Combinatorics, the study of arrangements of objects, is an important part of discrete mathematics.

Enumeration, the counting of objects with certain properties, is an important part of combinatorics.

We must count objects to solve many different types of problems.

CHAPTER 6 Counting

Basic Counting Principles

More Complex Counting Problems

The Subtraction Rule

Tree Diagrams

6.1 The Basics of Counting

FPT UNIVERSITY
DREAM OF INNOVATION

Basic Counting Principles

CHAPTER 6 Counting

We first present two basic counting principles, the **product rule** and the **sum rule**.

THE PRODUCT RULE Suppose that a procedure can be broken down into a sequence of two tasks. If there are n_1 ways to do the first task and for each of these ways of doing the first task, there are n_2 ways to do the second task, then there are $n_1 n_2$ ways to do the procedure.

EXAMPLE 1

There are 32 microcomputers in a computer center. Each microcomputer has 24 ports. How many different ports to a microcomputer in the center are there?

Solution:

EXAMPLE 2

How many different license plates can be made if each plate contains a sequence of three uppercase English letters followed by three digits (and no sequences of letters are prohibited, even if they are obscene)?

Solution:

EXAMPLE 3

Counting One-to-One Functions How many one-to-one functions are there from a set with m elements to one with n elements?

Solution:

EXAMPLE 4

Counting Subsets of a Finite Set Use the product rule to show that the number of different subsets of a finite set S is $2^{|S|}$.

Solution:

Basic Counting Principles

THE SUM RULE If a task can be done either in one of n_1 ways or in one of n_2 ways, where none of the set of n_1 ways is the same as any of the set of n_2 ways, then there are $n_1 + n_2$ ways to do the task.

EXAMPLE 5

Suppose that either a member of the mathematics faculty or a student who is a mathematics major is chosen as a representative to a university committee. How many different choices are there for this representative if there are 37 members of the mathematics faculty and 83 mathematics majors and no one is both a faculty member and a student?

Solution:

EXAMPLE 6

A student can choose a computer project from one of three lists. The three lists contain 23, 15, and 19 possible projects, respectively. No project is on more than one list. How many possible projects are there to choose from?

Solution:

More Complex Counting Problems

Many counting problems cannot be solved using just the sum rule or just the product rule.

EXAMPLE 7

In a version of the computer language BASIC, the name of a variable is a string of one or two alphanumeric characters, where uppercase and lowercase letters are not distinguished. (An *alphanumeric* character is either one of the 26 English letters or one of the 10 digits.) Moreover, a variable name must begin with a letter and must be different from the five strings of two characters that are reserved for programming use. How many different variable names are there in this version of BASIC?

Solution:

More Complex Counting Problems

EXAMPLE 8

Each user on a computer system has a password, which is six to eight characters long, where each character is an uppercase letter or a digit. Each password must contain at least one digit. How many possible passwords are there?

Solution:

More Complex Counting Problems

EXAMPLE 9

Counting Internet Addresses In the Internet, which is made up of interconnected physical networks of computers, each computer (or more precisely, each network connection of a computer) is assigned an *Internet address*. In Version 4 of the Internet Protocol (IPv4), now in use, an address is a string of 32 bits. It begins with a *network number* (*netid*). The netid is followed by a *host number* (*hostid*), which identifies a computer as a member of a particular network.

Bit Number	0	1	2	3	4	8	16	24	31
Class A	0	netid				hostid			
Class B	1	0	netid				hostid		
Class C	1	1	0	netid				hostid	

Several restrictions on addresses:
- 1111111 is not available as the netid of a Class A network, and
- the hostids consisting of all 0s and all 1s are not available for use in any network.

Solution:

The Subtraction Rule (Inclusion–Exclusion for Two Sets)

THE SUBTRACTION RULE If a task can be done in either n_1 ways or n_2 ways, then the number of ways to do the task is $n_1 + n_2$ minus the number of ways to do the task that are common to the two different ways.

The number of elements in the union of two sets.

$$|A_1 \cup A_2| = |A_1| + |A_2| - |A_1 \cap A_2|.$$

EXAMPLE 10

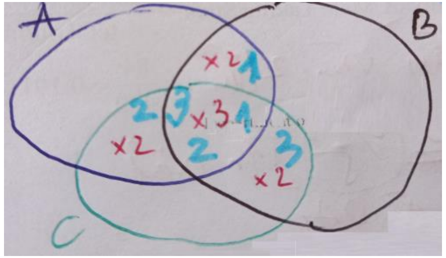
How many bit strings of length eight either start with a 1 bit or end with the two bits 00?

Solution:

The Subtraction Rule

EXAMPLE 11

A computer company receives 350 applications from computer graduates for a job planning a line of new Web servers. Suppose that 220 of these applicants majored in computer science, 147 majored in business, and 51 majored both in computer science and in business. How many of these applicants majored neither in computer science nor in business?



Tree Diagrams

Counting problems can be solved using **tree diagrams**.
To use trees in counting, we use a branch to represent each possible choice.
We represent the possible outcomes by the leaves, which are the endpoints of branches not having other branches starting at them.

EXAMPLE 12 How many bit strings of length four do not have two consecutive 1s?

Solution:

```
run:
1 0 1 0
1 0 0 1
1 0 0 0
0 1 0 1
0 1 0 0
0 0 1 0
0 0 0 1
0 0 0 0
The number is 8
```

Tree Diagrams

EXAMPLE 13

A playoff between two teams consists of at most five games. The first team that wins three games wins the playoff. In how many different ways can the playoff occur?

Solution:

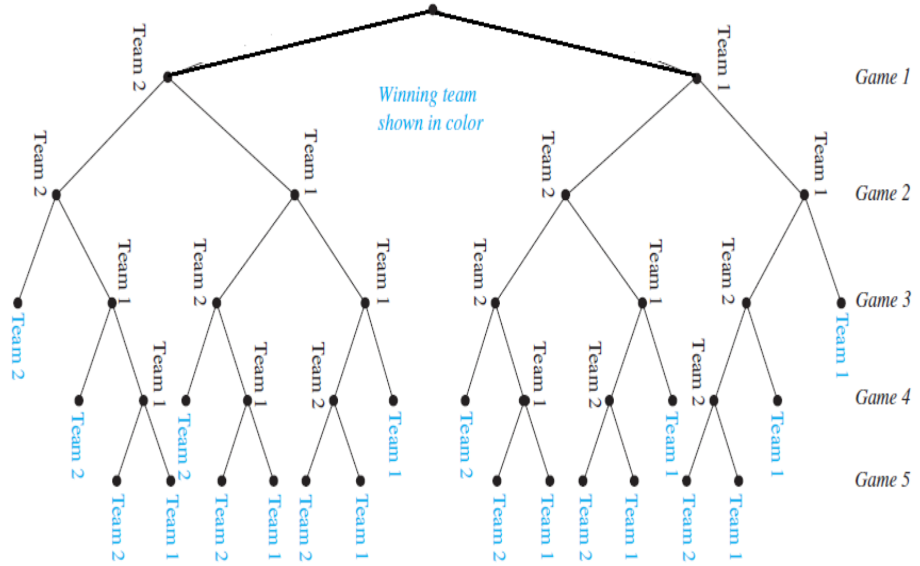


FIGURE **Best Three Games Out of Five Playoffs.**

```
run:
  1 1 1 0 0    0 1 1 1 0
  1 1 0 1 0    0 1 1 0 1
  1 1 0 0 1    0 1 1 0 0
  1 1 0 0 0    0 1 0 1 1
  1 0 1 1 0    0 1 0 1 0
  1 0 1 0 1    0 1 0 0 1
  1 0 1 0 0    0 0 1 1 1
  1 0 0 1 1    0 0 1 1 0
  1 0 0 1 0    0 0 1 0 1
  1 0 0 0 1    0 0 0 1 1

     The number is 20
```