Design Problem 1

Thursday, October 29, 2020 5:51 PM

"I pledge my honor I have abided by the Stevens Honor system."

- Alex Gaskins

Design Problem #1. Gymnastic Rings

In gymnastics, the rings are an event that require a great deal of upper body strength. Each ring is supported by a strap that is attached to the metal frame. The metal frame is supported by several cables anchored to the floor, as shown below in Figure 1. In Figure 2, dimensions are given in meters.

Figure 1. Gymnastics Ring Frame Apparatus (Gymnova)

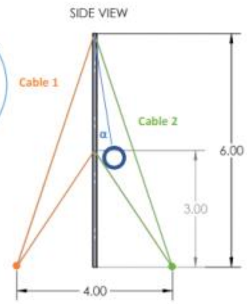


Figure 2. Right side view of Gymnastics Ring Frame, with strap angle α

As shown in the right side view of the frame, each cable is attached at the top of the frame (height = 6 m), looped around a small pulley at the floor and then attached at the middle of the frame (height = 3 m). There is a one cable at the front of the frame (Cable 1) and another cable at the back of the frame (Cable 2). This support system is repeated on the left side of the frame. The legs of the frame simply rest on the floor, and the small foot at the base of the leg allow for a moderate reaction moment to be generated. See inset in Figure 1.

Figure 2. Side view of the change in strap angle α during a backward roll.

Problem Statement:

You will perform an analysis of the tension in the support cables of the metal frame as a gymast performs a backward roll on the rings. As the gymnast performs a backward roll on the rings, the angle of the straps will change, causing a change in the tension in the support cables, Cable 1 and Cable 2. See Figure 2.

Assumptions:

1. The gymnast has a mass of 51 kg.
2. The tension in each cable is 500 N when α = 0°, i.e. the gymast is hanging straight down.
3. The vertical reaction force at the frame leg remains constant throughout this motion.
4. The rings remain the same distance apart during the exercise.

Write an expression for the tension in each support cable as a function of the angle of the straps from α = 0° to 10°. Then generate a plot of the Cable Tension vs. Strap Angle (α) in Excel or Matlab. Be sure to label your axes appropriately, including units.

Remember that there are **two** straps (left/right) and **two** sets of cables (left/right) on either side of the ring frame!

Using your plot, answer the following questions:

a. In which cable does the maximum tension occur? What is this maximum tension? At what strap angle does this occur?
b. Do the tensions in Cables 1 and 2 behave as you expected? Why or why not?

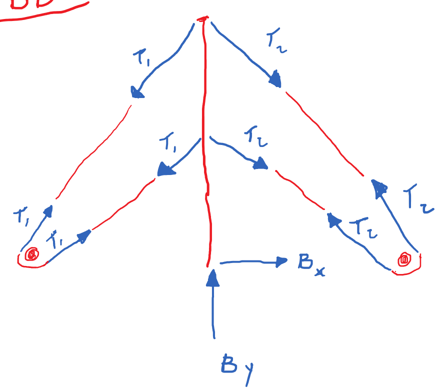
Submission Instructions:

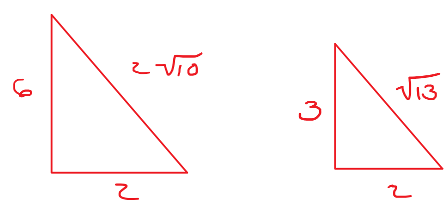
Create a report using Word or Powerpoint that includes the following:

1. **Hand calculations** (FBDs + Equilibrium equations) used to express the tension in each support cable as a function of the strap angle. You must show your work clearly in order to receive full credit.
2. **Plot** of Cable Tension vs. Strap Angle (α), using Excel or Matlab.
3. **Answers** to the follow-up questions.

Combine into a single document and upload as PDF to Gradescope.

FBD:





$W = 51(9.8) = 500\text{ N.}$

$\Sigma F_y = 0$

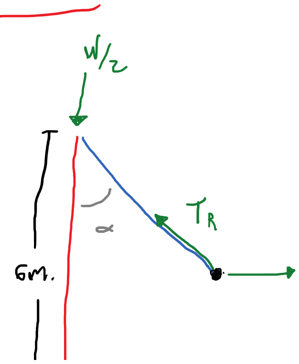
$B_y - \frac{W}{2} - 2T\left(\frac{6}{2\sqrt{10}}\right) - 2T\left(\frac{3}{\sqrt{13}}\right) = 0$

$B_y - \left(\frac{500}{2}\right) - 2(500)\left(\frac{6}{2\sqrt{10}}\right) - 2(500)\left(\frac{3}{\sqrt{13}}\right) = 0$

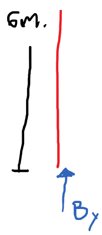
$B_y = 2030.73\text{ N.}$

$B_x = 0\text{ N.}$

Ring FBD:



$F = 250\tan(\alpha)$



$\bullet \longrightarrow F = 250 \tan(\alpha)$

When α changes:

$W = 500$ N.

$$\sum F_y = 0$$

$$\frac{W}{2} - T_A \cos(\alpha) = 0$$

$$T_A \cos(\alpha) = 250$$

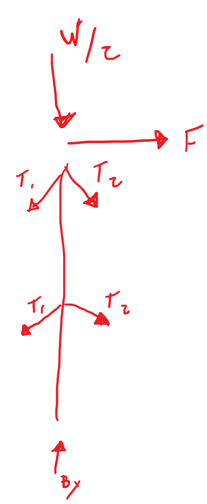
$$T_A = \frac{250}{\cos(\alpha)}$$

$$\sum F_x = 0$$

$$F - T_A \sin(\alpha) = 0$$

$$F - \left(\frac{250}{\cos(\alpha)}\right) \sin(\alpha) = 0$$

$$F = 250 \tan(\alpha)$$



$$\sum F_y = 0$$

$$B_y - \frac{W}{2} - T_1\left(\frac{6}{2\sqrt{10}}\right) - T_2\left(\frac{6}{2\sqrt{10}}\right) - T_1\left(\frac{3}{\sqrt{13}}\right) - T_2\left(\frac{3}{\sqrt{13}}\right) = 0$$

$$2030.73 - 250 = \frac{6}{2\sqrt{10}}(T_1 + T_2) + \frac{3}{\sqrt{13}}(T_1 + T_2)$$

$$1780.73 = \left(\frac{6}{2\sqrt{10}} + \frac{3}{\sqrt{13}}\right)(T_1 + T_2)$$

$$T_1 + T_2 = 1000$$

$$\sum F_x = 0$$

$$\Sigma F_x = 0$$

$$F - T_1\left(\frac{1}{\sqrt{10}}\right) + T_2\left(\frac{1}{\sqrt{10}}\right) - T_1\left(\frac{2}{\sqrt{13}}\right) + T_2\left(\frac{2}{\sqrt{13}}\right) = 0$$

$$250\tan(\alpha) = \left(\frac{1}{\sqrt{10}} + \frac{2}{\sqrt{13}}\right)(T_1 - T_2)$$

$$T_1 - T_2 = 287.05\tan(\alpha)$$

<u>For T_1:</u>

$$T_2 = 1000 - T_1$$

$$2T_1 - 1000 = 287.05\tan(\alpha)$$

$$\boxed{T_1 = 143.525\tan(\alpha) + 500}$$

<u>For T_2:</u>

$$T_1 = 1000 - T_2$$

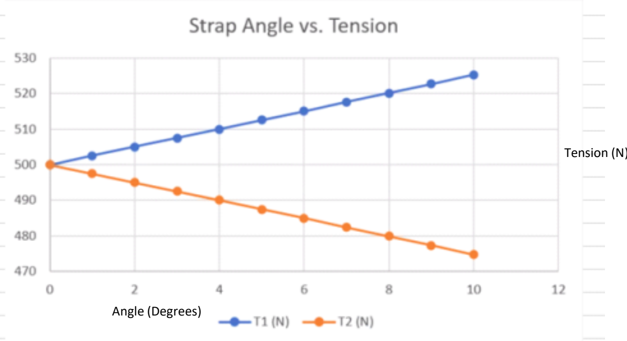
$$(1000 - T_2) - T_2 = 287.05\tan(\alpha)$$

$$-2T_2 = 287.05\tan(\alpha) - 1000$$

$$\boxed{T_2 = 500 - 143.525\tan(\alpha)}$$

<u>Graph:</u>

	A	B	C	D	E
1	Angle(α)	T1 (N)	T2 (N)		Angle(α) in Degrees
2	0	500	500		0
3	0.01745	502.505	497.495		1
4	0.03491	505.012	494.988		2
5	0.05236	507.522	492.478		3
6	0.06981	510.036	489.964		4
7	0.08727	512.557	487.443		5
8	0.10472	515.085	484.915		6
9	0.12217	517.623	482.377		7
10	0.13963	520.171	479.829		8
11	0.15708	522.732	477.268		9
12	0.17453	525.307	474.693		10



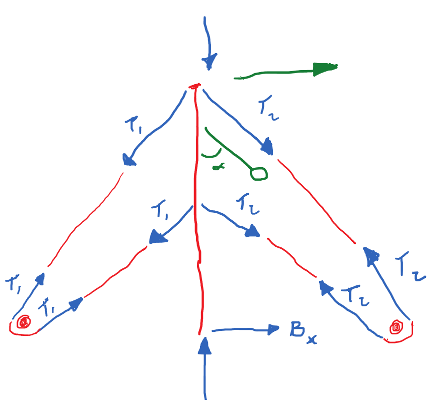
a. In which cable does the maximum tension occur? What is this maximum tension? At what strap angle does this occur?

The maximum tension occurs in T_1. From my graph, which displayed a function of alpha (α) between 0 and 10 degrees. According to the data, the maximum tension is 525.307 N, which occurs at $\alpha = 10°$.

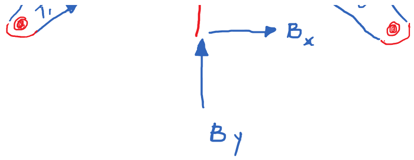
68	1.15192	822.362	177.638	66
69	1.16937	838.124	161.876	67
70	1.18682	855.237	144.763	68
71	1.20428	873.895	126.105	69
72	1.22173	894.332	105.668	70
73	1.23918	916.827	83.1731	71
74	1.25664	941.725	58.2755	72
75	1.27409	969.449	30.5509	73
76	1.29154	1000.53	-0.5312	74
77	1.309	1035.64	-35.643	75
78	1.32645	1075.65	-75.647	76
79	1.3439	1121.68	-121.68	77
80	1.36136	1175.23	-175.23	78
81	1.37881	1238.37	-238.37	79
82	1.39626	1313.97	-313.97	80
83	1.41372	1406.18	-406.18	81
84	1.43117	1521.23	-521.23	82
85	1.44862	1668.92	-668.92	83

In a more general aspect, I found that the largest value of T_1 before it exceeded 1000 N. was 969.449 N, which occurred at $\alpha = 73°$.

b. Do the tensions in Cables 1 and 2 behave as you expected? Why or why not?



Yes, I expected T_1 to increase and T_2 to decrease as α increased. From simply looking at the drawing, one can see that as the force made by the gymnast increase in the positive x-direction, there is more force



made by the [illegible] positive x-direction, there is more force being pulled by the cable on the left (T_1) in order to prevent the frame from bending to the right as a result of the horizontal force being made by the gymnast.