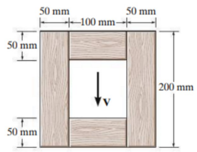
Homework 12

Sunday, November 22, 2020 8:57 PM

"I pledge my honor I have abided by the Stevens Honor system."

\- Alex Gaskins

12–6. The wood beam has an allowable shear stress of $\tau_{allow} = 7$ MPa. Determine the maximum shear force V that can be applied to the cross section.



$A_1 = 50(100) = 5000\text{ mm}^2$

$A_2 = 50(100) = 5000\text{ mm}^2$

$A_3 = 50(100) = 5000\text{ mm}^2$

$I = \frac{1}{12}bh^3 - \frac{1}{12}b'h'^3 = \frac{1}{12}(200)(200)^3 - \frac{1}{12}(100)(100)^3$

$I = 125\times10^6\text{ mm}^4$

$M_{tot} = \Sigma M$

$M_{tot} = A_1\bar{y}_1 + A_2\bar{y}_2 + A_2\bar{y}_2$

$M_{tot} = 5000(75 + 50 + 50)$

$M_{tot} = 875000\text{ mm}^3$

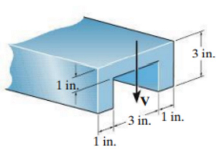
$\gamma = \frac{MV}{It}$

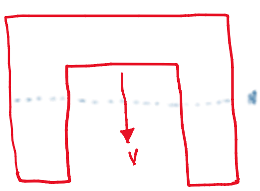
$V = \frac{\tau I t}{M}$

$V = \frac{(7)(125\times10^6)(50+50)}{875000}$

$\boxed{V_{max} = 100\text{ kN.}}$

12–10. If the applied shear force $V = 18$ kip, determine the maximum shear stress in the member.





$\bar{y} = \frac{\Sigma A_y}{\Sigma A}$

$\bar{y} = \frac{b_1h_1(y_1) + b_2h_2(y_2) + b_3h_3(y_3)}{b_1h_1 + b_2h_2 + b_3h_3}$

$\bar{y} = \frac{.5(1)(5) + 2(2)(1)(2)}{1(5) + 2(1)(2)}$

$\bar{y} = 1.1667\text{ in.}$

$I = \Sigma \frac{1}{12}bh^3 + Ad^2$

$I = \frac{1}{12}(5)(1)^3 + (5)(1)(1.1667 - .5^2) + 2\left(\frac{1}{12}\right)(1)(2)^3 + 2(2)(1)(2 - 1.1667)^2$

$I = 6.75\text{ in.}^4$

$$Q_{max} = \Sigma \bar{y}' A' = 2(.91665)(1.8333)(1)$$

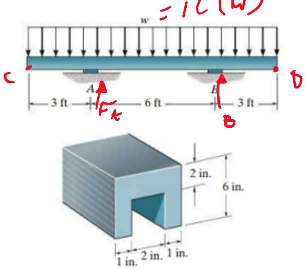
$$Q_{max} = 3.361 \text{ in.}^3$$

$$\tau_{max} = \frac{V_{max}}{It} = \frac{18(3.361)}{6.75(2)}$$

$$\boxed{\tau_{max} = 4.48 \text{ ksi}}$$

12–23. If $w = 800$ lb/ft, determine the absolute maximum shear stress in the beam. The supports at A and B are smooth.

$= 12(W)$



$$\Sigma F_y = 0$$

$$F_A + F_B - 12(W) = 0$$

$$F_A = 6(W)$$

$$\Sigma M_B = 0$$

$$F_B(6) - 9(W)(4.5) + 3(W)(1.5) = 0$$

$$F_B = 6(W)$$

$$V_D = 0 \quad V_{B^+} = 3W \quad V_{B^-} = -3W \quad V_{A^+} = 3W \quad V_{A^-} = -3W \quad V_C = 0$$

$$V_{max} = 3(W)$$

$$W = 800$$

$$V_{max} = 2400 \text{ lbs.}$$

$$A_1 = 4(1) = 4$$

$$A_2 = 4(1) = 4$$

$$A_3 = 4(2) = 8$$

$$\bar{y} = \frac{A_1\bar{y} + A_2\bar{y} + A_3\bar{y}_3}{A_1 + A_2 + A_3} = \frac{(4)(2)(1) + (4)(2)(1) + ((4+1)(2)(4))}{4 + 4 + 8}$$

$$\bar{y} = 3.5 \text{ in.}$$

Moment of Inertia on N:

$$I = 2\left(\frac{b_1 h_1^3}{12} + (b_1 h_1)(\bar{y} - \bar{y}_1)^2\right) + \left(\frac{b_3 h_3^3}{12} + (b_3 h_3)(\bar{y}_3 - \bar{y})^2\right)$$

$$I = 2(14.33) + 2.67 + 18$$

$$I = 49.33 \text{ in.}^4$$

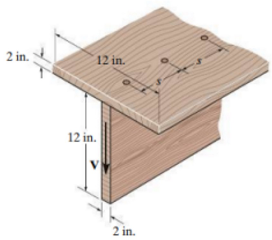
$$Q_{max} = A'\bar{y}' = 2(3.5)(1.75)$$

$$Q_{max} = 12.25 \text{ in.}^3$$

$$\tau_{max} = \frac{V_{max} Q_{max}}{It} = \frac{(2400)(12.25)}{(49.33)(2)}$$

$$\boxed{\tau_{max} = 298 \text{ psi}}$$

12–42. The T-beam is constructed as shown. If each nail can support a shear force of 950 lb, determine the maximum shear force V that the beam can support and the corresponding maximum nail spacing s to the nearest $\frac{1}{8}$ in. The allowable shear stress for the wood is $\tau_{allow} = 450$ psi.



$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{\bar{y}_1 A_1 + \bar{y}_2 A_2}{A_1 + A_2} = \frac{\bar{y}_1 (b_1 h_1) + \bar{y}_2 (b_2 h_2)}{(b_1 h_1) + (b_2 h_2)}$$

$$\bar{y} = \frac{(12+1)(2(12)) + (6)(12(2))}{(2(12)) + (12(2))}$$

$$\bar{y} = 9.5 \text{ in.}$$

Moment of Inertia for T-beam:

$$I = \frac{b_1 h_1^3}{12} + A_1 d_1^2 + \frac{b_2 h_2^3}{12} + A_2 d_2^2 = \frac{12(2)^3}{12} + 12(2)(13-9.5)^2 + \frac{2(12)^3}{12} + 2(12)(9.5-6)^2$$

$$I = 884 \text{ in.}^4$$

Section 1:

$$Q_{A_1} = \bar{y}_1' A_1 = (3.5)(2)(12)$$

$$Q_{A_1} = 84 \text{ in.}^3$$

Section 2:

$$Q_{A_2} = \bar{y}_2' A_2 = (4.75)(2)(12)$$

$$Q_{A_2} = 90.25 \text{ in.}^3$$

$$\sigma_{allow} = \frac{F}{s} = \frac{950}{s} = \frac{VQ_{A_1}}{I} = \frac{8815.51(84)}{884}$$

$$\boxed{s = 1.134 \text{ in.} \approx 1\tfrac{1}{8} \text{ in.}}$$

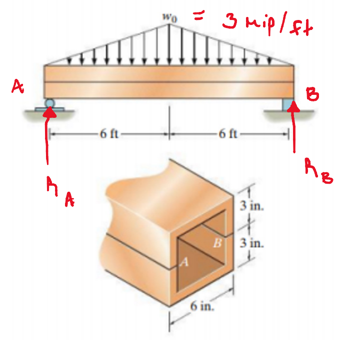
Max nail

$$\tau_{allow} = \frac{VQ_{max}}{It}$$

$$450 = \frac{V(90.25)}{884(2)}$$

$$\boxed{V = 8.82 \text{ kip.}}$$

12–45. The member consists of two plastic channel strips 0.5 in. thick, glued together at A and B. If the distributed load has a maximum intensity of $w_0 = 3$ kip/ft, determine the maximum shear stress resisted by the glue.



$R_A = R_B$

$\sum F_y = 0$

$R_A + R_B = 2\left(\frac{1}{2}\right)(3)(6)$

$R_A + R_B = 18$

$2R_A = 18$

$R_A = R_B = 9$ kip.

$V_{max} = R_A$

$V_{max} = 9$ kip.

$$I = \frac{1}{12}(6)(6)^3 - \frac{1}{12}(5)(5)^3$$

$I = 55.92 \text{ in.}^4$

$y_1 = 2.5 + \frac{.5}{2}$ $\qquad$ $y_2 = \frac{2.5}{2}$

$y_1 = 2.75$ in. $\qquad$ $y_2 = 1.25$ in.

$$Q = \sum \bar{A} y = \bar{A}_1 y_1 + \bar{A}_2 y_2 + \bar{A}_3 y_2$$

$Q = 11.375 \text{ in.}^3$

$$\tau_{max} = \frac{V_{max} Q}{It} = \frac{(9)(11.375)}{(55.92)(2(.5))}$$

$$\boxed{\tau_{max} = 1.83 \text{ ksi.}}$$