SaUCy

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Text of abstract ....

1 INTRODUCTION

Proving that a cryptographic protocol carries out a given task securely is an essential component in cryptography. Traditionally, such a protocol is analyzed in the *standalone* setting, in which a single execution takes place in isolation. In reality, however, the protocol may be running concurrently with arbitrary other protocols, and indeed, security guarantees in the standalone setting do not always translate into security guarantees in the concurrent setting. In order to provide meaningful security guarantees in the concurrent setting, the Universal Composability (UC) framework by Canetti [Canetti 2001] allows the security properties of a protocol to be defined in such a way that security is maintained under general concurrent composition with arbitrary other protocols. In other words, a UC-secure protocol maintains its security when dropped into *any context*. Importantly, this allows for complex cryptographic protocols to be designed and analyzed in a modular fashion from simpler building blocks.

Since universally composable security is such a powerful guarantee, it is perhaps not surprising that elaborating such proofs can be quite involved, and thus, error-prone. However, as UC proofs are primarily "pen-and-paper" proofs, this makes verifying them an unwieldy task. In addition, the numerous variations of UC (e.g., guc [Canetti et al. 2007], juc [Canetti and Rabin 2003], symbolic UC [Böhl and Unruh 2016], RSIM [Backes et al. 2007], GNUC [Hofheinz and Shoup 2015]) make it hard to keep track of the precise semantics of security claims. In this paper, we design and implement a specialized programming language called the Interactive Lambda Calculus (ILC) for building a concrete implementation of the UC framework, and for elaborating algorithmic entities (i.e., ideal functionalities and simulators) used in UC proofs. In particular, the type system of ILC enforces that well-typed programs are confluent, i.e., they either diverge or evaluate to a single unique value, which makes it easier to reason about the *indistinguishability* of two programs in ILC.

2 OVERVIEW

In order to prove that a cryptographic protocol carries out a given task securely, we first formalize the protocol, henceforth referred to as the real protocol, and its execution in the presence of an adversary and in a given computational environment. We then formalize an ideal protocol that is secure by definition for carrying out the task. In the ideal protocol, parties do not communicate with each other, rather, they rely on an incorruptible trusted party called the *ideal functionality* to meet the requirements of the task at hand. Finally, to show that the real protocol carries out the task securely, we show that running it "emulates" running the ideal protocol for that task, in the sense that an outside observer called the *environment*, which interacts with both the real and ideal protocols, cannot distinguish them apart.

As in [Goldwasser et al. 1989], a protocol is represented as a system of interactive Turing machines (ITMs), in which each ITM represents the program to be run within each party. Each ITM has an

input and output tapes to model inputs received from and outputs given to other ITMs. Additionally, each ITM has a communication tape to model messages sent to and received from the network.

Let π denote the real protocol followed by a set of parties, and let $\mathcal{A}$ denote an adversary that aims to break the security of π. If $\mathcal{A}$ is a *passive* (or *semi-honest*) adversary, then it can listen to all communications between the parties, and can observe the internal state of corrupted parties. If $\mathcal{A}$ is an *active* (or *malicious*) adversary, then it can additionally take full control of parties and alter messages en route arbitrarily. The adversary communicates with the environment $\mathcal{Z}$ to provide details of what it observes, and also to receive instructions on how to proceed. Note that parties cannot directly communicate with each other, rather, all communication passes through $\mathcal{A}$. If the network is synchronous, then $\mathcal{A}$ is not allowed to interfere with network traffic. If the network if asynchronous, $\mathcal{A}$ is allowed to delay and reorder messages arbitrarily.

Let ϕ denote the ideal protocol followed by a set of parties relying on the ideal functionality $\mathcal{F}$, and let $\mathcal{S}$ denote an ideal adversary, also known as a *simulator*, that aims to break the security of ϕ. Here, the parties are *dummy parties*, since they hand received inputs directly to $\mathcal{F}$ for processing, and output whatever is directly returned by $\mathcal{F}$. Clearly, since the dummy parties do nothing, and $\mathcal{F}$ is secure by definition, it makes sense to define ϕ as secure.

The goal of the environment $\mathcal{Z}$ is to distinguish between the real protocol and the ideal protocol. Since in the real protocol, $\mathcal{Z}$ interacts with the adversary $\mathcal{A}$, in the ideal protocol, $\mathcal{Z}$ interacts with the simulator $\mathcal{S}$. The job of $\mathcal{S}$ is to pretend to be $\mathcal{A}$ with the aid of $\mathcal{F}$. The amount of help $\mathcal{F}$ is able to provide is specified in $\mathcal{F}$ itself.

3 ILC

Definition 3.1 (Protocol Emulation). Let π and ϕ be probabilistic polynomial time (p.p.t) protocols. We say that π UC-emulates ϕ if for any p.p.t. adversary $\mathcal{A}$ there exists a p.p.t. ideal-process adversary $\mathcal{S}$ such that for any balanced PPT environment $\mathcal{Z}$ we have:

$$\text{EXEC}_{\phi, \mathcal{S}, \mathcal{Z}} \approx \text{EXEC}_{\pi, \mathcal{A}, \mathcal{Z}}.$$

Definition 3.2 (Protocol Emulation w.r.t. the Dummy Adversary). Let π and ϕ be probabilistic polynomial time (p.p.t) protocols. We say that π UC-emulates ϕ if for the dummy adversary $\mathcal{D}$ there exists a p.p.t. ideal-process adversary $\mathcal{S}$ such that for any balanced PPT environment $\mathcal{Z}$ we have:

$$\text{EXEC}_{\phi, \mathcal{S}, \mathcal{Z}} \approx \text{EXEC}_{\pi, \mathcal{D}, \mathcal{Z}}.$$

Definition 3.3 (Probability Distribution Ensemble). An *ensemble* of probability distributions is a family of probability distributions $\{X_{\lambda, z}\}_{\lambda \in \mathbb{N}, z \in \{0,1\}^*}$ with index set $\mathbb{N} \times \{0, 1\}^*$. The ensembles considered in this work are binary probability distribution ensembles, which describe single bit outputs of computations, where $\lambda \in \mathbb{N}$ represents the security parameter, and $z \in \{0, 1\}^*$ represents input.

Definition 3.4 (Indistinguishability). Let X and Y be two binary probability distribution ensembles. We say that X and Y are indistinguishable (written $X \approx Y$) if for any $c, d \in \mathbb{N}$, there exists $\lambda_0 \in \mathbb{N}$ such that for all $\lambda > \lambda_0$ and all $z \in \cup_{\lambda \leq \lambda^d} \{0, 1\}^\lambda$,

$$|\Pr[X_{\lambda, z} = 1] - \Pr[Y_{\lambda, z} = 1]| < \lambda^{-c}.$$

Definition 3.5 (Bit Producing ILC Term). Let τ be an ILC term. We say that τ is bit producing if it is closed except for an infinite bitstream free variable σ:Inf and σ:Inf $\vdash \tau$:Bit.

The denotation $[\![\tau]\!]\sigma$, in which a particular σ is given, evaluates to a value of type Bit, and the denotation $[\![\tau]\!]$, in which no σ is specified, evaluates to a binary probability distribution ensemble over types or values?

Definition 3.6 (Indistinguishability of Bit Producing ILC Terms). Let τ_1 and τ_2 be bit producing ILC terms. We say that τ_1 and τ_2 are indistinguishable terms if the binary probability distribution ensembles $[\![\tau_1]\!]$ and $[\![\tau_2]\!]$ are indistinguishable.

Definition 3.7 (Protocol Emulation in ILC). Let $(\pi_1, \mathcal{F}_1)$ and $(\pi_2, \mathcal{F}_2)$ be two protocol-functionality pairs. We say that $(\pi_1, \mathcal{F}_1)$ UC-emulates $(\pi_2, \mathcal{F}_2)$ iff for all adversaries $\mathcal{A}$, there exists an ideal-process adversary $\mathcal{S}$ such that for any environment $\mathcal{Z}$, $\textsc{ExecUC}_{\mathcal{Z}, \mathcal{A}, \pi_1, \mathcal{F}_1}$ and $\textsc{ExecUC}_{\mathcal{Z}, \mathcal{S}, \pi_2, \mathcal{F}_2}$ are bit producing and indistinguishable terms.

Definition 3.8 (Protocol Emulation in ILC). Let π and ϕ be probabilistic polynomial time (p.p.t.) protocols. We say that π UC-emulates ϕ if for any p.p.t. adversary $\mathcal{A}$, there exists a p.p.t. ideal-process adversary $\mathcal{S}$ such that for any balanced p.p.t. environment $\mathcal{Z}$, $\textsc{ExecUC}_{\phi, \mathcal{S}, \mathcal{Z}}$ and $\textsc{ExecUC}_{\pi, \mathcal{A}, \mathcal{Z}}$ are indistinguishable bit producing terms.

Definition 3.9 (Protocol Emulation in ILC). Let π and ϕ be probabilistic polynomial time (p.p.t.) protocols. We say that π UC-emulates ϕ if for any p.p.t. adversary $\mathcal{A}$, there exists a p.p.t. ideal-process adversary $\mathcal{S}$ such that for any balanced p.p.t. environment $\mathcal{Z}$, $\textsc{ExecUC}_{\phi, \mathcal{S}, \mathcal{Z}}$ and $\textsc{ExecUC}_{\pi, \mathcal{A}, \mathcal{Z}}$ are indistinguishable bit producing terms.

Definition 3.10 (Balanced Environment). An environment $\mathcal{Z}$ is balanced if the overall length of inputs given by $\mathcal{Z}$ to the parties of the main instance π is at most k times the length of the input to the adversary.

4 METATHEORY

(1) Type soundness
(2) Confluence

5 IMPLEMENTATION

(1) Bidirectional type checker
(2) Replication

6 EXPERIMENTS

(1) Impossibility of UC commitments using standard assumptions [Canetti and Fischlin 2001].
(2) UC commitments construction using CRS

Functionality $\mathcal{F}_{\text{COM}}$

$\mathcal{F}_{\text{COM}}$ proceeds as follows, running with parties $P_1, \ldots, P_n$ and an adversary S.

(1) Upon receiving a value (Commit, sid, P_i, P_j, b) from P_i, where $b \in \{0, 1\}$, record the value b and send the message (Receipt, sid, P_i, P_j) to P_j and S. Ignore any subsequent Commit messages.
(2) Upon receiving a value (Open, sid, P_i, P_j) from P_i, proceed as follows: If some value b was previously recorded, then send the message (Open, sid, P_i, P_j, b) to P_j and S and halt. Otherwise halt.

```
let F_com = lam S .
  let ('Commit, sid, P_i, P_j, b) = rd ?p2f in
    req mem b {0,1} in
    wr (('Receipt, sid, P_i, P_j), {P_j, S}) → ?f2p ;
    let ('Open, sid, P_i, P_j) = rd ?p2f in
```

```
                              execUC(ℰ, π, 𝒜, ℱ)
ν z2p z2f z2a p2f p2a a2f.
// The environment chooses SID, conf, and corrupted parties
let (Corrupted, SID, conf) = ℰ{z2p, z2a, z2f}
// The protocol determines conf′
let conf′ = π.cmap(SID, conf)
| 𝒜{SID, conf, Corrupted, a2z, a2p, a2f}
| ℱ{SID, conf′, Corrupted, f2z, f2p, f2a}
// Create instances of parties on demand
let partyMap = ref empty
let newPartyPID = do
    ν f2pp z2pp.
    @partyMap[PID].f2p := f2pp
    @partyMap[PID].z2p := z2pp
    | forever do {m ← pp2f; (PID, m) → f2p}
    | forever do {m ← pp2z; (PID, m) → z2p}
    | π{SID, conf, p2f/pp2z, p2z/pp2z}
let getParty PID =
    if PID ∉ partyMap then newParty PID
    return @partyMap[PID]
| forever do
    (PID, m) ← z2p
    if PID ∈ Corrupted then Z2P(PID, m) → p2a
    else m → (getParty PID).z2p
| forever do
    (PID, m) ← f2p
    if PID ∈ Corrupted then F2P(PID, m) → p2a
    else m → (getParty PID).f2p
| forever do
    | A2P2F(PID, m) ← a2p
      if PID ∈ Corrupted then (PID, m) → p2f
    | A2P2Z(PID, m) ← a2p
      if PID ∈ Corrupted then (PID, m) → p2z
```

Fig. 1. Definition of the SaUCy execution model. The environment, are run as concurrent processes. A new instance of the protocol π is created, on demand, for each party PID. Messages sent to honest parties are routed according to their PID; messages sent to corrupted parties are instead diverted to the adversary.

```
    wr (('Open, sid, P_i, P_j, b), {P_j, S}) → ?f2p
in
  nu f2p, p2f .
    | ▷ (F_com S)
```

7 RELATED WORK

EasyCrypt [Barthe et al. 2011], CertiCrypt [Barthe et al. 2009], CryptoVerif [Blanchet 2007], ProVerif [Blanchet 2005], RF* [Barthe et al. 2014], Cryptol [Lewis and Martin 2003], code-based game-playing proofs [Bellare and Rogaway 2006], symbolic UC [Böhl and Unruh 2016]

8 CONCLUSION

9 FUTURE WORK

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A APPENDIX

Value Types	$A, B ::= x$	Value variable
	$\mid$ unit	Unit value
	$\mid$ nat	Natural number
	$\mid A \times B$	Product
	$\mid A + B$	Sum type
	$\mid\ !A$	Intuitionistic type
	$\mid \mathbf{Rd}\, A$	Read channel
	$\mid \mathbf{Wr}\, A$	Write channel
	$\mid \mathbf{U}\, C$	Thunk type
Computation Types	$C, D ::= A \rightarrow C$	Value-consuming computation
	$\mid \mathbf{F}\, A$	Value-producing computation
Linear Typing Contexts	$\Delta ::= \cdot \mid \Delta, x : A$	
Intuitionisitic Typing Contexts	$\Gamma ::= \cdot \mid \Gamma, x : A$	

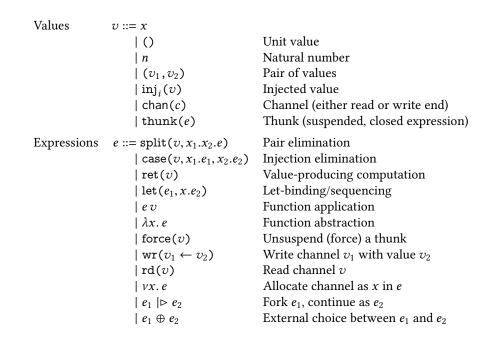Fig. 2. Syntax of types and typing contexts

Values $v ::= x$
                    $\mid$ ()   Unit value
                    $\mid n$ Natural number
                    $\mid (v_1, v_2)$ Pair of values
                    $\mid \mathrm{inj}_i(v)$ Injected value
                    $\mid \mathrm{chan}(c)$ Channel (either read or write end)
                    $\mid \mathrm{thunk}(e)$ Thunk (suspended, closed expression)

Expressions $e ::= \mathrm{split}(v, x_1.x_2.e)$ Pair elimination
                    $\mid \mathrm{case}(v, x_1.e_1, x_2.e_2)$ Injection elimination
                    $\mid \mathrm{ret}(v)$ Value-producing computation
                    $\mid \mathrm{let}(e_1, x.e_2)$ Let-binding/sequencing
                    $\mid e\, v$ Function application
                    $\mid \lambda x.\, e$ Function abstraction
                    $\mid \mathrm{force}(v)$ Unsuspend (force) a thunk
                    $\mid \mathrm{wr}(v_1 \leftarrow v_2)$ Write channel v_1 with value v_2
                    $\mid \mathrm{rd}(v)$ Read channel v
                    $\mid \nu x.\, e$ Allocate channel as x in e
                    $\mid e_1 \mathbin{\vert\triangleright} e_2$ Fork e_1, continue as e_2
                    $\mid e_1 \oplus e_2$ External choice between e_1 and e_2

Fig. 3. Syntax of values and expressions

Modes $m, n, p ::= \mathrm{W} \mid \mathrm{R} \mid \mathrm{V}$ (Write, Read and Value)

$\boxed{m \parallel n \Rightarrow p}$ The parallel composition of modes m and n is mode p.

$$\dfrac{m \parallel n \Rightarrow p}{n \parallel m \Rightarrow p}\ \text{sym} \qquad \dfrac{}{\mathrm{W} \parallel \mathrm{V} \Rightarrow \mathrm{W}}\ \text{wv} \qquad \dfrac{}{\mathrm{W} \parallel \mathrm{R} \Rightarrow \mathrm{W}}\ \text{wr} \qquad \dfrac{}{\mathrm{R} \parallel \mathrm{R} \Rightarrow \mathrm{R}}\ \text{rr}$$

$\boxed{m \mathbin{;} n \Rightarrow p}$ The sequential composition of modes m and n is mode p.

$$\dfrac{}{\mathrm{V} \mathbin{;} n \Rightarrow n}\ \text{v*} \qquad \dfrac{}{\mathrm{W} \mathbin{;} \mathrm{V} \Rightarrow \mathrm{W}}\ \text{wv} \qquad \dfrac{}{\mathrm{R} \mathbin{;} n \Rightarrow \mathrm{R}}\ \text{r*} \qquad \dfrac{}{\mathrm{W} \mathbin{;} \mathrm{R} \Rightarrow \mathrm{W}}\ \text{wr}$$

Note that in particular, the following mode compositions are *not derivable*:

- $\mathrm{W} \parallel \mathrm{W} \Rightarrow p$ is *not* derivable for any mode p
- $\mathrm{W} \mathbin{;} \mathrm{W} \Rightarrow p$ is *not* derivable for any mode p

Fig. 4. Syntax of modes; sequential and parallel mode composition.

$\boxed{\Delta; \Gamma \vdash e : C \rhd m}$ Under Δ and Γ, expression e has type C and mode m.

$$\frac{\Delta; \Gamma \vdash v : A}{\Delta; \Gamma \vdash \mathtt{ret}(v) : \mathbf{F}\, A \rhd \mathsf{V}} \ \mathrm{ret}$$

$$\frac{\begin{array}{c} m_1 \,;\, m_2 \Rightarrow m_3 \\ \Delta_1; \Gamma \vdash e_1 : \mathbf{F}\, A \rhd m_1 \\ \Delta_2, x : A; \Gamma \vdash e_2 : C \rhd m_2 \end{array}}{\Delta_1, \Delta_2; \Gamma, x : A \vdash \mathtt{let}(e_1, x.e_2) : C \rhd m_3} \ \mathrm{let}$$

$$\frac{\cdot; \Gamma \vdash v : A}{\cdot; \Gamma \vdash \mathtt{ret}(v) : \mathbf{F}\,(!A) \rhd \mathsf{V}} \ \mathrm{ret!}$$

$$\frac{\Delta_1; \Gamma \vdash v : !A \qquad \Delta_2; \Gamma, x : A \vdash e : C \rhd m}{\Delta_1, \Delta_2; \Gamma, x : A \vdash \mathtt{let!}(v, x.e) : C \rhd m} \ \mathrm{let!}$$

$$\frac{\Delta; \Gamma \vdash e : C \rhd m}{\Delta; \Gamma \vdash \lambda x.\, e : A \to C \rhd m} \ \mathrm{lam}$$

$$\frac{\Delta_1; \Gamma \vdash v : A \qquad \Delta_2; \Gamma \vdash e : A \to C \rhd m}{\Delta_1, \Delta_2; \Gamma \vdash e\, v : C \rhd m} \ \mathrm{app}$$

$$\frac{\Delta, x : \big(\mathbf{Rd}\, A \times !(\mathbf{Wr}\, A)\big); \Gamma \vdash e : C \rhd m}{\Delta; \Gamma \vdash \nu x.\, e : C \rhd m} \ \mathrm{nu}$$

$$\frac{\Delta; \Gamma \vdash v : \mathbf{Rd}\, A}{\Delta \vdash \mathtt{rd}(v) : \mathbf{F}\,(A \times (\mathbf{Rd}\, A)) \rhd \mathsf{R}} \ \mathrm{rd}$$

$$\frac{\Delta_1; \Gamma \vdash v_1 : \mathbf{Wr}\, A \qquad \Delta_2; \Gamma \vdash v_2 : A}{\Delta_1, \Delta_2 \vdash \mathtt{wr}(v_1 \leftarrow v_2) : \mathbf{F}\, \mathsf{unit} \rhd \mathsf{W}} \ \mathrm{wr}$$

$$\frac{\begin{array}{c} m_1 \parallel m_2 \Rightarrow m_3 \\ \Delta_1; \Gamma \vdash e_1 : C \rhd m_1 \\ \Delta_2; \Gamma \vdash e_2 : D \rhd m_2 \end{array}}{\Delta_1, \Delta_2 \vdash e_1 \mathrel{|\rhd} e_2 : D \rhd m_3} \ \mathrm{fork}$$

$$\frac{\begin{array}{c} \Delta_1; \Gamma \vdash e_1 : C \rhd \mathsf{R} \\ \Delta_2; \Gamma \vdash e_2 : C \rhd \mathsf{R} \end{array}}{\Delta_1, \Delta_2 \vdash e_1 \oplus e_2 : C \rhd \mathsf{R}} \ \mathrm{choice}$$

$$\text{Channels} \qquad \Sigma ::= \varepsilon \mid \Sigma, c$$

$$\text{Process pool} \qquad \pi ::= \varepsilon \mid \pi, e$$

$$\text{Configurations} \qquad C ::= \langle \Sigma; \pi \rangle$$

$$\text{Evaluation contexts} \quad E ::= \text{let}(E, x.e)$$
$$\mid E\,v$$
$$\mid \bullet$$

$$\text{Read contexts} \qquad R ::= \text{rd}(\text{chan}(c)) \oplus R$$
$$\mid R \oplus \text{rd}(\text{chan}(c))$$
$$\mid \bullet$$

$\boxed{e \longrightarrow e'}$ Expression e_1 reduces to e_2.

$$\frac{}{\text{let}(\text{ret}(v), x.e) \longrightarrow [v/x]e}\ \text{let} \quad \frac{}{(\lambda x.\,e)\,v \longrightarrow [v/x]e}\ \text{app} \quad \frac{}{\text{force}(\text{thunk}(e)) \longrightarrow e}\ \text{force}$$

$$\frac{}{\text{split}((v_1, v_2), x.y.e) \longrightarrow [v_1/x][v_2/y]e}\ \text{split} \quad \frac{}{\text{case}(\text{inj}_i(v), x_1.e_1, x_2.e_2) \longrightarrow e_i[v/x_i]}\ \text{case}$$

$\boxed{C_1 \equiv C_2}$ Configurations C_1 and C_2 are equivalent.

$$\frac{\pi_1 \equiv_{\text{perm}} \pi_2}{\langle \Sigma; \pi_1 \rangle \equiv \langle \Sigma; \pi_2 \rangle}\ \text{permProcs}$$

$\boxed{C_1 \longrightarrow C_2}$ Configuration C_1 reduces to C_2.

$$\frac{e \longrightarrow e'}{\langle \Sigma; \pi, E[e] \rangle \longrightarrow \langle \Sigma; \pi, E[e]' \rangle}\ \text{local} \quad \frac{}{\langle \Sigma; \pi, E[e_1 \mathrel{|\!\rhd} e_2] \rangle \longrightarrow \langle \Sigma; \pi, e_1, E[e_2] \rangle}\ \text{fork}$$

$$\frac{C_1 \equiv C_1' \qquad C_1' \longrightarrow C_2 \qquad C_2 \equiv C_2'}{C_1 \longrightarrow C_2'}\ \text{congr}$$

$$\frac{c \notin \Sigma}{\langle \Sigma; \pi, E[\nu x.\,e] \rangle \longrightarrow \langle \Sigma, c; \pi, E[[(\text{chan}(c), \text{chan}(c))/x]e] \rangle}\ \text{nu}$$

$$\frac{}{\langle \Sigma; \pi, E_1[R[\text{rd}(\text{chan}(c))]], E_2[\text{wr}(\text{chan}(c) \leftarrow v)] \rangle \longrightarrow \langle \Sigma; \pi, E_1[v], E_2[()] \rangle}\ \text{rw}$$