

Exploratory analyses of stochastic V1 model with EPI

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1 Introduction

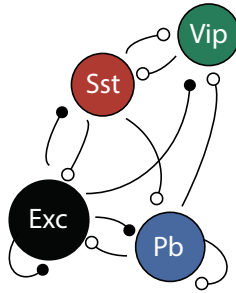
Last spring, I presented new work on the 4 neuron-type V1 model intended to replace what's currently in section 3.3 of our EPI paper. This new line of analyses focused on inferring the full distribution of inputs perturbations (dh) relative to a baseline input (b) that keep each neuron-type stable. From our meeting and afterwards, I realized that what I presented had a couple of important shortcomings: the baseline input we used b was not well-motivated (arbitrarily chosen) and the analyses we do on the deterministic model are easy with simpler methods.

In this write-up, I show that we have improved this work in the following ways:

1. **Stochasticity:** The V1 model is now stochastic. EPI's superior ability to handle stochastic models can be emphasized.
2. **Well-motivated b :** The baseline input b is motivated by a mode of an EPI distribution conditioned on S-V flipping – an emergent property suggested by Ken in the spring.
3. **Complex, nonlinear, fine posteriors:** The obtained stability distributions are nonlinear/complex and match expectations from ABC-obtained parameter sets.
4. **Beats competitors:** TODO: We beat competing approaches to biological model analysis.

2 V1 model

Figure 1: V1 model comprised of an excitatory (black), parvalbumin-expressing (blue), somatostatin-expressing (red) and VIP-expressing (green) populations. Connections are shown between populations where the effective connectivity is appreciable ($W_{\alpha,\beta} > 0.05$ in eq. 4).



We consider a four-dimensional circuit model (Fig. 1) with dynamical state given by the firing rate x of each neuron-type population $x = [x_E, x_P, x_S, x_V]^T$ (Fig. 1A). Given a time constant of

$\tau = 20$ ms and a power $n = 2$, the dynamics are driven by the rectified and exponentiated sum of recurrent inputs Wx , external inputs h , and external noise $\epsilon \sim \mathcal{N}(0, \sigma_\epsilon^2)$

$$\tau \frac{dx}{dt} = -x + [Wx + h + \epsilon]_+^n. \quad (1)$$

We considered fixed effective connectivity weights W approximated from experimental recordings of publicly available datasets of mouse V1. Specifically, Billeh et al. [3] produce estimates of the synaptic strength (in mV)

$$M = \begin{bmatrix} 0.36 & -0.48 & -0.31 & -0.28 \\ 1.49 & -0.68 & -0.50 & -0.18 \\ 0.86 & -0.42 & -0.15 & -0.32 \\ 1.31 & -0.41 & -0.52 & -0.37 \end{bmatrix} \quad (2)$$

and connection probability

$$C = \begin{bmatrix} 0.16 & 0.411 & 0.424 & 0.087 \\ 0.395 & .451 & 0.857 & 0.02 \\ 0.182 & 0.03 & 0.082 & 0.625 \\ 0.105 & 0.22 & 0.77 & 0.028 \end{bmatrix}. \quad (3)$$

Multiplying these connection probabilities and synaptic efficacies gives us an effective connectivity matrix:

$$W = C \odot M = \begin{bmatrix} 0.0576 & -0.197 & -0.131 & -0.0244 \\ 0.589 & -0.307 & -0.429 & -0.00360 \\ 0.157 & -0.0126 & -0.0123 & -0.200 \\ 0.138 & -0.0902 & -0.400 & -0.0104 \end{bmatrix}. \quad (4)$$

3 Finding parameters that produce S-V flipping.

TODO Summarize S-V flipping phenomena and why it's a good EP (can't really fit the whole data-set).

TODO Motivate the input model.

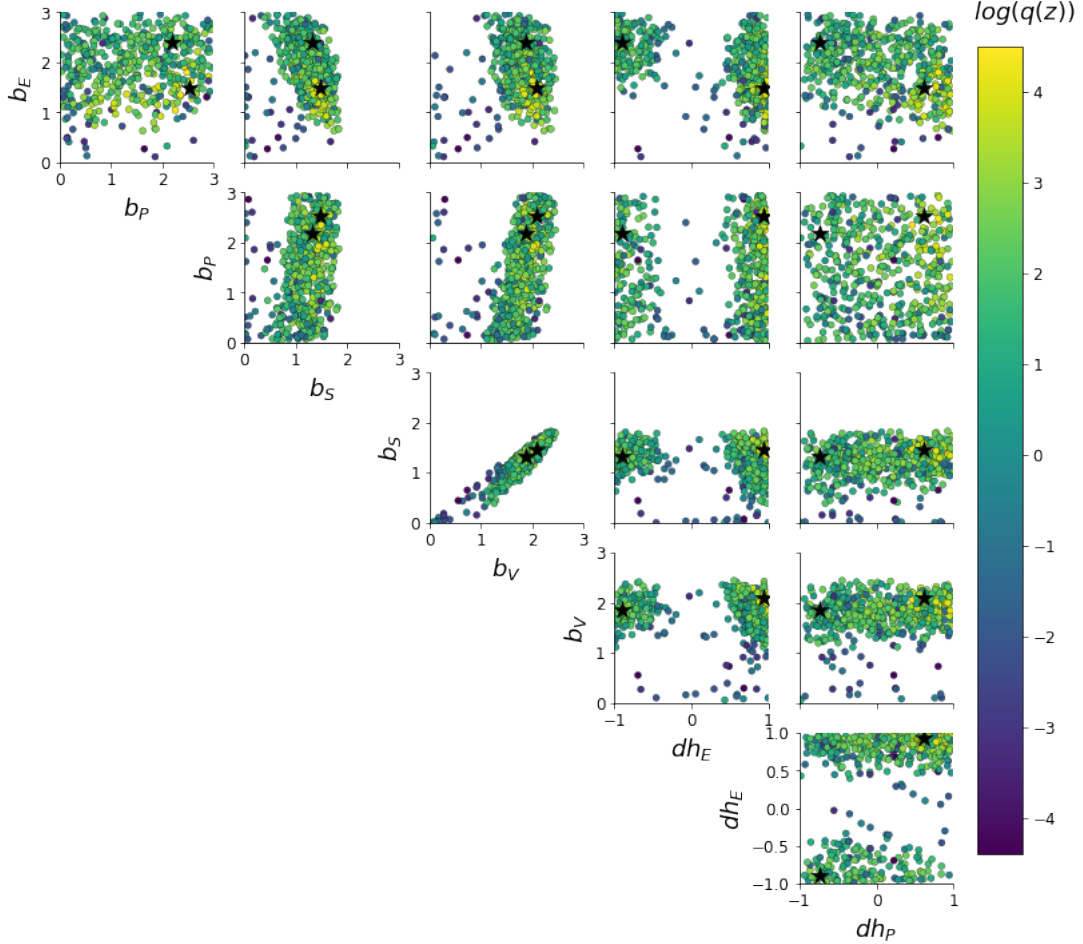
$$\begin{bmatrix} h_E \\ h_P \\ h_S \\ h_V \end{bmatrix} = \begin{bmatrix} b_E \\ b_P \\ b_S \\ b_V \end{bmatrix} + \begin{bmatrix} dh_E \\ dh_P \\ 0 \\ 0 \end{bmatrix} \quad (5)$$

Since we are unsure what either the baseline input b and contrast-dependent change in input dh should be, we treat them as free parameters when running EPI.

$$z = [b_E \ b_P \ b_S \ b_V \ dh_E \ dh_P]^\top \quad (6)$$

We consider positive baseline inputs $b_\alpha \in [0, 3]$ and small contrast-dependent changes in input $|dh_\alpha| \leq 1$. To find parameters resulting in S-V flipping, we focus on models driven by nominal amounts of external noise $\sigma_\eta = 0.05$. However, in the next section we take a parameter choice at the mode of the obtained posterior, and use EPI again to examine parametric stability in various noise regimes.

Figure 2: EPI posterior $q_\theta(z \mid \mathcal{B}_{S-V})$ for S-V flipping. The obtained posterior is visualized as 500 samples from the neural network colored by log-probability. Stars indicate the modes of the posterior for $dh_E < 0$ and $dh_E > 0$.



We formulate the emergent property of S-V flipping with a statistic measuring the difference in steady state (x_{ss}) between the S- and V-population at a given input:

$$d_{S-V}(h) = x_{ss,S}(h) - x_{ss,V}(h). \quad (7)$$

For S and V to flip their steady states, the difference between the S- and V-population rates at $h = b$ must have opposite sign from $h = b + dh$. Therefore, we stipulate the emergent property of S-V flipping to require the product between $d_{S-V}(b)$ and $d_{S-V}(b + dh)$ to be appreciably negative. Second we stipulate that the differences between S and V in each input condition cancel out on average ($d_{S-V}(b) + d_{S-V}(b + dh)$ is 0 on average). The means and variances of the emergent property statistics were sensibly chosen based on some inexpensive model simulations within the parameter bounds.

$$\mathcal{B}_{S-V} \triangleq \mathbb{E} \begin{bmatrix} d_{S-V}(b)d_{S-V}(b + dh) \\ (d_{S-V}(b)d_{S-V}(b + dh) - (-0.25))^2 \\ d_{S-V}(b) + d_{S-V}(b + dh) \\ (d_{S-V}(b) + d_{S-V}(b + dh))^2 \end{bmatrix} = \begin{bmatrix} -0.25 \\ 0.125^2 \\ 0 \\ 0.125^2 \end{bmatrix} \quad (8)$$

We ran EPI to inspect the structure of the posterior distribution of z conditioned on S-V flipping shown in Fig 2. It is clear from this visualization that S-V flipping in this V1-model is sensitive with respect to some parameter settings and robust with respect to others. Additionally, the posterior is bimodal: there is one mode in each of the hyperplanes $dh_E < 0$ and $dh_E > 0$.

The structure of the V1 model S-V flipping posterior yields the following insights:

1. The marginal distributions of $q_\theta(h_P | \mathcal{B}_{S-V})$ and $q_\theta(dh_P | \mathcal{B}_{S-V})$ are approximately uniform along their allowed range. The approximate uniformity of the P – *population* parameter marginal distributions shows that the P-population plays little role in S-V flipping.
2. The pairwise marginal distribution of $q_\theta(h_S, h_V | \mathcal{B}_{S-V})$ shows strong correlation between these parameters. The strong correlation between h_S and h_V in the posterior reveals that S-V flipping is sensitive with respect to the baseline inputs to the S- and V-populations. **Augment this to show that multidim info from Hessian is useful.** The Hessian provided by EPI at the modes indicates that coordinated increases of h_S and h_V by a ratio of $\frac{h_S}{h_V} = 0.701$ will preserve S-V flipping, while changes in an orthogonal dimension will disrupt it.
3. This distribution is multimodal.

I recommend substituting this work for section 3.3 in the bioRxiv manuscript. I'd be happy to write up a draft of such a section.

References

- [1] Sean R Bittner, Agostina Palmigiano, Alex T Piet, Chunyu A Duan, Carlos D Brody, Kenneth D Miller, and John P Cunningham. Interrogating theoretical models of neural computation with deep inference. *bioRxiv*, page 837567, 2019.
- [2] (2018) Allen Institute for Brain Science. Layer 4 model of v1. available from: <https://portal.brain-map.org/explore/models/l4-mv1>.
- [3] Yazan N Billeh, Binghuang Cai, Sergey L Gratiy, Kael Dai, Ramakrishnan Iyer, Nathan W Gouwens, Reza Abbasi-Asl, Xiaoxuan Jia, Joshua H Siegle, Shawn R Olsen, et al. Systematic integration of structural and functional data into multi-scale models of mouse primary visual cortex. *bioRxiv*, page 662189, 2019.