Problem of the Week 4

Monday, November 16, 2020 12:34 PM

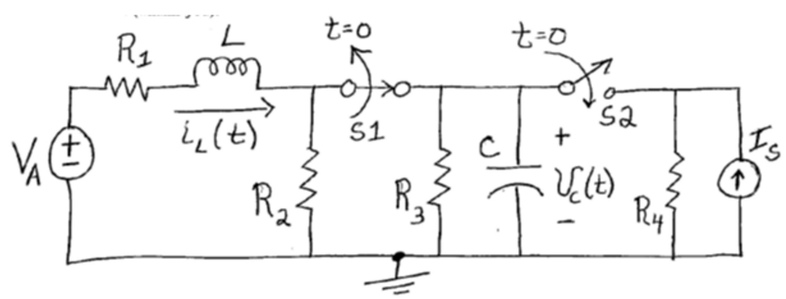
"I pledge my honor I have abided by the Stevens Honor system." -Alex Gaskins

Problem One: Consider the circuit shown for Problem One in the file above. **(45 points)**

The voltage source, V_A, and the current source, I_S, are each a constant dc source. The circuit has been operating for a long time with Switch One (S1) in the closed position prior to t = 0 and Switch Two (S2) in the open position prior to t = 0. Note; I do not provide numerical values for this problem. Thus, your answer should be expressed in terms of the given circuit elements. Please Note; in this problem, if you have an Req in your answer, please clearly define the Req term. (Thank-you).

1. Determine the expression for $i_L(t = 0^-)$. **(5 points)**
2. Determine the expression for $V_C(t = 0^-)$. **(5 points)**
3. Write the first order differential equation for $i_L(t)$ for t > 0, in standard form. **(5 points)**
4. Determine the complete solution for $i_L(t)$ for t > 0. (Please solve for all constant(s) and define τ) **(10 points)**
5. Write the first order differential equation for $V_C(t)$ for t > 0, in standard form. **(5 points)**
6. Determine the complete solution for $V_C(t)$ for t > 0. (Please solve for all constant(s) and define τ) **(15 points)**

PROBLEM ONE;



1.) $i_L(t=0^-)$

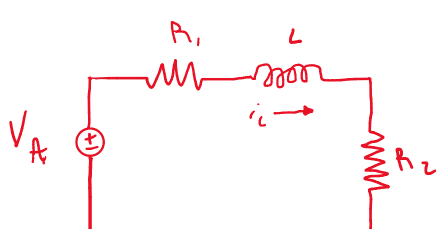
$$R_{eq} = \frac{R_2 R_3}{R_2 + R_3}$$

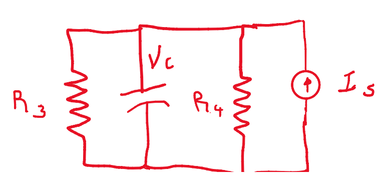
$$\boxed{i_L(t=0^-) = \frac{V_A}{R_1 + R_{eq}}}$$

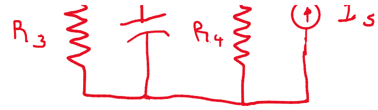
2.) $V_c(t=0^-)$

V_c is charged so $\boxed{V_c = V_A}$

3.)







$$-V_A + R_1 i_L + L\frac{di_L}{dt} + R_2 i_L = 0$$

$$L\frac{di_L}{dt} + (R_1+R_2)i_L = V_A$$

$$\boxed{\frac{L}{(R_1+R_2)}\frac{di_L}{dt} + i_L = \frac{V_A}{(R_1+R_2)}}$$

4.)

$$i_L = Ke^{-t/\tau} + \frac{V_A}{(R_1+R_2)}$$

$$t > 0$$

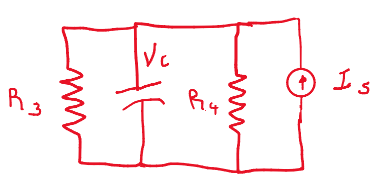
$$\tau = \frac{L}{R_1+R_2}$$

$$R_{eq} = \frac{R_2R_3}{R_2+R_3}$$

$$\frac{V_A}{(R_1+R_{eq})} = K + \frac{V_A}{(R_1+R_2)}$$

$$\boxed{i_L = \left(\frac{V_A}{R_1+R_{eq}} - \frac{V_A}{R_1+R_2}\right)e^{-\frac{(R_1+R_2)t}{L}} + \frac{V_A}{(R_1+R_2)}}$$

5.)



$$R_{eq} = \frac{R_3R_4}{R_3+R_4}$$

$$-I_s + \frac{V_c}{R_{eq}} + C\frac{dV_c}{dt} = 0$$

$$\boxed{R_{eq}C\frac{dV_c}{dt} + V_c = I_sR_{eq}}$$

6.)

6.)

$$V_c = Ke^{-t/\tau} + I_s R_{eq} \qquad t > 0$$

$$\tau = R_{eq}C$$

$$V_A = K + I_s R_{eq} \qquad R_{eq} = \frac{R_3 R_4}{R_3 + R_4}$$

$$K = V_A - I_s R_{eq}$$

$$\boxed{V_c = (V_A - I_s R_{eq})\, e^{-\frac{t}{R_{eq}C}} + I_s R_{eq}}$$

Problem Two: Consider the circuit shown for Problem Two in the file above. **(30 points)**

The circuit has been operating for a long time with the switch in the open position prior to t = 0. The voltage source, V_S, is a constant dc source.

1. Write the second-order differential equation in standard form for the capacitor voltage $V_c(t)$ for t > 0. (Note; the only variable which should appear in your equation is $V_c(t)$). Please note; I do not provide numerical values for the resistor, capacitor, inductor, and the source. Thus, your answer should be expressed in terms of the given variables). **(10 points)**
2. You are now given that this second-order circuit is **critically damped**. In addition, the following two initial conditions are given;

   $v_C(t = 0^+) = 0$ Volts

   $i(t = 0^+) = 0$ Amps

   Determine the complete solution to this second-order circuit. As part of your solution, determine the two constants; K_1 and K_2. (You do not have to solve for the value of "s"). **(15 points total)**

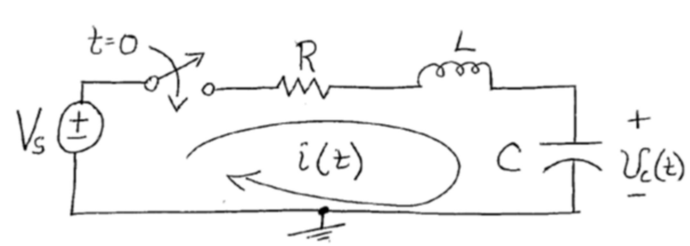
3. For this part of the problem, you are now given the following values;

   L = 100 mH

   C = 0.01 µF

   Determine the value of the resistor R for the second-order system to be critically damped. **(5 points)**

PROBLEM TWO;



1.)

$$-V_s + Ri(t) + L\frac{di(t)}{dt} + V_c(t) = 0$$

1.)

$$-V_s + R\,i(t) + L\frac{d\,i(t)}{dt} + V_c(t) = 0$$

$$i = C\frac{dV_c}{dt}$$

$$i' = C\frac{d^2V_c}{dt^2}$$

$$LC\frac{d^2V_c}{dt^2} + RC\frac{dV_c}{dt} + V_c = V_s$$

$$\boxed{\frac{d^2V_c}{dt^2} + \frac{R}{L}\frac{dV_c}{dt} + \frac{1}{LC}V_c = \frac{V_s}{LC}}$$

2.)

Critically damped

$$z = 1$$

$$\alpha = \frac{R}{2L} \qquad \omega_0 = \frac{1}{\sqrt{LC}}$$

$$z = \frac{\alpha}{\omega_0}$$

$$z = \frac{R\sqrt{LC}}{2L} = \frac{R\sqrt{C}}{2\sqrt{L}}$$

$$\frac{R\sqrt{C}}{2\sqrt{L}} = 1$$

$$L = \frac{R^2C}{4}$$

$$V_{c_h} = k_1e^{st} + k_2te^{st}$$

$$V_{c_p} = V_s$$

$$V_c = k_1 e^{st} + k_2 t e^{st} + V_s$$

$$0 = k_1 + V_s$$

$$k_1 = -V_s$$

$$\frac{dV_c}{dt} = k_1 s e^{st} + k_2 t s e^{st} + k_2 e^{st}$$

$$0 = k_1 s + k_2$$

$$k_2 = V_s s$$

$$\boxed{V_c = -V_s e^{st} + V_s s t e^{st} + V_s}$$

3.)

$$L = \frac{R^2 C}{4}$$

$$R^2 C = 4L$$

$$R = \sqrt{\frac{4L}{C}}$$

$L = 100$ mH.

$C = .01$ µF.

$$R = \sqrt{\frac{4(100 \times 10^{-3})}{(.01 \times 10^{-6})}}$$

$$\boxed{R = 6.3245\ k\Omega}$$