1. General Initial Value ODE (1x1) Problem

$$\frac{dy}{dt} = f(y,t), \quad y(t_0) = y_0 \tag{1.1}$$

where

y= dependent variable

t = independent variable

$f(y,t)$ = derivative function

t_0 = initial value of the independent variable

y_0 = initial value of the dependent variable

The analytical solution:

$$\mathrm{y(t)} = y_0 \exp\left(\frac{\lambda}{\alpha}(1 - \exp(-\alpha t))\right), \quad y(0) = y_0 \tag{1.2}$$

2. General Initial Value ODE (2x2) Problem

$$\frac{dy_1}{dt} = a_{11}y_1 + a_{12}y_2 \quad y_1(0) = y_{10}$$

$$\frac{dy_2}{dt} = a_{21}y_1 + a_{22}y_2 \quad y_2(0) = y_{20} \tag{2.1}$$

3. Heat Transfer Equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

$$u(x,0) = u_0(x) = \sin(\pi x)$$

$$u(0,t) = u(1,t) = 0 \tag{3.1}$$

The analytical solution:

$$\mathrm{u(x,t)} = \sin(\pi x)e^{-\pi^2 t} \tag{3.2}$$

4. Nonlinear Partial Differential Equation

$$\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2}$$

$$-\mathrm{k}\frac{\partial u(0,t)}{\partial x} = \sigma(au_a^4 - eu^4(0,t))$$

$$L_s\rho_s C_{ps}\frac{du_s}{dt} = -k\frac{\partial u(L,t)}{\partial x}$$

where, in SI (MKS) units,

u = insulation temperature, K

t = time, s

x = position in the insulation, m

u_a = ambient (flame) temperature, K

α = insulation thermal diffusivity, m^2/s

k = insulation thermal conductivity, $\frac{J \cdot m}{s \cdot m^2 \cdot K}$

σ = Stefan–Boltzmann constant = $5.67 \times 10^{-8} \frac{J}{s \cdot m^2 \cdot K}0$

L = insulation half thickness, m

a = absorptivity

e = emissivity

u_s = steel temperature, K

L_s = steel half thickness, m

ρ_s = steel density, kg/m^3

C_{ps} = steel specific heat, J/kg · K