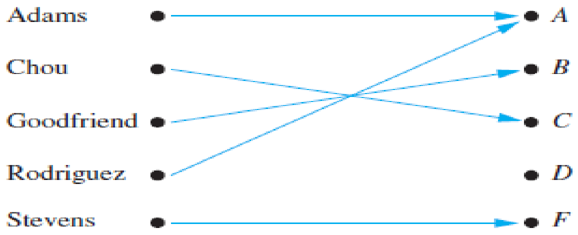
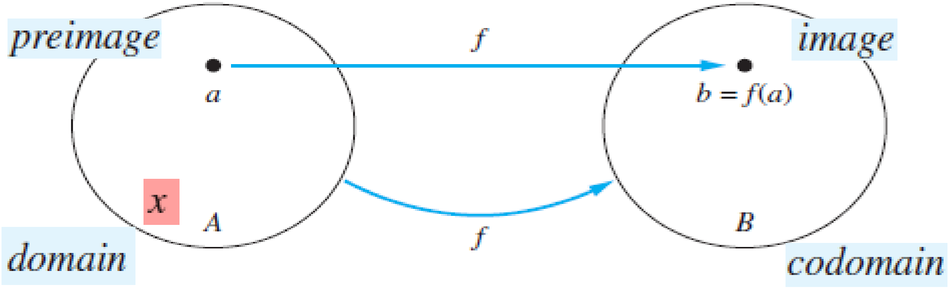
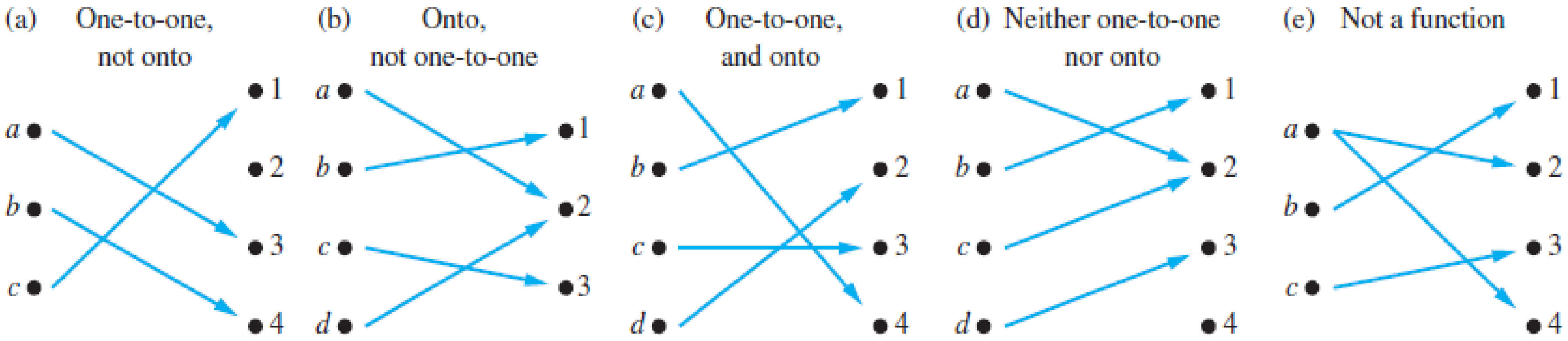
2.2 Functions

Assignment of Grades in a Discrete Mathematics Class.



The Function f Maps A to B.





Examples of Different Types of Correspondences.

Suppose that $f : A \to B$.

To show that f is injective Show that if $f(x) = f(y)$ for arbitrary $x, y \in A$ with $x \neq y$, then $x = y$.

To show that f is not injective Find particular elements $x, y \in A$ such that $x \neq y$ and $f(x) = f(y)$.

To show that f is surjective Consider an arbitrary element $y \in B$ and find an element $x \in A$ such that $f(x) = y$.

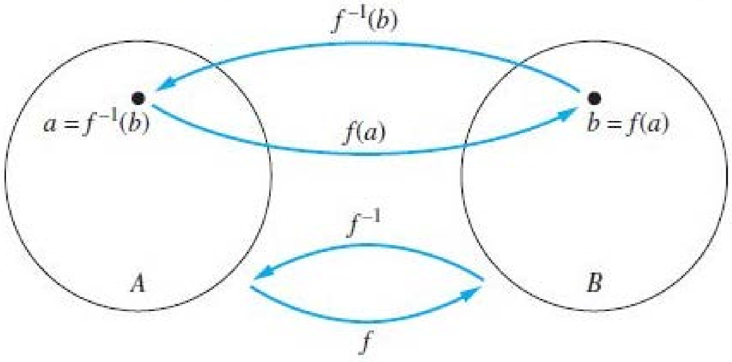
To show that f is not surjective Find a particular $y \in B$ such that $f(x) \neq y$ for all $x \in A$.

EXAMPLE Determine whether the functions are one-to-one, onto, or both?

Correspondences	One-to-one	Onto	Note

Inverse Functions

The Function f^{-1} Is the Inverse of Function f.

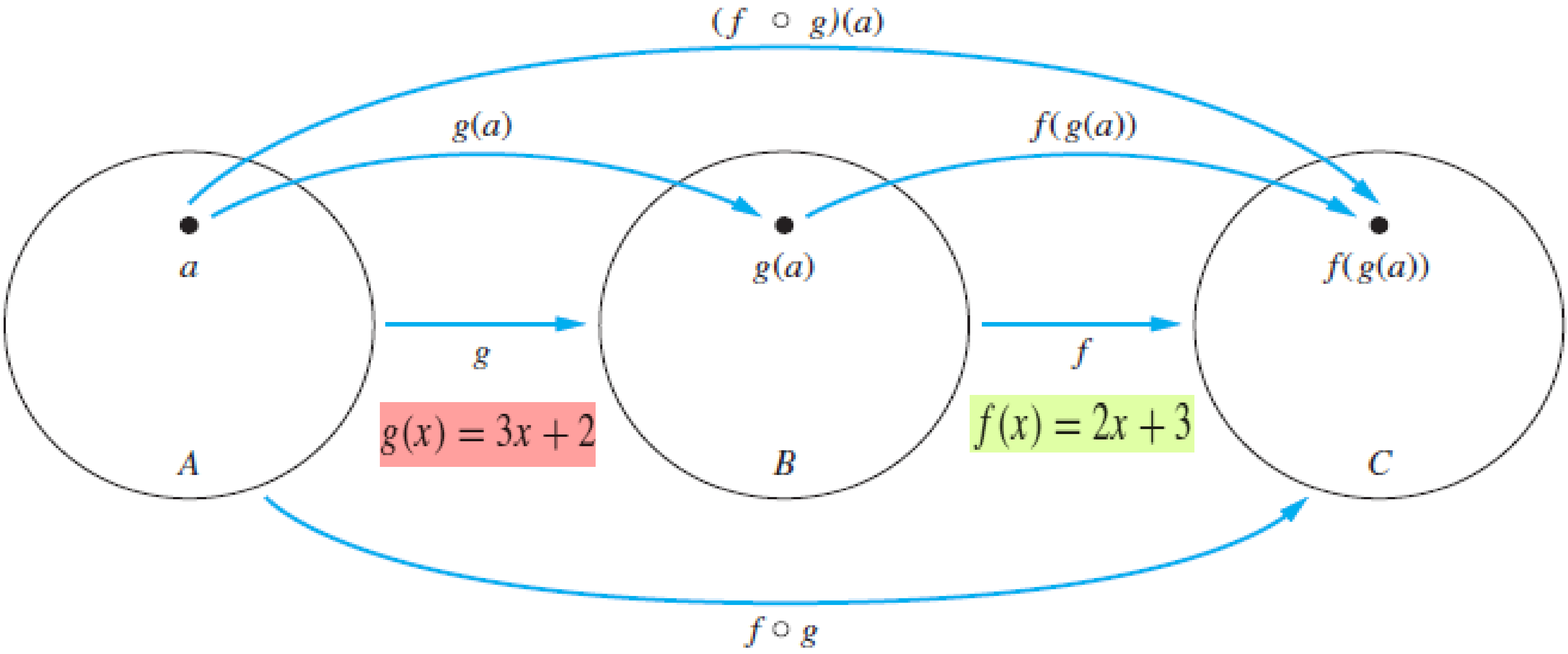


Let f be a one-to-one correspondence from the set A to the set B.
The inverse function of f is the function that assigns to an element b belonging to B the unique element a in A such that f (a) = b.
If a function f is not a one-to-one correspondence, we cannot define an inverse function of f .

Let f be the function from $\mathbf{R}$ to $\mathbf{R}$ with $f(x) = x^2$. Is f invertible?

Let $f : \mathbf{Z} \to \mathbf{Z}$ be such that $f(x) = x + 1$. Is f invertible, and if it is, what is its inverse?

Compositions of Functions



$g(x) = 3x + 2$

$f(x) = 2x + 3$

$$(f \circ g)(x) = f(g(x)) = f(3x+2) = 2(3x+2) + 3 = 6x + 7$$

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2.3 Sequences and Summations

DEFINITION

A *sequence* is a function from a subset of the set of integers (usually either the set $\{0, 1, 2, \ldots\}$ or the set $\{1, 2, 3, \ldots\}$) to a set S. We use the notation a_n to denote the image of the integer n. We call a_n a *term* of the sequence.

$$1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots.$$

Fibonacci sequence: *0, 1, 1, 2, 3, 5, 8, 13,…*

Ex. If $a_n = 2n$, find $\{a_n\}$

arithmetic progression

An *arithmetic progression* is a sequence of the form

$$a, a+d, a+2d, \ldots, a+nd, \ldots$$

where the *initial term* a and the *common difference* d are real numbers.

1. Let $a=5$ and $d=2$. Find the first 5 terms of $\{a_n\}$

2. Find a and d for the sequence $\{7, 4, 1, -2, \ldots\}$

$$-1, 3, 7, 11, \ldots$$

DEFINITION

A *geometric progression* is a sequence of the form

$$a, ar, ar^2, \ldots, ar^n, \ldots$$

where the *initial term* a and the *common ratio* r are real numbers.

1. Let $a = 4$ and $r = 3$. Find the first 5 terms of the geometric progression.

2. Find a and r for $\{a_n\} = \{3, \frac{3}{2}, \frac{3}{4}, \frac{3}{8} \ldots\}$

$1, -1, 1, -1, 1, \ldots$

$2, 10, 50, 250, 1250, \ldots$

$6, 2, \frac{2}{3}, \frac{2}{9}, \frac{2}{27}, \ldots$

Summations

$$\sum_{j=m}^{n} a_j \,, \qquad \sum\nolimits_{j=m}^{n} a_j \,, \qquad \text{or} \qquad \sum\nolimits_{m \le j \le n} a_j$$

$$a_m + a_{m+1} + \cdots + a_n$$

$$\sum_{s \in S} f(s)$$

Some Useful Summation Formulae.

Sum	*Closed Form*
$\sum_{k=1}^{n} k$	$\frac{n(n+1)}{2}$
$\sum_{k=1}^{n} k^2$	$\frac{n(n+1)(2n+1)}{6}$
$\sum_{k=1}^{n} k^3$	$\frac{n^2(n+1)^2}{4}$
$\sum_{k=0}^{n} ar^k \ (r \neq 0)$	$\frac{ar^{n+1} - a}{r-1}, r \neq 1$

Summations

$\sum_{j=m}^{n} a_j$, $\sum_{j=m}^{n} a_j$, or $\sum_{m \le j \le n} a_j$

$a_m + a_{m+1} + \cdots + a_n$

$\sum_{s \in S} f(s)$

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EXAMPLE

Find $\sum_{k=50}^{100} k^2$.

$= 297{,}925.$

10 20 40 80 160 320 640

5 6 7 8 ,... 100

3 6 11 18 27 38 51

-2 5 24 61 122

Sum	Closed Form
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$\sum_{k=1}^{n} k^3$	$\frac{n^2(n+1)^2}{4}$
$\sum_{k=0}^{n} ar^k \ (r \neq 0)$	$\frac{ar^{n+1}-a}{r-1}, r \neq 1$

Cardinality

- Cardinality = number of elements in a set.
- The sets A and B have the same cardinality if and only if there is a one-to-one correspondence from A to B
- A set that is either finite or has the same cardinality as the set of positive integers is called ***countable***.
- A set that is not countable is called ***uncountable.***

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Excel: The set of all finite strings over the alphabet of lowercase letters is countable.

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