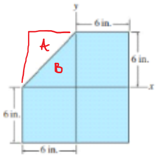
# Homework 9

Wednesday, November 4, 2020 3:05 PM

*"I pledge my honor I have abided by the Stevens Honor system."*

\- Alex Gaskins

***6–36.** Locate the centroid $(\bar{x}, \bar{y})$ of the area.




B $(12)(12) = 144$

A $-\left(\frac{1}{2}\right)(6)(6) = -18$

$\Big\}$ 126 in.$^2$

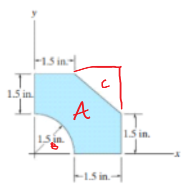
| $x$ | $y$ | $\bar{x}A$ | $\bar{y}A$ |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| -4 | 4 | 72 | -72 |

$$\bar{x} = \frac{\Sigma \bar{x} A}{\Sigma A} = \frac{72}{126} = .5714 \text{ in. along } x\text{-axis}$$

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{-72}{126} = -.5714 \text{ along } y\text{-axis}$$

$\boxed{\text{Centroid: } (.5714, -.5714)}$

**6–42.** Locate the centroid $(\bar{x}, \bar{y})$ of the area.




A: $(3)(3)$

B: $-\frac{\pi}{4}(1.5)^2$

C: $-\frac{1}{2}(1.5)(1.5)$

$\Big\}$ 6.1079 in.$^2$

| $\bar{x}$ | $\bar{y}$ | $\bar{x}A$ | $\bar{y}A$ |
|---|---|---|---|
| 1.5 | 1.5 | $3(3)(1.5)$ 13.5 | $3(3)(1.5)$ 13.5 |
| $\frac{2}{\pi}$ | $\frac{2}{\pi}$ | $-\frac{\pi}{4}(1.5)^2\left(\frac{2}{\pi}\right)$ -1.125 | $-\frac{\pi}{4}(1.5)^2\left(\frac{2}{\pi}\right)$ -1.125 |
| 2.5 | 2.5 | $-\frac{1}{2}(1.5)(1.5)(2.5)$ -2.8125 | $-\frac{1}{2}(1.5)(1.5)(2.5)$ -2.8125 |

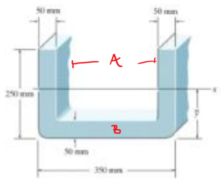
$\Sigma \bar{x} A = 9.5625$ $\Sigma \bar{y} A = 9.5625$

$$\bar{x} = \frac{\Sigma \bar{x} A}{\Sigma A} = \frac{9.5625}{6.1079} = 1.57 \text{ in.}$$

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{9.5625}{6.1079} = 1.57 \text{ in.}$$

$\boxed{\text{Centroid: } (1.57 \text{ in.}, 1.57 \text{ in.})}$

**6–83.** Determine the location $\bar{y}$ of the centroid of the cross-sectional area of the channel, and then calculate the moment of inertia of this area about this axis.




A: $2(250)(50) = 25000$

B: $250(50) = 12500$

$\bar{y}$: $\left(\frac{250}{2}\right) = 125$, $\left(\frac{50}{2}\right) = 25$

$\bar{y}A$: $3(125 \times 10^6)$, $.3125 \times 10^6$

$\Sigma = 3.4375 \times 10^6$

$$\bar{Y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{3.4375 \times 10^4}{37500} = 91.67\text{ mm.}$$

$$d_1 = \bar{Y}_1 - \bar{Y} = 125 - 91.67 = 33.33\text{ mm.}$$

$$d_2 = \bar{Y} - \bar{Y}_2 = 91.67 - 25 = 66.67\text{ mm.}$$

$$I_{x_1} = 2 \times \left(\frac{50 + 250^3}{12}\right) + 25000(33.33)^2$$

$$I_{x_1} = 157.99 \times 10^6\text{ mm.}^4$$

$$I_{x_2} = \left(\frac{250 + 50^3}{12}\right) + 12500(66.67)^2$$

$$I_{x_2} = 56.16 \times 10^6\text{ mm.}^4$$

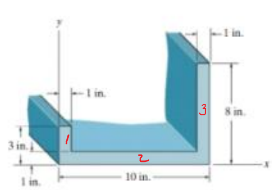
$$I_{x'} = (I_{x_1}) + (I_{x_2})$$

$$I_{x'} = 157.99 \times 10^6 + 56.16 \times 10^6$$

$$\boxed{I_{x'} = 214.15 \times 10^6\text{ mm.}^4}$$

**6–86.** Determine the moment of inertia of the cross-sectional area of the beam about the $y$ axis.

$$I_y = \frac{h_1 b_1^3}{12} = \frac{2(1)^3}{12} = .166\text{ in.}^4$$




$$I_1 = (I_y)_1 + A_1 (x_c)_1^2$$

$$I_1 = .166 + 2(.5)^2$$

$$I_1 = .666\text{ in.}^4$$

$$A_1 = 2\text{ in.}^2$$

$$x_c = \frac{b_1}{2} = \frac{1}{2}$$

$$I_{y_2} = \frac{10^3}{12}$$

$$I_{y_2} = 83.33\text{ in.}^4$$

$$I_2 = 82.33 + 10(5)^2$$

$$I_2 = 333.33\text{ in.}^4$$

2:

$$I_{y_2} = \frac{10^3}{12}$$

$$I_{y_2} = 83.33\text{ in.}^4$$

$$b' = b_2 h_2$$

$$b' = 10\text{ in.}^2$$

$$x_{c2} = \frac{b'}{2}$$

$$x_{c2} = \frac{10}{2}$$

$$x_{c2} = 5\text{ in.}$$

3:

$$I_{y3} = \frac{h_3 b_3^3}{12}$$

$$I_{y3} = \frac{7(1)^3}{12}$$

$$I_{y3} = .5833\text{ in.}^4$$

$$A_3 = b_3 h_3$$

$$A_3 = 7\text{ in.}^2$$

$$(x_c)_3 = b_2 - \frac{b_3}{2}$$

$I_2 = 82.33 + 10(5)$

$I_2 = 333.33 \text{ in.}^4$

$x_{c2} = 5 \text{ in.}$

$(x_c)_3 = b_2 - \frac{b_3}{2}$

$(x_c)_3 = 9.5 \text{ in.}$

$I_y = I_1 + I_2 + I_3$

$I_y = .666 + 333.33 + 632.33$

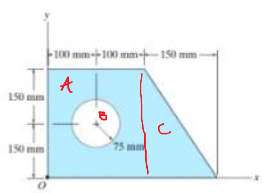
$\boxed{I_y = 966.32 \text{ in.}^4}$

$I_3 = (I_{y3}) + A_3 (x_c)_3^2$

$I_3 = .5833 + 7(9.5)^2$

$I_3 = 632.33 \text{ in.}^4$

*6-88. Determine the moment of inertia $I_y$ of the area about the $y$ axis.




| | A mm.² | $\bar{x}$ mm. | $\bar{y}$ mm. | $\bar{x}$ A mm.³ | $\bar{y}$ A mm.³ |
|---|---|---|---|---|---|
| A | 60,000 | 100 | 150 | 60,000 | 9,000,000 |
| B | -17,671.5 | 100 | 150 | | |
| C | 22,500 | | | | |

$$I_{total} = \begin{cases} I_{rectangle} = \frac{1}{12}(300)(200)^3 + (60,000)(100)^2 \\ -I_{circle} = \frac{1}{4}(75)^4 + (17,671.5)(100)^2 \\ I_{triangle} = \frac{1}{12}(300)(150)^3 + (22,500)(200)^2 \end{cases}$$

$\boxed{I_{total} = 1.5828 \times 10^9 \text{ mm.}^4}$