

3.

Non echelon form:

$$\begin{array}{cccc} 2 & 1 & 0 & 6 \end{array}$$

$$\begin{array}{cccc} 6 & 2 & 1 & 8 \end{array}$$

$$\begin{array}{cccc} 7 & 0 & 0 & 3 \end{array}$$

(4)

Echelon but not reduced:

$$\begin{array}{cccc} 1 & 0 & 0 & 0 \end{array}$$

$$\begin{array}{cccc} 2 & 1 & 0 & 0 \end{array}$$

$$\begin{array}{cccc} 0 & 0 & 0 & 1 \end{array}$$

Reduced echelon form:

$$\begin{array}{cccc} 1 & 0 & 0 & 0 \end{array}$$

$$\begin{array}{cccc} 0 & 1 & 0 & 0 \end{array}$$

$$\begin{array}{cccc} 0 & 0 & 0 & 1 \end{array}$$

9/

$$x_1 + x_2 + x_3 = 0$$

⑦
Echelon but not reduced:

$$\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array}$$

Reduced echelon form:

$$\begin{array}{cccc} 1 & 0 & 4 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{array}$$

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$$x_1 + x_2 + 1_7 - x_4 = 3$$

$$3x_1 + 5x_2 - 2x_3 + x_4 = 1$$

$$-3x_1 - 2x_2 + 7x_3 - 5x_4 = 7$$

$$x_1 + 3x_2 - 4x_3 + 5x_4 = -5$$

| | | | | | | | | | | |
|------|------|------|------|------|--------------------------|-----|------|------|------|------|
| 1 | 1 | 1 | -7 | 3 | | 1 | 1 | 1 | -7 | 3 |
| 3 | 5 | -2 | 1 | 1 | $\xrightarrow{3x_1+x_2}$ | 0 | 2 | -5 | 4 | -8 |
| -3 | -2 | 0 | -5 | 7 | $\xrightarrow{3x_1+x_3}$ | 0 | -4 | 9 | -8 | 16 |
| 1 | 3 | -4 | 3 | -5 | $\xrightarrow{-x_1+x_4}$ | 0 | 2 | -5 | 4 | -8 |

| | | | | | | | | | | | |
|--------------------------------|-----|------|----------------|------|------|---------------------------------|-----|-----|----------------|------|------|
| $\xrightarrow{\frac{7}{2}x_2}$ | 1 | 1 | 1 | -7 | 3 | $\xrightarrow{\frac{4}{2}x_2}$ | 1 | 1 | 1 | -7 | 3 |
| | 0 | 1 | $-\frac{5}{2}$ | 2 | -4 | $\xrightarrow{-\frac{5}{2}x_2}$ | 0 | 1 | $-\frac{5}{2}$ | 2 | -4 |
| | 0 | -4 | 9 | -7 | 16 | | 0 | 0 | 1 | 0 | 0 |
| | 0 | 2 | -5 | 4 | -8 | | 0 | 0 | 0 | 0 | 0 |

| | | | | | | | | | | | |
|------------------------------------|-----|-----|-----|------|------|--------------------------------|-----|-----|-----|------|------|
| $\xrightarrow{\frac{5}{2}x_2+x_3}$ | 1 | 1 | 1 | -7 | 3 | $\xrightarrow{-x_1+x_3}$ | 1 | 0 | 0 | -3 | 0 |
| | 0 | 1 | 0 | 2 | -4 | $\xrightarrow{\frac{1}{3}x_3}$ | 0 | 1 | 0 | 2 | -4 |
| | 0 | 0 | 1 | 0 | 0 | | 0 | 0 | 1 | 0 | 0 |
| | 0 | 0 | 0 | 0 | 0 | | 0 | 0 | 0 | 0 | 0 |

| | | | | | | | |
|--------------------------|-----|-----|-----|------|------|---------------------|---------------------|
| $\xrightarrow{4x_3+x_4}$ | 1 | 0 | 0 | -3 | 7 | $\rightarrow x_3=0$ | $\rightarrow x_3=0$ |
| | 0 | 1 | 0 | 2 | -4 | $x_2+x_4=-4$ | $x_2=-4$ |
| | 0 | 0 | 1 | 0 | 0 | $x_1-x_4=7$ | $x_1=7$ |

$$\begin{array}{ccccc|ccccc} -3 & -2 & 10 & -5 & 7 & 2x_1 + x_3 & 0 & -4 & 9 & -8 & 16 \\ & & & & & -x_1 + x_4 & 0 & 2 & -5 & 4 & -8 \end{array}$$

$$\begin{array}{ccccc|ccccc} \frac{2}{5}x_2 \rightarrow & 1 & -1 & 1 & -1 & 3 & 4x_2 + x_3 & 1 & 1 & 1 & -1 & 3 \\ & 0 & 1 & \frac{-5}{2} & 2 & -4 & -2x_2 + x_4 & 0 & 1 & \frac{-5}{2} & 2 & -4 \\ & 0 & -4 & 9 & -7 & 16 & & 0 & 0 & 1 & 0 & 0 \\ & 0 & 2 & -5 & 4 & -8 & & 0 & 0 & 0 & 0 & 0 \end{array}$$

$$\begin{array}{ccccc|ccccc} \frac{5}{2}x_2 + x_3 \rightarrow & 1 & 1 & 1 & -1 & 3 & -x_2 + x_1 & 1 & 0 & 1 & 0 & 1 \\ & 0 & 1 & 0 & 2 & -4 & \frac{1}{3}x_3 & 0 & 1 & 0 & 2 & -4 \\ & 0 & 0 & 1 & 0 & 0 & & 0 & 0 & 1 & 0 & 0 \\ & 0 & 0 & 0 & 0 & 0 & & 0 & 0 & 0 & 1 & 0 \end{array}$$

$$\begin{array}{ccccc|ccccc} -3x_3 + x_4 \rightarrow & 1 & 0 & 0 & -3 & 7 & \rightarrow x_3 = 0 & \rightarrow x_3 = 0 \\ & 0 & 1 & 0 & 2 & -4 & x_2 + x_4 = -4 & x_2 = -4 \\ & 0 & 0 & 1 & 0 & 0 & x_1 - x_4 = 7 & x_1 = 7 \\ & 0 & 0 & 0 & 0 & 0 & x_4 = 0 & x_4 = 0 \end{array}$$

$$\text{So } x_1 = 7; x_2 = -4; x_3 = 0; x_4 = 0$$