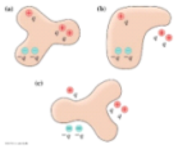
Exam 2 "I pledge my honor I have abided by the Stevens Honor system." -Alex Gaskins

Friday, October 30, 2020 3:29 PM

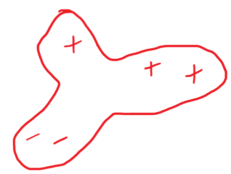
1. (3 pts) What is the electric flux through each of the surfaces in the figure? Give your answers in multiples of q/ϵ_0 and explain your reasoning.



$$\text{Electric Flux} = \frac{q_{enclosed}}{\varepsilon_0}$$

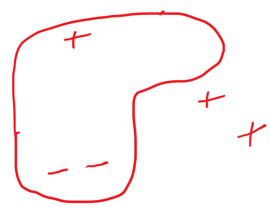
A.) $3q - 2q = q$

$$\boxed{\frac{q}{\varepsilon_0}}$$



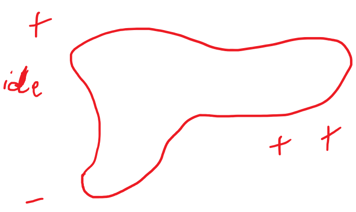
B.) $q - 2q = -q$

$$\boxed{-\frac{q}{\varepsilon_0}}$$



C.) No charge inside

$$\boxed{0}$$



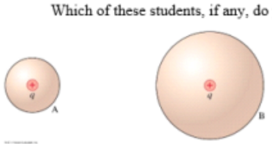
2. (1pt) the two spheres I the figure surround equal charges. Three students are discussing the situation.

Student 1: the fluxes through spheres A and B are equal because they enclose equal charges.

Student 2: But the electric field on sphere B is weaker than the electric field on sphere A. the flux depends on the electric field strength, so the flux through A is larger than the flux through B.

Student 3: I thought we learned that flux was about surface area. Sphere B is larger than sphere A, so I think the flux through B is larger than the flux through A.

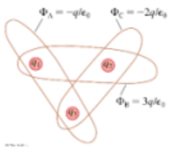
Which of these students, if any, do you agree with. Explain.



student 1 is correct. Gaussians are artificially created areas, and do not matter in this case, each gaussian only has one charge. ... they have the as each thus they

they are the same charges, thus they are the same flux in this case.

3. (1 pt) The figure shows three gaussian surfaces and the electric flux through each. What are the three charges q1, q2, and q3?



$\phi_A = \frac{-q}{\varepsilon_0}$ $\quad -1 = q_1 + q_3$

$\phi_B = \frac{3q}{\varepsilon_0}$ $\quad -2 = q_2 + q_3$

$\phi_C = \frac{-2q}{\varepsilon_0}$ $\quad 3 = q_1 + q_2$

$q_1 = 3 - q_2$

$-1 = (3 - q_2) + q_3$

$q_3 = q_2 - 4$

$-2 = q_2 + (q_2 - 4)$

$-2 = 2q_2 - 4$

$$\boxed{\begin{aligned} q_2 &= 1 \text{ C} \\ q_3 &= -3 \text{ C} \\ q_1 &= 2 \text{ C} \end{aligned}}$$

4. (2 pt) a 2.0 cm by 3.0 cm rectangle lies in the xy plane. What is the magnitude of the electric flux through the rectangle if

a. $\vec{E} = (100\hat{\imath} - 200\hat{k})N/C$

b. $\vec{E} = (100\hat{\imath} - 200\hat{\jmath})N/C$

$A = (.02)(.03)$

$A = .0006 \text{ m}^2$ in k direction

Flux is perpendicular to surface area

A.) $E = (100i - 200k)$ N/C $\qquad \phi = 0°$

Flux $= EA\cos(\phi)$

$= (100i - 200k)(.0006k) = -200(.0006)$

$= |-.12|$

$$\boxed{= .12 \ \frac{N \cdot m^2}{C}}$$

B.) $E = (100i - 200j) \ N/C$

$Flux = EA\cos(\theta)$

$= (100i - 200j)(.0006k) = (0)(.0006)$

$$\boxed{= 0 \ \frac{N \cdot m^2}{C}}$$

5.__(3 pts) a long, thin straight wire with linear charge density λ runs down the center of a thin, hollow metal cylinder of radius R. the cylinder has a net linear charge density 2 λ. Assume λ is positive. Find expressions for the electric field strength

a. inside the cylinder, r<R

b. outside the cylinder, r>R. in what direction does the electric field point in each of the cases.

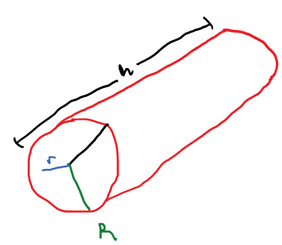
c. what is charge on the inner surface of the cylinder, the outer surface of the cylinder

A.)

$\int E \, dA = \frac{q}{\varepsilon_0}$ $\qquad q = \lambda h$

$EA = \frac{\lambda h}{\varepsilon_0}$ $\qquad r = h$

$A = 2\pi r^2$

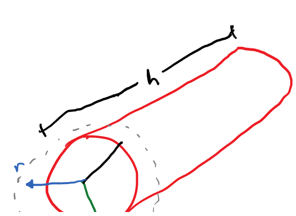


$E = \frac{\lambda h}{A \varepsilon_0}$

$$\boxed{E = \frac{\lambda}{2\pi r \varepsilon_0}}$$

B.)

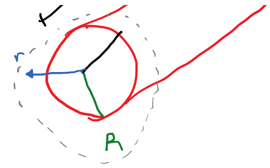
$\int E \, dA = \frac{q}{}$ $\qquad q = 2\lambda h$



$$\int E\, dA = \frac{q}{\varepsilon_0} \qquad q = 2\lambda h$$

$$EA = \frac{2\lambda h}{\varepsilon_0} \qquad r = h$$

$$A = 2\pi r^2$$



$$E = \frac{\lambda h}{A\, \varepsilon_0}$$

$$= \frac{\lambda}{\pi r\, \varepsilon_0}$$

The E-field is directed outward in both cases.

$$E = \frac{\lambda}{\pi r\, \varepsilon_0} + \frac{\lambda}{2\pi r\, \varepsilon_0}$$

$$\boxed{E = \frac{3\lambda}{2\pi r\, \varepsilon_0}}$$

C.) Net charge density $= 2\lambda$

so $q_{total} = 2\lambda h$

$q_{inner} = \lambda h$

$$q_{total} = q_{outten} + q_{inner}$$

$$2\lambda h = q_{outter} + \lambda h$$

$$q_{outter} = 2\lambda h - \lambda h$$

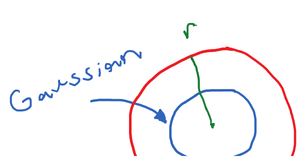
$$q_{outter} = \lambda h$$

6. (6 pts) There is dielectric sphere of radius 0.1 m at the center of a thin metal shell of inner radius 0.2m and outer radius of 0.3 m. The inner sphere has an unknown charge uniformly distributed uniformly through out it. The outer shell was charged with 3 nC at the beginning of the experiment. The electric field at a radius of 0.05 m inside the dielectric sphere is 4.48x10^2 N/C.

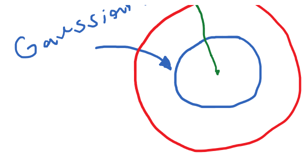
a. What is the total charge in the dielectric sphere?
b. What is the electric field at a distance of 0.15 m as measured from the center of the dielectric sphere?
c. What is the charge on the inner surface of the thin metal shell? Where did this charge come from?
d. What is the electric field at a distance of 0.25 m? Explain.
e. What is the charge on the surface of the thin metal shell, what is the electric field at this point.
f. What is the electric field at a point of 0.4 m?

A.) $\int E\, ds = \frac{q}{\varepsilon_0}$

$EA = \frac{q}{-}$



A.) $\int E \, ds = \frac{q}{\varepsilon_0}$



$$EA = \frac{q}{\varepsilon_0}$$

$$q = E 4\pi r^2 \varepsilon_0$$

$$q = (4.48 \times 10^2) 4\pi (.05)^2$$

$$q = 124.61 \times 10^{-12} \text{ C}$$

$$\lambda = \frac{q}{\frac{4}{3}\pi r^3}$$

$$\lambda = \frac{124.61 \times 10^{-12}}{\frac{4}{3}\pi (.05)^3}$$

$$\lambda = 2.38 \times 10^{-7} \frac{\text{C}}{\text{m}^3}$$

$$q = \lambda \left(\frac{4}{3}\pi r^3\right)$$

$$q = (2.38 \times 10^{-7}) \left(\frac{4}{3}\pi (.1)^3\right)$$

$$q = 9.97 \times 10^{-10} \text{ C}$$

$$\boxed{q = .997 \text{ nC}}$$

B.) $\int E \, ds = \frac{q}{\varepsilon_0}$

$$EA = \frac{q}{\varepsilon_0} \qquad q = .997 \text{ nC}$$

$$E = \frac{q}{4\pi r^2 \varepsilon_0} \qquad k = \frac{1}{4\pi\varepsilon_0}$$

$$E = \frac{kq}{r^2} = \frac{(8.99 \times 10^9)(.997 \times 10^{-9})}{(.15)^2}$$

$$\boxed{E = 398.36 \frac{\text{N}}{\text{C}}}$$

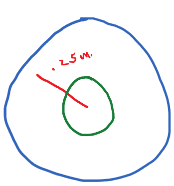
C.) Charge on inner sphere $= -q$

As a result of polarization in the shell because of the dielectric sphere, where charge $q = .997$ nC.

Thus, charge on inner sphere =

$$\boxed{-.997 \text{nC}}$$

D.)



$$\int E \, ds = \frac{q_{enclosed}}{\varepsilon_0}$$

$$q_{enclosed} = .997 \text{ nC} - .997 \text{ nC} = 0$$

$$\frac{0}{\varepsilon_0} = \boxed{0 \frac{\text{N}}{\text{C}}}$$

(causes polarization)

E.) Inner surface of shell:

$$q = -.997 \text{ nC}$$

Due to polarization, the outter surface has same magnitude but opposite polarity.

The outter surface was charged with 3nC at the beginning, so:

$$q = 3\text{nC} + .997 \text{ nC}$$

$$\frac{0}{\varepsilon_0} = \boxed{0\ \frac{N}{C}}$$

Because of dielectric sphere, net charge (causes polarization) enclosed is 0, so the electric field is also 0.

$$q = 3\,nC + .997\,nC$$

$$\boxed{q = 3.997\,nC}$$

F.) $q_{total} = 3.997\,nC$

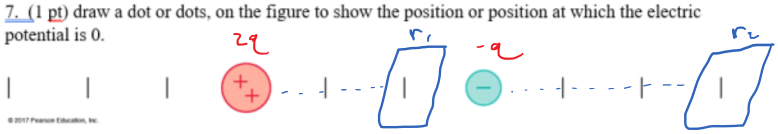
$$E = \frac{kq}{r^2} = \frac{(8.99\times10^9)(3.997\times10^{-9})}{(.4)^2}$$

$$\boxed{E = 224.58\ \frac{N}{C}}$$

$$E = \frac{kq}{r^2} = \frac{(8.99\times10^9)(3.997\times10^{-9})}{(.3)^2}$$

$$\boxed{E = 399.26\ \frac{N}{C}}$$

7. (1 pt) draw a dot or dots, on the figure to show the position or position at which the electric potential is 0.



$$V = \sum \frac{kq}{r}\ ; \quad V = 0$$

$$V_1 = \frac{2kq}{r_1} \qquad V_2 = \frac{-kq}{(3x - r_1)}$$

$$0 = \frac{2kq}{r_1} - \frac{kq}{(3x - r_1)}$$

$$\frac{2}{r_1} = \frac{1}{(3x - r_1)}$$

$$3r_1 = 6x$$

$$\boxed{r_1 = 2x}$$

$$V_1 = \frac{2kq}{(3x + r_2)} \qquad V_2 = \frac{-kq}{r_2}$$

$$\frac{2kq}{(3x + r_2)} - \frac{kq}{r_2} = 0$$

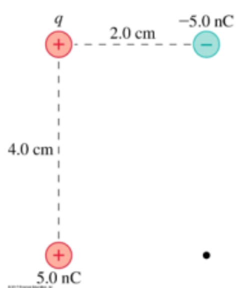
$$\frac{2}{(3x + r_2)} = \frac{1}{r_2}$$

$$\boxed{r_2 = 3x}$$

8. . (2pts) the electric potential at the dot in the figure is 3140 V.

a. what is the charge q?

b. if a 5 nC charge is brought in from infinity to the point shown in the figure what was the work done.



A.) $V = \sum E_i$

$$3140 = \frac{kq}{\sqrt{(.02)^2 + (.04)^2}} - \frac{k5}{.04} + \frac{k5}{.02}$$

$$\frac{kq}{\sqrt{(.02)^2 + (.04)^2}} = 2015$$

$$q = 1.1\times10^{-10}$$

$q = 11\ nC.$

B.)

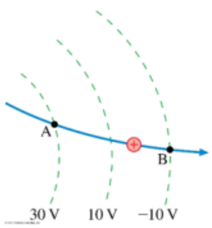
$W = q \Delta V$

$W = (5 \times 10^{-9})(3140 - 0)$

$\boxed{W = 1.57 \times 10^{-5}\ J.}$

9. (2 pts)A proton's speed as it passes point A is 50,000 m/s. it follows the trajectory shown in the figure.

a. What is the protons speed at point B.
b. If the particle is an electron at point A with the same velocity, what is the electrons speed at point B.



A.)

$E_p = qV$

$q = 1.602 \times 10^{-19}\ C.$

$m = 1.67 \times 10^{-27}\ kg.$

$qV_1 + \frac{1}{2} m V_A^2 = qV_2 + \frac{1}{2} m V_B^2$

$\frac{1}{2} m V_B^2 = \frac{1}{2} m V_A^2 + q(V_1 - V_2)$

$V_B = \sqrt{V_A^2 + \frac{2q(V_1 - V_2)}{m}}$

$V_B = \sqrt{(50{,}000)^2 + \frac{2(1.602 \times 10^{-19})(40)}{(1.67 \times 10^{-27})}}$

$\boxed{V_B = 100{,}867.49\ m/s}$

B.)

$E_p = -qV$

$q = 1.602 \times 10^{-19}\ C.$

$m = 9.11 \times 10^{-31}\ kg.$

$-qV_1 + \frac{1}{2} m V_A^2 = -qV_2 + \frac{1}{2} m V_B^2$

$\frac{1}{2} m V_B^2 = \frac{1}{2} m V_A^2 - q(V_1 - V_2)$

$V_B = \sqrt{V_A^2 - \frac{2q(V_1 - V_2)}{m}}$

$V_B = \sqrt{(50{,}000)^2 - \frac{2(1.602 \times 10^{-19})(40)}{(9.11 \times 10^{-31})}}$

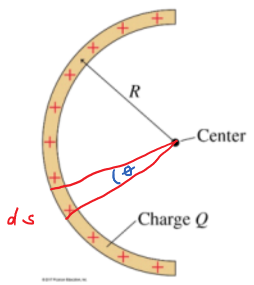
$\boxed{V_B = 3.75 \times 10^6 i \ m/s}$ $i = \sqrt{-1}$

This signifies that the electron is traveling in the opposite direction.

10. (2 pts) the figure shows a thin rod with charge Q that has been bent in a semicircle of radius R.

A. Find an expression for the electric potential at the center.

b. Describe the electric field at the center.



A.)

$dQ = \lambda \, ds$

$dQ = \frac{Q}{\pi R} ds$

$ds = R \, d\theta$

$$\Delta V = \frac{1}{4\pi\varepsilon_0}\left[\frac{\frac{Q}{R\pi} d\theta \cancel{R}}{\cancel{R}}\right]$$

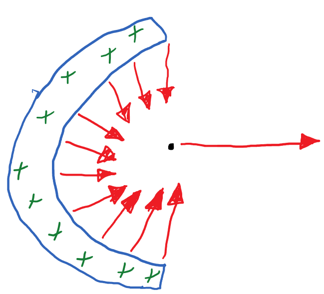
$$= \int_0^{\pi}\left[\frac{kQ}{R\pi}\right] d\theta = \frac{kQ}{R\pi}\int_0^{\pi} d\theta$$

$$= \left[\frac{kQ}{R\pi} x\right]_0^{\pi}$$

$$= \frac{kQ}{R\cancel{\pi}}\cancel{\pi} - 0$$

$$\boxed{V = \frac{kQ}{R}}$$

B.)

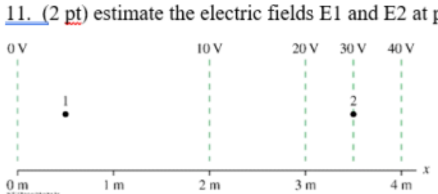


The electric field from the positively charged rod points outwards.

The y components cancel each other out and the x components add up.

Thus, the net electric field is directed in the positive x-direction.

11. (2 pt) estimate the electric fields E1 and E2 at points 1 and 2 in the figure. Show your work.



$E = -\frac{\Delta V}{\Delta x}$; $E = \frac{(V_f - V_i)}{(x_f - x_i)}$

For E_1:

$$E_1 = -\frac{(0-10)}{(0-2)}$$

$$\boxed{E_1 = -5\ \frac{V}{m}}$$

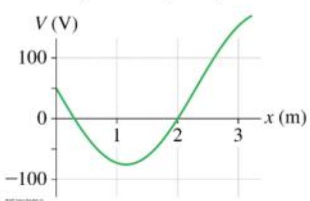
For E_2:

$$E_2 = -\frac{(20-40)}{(3-4)}$$

$$\boxed{E_2 = -20\ \frac{V}{m}}$$

12. (2pts) an electron is released from rest at x=2 m in the potential shown in the figure.

a. Does it move? Explain.
b. Repeat for a proton placed at x=2 m.



Electric field moves from higher to lower potential.

$\Delta E_p = q\Delta V$ $\qquad$ $F = qE$

A.) It moves in the positive x-direction after being placed at x = 2 m. Where the graph is shown to be increasing above the x-axis, which represents increasing electric potential. For an electron, if electric potential increases, potential energy decreases. In this case, potential energy decreases and stability increases as the electron moves in the positive direction.

(Moves in opposite direction of E-field)

$F = -q(-E)$

B.) The proton will move to the left, since electric potential to the left of x = 2 m. is less than zero. The direction of the electric field is in the direction of decreasing electric potential. Proton will travel in the same direction as the electric field.

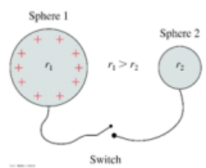
$-F = q(-E)$

13. (3pts) the two metal spheres in the figure are connected by a metal wire with a switch in the middle. Initially the switch is open. Sphere 1 with the larger radius, is given a positive charge. Sphere 2, with the smaller radius, is neutral. Then the switch is closed. Afterward sphere 1 has charge Q1, is at potential V1, and the electric field strength is E1. The values for sphere 2 are Q2, V2 and E2.

a. is V1 larger than, smaller than or equal to V2. Explain.

b. is Q1 larger than, smaller than or equal to Q2. Explain.

c. is E1 larger than, smaller than, or equal to E2. Explain.



A.) After the switch is closed, charge transfer will take place until the potential of both becomes equal.

$$V_1 = V_2$$

B.) If $V_1 = V_2$ and $V = \frac{kq}{r}$

$$\frac{\cancel{k}q_1}{r_1} = \frac{\cancel{k}q_2}{r_2}$$

$$Q_1 = Q_2\left(\frac{r_1}{r_2}\right)$$

$Q_1 > Q_2$ when $r_1 > r_2$

C.) $E_1 = \frac{Q_1}{(r_1)^2}$ $E_2 = \frac{Q_2}{(r_2)^2}$

$$\frac{E_1}{E_2} = \frac{Q_1}{Q_2}\left(\frac{(r_2)^2}{(r_1)^2}\right)$$

From part B:

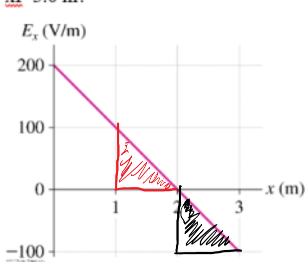
$$Q_1 = Q_2\left(\frac{r_1}{r_2}\right)$$

$$\frac{E_1}{E_2} = \frac{\left(Q_2\left(\frac{r_1}{r_2}\right)\right)}{Q_2}$$

$$\frac{E_1}{E_2} = \frac{r_2}{r_1}$$

$E_1 < E_2$ when $r_1 > r_2$

14. (1pt) The figure is the graph of Ex. What is the potential difference between xi=1.0 m and xf=3.0 m?



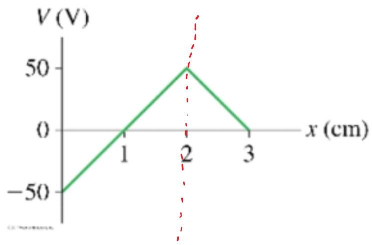
$$\Delta V = \int -E\,dx$$

ΔV = Area under curve $A = \frac{1}{2}bh$

$$\Delta V = \left(\tfrac{1}{2}(100)\right) - \left(\tfrac{1}{2}(100)\right)$$

$$\boxed{\Delta V = 0\ V}$$

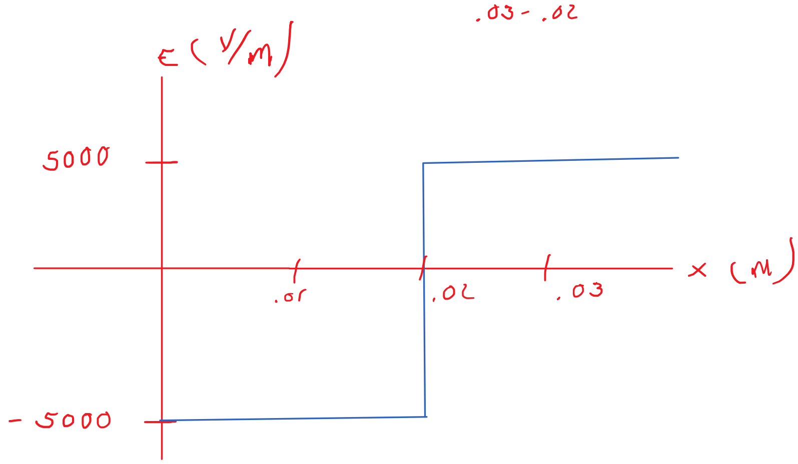
15. (1pt) The figure is the graph of V vs. x. Draw the corresponding graph of Ex vs. x.



$$E_x = \frac{-\Delta V}{\Delta x}$$

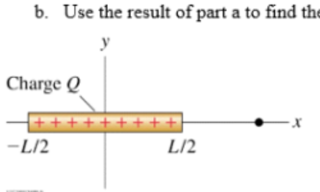
$$\frac{-(50-(-50))}{.02-0} = -5000 \ \frac{V}{m}$$

$$\frac{-(0-50)}{.03-.02} = 5000 \ \frac{V}{m}$$



Bonus (2pts)

a. What is the potential at the dot in the figure.
b. Use the result of part a to find the electric field at the dot



A.) $\lambda = \frac{Q}{L}$ $\quad dV = \int \frac{k \lambda \, dr}{r}$

$$\int_d^{d+L} dV = \int_d^{d+L} \frac{k\lambda}{r} dr$$

$$V = k\lambda \int_d^{d+L} \frac{1}{r} dr$$

$$V = k\lambda \left[\ln(r)\right]_d^{d+L}$$

$$V = k\lambda \left(\ln(d+L) - \ln(d)\right)$$

$$V = k\lambda \left(\ln\left(\frac{d+L}{d}\right)\right)$$

$$\boxed{V = \frac{kQ}{L}\ln\left(1 + \frac{L}{d}\right)}$$

B.)

$$dE = \int_d^{d+L} -\frac{dV}{dr} = \left[-\frac{k\lambda}{r}\right]_d^{d+L}$$

$$E = -k\lambda\left[\frac{1}{r}\right]_d^{d+L}$$

$$E = -k\lambda\left(\frac{1}{d+L} - \frac{1}{d}\right)$$

$$\frac{d - d + L}{d(d+L)} = \frac{L}{d(d+L)}$$

$$E = -k\lambda\left(\frac{L}{d(d+L)}\right)$$

$$E = \frac{-kQ}{\cancel{L}}\left(\frac{\cancel{L}}{d(d+L)}\right)$$

$$\boxed{E = -\frac{kQ}{d(d+L)}}$$