Homework 5

Wednesday, November 11, 2020 1:53 PM

"I pledge my honor I have abided by the Stevens Honor system."

-Alex Baskins

***P4.3.** The initial voltage across the capacitor shown in **Figure P4.3** is $v_C(0+) = -10$ V. Find an expression for the voltage across the capacitor as a function of time. Also, determine the time t_0 at which the voltage crosses zero.

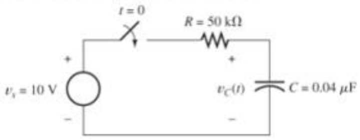


Figure P4.3

$i_c = C\frac{dV_c}{dt}$

$-10 + 50{,}000\, i_c + V_c = 0$

$50{,}000\, C\frac{dV_c}{dt} + V_c = 10$

$50{,}000(.04\times10^{-6}) = .002$

$\frac{dV_c}{dt} + \frac{V_c}{.002} = 5000$

$\frac{dV_c}{dt} + 500V_c = 0$

$y_c = Ke^{-st}$

$y_c = Ke^{-500t}$

$y_p = V_s$

$y_p = 10$

$y = y_c + y_p$

$V_c = Ke^{-500t} + 10$

when $t = 0$, $V = -10$

$-10 = Ke^{-500(0)} + 10$

$K = -20$

$\boxed{V_c = 10 - 20e^{-500t}}$

When $V_c = 0$:

$0 = 10 - 20e^{-500t}$

$e^{500t} = \frac{1}{2}$

$\boxed{t = .00139\,s.}$

P4.13. Derive an expression for $v_C(t)$ in the circuit of **Figure P4.13** and sketch $v_C(t)$ to scale versus time.

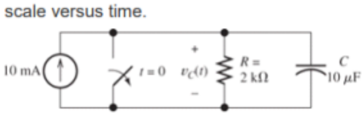


Figure P4.13

$i_c = C\frac{dV_c}{dt}$

$10\times10^{-3} = .01\,A.$

$RC = .02$

$RC\frac{dV_c}{dt} + V_c = .01$

$\frac{dV_c}{dt} + 50V_c = \frac{1}{2}$

$\frac{dV_c}{dt} + 50V_c = 0$

$V_c = Ke^{-50t}$

When $t = 0$, $K = V_c$

$V_c \to V_{source}$

where $i_c = 0$

so $K = V_c = IR$

$V_c = .01(2000)$

$\boxed{V_c = 20e^{-50t}}$

$V_c = .01(2000)$

$V_c = 20$

P4.24. The circuit shown in Figure P4.24 has been set up for a long time prior to t=0 with the switch closed. Find the value of v C prior to t=0. Find the steady-state value of v C after the switch has been opened for a long time.

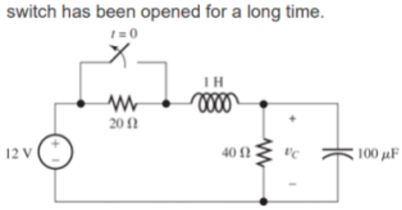
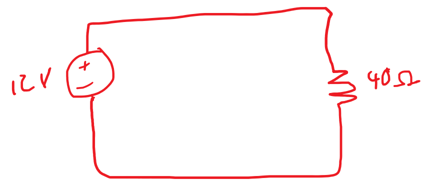


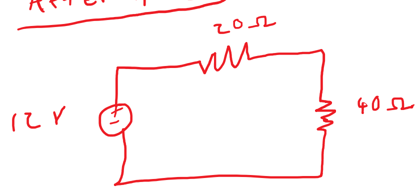
Figure P4.24

Before t=0:



$\boxed{V_c = 12\,V}$

After t=0:



$V_c = \frac{40}{60}\,12$

$\boxed{V_c = 8\,V}$

P4.25. Solve for the steady-state values of i 1 , i 2 , i 3 , i 4 , and v C for the circuit shown in Figure P4.25, assuming that the switch has been closed for a long time.

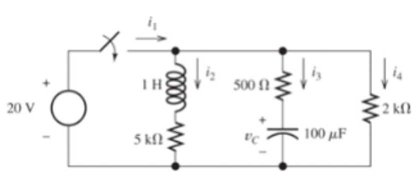


Figure P4.25

When $V_c = 20V$ (Fully charged), $i_c = 0A$.

$i_3 = i_c$

$\boxed{i_3 = 0\,A.}$

Req for remaining resistors:

$R_{eq} = \frac{10}{7}\,k\Omega \quad \left(\frac{5(2)}{5+2}\right)$

For i_1:

For i_2:

For i_3:

For i_1:

$$V_s = i_1 R_{eq}$$

$$20 = i_1 \left(\frac{10}{7} \times 10^3\right)$$

$$\boxed{i_1 = .014 \text{ A.}}$$

For i_2:

$$i_2 = \frac{2000}{7000}(.014)$$

$$\boxed{i_2 = .004 \text{ A.}}$$

For i_3:

$$i_3 = \frac{5000}{7000}(.014)$$

$$\boxed{i_3 = .010 \text{ A.}}$$

***P4.34.** Consider the circuit shown in **Figure P4.34**. The initial current in the inductor is $i_L(0-) = -0.2$ A. Find expressions for $i_L(t)$ and $v(t)$ for $t \geq 0$ and sketch to scale versus time.

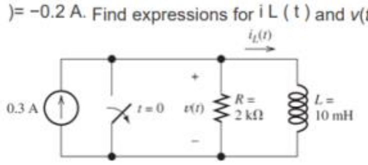
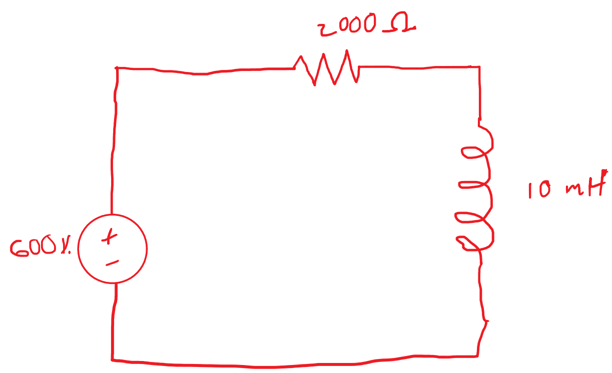


Figure P4.34

$$V_s = 2000(.3)$$

$$V_s = 600 \text{ V.}$$

$$V_L = L\frac{di_L}{dt} \qquad 10 \times 10^{-3} = .01$$



$$-600 + 2000 i_L + .01\frac{di_L}{dt} = 0$$

$$\frac{di_L}{dt} + 200000 i_L = 60000$$

$$y_c = K e^{-200000t}$$

$$y_p = i_s$$

$$y_p = .3$$

$$i(t) = .3 + K e^{-2000000t}$$

At $t = 0$, $i(t) = -.2$ A

$$-.2 = .3 + K$$

$$K = -.5$$

$$\boxed{i(t) = .3 - .5 e^{-200000t}}$$

$$v(t) = L\frac{di}{dt}$$

$$v(t) = 200000(.5) e^{-200000t}$$

$$\boxed{v(t) = 100000 e^{-200000t}}$$

$v(t) = 200000(.5)e$

$$\boxed{V(t) = 100000\, e^{-200000t}}$$

P4.39. The circuit shown in Figure P4.39 is operating in steady state with the switch closed prior to t=0. Find expressions for i L (t) for t<0 and for t≥0. Sketch i L (t) to scale versus time.

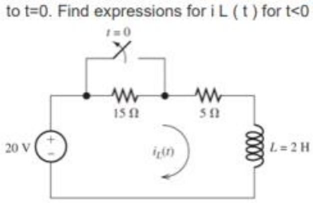


Figure P4.39

$R_{eq} = 20\,\Omega$

$-20 + 20 i_L + 2\frac{di_L}{dt} = 0$

$\frac{di_L}{dt} + 10 i_L = 10$

$y_c = ke^{-10t}$

$y_p = i_s$

$i_s = 1$ A.

$i(t) = ke^{-10t} + 1$

Steady state current:

$\frac{20}{5} = 4$ A.

$4 = k + 1$

$k = 3$

$\boxed{i(t) = 3e^{-10t} + 1}$ for $t \geq 0$

$i(t) = 4$ for $t < 0$