Workshop 8

Tuesday, October 27, 2020 9:55 AM

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1. Determine the inverse Laplace transforms, $f(t) = \mathcal{L}^{-1}\{F(s)\}$.

(b) $F_1(s) = \dfrac{3s-15}{2s^2-4s+10}$ (c) $F_2(s) = \dfrac{3s^2+5s+3}{s^4+s^3}$

B.) $\frac{1}{2}\left(\frac{3s-15}{s^2-2s+5}\right)$

$s^2-2s+1 = -5+1$

$(s-1)^2+4$

$-\dfrac{\frac{12}{2}}{(s-1)^2+4}$

$3s-15 = 3(s-1)-12$

$\frac{3}{2}\left(\frac{s-1}{(s-1)^2+4}\right) = \frac{3}{2}e^t\cos(2t) \quad - \quad \frac{6}{2}\left(\frac{2}{(s-1)^2+4}\right) = 3e^t\sin(2t)$

$\frac{3}{2}e^t\cos(2t) - 3e^t\sin(2t)$

C.) $\dfrac{s^3(3s^2+5s+3)(s+1)}{s^3(s+1)} = \dfrac{As^3(s+1)}{s} + \dfrac{Bs^3(s+1)}{s^2} + \dfrac{Cs^3(s+1)}{s^3} + \dfrac{Ds^3(s+1)}{s+1}$

$3s^2+5s+3 = As^2(s+1) + Bs(s+1) + C(s+1) + Ds^3$

$A=1$

$B=2$

$C=3$

$D=-1$

$= \frac{1}{s} + \frac{2}{s^2} + \frac{3}{s^3} - \frac{1}{s+1}$

$\mathcal{L}^{-1} = 2t + \frac{3t^2}{2} - e^{-t}$

2. Use Laplace Transforms to solve the following initial value problems. Provide answers to each of the four steps listed here.

S1: Solve for $Y(s) = \mathcal{L}\{y(t)\}$;

S2: Set up the partial fractions decomposition (PF) for $Y(s)$;

S3: Without solving for the coefficients, transform the PF to $y(t)$;

S4: Solve for the unknown PF coefficients.

(b) $y'' + 4y' + 20y = 3te^{-t}$ $\quad y(0)=1,\ y'(0)=-1$

$L_y(y''+4y'+20y) = L_y(3te^{-t})$

$y'' = s^2[L_y] - sy - y'$

$y' = s[L_y] - y \quad (4)$

$y = [L_y] \quad (20)$

$\Big\}\ s^2[L_y] - sy - y' + (s[L_y]-y)(4) + 20[L_y]$

$L_y(3te^{-t}) = 3(L_y(te^{-t}))$

$L_y(e^{at}) = \frac{a}{s+a} = \frac{1}{s+1}$

$$L_y(e^{at}) = \frac{}{s+a} - \frac{}{s+1}$$

$$\frac{d}{ds}(s+1)^{-1} = \frac{1}{(s+1)^2}$$

$$= \frac{3}{(s+1)^2}$$

$$s^2[L_y] - sy - y' + (s[L_y] - y)(4) + 20[L_y] = \frac{3}{(s+1)^2}$$

$$L_y = \frac{s^3+5s^2+7s+6}{(s^2+2s+1)(s^2+4s+20)}$$

$$y = L^{-1}\left(\frac{s^3+5s^2+7s+6}{(s^2+2s+1)(s^2+4s+20)}\right)$$

$$\frac{s^3+5s^2+7s+6}{(s+1)^2(s^2+4s+20)} = \frac{A}{s+1} + \frac{B}{(s+1)^2} + \frac{Ds+C}{s^2+4s+20}$$

$s+1=0$
$s=-1$

$$s^3+5s^2+7s+6 = A(s+1)(s^2+4s+20) + B(s^2+4s+20) + (Ds+C)(s+1)^2$$

$$(-1)^3 + 5(-1)^2 - 7 + 6 = 17B$$

$$B = \frac{3}{17}$$

$$20A + \frac{60}{17} + C = 6; \quad 24A + \frac{12}{17} + D + 2C = 7$$

$$5A + \frac{3}{17} + 2D + C = 5; \quad A + D = 1$$

$$A = -\frac{6}{289} \quad C = \frac{834}{289} \quad D = \frac{295}{289}$$

$$L^{-1}\left(-\frac{6}{289(s+1)} + \frac{3}{17(s+1)^2} + \frac{295s+834}{289(s^2+4s+20)}\right) \longrightarrow \frac{s+2}{(s+2)^2+16}$$

$$\frac{6}{289}e^{-t} \qquad \frac{3te^{-t}}{17} \qquad \frac{295}{289}e^{-2t}\cos(4t) + \frac{61}{289}e^{-2t}\sin(4t)$$

$$\frac{6}{289}e^{-t} + \frac{3te^{-t}}{17} + \frac{295}{289}e^{-2t}\cos(4t) + \frac{61}{289}e^{-2t}\sin(4t)$$

$$\frac{51te^{-t} + 6e^{-t} + 295e^{-2t}\cos(4t) + 61e^{-2t}\sin(4t)}{289}$$

3. **Laplace transforms for *hyperbolic trig* functions.** Derive expressions for $\mathcal{L}\{\cosh(\beta t)\}$ and $\mathcal{L}\{\sinh(\beta t)\}$, where,

$$\cosh(\beta t) = \frac{1}{2}\left(e^{\beta t} + e^{-\beta t}\right), \qquad \sinh(\beta t) = \frac{1}{2}\left(e^{\beta t} - e^{-\beta t}\right)$$

Compare your results to the the Laplace transforms for $\cos(\beta t)$ and $\sin(\beta t)$.

$$L(\cosh(Bt)) \quad L[e^{Bt}] = \frac{B}{s-B}, \quad L[e^{-Bt}] = \frac{B}{s+B} \quad \frac{1}{2}\left(\frac{B}{s-B} + \frac{B}{s+B}\right)$$

$$L(\cos(Bt)) = \frac{s}{s^2+B^2}$$

$-1 \qquad 2$

$$L(\cos(Bt)) = \frac{s}{s^2+B^2}$$

$$L(\sinh(Bt)) = \left.\begin{matrix} L[e^{Bt}] = \dfrac{B}{s-B} \\ L[e^{-Bt}] = \dfrac{B}{s+B} \end{matrix}\right\} \frac{B}{2(s-B)} - \frac{B}{2(s+B)} ; \quad \frac{B(s+B)}{2(s-B)(s+B)} - \frac{B(s-B)}{2(s-B)(s+B)} = \frac{B^2}{(s-B)(s+B)}$$

$$\frac{Bs + B^2 - (Bs - B^2)}{2(s-B)(s+B)}$$

$$L(\sin(Bt)) = \frac{B}{s^2+B^2}$$

4. Consider the following first-order initial value problem (IVP),

$$\frac{dy}{dt} + \frac{1}{2}y = f(t), \qquad y(0) = 0.$$

where $f(t)$ is given as the piecewise function:

$$f(t) = \begin{cases} 4, & 0 \le t < 2 \\ 0, & 2 \le t < \infty \end{cases}$$

Find the (unique) solution, $y(t)$, to this IVP, evaluate $y(4)$ and and sketch the graph of $y(t)$ on the interval $0 \le t \le 4$.

***S1*:** Solve the initial value problem on the interval $0 \le t \le 2$.

***S2*:** Evaluate $y(2)$ (label this value as y_2).

***S3*:** Solve the IVP, $L[y] = f(t)$ for $t > 2$ with initial condition $y(2) = y_2$.

***S4*:** Express your solution in piecewise form and sketch its graph on $0 \le t \le 4$.

y_1: $$s[L_y] - y(0) + \frac{[L_y]}{2} = \frac{4}{s}$$

$y(0) = 0$

$$L_y = \frac{4}{s(s+\frac{1}{2})}(2) = \frac{8}{2s^2+1}$$

$$L^{-1}\left[\frac{8}{2s^2+1}\right]$$

$$\frac{8}{s(2s+1)} = \frac{A}{s} + \frac{B}{2s+1}$$

$$8 = A(2s+1) + \frac{2Bs(s+1)}{2s+1}$$

$$8 = 2As + A + Bs$$

$A = 8 \qquad 2As + Bs = 0$

$A = 8$

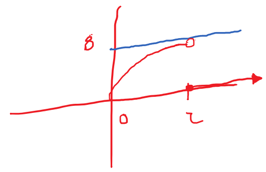
$B = -16 \qquad L^{-1}\left[\frac{8}{s} - \frac{16}{2s+1}\right]$

y_2: $$s[L_y] + \frac{[L_y]}{2} = 0$$

$L_y = 0$

$y = 8 - 8e^{-t/2}, \quad 0 \le t < 2$

$y = 0, \quad 2 \le t < \infty$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