

1) A party planning committee is being selected randomly in an office. Jim (male) and Pam (female) are married with a child and only one of them will be on the committee. Dwight (male) does not like Jim and will not be on the committee if Jim is on the committee. The committee must contain 2 males and 3 females. There are a total of 7 males and 8 females in the office.

- a) In how many ways can the committee be formed?  
 b) What is the probability that Dwight is on the committee?

a.) 7 males and 8 females  
 2 m and 3 f on committee

$$\# \text{ of ways to form committee if Dwight is on it} = {}^5C_2 \times {}^8C_3 = \frac{5!}{1!(4!)} \times \frac{8!}{3! \times 5!} = 280 \text{ ways}$$

$$\# \text{ of ways to form committee if Jim is on it} = {}^5C_1 \times {}^7C_3 = \frac{5!}{1!(4!)} \times \frac{7!}{3! \times 4!} = 175 \text{ ways}$$

$$\# \text{ of ways if neither are on it} = {}^5C_2 \times {}^8C_3 \\ \frac{5!}{2!(3!)} \times \frac{8!}{3! \times 5!} = 560 \text{ ways}$$

$$\text{Total number of ways} = 280 + 175 + 560 = \boxed{1015 \text{ ways}}$$

b.) Probability of Dwight on committee =  $\frac{280}{1015} = \boxed{0.276}$

(10 + 10 = 20 points)

2) A tournament is played by 3 teams - NY, NJ, and MA. 50% of the people in a poll wanted NJ to win, 30% wanted NY to win, and 20% wanted MA to win. 50% of NJ supporters and 40% of MA supporters said they would watch the game. If a total of 55% of poll respondents watch the game,

- a) What percentage of NY supporters watch the game?  
 b) If a poll respondent does NOT watch the game, what is the probability that she is a MA supporter?

a.) 0.55 total =  $(0.5 \times 0.5) + (0.3 \times p_{NY}) + (0.2 \times 0.4)$

$$p(NY) = \frac{0.22}{0.3} = 0.7333 = \boxed{73.33\%}$$

b.) Using Baye's Theorem:

$$P\left(\frac{MA}{W'}\right) = \left(P(MA)\left[P\left(\frac{W'}{MA}\right)\right]\right) \frac{1}{P(W')}$$

$$(0.2)(0.6)\left[\frac{1}{0.48}\right] = 0.277 = 27.7\%$$

(10 points)

3) Browser choices of 100 students in a class are being studied. 45 students use google chrome, 40 students use safari, and 35 students use internet explorer. 15 students use google chrome and safari, 10 students use google chrome and explorer, and 10 students use safari and internet explorer. 5 students use all three browsers. How many students use none of these 3 browsers?

$$45 + 40 + 35 - 15 - 10 - 10 + 5 = 90$$

90 students use all three

For 100 students,

$$100 - 90 = 10 \text{ students use none}$$

(10+10 points)

4) The random variable X has the probability density function

$$f(x) = k \quad 0 < x < 1$$

$$= x^3 / 7.5 \quad 1 \leq x \leq 2$$

$$= 0 \text{ otherwise}$$

a) Find the value of k.

b) Find the 70<sup>th</sup> percentile value of X

$$\text{a.) } \int f(x) dx = 1$$

$$\int_0^x k dx + \int_x^2 \frac{x^3}{7.5} dx = 1$$

$$kx \Big|_0^1 + \frac{1}{7.5} \left( \frac{x^4}{4} \Big|_1^2 \right) = 1$$

$$k + \frac{1}{7.5} \left( \frac{16}{4} - \frac{1}{4} \right) = 1$$

$$k + \frac{15}{30} = 1$$

$$k = \frac{1}{2}$$

$$k = \frac{1}{2}$$

b.)  $\int_{-\infty}^x f(x) dx = 0.70$

$$\int_0^1 \frac{1}{2} dx + \int_1^x f(x) dx = 0.70$$

$$\int_1^x f(x) dx = 0.70 - 0.50 = 0.20$$

$$\int_1^x \frac{x^3}{7.5} dx = 0.20 = \frac{1}{7.5} \left( \frac{x^4}{4} \Big|_1^x \right)$$

$$\frac{x^4 - 1}{30} = 0.20$$

$$x^4 - 1 = 0.2(30) = 6$$

$$x = \sqrt[4]{7} \approx 1.63$$

5) X and Y are two random variables with the following joint probability mass function:

$$p(x,y) = (x+2y)/42 \text{ For } X=0,1,2,3 \text{ and } Y=0,1,2$$

$$= 0 \text{ otherwise}$$

a) Find  $P(X < 2 | Y=1)$

b) Find  $E(12 - 2Y - Y^2)$

c) Find  $V(1-3Y)$

d) Are X and Y independent?

$$\begin{aligned} \text{a.) } p(x < 2 | Y=1) &= P(X < 2, Y=1) / P(Y=1) \\ &= (P(X=0, Y=1) + P(X=1, Y=1)) / P(Y=1) \\ &= \left( \left( \frac{2}{42} \right) + \left( \frac{3}{42} \right) \right) / \left( \underbrace{\frac{3+4+1}{42}}_{2/1} \right) \end{aligned}$$

$$= \boxed{\frac{5}{14} = 0.36}$$

b.)  $E(Y) = \sum_y y P(y) = \sum_0^2 y \left( \frac{3+4y}{21} \right)$

$$E(Y) = 0 \left( \frac{3}{21} \right) + 1 \times \left( \frac{3+4}{21} \right) + 2 \times \left( \frac{3+8}{21} \right)$$

$$= \frac{7}{21} + \frac{22}{21} = \frac{29}{21}$$

$$E(X^2) = \sum_y y^2 P(y) = \sum_0^2 y^2 \left( \frac{3+4y}{21} \right)$$

$$= \frac{3}{21}(0) + 1 \times \frac{7}{21} + 4 \times \left[ \frac{11}{21} \right]$$

$$= \frac{7}{21} + \frac{44}{21}$$

$$E(X^2) = \frac{51}{21}$$

$$E(12 - 2Y - Y^2) = 12 - 2(E(Y) - E(Y^2))$$

$$= 12 - 2 \left( \frac{29}{21} \right) - \frac{51}{21}$$

$$= 12 - \left( \frac{58}{21} + \frac{51}{21} \right)$$

$$= \boxed{6.81}$$

c.)

$$V(Y) = E(Y^2) - E(Y)^2$$

$$= \frac{51}{21} - \frac{29}{21} = \frac{22}{21}$$

$$V(1-3Y) = 9 \left[ \frac{22}{21} \right] = \frac{198}{21}$$

$$V(1-3Y) = 9.43$$

d.)  $P(x, y) \neq P(x)[P(y)]$

$$P(x=0, y=0) = \frac{0+2(0)}{42} = 0$$

$$P(x=0) = \frac{2}{14} \quad P(y=0) = \frac{3}{21}$$

$\therefore x$  and  $y$  are not independent