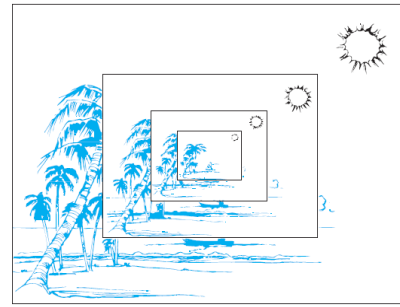


Recursive

Sometimes it is difficult to define an object explicitly.
However, it may be easy to define this object in terms of itself. This process is called **recursion**.



$$f(0) = 3,$$

$$f(n + 1) = 2f(n) + 3.$$

$$f(1) =$$

$$f(2) =$$

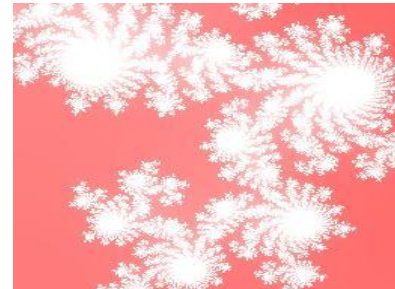
$$f(3) =$$

$$f(4) =$$

Recursively

BASIS STEP:

RECURSIVE STEP:



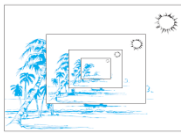
Give a recursive definition of a^n ,
where a is a nonzero real number
and n is a nonnegative integer.

Recursively

BASIS STEP:

RECURSIVE STEP:

Give a recursive algorithm for computing the greatest common divisor of two nonnegative integers a and b not both zero.



Give a recursive algorithm for computing $n!$,
where n is a nonnegative integer.

Give a recursive definition of

$$\sum_{k=0}^n a_k.$$

$$4! =$$

$$3! =$$

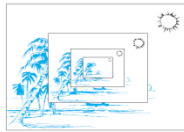
$$2! =$$

$$1! =$$

$$0! =$$

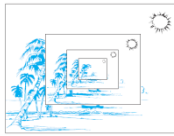
Devise a recursive algorithm for computing $b^n \bmod m$,
where b , n , and m are integers with
 $m \geq 2$, $n \geq 0$, and $1 \leq b$.

Recursive

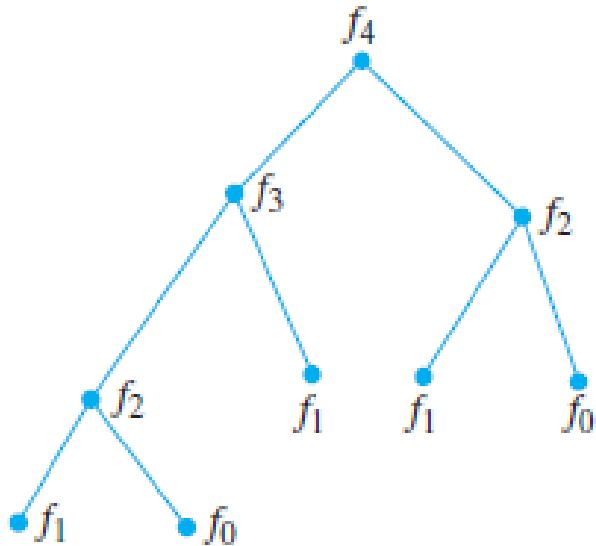


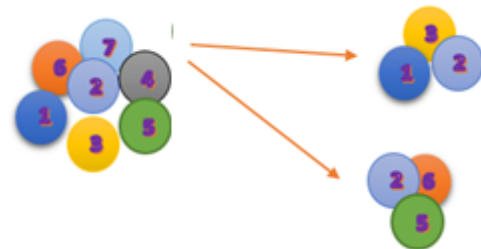
Some binary-recursions

Recursive



Fibonacci numbers of 0, 1, 1, 2, 3, ...





The number of k -combinators
from the set of $n+1$ elements

$$\underbrace{\binom{n+1}{k}}_{\substack{\text{number of} \\ k\text{-element} \\ \text{subsets of} \\ S=\{1,\dots,n,n+1\}}} = \underbrace{\binom{n}{k-1}}_{\substack{\text{number of} \\ k\text{-1-element} \\ \text{subsets of} \\ S \text{ without } n+1 \\ \text{+}^+ \text{+} \\ n+1}} + \underbrace{\binom{n}{k}}_{\substack{\text{number of} \\ k\text{-element} \\ \text{subsets of} \\ S \text{ without } n+1 \\ \text{-}^- \text{-} \\ n+1}}.$$

A Recursive Linear Search Algorithm.

A Recursive Binary Search Algorithm.

The Merge Sort

The Merge Sort of 8, 2, 4, 6, 9, 7, 10, 1, 5, 3.

