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$$I = \iint\limits_{D \in Oxy}$$

Tọa độ cực: $x = r\cos\varphi$, $0 \le r \le 2$

$y = r\sin\varphi$. $\pi \le \varphi \le \frac{3\pi}{2}$

$$I = \int_{\pi}^{3\pi/2} d\varphi \int_{0}^{2} r^3\cos^3\varphi \cdot r\sin\varphi \cdot r^3\, dr$$

$$= \int_{\pi}^{3\pi/2} d\varphi \cdot \frac{r^8}{8}\Big|_0^2 \cos^3\varphi \cdot \sin\varphi\, d\varphi$$

$$= \int_{\pi}^{3\pi/2} 32 \cdot \cos^3\varphi \cdot \sin\varphi\, d\varphi$$

$$= -32\int_{\pi}^{3\pi/2} \cos^3\varphi\, d(\cos\varphi) = -32 \cdot \frac{(\cos^4\varphi)}{4}\Big|_{\pi}^{3\pi/2}$$

$$= -8 \cdot (0 - 1)$$

$$= 8$$

$= \lim\limits_{n\to+\infty} \dfrac{\sqrt{n}.n+\sqrt{n}}{\sqrt{n^2+3}} = \lim\limits_{n\to+\infty} \dfrac{\sqrt{n}+1}{\sqrt{1+\frac{3}{n^2}}} = +\infty$

Mà $g(x) = \dfrac{1}{n}$ $\cancel{\lim\limits_{n\to+\infty} \dfrac{1}{n} = +\infty}$

$\sum\limits_{n=1}^{\infty} \dfrac{1}{n}$ phân kỳ $\Rightarrow f(x)$ phân kỳ.

Câu 4. $\sum\limits_{n=1}^{+\infty} \dfrac{n3^n(x+5)^n}{n^2+5n+6}$.

NX với $x=-5$ ta được chuỗi số $\sum\limits_{n=1}^{\infty} 0$ hội tụ.

Xét $x \neq -5$.

Ta có: $\left|\dfrac{u_{n+1}(x)}{u_n(x)}\right| = \left|\dfrac{(n+1)3^{n+1}.(x+5)^{n+1}}{(n+1)^2+(n+1)5+6}\right| : \left|\dfrac{n.3^n(x+5)^n}{n^2+5n+6}\right|$

$= \left|\dfrac{(n+1).3^{n+1}.(x+5)^{n+1}(n^2+5n+6)}{((n+1)^2+5(n+1)+6).n.3^n(x+5)^n}\right|$

$= \left|\dfrac{(3.3^n.n+3.3^n)(x+5)}{n.3^n}\right| = \left|\dfrac{(3n+3)(x+5)}{n}\right|$

$\Rightarrow q \overset{n \to +\infty}{=} 3|x+5|$

Giả $3|x+5| < 1$

$\Leftrightarrow -\frac{1}{3} < x+5 < \frac{1}{3}$

$\Leftrightarrow -\frac{16}{3} < x < -\frac{14}{3}$ => chuỗi hội tụ theo D'lambert

Xét $\left.\begin{array}{l} x > -\frac{14}{3} \\ x < -\frac{16}{3} \end{array}\right\}$ => chuỗi phân kỳ.

Xét tại $x = -\frac{16}{3}$ ta có chuỗi

$$\sum_{n=1}^{+\infty} \frac{n \cdot 3^n \left(-\frac{1}{3}\right)^n}{n^2+5n+6} = \sum_{n=1}^{+\infty} \frac{n \cdot (-1)^n}{n^2+5n+6}.$$

Dấu hiệu Lebniz => chuỗi hội tụ.

Xét tại $x = -\frac{14}{3}$

$$\sum_{n=1}^{+\infty} \frac{n \cdot (1)^n}{n^2+5n+6} \Rightarrow \sum_{n=1}^{+\infty}$$ chuỗi phân kỳ do cùng tính chất $\frac{1}{n}$

$\Rightarrow y_p = A\cos x + B\sin x$

$y' = -A\sin x + B\cos x$

$y'' = -A\cos x - B\sin x$

$(\Leftrightarrow)$ $\cancel{A}\, y'' - y = \cos x + \sin y$

$(\Leftrightarrow)$ $-A\cos x - B\sin x + A\sin x - B\cos x = \cos x + \sin x$

$(\Leftrightarrow)$ $(-A - B)\cos x + (A - B)\sin x = \cos x + \sin x$

$(\Leftrightarrow)$ $\begin{cases} -A - B = 1 \\ A - B = 1 \end{cases} \Rightarrow \begin{cases} A = 0 \\ B = -1 \end{cases}$

$\Rightarrow y_p = -\sin x$

$\Rightarrow y = C_1 \cdot e^{x} + C_2 \cdot e^{-x} - \sin x$

$\frac{1}{2}\sin x \cdot \cos x$ $\quad 2\sin x \cdot \cos x$

2

$\sin 2x$

4

Câu 3. $\sum_{n=1}^{+\infty} f(x) = \sum_{n=1}^{+\infty} \frac{\sqrt{n} + 1}{\sqrt{n^2 + 3}}$

Xét $g(x) = \frac{1}{\sqrt{n}}$

$\lim_{n \to +\infty} \frac{(\sqrt{n} + 1)\cdot \sqrt{n}}{\sqrt{n^2 + 3}}$

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$\Rightarrow y = y_0 + y_p$

$= C_1 \cos x + C_2 \cos x + x \cdot \left(-\frac{1}{2} \cos x + \dots \right.$

Câu 3.

$$\sum_{n=2}^{+\infty} \sqrt{\frac{n+4}{n^4+4}} = \sum_{n=2}^{+\infty} \frac{\sqrt{n+4}}{\sqrt{n^4+4}}$$

Xét $g(x) = \frac{1}{x^{3/2}}$

$$\lim_{n \to +\infty} \frac{\sqrt{n+4} \cdot \sqrt{n^3}}{\sqrt{n^4+4}} = \lim_{n \to +\infty} \frac{\sqrt{n^4+4n^3}}{\sqrt{n^4+4}} = 1$$

Mà $\sum_{n=2}^{+\infty} \frac{1}{x^{3/2}}$ hội tụ $\Rightarrow \sum_{n=2}^{+\infty} f(x)$ hội tụ.

~~Câu 4~~. Câu 5.

$$I = \iint_S x^3 y (z-2)^2 \, dx dy$$

$$\Rightarrow I = \iint_{D \in Oxy} x^3 \cdot y \cdot \left(+ \sqrt{x^2+y^2} \right)^2 dx dy$$

Đề Gr.

Câu 5

$$I = \iint_S 3x^2 y^2 z \, dx\, dy$$

$$I = +\iint_{D_{xy}} 3x^2 y^2 \cdot (x^2+y^2) \, dx\, dy$$

Tọa độ cực: $x = r\cos\varphi \quad 0 \le r \le \sqrt{3}$

$y = r\sin\varphi \quad 0 \le \varphi \le \pi/2$

$$I = \int_0^{\pi/2} d\varphi \int_0 3 . r^2 \sin^2\varphi . r^2 \cos^2\varphi . r^3 \, dr$$

$$I = \int_0^{\pi/2} d\varphi . \frac{3.r^8}{8} \Big|_0^{\sqrt{3}} . \sin^2\varphi . \cos^2\varphi \, d\varphi$$

$$= \cancel{6561} \; \frac{243}{8} \int_0^{\pi/2} \sin^2\varphi . \cos^2\varphi .$$

$$= \frac{\cancel{19683}}{\cancel{8}} \frac{243}{8} \int_0^{\pi/2} \sin^2\varphi - \sin^4\varphi \, d\varphi .$$

$$= \frac{\cancel{19683}}{8} \frac{243}{8} \int_0^{\pi/2} \frac{1 - \cos 2\varphi}{2} - \left(\frac{\cos 4x - 4\cos 2x + 3}{8} . \right.$$

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$= \frac{1}{8}$

ĐỀ SỐ 4.

Câu 1.

$$xy' + 3y = \frac{\sin x}{x^2}$$

$$\Leftrightarrow x \cdot y' = \frac{\sin x}{x^2} - 3y$$

$$\Leftrightarrow y' = \frac{\sin x}{x^3} - \frac{3y}{x}$$

$$\Leftrightarrow y' + \frac{3y}{x} = \frac{\sin x}{x^3}$$

$$y = C \cdot e^{-\int \frac{3}{x} dx} + e^{-\int \frac{3}{x} dx} \cdot \int \frac{\sin x}{x^3} \cdot e^{\int \frac{3}{x} dx} dx$$

$$= C \cdot x^{-3} + x^{-3} \cdot \int \frac{\sin x}{x^3} \cdot x^3 \, dx$$

$$= C \cdot x^{-3} - x^{-3} \cdot \cos x.$$

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Câu 2.

$y'' + y = 2\cos x + \sin x$

$= e^{0x}.(2\cos x + \sin x)$, $a = 0$, $b = 1$.

PTĐT:

~~$y'' + y$~~ $k^2 + 1 = 0$

$\Leftrightarrow k = \pm i$

$\Rightarrow y_0 = C_1 \cos x + C_2 \sin x$

$k = \pm i$ là nghiệm của pt đặc trưng

$\Rightarrow y_p = x.(A\cos x + B\sin x)$

$y' = A\cos x + B\sin x + x.(B\cos x - A\sin x)$

$y'' = -A\sin x + B\cos x + (B\cos x - A\sin x)$
$+ x.(-B\sin x - A\cos x)$

$= -2A\sin x + 2B\cos x - x(B\sin x + A\cos x)$

$y'' + y = 2(\cos x + \sin x)$

$= -2A\sin x + 2B\cos x - x(B\sin x + A\cos x) + x(A\cos x + B\sin x)$

$= 2\cos x + \sin x$

$\Rightarrow B = 1$, $A = -\frac{1}{2}$

HẢI TIẾN

Câu 1. ĐỀ 3.

$$xy' = \frac{\cos x}{x} - 2y$$

$$(=)\ y' + \frac{2y}{x} = \frac{\cos x}{x^2}$$

$$\Rightarrow y = C.e^{-\int \frac{2}{x}dx} + e^{-\int \frac{2}{x}dx} \cdot \int \frac{\cos x}{x^2} \cdot e^{\int \frac{2}{x}dx}$$

$$= C.e^{-2\ln x} + e^{-2\ln x} \cdot \int \frac{\cos x}{x} \cdot e^{2\ln x} dx$$

$$= C.x^{-2} + x^{-2} \cdot \int \frac{\cos x}{x} \cdot x^2 dx$$

$$= C.x^{-2} + x^{-2} \cdot \int \cos x . x \, dx$$

$$= C.x^2 + x^{-2}(x \sin x + \cos x)$$

$$= Cx^2 + \frac{\sin x}{x} + \frac{\cos x}{x^2}$$

Câu 2. $y'' - y = \cos x + \sin x$

$$y' - y = e^{0x}.(\cos(1x) + \sin(1x))$$

$$a = 0,\ b = 1$$

PTĐT: $k^2 - 1 = 0 \Rightarrow k = \pm 1 \Rightarrow y_0 = C_1 \cdot e^{x} + C_2 \cdot e^{-x}$

$\Rightarrow k = 0 + 1i$ không phải là nghiệm