The Foundations: Logic and Infererence

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3.1. PREDICATES

3.2. QUANTIFIERs

3.3. NEGATING

3.4. TRANSLATING

PROPOSITIONAL LOGIC IS NOT ENOUGH

Propositional logic cannot adequately express the meaning of all mathematical statements.

This can't be represented in propositional logic. We need a language that talks about objects, their properties, and their relations. This is called predicate logic.

Predicate logic uses the following new features:

- Variables:
- Predicates:
- Quantifiers -

3.1. PREDICATES

"$x > 3$," "$x = y + 3$," "$x + y = z$," "computer x is under attack by an intruder,"

"$x > 3$,"

- Variables:
- Predicates:

Once a value has been assigned to the variable, the statement $P(x)$ becomes a proposition and has a truth value.

Example. What are the truth values of $P(1)$ and $P(2)$?

propositional function P

For example, let $P(x)$ denote "$x > 0$" and the domain be the integers. Then:

$P(-3)$ is ____

$P(0)$ is ____

$P(3)$ is ____

EXAMPLES OF PROPOSITIONAL FUNCTIONS

Let "$x - y = z$" be denoted by $Q(x, y, z)$

$Q(x, 3, z)$

$Q(2, -1, 3)$

$Q(3, 4, 7)$

COMPOUND EXPRESSION

Connectives from propositional logic carry over to predicate logic.

If $P(x)$ denotes "$x > 0$," find these truth values:

$P(3) \vee P(-1)$ $\quad T \vee F \quad$ $Solution: T$

$P(3) \wedge P(-1)$

$P(3) \rightarrow P(-1)$

$P(3) \rightarrow \neg P(-1)$

Expressions with variables are not propositions and therefore do not have truth values.

When used with quantifiers

3.2. QUANTIFIERs

The two most important quantifiers are:

- *Universal Quantifier*, "For all," symbol: $\forall$
- *Existential Quantifier*, "There exists," symbol: $\exists$

$\forall x\, P(x)$ is read as "For all x, $P(x)$" or "For every x, $P(x)$"

$\exists x\, P(x)$ is read as "There exists an element x in the domain such that $P(x)$." This might also be read as "For some x, $P(x)$", or as "There is an x such that $P(x)$".

Statement	***When True?***	***When False?***
$\forall x P(x)$	$P(x)$ is true for every x.	There is an x for which $P(x)$ is false.
$\exists x P(x)$	There is an x for which $P(x)$ is true.	$P(x)$ is false for every x.

We can also think of $\forall x P(x)$ being true if $P(x_1) \wedge P(x_2) \wedge \cdots \wedge P(x_n)$ is true for all $n \in U$

If $P(x)$ denotes "$x \geq 0$" and U is the positive integers,

Is the statement $\forall x P(x)$ true or false?

$P(x)$ is "$x^2 > 0$"

We can think of this as $\exists x\, P(x)$ is true if $P(x_1) \vee P(x_2) \vee \cdots \vee P(x_n)$ is true.

Suppose that $Q(x)$ is "$x^2 < 0$" where U is all integers. Is the statement $\exists x\, Q(x)$ true or false?

Let $P(x)$ denote the statement $x + 1 = 2x$. What is the truth value of $\exists x\, P(x)$ for the domain of all real numbers?

DE MORGAN'S LAWs for QUANTIFIERs

$$\neg\forall x P(x) \equiv \exists x \neg P(x)$$
$$\neg\exists x P(x) \equiv \forall x \neg P(x)$$

What are the negations of the statements "There is an honest politician" and "All Americans eat cheeseburgers"?

Let $C(x)$ denote "x eats cheeseburgers". Then "All Americans eat cheeseburgers" is represented by $\forall x C(x)$ with a domain of all Americans. The negation is $\neg\forall x C(x)$ which is equivalent to $\exists x \neg C(x)$, or "Some Americans do not each cheeseburgers" or "Not every American eats cheeseburgers".

3.4. TRANSLATING FROM ENGLISH TO LOGIC

First decide on the domain U.

"Every student in this class has taken a course in Java."

"Some student in this class has taken a course in Java."

"Every student in this class has visited Canada or Mexico."

3.4. TRANSLATING FROM ENGLISH TO LOGIC

First decide on the domain U.

"Some student in this class has taken a course in Java."

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TRANSLATING FROM ENGLISH TO LOGIC

"All men are mortal."
"Socrates is a man."
Does it follow that "Socrates is mortal?"

TRANSLATING

$U = \{fleegles, snurds, thingamabobs\}$

$F(x)$: x is a fleegle
$S(x)$: x is a snurd
$T(x)$: x is a thingamabob

Translate the following:

1. "Everything is a fleegle"
2. "Nothing is a snurd."
3. "All fleegles are snurds."
4. "Some fleegles are thingamabobs."
5. "No snurd is a thingamabob.
6. "If any fleegle is a snurd then it is also a thingamabob.