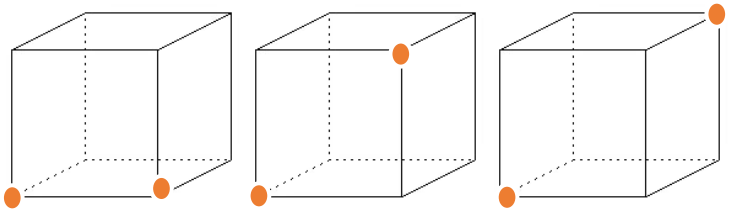
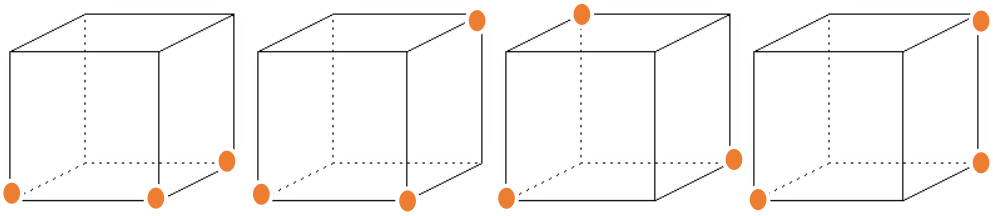
Colton Cunov
Artificial Neural Networks
B. Mehlig

1) *How many linearly separable 3-dimensional Boolean functions are there? To answer this question, use graphical representation of the different functions, and symmetries to reduce the number of cases to be considered. You must also upload a one-page pdf file with the explanation of how you arrived at the answer. Online sources are not accepted as a valid answer.*

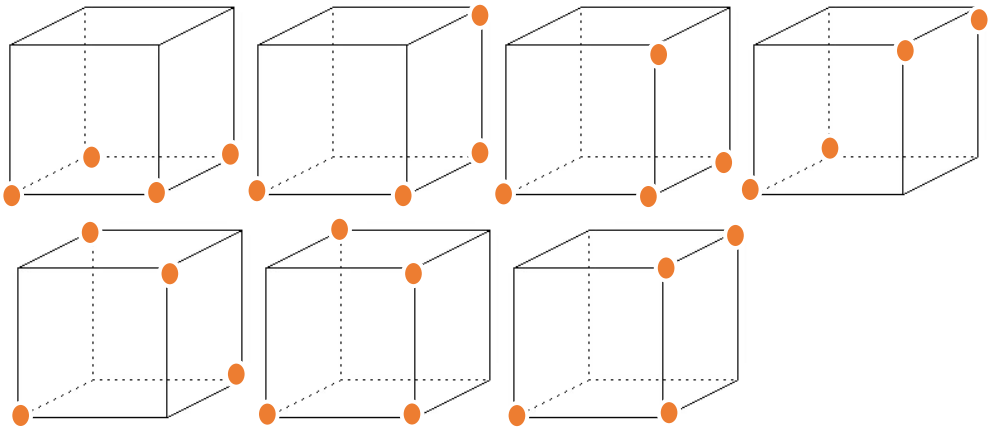
k=0 (1): we have 1 possible function and it is linearly separable.
k=1 (8): we have 8 possible functions, and only one symmetry. They are all linearly separable.
k=2 (12): we have 3 symmetries as shown below, only the first one is linearly separable. There are 12 linearly separable functions in this case.



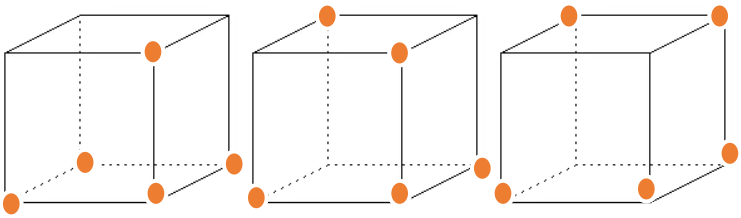
k=3 (24): we have 4 symmetries as shown below. Only the first set is linearly separable, and there are 24 different functions in this symmetry, so we have 24 linearly separable functions in this case.



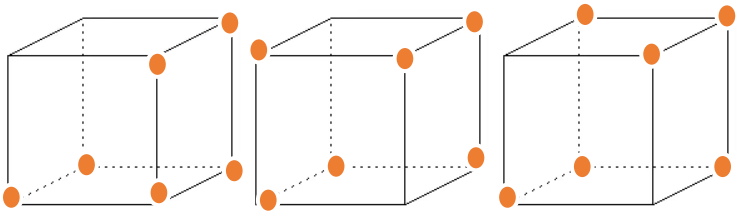
k=4 (14): we have 7 symmetries as shown below. In the 1st set we have 6 different functions and they are linearly separable. The 2nd set is not linearly separable. The 3rd set is linearly separable and there are 8 different functions. The 4th, 5th, 6th, and 7th sets are not linearly separable.



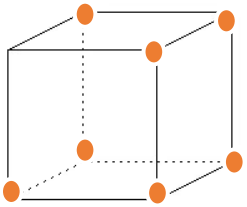
k=5 (24): we have only one symmetry that is linearly separable (the first one). There are 24 functions in this set.



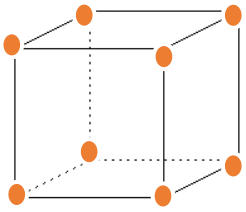
k=6 (12): we have two linearly separable symmetries, the 1st one below. There are 12 functions in this case.



k=7 (8): we have only one symmetry, with 8 possible functions, and it is linearly separable.



k=8 (1): we have only one possible function, which is linearly separable.



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Linear separability of 4-dimensional Boolean functions (2020)

```
X = readmatrix('input_data_numeric.csv');
X(:,1)=[];
boolean_functions = readmatrix('boolean_functions.txt');
t = boolean_functions(6,:);
n = .02;
W = -.2 + .4.*rand(1,4);
T = -1 + 2*rand;

converged = 0;
H = zeros(1,10^5);
for iRun = 1:10
    iLearned = 1;
    O = zeros(1,16);
    sigO = zeros(1,16);
    while converged == 0 && iLearned < 10^5
        % calc output for all patterns
        for mu=1:16
            dotProduct = dot(X(mu,:),W);
            O(mu) = tanh(dotProduct - T);
            if O(mu) >= 0
                sigO(mu) = 1;
            else
                sigO(mu) = -1;
            end
            H(iLearned) = H(iLearned) + (t(mu)-O(mu))^2;
        end
        H(iLearned) = H(iLearned)/2;
        % check for convergence
        if isequal(sigO,t)
            converged = 1;
            break
        end
        % pick random pattern and update weights, thresh
        muRand = randi(16,1);
        dotProduct = dot(X(muRand,:),W);
        gPrime = 1 - tanh(dotProduct-T)^2;
        gradient = gPrime * (t(muRand)-O(muRand));
        for i=1:4
            W(i) = W(i) + n*gradient*X(muRand,i);
        end
        T = T + -n*gradient;
        iLearned = iLearned + 1;
    end
end
H(H==0) = [];
converged
plot(H(1:end))
```

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Two-layer perceptron (2020)

```
clear all
M1 = 8;
M2 = 4;
step = .02;
numRuns = 5000;

% training variables
train = readmatrix('training_set.csv');
X = train(:,1:2);
t = train(:,3);
pVal = length(X);
w{1} = -.2 + .4.*rand(M1,2);
w{2} = -.2 + .4.*rand(M2,M1);
w{3} = -.2 + .4.*rand(1,M2);
theta{1} = zeros(M1,1);
theta{2} = zeros(M2,1);
theta{3} = 0;
B{1} = zeros(M1,pVal);
B{2} = zeros(M2,pVal);
B{3} = zeros(1,pVal);
V{1} = zeros(M1,pVal);
V{2} = zeros(M2,pVal);
V{3} = zeros(1,pVal);
sigO = zeros(1,pVal);
err{1} = zeros(M1);
err{2} = zeros(M2);
err{3} = zeros(1);
C = zeros(1,numRuns);

% testing variables
test = readmatrix('validation_set.csv');
testInput = test(:,1:2);
testTarget = test(:,3);
pValTest = length(testInput);
wGood{1} = zeros(M1,2);
wGood{2} = zeros(M2,M1);
wGood{3} = zeros(1,M2);
thetaGood{1} = theta{1};
thetaGood{2} = theta{2};
thetaGOod{3} = theta{3};
BTest{1} = B{1};
BTest{2} = B{2};
BTest{3} = B{3};
VTest{1} = V{1};
VTest{2} = V{2};
VTest{3} = V{3};
sigOTest = sigO;
CTest = zeros(1,numRuns);

numIter = 0;
for iRun=1:numRuns
    % calc Bs, Vs, and final output sigO
    for mu=1:pVal
```

```
    B{1}(:,mu) = w{1} * X(mu,:)' - theta{1};
    V{1}(:,mu) = tanh(B{1}(:,mu));
    for L=2:3
        B{L}(:,mu) = w{L} * V{L-1}(:,mu) - theta{L};
        V{L}(:,mu) = tanh(B{L}(:,mu));
    end
    if V{3}(:,mu) >= 0
        sigO(:,mu) = 1;
    else
        sigO(:,mu) = -1;
    end
    C(iRun) = C(iRun) + abs(sigO(:,mu) - t(mu));
end
C(iRun) = C(iRun)/(2*pVal);

for numFed=1:pVal
    numIter = numIter + 1;
    iRand = randi(pVal,1);
    % calc local fields, Vs
    B{1}(:,iRand) = w{1} * X(iRand,:)' - theta{1};
    V{1}(:,iRand) = tanh(B{1}(:,iRand));
    for L=2:3
        B{L}(:,iRand) = w{L} * V{L-1}(:,iRand) - theta{L};
        V{L}(:,iRand) = tanh(B{L}(:,iRand));
    end
    % calc errors
    err{3} = (t(iRand)-V{3}(:,iRand)) * (1 - tanh(B{3}(:,iRand))^2);
    for L=flip(1:2)
        err{L} = w{L+1}' * err{L+1} .* (1 - tanh(B{L}(:,iRand)).^2);
    end
    % update weights and biases
    w{1} = w{1} + step * err{1} * X(iRand,:);
    w{2} = w{2} + step * err{2} * V{1}(:,iRand)';
    w{3} = w{3} + step * err{3} * V{2}(:,iRand)';
    theta{1} = theta{1} - step * err{1};
    theta{2} = theta{2} - step * err{2};
    theta{3} = theta{3} - step * err{3};
end

% calc C for test data
for i=1:pValTest
    BTest{1}(:,i) = w{1} * testInput(i,:)' - theta{1};
    VTest{1}(:,i) = tanh(BTest{1}(:,i));
    for L=2:3
        BTest{L}(:,i) = w{L} * VTest{L-1}(:,i) - theta{L};
        VTest{L}(:,i) = tanh(BTest{L}(:,i));
    end
    if VTest{3}(:,i) >= 0
        sigOTest(:,i) = 1;
    else
        sigOTest(:,i) = -1;
    end
    CTest(iRun) = CTest(iRun) + abs(sigOTest(:,i) - testTarget(i));
end
CTest(iRun) = CTest(iRun)/(2*pValTest);
```

```
    if mod(iRun,10) == 0
        plot(CTest)
        drawnow
    end
    if CTest(iRun) < .12 % store good weights and biases
        for L=1:3
            wGood{L} = w{L};
            thetaGood{L} = theta{L};
        end
    end
    if CTest(iRun) < .115 % stopping criteria
        break
    end
end
```