

Stevens Institute of Technology
Department of Electrical and Computer Engineering

Spring Semester 2022

CPE 462 Introduction to Image Processing

Homework 1: Due Feb. 10.

1.1 Determine if $y[n] = 3x[n] + 7$ is linear? time-invariant?

Linearity:

$$y_1[n] = 3x_1[n] + 7$$

$$y_2[n] = 3x_2[n] + 7$$

$$T\{ax_1[n] + bx_2[n]\} = 3(ax_1[n] + bx_2[n]) + 7$$

$$aT\{x_1[n]\} + bT\{x_2[n]\} = (3ax_1[n] + 7a) + (3bx_2[n] + 7b)$$

$$T\{ax_1[n] + bx_2[n]\} \neq aT\{x_1[n]\} + bT\{x_2[n]\}$$

∴ Non-Linear

Time-Invariance:

$$x_1[n] = x[n - n_0], \text{ then } y_1[n] = T\{x_1[n]\} = y[n - n_0]$$

$$3x_1[n] + 7 = 3x[n - n_0] + 7$$

∴ Time-Invariant

1.2 Prove that convolution is commutative, i.e. $x[n] * h[n] = h[n] * x[n]$.

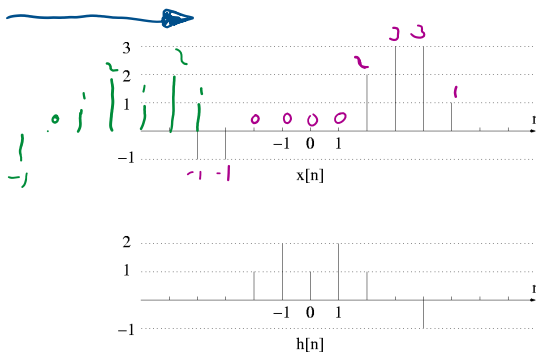
$$y[n] = \sum_{-\infty}^{\infty} (x[k]h[n-k])$$

$$x[n] * h[n] = \int_{-\infty}^{\infty} (x[k]h[n-k])dk$$

Let $l = n - k$

$$x[n] * h[n] = -\int_{-\infty}^{\infty} (x[n-l]h[l])dl = -\int_{-\infty}^{\infty} (h[l]x[n-l])dl = h[n] * x[n]$$

1.3.1 Calculate the 1-D convolution $x[n] * h[n]$ using graphic approach, provide necessary intermediate steps.

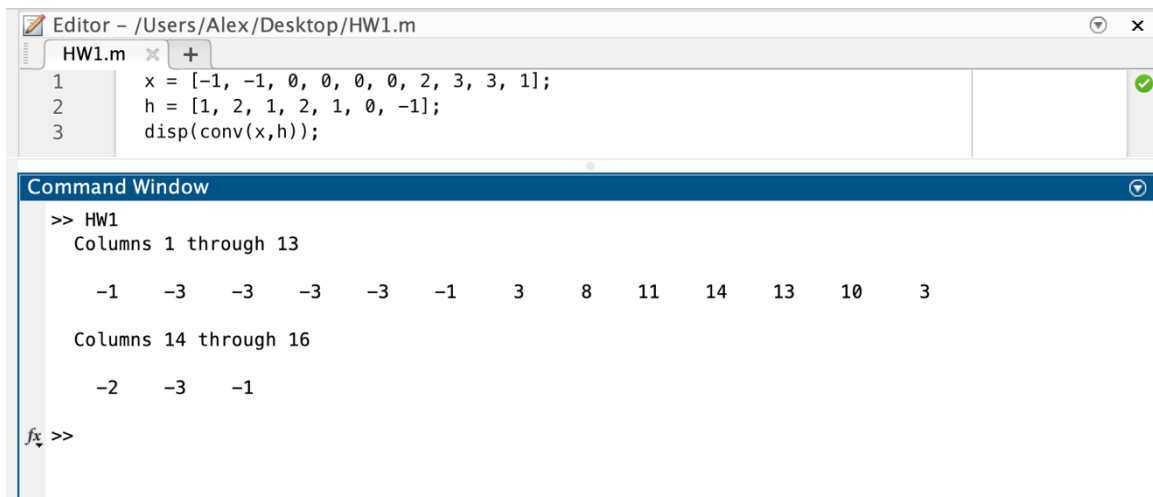


Marsh:
 using $h[n]$

$1(-1) = -1$, $2(-1)+1(-1) = -3$, $1(-1)+2(-1)+1(0) = -3$, $2(-1)+1(-1)+2(0)+1(0) = -3$, $1(-1)+2(-1)+1(0)+2(0)+1(0) = -3$,
 $0(-1)+1(-1)+2(0)+1(0)+2(0)+1(0) = -1$, $-1(-1)+0(-1)+1(0)+2(0)+1(0)+2(0)+1(2) = 3$, $-1(-1)+0(0)+1(0)+2(0)+1(0)+2(2)+1(3) = 8$,
 $-1(0)+0(0)+1(0)+2(0)+1(2)+2(3)+1(3) = 11$, $-1(0)+0(0)+1(0)+2(2)+1(3)+2(3)+1(1) = 14$, $-1(0)+0(0)+1(2)+2(3)+1(3)+2(1) = 13$,
 $-1(0)+0(2)+1(3)+2(3)+1(1) = 10$, $-1(2)+0(3)+1(3)+2(1) = 3$, $-1(3)+0(3)+1(1) = -2$, $-1(3)+0(1) = -3$, $-1(1) = -1$

$y[n] = \{-1, -3, -3, -3, -3, -1, 3, 8, 11, 14, 13, 10, 3, -2, -3, -1\}$

1.3.2 Confirm your results using MATLAB [conv](#) function. Show your steps by saving your script using diary command or some screen shots. (Use help to find more about [conv](#) and diary in MATLAB.)



The screenshot shows the MATLAB Editor window with a script named HW1.m. The script defines two vectors, x and h, and uses the conv function to compute their convolution. The Command Window displays the output of the script, showing the resulting vector y[n] in two columns.

```

Editor - /Users/Alex/Desktop/HW1.m
HW1.m
1 x = [-1, -1, 0, 0, 0, 0, 2, 3, 3, 1];
2 h = [1, 2, 1, 2, 1, 0, -1];
3 disp(conv(x,h));

Command Window
>> HW1
Columns 1 through 13
    -1    -3    -3    -3    -3    -1     3     8    11    14    13    10     3

Columns 14 through 16
    -2    -3    -1

fx >>
  
```