Exam 2

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"I pledge my honor I have abided by the Stevens Honor system."

- Alex Gaskins

5.4 (*a*) The resistivity of a p-type GaAs material at $T = 300$ K is required to be $\rho = 0.35\,(\Omega\text{-cm})$. Determine the acceptor impurity concentration that is required. What is the hole mobility corresponding to this impurity concentration? (*b*) An n-type GaAs material is required to have a conductivity of $\sigma = 120\,(\Omega\text{-cm})^{-1}$. What donor impurity concentration is required and what is the corresponding electron mobility?

$$\rho = \frac{1}{e\mu_p N_a} \qquad x = \mu_p N_a$$

$$0.35 = \frac{1}{(1.6\times10^{-19})\,x} \qquad x = 1.786\times10^{19}$$

a.) From figure 5.2 and 5.3, $\boxed{N_a \approx 8\times10^{16}/cm^3}$

$$\mu_p = \frac{1.786\times10^{19}}{8\times10^{16}} = \boxed{220\ cm^2/V{\cdot}s}$$

b.) $\sigma = e\mu_n N_d = 120/\Omega\ cm$

$7.5\times10^{20} = \mu_n N_d$ where $\boxed{N_d \approx 2\times10^{17}/cm^3}$

$$\mu_n = \frac{7.5\times10^{20}}{2\times10^{17}} = \boxed{3800\ cm^2/V{\cdot}s}$$

5.14 In a particular semiconductor material, $\mu_n = 1000$ cm²/V-s, $\mu_p = 600$ cm²/V-s, and $N_C = N_V = 10^{19}$ cm^{-3}. These parameters are independent of temperature. The measured conductivity of the intrinsic material is $\sigma = 10^{-6}\,(\Omega\text{-cm})^{-1}$ at $T = 300$ K. Find the conductivity at $T = 500$ K.

$\sigma = e(\mu_n + \mu_p)n_i$ | $n_i^2 = N_c N_v \exp\left(-\frac{E_g}{kT}\right)$

Conductivity

$$n_i = \frac{\sigma}{e(\mu_n + \mu_p)} = \frac{10^{-6}}{(1.6\times10^{-19})(1000+600)} = 3.9\times10^{9}/cm^3$$

$$E_g = kT \ln\left(\frac{N_c N_v}{n_i^2}\right) = (26\times10^{-3}) \ln\left(\frac{(10^{19})(10^{19})}{(3.9\times10^{9})^2}\right) = 1.122\ eV$$

$$kT = 26\times10^{-3}\left(\frac{500}{300}\right) = 0.0432\ eV$$

$$n_i^2 = (10^{19})(10^{19}) \exp\left(\frac{-1.122}{0.0432}\right) = 2.29\times10^{13}/cm^3$$

$$\sigma = (1.6\times10^{-19})(1000+600)(2.29\times10^{13}) = \boxed{5.86\times10^{-3}/\Omega\cdot cm}$$

5.16 An n-type silicon material at $T = 300$ K has a conductivity of 0.25 $(\Omega\text{-cm})^{-1}$. (*a*) What is the donor impurity concentration and the corresponding electron mobility? (*b*) Determine the expected conductivity of the material at (*i*) $T = 250$ K and (*ii*) $T = 400$ K.

a.) $\sigma = e\mu_n N_d$

$$0.25 = (1.6\times10^{-19})\mu_n N_d$$

From figure 5.3, $\boxed{N_d \approx 1.2\times10^{15}/cm^3}$

$$\frac{\left(\frac{0.25}{1.6\times10^{-19}}\right)}{1.2\times10^{15}} = \mu_n = \boxed{1300\ cm^2/V.s}$$

b.) [i] $T = 250K = -23°C$

5.2 yields $\mu_n \approx 1800\ cm^2/V\cdot s$

$$\sigma = (1.6\times10^{-19})(1800)(1.2\times10^{15}) = \boxed{0.346/\Omega\cdot cm}$$

[ii] $T = 400\ K = 127\,^\circ C$

5.2 yields $\mu_n \approx 670\ cm^2/V\cdot s$

$$\sigma = (1.6\times10^{-19})(670)(1.2\times10^{15}) = \boxed{0.129/\Omega\cdot cm}$$

5.32 The hole concentration in p-type GaAs is given by $p(x) = 10^{16}(1 + x/L)^2\ \text{cm}^{-3}$ for $-L \le x \le 0$ where $L = 12\ \mu\text{m}$. The hole diffusion coefficient is $D_p = 10\ \text{cm}^2/\text{s}$. Calculate the hole diffusion current density at (*a*) $x = 0$, (*b*) $x = -6\ \mu\text{m}$, and (*c*) $x = -12\ \mu\text{m}$.

$$J_p = -eD_p\frac{dp}{dx} = -eD_p\frac{d}{dx}\left[10^{16}\left(1+\frac{x}{L}\right)^2\right]$$

$$= -eD_p\left[\frac{10^{16}}{L}\left(2\left(1+\frac{x}{L}\right)\right)\right]$$

a.) $x = 0\ \mu m$

$$J_p = \frac{-(1.6\times10^{-19})(10)(10^{16})(2)}{12\times10^{-4}} = \boxed{-26.7\ A/cm^2}$$

b.) $x = -6\ \mu m$

$$J_p = \frac{-(1.6\times10^{-19})(10)(10^{16})(2)(1-\frac{6}{12})}{12\times10^{-4}} = \boxed{-13.3\ A/cm^2}$$

c.) $x = -12\ \mu m$

$$J_p = \frac{-(1.6\times10^{-19})(10)(10^{16})(2)(1-\frac{12}{12})}{12\times10^{-4}} = \boxed{0\ A/cm^2}$$

5.38 In n-type silicon, the Fermi energy level varies linearly with distance over a short range. At $x = 0$, $E_F - E_{Fi} = 0.4$ eV and, at $x = 10^{-3}$ cm, $E_F - E_{Fi} = 0.15$ eV. (*a*) Write the expression for the electron concentration over the distance. (*b*) If the electron diffusion coefficient is $D_n = 25$ cm²/s, calculate the electron diffusion current density at (*i*) $x = 0$ and (*ii*) $x = 5 \times 10^{-4}$ cm.

$$n = n_i \exp\left(\frac{E_F - E_{Fi}}{kT}\right)$$

a.)

$$E_F - E_{Fi} = ax + b$$

$$E_F - E_{Fi} = a(0) + b$$

$$b = 0.4$$

$$E_F - E_{Fi} = a(10^{-3}) + 0.4 = 0.15$$

$$a = -2.5 \times 10^{2} \text{ eV/cm}$$

$$E_F - E_{Fi} = 0.4 - 2.5 \times 10^{2} x$$

$$n = n_i \exp\left(\frac{0.4 - 2.5 \times 10^{2} x}{kT}\right)$$

b.)

$$J_n = eD_n \frac{dn}{dx} = e D_n n_i \left[\frac{-2.5 \times 10^{2}}{kT}\right] \exp\left[\frac{0.4 - 2.5 \times 10^{2} x}{kT}\right]$$

$$T = 300K$$

$$J_n = \frac{-(1.6 \times 10^{-19})(25)(1.5 \times 10^{10})(2.5 \times 10^{2})}{0.0259} \exp\left[\frac{0.4 - 2.5 \times 10^{2} x}{kT}\right]$$

$$J_n = -5.79 \times 10^{-4} \exp\left[\frac{0.4 - 2.5 \times 10^{2} x}{kT}\right]$$

[i] $x = 0 \mu m$

$$J_n = -5.79 \times 10^{-4} \exp\left[\frac{0.4 - 2.5 \times 10^{2}(0)}{kT}\right]$$

$$\boxed{J_n = -2.95 \times 10^{3} \text{ A/cm}^2}$$

[ii] $x = 5\,\mu m$

$$J_n = -5.79 \times 10^{-4} \exp\left[\frac{0.4 - 2.5\times10^{2}(5)}{kT}\right]$$

$$\boxed{J_n = -23.7 \ \ A/cm^2}$$

6.2 GaAs, at $T = 300$ K, is uniformly doped with acceptor impurity atoms to a concentration of $N_a = 2 \times 10^{16}$ cm^{-3}. Assume an excess carrier lifetime of 5×10^{-7}s. (*a*) Determine the electron–hole recombination rate if the excess electron concentration is $\delta n = 5 \times 10^{14}$ cm^{-3}. (*b*) Using the results of part (*a*), what is the lifetime of holes?

$$p_o = N_a = 2\times10^{16}/cm^3$$

$$n_o = \frac{n_i^2}{p_o} = \frac{(1.8\times10^{6})^2}{2\times10^{16}} = 1.62\times10^{-4}/cm^3$$

a.) $$R' = \frac{\Delta n}{\tau_{n0}} = \frac{5\times10^{14}}{5\times10^{-7}} = 10^{21}/cm\cdot s$$

b.) $$R_p = \frac{p_o}{\tau_{pt}} = \frac{n_o}{\tau_{nt}} = \frac{n_o}{\tau_{no}}$$

$$\tau_{pt} = \frac{p_o}{n_o}(\tau_{no}) = \frac{(2\times10^{16})}{(1.62\times10^{-4})}(5\times10^{-7})$$

$$\boxed{\tau_{pt} = 6.17\times10^{13}\ s}$$

6.12 Consider a silicon sample at $T = 300$ K that is uniformly doped with acceptor impurity atoms at a concentration of $N_a = 10^{16}$ cm^{-3}. At $t = 0$, a light source is turned on generating excess carriers uniformly throughout the sample at a rate of $g' = 8 \times 10^{20}$ cm^{-3} s^{-1}. Assume the minority carrier lifetime is $\tau_{n0} = 5 \times 10^{-7}$ s, and assume mobility values of $\mu_n = 900$ cm^2 /V-s and $\mu_p = 380$ cm^2 /V-s. (*a*) Determine the conductivity of the silicon as a function of time for $t \geq 0$. (*b*) What is the value of conductivity at (*i*) $t = 0$ and (*ii*) $t = \infty$?

$$p_o = N_a = 10^{16}/cm^3$$

$$n_o = \frac{n_i^2}{p_o} = \frac{(1.5\times10)^2}{10^{16}} = 2.25\times10^4/cm^3$$

a.) $$\sigma = e\mu_n(n_o + \Delta n) + e\mu_p(p_o + \Delta p)$$

$$= e\mu_p p_o + e(\mu_n + \mu_p)\Delta n$$

$$\Delta n = \Delta p = g'\tau_{no}\left(1 - e^{-t/\tau_{no}}\right)$$

$$= (8\times10^{20})(5\times10^{-7})\left(1 - e^{-t/\tau_{no}}\right)$$

$$= 4\times10^{14}\left(1 - e^{-t/\tau_{no}}\right)$$

$$\sigma = (1.6\times10^{-19})(380)(10^{16}) + (1.6\times10^{-19})(900+380)\left[4\times10^{14}\left(1 - e^{-t/\tau_{no}}\right)\right]$$

$$\sigma = 0.61 + 0.082\left(1 - e^{-t/\tau_{no}}\right)/\Omega\cdot cm$$

b.)

[i] $$\sigma(0) = 0.61/\Omega\cdot cm$$

[ii] $$\sigma(\infty) = 0.69/\Omega\cdot cm$$

6.14 A bar of silicon at $T = 300$ K has a length of $L = 0.05$ cm and a cross-sectional area of $A = 10^{-5}$ cm^2. The semiconductor is uniformly doped with $N_d = 8 \times 10^{15}$ cm^{-3} and $N_a = 2 \times 10^{15}$ cm^{-3}. A voltage of 10 V is applied across the length of the material. For $t < 0$, the semiconductor has been uniformly illuminated with light, producing an excess carrier generation rate of $g' = 8 \times 10^{20}$ cm^{-3} s^{-1}. The minority carrier lifetime is $\tau_{p0} = 5 \times 10^{-7}$ s. At $t = 0$, the light source is turned off. Determine the current in the semiconductor as a function of time for $t \geq 0$.

$$V = IR \qquad I = \frac{V}{R} \qquad R = \frac{L}{\sigma A}$$

$$I = \frac{\sigma A}{L}(V)$$

$$N_I = N_d + N_a = 8\times10^{15} + 2\times10^{15} = 10^{16}/\text{cm}^3$$

$$\mu_n \approx 1300 \text{ cm}^2/\text{V}\cdot\text{s} \qquad \mu_p \approx 400 \text{ cm}^2/\text{V}\cdot\text{s}$$

$$\sigma = e\mu_n n_0 + e(\mu_n + \mu_p)\Delta p$$

$$\Delta p = g' \tau_{p0} e^{-t/\tau_{p0}} = (8\times10^{20})(5\times10^{-7})e^{-t/\tau_{p0}}$$

$$\Delta p = 4\times10^{14} e^{-t/\tau_{p0}} / \text{cm}^3$$

$$\sigma = (1.6\times10^{-19})(1300)(8\times10^{15} - 2\times10^{15}) + (1.6\times10^{-19})(1300+400)(4\times10^{14} e^{-t/\tau_{p0}})$$

$$\sigma = 1.25 + 0.11 e^{-t/\tau_{p0}}$$

$$I = \frac{(1.25 + 0.11 e^{-t/\tau_{p0}})(10^{-5})(10)}{0.05}$$

$$\boxed{I = 2.5\times10^{-3} + (2.2\times10^{-4})e^{-t/\tau_{p0}} \text{ A}}$$

6.20 The $x = 0$ end of an $N_a = 1 \times 10^{14}$ cm^{-3} doped semi-infinite ($x \geq 0$) bar of silicon maintained at $T = 300$ K is attached to a "minority carrier digester" which makes $n_p = 0$ at $x = 0$ (n_p is the minority carrier electron concentration in a p-type semiconductor). The electric field is zero. (*a*) Determine the thermal-equilibrium values of n_{p0} and p_{p0}. (*b*) What is the excess minority carrier concentration at $x = 0$? (*c*) Derive the expression for the steady-state excess minority carrier concentration as a function of x.

$$p_p = p_{po} + \Delta p = n_{po} + 0 = p_{po}$$

$N_a = 10^{14}/cm^3$ $\quad$ $p_p = N_a = 10^{14}/cm^3$

a.) $n_{po} \, p_{po} = n_i^2$

$$n_{po} = \frac{n_i^2}{p_{po}} = \frac{(1.5 \times 10^{14})^2}{10^{14}} = \boxed{2.25 \times 10^6/cm^3}$$

b.) $\Delta n = n_p - n_{po}$

$x = 0 \quad n_p = 0$

$$\Delta n = 0 - 2.25 \times 10^6 = \boxed{-2.25 \times 10^6/cm^3}$$

(minority carrier)

c.) $x = 0 \quad E = 0 \quad g' \approx 0 \quad \frac{\delta(n_p)}{\Delta t} = 0$

$$D_n \frac{d^2(\Delta n)}{dx^2} - \frac{\Delta n}{\tau_{no}} = 0$$

$$L^2 = D_n \tau_{no}$$

$$\frac{d^2(\Delta n)}{dx^2} - \frac{\Delta n}{L^2} = 0$$

Auxillary Solution:

$$n_p = A \exp\left(\frac{-x}{L_n}\right) + B \exp\left(\frac{x}{L_n}\right)$$

$B = 0; \quad x > 0$

$A = 0; \quad x < 0$

$$\boxed{\begin{aligned} \Delta n &= \Delta n(0) \exp\left(\frac{-x}{L_n}\right); \quad x \geq 0 \\ \Delta n &= \Delta n(0) \exp\left(\frac{x}{L_n}\right); \quad x \leq 0 \end{aligned}}$$

6.32 Consider n-type silicon doped at $N_d = 5 \times 10^{15}$ cm^{-3}. It is found that $E_{Fn} - E_F = 1.02 \times 10^{-3}$ eV. (*a*) What is the excess carrier concentration? (*b*) Determine $E_{Fn} - E_{Fi}$. (*c*) Calculate $E_{Fi} - E_{Fp}$.

a.) $E_{Fn} - E_F = (E_{Fn} - E_{Fi}) - (E_F - E_{Fi})$

$$= kT\left[\ln\left(\frac{n_0 + \Delta n}{n_i}\right)\right] - kT\left[\ln\left(\frac{n_0}{n_i}\right)\right]$$

$$= kT\left[\ln\left(\frac{n_0 + \Delta n}{n_0}\right)\right]$$

$$1.02 \times 10^{-3} = (0.0259) \ln\left[\frac{5 \times 10^{15} + \Delta n}{5 \times 10^{15}}\right]$$

$$5 \times 10^{15} + \Delta n = 5 \times 10^{15} \exp\left[\frac{1.02 \times 10^{-3}}{0.0259}\right]$$

$$\boxed{\Delta n = 2 \times 10^{14} / cm^3}$$

b.) $$E_{F_n} - E_{F_i} = kT\left[\ln\left(\frac{n_0 + \Delta n}{n_i}\right)\right]$$

$$= (0.0259)\ln\left[\frac{5\times10^{15} + 2\times10^{14}}{1.5\times10^{10}}\right]$$

$$= \boxed{0.33\ eV}$$

c.) $$E_{F_i} - E_{F_p} = kT\left[\ln\left(\frac{\Delta p}{n_i}\right)\right]$$

$$= (0.0259)\ln\left[\frac{2\times10^{14}}{1.5\times10^{10}}\right]$$

$$= \boxed{0.25\ eV}$$