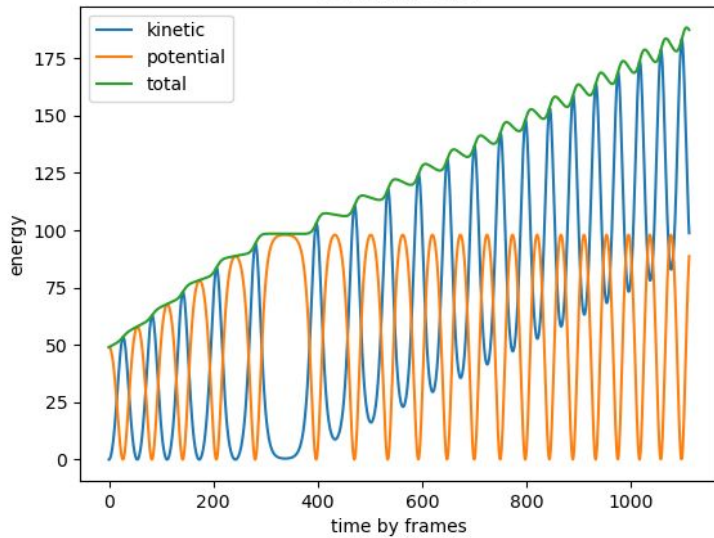


(I used 1200 data incorrectly which is 60 seconds in simulation time but I think it can also show the difference between the ODEs, please accept my work and take a look at it!, I deleted all data before I realized so I couldn't draw them again with only first $\frac{1}{3}$)

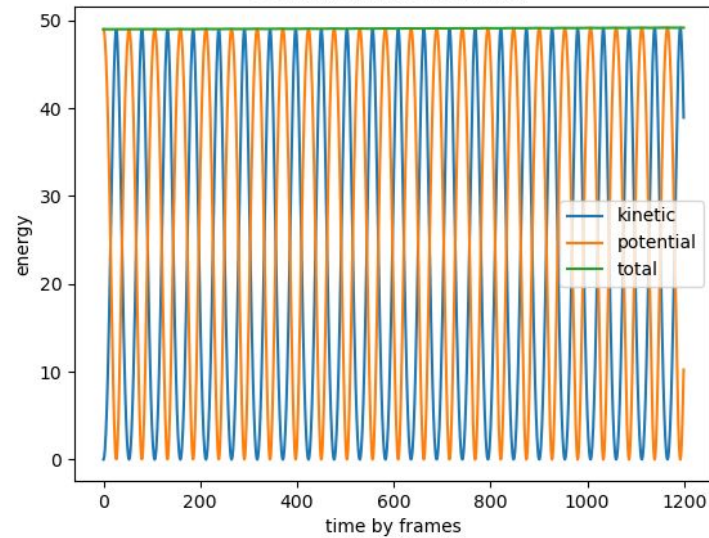
Setting A.

In this setting, Euler will have some error by “generating”, the pendulum will exceed the original position during the phrase. other settings have a similar performance which the pendulum keep swinging at almost the same rate and angular.

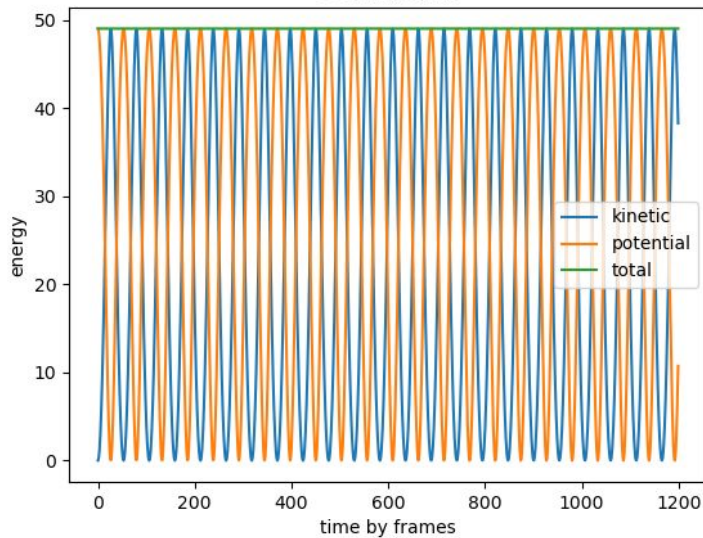
Setting A, Euler



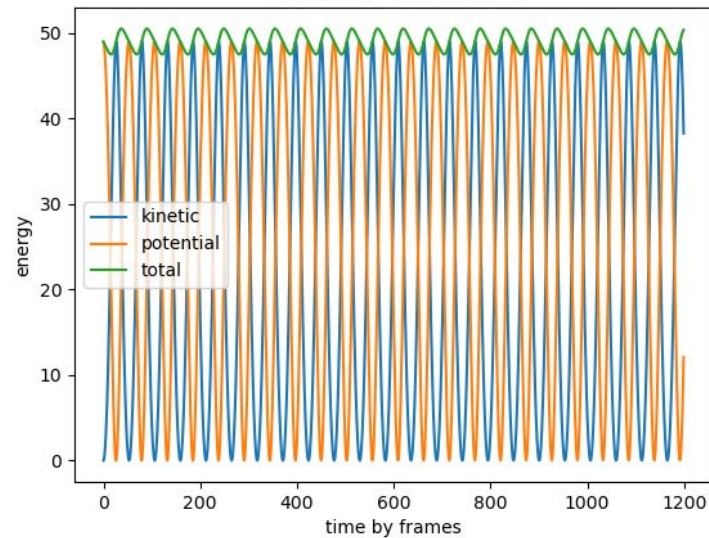
Setting A, improved-Euler



Setting A, rk



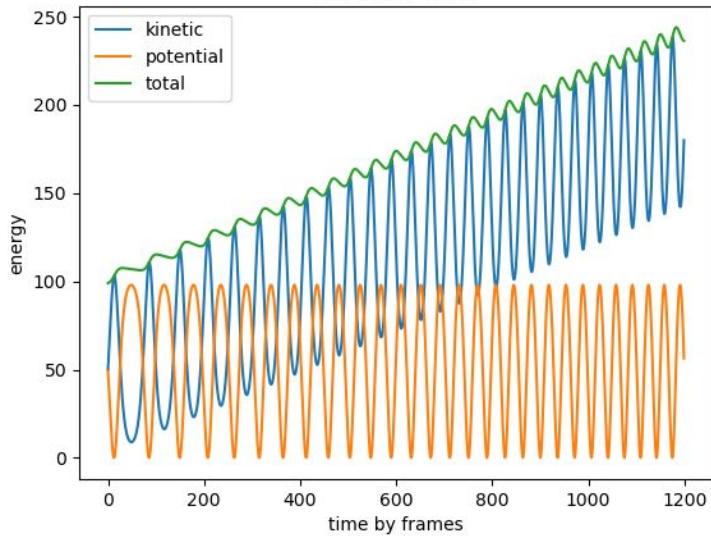
Setting A, semi-implicit



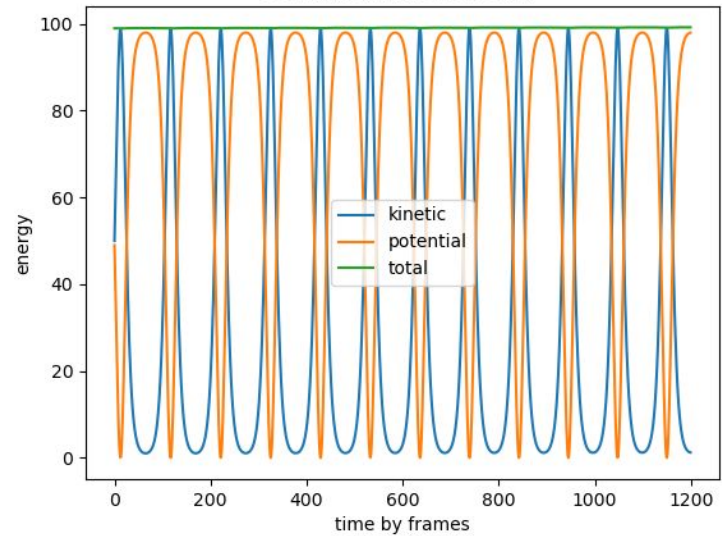
Setting B.

Only euler method is still “generating” energy and the pendulum will swing over the vertical line. Other method with initial speed will also exceed the horizontal line be then the total energy is still the same so on left side and right side it will have the same height.

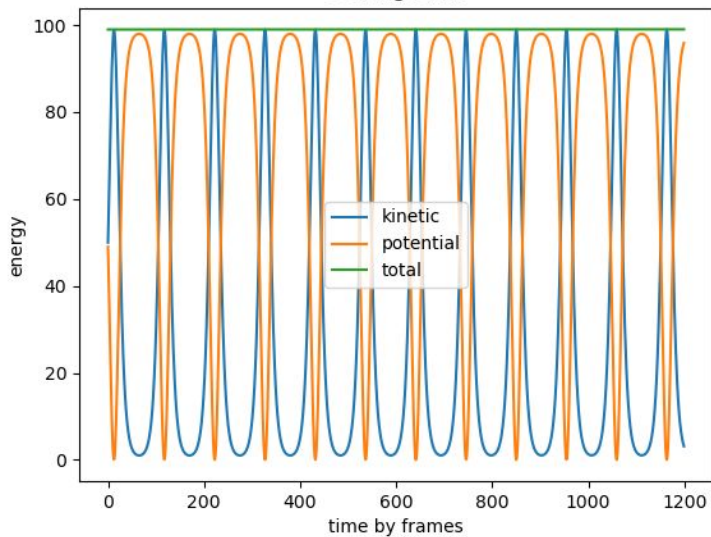
Setting B, euler



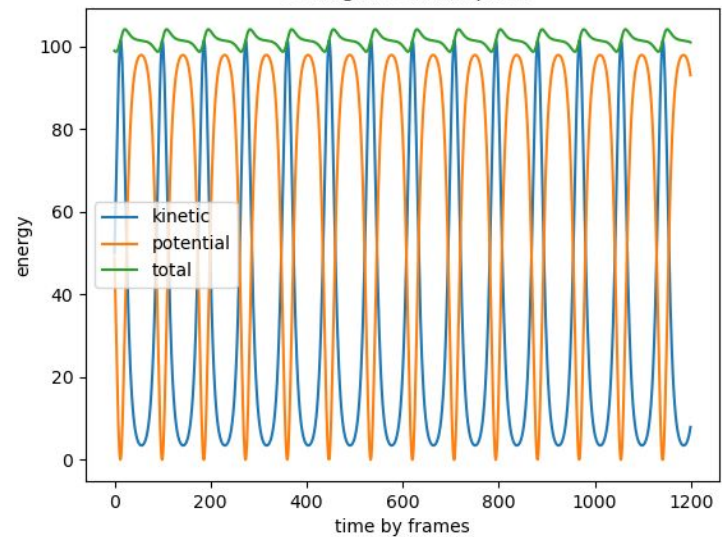
Setting B, improved-euler



Setting B, rk

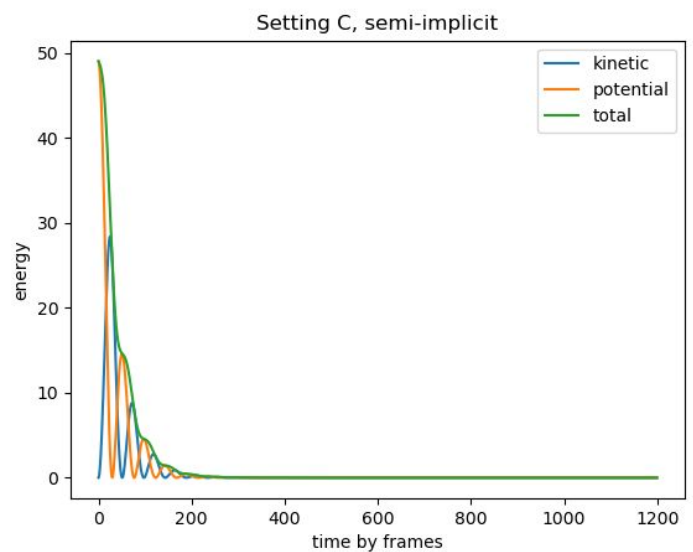
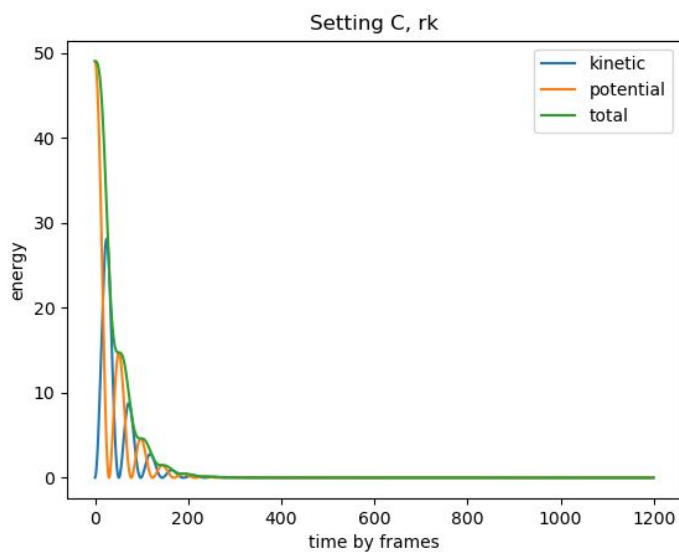
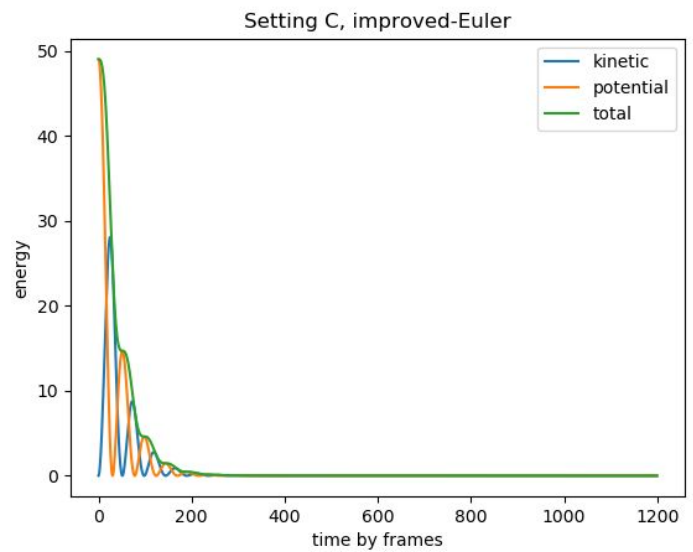
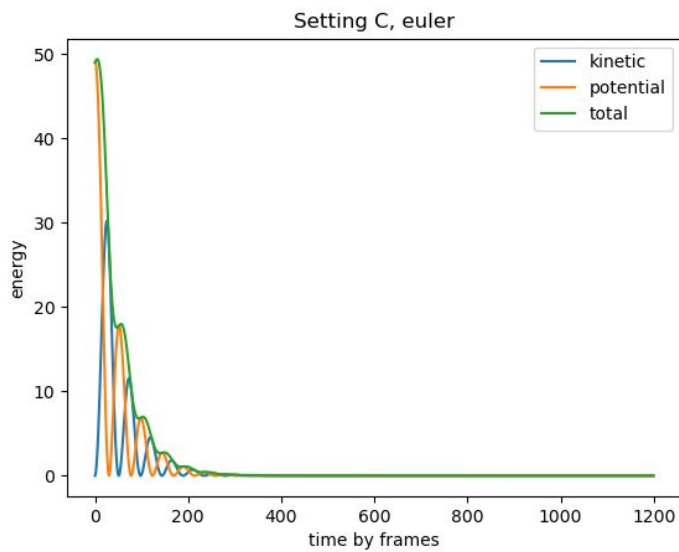


Setting B, semi-implicit

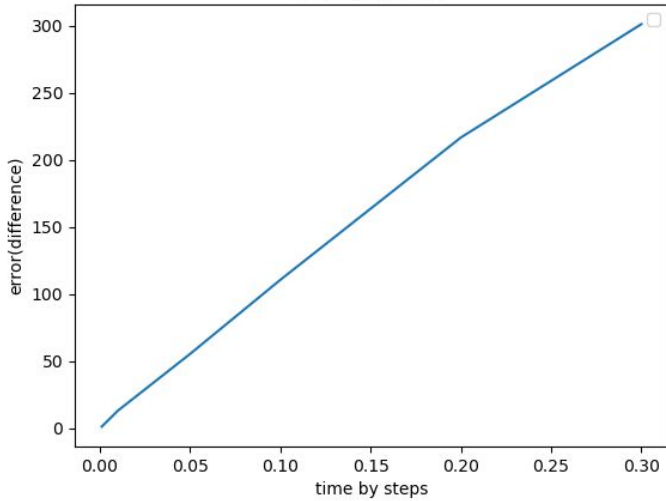


Setting C.

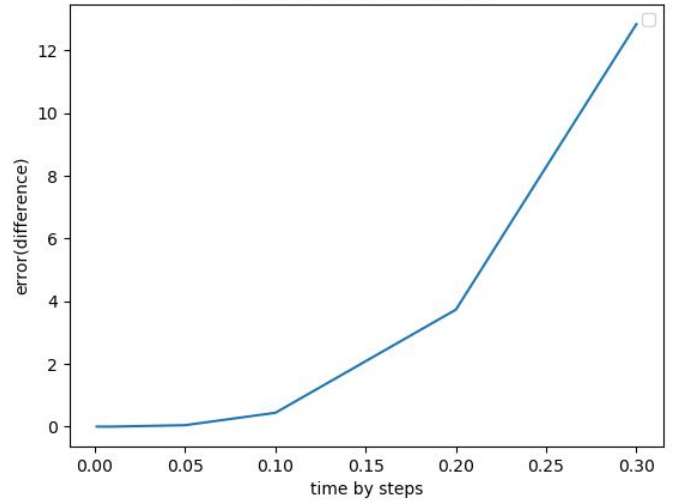
They are acting pretty similar. All of them slowly lower the height during swinging, each swinging will lose them some energy because of the friction, and finally almost stop (still very little swinging can't be recognized by eyes)



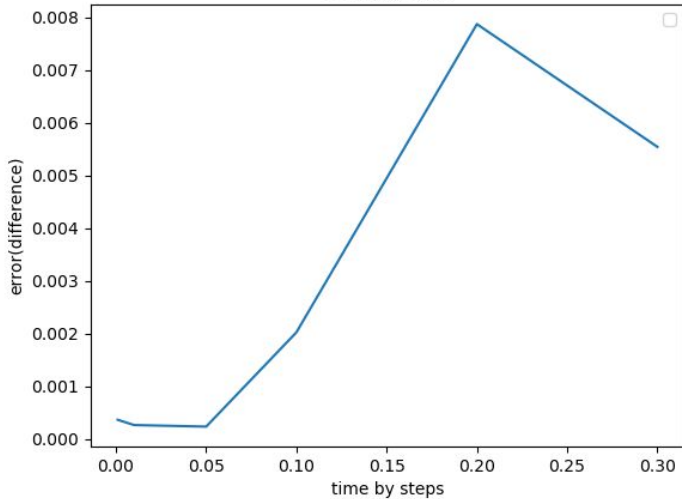
Setting A, Euler



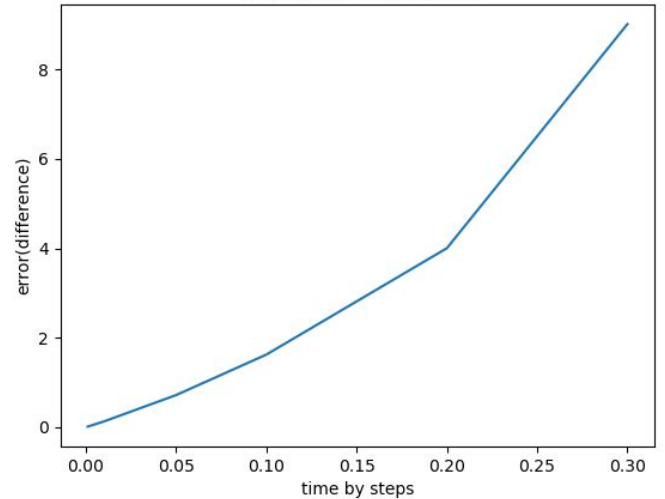
Setting A, Improved-Euler



Setting A, rk



Setting A, semi-implicit



Error.

Q: Why is the difference in a total energy a reasonable measure of error?

A: Because in a system like setting A, the total energy should be consistent, so if there's any change on the total energy that must be an error. and also the value is measurable.

Q: What empirical conclusions can you draw about the four methods?

A: When we are using really small time step, the difference between methods are not very large, but if we want to use larger time step, Euler is obviously not a good choice. RK have a very good performance but also takes a lot of space and calculation.

Improved-euler and semi-implicit is also acceptable. In fact, they didn't change a lot on Euler method, but only some minor change on each step, improves the accuracy a lot.