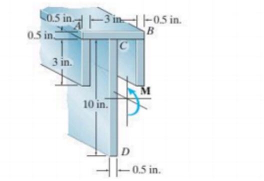
Homework 11

Tuesday, November 17, 2020 8:20 PM

"I pledge my honor I have abided by the Stevens Honor system."

\- Alex Gaskins

***11–48.** Determine the moment M that will produce a maximum stress of 10 ksi on the cross section.



$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{y_1 A + 2(y_2 A_2) + y_3 A_3}{A_1 + 2A_2 + A_3} = \frac{10.25(2) + 2(8.5(1.5)) + (50(.5))}{2 + 6 + 5}$$

$$\bar{y} = 7.1 \text{ in.}$$

$$I = \frac{1}{12} b_1 h_1^3 + A_1 (y_1 - \bar{y})^2 + 2\left(\frac{1}{12} b_2 h_2^3 + A_2 (y_2 - \bar{y})^2\right) + \frac{1}{12} b_3 h_3^3 + A_3 (y_3 - \bar{y})^2$$

$$I = \frac{1}{12}(4)(.5)^3 + 2(10.25 - 7.1)^2 + 2\left(\left(\frac{1}{12}(.5)(3)^3 + 3(.5)(8.5 - 7.1)^2\right)\right) + \frac{1}{12}(.5)(10)^3 + (5)(5 - 7.1)^2$$

$$I = 91.73 \text{ in.}^4$$

$$\sigma_{max} = \frac{Mc}{I}$$

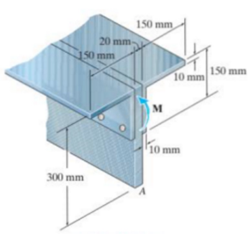
$$M = \frac{\sigma I}{c}$$

$$M = \frac{10(91.73)}{7.1}$$

$$M = \frac{129.2 \text{ kip} \cdot \text{in}}{12}$$

$$\boxed{M = 10.8 \text{ kip} \cdot \text{ft}}$$

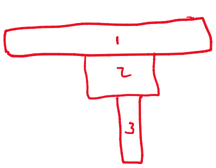
11–54. If the built-up beam is subjected to an internal moment of $M = 75$ kN · m, determine the maximum tensile and compressive stress acting in the beam.



$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3}$$

$$\bar{y} = \frac{(3000)(75) + (5600)(220) + (3200)(295)}{3000 + 5600 + 3200}$$

$$\bar{y} = 203.47 \text{ mm.}$$



$$A_1 = 150(20) = 3000 \text{ mm.}^2$$

$$A_2 = 140(40) = 5600 \text{ mm}^2$$

$$A_3 = 320(10) = 3200 \text{ mm.}^2$$

$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2$$

$$I_1 = \frac{1}{12}(20)(150)^3 + 3000(203.47)^2$$

$$I_1 = 5.514 \times 10^7 \text{ mm.}^4$$

$$= \frac{1}{12}(40)(140)^3 + 5600(203.47 - 220)^2$$

$A_3 = 320(10) = 3200 \ldots$

$I_2 = \frac{1}{12}(40)(140)^3 + 5600(203.47-220)^2$

$I_2 = 1.068 \times 10^7 \text{ mm.}^4$

$I_{total} = I_1 + I_2 + I_3$

$I_{total} = \dfrac{(5.514 + 1.068 + 2.684) \times 10^7}{1000^4}$

$I_{total} = 92.65 \times 10^{-6} \text{ m.}^4$

$I_3 = \frac{1}{12}(320)(10)^3 + 3200(203.47-205)^2$

$I_3 = 2.684 \times 10^7 \text{ mm.}^4$

$\sigma_T = \dfrac{MC_1}{I} = \dfrac{75000(.20347)}{92.65 \times 10^{-6}}$

$\sigma_T = 164.71 \times 10^6 \frac{N}{m^2} = \boxed{164.71 \text{ MPa}}$

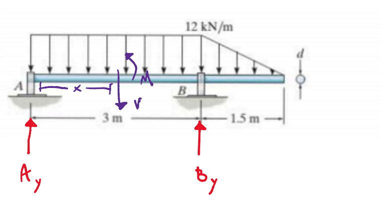
Tensile

$\sigma_c = \dfrac{MC_2}{I} = \dfrac{75000(.3-.20347)}{92.652 \times 10^{-6}}$

$\sigma_c = 78.14 \times 10^6 \frac{N}{m^2} = \boxed{78.14 \text{ MPa}}$

Compressive

11–67. The shaft is supported by smooth journal bearings at A and B that only exert vertical reactions on the shaft. If $d = 90$ mm, determine the absolute maximum bending stress in the beam, and sketch the stress distribution acting over the cross section.



$\circlearrowleft \sum M_A = 0$

$B_y(3) - (36)\left(\frac{3}{2}\right) - (1.5)(6)\left(\frac{10.5}{3}\right) = 0$

$3B_y = 85.5$

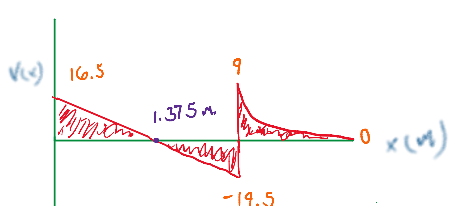
$B_y = 28.5 \text{ kN.}$

$\uparrow \sum F_y = 0$

$A_y + B_y - 36 - 6(1.5) = 0$

$A_y + 28.5 = 45$

$A_y = 16.5 \text{ kN.}$



11.35

$M(x) = 16.5x - \frac{12}{2}(x-0)^2$

$M(x) = 16.5x - 6x^2$

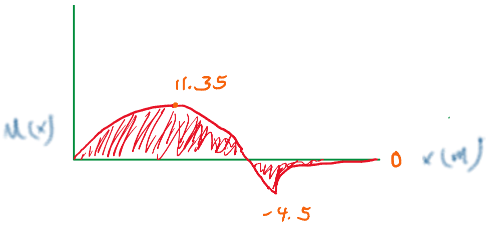
$V(x) = 16.5 - 12(x-0)$

For Point A:

$x = 0$

$V(0) = 16.5 - 12(0)$

For Point A:



For Point A:

$V = 0;\ M_{max},\ x = 1.375$

$M_{max} = 16.5(1.375) - 6(1.375)^2$

$M_{max} = 11.34\ kN\cdot m$

For Point B:

$x = 3$

$M(3) = 16.5(3) - 6(3)^2$

$M(3) = -4.5\ kN\cdot m$

For Point A:

$x = 0$

$V(0) = 16.5 - 12(0)$

$V(0) = 16.5\ kN.$

For Point B:

$x = 3$

$V(3) = 16.5 - 12(3)$

$V(3) = -19.5\ kN.$

When $V = 0$

$0 = 16.5 - 12x$

$x = \frac{16.5}{12}$

$x = 1.375$

Shear Force:

$V(x) = 16.5 - 12(3) - 6(x-3) + 28.5$

B: $V(3) = 9\ kN.$ C: $V(4.5) = 0$

Bending Moment:

$M(x) = 16.5x - 12(3)(x - 1.5) - 6(x-3)\left(x - \frac{10.5}{3}\right) + 28.5(x-3)$

B: $M(3) = -4.5\ kN\cdot m$ C: $M(4.5) = 0$

$I = \frac{d^4 \pi}{64} = \frac{(.09)^4 \pi}{64}$

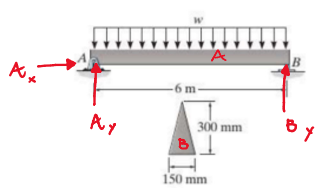
$I = 3.22 \times 10^{-6}\ m^4$

$\sigma_{max} = \frac{M_{max}\left(\frac{d}{2}\right)}{I}$

$\sigma_{max} = \frac{(11.34 \times 10^3)\left(\frac{.09}{2}\right)}{3.22 \times 10^{-6}}$

$\sigma_{max} = 158 \times 10^6\ Pa = \boxed{158\ MPa}$

***11–76.** If the intensity of the load w = 15 kN/m, determine the absolute maximum tensile and compressive stress in the beam.



$\Sigma M_A = 0$

$B_y(6) - W(6)\left(\frac{6}{2}\right) = 0$

$B_y(6) - 15(6)\left(\frac{6}{2}\right) = 0$

$B_y = 45\ kN.$

$\Sigma F_y = 0$

$A_y + B_y - 15(6) = 0$

$A_y + 45 = 90$

$A_y = 45\ kN.$

$M_{max} = \frac{WL^2}{8}$

$M_{max} = \frac{15(6)^2}{8}$

$M_{max} = 67.5\ kN\cdot m$

B:

$I = \frac{1}{36} bh^3$

$I = \frac{1}{36}(150)(300)^3$

$$M_{max} = \frac{\quad}{8}$$

$$M_{max} = 67.5 \text{ kN.m}$$

$$I = \frac{\quad}{36}$$

$$I = \frac{1}{36}(150)(300)^3$$

$$I = \frac{112.5 \times 10^6}{1000^4}$$

$$I = 1.125 \times 10^{-4} \text{ m.}^4$$

A:

$$\sigma_c = \frac{M_{max}(y_c)}{I} = \frac{(67.5 \times 10^5)\left(\frac{400}{3} \times 10^{-3}\right)}{(1.125 \times 10^{-4})}$$

$$\boxed{\sigma_c = 120 \text{ MPa}}$$

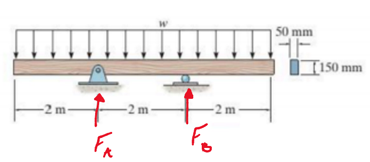
compressive

$$\sigma_T = \frac{M_{max}(y_2)}{I} = \frac{(67.5 \times 10^5)\left(\frac{300}{3} \times 10^{-3}\right)}{(1.125 \times 10^{-4})}$$

$$\boxed{\sigma_T = 60 \text{ MPa}}$$

Tensile

11–94. The beam has a rectangular cross section as shown. Determine the largest intensity w of the uniform distributed load so that the bending stress in the beam does not exceed $\sigma_{max} = 10$ MPa.

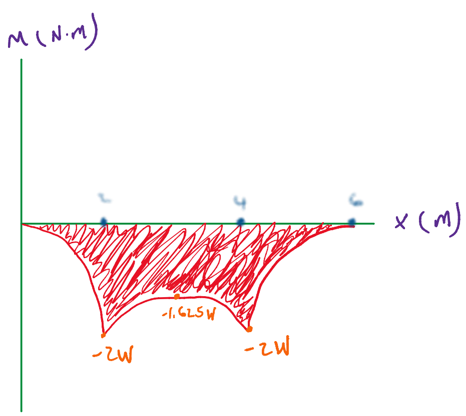


$$F_A + F_B = 6w$$

$$F_B(2) - w(4)(2) + w(2) = 0$$

$$F_B = 3w$$

$$F_A = 3w$$



$$M_{max} = -2w$$

$$M_1 = \frac{-wx^2}{2}$$

$x = 2$ m.

$$M_1 = -2w$$

$$M_2 = F_A(x-2) - \frac{wx^2}{2}$$

$x = 3.5$ m.

$$M_2 = -1.625w$$

$$M_3 = -\frac{wx^2}{2}$$

$x = 2$

$$M_3 = -2w$$

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(.05)(.15)^3$$

$$I = 1.406 \times 10^{-5} \text{ m.}^4$$

$$\sigma_{max} = \frac{M_{max}(y)}{I}$$

$$10 \times 10^{6} = \frac{2w(.075)}{(1.40625 \times 10^{-5})}$$

$$w = \frac{140.625}{.15}$$

$$\boxed{w = 937.5 \ \frac{N}{m}}$$