Quiz 3

Monday, November 23, 2020 9:52 AM

"I pledge my honor I have abided by the Stevens Honor system."

-Alex Haskins

- Standard Form; First Order Differential Equation

$$\tau \cdot \frac{dx(t)}{dt} + x(t) = f(t)$$

- Second-order systems;

$$\frac{d^2x(t)}{dt^2} + 2\alpha\frac{dx(t)}{dt} + \omega_o^2 \cdot x(t) = f(t)$$

$$\zeta = \frac{\alpha}{\omega_o}$$

$$S_1 = -\alpha + \sqrt{\alpha^2 - \omega_o^2}$$

$$S_2 = -\alpha - \sqrt{\alpha^2 - \omega_o^2}$$

For $\zeta > 1$; $x_c(t) = K_1 e^{S_1 t} + K_2 e^{S_2 t}$

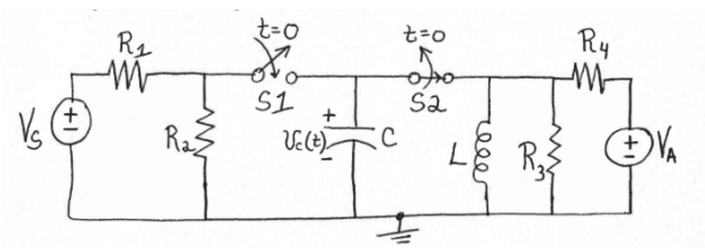
For $\zeta = 1$; $x_c(t) = K_1 e^{S_1 t} + K_2 t e^{S_1 t}$

For $\zeta < 1$; $x_c(t) = K_1 e^{-\alpha t}\cos(\omega_n t) + K_2 e^{-\alpha t}\sin(\omega_n t)$

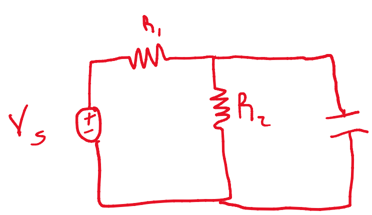
$$\omega_n = \sqrt{\omega_o^2 - \alpha^2}$$

Problem One: Consider the circuit shown below. The input voltage source, V_s, and the input voltage source, V_A, are each a constant dc source. The circuit has been operating for a long time with Switch One (S1) open prior to t = 0 and Switch Two (S2) closed prior to t = 0. You are to solve for the capacitor voltage, $v_c(t)$, for t > 0.

Note; I do not provide numerical values for this problem. Thus your answer should be expressed in terms of the given network elements.



a) Write the first order differential equation for the capacitor voltage, $V_c(t)$ for t > 0, in standard form. Do not enter your answer in the online answer box; please enter your answer in your worksheet. Please clearly label your answer. **(20 points)**



$t > 0$

$$i_c = C\frac{dV_c}{dt}$$

$$-V_s + (R_1 + R_2)\, i_c + V_c = 0$$

$$\boxed{(R_1 + R_2)\, C\frac{dV_c}{dt} + V_c = V_s}$$

$$\boxed{(R_1 + R_2) C \frac{dV_c}{dt} + V_c = V_s}$$

$$\tau = (R_1 + R_2) C$$

$$V_c = K e^{-t/\tau} + V_s$$

$$V_c = K e^{-t/(R_1+R_2)C} + V_s$$

At $t = 0^-$, $V_c = V_A$

$$V_A = K e^0 + V_s$$

$$K = V_A - V_s$$

$$V_c = (V_A - V_s) e^{-\frac{t}{(R_1+R_2)C}} + V_s$$

b) What is the expression for the time constant for this system? Do not enter your answer in the online answer box; please enter your answer in your worksheet. Please clearly label your answer. **(10 points)**

$$\tau = (R_1 + R_2) C$$

From part a

c) Determine the homogeneous solution. Do not enter your answer in the online answer box; please enter your answer in your worksheet. Please clearly label your answer. (**5 points)**

$$V_c = K e^{-\frac{t}{(R_1+R_2)C}}$$

From part a

d) Determine the particular solution. Do not enter your answer in the online answer box; please enter your answer in your worksheet. Please clearly label your answer. (**5 points)**

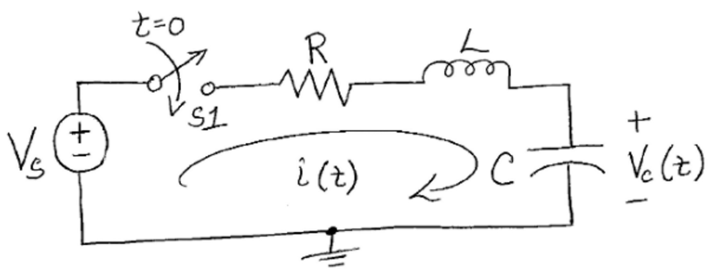
$y_p = V_s$ (V_c approaches V_s)

e) Determine the complete solution (please solve for all constant(s)). Do not enter your answer in the online answer box; please enter your answer in your worksheet. Please clearly label your answer. Hint; to solve for the constant K, determine Vc(t=0⁻). Then, as we know, Vc(t=0⁺) equals Vc(t=0⁻). You can do this. **(20 points)**

$$\boxed{V_c = (V_R - V_s)\, e^{-\frac{t}{(R_1+R_2)C}} + V_s}$$

(From part a)

Problem Two; Consider the circuit shown below. The circuit has been operating for a long time with Switch One (S1) open prior to t = 0. The voltage source, V_S, is a constant dc source.



a) Write the second-order differential equation in standard form for the capacitor voltage $v_c(t)$ for t > 0. (Note; the only variable which should appear in your equation is $v_c(t)$). My suggestion is to use mesh analysis. Please note; I do not provide numerical values for the resistor, capacitor, inductor, and the source. Thus, your answer should be expressed in terms of the given variables. Do not enter your answer in the online answer box; please enter your answer in your worksheet. Please clearly label your answer. **(15 points)**

$$-V_s + R\,i(t) + L\frac{di(t)}{dt} + V_c = 0$$

$$LC\frac{d^2V_c}{dt^2} + RC\frac{dV_c}{dt} + V_c = V_s \qquad i(t) = C\frac{dV_c}{dt}$$

$$\boxed{\frac{d^2V_c}{dt^2} + \frac{R}{L}\frac{dV_c}{dt} + \frac{1}{LC}V_c = \frac{V_s}{LC}} \qquad \frac{di(t)}{dt} = C\frac{d^2V_c}{dt^2}$$

b) Determine the requirement on the ratio of C/L for the second-order system to be under-damped. Do not enter your answer in the online answer box; please enter your answer in your worksheet. Please clearly label your answer. **(5 points)**

Under damped

$\zeta < 1$ $\qquad\qquad$ $\frac{1}{}$

$z < 1$

$2\alpha = \frac{R}{L}$

$\alpha = \frac{R}{2L}$

$\omega_0^2 = \frac{1}{LC}$

$\omega_0 = \frac{1}{\sqrt{LC}}$

$z = \frac{\alpha}{\omega_0}$

$L = \sqrt{L} \times \sqrt{L}$

$z = \frac{R\sqrt{LC}}{2L} = \frac{R\sqrt{C}}{2\sqrt{L}}$

$\frac{R\sqrt{C}}{2\sqrt{L}} < 1$

$\frac{\sqrt{C}}{\sqrt{L}} < \frac{2}{R}$

$$\boxed{\frac{C}{L} < \frac{4}{R^2}}$$

c) You are now given that the system is critically-damped (and not under-damped). Determine the complete solution. Since there are no numerical values given, you do not have to solve for the constants. (You do not have to solve for any "K" value(s) and any "s" value(s)). Do not enter your answer in the online answer box; please enter your answer in your worksheet. Please clearly label your answer. **(10 points)**

Critically damped

$z = 1$

$V_{c_h} = k_1 e^{st} + k_2 t e^{st}$

$V_{c_p} = V_s$

$$\boxed{V_c = k_1 e^{st} + k_2 t e^{st} + V_s}$$

d) You are now given the following two initial condition. (These initial conditions are established by another circuit that is not shown). The system is still critically-damped.

$$V_c(t = 0^+) = 15 \text{ Volts}$$

$$i(t = 0^+) = 0.5 \text{ Amps}$$

For this part of the problem only, C = 1 μF and Vs = 5 Volts. Solve for K_1 and K_2. Please note; you do not have to solve for the parameter "s". Do not enter your answer in the online answer box; please enter your answer in your worksheet. Please clearly label your answer. **(10 points)**

$$V_c = k_1 e^{st} + k_2 t e^{st} + V_s$$

$$i(t) = C\frac{dV}{dt}$$

$$\frac{dV_c}{dt} = k_1 s e^{st} + k_2 t s e^{st} + k_2 e^{st}$$

$$\frac{1}{C} i(t) = \frac{1}{1\times 10^{-6}} (.5)$$

$$\frac{dV_c}{dt} = 5\times 10^5$$

$$15 = k_1 + V_s$$

$$k_1 = 15 - V_s$$

$$k_1 = 10$$

$$5\times 10^5 = k_1 s + k_2$$

$$k_2 = (5\times 10^5 - 10s)$$

$$\boxed{V_c = 10 e^{st} + (5\times 10^5 - 10s) t e^{st} + 5}$$