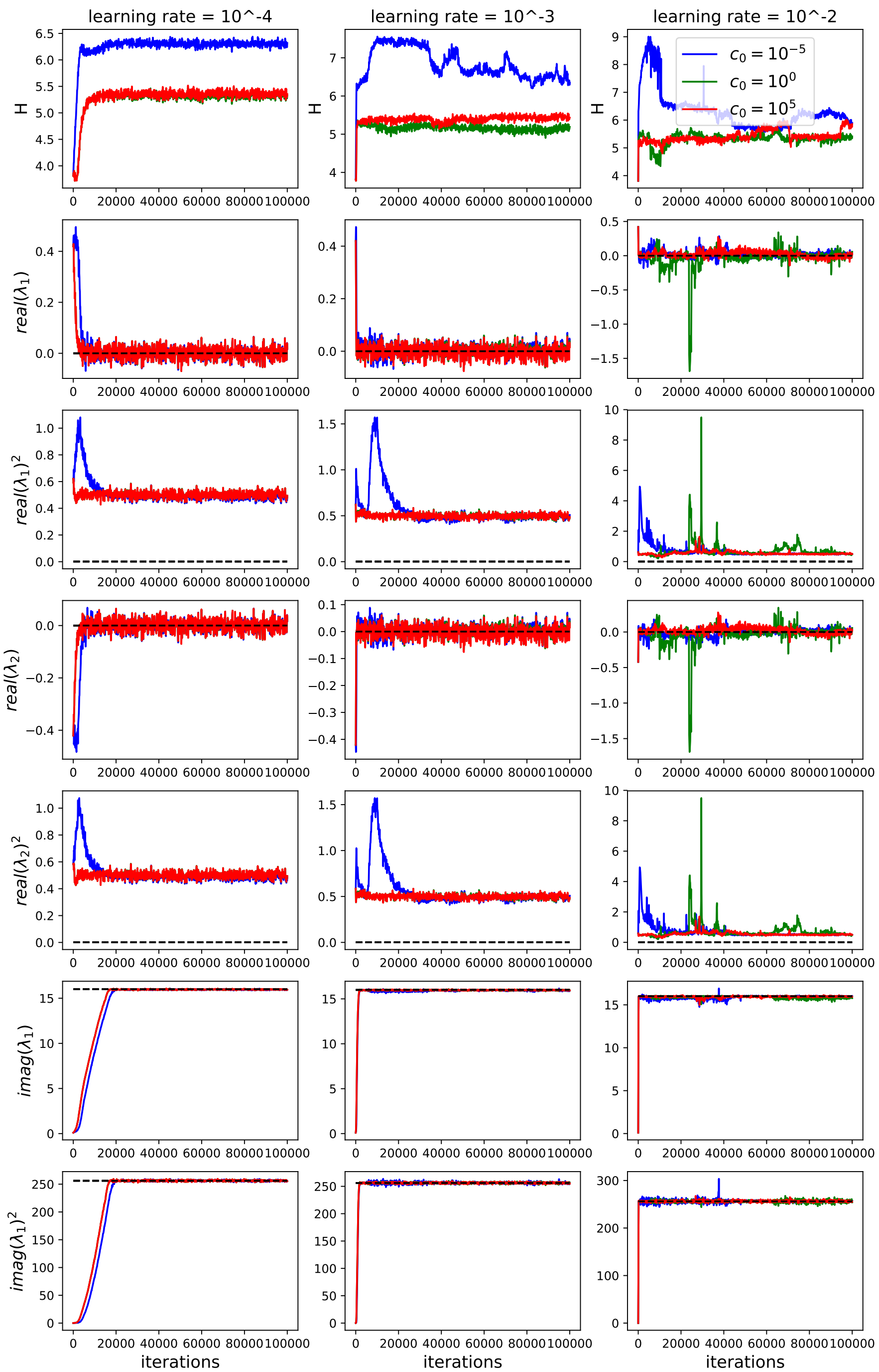
2 planar layers



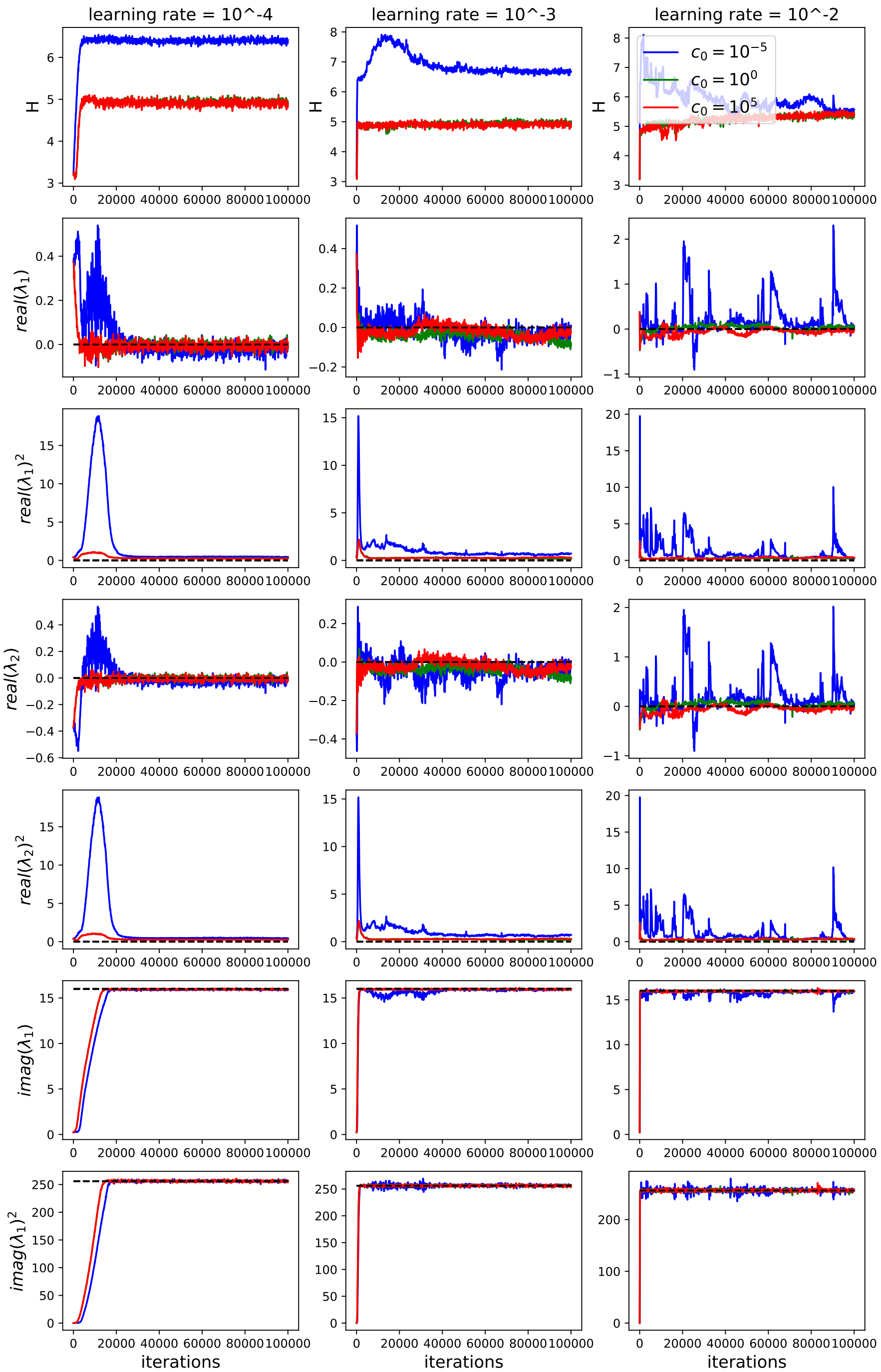
4 planar layers



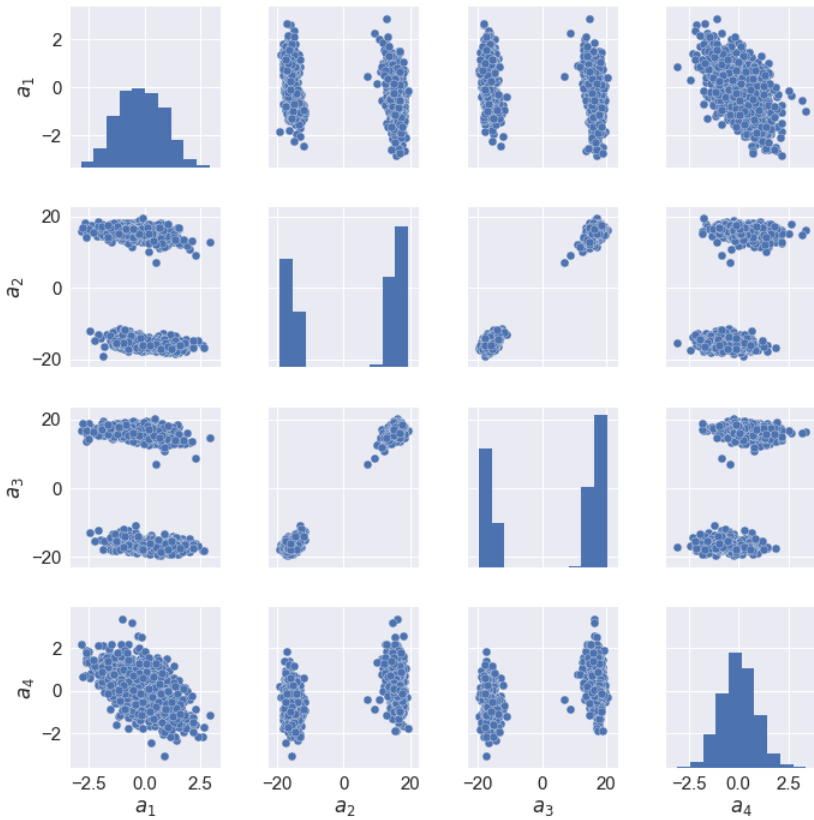
8 planar layers



10 planar layers

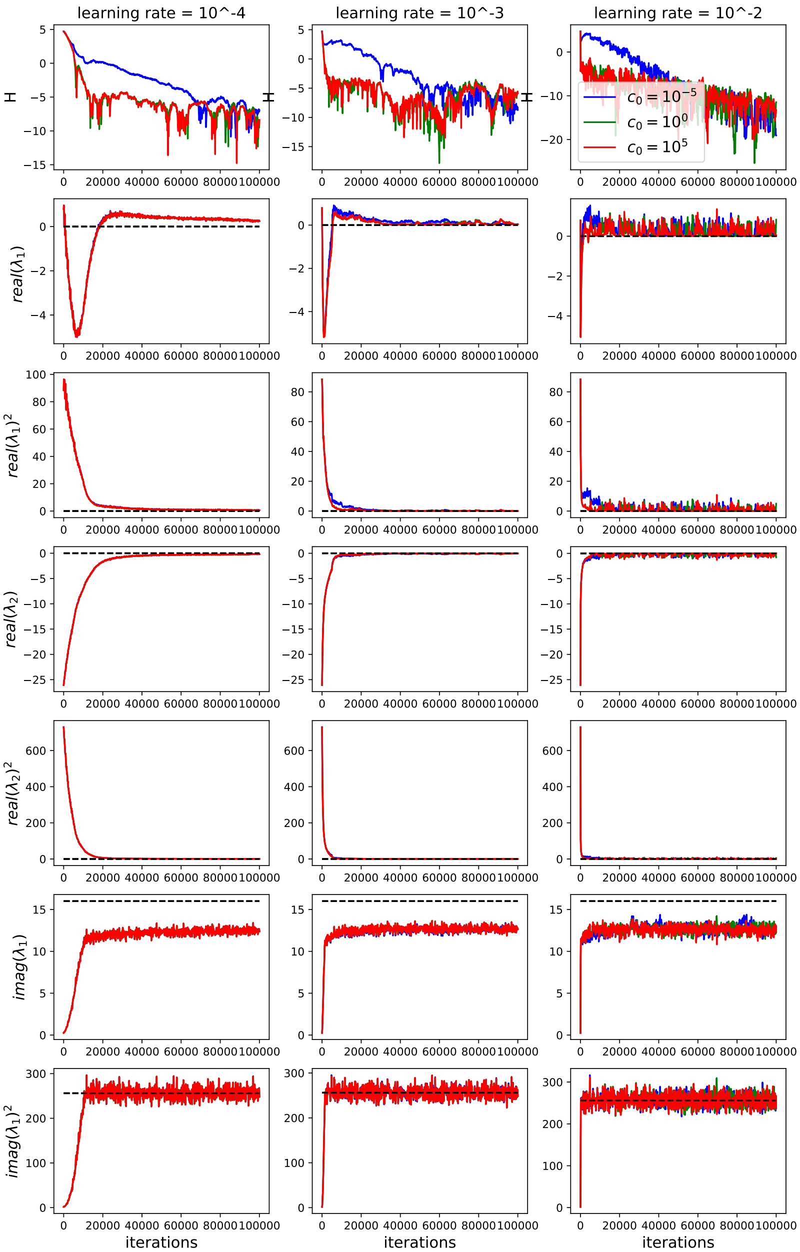


A: 10 planar layers: $c_0 = 1$, learning rate $= 10^{-3}$

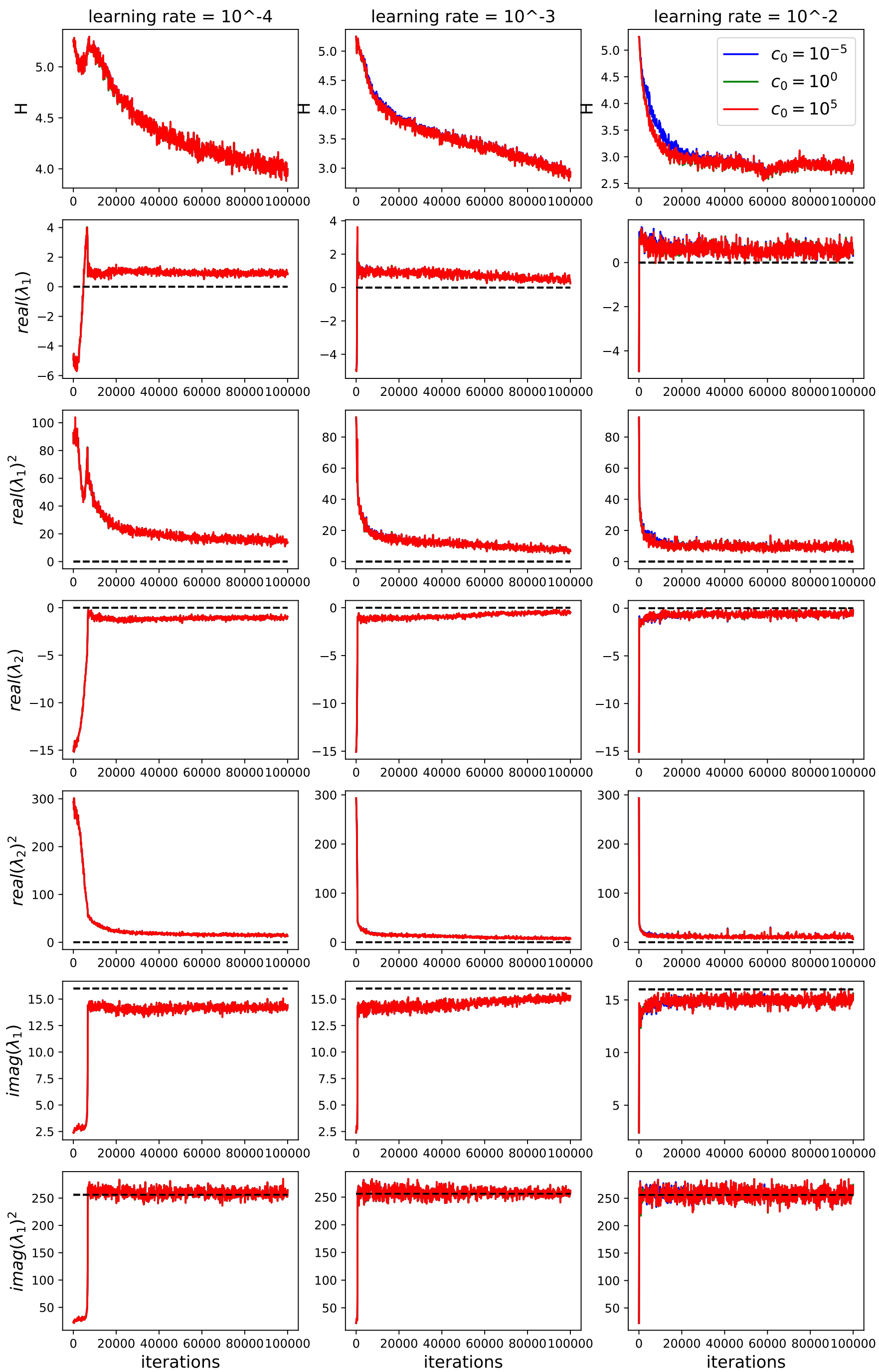


3: Learning W

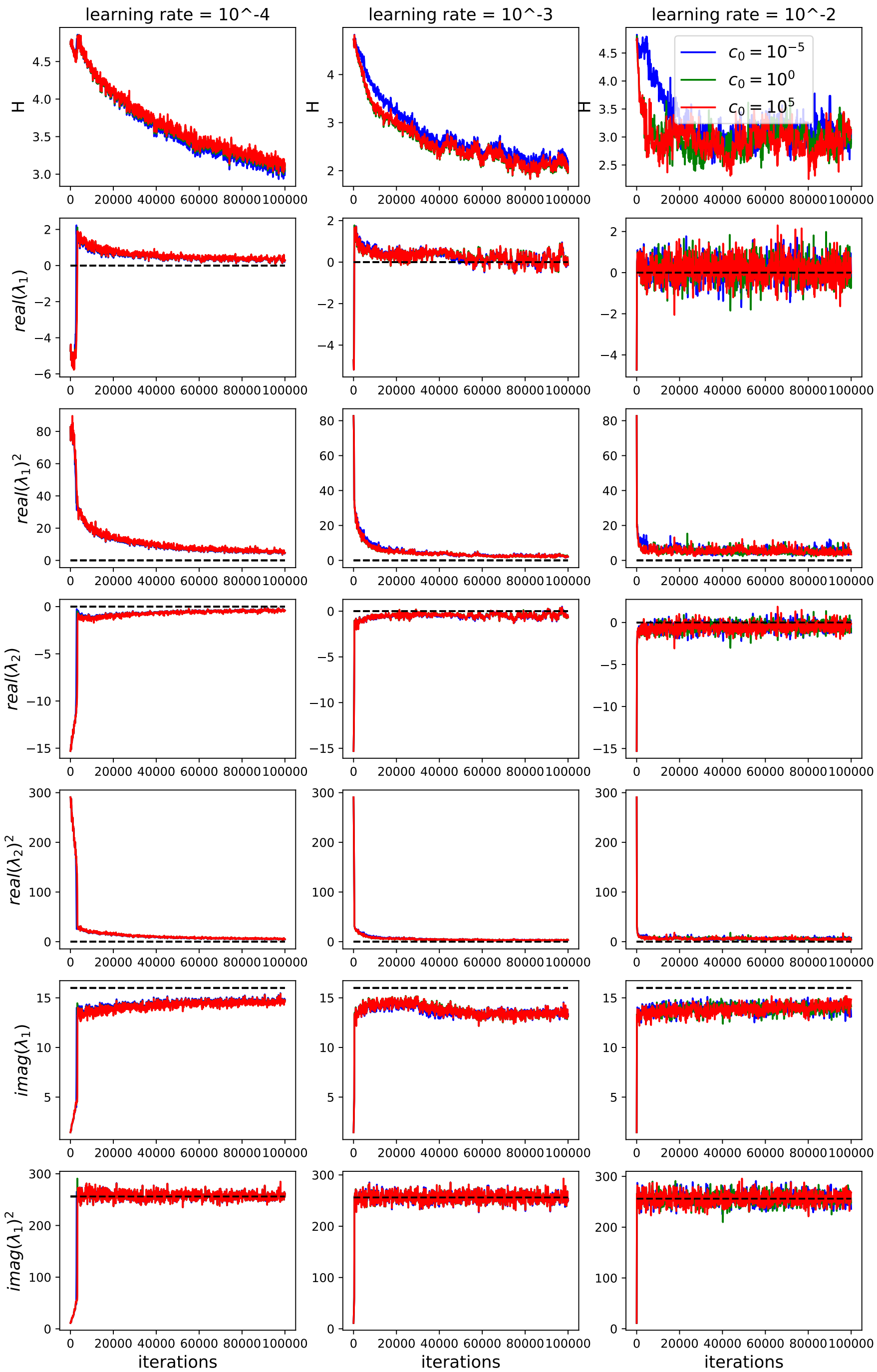
W: 1 affine layer



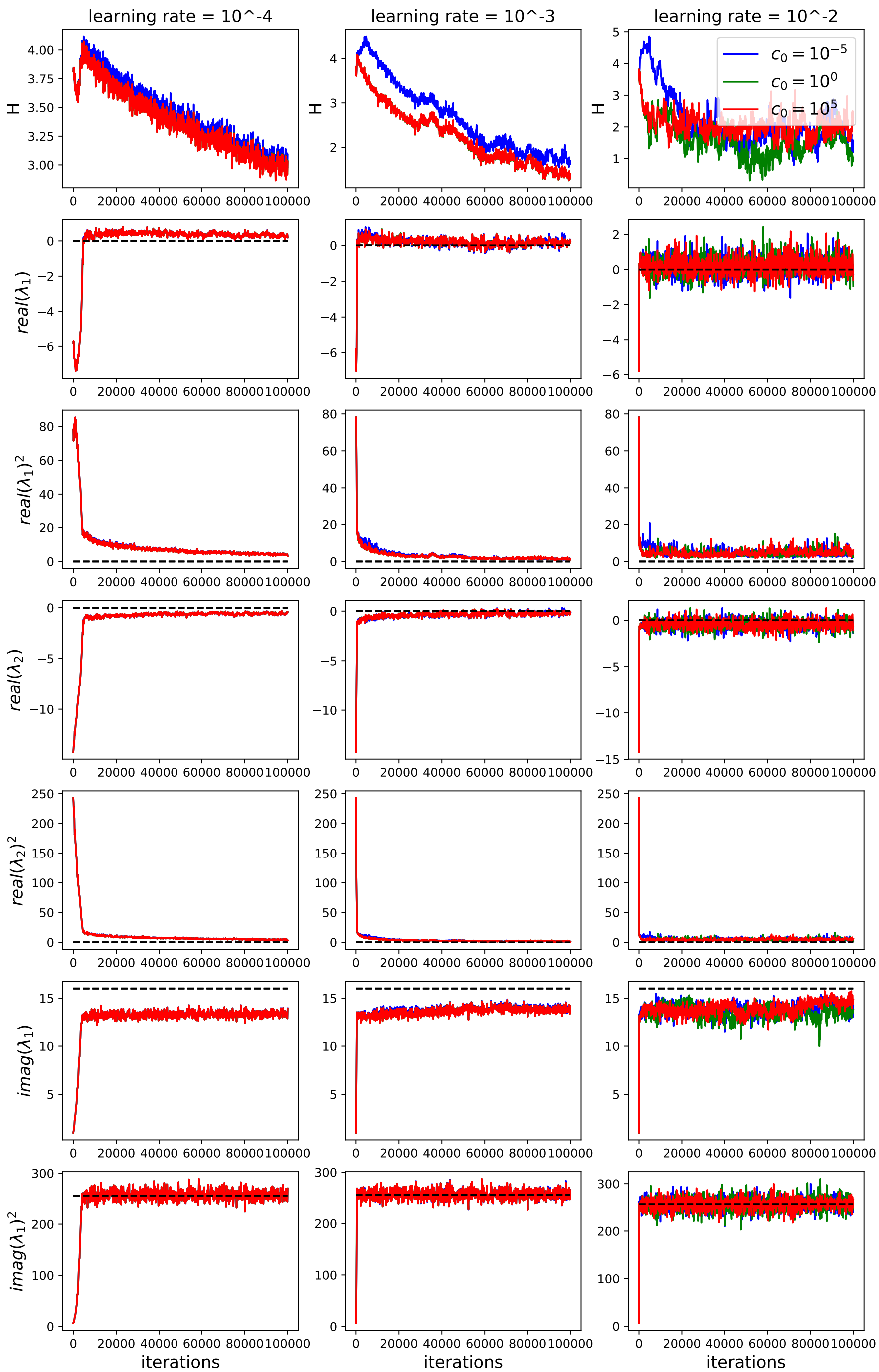
W: 2 planar layers



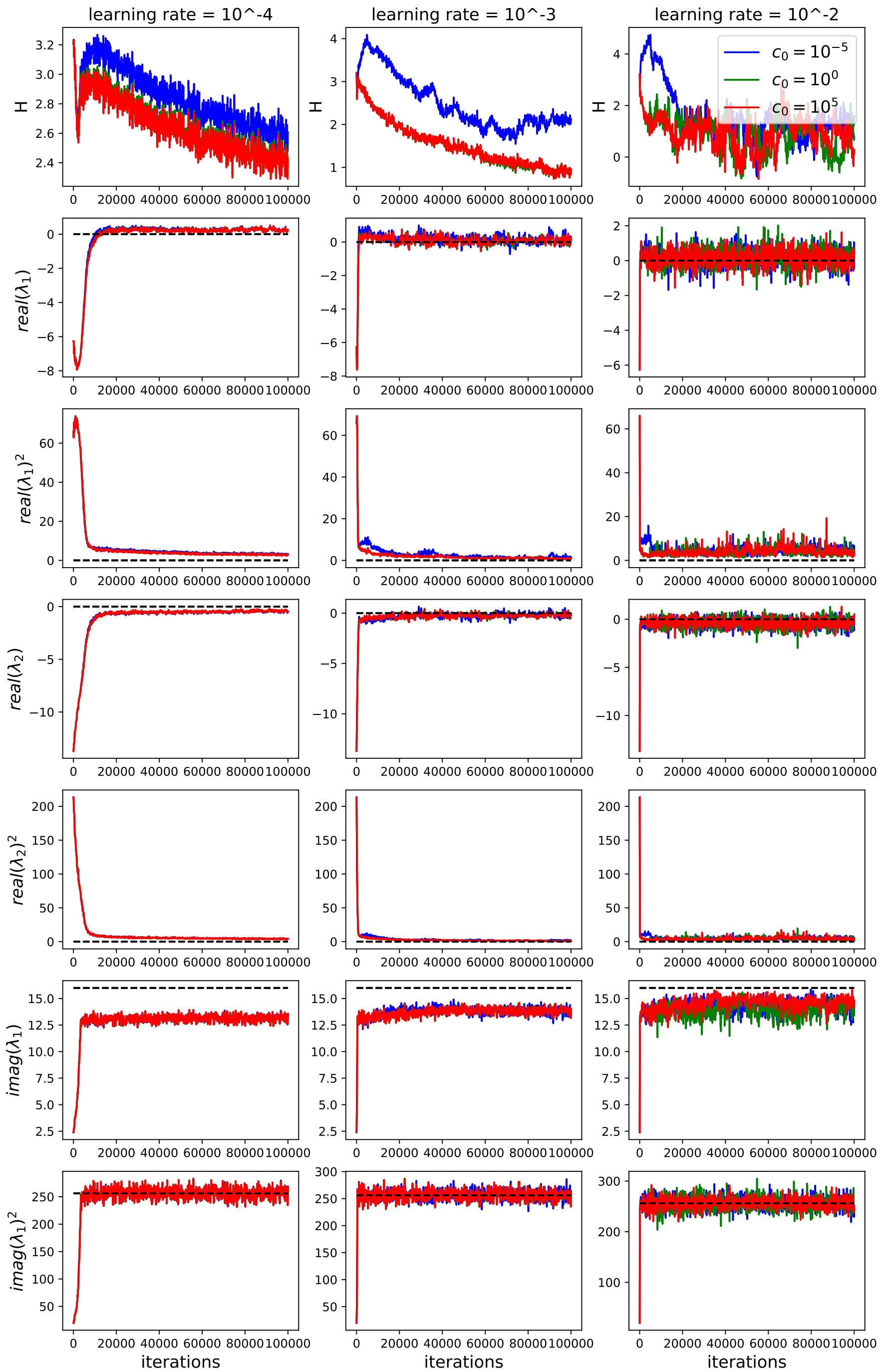
W: 4 planar layers



W: 8 planar layers



W: 10 planar layers



W: 10 planar layers: $c_0 = 1$, learning rate $= 10^{-3}$