

## 2.1 (1A) Prove the multiplication property of DTFT

$$x[n]y[n] \xleftrightarrow{\text{DTFT}} \frac{1}{2\pi} \int_{2\pi} X(e^{j\theta})Y(e^{j(\omega-\theta)})d\theta$$

**DTFT  $x[n]$ :**

$$x[n] = \frac{1}{2\pi} \int_{2\pi} X(e^{j\omega})e^{j\omega n} d\omega$$

**For  $\omega \rightarrow \theta$ :**

$$x[n] = \frac{1}{2\pi} \int_{2\pi} X(e^{j\theta})e^{j\theta n} d\theta$$

**For the product of  $x[n]$  and  $y[n]$ :**

$$x[n]y[n] \leftrightarrow \sum_{n=-\infty}^{\infty} x[n]y[n]e^{-j\omega n}$$

$$x[n]y[n] \leftrightarrow \sum_{n=-\infty}^{\infty} \left[ \frac{1}{2\pi} \int_{2\pi} X(e^{j\theta})e^{j\theta n} d\theta \right] y[n]e^{-j\omega n}$$

$$x[n]y[n] \leftrightarrow \frac{1}{2\pi} \sum_{n=-\infty}^{\infty} y[n] \left[ \int_{2\pi} X(e^{j\theta})e^{j\theta n} e^{-j\omega n} d\theta \right]$$

$$x[n]y[n] \leftrightarrow \frac{1}{2\pi} \int_{2\pi} X(e^{j\theta}) \left[ \sum_{n=-\infty}^{\infty} y[n]e^{-j(\omega-\theta)n} \right] d\theta$$

$$\text{Frequency Shift } \sum_{n=-\infty}^{\infty} y[n]e^{-j(\omega-\theta)n} = Y(e^{j(\omega-\theta)})$$

$$\therefore x[n]y[n] \leftrightarrow \frac{1}{2\pi} \int_{2\pi} X(e^{j\theta})[Y(e^{j(\omega-\theta)})]d\theta$$

**2.2 (1C) Let  $x[n] = \delta[n] + 2\delta[n-1] - \delta[n-2] + \delta[n-3]$ ,  $h[n] = \delta[n] + \delta[n-1]$ . If  $y[n] = x[n] * h[n]$ , calculate the DTFT of  $y[n]$ . (Hint: use convolution property of DTFT.)**

$$x[n] = \delta[n] + 2\delta[n-1] - \delta[n-2] + \delta[n-3]$$

$$X[\theta] = (1 + 2\theta^{-1} - \theta^{-2} + \theta^{-3})$$

$$X[e^{j\omega}] = (1 + 2e^{-1j\omega} - e^{-2j\omega} + e^{-3j\omega})$$

$$h[n] = \delta[n] + \delta[n-1]$$

$$H[\theta] = (1 + \theta^{-1})$$

$$H[e^{j\omega}] = (1 + e^{-1j\omega})$$

$$y[n] = x[n] * h[n]$$

$$Y[\theta] = X[\theta]H[\theta]$$

$$Y[e^{j\omega}] = X[e^{j\omega}]H[e^{j\omega}]$$

$$(1 + 2e^{-1j\omega} - e^{-2j\omega} + e^{-3j\omega})(1 + e^{-1j\omega}) = (1 + 2e^{-1j\omega} - e^{-2j\omega} + e^{-3j\omega}) + (e^{-j\omega} + 2e^{-2j\omega} - e^{-3j\omega} + e^{-4j\omega})$$

$$= (1 + 3e^{-j\omega} + e^{-2j\omega} + e^{-4j\omega})$$

$$y[n] = 1 + 3\delta[n - 1] + \delta[n - 2] + \delta[n - 4]$$

2.3 (3B) Load and display an image using OpenCV and C++ in Visual Studio or other development environment.

