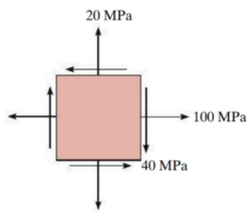
Homework 14

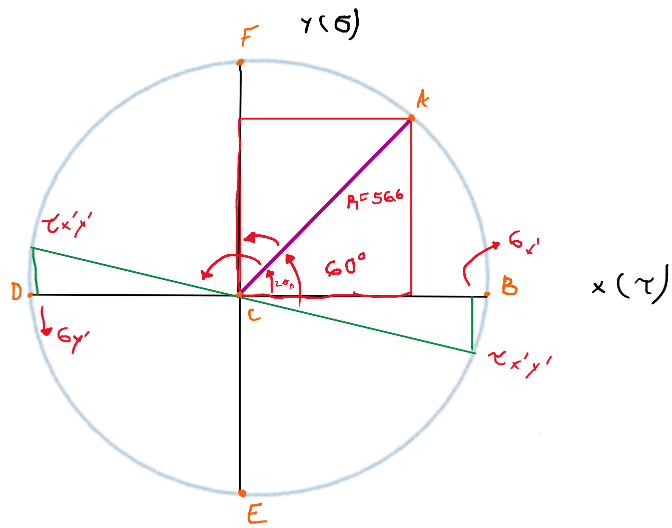
Tuesday, December 8, 2020 2:09 PM

"I pledge my honor I have abided by the Stevens Honor system."

- Alex Gaskins

14–55. Determine (a) the principal stresses and (b) the maximum in-plane shear stress and average normal stress. Sketch a properly oriented element for each case.





$\tau_x = 100$ MPa

$\tau_y = 20$ MPa

$\tau_{xy} = -40$ MPa

$$\tau_{avg} = \frac{\tau_x + \tau_y}{2}$$

$$\tau_{avg} = \frac{100 + 20}{2} = 60 \text{ MPa}$$

Center Point: (60, 0)

$$R = \sqrt{\left(\frac{\tau_x - \tau_y}{2}\right)^2 + (\tau_{xy})^2}$$

$$R = \sqrt{\left(\frac{100 - 20}{2}\right)^2 + (-40)^2}$$

$$R = 56.6 \text{ MPa}$$

A.)

Point B and D

$\tau_1 > \tau_2$

$\tau_1 = 60 + R = 60 + 56.6 = 116.6$ MPa

$\tau_2 = 60 - R = 60 - 56.6 = 3.4$ MPa

$2\theta_{p_1}$ = Angle from $\overline{CA}$ to $\overline{CB}$ for θ_{p_1} at $|\tau_1|$

$$2\theta_1 = \tan^{-1}\left(\frac{40}{100-60}\right)$$

$$2\theta_1 = \tan^{-1}(1)$$

$$2\theta_1 = 45^\circ$$

$$\theta_1 = 22.5^\circ$$

$2\theta_2$ = Angle from CA to CD

$$2\theta_2 = 180 - 2\theta_1$$

$$\theta_2 = 67.5^\circ$$

B.)

$$\tau_{max\,(in\text{-}plane)} = \pm 56.6 \text{ MPa}$$

θ_3 = Angle from $\overline{CA}$ to $\overline{CF}$

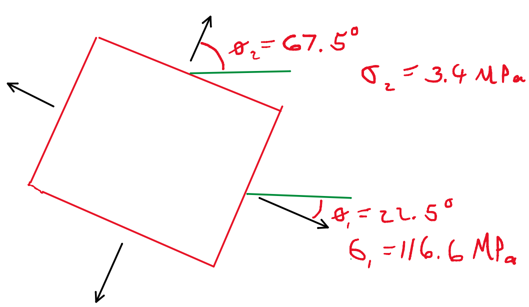
$$2\theta_3 = \tan^{-1}\left(\frac{100-60}{40}\right)$$

$$2\theta_3 = \tan^{-1}(1)$$

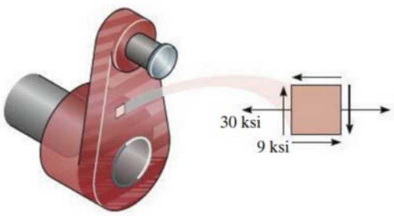
$$2\theta_3 = 45^\circ$$

$$\theta_3 = 22.5^\circ$$

$\sigma_{avg} = 60$ MPa at center of circle



*14–56. Determine (a) the principal stress and (b) the maximum in-plane shear stress and average normal stress. Sketch a properly oriented element for each case.



$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{30+0}{2}$$

$$\sigma_{avg} = 15 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$R = \sqrt{\left(\frac{30-0}{2}\right)^2 + (-9)^2}$$

$$R = \tau_{max} = \pm 17.5 \text{ ksi}$$

A.)

$$\sigma_1 = \sigma_{avg} + R \qquad \sigma_2 = \sigma_{avg} - R$$

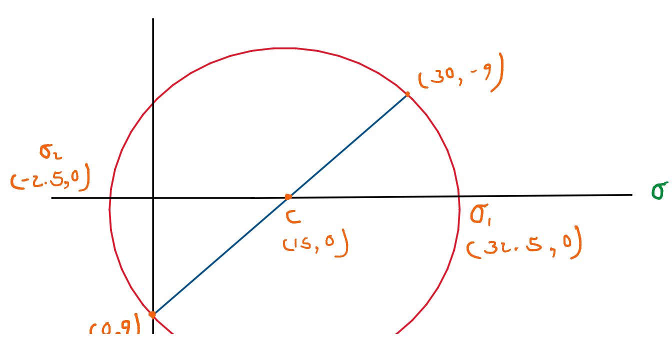
$$\sigma_1 = 15 + 17.5 \qquad \sigma_2 = 15 - 17.5$$

$$\sigma_1 = 32.5 \text{ ksi} \qquad \sigma_2 = -2.5 \text{ ksi}$$

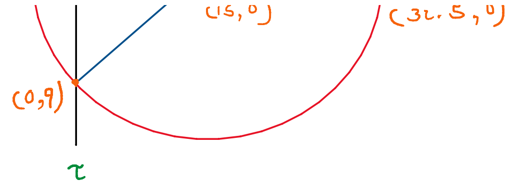
$$\tan(2\theta_p) = \frac{\tau_{xy}}{\left(\frac{\sigma_x - \sigma_y}{2}\right)}$$

$$\tan(2\theta_p) = \frac{-9}{15}$$

$$\theta_p = -15.5^\circ$$



B.)



B.)

$$\tan(2\theta_s) = \left(-\frac{\sigma_x - \sigma_y}{2\tau_{xy}}\right)$$

$$\tan(2\theta_s) = \left(-\frac{30-0}{2(-9)}\right)$$

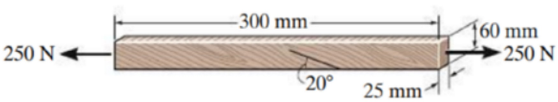
$$\theta_s = 29.5°$$

$$\tau_{max(in\text{-}plane)} = \left(-\frac{\sigma_x - \sigma_y}{2}\right)\sin(2\theta_s) + \tau_{xy}\cos(2\theta_s)$$

$$\tau_{max(in\text{-}plane)} = \left(-\frac{30-0}{2}\right)\sin(2(29.5)) + (-9)\cos(2(29.5))$$

$$\tau_{max(in\text{-}plane)} = -17.5 \text{ ksi}$$

14–61. The grains of wood in the board make an angle of 20° with the horizontal as shown. Determine the normal and shear stresses that act perpendicular and parallel to the grains if the board is subjected to an axial load of 250 N.



$$\sigma = \frac{250}{(60(25))}$$

$$\sigma = 166.67 \text{ kPa}$$

$$\theta = 90 - 20$$
$$\theta = 70°$$

$$\sigma_\theta = \frac{\sigma_x + \sigma_y}{2} + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos(2\theta) + \tau_{xy}\sin(2\theta)$$

$$\sigma_\theta = \frac{166.67}{2} + \frac{166.67}{2}\cos(140) + 0$$

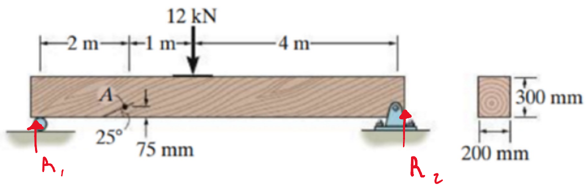
$$\boxed{\sigma_\theta = 19.50 \text{ kPa}}$$

$$\tau_\theta = \left(-\frac{\sigma_x - \sigma_y}{2}\right)\sin(2\theta) + \tau_{xy}\cos(2\theta)$$

$$\tau_\theta = -\frac{166.67}{2}\sin(140)$$

$$\boxed{\tau_\theta = -53.57 \text{ kPa}}$$

14–23. The wood beam is subjected to a load of 12 kN. If grains of wood in the beam at point A make an angle of 25° with the horizontal as shown, determine the normal and shear stress that act perpendicular and parallel to the grains due to the loading.



$$I = \frac{1}{12} b h^3$$

$$I = \frac{(200)(300)^3}{12}$$

$$I = 450 \times 10^6 \ mm^4$$

$$\Sigma M_A = 0$$

$$R_1(7) - 12(4) = 0$$

$$R_1 = 6.857 kN.$$

$$\Sigma F_y = 0$$

$$R_2 + 6.857 - 12 = 0$$

$$R_2 = 5.143 kN.$$

At point A:

$$6.857 - V_A = 0$$

$$V_A = 6.857 kN.$$

$$\Sigma M_A = 0$$

$$6.857(2) - M_A = 0$$

$$M_A = 13.714 kN \cdot m$$

$$\sigma_A = \frac{M_A y_A}{I} = \frac{(13.714 \times 10^6)(75)}{(450 \times 10^6)}$$

$$\sigma_A = 2.2857 \ MPa$$

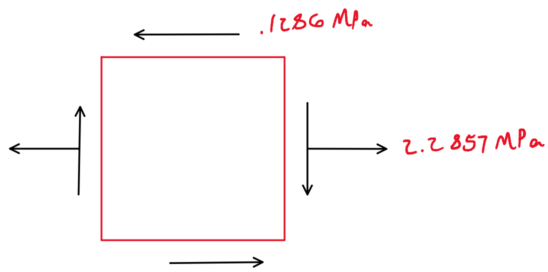
$$Q = A y = 200(75)(112.5)$$

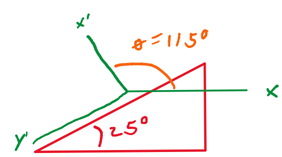
$$Q = 1.685 \times 10^6 \ mm^3$$

$$\tau_A = \frac{V_A Q_A}{I t}$$

$$\tau_A = \frac{(6.857 \times 10^3)(1.6875 \times 10^6)}{(450 \times 10^6)(200)}$$

$$\tau_A = .1286 \ MPa$$





$$\sigma_{x'} = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos(2\theta) + \tau_{xy}\sin(2\theta)$$

$$\sigma_{x'} = \left(\frac{2.2857 + 0}{2}\right) + \left(\frac{2.2857 - 0}{2}\right)\cos(230) - .1286 \sin(230)$$

$$\boxed{\sigma_{x'} = .5068 \ MPa}$$

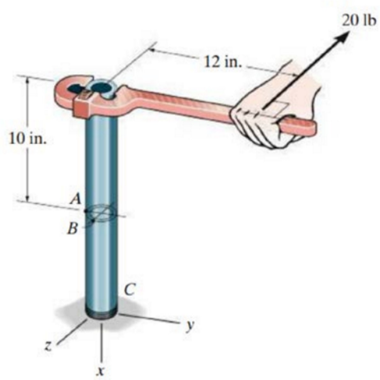
$$\boxed{\sigma_{x'} = .5068 \text{ MPa}}$$

$$\tau_{x'y'} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin(2\theta) + \tau_{xy}\cos(2\theta)$$

$$\tau_{x'y'} = -\left(\frac{2.2857 - 0}{2}\right)\sin(230) - .1286\cos(230)$$

$$\boxed{\tau_{x'y'} = .9582 \text{ MPa}}$$

R14–1. The steel pipe has an inner diameter of 2.75 in. and an outer diameter of 3 in. If it is fixed at *C* and subjected to the horizontal 20-lb force acting on the handle of the pipe wrench, determine the principal stresses in the pipe at point *A*, which is located on the surface of the pipe.



$$(Q_A)_z = \Sigma \bar{y}' A'$$

$$(Q_A)_z = \frac{4(1.5)}{3\pi}\left[\frac{1}{2}\pi(1.5)^2\right] - \frac{4(1.375)}{3\pi}\left[\frac{1}{2}\pi(1.375)^2\right]$$

$$(Q_A)_z = .51693 \text{ in.}^3$$

$$I = \frac{\pi}{4}(1.5^4 - 1.375^4) = 1.1687 \text{ in.}^4$$

$$J = \frac{\pi}{2}(1.5^4 - 1.375^4) = 2.3374 \text{ in.}^4$$

$$\sigma_A = \frac{M_y z}{I_y}$$

$$\sigma_A = \frac{200(0)}{1.1687}$$

$$\sigma_A = 0$$

$$\tau_A = \frac{20(.51693)}{1.1687(2)(.125)} - \frac{240(1.5)}{2.3374}$$

$$\tau_A = -118.6 \text{ psi}$$

$$\sigma_{\text{in-plane}} = \frac{\sigma_x + \sigma_z}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_z}{2}\right)^2 + \tau_{xz}^2}$$

$$\sigma_{\text{in-plane}} = 0 \pm \sqrt{0 + (-118.6)^2}$$

$$\sigma_1 = 118.6 \text{ psi} \qquad \sigma_2 = -118.6 \text{ psi}$$

$$\sigma_B = \frac{200(1.5)}{1.1687}$$

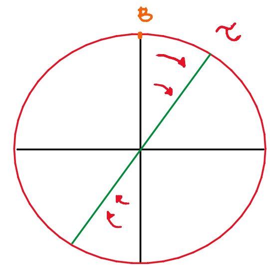
$$\sigma_B = \frac{200(1.5)}{1.1687}$$

$$\sigma_B = 256.7 \text{ psi}$$

$$\tau_B = \frac{240(1.5)}{2.3374}$$

$$\tau_B = 154 \text{ psi}$$

$\sigma_1 = 118.6$ psi $\sigma_2 = -118.6$ psi



$$\sigma_p = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_p = \frac{256.7 + 0}{2} \pm \sqrt{\left(\frac{256.7 - 0}{2}\right)^2 + (-154)^2}$$

$$\boxed{\sigma_{p_1} = 329 \text{ psi} \qquad \sigma_{p_2} = -72.1 \text{ psi}}$$