Homework 5

Sunday, March 21, 2021 9:53 PM

"I pledge my honor I have abided by the Stevens Honor system."

- Alex Gaskins

1. A steady flow compressor is used to compress helium from 15 psia and 70°F at the inlet to 200 psia and 600°F at the outlet. The outlet area and velocity are 0.01ft² and 100 ft/s, respectively, and the inlet velocity is 50 ft/s. Determine the mass flow rate and inlet area. [10]

$$R_{air} = 53.35 \frac{ft \cdot lbf}{lb \cdot R}$$

$$\rho_1 = \frac{P_1}{R_{air} \cdot T_1} = \frac{2160}{53.35 \cdot 529.67} = .0764 \text{ lb/ft}^3$$

$$\rho_2 = \frac{P_2}{R_{air} \cdot T_2} = \frac{28800}{53.35 \cdot 1059.67} = .509 \text{ lb/ft}^3$$

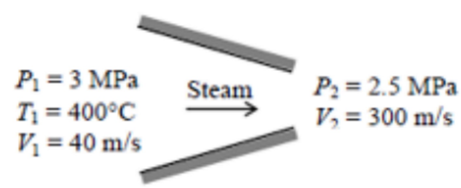
$$V_2 = A \cdot \bar{V}_2 = .01(100) = 1 \text{ ft}^3/s$$

$$\dot{m} = \rho_2 \cdot V_2 = .509(1) = .509 \text{ lb/s}$$

$$A_i = \frac{\dot{m}}{\rho_1 \cdot \bar{V}_1} = \frac{.509}{.0764(50)} = .133 \text{ ft}^2$$

$$\boxed{\begin{array}{l} \text{flow rate} = .509 \text{ lb/s} \\ \text{inlet area} = .133 \text{ ft}^2 \end{array}}$$

2. Steam flows adiabatically through the nozzle shown in the diagram. What is the exit temperature and the ratio of the inlet and outlet area (A_1/A_2)? [20]



$$A_1 v_1 = A_2 v_2 = \frac{A_1}{A_2} = \frac{V_2}{V_1} = \frac{300}{40} = 7.5$$

$$\frac{P_1 v_1'}{T_1} = \frac{P_2 v_2'}{T_2} \; ; \; \frac{P_1 A_1 V_1}{T_1} = \frac{P_2 A_2 V_2}{T_2}$$

$$\frac{3(40)A_1}{673} = 2.5(300)A_2\left(\frac{1}{T_2}\right)$$

$$\frac{1}{T_2} = \frac{3(40)}{673(2.5)(300)}\left(\frac{A_1}{A_2}\right)$$

$$\frac{1}{T_2} = \frac{3(40)}{673(2.5)(300)}\left(\frac{A_1}{A_2}\right)$$

$$\boxed{T_2 = 287.68^\circ C}$$

3. Air flows through an adiabatic turbine that is in steady operation. The air enters at 150 psia, 900°F, and 350 ft/s and leaves at 20 psia, 300°F, and 700 ft/s. The inlet area of the turbine is 0.1 ft². Determine the mass flow rate of the air and the power output of the turbine. [20]

$$v_1 = \frac{RT_1}{P_1} = \frac{.3704\ (1360)}{150} = 3.358\ \frac{ft^3}{lbm}$$

$$\dot{m} = \frac{A_1 V_1}{v_1} = \frac{.1\,(350)}{3.358} = 10.42\ \frac{lbm}{s}$$

$$E_i = E_f$$

$$\dot{m}\left(h_1 + \frac{V_1^2}{2}\right) = W_{out} + \dot{m}\left(h_2 + \frac{V_2^2}{2}\right) + Q_{out}$$

$$W_{out} = \dot{m}\left(h_2 - h_1 + \frac{V_2^2 - V_1^2}{2}\right) = -m\left(c_p(T_2 - T_1) + \frac{V_2^2 - V_1^2}{2}\right)$$

$$W_{out} = -10.42\left(.250\,(300-900) + \frac{700^2 - 350^2}{2\,(25{,}037)}\right) = 1486.5\ \frac{Btu}{s}$$

$$P = 1486.5\ (1.055) = 1568\ kW$$

$$\boxed{\begin{array}{l} \text{flow rate} = 10.42\ lbm/s \\ \text{power output} = 1568\ kW \end{array}}$$

4. Refrigerant-134a enters the expansion (throttling) valve of a refrigeration system at 120 psia as a saturated liquid and leaves at 20 psia. Determine the temperature and internal energy changes across the valve. [20]

$$E_i = E_f\ ;\ h_1 = h_2$$

Using table A-11 and 13

120 psia $\rightarrow$

$h_1 = 41.79 \frac{Btu}{lbm}$

$u_1 = 41.49 \frac{Btu}{lbm}$

$T_1 = 90.49\,°F$

20 psia $\rightarrow$

$h_2 = h_1 = 41.79 \frac{Btu}{lbm}$

$u_2 = 38.96 \frac{Btu}{lbm}$

$T_2 = -2.43\,°F$

$\Delta T = T_2 - T_1 = -2.43 - 90.49 = -92.9\,°F$

$\Delta u = u_2 - u_1 = 38.96 - 41.49 = -2.53 \frac{Btu}{lbm}$

$$\boxed{\begin{array}{l} \Delta T = -92.9\,°F \\ \Delta u = -2.53 \frac{Btu}{lbm} \end{array}}$$

5. The volume flow rates of an ideal gas at the exit of an adiabatic compressor and its inlet are denoted as $\dot{\forall}_{out}$ and $\dot{\forall}_{in}$ respectively ($\dot{\forall}_{out} \neq \dot{\forall}_{in}$). The compressor is operating steadily. [30 - 3 points for each]
   a. How does the pressure vary from the inlet to the outlet of a compressor?

      It increases from the inlet to the outet.

   b. Does this device require a work input or does it produce a work output?

      It requires an external work source.

   c. How does the temperature vary from the inlet to the outlet? Use the energy conservation equation to justify your answer.

      It decreases from the inlet to the outet.

      $\Delta U = \Delta W$

      $U_f - U_i = \Delta W$

      Work done is positive

      $U_f < U_i$

   d. Is the mass flow rate constant? Why? Use an equation to justify this answer.

Yes, by conservation of mass.

$$\Delta E = E_{in} - E_{out} \qquad m_{cv} = m - m_o$$

$$\dot{E}_{in} = \dot{E}_{out} \qquad m = m_o$$

$$\dot{W}_{in} + \dot{m}h_1 = \dot{Q}_{out} + \dot{m}h_2$$

e. What is the relationship between mass flow rate and volume flow rate?

$$\dot{m} = \rho \dot{V} \qquad [\rho_1 A_1 v_1 = \rho_2 A_2 v_2]$$

Mass flow rate is dependent on volume flow rate.

f. Does the density increase or decrease from the inlet to the outlet? Justify this with equations. You may assume that $(P/T)_{out} > (P/T)_{in}$.

$$\rho = \frac{\dot{m}}{\dot{V}}$$

To account for a steady flow, density increases as volume decreases

Density increases from the inlet to the outlet.

g. How does the volume flow rate at the outlet compare to that at the inlet? (greater, lesser, same) Justify your answer.

$$\rho = \frac{\dot{m}}{\dot{V}}$$

If density increases, then $\dot{V}$ is lesser at the outlet than at the inlet.

h. Is mass flow rate always constant in a steady flow device?

Yes

$$\dot{m} = \dot{m}_o$$

According to CV

i. Is volume flow rate always constant in a steady flow device?

$\dot{V} = AV$ volume flow rate is not always constant.

[Dependent on volume and area]

j. Under what conditions is the volume flow rate a constant in a steady flow device?

when $A_1 V_1 = A_2 V_2$

[Constant area and CV (control volume)]