

3.1 (1A) Given: $\mathbf{x}_1[\mathbf{n}] = \delta[\mathbf{n}] - 2\delta[\mathbf{n} - 1]$

$$\mathbf{x}_2[\mathbf{n}] = 2\delta[\mathbf{n}] + \delta[\mathbf{n} - 1] - \delta[\mathbf{n} - 2]$$

3.1.1 Compute the linear convolution $\mathbf{x}_1[\mathbf{n}] * \mathbf{x}_2[\mathbf{n}]$

$$x_1[n] = [1; n = 0], [-2; n = 1] \text{ where } L_1 = 2$$

$$x_2[n] = [2; n = 0], [1; n = 1], [-1; n = 2] \text{ where } L_2 = 3$$

$$y[k] = h[n] * x[n] = \sum_{i=-k}^L x[i]h[k-i]$$

$$L = [2 - 1] + [3 - 1] = 3$$

$$y[0] = x_2[n] \bullet x_1[4 - n] :$$

$$(x_2[0] \bullet x_1[0]) + (x_2[1] \bullet x_1[3]) + (x_2[2] \bullet x_1[2]) + (x_2[3] \bullet x_1[1]) = 2 \bullet 1 + 1 \bullet 0 + -1 \bullet 0 + 0 \bullet -2 = 2$$

$$y[1] = x_2[n] \bullet x_1[1 - n] :$$

$$(x_2[0] \bullet x_1[1]) + (x_2[1] \bullet x_1[0]) + (x_2[2] \bullet x_1[3]) + (x_2[3] \bullet x_1[2]) = 2 \bullet -2 + 1 \bullet 1 + -1 \bullet 0 + 0 \bullet 0 = -3$$

$$y[2] = x_2[n] \bullet x_1[2 - n] :$$

$$(x_2[0] \bullet x_1[2]) + (x_2[1] \bullet x_1[1]) + (x_2[2] \bullet x_1[0]) + (x_2[3] \bullet x_1[3]) = 2 \bullet 0 + 1 \bullet -2 + -1 \bullet 1 + 0 \bullet 0 = -3$$

$$y[3] = x_2[n] \bullet x_1[3 - n] :$$

$$(x_2[0] \bullet x_1[3]) + (x_2[1] \bullet x_1[2]) + (x_2[2] \bullet x_1[1]) + (x_2[3] \bullet x_1[0]) = 2 \bullet 0 + 1 \bullet 0 + -1 \bullet -2 + 0 \bullet 1 = 2$$

$$y[n] = \{2, -3, -3, 2\}$$

3.1.2 Compute 4-point DFT: $\mathbf{X}_1[\mathbf{k}] = \mathbf{DFT}\{\mathbf{x}_1[\mathbf{n}]\}$ and $\mathbf{X}_2[\mathbf{k}] = \mathbf{DFT}\{\mathbf{x}_2[\mathbf{n}]\}$.

Note:

- DFT should be computed in numbers, following the DFT example provided
- You can use Matlab to help in calculation, without using Matlab DFT function
- Show all your steps.

$$x_1[k] = \sum_{n=0}^{N-1} x_1[n] \cdot W_N^{kn} \text{ where } N = 4 \text{ and } W_N^{kn} = e^{-j\frac{2\pi kn}{N}}$$

$$x_1[0] = x_1[0] + x_1[1] + x_1[2] + x_1[3]$$

$$x_1[0] = 1 - 2 = -1$$

$$x_1[1] = 1 + (-2)W_4^1 = 1 + (-2)(-j) = 1 + 2j$$

$$x_1[2] = 1 + (-2)W_4^2 = 1 + (-2)(-1) = 3$$

$$x_1[3] = 1 + (-2)W_4^3 = 1 + (-2)(j) = 1 - 2j$$

$$x_2[k] = \sum_{n=0}^3 x_2[n] \cdot W_4^{kn}$$

$$x_2[0] = 2 + 1 - 1 = 2$$

$$x_2[1] = 2 + W_4^1 - W_4^2 + 0 = 2 - j + 1 = 3 - j$$

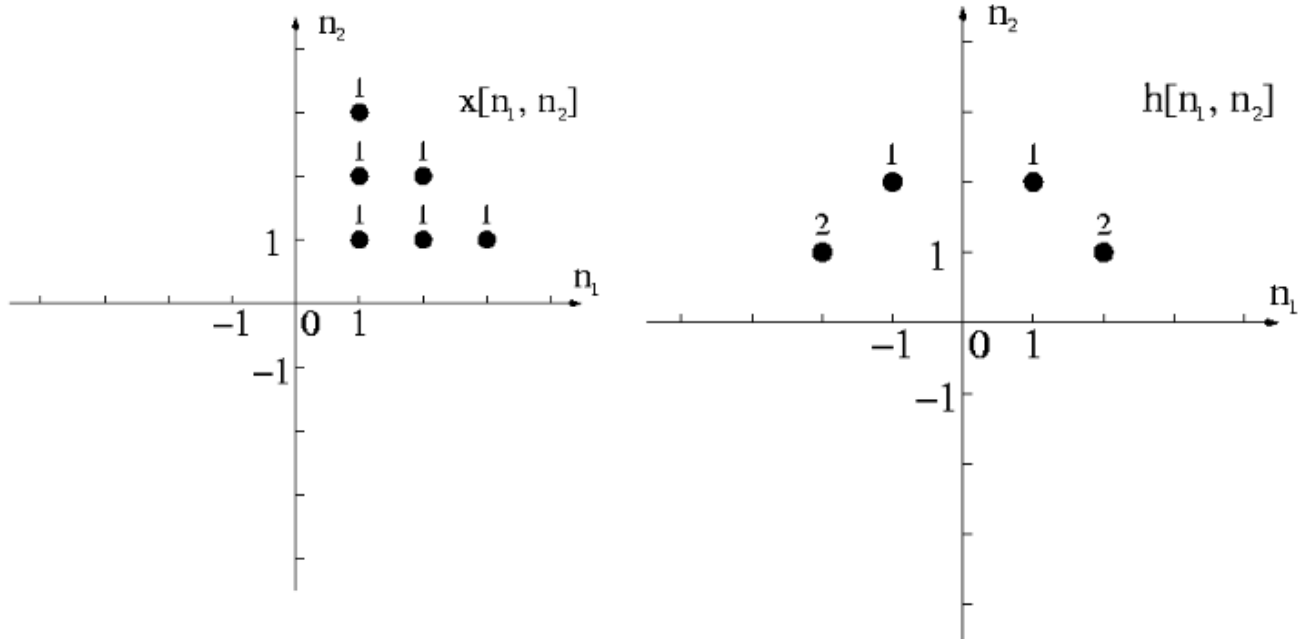
$$x_2[2] = 2 + W_4^2 - W_4^4 + 0 = 2 - 1 - 1 = 0$$

$$x_2[3] = 2 + W_4^3 - W_4^6 = 2 + j + 1 = 3 + j$$

$$x_1[k] = \{-1, 1 + 2j, 3, 1 - 2j\}$$

$$x_2[k] = \{2, 3 - j, 0, 3 + j\}$$

3.2 (1A) Calculate the 2-D convolution $\mathbf{x[n,n]**h[n,n]}$, show all the necessary intermediate steps.



$$y[n, n] = x[n, n] * h[n, n]$$

$$x[1, 1] = x[1, 2] = x[1, 3] = 1$$

$$x[2, 1] = x[2, 2] = 1$$

$$x[3, 1] = 1$$

$$h[-2, 1] = h[2, 1] = 2$$

$$h[-1, 2] = h[1, 2] = 1$$

$$y[-1, 2] = x[1, 1]h[-2, 1] = 2$$

$$y[-1, 3] = x[1, 2]h[-2, 1] = 2$$

$$y[-1, 4] = x[1, 3]h[-2, 1] = 2$$

$$y[0, 2] = x[2, 1]h[-2, 1] = 2$$

$$y[0, 3] = x[1, 1]h[-1, 2] + x[2, 2]h[-2, 1] = 3$$

$$y[0, 4] = x[1, 2]h[-1, 2] = 1$$

$$y[0, 5] = x[1, 3]h[-2, 1] = 2$$

$$y[1, 2] = x[3, 1]h[-2, 1] = 2$$

$$y[1, 3] = x[2, 1]h[-1, 2] = 1$$

$$y[1, 4] = x[2, 2]h[-1, 2] = 1$$

$$y[2, 3] = x[1, 1]h[1, 2] + x[3, 1]h[-1, 2] = 2$$

$$y[2, 4] = x[1, 2]h[1, 2] = 1$$

$$y[2,5] = x[1,3]h[1,2] = 1$$

$$y[3,2] = x[1,1]h[2,1] = 2$$

$$y[3,3] = x[1,2]h[2,1] + x[2,1]h[1,2] = 3$$

$$y[3,4] = x[1,3]h[2,1] + x[2,2]h[1,2] = 3$$

$$y[4,2] = x[2,1]h[2,1] = 2$$

$$y[4,3] = x[2,2]h[2,1] + x[3,1]h[1,2] = 3$$

$$y[5,2] = x[3,1]h[2,1] = 2$$

$$y[n,n] = \{2, 2, 2, 2, 3, 1, 2, 2, 1, 1, 2, 1, 1, 2, 3, 3, 2, 3, 2\}$$