

PEP151 Homework 3

Problem 1.

Sirius is the brightest star in the night sky. It has a surface temperature of $T = 9940$ K.

a) (2 pts) At what wavelength (in nm) is its radiation the most intense? Which part of the electromagnetic spectrum does that correspond to?

Blackbody radiation:

$$\lambda_{\max} = \frac{2.898 \times 10^6}{T} = \frac{2.898 \times 10^6}{9940} = 291.55 \text{ nm}$$

In accordance with the electromagnetic spectrum, this falls in the Ultraviolet range.

b) (2 pts) Given that the radius of Sirius is 1.71 times the radius of the Sun, and the Sun's surface temperature is 5777 K, what is the ratio of their luminosities $L_{\text{Sirius}}/L_{\text{Sun}}$?

$$T_{\text{Sirius}} = 9940 \text{ K}$$

$$T_{\text{Sun}} = 5777 \text{ K}$$

$$L = R \times T^4$$

$$R_{\text{Sirius}} = 1.71[R_{\text{Sun}}]$$

$$\frac{L_{\text{Sirius}}}{L_{\text{Sun}}} = \frac{[1.71R_{\text{Sun}}]^2 \times (9940)^4}{R_{\text{Sun}}^2 \times (5777)^4} = 25.6$$

The Ratio is 25.6:1

Problem 2.

This problem is about stellar parallax. Feel free to review the relevant lecture slides.

a) (2 pts) From Earth (with the standard baseline of 1 AU), if we observe the parallax angle of Vega to be 0.13 arcseconds, what is the distance to Vega in parsecs? What is that distance in light years?

Stellar Parallax:

$$d = \frac{1}{p}$$

where d = distance in parsecs and p = parallax angle in arcseconds

$$d = \frac{1}{0.13} = 7.69 \text{ parsecs}$$

$$1 \text{ parsec} = 3.26 \text{ light years}$$

$$d = 7.69 * 3.26 = 25.07 \text{ light years}$$

b) (2 pts) If through some other method we know that the distance to Arcturus is 36.7 light years, what do you expect its parallax angle (in arcseconds) to be, as observed from Earth (with the standard 1AU baseline)?

Given a distance of 36.7 light years, we can find the distance in parsecs:

$$d_{\text{parsecs}} = \frac{36.7}{3.26} = 11.25 \text{ parsecs}$$

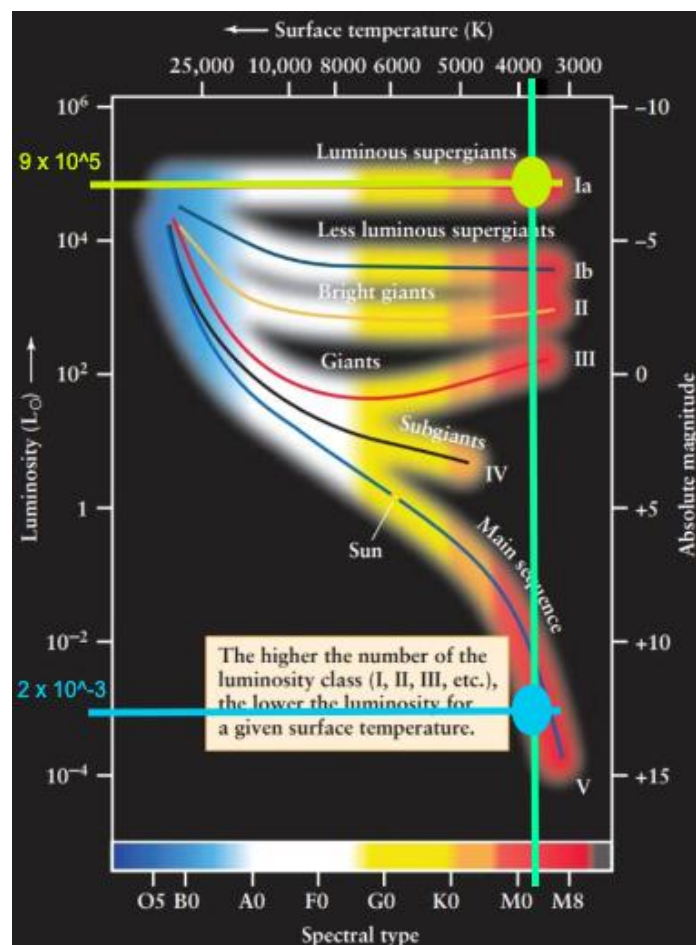
$$d = \frac{1}{p}, \text{ so } p = \frac{1}{d}$$

$$p = \frac{1}{d} = \frac{1}{11.25} = 0.089 \text{ arcseconds}$$

Problem 3.

Consider two stars Gliese 667C and Betelgeuse. From spectroscopic observations we conclude that they both have the same spectral type M1, and therefore the same surface temperature of ~ 3700 K. Further spectral feature analyses show that Gliese 667C has luminosity class of V (i.e. on the main sequence), and Betelgeuse has luminosity class of Ia (luminous supergiant).

a) (3 pts) On the HR diagram below, pinpoint the locations of Gliese 667C and Betelgeuse by drawing a vertical line through the M1 spectral type, and find its intersection with the luminous supergiant and main-sequence luminosity class. From the locations of Gliese 667C and Betelgeuse on the HR diagram, draw horizontal lines towards the luminosity axis to determine the luminosities of these two stars. Write down the luminosities of Gliese 667C and Betelgeuse on the luminosity axis – a ballpark estimate from eyeballing the numbers on the axis is sufficient. You need to show this HR diagram with your drawings on it in your submission.



Using color-coded straight lines, I estimated the luminosity of Betelgeuse (green) to be $9e5$, and the luminosity of Gliese 667C (blue) to be $2e-3$.

b) (2 pts) Based on the information in this question, what is the ratio of Gliese 667C's radius to that of Betelgeuse, R_{Gli} / R_{Bet} ?

Luminosity can be mathematically defined using the Stefan-Boltzmann Law:

$$L = [4\pi R^2][\sigma T^4] \text{ where } \sigma \text{ (Stefan-Boltzmann Constant)} = 5.67 \times 10^{-8} \frac{W}{m^2 K}$$

Using this formula, a ratio of the radii of the two stars can be acquired:

$$\frac{L_{Gli}}{L_{Bet}} = \frac{[4\pi R_{Gli}^2][\sigma T^4]}{[4\pi R_{Bet}^2][\sigma T^4]}$$

$$\therefore \frac{L_{Gli}}{L_{Bet}} = \frac{R_{Gli}^2}{R_{Bet}^2} = \left(\frac{R_{Gli}}{R_{Bet}}\right)^2$$

With the estimated luminosity values obtained in part a, the ratio can be solved for:

$$\frac{R_{Gli}}{R_{Bet}} = \sqrt{\frac{2 \times 10^{-3}}{9 \times 10^5}} = 4.71 \times 10^{-5}$$