Homework 31-33

Saturday, May 8, 2021 4:52 PM

"I pledge my honor I have abided by the Stevens Honor system."

- Alex Haskins

Problem 31.1. Assume you have 20 numbered red balls and 10 numbered black balls.

1. How many ways are there to select two balls (of either color)?
2. How many ways are there to select one red ball and one black ball?
3. How many ways are there to select 3 red balls, or 3 black balls?
4. How many ways are there to select 3 red balls and 3 black balls?
5. How many ways are there to arrange all 30 balls in a row?
6. How many ways are there to arrange all 30 balls in a row in such a way that all black balls are adjacent and all red balls are adjacent (i.e. either first 10 balls are black and last 20 are red, or first 20 are red and last 10 are black)?

For (a), (b), (c), (d) we assume that order in which you select balls doesn't matter.

1.) Red or black can be chosen

$20 + 10 = 30$

$$\frac{30(30-1)}{2} = \boxed{435 \text{ ways}}$$

2.)

$$20 \times (10 \times) = 20(10) = \boxed{200 \text{ Ways}}$$

3.)

$$= 20x + 10x$$

$$= \frac{20(19)(3)}{3(2)} + \frac{10(9)(8)}{3(2)}$$

$= (60(19) + 120) = \boxed{1260 \text{ ways}}$

4.)

$$20 \times (10 \times) = \frac{20(19)(18)}{3(2)}$$

$$= (60(19)(120)) = \boxed{136800 \text{ ways}}$$

5.)

$$\frac{30!}{20!(10!)} = \boxed{30045015 \text{ ways}}$$

6.) $20!(10!)$ red, black or black, red

$$20!(10!) + 10!(20) = \boxed{2(20!(10!)) \text{ ways}}$$

Problem 32.1. Assume you want to create a piano melody. For simplicity, assume we can't press more than one key at a time (i.e. no harmonies) and you press each key for exactly one second. Piano has 88 keys.

a) How many 1 minute long piano melodies are there?

b) We say that a piano melody is **simple** if it uses at most 10 keys.

Assume you want to pick 10 keys that you want to use for a simple melody. How many possible choices of those 10 keys are there?

c) Given a fixed set of 10 keys, how many 1 minute long melodies are there that use only those 10 keys?

d) *(tricky)* Assume the answer for part (b) is B and the answer for part (c) is C. Is it true that the total number of 1 minute long simple melodies is $B \cdot C$? Explain your

d) *(tricky)* Assume the answer for part (b) is B and the answer for part (c) is C. Is it true that the total number of 1 minute long simple melodies is $B \cdot C$? Explain your answer!

A.) Each key has 88 choices

$\boxed{88^{60} \text{ melodies}}$

B.) 88 choose 10 = $\boxed{\binom{88}{10}}$

c.) 10 keys

$\boxed{10^{60} \text{ melodies}}$

D.) B = selection of 10 keys

C = melodies using 10 keys

Yes, B·C gives all melodies using 10 keys where B selects 10 keys and C gives the number of 10 keys in one minute.

Problem 33.1. Assume you manage a start-up that has 4 interns (say Alice, Bob, Melody and Zac). Assume you want to meet them several times one-on-one in alphabetical order (for example first meet Alice x times, then Bob y times, then Melody z times and finally Zac w times). Assume also you want to have 10 total meetings (that is $x + y + z + w = 10$).

a) Assume you are OK not meeting some interns (that is $x, y, z, w \geq 0$) and you want to hold total 10 meetings. How many ways can you schedule those 10 meetings?

b) *(tricky)* Assume you want to meet each of them at least once ($x, y, z, w > 0$). How many ways can you host those 10 meetings?

c) Assume now that you have a total 6 applicants, and you want to select 4 of them to be your interns. How many ways can you do that?

d) Assume the answer to part (b) is B and the answer to part (c) is C. What is the number of ways you can select 4 out of 6 applicants and then schedule 10 meetings in such a way that you meet each out of four selected applicants at least once? Write your answer in terms of B, C.

A.) $n = 10 \quad r = 4$

$$\frac{n + r - 1}{r - 1} = \binom{13}{3} = \boxed{286 \text{ ways}}$$

B.) $x, y, z, w \geq 1$

$a = x - 1 \quad b = y - 1 \quad c = z - 1 \quad d = w - 1$

$a + 1 + b + 1 + c + 1 + d + 1 = 10$

$a + b + c + d = 6$

$$\binom{6+4-1}{4-1} = \binom{9}{3} = \boxed{84 \text{ ways}}$$

C.) 4 out of 6 interns

$$\binom{6}{4} = \boxed{15 \text{ ways}}$$

D.) 4 out of 6 interns

scheduling 10 meetings with at least one person

B·C

$$\binom{6}{4}\binom{6+4-1}{4-1} = \binom{6}{4}\binom{9}{3} = \boxed{1260 \text{ ways}}$$