DISCRETE MATHEMATICS–MAD101

- Textbook: Discrete Mathematics and Its Applications [7th Edition] - Kenneth H. Rosen
- 03 Assignments: 30% and 03 Progress tests: 30%
- Final exam (50 multiple choice questions in 60 minutes): 40%
- To pass:
  - Must attend more than 80% of contact hours (absent at most 6 slots)
  - Every on-going assessment component is positive
  - FE >= 4, Average(3A,3P,FE) >= 5
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CIP (10 weeks)

Slot	Chapter	Topic -Content	Assessment	Duration-#MC
1	The Foundations: Logic and Proofs	Introduction & 1.1 Propositional Logic		
2		1.2 Propositional Equivalences		
3		1.3 Predicates and Quantifiers		
		1.4 Nested Quantifiers		
4		1.5 Rules of Inference		
5	Basic Structures: Sets, Functions, Sequences	2.1 Sets	Assignment-1	~ 60'–30q
6		2.2 Set operations		
7		2.3 Functions		
8		2.4 Sequences and Summations		
9	The Fundamentals: Algorithms, the Integers	3.1 Algorithms		
10		3.2 The Growth of Functions		
11		3.3 Compexity of Algorithms		
		3.4 The Integers and Division	ProgressTest-1	~ 60'–30q
12		3.5 Primes, Greatest Common Divisors		
13		3.6 Intergers and Algorithms		
14	Induction and Recursion	4.1 Mathematical Induction		
15		4.3 Recursive Definitions		
16		4.4 Recursive Algorithms		
17	Counting	5.1 The Basics of Counting		
18		7.1,7.3 Advanced Counting	Assignment-2	~ 60'–30q
19	Graphs	9.1 Graphs Terminology		
		9.2 Special Types of Graphs		
20		9.3 Representing Graphs, Isomorphism		
21		9.4 Connectivity		
22		9.5 Euler and Hamilton Paths		
23		9.6 Shortest-Path Problems	ProgressTest-2	~ 60'–30q
24	Trees	10.1 Introduction to Trees		
25		10.2 Applications of Trees		
26		10.3 Tree Traversal		
27		10.4 Spanning Trees		
28		10.5 Minimum Spanning Trees	Assignment-3	~ 60'–30q
29	Review all chaps by solving ~ 50q MC			
30	Review (cont'd) all chaps		ProgressTest-3	60'–50q

DISCRETE MATH - introduction

- Discrete mathematics is the part of mathematics devoted to the study of discrete (as opposed to continuous) objects.
- Example of of discrete objects are integers, steps taken by a computer program, distinct paths to travel from point A to point B, etc.
- A course in discrete math provides the mathematical background needed for all subsequent courses in computer science.

DISCRETE MATH – introduction

- How many ways are there to choose a valid password on a computer system?
- Is there a link between two computers in a network?
- How can a list of integers be sorted so that the integers are in increasing order?
- How many steps are required to do such a sorting?
- How can I identify spam email messages?
- What is the shortest path between two cities using a transportation system?
- How can I encrypt a message so that no uninterested recipient can read it?

The Foundations: Logic and Infererence

1. **Propositional logic** **[slot 1]**
2. **Propositional equivalences** **[2]**
3. Predicates and quantifiers **[3]**
4. Nested quantifiers and negations **[3]**
5. Rules of Inference for Propositional Logic **[4]**
6. Rules of Inference for Quantified Statements **[4]**

1.1. PROPOSITIONs

1.2. CONNECTTIVEs

1.3. TRUTH TABLES

1.4. TRANSLATING

1.1. PROPOSITIONs

A proposition is a declarative sentence that is either true or false.

Examples of propositions:

a) The Moon is made of green cheese.
b) Trenton is the capital of New Jersey.
c) Toronto is the capital of Canada.
d) $1 + 0 = 1$
e) $0 + 0 = 2$

1.1. PROPOSITIONs

A *proposition* is a declarative sentence that is either true or false.

Examples that are not propositions.

a) Sit down!

b) What time is it?

c) $x + 1 = 2$

d) $x + y = z$

CONSTRUCT PROPOSITIONs

Propositional Variables: $p, q, r, s, \ldots$

The proposition that is always true is denoted by T and the proposition that is always false is denoted by F.

Compound Propositions; constructed from logical connectives and other propositions

- Negation $\neg$
- Conjunction $\wedge$
- Disjunction $\vee$
- Implication $\rightarrow$
- Biconditional $\leftrightarrow$

1.2. CONNECTTIVEs: **NEGATION**

The *negation* of a proposition p is denoted by $\neg p$ (not p) and has this truth table:

p	$\neg p$

Example: If p denotes "The earth is round.", then $\neg p$ denotes "It is not the case that the earth is round," or more simply "The earth is not round."

Practice on NEGATION

Determine if the following statements are propositions. If so, find the negations.

a. My dog is the cutest dog.

My dog is not the cutest dog

b. Discrete math is the hardest class you will ever take.

Discrete math is not the hardest class you will ever take.

c. Is class over yet?

This is not a proposition so it cannot be negated

CONNECTTIVEs: **CONJUNCTION**

The conjunction of propositions p and q is denoted by $p \wedge q$ "p *and* q" and has this truth table:

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Example: If p denotes "I am at home." and q denotes "It is raining." then $p \wedge q$ denotes "I am at home and it is raining." However, we may also use "but" instead of "and". "I am home, but it is raining".

CONNECTTIVEs: **DISJUNCTION**

The disjunction of propositions p and q is denoted by $p \vee q$ (p or q) and has this truth table:

p	q	$p \vee q$

Example: If p denotes "I am at home." and q denotes "It is raining." then $p \vee q$ denotes "I am at home or it is raining." But "or" has it's own issues…

"OR" in ENGLISH

In English "or" has two distinct meanings.

- "Inclusive Or" - In the sentence "Students who have taken CS202 or Math120 may take this class," we assume that students need to have taken one of the prerequisites, but may have taken both. This is the meaning of disjunction. For $p \vee q$ to be true, either one or both of *p* and *q* must be true.

- "Exclusive Or" - When reading the sentence "Soup or salad comes with this entrée," we do not expect to be able to get both soup and salad. This is the meaning of Exclusive Or (Xor). In $p \oplus q$, one of *p* and *q* must be true, but not both. That is, exactly one condition must be true. The truth table for $\oplus$ is:

p	q	$p \oplus q$
T	T	
T	F	
F	T	
F	F	

CONNECTIVEs: **IMPLICATION**

If p and q are propositions, then $p \rightarrow q$ is a conditional statement or implication which is read as "if p, then q" and has this truth table:

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

More on **IMPLICATION**

In $p \to q$, p is the hypothesis (antecedent or premise) and q is the conclusion (or consequence).

In $p \to q$ there does not need to be any connection between the antecedent or the consequent. The "meaning" of $p \to q$ depends only on the truth values of p and q.

These implications are perfectly fine, but would not be used in ordinary English.

- "If the moon is made of green cheese, then I have more money than Bill Gates. "

"If Juan has a smartphone, then $2 + 3 = 6$"

"If Juan has a smartphone, then $2 + 3 = 5$"

IMPLICATIONS: CONVERSE, INVERSE, CONTRA-POSITIVE

From $p \rightarrow q$ we can form new conditional statements .

- $q \rightarrow p$ is the converse of $p \rightarrow q$
- $\neg p \rightarrow \neg q$ is the inverse of $p \rightarrow q$
- $\neg q \rightarrow \neg p$ is the contrapositive of $p \rightarrow q$

Example: Find the converse, inverse, and contrapositive of "It raining is a sufficient condition for my not going to town."

converse: If I do not go to town, then it is raining.

inverse: If it is not raining, then I will go to town.

contrapositive: If I go to town, then it is not raining.

Pratice on IMPLICATION

Give the converse, inverse and contrapositive of the conditional statement:

Prof. B is happy when you get your homework done on time.

Converse $q \to p$

If Prof. B is happy, then you got your homework done on time

Inverse $\neg p \to \neg q$

If you did not get your homework done on time, Prof. B is not happy

Contrapositive $\neg q \to \neg p$

If Prof. B is not happy, then you did not get your homework done on time.

CONNECTTIVEs: **BICONDITIONAL**

If p and q are propositions with the same truth value, then we can form the *biconditional* proposition (or bi-implication) $p \leftrightarrow q$, read as "p iff q." Recall that "iff" in mathematics means "if and only if". The biconditional $p \leftrightarrow q$ denotes the proposition with this truth table:

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

If p denotes "I am at home." and q denotes "It is raining." then $p \leftrightarrow q$ denotes "I am at home if and only if it is raining."

CONNECTTIVEs: **BICONDITIONAL**

So $p \leftrightarrow q \equiv (p \rightarrow q) \land (q \rightarrow p)$. Recall that " $\equiv$ " means equivalent.

p	q	$p \rightarrow q$	$q \rightarrow p$	$(p \rightarrow q) \land (q \rightarrow p)$	$p \leftrightarrow q$
T	T				T
T	F				F
F	T				F
F	F				T

Order of operations

Operator	Precedence
$\neg$	1
$\wedge$	2
$\vee$	3
$\rightarrow$	4
$\leftrightarrow$	5

1.3. TRUTH TABLES for COMPOUND PROPOSITIONS

Construct a truth table for $p \vee q \rightarrow \neg r$

1. First, construct columns for each proposition; p, q, r (these may be referred to as atomic propositions)

2. Next, create a column for each compound proposition; $p \vee q$, $\neg r$

3. Lastly, create a column for the final compound proposition

Create this truth table $p \vee q \rightarrow \neg r$

p	q	r	$p \vee q$	$\neg r$	$p \vee q \rightarrow \neg r$
T	T	T			
T	T	F			
T	F	T			
T	F	F			
F	T	T			
F	T	F			
F	F	T			
F	F	F			

TRUTH TABLE: Practice

Create a truth table for $(p \vee \neg q) \rightarrow (p \wedge q)$.

p	q	$\neg q$	$p \vee \neg q$	$p \wedge q$	$(p \vee \neg q) \rightarrow (p \wedge q)$
T	T				
T	F				
F	T				
F	F				

1.4. TRANSLATING ENGLISH SENTENCES

Steps to convert an English sentence to a statement in propositional logic using:

"If I go to Harry's or to the country, I will not go shopping."

Step 1: Identify atomic propositions and represent using propositional variables.

Step 2: Determine appropriate logical connectives

TRANSLATING ENGLISH SENTENCES

following ways to express this conditional statement:

$p \rightarrow q$

"if p, then q"	"p implies q"
"if p, q"	"p only if q"
"p is sufficient for q"	"a sufficient condition for q is p"
"q if p"	"q whenever p"
"q when p"	"q is necessary for p"
"a necessary condition for p is q"	"q follows from p"
"q unless $\neg p$"	

There are some other common ways to express $p \leftrightarrow q$:

"p is necessary and sufficient for q"
"if p then q, and conversely"
"p iff q."

TRANSLATING ENGLISH SENTENCES

Convert each sentence into propositional logic.

a. You can access the Internet from campus only if you are a computer science major or you are not a freshman.

$p \rightarrow (q \vee \neg r)$

b. The automated reply cannot be sent when the file system is full.

$q \rightarrow \neg p$

The Foundations: Logic and Infererence

1. Propositional logic [1]
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2.1. TAUTOLOGIES,

2.2. LOGICAL EQUIVALENCES

2.3. CONSTRUCTING NEW LOGICAL EQUIVALENCE

2.1. TAUTOLOGIES, CONTRADICTIONS and CONTINGENCIES

Statements that produce propositions with the same truth value as a given compound proposition are used in the construction of mathematical arguments.

A tautology is a proposition which is always true. Example: $p \vee \neg p$

p	$\neg p$	$p \vee \neg p$	$p \wedge \neg p$
T	F	T	F
F	T	T	F

A contradiction is a proposition which is always false. Example: $p \wedge \neg p$

A contingency is a proposition which is neither a tautology nor a contradiction. Example: p.

2.2. LOGICAL EQUIVALENCES

Two compound propositions p and q are logically equivalent if $p \leftrightarrow q$ is a tautology, or if they have the same truth value in all possible cases.

We write this as $p \equiv q$ where p and q are compound propositions.

This truth table shows that $\neg p \vee q$ is equivalent to $p \to q$.

p	q	$\neg p$	$\neg p \vee q$	$p \to q$

2.2. LOGICAL EQUIVALENCES

if $\neg(p \wedge q)$ and $\neg p \vee \neg q$ are logically equivalent.

p	q	$p \wedge q$	$\neg(p \wedge q)$	$\neg p$	$\neg q$	$\neg p \vee \neg q$
T	T			F	F	
T	F			F	T	
F	T			T	F	
F	F			T	T	

DE MORGAN'S LAWS

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

Augustus De Morgan
1806-1871

Use De Morgan's laws to express the negation of:

"Professor Brehm has a dog and she has a pool".

Therefore, we can express the negation of our original statement as:

"Professor Brehm does not have a dog or she does not have a pool".

TABLE 6 Logical Equivalences.

Equivalence	*Name*
$p \wedge \mathbf{T} \equiv p$ $p \vee \mathbf{F} \equiv p$	Identity laws
$p \vee \mathbf{T} \equiv \mathbf{T}$ $p \wedge \mathbf{F} \equiv \mathbf{F}$	Domination laws
$p \vee p \equiv p$ $p \wedge p \equiv p$	Idempotent laws
$\neg(\neg p) \equiv p$	Double negation law
$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$	Commutative laws

Equivalence	*Name*
$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative laws
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive laws
$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	De Morgan's laws
$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$	Absorption laws
$p \vee \neg p \equiv \mathbf{T}$ $p \wedge \neg p \equiv \mathbf{F}$	Negation laws

Logical Equivalences Involving Conditional Statements.

1 $p \to q \equiv \neg p \vee q$

2 $p \to q \equiv \neg q \to \neg p$

3 $p \vee q \equiv \neg p \to q$

4 $p \wedge q \equiv \neg(p \to \neg q)$

5 $\neg(p \to q) \equiv p \wedge \neg q$

6 $(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$

7 $(p \to r) \wedge (q \to r) \equiv (p \vee q) \to r$

8 $(p \to q) \vee (p \to r) \equiv p \to (q \vee r)$

9 $(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$

Logical Equivalences Involving Biconditional Statements.

10 $p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$

11 $p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$

12 $p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$

13 $\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$

2.3. CONSTRUCTING NEW LOGICAL EQUIVALENCE

We can show that two expressions are logically equivalent by developing a series of logically equivalent statements.

To prove that $A \equiv B$ we produce a series of equivalences beginning with A and ending with B.

$$
\begin{aligned}
A &\equiv A_1 \\
A_1 &\equiv A_2 \\
&\vdots \\
A_n &\equiv B
\end{aligned}
$$

EQUIVALENCE PROOFs

Show that $\neg(p \vee (\neg p \wedge q)) \equiv \neg p \wedge \neg q$

$\neg(p \vee (\neg p \wedge q))$

$\equiv$	$\neg p \wedge \neg(\neg p \wedge q)$	by the second De Morgan law
$\equiv$	$\neg p \wedge [\neg(\neg p) \vee \neg q]$	by the first De Morgan law
$\equiv$	$\neg p \wedge (p \vee \neg q)$	by the double negation law
$\equiv$	$(\neg p \wedge p) \vee (\neg p \wedge \neg q)$	by the second distributive law
$\equiv$	$F \vee (\neg p \wedge \neg q)$	because $\neg p \wedge p \equiv F$
$\equiv$	$(\neg p \wedge \neg q) \vee F$	by the commutative law for disjunction
$\equiv$	$(\neg p \wedge \neg q)$	by the identity law for **F**

EQUIVALENCE PROOFs

Show using an equivalence proof that $(p \land q) \longrightarrow (p \lor q)$ is a tautology.

$$(p \land q) \rightarrow (p \lor q)$$

$\equiv \neg(p \land q) \lor (p \lor q)$	by truth table for $\rightarrow$
$\equiv (\neg p \lor \neg q) \lor (p \lor q)$	by the first De Morgan law
$\equiv (\neg p \lor p) \lor (\neg q \lor q)$	by associative and commutative laws for disjunction
$\equiv T \lor T$	by truth tables
$\equiv T$	by the domination law

Use logical equivalences to show $\neg(\neg p \lor q)$ and $\neg q \land p$ are equivalent.

$$\neg(\neg p \lor q)$$

$\equiv \neg\neg p \land \neg q$	The first De Morgan's Law
$\equiv p \land \neg q$	Double Negation Law
$\equiv \neg q \land p$	Commutative Law