Name:

Pledge:

EE/CPE 345: Modeling and Simulation
Midterm Exam – Spring 2019

1. **(15 points)** To transmit a packet from source to destination in the network depicted below, a packet needs to be forwarded on multiple transmission lines. Each transmission line has a propagation delay that is exponentially distributed with a mean of 100 ms. Assume that the packet is instantly forwarded on the next transmission line and there is no queueing of packets at routers (infinite bandwidth assumption).
   a. What is the distribution of the end-to-end propagation delay experienced by a packet in this network?
   b. Determine the mean and variance for this distribution
   c. What is the probability that the delay experienced by the packet will be greater than 300 ms?

Solution:

a. Sum of three exponential iid variables → Erlang of order 3 (5pt)
b. Exponential mean E[X]= 100 ms → λ= 1/E[X] = 0.01–rate (5pt)
   Erlang mean E[Y] = 3E[X] = 300 ms, where Y=X1+X2+X3
   Var[Y] = 3 var[X] =$3/\lambda^2$ = $3E[X]^2$ = 30000ms = 30 s
c. (5pt)
   P(Y>300 ms) = $\sum_{i=0}^{2} \frac{(\lambda * 300)^i e^{-300\lambda}}{i!} = 1 - e^{-3} - 3e^{-3} - \frac{9}{2} e^{-3} = 1 - 0.05(1 + 3 + 4.5) = 0.575$

2. **(20 points)** Consider you are simulating the ARQ system studied in class. The objective is to determine the delay due to retransmissions.
   a. Sketch the system model using a block diagram
   b. Determine the entities and their attributes (as relevant to the simulation)
   c. Determine the system state
   d. Determine the events

Solution:
There is no unique representation of the system, depending on the assumptions made the system model may slightly be different (for example a more accurate model will assume that source generates packets continuously, which are then queued waiting for transmission).
Here we assume no queueing of packets (as in our current class example) – source sends new packet only if previous packet correctly received.

a. (5pt) Source Destination

b. (5pt)

source – number of packets to generate (inter-arrival times not relevant for this model since source generates a new packet only when previous one ACK-ed)

destination – no attributes for our current implementation

channel (not an entity, it is connection but has attributes): probability of bit error (you could also consider propagation delay)

c. State variable for this simplified model (5pt)
S = {b,nb}: Source state – blocked or not blocked (it is blocked until packet is received correctly and ACK-ed – ACK changes the state of the source).
D= {1=e, ne}: Destination State – packet received in error/ packet received correctly (not errored)

State variable (S,D)

d. Events: (5pt)
Packet arrival at destination
ACK/NACK arrival at source

3. **(20 points)** You are running a simulation for a MacDonald manager to determine what queueing architecture would be the most efficient. Currently this MacDonald has two servers and customers wait in separate queues to be served (one queue for each server). The manager is asking the question if having a common queue that is served by the two servers in parallel would be a better alternative. Customers arrive with Poisson arrivals with rate λ, and each server has an exponentially distributed service time with rate μ.
   a. What queue type is the current system?
   b. What queue type is the alternate system?
   c. For λ = 1 customer/min and μ = 1 customer per minute, compute the average delay in the system for the two scenarios and determine which one is better. Assume that for the first system customers join the two queues randomly with equal probability.
   d. **(Extra credit 5pt)**. Provide an intuitive/statistical explanation for your results.

Solution:

a. 2 M/M/1 queues working independently (5pt)
b. M/M/2 queue (5pt)
c. For M/M/1: arrivals λ /2 = 0.5 customers per min. → ρ1 = 0.5

   w_1= 1/(μ-λ) = 2 min (4pt)

   For M/M/2 → ρ2= λ/(2μ) = 0.5

   P0=

$$\left\{\left[\sum_{n=0}^{c-1}\frac{(\lambda/\mu)^n}{n!}\right]+\left[\left(\frac{\lambda}{\mu}\right)^c\left(\frac{1}{c!}\right)\left(\frac{c\mu}{c\mu-\lambda}\right)\right]\right\}^{-1}=\left\{\left[\sum_{n=0}^{c-1}\frac{(c\rho)^n}{n!}\right]+\left[(c\rho)^c\left(\frac{1}{c!}\right)\left(\frac{1}{1-\rho}\right)\right]\right\}^{-1}$$

   $=[1/1+1/1+1/2*2/(2-1)]^{-1} = 1/3 = 0.33$

   → $w2 = 1/\lambda*[c\rho+ (c\rho)^{c+1}P_0/(cc!(1-\rho)^2)]=1*[1+1*0.33/(4*0.25)]=1.33$ min (5pt)

   Comparing w1 and w2 → the new architecture with a common queue is better (1pt)

d. Having a common queue allows for a better statistical multiplexing of traffic. For example, having two queues, it may happen that one queue is completely empty, while the other queue has customers waiting in queue, which will not happen when the queue is common (5pt)

4. (10 points) Complete the table for a bank teller simulation starting at time T=0 with an arrival, given that some previously generated random variables for inter-arrival times are 2, 1, 5, 2, and for departure times are 1, 4, 3, 2. (no need to use all the values) The state of the system is characterized by (number of packets in line, busy/idle for server). Busy state for server is marked as "1". The events in FEL are denoted as (type of event, time). For arrival events, type =1, for departure events, type =2.

Clock	**Arrival Time**	**Departure time**	**System state**	**FEL**
T=0	0		(0,1)	(2,1) (1,2)
T=1		1	(0,0)	(1,2)
T=2	1		(0,1)	(1,3) (2,6)
T=3	3		(1,1)	(2,6) (1,8)
T=6		6	(0,1)	(1,8) (2,9)