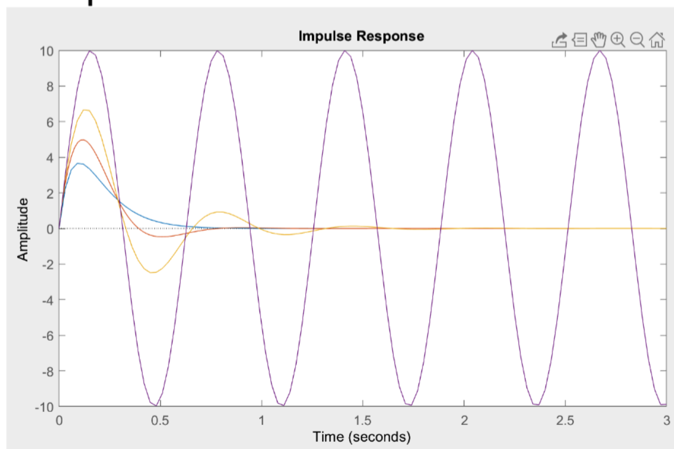
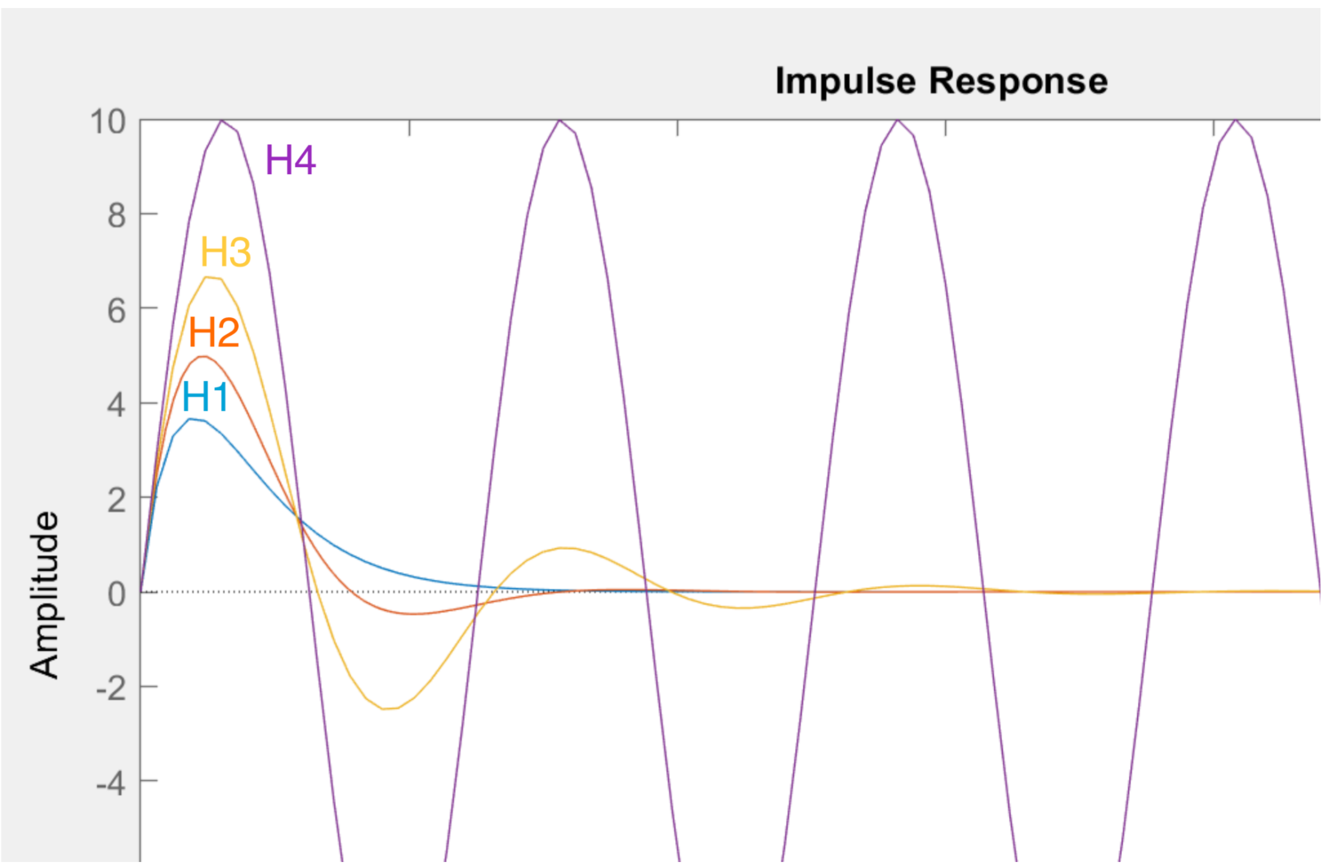
- Q1: Read the paper and a summary (a minimum of half a page) to describe what you have learned from it. (20 points)

Robert N. Clark, "The Routh-Hurwitz Stability Criterion, Revisited," IEEE Control Systems Magazine, Vol. 12, No. 3, June 1992.

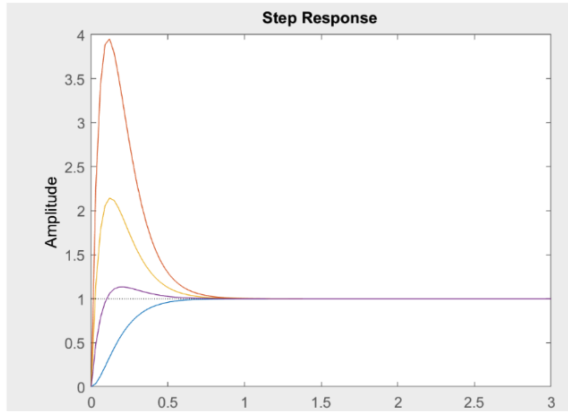
The discovery of the Routh-Hurwitz Stability Criterion was significant for control system analysis and output predictions. However, it was not an overnight process. It began in the mid-nineteenth century, with engineers and mathematicians expressing interest towards the stability of more complex dynamic systems. One of these brilliant minds was James C. Maxwell, who gained an interest in dynamic system stability after working with an automatic control system project concerning the definition of the ohm. His interest led him to become the first to publish a dynamic analysis of this feedback system using differential equations, where he found a relation between system responses and the coefficients of the linearized equations. However, it was an imperfect discovery, with much of the relation still being unknown past lower order differential equations due to their complex nature. That was the case, until Maxwell's plea for extraneous insight was answered by the brilliant findings of E. J. Routh, who wrote an essay on how the number of roots of the characteristic polynomial lying in the right half plane could be determined from the coefficients of the polynomial. His findings were unknowingly reestablished through a similar breakthrough made by A. Hurwitz at the end of the 19th century, with the conditions being identical to those given by Routh for no right half plane roots, both of which excluded roots on the jω axis if the test functions were simply computed from the formulae of the test functions. This breakthrough further lessened the gap between theoretical uncertainty and reliance on trial and error when it came to dynamic system analysis, as outputs could be predicted with greater accuracy under more conditions, with respect to the significance of polynomial coefficients in determining changes in polarity of output functions and the impact of transfer function roots on system stability.

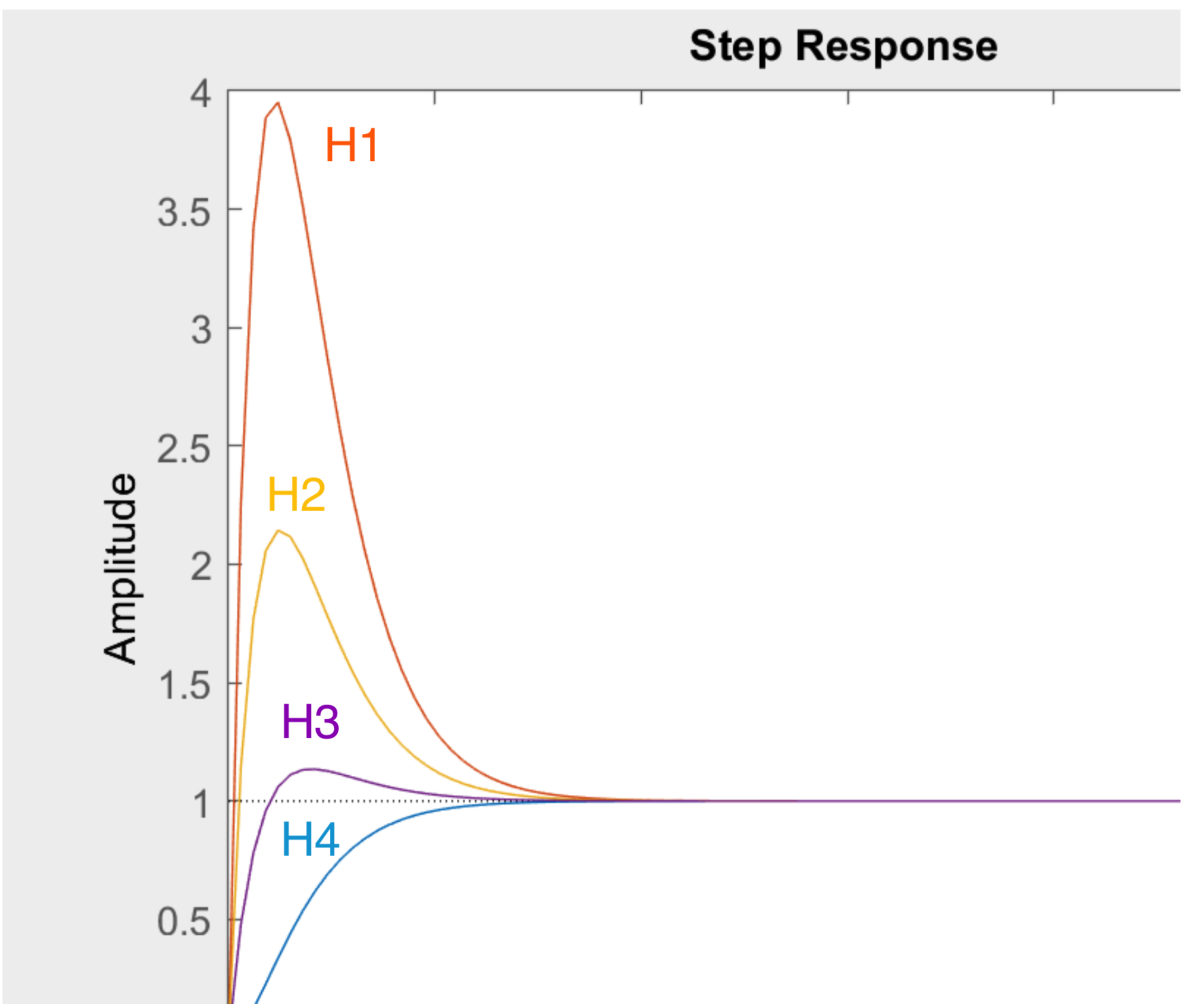
- Q2: Do NOT use Matlab or other tools, answer the following questions and explain your analysis procedure
  - A. map the below four impulse response curves with the four transfer functions (5 points)
    - sysH1=100/(s^2+2*1*10*s+100);
    - sysH2=100/(s^2+2*0.6*10*s+100);
    - sysH3=100/(s^2+2*0.3*10*s+100);
    - sysH4=100/(s^2+2*0*10*s+100);



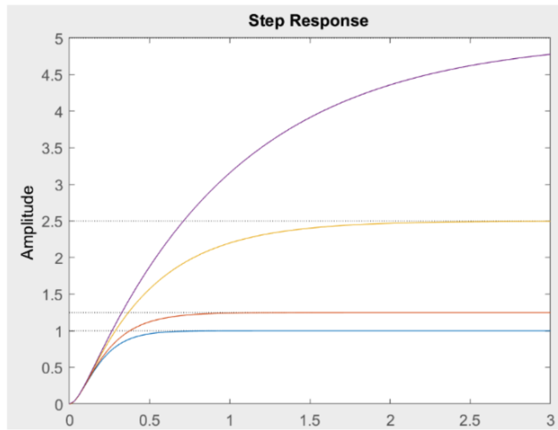


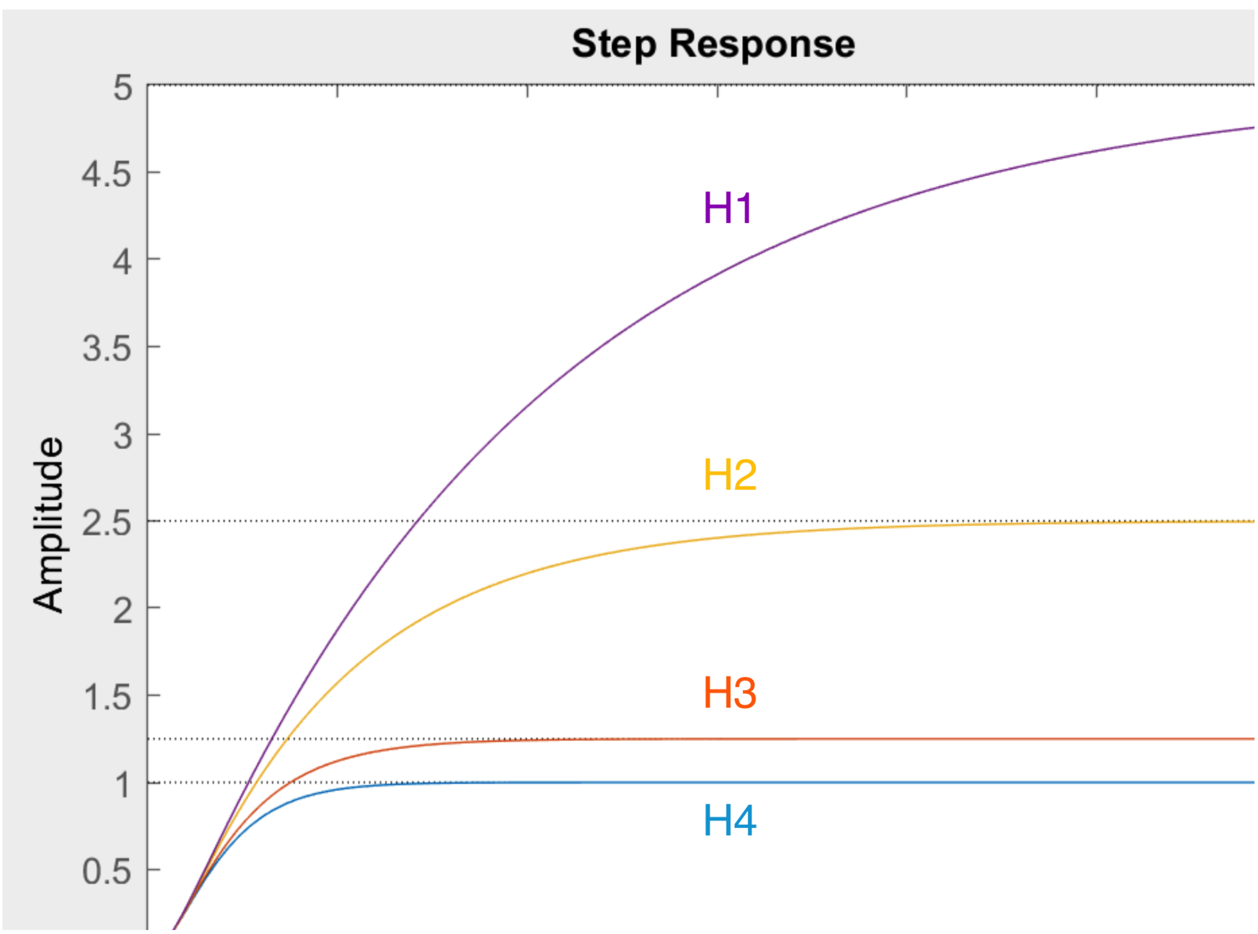
- B. map the below four step response curves with the four transfer functions (5 points)
  - sysH1=100/(s^2+2*1*10*s+100);
  - sysH2=100*(s+1)/(s^2+2*1*10*s+100);
  - sysH3=50*(s+2)/(s^2+2*1*10*s+100);
  - sysH4=20*(s+5)/(s^2+2*1*10*s+100);





- C. map the below four step response curves with the four transfer functions (5 points)
  - sysH1=100/(s^2+2*1*10*s+100);
  - sysH2=100/(s^2+2*1*10*s+80);
  - sysH3=100/(s^2+2*1*10*s+40);
  - sysH4=100/(s^2+2*1*10*s+20);





- Q3: Find the time function corresponding to F(s)= $\frac{10}{s(s+2)(s+10)}$ using partial-fraction expansion, and find its final value. (20 points)

$$F(s) = \frac{10}{s(s+2)(s+10)} = \frac{A}{s} + \frac{B}{(s+2)} + \frac{C}{(s+10)}$$

$$A = \frac{10}{(s+2)(s+10)} = \frac{10}{((0)+2)((0)+10)} = \frac{1}{2} \qquad B = \frac{10}{s(s+10)} = \frac{10}{(-2)((-2)+10)} = -\frac{5}{8}$$

$$C = \frac{10}{s(s+2)} = \frac{10}{(10)((10)+2)} = \frac{1}{8}$$

$$F(s) = \frac{1}{2s} - \frac{5}{8(s+2)} + \frac{1}{8(s+10)}$$

$$f(t) = [\frac{1}{2} - \frac{5}{8}e^{-2t} + \frac{1}{8}e^{-10t}]u(t)$$

$$\lim_{t \to \infty} f(t) = \frac{1}{2} - \frac{5}{8}e^{-2(\infty)} + \frac{1}{8}e^{-10(\infty)} = \frac{1}{2}$$

- Q4: Use Routh's stability criterion to determine how many roots with positive real parts the following two equations have: (25 points)
  - $s^4 + 8s^3 + 32s^2 + 80s + 100 = 0$

s^4 | 1 32 100
s^3 | 8 80 0
vvvvvv
$\frac{8(32)-1(80)}{8}$
vvvvvv
s^2 | **22** 100

s^3 | 8 80 0
s^2 | 22 100
vvvvvv
$\frac{22(80)-8(100)}{22}$
vvvvvv
s^1 | **43.63** 0

s^2 | 22 100
s^1 | 43.63 0
vvvvvv
$\frac{43.63(100)-22(0)}{43.63}$
vvvvvv
s^0 | **100**

=

s^4 |	1	32	100
s^3 |	8	80	0
s^2 |	22	100	
s^1 |	43.63	0	
s^0 |	100		

No sign change in the first column indicates that the number of positive real roots is **zero**

- $s^5 + 10s^4 + 30s^3 + 80s^2 + 344s + 480 = 0$

s^5 |	1	30	344
s^4 |	10	80	480
s^3 |	22	296	0
s^2 |	-54.54	480	
s^1 |	489.62	0	
s^0 |	480		

$\frac{10(30)-1(80)}{10} = 22$ $\frac{10(344)-1(480)}{10} = 296$

$\frac{22(80)-10(296)}{22} = -54.54$ $\frac{22(480)-10(0)}{22} = 480$

$\frac{-54.54(296)-22(480)}{-54.54} = 489.62$ $\frac{489.62(480)-(-54.54(0))}{489.62} = 480$

Two sign changes in the first column indicates that the number of positive real roots is **two**

- Q5: Use Routh's stability criterion to determine the range of K within which the system with the characteristic equation $1+K\dfrac{s+4}{s(s+0.5)(s+1)(s^2+0.4s+4)}=0$ is stable. (20 points)

$s(s + 0.5)(s + 1)(s^2 + 0.4s + 4) + K(s + 4) = 0$

$(s^2 + 0.55)(s^3 + 1.4s^2 + 4.4s + 4) + K(s + 4) = 0$

$s^5 + 1.9s^4 + 5.1s^3 + 6.2s^2 + (2 + K)s + 4K = 0$

s^5 |	1	5.1	(2+K)
s^4 |	1.9	6.2	4K
s^3 |	1.84	α	0
s^2 |	β	4K	
s^1 |	ω	0	
s^0 |	4K		

$\frac{1.9(5.1)-6.2(1)}{1.9} = 1.84$ $\qquad \frac{1.9(2+K)-4K}{1.9} = \frac{3.8-2.1K}{1.9} = \alpha$

$\frac{1.84(6.2)-1.9(\alpha)}{1.84} = \frac{7.608+2.1K}{1.84} = \beta$ $\qquad \frac{1.84(4K)-1.9(0)}{1.84} = 4K$

$\frac{\beta(\alpha)-1.84(4K)}{\beta} = \omega$ $\qquad \frac{\omega(4K)-\beta(0)}{\omega} = 4K$

Check for K>0 in cases in first column (β, ω, $4K$):

$\beta > 0 \rightarrow \frac{7.608+2.1K}{1.84} > 0$

$2.1K > -7.608$

$K > -3.62$

$\omega > 0 \rightarrow \frac{[\frac{7.608+2.1K}{1.84}][\frac{3.8-2.1K}{1.9}]-1.84(4K)}{[\frac{7.608+2.1K}{1.84}]} > 0$

$4.41K^2 + 33.73K - 28.91 < 0$

$(K - 0.78)(K + 8.43) < 0$

$K < 0.78, K < -8.43$

$4K > 0$

$K > 0$

Only $K > 0$ and $K < 0.78$ are greater than zero, which indicates stability.

$0 < K < 0.78$