"I pledge my honor I have abided by the Stevens Honor system."
- Alex [signature]

Exam 2

Wednesday, October 27, 2021 1:48 PM

1. A student created random variable X from the binomial distribution with 10 trials and probability of success 0.4. If this student randomly selects 60 numbers from this distribution, what is the probability that the sum of these 60 numbers is less than 245?

$P(\text{Success}) = 0.4(10) = 4$

Binomial

$\text{Variance} = 10(0.4)(0.6) = 2.4$

$\sqrt{2.4} = 1.5492$

$\sigma = \dfrac{1.5492}{\sqrt{60}} = 0.2$

$n = 60 \qquad \text{sum} < 245 = \dfrac{245}{60} = 4.0833$

$P(x < 4.0833) = \dfrac{(4.0833 - 4)}{1.5492} / \sqrt{60} = 0.4165$

$P(z < 0.4165) = \boxed{0.6615}$

2. In a professional gathering of entrepreneurs, the probability that a randomly chosen person is from corporation, partnership, or a non-profit is 0.24, 0.30, and 0.46 respectively.

a) If a group of 10 people are chatting, what is the probability that there are 3 partnership and at least 6 non-profit persons?

b) If a group of 10 people are chatting, what is the probability that there are 3 partnership persons?

c) What is the probability that you need to meet at least 6 persons to meet 3 partnership members?

A.) $P(x_2 = 3 ; x_3 \geq 6)$

Multinomial

$= \dfrac{10!}{3!6!}[0.24][0.30]^3[0.46]^6 + \dfrac{10!}{3!7!}[0.24]^0[0.3]^3[0.46]^7$

$= \boxed{0.0657}$

B.) Binomial

$P(x_2 = 3) = {10 \choose 3}[0.3]^3[0.7]^{10-3} = \boxed{0.2668}$

C.) Negative Binomial

$P(Y \geq 6) = 1 - P(Y \leq 5) = 1 - \left[{3-1 \choose 3-1}(0.3)^3(0.7) + {4-1 \choose 3-1}(0.3)^3(0.7)^1 + {5-1 \choose 3-1}(0.3)^3(0.7)^2\right]$

$= \boxed{0.8369}$

3. The lifetime of a car battery can be modeled as a Weibull distribution with a=1.

a) If the probability that a battery works longer than 10 years is 0.4, find the value of the parameter λ?

b) What is the time to which 80% of the batteries work?

Weibull $1-e^{-(\lambda t)^{a}}$

A.) $P(x>10)=0.4$

$= 1 - P(x \leq 10) = 1 - \left(1 - e^{-(\lambda(10))^{1}}\right) = 0.4$

$1 - 1 + e^{-10\lambda} = 0.4$

$$\frac{-10\lambda}{-10} = \frac{\ln(0.4)}{-10}$$

$$\boxed{\lambda = 0.09163}$$

B.)

$P(x \leq t) = 0.80$

$1 - \left(1 - e^{-(0.09163)(t)^{1})}\right) = 0.8$

$t = \boxed{2.43 \text{ years}}$

4. The time for schools to change their syllabus to a new one in years is found to have an exponential distribution with a mean of 3 years.

a) A school changed its syllabus last year. Find the probability that the school will keep their current syllabus more than 4 years.

b) Among 100 schools, what is the probability that more than 30 schools will keep their current syllabus for less than 4 years?

Exp. distribution mean of 3 years

A.) $\lambda = \frac{1}{3}$

$$P(x>4) = 1 - \left[-e^{-x/3}\right]_0^4 = 1 - \left[-e^{-4/3} + 1\right]$$

$$= e^{-4/3} = 0.2636 = \boxed{26.36\%}$$

B.)

$$P(x<4) = 1 - 0.2636 = 0.7364$$

$$\mu = np = 100(0.7364) = 73.64$$

$$\sigma = \sqrt{np(1-p)} = \sqrt{73.64(0.2636)} = 4.405$$

$$P(x>30) = P\left(\frac{x-\mu}{\sigma} > \frac{30-73.64}{4.405}\right)$$

$$P(z>-9.905)$$

$$\approx \boxed{0.9999}$$

5. The mean daily flow in a river is modeled as a lognormal distribution. If the probability that the flow is greater 1400 cfs is 60% and the flow is greater than 100 cfs is 90%, find the mean of the log-transformed flow of the river? [use log to the base 10]

$0.6 \rightarrow 1400$

$0.9 \rightarrow 100$

log (x)	3.146	2
P (x)	0.6	0.9

$E(x) = x[P(x)]$

$E(x) = [3.146(0.6) + 2(0.9)]$

$E(x) = 3.688$

6. There are 9 engineering students and 6 arts students in a group. If you pick a committee of 5 randomly from this, what is the probability that you will have exactly 3 engineering students?

Binomial Dist.

$$\binom{5}{3}\left(\frac{9}{15}\right)^3\left(1-\frac{9}{15}\right)^2 = \boxed{0.3456}$$