17/

$\int f(x)\,g'(x)\,dx = f(x)g(x) - \int g(x)\,f'(x)\,dx$

$\int_a^b f(x)\,g'(x)\,dx = f(x)\,g(x)\Big]_a^b - \int_a^b g(x)\,f'(x)\,dx$

ex1:

$\int x\cos x\,dx = x\sin x - \int \sin x\,dx$

$= x\sin x + \cos x + c$

ex2:

$\int_0^{\pi} x^2 \sin x\,dx = -x^2\cos x\Big]_0^{\pi} + \int_0^{\pi} 2x\cos x\,dx$

$= -x^2\cos x\Big]_0^{\pi} + 2\left(x\sin x\Big]_0^{\pi} + \cos x\Big]_0^{\pi}\right)$

$= \pi^2 \cancel{(+2)} - 4$

18/

1) $\int \cos x\,dx = \sin x + c$

2) $\int \sin x\,dx = -\cos x + c$

3) $\int \sec^2 x\,dx = \tan x + c$

4) $\int \csc^2 x\,dx = -\cot x + c$

5) $\int \sec x \tan x\,dx = \sec x\,dx$

6) $\int \csc x \cot x\,dx = -\csc x + c$

7) $\int \tan x\,dx = \ln|\sec x| + c$

8) $\int \cot x\,dx = \ln|\sin x| + c$

9) $\int \sec x\,dx = \ln|\sec x + \tan x| + c$

10) $\int \csc x\,dx = \ln|\csc x - \cot x| + c$

19/

-) To use trigonometric substitution, we rewrite integrals as trigonometric integrals

-) ex: $\int \sqrt{9-x^2}\,dx$

let $x^2 = 3\sin\theta$ ($-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$)

$\Rightarrow \int \sqrt{9-x^2} = \int \sqrt{9-9\sin^2\theta} \Rightarrow \int \sqrt{1-\sin^2\theta} = 3\int \cos\theta = 3\sin x\theta + C$

let $x = 3\sin\theta \Rightarrow dx = 3\cos\theta\, d\theta$

$\Rightarrow \int \sqrt{9-x^2} = \int \sqrt{9-9\sin^2\theta}\; 3\cos\theta\, d\theta$

$= \int 3\sqrt{1-\sin^2\theta}\; 3\cos\theta\, d\theta$

$= 9\int \cos^2\theta\, d\theta$

$= 9 \int \frac{1}{2} + \frac{1}{2}\cos 2\theta\, d\theta$

$= \frac{9}{2}\theta + \frac{9}{4}\sin(2\theta) + C$

20/

-) To use partial fractions, we decompose rational functions into sums of simpler, more easily intergrated rational functions

$\int \frac{x^2+3x+5}{x+1}\,dx = \int (x+2+\frac{3}{x+1})\,dx$

$= \frac{x^2}{2} + 2x + 3\ln|x+1| + C$

21/

- Midpoint rule: $\int_a^b f(x)\,dx \approx \Delta x\,[f(\bar{x}_1) + f(\bar{x}_2) + \dots + f(\bar{x}_n)]$

- Trapezoidal rule: $\int_a^b f(x)\,dx \approx \Delta x\,[f(x_0) + 2f(x_1) + \dots + 2f(x_{n-1}) + f(x_n)]$

- Simpson rule: $\int_a^b f(x)\,dx \approx \frac{\Delta x}{3}[f(x_0) + 4f(x_1) + 2f(x_2) + \dots + 4f(x_{n-1})$

23/ b) $\int \frac{1}{x^4+1}dx = \int \frac{1}{(x^2-x\sqrt{2}+1)(x^2+x\sqrt{2}+1)}dx = \int \frac{x+\sqrt{2}}{\sqrt{8}(x^2-x\sqrt{2}+1)} - \frac{x-\sqrt{2}}{2\sqrt{8}(x^2+x\sqrt{2}+1)}$

$\Rightarrow \frac{1}{x^4+1}$ can be integrated using partial fraction

$\Rightarrow$ The statement is false

1) $[x_a)]$

Simpson's rule is the best

ex: $\int_1^2 \frac{1}{x}dx = \cancel{(x)}\ 0.6931$

$M_4 \approx \cancel{0.3133}\ 0.9193$

$T_4 \approx \cancel{0.4530}\ 0.6132$

$S_4 \approx \cancel{0.4801}\ 0.6933$

22/ Improper integrals:

- Type 1: $\int_a^{\infty} f(x)\,dx = \lim_{t\to\infty} \int_a^t f(x)\,dx$

$\int_{-\infty}^{b} f(x)\,dx = \lim_{t\to-\infty} \int_t^b f(x)\,dx$

ex: $\int_1^{\infty} x^2\,dx = \lim_{t\to\infty} \int_1^t x^2\,dx = \lim_{t\to\infty} t^2-1 = \infty$

- Type 2: $\int_a^b f(x)\,dx = \lim_{t\to b^-} \int_a^t f(x)\,dx$

$\int_a^b f(x)\,dx = \lim_{t\to a^+} \int_t^b f(x)\,dx$

ex: $\int_1^2 x^2\,dx = \lim_{t\to 1^+} \int_t^2 x^2\,dx = \lim_{t\to 1^+} 4-t^2 = 3$

23/

a) $\int e^x \sin(x)\,dx = -e^x\cos x + \int e^x \cos x\,dx$

$= -e^x\cos x + e^x \sin x - \int e^x \sin x\,dx$

$\Rightarrow \int e^x \sin x\,dx = \frac{1}{2}e^x(\sin x - \cos x) + C$

So the statement is false

(b) Since $\frac{1}{x^4+1}$ ~~cannot be transformed into (t.dt)~~ $\Rightarrow$ It cannot

$\frac{1}{x^4+1}$ in cannot be further decomposed

T.T BOOK

(be integrated using partial fractions. So the statement is true)

c) ~~the statement is true be)~~

$\int_1^4 x^{\theta} = 7.5$

$L_2 = 5.25$

$L_3 = 6$

$L_4 = 6.375$

$\Rightarrow$ In numerical integration, increasing the number of points decreases the error

So the statement is true

d) The statement is false because definite integral, when integrated by parts can yield a real number

Ex: $\int_0^{\pi} x^2 \sin x \, dx = -x^2 \cos x]_0^{\pi} + \int_0^{\pi} 2x \cos x \, dx$

$= -x^2 \cos x]_0^{\pi} + 2 (x \sin x]_0^{\pi} + \cos x]_0^{\pi})$

$= \pi^2 - 4$ (Not an integral)

2 Q1

a) $\int x^2 \sin(4x) \, dx$

$du = \sin(4x) dx \Rightarrow u = \frac{-\cos(4x)}{4}$

$v = x^2 \Rightarrow dv = 2x$

$\int x^2 \sin(4x) dx = \frac{-1}{4} [x^2 \cos(4x)] + \frac{1}{2} \int \cos(4x) \, x \, dx$

$= \frac{-1}{4} [x^2 \cos(4x)] + \frac{1}{2} [\frac{x \sin(4x)}{4} + \frac{\cos(4x)}{16}] + c$

$= \frac{-1}{4} x^2 \cos(4x) + \frac{1}{32} [4x \sin(4x) + \cos(4x)] + c$

T.T BOOK

So $\int x^2 \sin(4x)\,dx = \frac{-1}{4} x^2 \cos(4x) + \frac{1}{32}\left[4x \sin(4x) + \cos(4x)\right] + C$

b) $\int \frac{1}{x^2\sqrt{x^2+16}}\,dx$

Let $x = 4\tan(u) \Rightarrow u = \arctan\left(\frac{x}{4}\right)$

$dx = 4\sec^2 u\,du$

$\Rightarrow \int \frac{1}{x^2\sqrt{x^2+16}}\,dx = \int \frac{\sec^2 u}{4\tan^2 u\,\sqrt{16\tan^2 u+16}}\,du = \frac{1}{16}\int \frac{\sec u}{\tan^2 u}\,du$

$= \frac{1}{16}\int \frac{\cos u}{\sin^2 u}\,du$

Let $v = \sin(u) \Rightarrow \frac{dv}{du} = \cos u \Rightarrow du = \frac{1}{\cos u}\,dv$

$\frac{1}{16}\int \frac{\cos u}{\sin^2 u}\,du = \frac{1}{16}\int \frac{1}{v^2}\,dv = \frac{-1}{16v} = \frac{-1}{16\sin u} = \frac{-1}{16\sin\left(\arctan\left(\frac{x}{4}\right)\right)}$

So $\int \frac{1}{x^2\sqrt{x^2+16}}\,dx = \frac{-1}{16\sin\left(\arctan\frac{x}{4}\right)} + C$

c) $\int \sqrt{x}\,\ln(x)\,dx$

$u = \ln(x) \Rightarrow du = \frac{1}{x}$

$dv = \sqrt{x} \Rightarrow v = \frac{2x^{\frac{3}{2}}}{3}$

$\int \frac{1}{x^2\sqrt{x^2+16}}\,dx = \frac{2x^{\frac{3}{2}}}{3}\ln(x) - \int \left(\cancel{\frac{2x^{\frac{3}{2}}}{3}}\right)\frac{2\sqrt{x}}{3}\,dx$

$= \frac{2x^{\frac{3}{2}}}{3}\ln(x) - \frac{2}{3}\cdot\frac{2x^{\frac{3}{2}}}{3}$

$= \frac{2x^{\frac{3}{2}}}{3}\ln(x) - \frac{4x^{\frac{3}{2}}}{9}$

$= \frac{2x^{\frac{3}{2}}(3\ln(x) - 2)}{9} + C$

So $\int \sqrt{x}\,\ln(x)\,dx = \frac{2x^{\frac{3}{2}}(3\ln(x)-2)}{9} + C$

d) $\int \frac{3x}{x^3+2x^2-5x-6}\,dx = 3\int \frac{x}{(x-2)(x+1)(x+3)}\,dx$

$= 3\int \frac{-3}{10(x+3)} + \frac{1}{6(x+1)} + \frac{2}{15(x-2)}\,dx$

$= 3\left[\frac{-3}{10}\ln|x+3| + \frac{1}{6}\ln|x+1| + \frac{2}{15}\ln|x-2|\right] + C$

$= \dfrac{-9\ln|x+3| + 5\ln|x+1| + 4\ln|x-2|}{10} + C$

e) $\int \dfrac{x^5}{(4x^2+4)^{\frac{5}{2}}}\,dx = \dfrac{1}{32}\int \dfrac{x^5}{(x^2+1)^{\frac{5}{2}}}$

let $x = \tan u \Rightarrow u = \arctan x$

$dx = \sec^2 u\,du$

$\int \dfrac{x^5}{(x^2+1)^{\frac{5}{2}}}\,dx = \dfrac{1}{32}\int \dfrac{\tan^5 u \cdot \sec^2 u}{\sec^5 u}\,du = \dfrac{1}{32}\int \dfrac{\tan^5 u}{\sec^3 u}\,du = \dfrac{1}{32}\int \dfrac{\sin^5 u}{\cos^2 u}\,du$

$= \dfrac{1}{32}\int \dfrac{(\cos^2 u - 1)^2}{\cos^2 u}\sin x\,du$

$v = \cos u \Rightarrow \dfrac{dv}{du} = -\sin(u) \Rightarrow du = \dfrac{-1}{\sin x}\,dv$

$\Rightarrow \dfrac{1}{32}\int \dfrac{(\cos^2 u-1)^2}{\cos^2 u}\sin u\,du = \dfrac{1}{32}\left(-\dfrac{\cos^3 u}{3} + 2\cos u + \dfrac{1}{\cos u}\right) + C$

$= \dfrac{1}{32}\left(\dfrac{\cos^3(\cos(\arctan$

$= \dfrac{-\cos^3(\arctan x) + 48\cos^2(\arctan x) + 96}{96\cos(\arctan x)} + C$

So $\int \dfrac{x^5}{(4x^2+4)^{\frac{5}{2}}}\,dx = \dfrac{-\cos^3(\arctan x) + 48\cos^2(\arctan x) + 96}{96\cos(\arctan x)} + C$

25/

a) $\int \sin^2 x \cos^2 x\,dx = \int \sin^2 x(1-\sin^2 x)\,dx = \int \sin^2 x - \sin^4 x\,dx$

$= \int \sin^2 x\,dx - \int \sin^4 x\,dx$

(1) $\int \sin^2 x\,dx = \int \dfrac{1-\cos(2x)}{2}\,dx = \dfrac{1}{2}\left[x - \dfrac{1}{2}\sin(2x)\right] = \dfrac{x}{2} - \dfrac{\sin x\cos x}{2}$ (1)

$u = \sin x \Rightarrow du = \cos x\,dx$

$\int \sin^2 x\,dx = \int u^2\,du = \dfrac{u^3}{3}$ $\int \dfrac{u^2}{\cos x}\,du =$

(2) $\int \sin^4 x\,dx = -\dfrac{\cos x\sin^3 x}{4} + \dfrac{3}{4}\int \sin^2 x\,dx = -\dfrac{\cos x\sin^3 x}{4} - \dfrac{3\sin x\cos x}{8} + \dfrac{3x}{8}$ (2)

$(1)-(2) = \dfrac{\cos(x)\sin^3(x)}{4} - \dfrac{\sin x\cos x}{8} + \dfrac{x}{8} + C$

T.T BOOK

So $\int \sin^2 x \cos^2 x = \frac{\cos x \sin^3 x}{4} - \frac{\sin x \cos x}{8} + \frac{x}{8} + c$

b) $\int x^3 \sqrt{x^2+2}\, dx$

$u = x^2+2 \Rightarrow \frac{du}{dx} = 2x \Rightarrow dx = \frac{1}{2x} du$

$\int x^3 \sqrt{x^2+2}\, dx = \frac{1}{2} \int u^{\frac{3}{2}} - 2\sqrt{u}\, du$

$= \frac{1}{2} \left(\frac{2u^{\frac{5}{2}}}{5} - \frac{4u^{\frac{3}{2}}}{3} \right) + c$

$= \frac{u^{\frac{5}{2}}}{5} - \frac{2u^{\frac{3}{2}}}{3} + c = \frac{(x^2+2)^{\frac{5}{2}}}{5} - \frac{2(x^2+2)^{\frac{3}{2}}}{3} + c$

$= \frac{(x^2+2)^{\frac{3}{2}}(3x^2-4)}{15} + c$

So $\int x^3 \sqrt{x^2+2}\, dx = \frac{(x^2+2)^{\frac{3}{2}}(3x^2-4)}{15} + c$

c) $\int \frac{3x^2+1}{x^4-2x^3-x^2+2x} dx = \int \frac{3x^2+1}{(x-2)(x-1)x(x+1)} dx$

$= \int \frac{-2}{3(x+1)} + \frac{1}{2x} - \frac{2}{x-1} + \frac{13}{6(x-2)} dx$

$= -\frac{2}{3} \ln|x+1| + \frac{1}{2} \ln|x| - 2 \ln|x-1| + \frac{13}{6} \ln|x-2| + c$

$= \frac{-4 \ln|x+1| + 3 \ln|x| - 12 \ln|x-1| + 13 \ln|x-2|}{6} + c$

So $\int \frac{3x^2+1}{x^4-2x^3-x^2+2x} dx = \frac{-4 \ln|x+1| + 3 \ln|x| - 12 \ln|x-1| + 13 \ln|x-2|}{6} + c$

d) $\int \frac{1}{x^4+4} dx = \int \frac{1}{(x^2-2x+2)(x^2+2x+2)} dx = \int \frac{x+2}{8(x^2+2x+2)} - \frac{x-2}{8(x^2-2x+2)} dx$

$= \frac{1}{8} \left(\int \frac{x+2}{x^2+2x+2} dx - \int \frac{x-2}{x^2-2x+2} dx \right)$

*) $\left(\int \frac{x+2}{x^2+2x+2} dx \right)$ $\int \frac{x+2}{x^2+2x+2} dx = \frac{\ln|x^2+2x+2|}{2} + \arctan(x+1) + c$ (1)

$\int \frac{x-2}{x^2+2x+2} dx = \frac{\ln|x^2-2x+2|}{2} - \arctan(x-1)$ (2)

$\frac{1}{8}((1)-(2)) = \frac{\ln|x^2+2x+2|}{16} + \frac{\arctan(x+1)}{8} - \frac{\ln(x^2-2x+2)}{16} + \frac{\arctan(x-1)}{8} + c$

8) $\int \frac{1}{x^4+4} = \frac{\ln(x^2+2x+2) - \ln(x^2-2x+2) + 2\arctan(x+1) + 2\arctan(x-1)}{16}$

f) $\int$ [illegible] dx)

26|

a) $\int_1^2 \sqrt{x^5+2}\, dx$

$M_4 \approx \cancel{16.702}\ 3.312$

$T_4 \approx 3.354$

$S_4 \approx 3.326$

b) $\int_0^{\sqrt{\pi}} e^{-\sin(x^2)}\, dx$

$M_4 \approx \cancel{8.712}\ 1.110$

$T_4 \approx \cancel{8.752}\ 3.309$

$S_4 \approx 3.574$

c) $\int_1^4 \frac{\ln\left(\frac{1}{x}\right)}{x}\, dx$

$M_4 \approx -0.982$

$T_4 \approx -0.917$

$S_4 \approx 0.952$

27|

a) $\int_1^{\infty} \frac{1}{x^p}\, dx = \lim_{t\to\infty} \int_1^t \frac{1}{x^p}\, dx = \lim_{t\to\infty} \left[\frac{x^{1-p}}{1-p}\right]_1^t = \frac{t^{1-p}}{1-p} - \frac{1}{1-p}$

$1-p$ is positive when $p<1 \Rightarrow$ lim does not exist $\Rightarrow$ This integral is diverge when $p<1$

$1-p$ is negative when $p>1 \Rightarrow \lim_{t\to\infty} \frac{t^{1-p}}{1-p} = 0 \Rightarrow$ converge when $p>1$

So $\int_1^{\infty} \frac{1}{x^p}\, dx$ diverge when $p<1$, converge when $p>1$

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