

■ Q1: Read below paper and write a summary (a minimum of half a page) to describe what you have learn from it. (20 points)

Oluwasegun Ayokunle Somefun, Kayode Akingbade, and Folasade Dahunsi, "The dilemma of PID tuning," Annual Reviews in Control, Vol. 52, pp. 65–74, 2021.

The article provided a detailed overview on control theory being applied to digital systems, where the importance of feedback became of concern in order to address "the need for guaranteed stable and accurate closed-loop performance of dynamical systems, especially in motor-control tasks." For the advancement of automated systems, feedback is a requirement, and a lot of these automation tasks still fundamentally rely on a form of Proportional Integral Derivative (PID) control. While this method is conventional, it has received criticism concerning the complexity and costs that come with tuning the PID control law as these advancements become more necessary. "PID control design (tuning) methods are presented from the design perspective of being either **plant-model based**, **plant-model free** or a **hybrid** of both perspectives." With a plant-model based control system, one of the most difficult issues that arise when it comes to PID tuning is the development of automatic methods for closed-loop input/output settling time. PID control is typically represented or designed as a continuous-time system but implemented as a discrete-time system, with the major drawback being that computers cannot interpret feedback with the same accuracy as the theoretical error-response models and analogous data based on feedback in neural networks, which are inherently much more complex in general. However, PID is still ideal for a closed-loop system, since it is an incorporation of the natural law of feedback nonetheless. Unfortunately, we find from this that another disadvantage that is difficult to address lies not in the ambiguity of the PID control law, but instead in the uncertainty of the system itself. Plant-model free systems provide more complexity when it comes to addressing a uniform tuning method that invokes consistent precision, with a notable feature of plant-model free methods being that they realistically assume that the true system is inherently nonlinear and uncertain. In the case of hybrid models, the varying mixture between both plant-model based and plant-model free systems causes a viable approach to lean towards "mixing different robust design objectives, such as specification of gain and phase margins, by employing constrained optimization and multi-objective optimization." While there is difficulty due to its complex nature, the involvement of plant-model based constraints can actually help provide more accuracy when it comes to PID tuning, as opposed to a strictly plant-model free system. We are still seeing new PID tuning methods being theorized, but as of now, it is difficult to keep up with the increasing complexity of modern control systems while maintaining some ambiguity in the nature of how these systems should be assessed. For now, our best bet is to continue improving how feedback is handled in these complex systems.

- Q2: Given a second-order system  $G = \frac{1}{s^2 + 2\gamma s + 1}$ , we add  $D = \frac{K(s+a)}{s+b}$  in series with  $G(s)$  in a unity feedback structure. (40 points)
- a. Ignoring the stability, what are the constraints on  $K$ ,  $a$ , and  $b$  so that the system is Type 1?
  - b. What are the constraints on  $K$ ,  $a$ , and  $b$  so that the system is stable and Type 1? (Hint: you may need to use Routh's Stability Criterion we learned from Lecture 2)

A.)

$$\frac{Y(s)}{R(s)} = \frac{K(s+a)}{(s^2 + 2\gamma s + 1)(s+b) + K(s+a)}; R - Y = \frac{(s^2 + 2\gamma s + 1)(s+b)}{(s^2 + 2\gamma s + 1)(s+b) + K(s+a)} R(s)$$

$$\lim_{s \rightarrow 0} = \frac{(s^2 + 2\gamma s + 1)(s+b)}{(s^2 + 2\gamma s + 1)(s+b) + K(s+a)} R(s)$$

To be Type 1:

$$b = 0, K \neq 0, a \neq 0$$

B.)

$$b = 0, R(s) = \frac{1}{s} \text{ and Type 1}$$

$$\lim_{s \rightarrow 0} = \frac{s^2(s^2 + 2\gamma s + 1)}{(s^2 + 2\gamma s + 1)(s+K)(s+a)} \left[ \frac{1}{s^2} \right]$$

$$\text{Satisfy equation } s^2 + 2\gamma s^2 + (K + 1)s + Ka = 0$$

Check stability

$$s^3 \mid \quad 1 \quad K + 1 \quad \frac{2\gamma(K+1) - 1(Ka)}{2\gamma} = (K + 1) - \frac{Ka}{2\gamma}$$

$$s^2 \mid \quad 2\gamma \quad Ka$$

$$s^1 \mid (K + 1) - \frac{Ka}{2\gamma} \quad 0 \quad \frac{((K+1) - \frac{Ka}{2\gamma})Ka - 2\gamma(0)}{(K+1) - \frac{Ka}{2\gamma}} = Ka$$

$$s^0 \mid \quad Ka$$

$$Ka > 0 \quad (K + 1) - \frac{Ka}{2\gamma} > 0 = Ka < 2\gamma(K + 1)$$

To be Type 1:

$$b = 0, Ka > 0, Ka < 2\gamma(K + 1)$$

- Q3: Given a second-order system  $G = \frac{1}{(s+1)(5s+1)}$ , and in a unity feedback structure. (40 points)
- a. Determine the system type and error constant with respect to tracking polynomial reference input of the system for
    - $D = k_P$
    - $D = k_P + k_D s$
    - $D = k_P + k_I/s + k_D s$
 With  $k_P = 19$ ,  $k_I = 0.5$ , and  $k_D = 4/19$
  - b. Verify your results using Matlab, by plotting unit-step and ramp response for tracking.

A.)

$$G(s) = \frac{1}{(s+1)(5s+1)}$$

$$(s+1)(5s+1) = 0; s = -1 \text{ and } s = -\frac{1}{5}$$

No poles at the origin, so this is a Type 0 system.<sup>2</sup>

$$\text{For } D(s) = k_p = 19; D(s)G(s) = \frac{19}{(s+1)(5s+1)}$$

**Error Constant:**

$$k_p = \lim_{s \rightarrow 0} G(s) = \lim_{s \rightarrow 0} \frac{19}{(s+1)(5s+1)} = 19 \quad \frac{E(s)}{R(s)} = \frac{1}{1+D(s)G(s)} = \frac{5s^2+6s+1}{5s^2+6s+20} \text{ -P}$$

$$\text{For } D(s) = k_p + k_D s = 19 + \frac{4}{19}s; D(s)G(s) = \frac{19 + \frac{4}{19}s}{(s+1)(5s+1)}$$

**Error Constant:**

$$k_p = 19 \quad k_d = \lim_{s \rightarrow 0} \frac{4}{19}s = 0 \quad \frac{E(s)}{R(s)} = \frac{1}{1+D(s)G(s)} = \frac{5s^2+6s+1}{5s^2 + \frac{118}{19}s + 20} \text{ -PD}$$

$$\text{For } D(s) = k_p + \frac{k_I}{s} + k_D s = 19 + \frac{0.5}{s} + \frac{4}{19}s; D(s)G(s) = \frac{19 + 0.5 + \frac{4}{19}s}{s(s+1)(5s+1)}$$

One pole at origin  $s=0$ , so this is a Type 1 system.

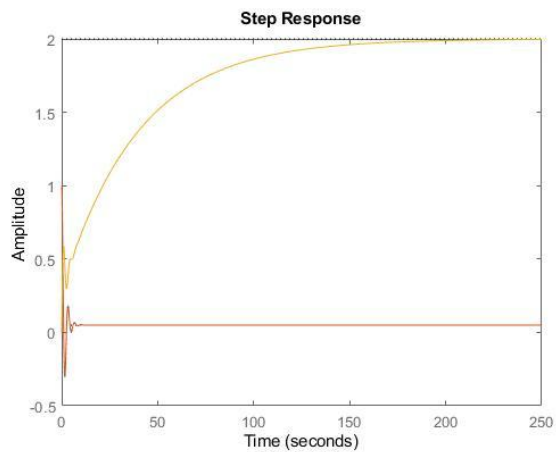
**Error Constant:**

$$\lim_{s \rightarrow 0} \frac{19 + 0.5 + \frac{4}{19}s}{s(s+1)(5s+1)} = 0.5 \quad \frac{E(s)}{R(s)} = \frac{1}{1+D(s)G(s)} = \frac{5s^3 + 6s^2 + s}{5s^3 + \frac{118}{19}s^2 + 20s + 0.5} \text{ -PID}$$

B.)

```
1 - num = [5, 6, 1];
2
3 - d1 = [5 6 20];
4 - d2 = [5 118/19 20];
5 - d3 = [5 118/19 20 0.5];
6
7 - s = tf("s");
8
9 - sys1 = tf(num,d1);
10 - sys2 = tf(num,d2);
11 - sys3 = tf(num,d3);
12
13 - step(sys1, sys2, sys3);|
14 %step(sys1/s, sys2/s, sys3/s);
```

Step:



Ramp:

