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1/ The slope of the tangent line:

$$\lim_{x\to -1} \frac{2x^2-5x-3}{x+1} = \cancel{(x^2-2x-3)} \lim_{x\to -1}(2x^2-2x-3) = 1$$

The equation of the tangent line through $(-1;3)$ is:

$y = x+4$

2/ The equation of the tangent line:

$y = 4x - 23$

3/ $$\lim_{h\to 0} \frac{\sqrt[4]{16+h}-2}{h} = \lim_{h\to 0} \frac{\sqrt[4]{16+h}-\sqrt[4]{16}}{h}$$

So function f is $f(x) = \sqrt[4]{x}$ and the number a is 16

4/ $$\lim_{x\to 5} \frac{2^x-32}{x-5} = \lim_{x\to 5}\frac{2^x-2^5}{x-5}$$

So function f is $f(x) = 2^x$ and the number a is 5

~~5/~~

~~a) $f'(0) = \frac{3-(-3)}{4,9-0,75} = \frac{7}{.}$~~

5/ a) $y'(0)$ doesn't exist

a) $f'(1) = 0$ d) $f'(3) = -1$ g) $f'(6) = 0$

c) $f'(1) = -\frac{3}{2}$ e) $f'(4) = \frac{-4}{3}$ h) $f'(7) = \frac{1}{4}$

6/

a) $(f+g)'(3) = -6+5 = -1$

b) $(fg)'(3) = 4.5 + (-6).2 = 20 - 12 = 8$

c) $\left(\frac{f}{g}\right)'(3) = \frac{(-6).2 - 5.4}{4} = \frac{-32}{4} = -8$

7/ $F(x) = (x^2 - x + 1)^3$

$F'(x) = 3(2x-1)(x^2-x+1)^2 = (6x-3)(x^2-x+1)^2$

8/ $f(x) = \frac{x}{\sqrt{7-3x}}$

$$f'(x) = \frac{1}{\sqrt{7-3x}} + \frac{\frac{3}{2\sqrt{7-3x}}}{7-3x} . x$$

$$= \frac{1}{\sqrt{7-3x}} + \frac{3x}{2\sqrt{7-3x}} . \frac{1}{7-3x} = \frac{2(7-3x)+3x}{2(\sqrt{7-3x})^3} = \frac{14-3x}{2(\sqrt{7-3x})^3}$$

9) $y = x \sin\frac{1}{x}$

$y' = \sin\frac{1}{x} + x.\left(\frac{-1}{x^2}\right).\cos\frac{1}{x}$

$= \sin\frac{1}{x} - \frac{1}{x}.\cos\frac{1}{x}$

10/ $h(x) = \sqrt{4+3f(x)}$

$h'(x) = \frac{3f'(x)}{2\sqrt{4+3f(x)}}$

$\Rightarrow h'(1) = \frac{3.4}{2\sqrt{4+3.7}} = \frac{12}{10} = \frac{6}{5}$

So $h'(1) = \frac{6}{5}$

11/ $1+x = \sin(xy^2)$

$\frac{d}{d(x)}[\sin(xy^2)] = \cancel{(y^2)\, 2xy\cos(xy^2)}\,\cancel{(2xy)}\,(2xyy'+y^2)\cos(xy^2)$

$\Rightarrow \frac{d}{d(x)}(1+x) = \frac{d}{d(x)}[\sin(xy^2)]$

$\Leftrightarrow 1 = (2xyy'+y^2)\cos(xy^2)$

$\Leftrightarrow \cancel{2xy.y'\cos(xy)}$

$\Leftrightarrow 2xy.y' + y^2 = \frac{1}{\cos(xy^2)}$

$\Leftrightarrow y' = \frac{1-y^2}{2xy\cos(xy^2)}$

$\Leftrightarrow \cancel{(2xy)}.\,y' = \frac{1-y^2\cos(xy^2)}{2xy\cos(xy^2)}$

So $\frac{d}{dx}(y) = \frac{1-y^2\cos(xy^2)}{2xy\cos(xy^2)}$

12/ $\sqrt{x+y} = 1+x^2y^2$

$\frac{d}{d(x)}(\sqrt{x+y}) = \frac{\cancel{(1+y')}\,1+y'}{2\sqrt{x+y}}$

$\frac{d}{d(x)}(1+x^2y^2) = 2xy^2 + 2x^2y\,y'$

$\frac{d}{d(x)}(\sqrt{x+y}) = \frac{d}{d(x)}(1+x^2y^2)$

$\Leftrightarrow \frac{1+y'}{2\sqrt{x+y}} = 2xy^2 + 2x^2yy'$

$\Rightarrow \frac{1+y'}{2\sqrt{x+y}} = \frac{4xy^2\sqrt{x+y} + 4x^2yy'\sqrt{x+y}}{2\sqrt{x+y}}$

$\Leftrightarrow 0 = \frac{4xy^2\sqrt{x+y} + 4x^2yy'\sqrt{x+y} - 1 - y'}{2\sqrt{x+y}}$

$\Rightarrow \frac{d}{dx}(y) = \frac{(4xy^2\sqrt{x+y} - 1)(1+4x^2y\sqrt{x+y})}{1 - 16x^5y^2 - 16x^4y^3}$.

14/

Let l is length of the rectangle

w is width of the rectangle

A is the area of the rectangle

t is time

$\frac{dl}{dt} = 8 cm/s$; $\frac{dw}{dt} = 3 cm/s$

$A = l.w$

$\frac{dA}{dt} = l.\frac{dw}{dt} + w.\frac{dl}{t}$

$\Rightarrow \frac{dA}{dt} = 3l + 8w$

When $l = 20$ and $w = 10$

$\Rightarrow \frac{dA}{dt} = 140$ (cm)

$\Rightarrow$ The area of the rectagle increasing when the length is 20cm and the width is 10cm with the speed of 140 cm/s

15/ $x^2+y^2=25$

$\Leftrightarrow \frac{dy}{dt}x^2 + \frac{dy}{dt}y^2 = \frac{dy}{dt}25$

T.T BOOK

$\Leftrightarrow 2x\dfrac{dx}{dt} + 2y = 0$

$\Leftrightarrow \dfrac{dx}{dt} = \dfrac{-y}{x}$

$\Leftrightarrow$ when $y = 4$

$\Rightarrow \dfrac{dx}{dt} = \dfrac{-4}{x} = \dfrac{4}{x^2}$

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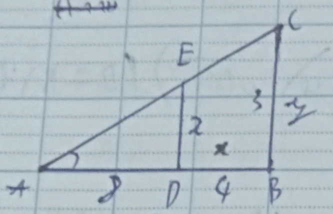
$(l = 12\,m)$

$(h = 2\,m)$

~~At m~~



At the specified moment,

If the man stands at DE then his shadow is at $BC = y$

The two triangles ADE and ABC are right and similar triangle

$\Rightarrow \dfrac{AD}{AB} = \dfrac{DE}{BC} \Rightarrow y = 3\ (m)$

If we consider the distance of the man from the building as x then the man to the spotlight is $12 - x$

$\dfrac{12-x}{12} = \dfrac{2}{y} \Leftrightarrow 1 - \dfrac{x}{12} = \dfrac{2}{y}$

$\Leftrightarrow \dfrac{d}{dx}\left(1 - \dfrac{x}{12}\right) = \dfrac{d}{dy}\left(\dfrac{2}{y}\right) \Leftrightarrow -\dfrac{1}{12}dx = \dfrac{-2}{y^2}dy$

$\Rightarrow \dfrac{-1}{12}\cdot\dfrac{dx}{dt} = \dfrac{-2}{y^2}\cdot\dfrac{dy}{dt}$

At the specified moment:

$\dfrac{-1}{12} \times 1{,}6 = \dfrac{-2}{3^2}\cdot\dfrac{dy}{dt} \Rightarrow \dfrac{dy}{dt} = 0{,}6\ (m/s)$

17)

$V_r = \frac{1}{2} \cdot \frac{4}{3}\pi r^3 = \frac{4}{6}\pi r^3$

$V_r' = 2\pi r^2 r'$

$\Leftrightarrow d = 50\,m \Rightarrow r = 2500\,cm\ ;\ dr = 0.05\,m$

$V_r' = 2\pi \times 2500 \times 0.05 \approx 1.96\ (m^3)$

The needed paint is approximately $1.96\,m^3$

18) $G(x) = \sqrt[3]{x^2 - x}$

$G'(x) = \frac{2x-1}{3\sqrt[3]{(x^2-x)^2}}$

$\frac{2x-1}{3\sqrt[3]{(x^2-x)^2}} = 0 \Leftrightarrow \left(x = \frac{1}{2} \right)$ $(D = \mathbb{R})$. (Since $3\sqrt[3]{(x^2-x)}$) $(D = \mathbb{R}\setminus\{0\}) \Rightarrow x = \frac{1}{2}\ ;\ x = 0$

Therefore, the critical numbers of the function are $x = \frac{1}{2}\ ;\ x = 0$

19) $f(x) = x^3 - 3x + 1$ $(D \Rightarrow$

$\Rightarrow f'(x) = 3x^2 - 3$

x	$-\infty$		-1		1		$+\infty$
$f'(x)$		$+$	0	$-$	0	$+$	
$f(x)$	$-\infty$	$\nearrow$	3	$\searrow$	-1	$\nearrow$	$+\infty$

$f(0) = 1\ ;\ f(3) = 19$

$\Rightarrow$ the absolute maximum value is 19

The absolute minimum value is -1

20) $f(x) = \frac{x}{x^2+4}$

$\Rightarrow f'(x) = \frac{-2x^2}{(x^2+4)^2} + \frac{1}{x^2+4} = \frac{-x^2+4}{(x^2+4)^2}$

x	$-\infty$		-2		2		$+\infty$
$f'(x)$		$-$	0	$+$	0	$-$	
$f(x)$	$-\infty$	$\searrow$	-0.25	$\nearrow$	0.25	$\searrow$	$-$

$f(0) = 0 ; f(3) = \frac{3}{13}$

$\Rightarrow$ The absolute maximum value is $(0.25 > \frac{3}{13})$ 0.25

The absolute minimum value is -0.25

21/

$f'(x) = \frac{f(8) - f(2)}{8-2}$

Since $3 \le f'(x) \le 5$

$\Rightarrow 3 \le \frac{f(8)-f(2)}{6} \le 5 \Leftrightarrow 18 \le f(8) - f(2) \le 30$

22/

Assume the contradiction that f has more than one fixed point. Choose two ~~points~~ fixed point $x ; y$

$\Rightarrow f(x) = x ; f(y) = y$

Mean value theorem: There exists some $c \in (x ; y)$ such that $f'(c) = \frac{f(x) - f(y)}{x - y} = \frac{x-y}{x-y} = 1$, contradicting the hypothesis

So if $f'(x) \neq 1$ for all real numbers x, then f has at most only one fixed point

23/

a) On $(2 ; 4) ; (6 ; 9)$ f is increasing because on those intervals, $f'(x) > 0$

a) ~~At (x = 7;) x ∈ (2; 4; 6) f(x) have)~~ and $x = 6$

At $x = 2$; $f'(x)$ changes from negative to positive → ~~(x = 4)~~ f has a local minimum at $x = 2$ and $x = 6$

At $x = 4$, $f'(x)$ changes from positive to negative → f has a local maximum at $x = 4$

c) ~~At~~ $[0; 1]$; $(3; 5)$ ~~(4; 5)~~ and $(7; 8)$, the graph is concave downward ~~(upward)~~ because f' is decreasing over those intervals ⇒ $f'' < 0$ ~~(⇒ f'' < 0)~~

At $(1; 3)$; ~~(2; 3)~~; $(5; 7)$; ~~(6; 7)~~; $(8; 9)$, the graph is concave upward because f' is increasing over those intervals ($f'' > 0$)

d) The inflection point of f are $x = 1$; ~~(x = 2)~~ $x = 3$; ~~(x = 4; x = 5)~~ ~~(x = 6)~~ $x = 7$; $x = 8$ ~~(x = 9)~~ because f is continuous there and change in concaveness

24/

Let a is the first number

b is the second number

$\Rightarrow a - b = 100$

$\Rightarrow a = 100 + b$

$P = a \cdot b \Rightarrow P = b^2 + 100b$

$P' = 2b + 100$

Product is a minimum

$\Rightarrow P' = 2b + 100 = 0 \Leftrightarrow b = -50 \Rightarrow a = 50$

So $a = 50$; $b = 50$

25/

The box with a square base and open top

$\Rightarrow V = l.w.h = 32000$

$\Rightarrow x^2 . y = 32000 \Rightarrow y = \frac{32000}{x^2}$ (1)

$\Rightarrow$ The amount of material used is $x^2 + 4xy$

$\Rightarrow A = x^2 + 4x . \frac{32000}{x^2} = \frac{x^4 + 4x \cdot 32000x^3}{x^2} = x^2 + 128000x^{-1}$

$\Rightarrow A' = 2x - 128000x^{-2}$

Minimize the amount of material used

$\Rightarrow A' = 2x - 128000x^{-2} = 0$

$\Leftrightarrow \quad 2x = 128000x^{-2}$

$\Leftrightarrow \quad x^3 = 64000$

$\Leftrightarrow \quad x = 40$ (cm)

$\Rightarrow \quad y = 20$ (cm)

$\Rightarrow A = 40.40.20 = 32000$

So the dimension of the box is: length: 40cm; width: 40cm; height: 20cm

26/

$V = l.w.h = 10$

~~(Squared base ⇒~~ $\cancel{2x^2 . y = 1)}$ $l = 2w$

$\Rightarrow V = 2w.w.h = 10 \Leftrightarrow w^2 h = 5 \Rightarrow h = \frac{5}{w^2}$

Cost: $C = 6wh + 6wh + 12wh + 12wh + 20w^2 = 36wh + 20w^2$

$\Rightarrow C = 180\, w^{-1} + 20 w^2$

$C' = -180\, w^{-2} + 40 w = 0$

$\Rightarrow \quad 180\, w^{-2} = 40 w$

$\Rightarrow \quad 180 = 40 w^3$

$\Rightarrow \quad w = \sqrt[3]{\frac{9}{2}}$

$\Rightarrow C_{min} = 180\, (\sqrt[3]{4.5})^{-1} + 20 (\sqrt[3]{4.5})^2 \approx \$163.54$

27/ $f(x) = x^5 + 2 = 0$, $x_1 = -1$

$f'(x) = 5x^4$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = -1 - \frac{-1+2}{5} = -1 - \frac{1}{5} = -\frac{6}{5}$

$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = -\frac{6}{5} - \frac{(\frac{-6}{5})^5 + 2}{5 \cdot (\frac{-6}{5})^4} = -1.1529$

28/ $f''(x) = 4 - 6x - 40x^3$, $f(0) = 2$, $f'(0) = 1$

$\Rightarrow f'(x) = 4x - \frac{6x^2}{2} - \frac{40x^4}{4} + c = 4x - 3x^2 - 10x^4 + c$

$\Rightarrow f(x) = 2x^2 - x^3 - 2x^5 + cx + d$

$f'(0) = 1 \Rightarrow c = 1$

$f(0) = 2 \Rightarrow d = 2$

$\Rightarrow f(x) = -2x^5 - x^3 + 2x^2 + x + 2$

29/ $a(x) = 10 + 3x - 3x^2$, $s(0) = 0$, $s(1) = 10$

$v'(x) = a(x) \Rightarrow v(x) = -(\frac{3}{3}) x^3 + \frac{3}{2} x^2 + 10x + c$

$s(x) = \frac{-x^4}{4} + \frac{1}{2} x^3 + 5x^2 + cx + d$

$s(0) = 0 \Rightarrow d = 0$

$g(2) = \cancel{7}0 \Rightarrow a = \cancel{40} - 5$

$\Rightarrow g(x) = \frac{-7}{4}x^4 + \frac{7}{2}x^3 + 5x^2 \cancel{(+)} - 5x$

3c/ ~~Anti~~derivative of f is F

$\Rightarrow f = F'$

Since $f(x_0) = 0 \Rightarrow F$ has a critical ~~n~~umber $= x_0$

On the interval $(-\infty; x_0)$, $f > 0 \Rightarrow F$ increases

$\Rightarrow F$ is a