Homework 28-30

Friday, April 30, 2021 10:44 AM

"I pledge my honor I have abided by the Stevens Honor system."

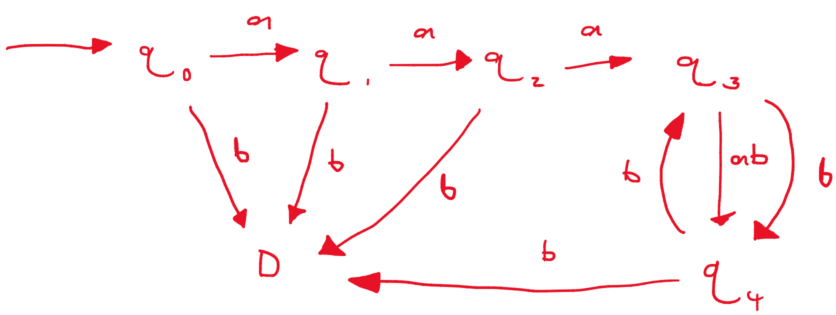
- Alex Haskins

Problem 29.1. Let language L be defined by $aaa((a \mid b)b)^*$

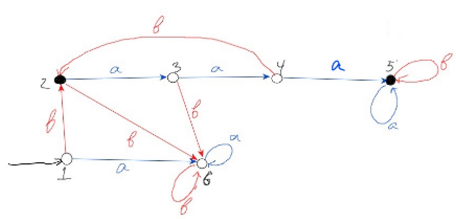
a) *(3 points)* Describe L (i.e. provide a "nice" criterion for a word w to be in L).

L starts with the string "aaa" and is succeeded by an even length sring which ends with b at even places, such as "ab", "bb", "abab", "bbb", "abbb", etc.

b) *(7 points)* Draw a deterministic FSA that defines the same language L.



Problem 29.2. *(10 points)*
Consider the language L_2 defined by the following FSA:



Write a regular expression that defines the same language L_2.

1 is the initial state; 2 and 5 are the final states

The expression represents the path from state 1 to state 2 or state 1 to state 5

<u>For $1 \to 2$:</u>

- Single "b"
- Followed by aab recursively } $b(aab)^*$

<u>For $1 \to 5$:</u>

- Single "b"
- Followed by 3 consecutive a's } $b(ba)^* a (a+b)^*$

- Single b
- Followed by 3 consecutive a's
- Followed by any number of a's and b's
- b followed by 2 consecutive a's
- Followed by "baa" recursively
- Followed by "a"
- Followed by any number of a's and b's

$\Big\}$ $baa(baa)^* a (a+b)^*$

$\boxed{\text{Regular expression: } b(aab)^* + baa\,(baa)^*\,a\,(a+b)^*}$

Problem 29.3. Let L_1 be the language that consist of words in the alphabet 0, 1 that start with 1, then have 00 or 11 repeating arbitrary number of times (in any order). For example it includes words like 10011001111, 1, 11111, etc.

Let L_2 be the language that consists of words in the alphabet 0, 1 that start with 1, then either have 00 repeating, or 11 repeating (11's and 00's don't mix). It only contains words like 11111, 10000, 1, 111111111 etc.

Construct each of the following:

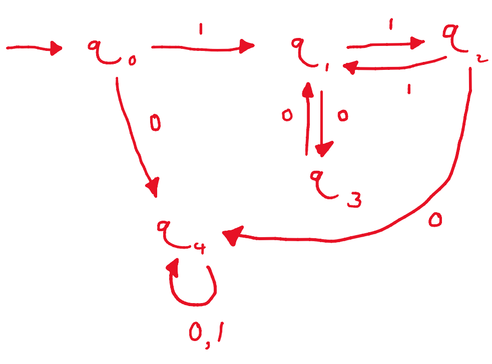
a) *(3 points)* a regular expression representing L_1;

$$L_1 = 1(00+11)^*$$

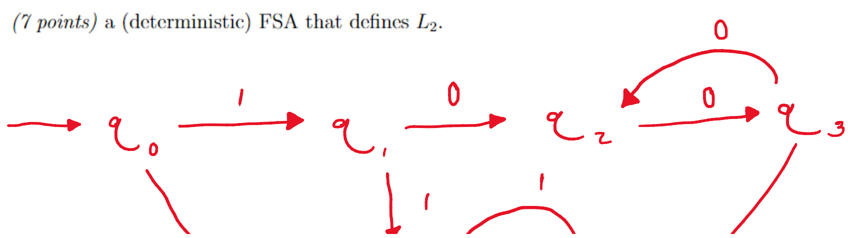
b) *(3 points)* a regular expression defining L_2;

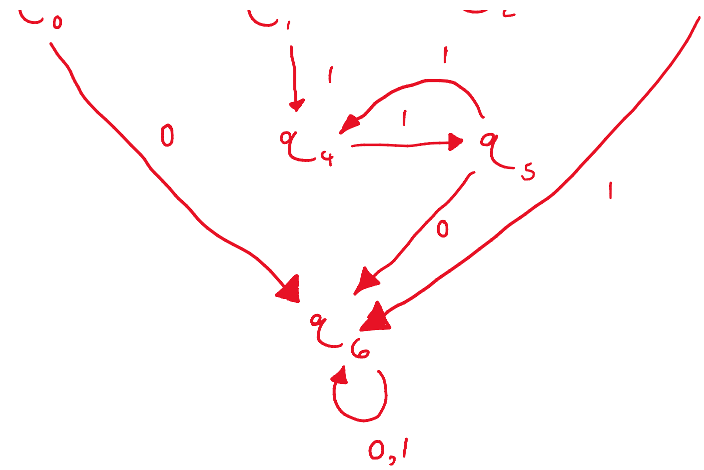
$$L_2 = 1(00)^* + 1(11)^*$$

c) *(5 points)* a (deterministic) FSA that defines L_1;



d) *(7 points)* a (deterministic) FSA that defines L_2.





Problem 30.1. Let

$$L = \{a^{k+2}b^k \mid k \in \mathbb{N}\} = \{\underbrace{aa\ldots a}_{k+2 \text{ times}}\underbrace{bb\ldots b}_{k \text{ times}} \mid k \in \mathbb{N}\}$$

a) *(10 points)* Prove that L is not a regular language.

Number of b's in L is dependent on the number of a's in L.

Using the pumping lemma, a string s from L, where $|s| \geq p$ can be formed, such that p is the pumping length.

The pumping length for this language is 4; the string of length less than 4 does not have any repetition where $p = 4$.

Assume then $s = aaaabb$ and is separated into x, y, z where $y \neq \epsilon$ & $|x, y| \leq p$

$$s = \underbrace{aa}_{x}\underbrace{a}_{y}\underbrace{abb}_{z} \qquad x = aa \quad y = a \quad z = abb$$

For $i = 0$, $xy^0z = xz = aaabb \notin L$

For $i = 1$, $xy^1z = aaaabb \notin L$

For $i = 2$, $xy^2z = aaaaabb \notin L$

$\boxed{\text{Thus, for all } i \geq 0,\ xy^iz \text{ doesn't exist in } L, \text{ so } L \text{ is not a regular language.}}$

b) *(3 points)* Prove that the compliment of L, $\Delta = \overline{L} = \{w \in \{a, b\}^* \mid w \notin L\}$ is not a regular language.

The pumping length p for language L is 2; the string of length less than 2 doesn't have any repetition [$p = 2$].

Take a string s from $\bar{L}$ where $|s| \geq p$.

Assume $s = aab$ where s is separated into x, y, z such that $y = \cancel{\epsilon}$ & $|x, y| \leq p$.

$s = \underbrace{a}_{x}\underbrace{a}_{y}\underbrace{b}_{z}$ $x = a$ $y = a$ $z = b$

For $i = 0$, $xy^0z = ab \in \bar{L}$

For $i = 1$, $xy^1z = aab \in \bar{L}$

For $i = 2$, $xy^2z = aaab \notin \bar{L}$

$\boxed{\text{Thus, for all } i \geq 0,\ xy^iz \text{ does not exist in } \bar{L}, \text{ so } \bar{L} \text{ is not a regular language.}}$