Stevens Institute of Technology
Department of Electrical and Computer Engineering

CpE 462 Introduction to Image Processing and Coding

Spring Semester 2022 Midterm Exam, Mar. 10, 6:30 – 10:30 PM

Instructions:

- Please provide necessary **intermediate steps** in your work. You will get zero credit if you only provide the final result without necessary steps.
- **All** calculations are to be done by **hand**, with the help of a calculator. Computer is only allowed for viewing lecture notes and course materials.
- Sign the following statement

I pledge on my honor that I have abided by the Stevens honor code

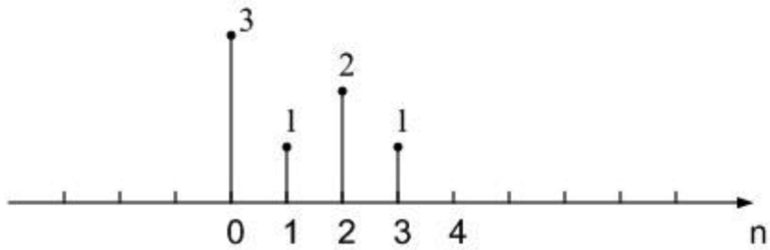
Alex Gaskins

Name (print): Alex Gaskins

Last four digits of your Student ID: 5143

Problem 1. (10 points) A discrete time signal can be expressed in many forms. Given the following signals:

- (Analytic form) $x_1[n] = 0.2^n$ **for** $n = \{0, 1, 2, 3\}$
- ($\delta[n]$ sequence) $x_2[n] = \delta[n] - 2\delta[n-1] - \delta[n-2] + 3\delta[n-3]$
- (Graphic form) $x_3[n]$ **as shown**



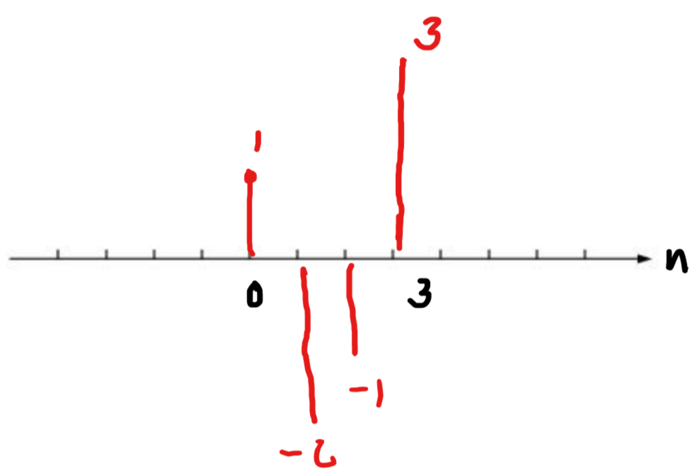
a) Express $x_1[n]$ in the form of $\delta[n]$ sequence.

$$0.2^n \; for \; n = \{0, 1, 2, 3\}$$
$$0.2^0 = 1$$
$$0.2^1 = 0.2$$
$$0.2^2 = 0.04$$
$$0.2^3 = 0.008$$
$$x_1[n] = \delta[n] + (0.2)\delta[n-1] + (0.04)\delta[n-2] + (0.008)\delta[n-3]$$

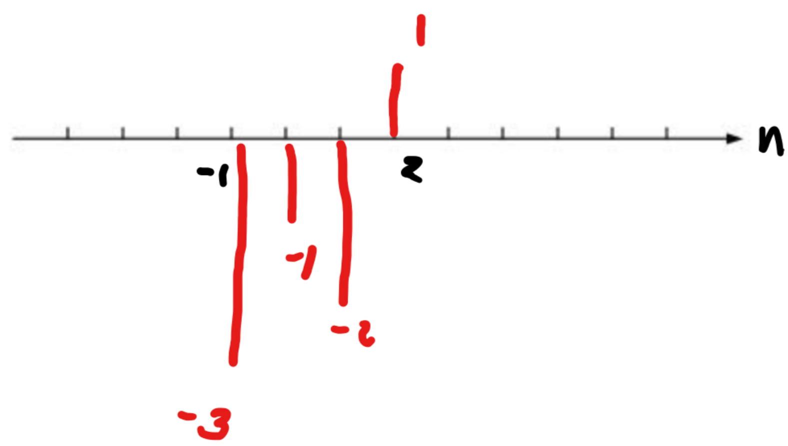
b) Express $x_3[n]$ in the form of $\delta[n]$ sequence.

$$x_3[n] = 3\delta[n] + \delta[n-1] + 2\delta[n-2] + \delta[n-3]$$

c) Plot $x_2[-n-2]$



Ans:



Problem 2 (20 points) Determine if the system $y[n] = \frac{1}{3}\left(x[n] + x[n-1] + x[n-2]\right) + 1$ is

a) Additive

$y_1[n] = \frac{1}{3}(x_1[n] + x_1[n-1] + x_1[n-2]) + 1$

$y_2[n] = \frac{1}{3}(x_2[n] + x_2[n-1] + x_2[n-2]) + 1$

$T\{ax_1[n] + bx_2[n]\} = \frac{1}{3}((ax_1[n] + bx_2[n]) + (ax_1[n-1] + bx_2[n-1]) + (ax_1[n-2] + bx_2[n-2])) + 1$

$T\{ax_1[n]\} + T\{bx_2[n]\} = (\frac{1}{3}(ax_1[n] + ax_1[n-1] + ax_1[n-2])) + (\frac{1}{3}(ax_1[n] + ax_1[n-1] + ax_1[n-2])) + 2$

$T\{ax_1[n] + bx_2[n]\} \neq T\{ax_1[n]\} + T\{bx_2[n]\}$

$\therefore$ *Not Additive*

b) Homogeneous

$T\{ax_1[n]\} = \frac{1}{3}(ax_1[n] + ax_1[n-1] + ax_1[n-2]) + 1$

$aT\{x_1[n]\} = \frac{1}{3}(ax_1[n] + ax_1[n-1] + ax_1[n-2]) + a$

$T\{ax_1[n]\} \neq aT\{x_1[n]\}$

$\therefore$ *Not Homogeneous*

c) Time-invariant

$T\{x[n-n_0]\} = \frac{1}{3}(x_1[n-n_0] + x_1[n-n_0-1] + x_1[n-n_0-2]) + 1$

$y[n-n_0] = \frac{1}{3}(x_1[n-n_0] + x_1[n-n_0-1] + x_1[n-n_0-2]) + 1$

$T\{x[n-n_0]\} = y[n-n_0]$

$\therefore \textit{Time Invariant}$

Problem 3 (20 points) Given $\boldsymbol{x_1[n] = 2\delta[n] + \delta[n-1] - \delta[n-2]}$, $\boldsymbol{x_2[n] = \delta[n] - 2\delta[n-1]}$,

a) Calculate the 1-D convolution $\boldsymbol{y[n] = x_1[n] * x_2[n]}$

$x_1[n] = [1; n = 0], [-2; n = 1]$ where $L_1 = 2$

$x_2[n] = [2; n = 0], [1; n = 1], [-1; n = 2]$ where $L_2 = 3$

$$y[k] = h[n] * x[n] = \sum_{i=-k}^{L} x[i]h[k-i]$$

$L = [2-1] + [3-1] = 3$

$y[0] = x_2[n] \bullet x_1[4-n]:$

$(x_2[0] \bullet x_1[0]) + (x_2[1] \bullet x_1[3]) + (x_2[2] \bullet x_1[2]) + (x_2[3] \bullet x_1[1]) =$

$2 \bullet 1 + 1 \bullet 0 + -1 \bullet 0 + 0 \bullet -2 = 2$

$y[1] = x_2[n] \bullet x_1[1-n]:$

$(x_2[0] \bullet x_1[1]) + (x_2[1] \bullet x_1[0]) + (x_2[2] \bullet x_1[3]) + (x_2[3] \bullet x_1[2]) =$

$2 \bullet -2 + 1 \bullet 1 + -1 \bullet 0 + 0 \bullet 0 = -3$

$y[2] = x_2[n] \bullet x_1[2-n]:$

$(x_2[0] \bullet x_1[2]) + (x_2[1] \bullet x_1[1]) + (x_2[2] \bullet x_1[0]) + (x_2[3] \bullet x_1[3]) =$

$2 \bullet 0 + 1 \bullet -2 + -1 \bullet 1 + 0 \bullet 0 = -3$

$y[3] = x_2[n] \bullet x_1[3-n]:$

$(x_2[0] \bullet x_1[3]) + (x_2[1] \bullet x_1[2]) + (x_2[2] \bullet x_1[1]) + (x_2[3] \bullet x_1[0]) =$

$2 \bullet 0 + 1 \bullet 0 + -1 \bullet -2 + 0 \bullet 1 = 2$

$$y[n] = \{2, -3, -3, 2\}$$

b) Calculate the **4-point DFT** of the output ***y*[*n*]**, i.e. ***Y*[*k*] for *k*=0, 1, 2, 3.** **(Show your steps)**

$y[n] = \{2, -3, -3, 2\}$

$y[k] = \sum_{n=0}^{3} y[n] \bullet W_4^{kn} = \sum_{n=0}^{3} y[n] \bullet e^{-j\frac{2\pi}{N}kn}$

$y[0] = 2 - 3 - 3 + 2 = -2$

$y[1] = 2 - 3e^{-j\frac{2\pi}{4}} - 3e^{-j\pi} + 2e^{-j\frac{2\pi}{4}3} = 2 + 3j + 3 + 2j = 5 + 5j$

$y[2] = 2 - 3e^{-j\pi} - 3e^{-j\frac{2\pi}{4}4} + 2e^{-j\frac{2\pi}{4}6} = 2 + 3 - 3 - 2 = 0$

$y[3] = 2 - 3e^{-j\frac{2\pi}{4}3} - 3e^{-j\frac{2\pi}{4}6} + 2e^{-j\frac{2\pi}{4}9} = 2 - 3j + 3 - 2j = 5 - 5j$

$y[k] = \{-2, 5 + 5j, 0, 5 - 5j\}$

Problem 4 (20 points) Calculate the **2×2 DCT** of the following 2-D signal

$$\boldsymbol{x[n_1, n_2]} = \begin{bmatrix} \mathbf{1} & \mathbf{-2} \\ \mathbf{1} & \boldsymbol{a} \end{bmatrix}$$

where ***a*** is a constant. (Your answer will contain ***a*** in it.)

$$c(k,n) = \begin{cases} \frac{1}{\sqrt{N}}, & k = 0,\ 0 \le n \le N-1 \\ \sqrt{\frac{2}{N}} cos\frac{\pi(2n+1)k}{2N}, & 1 \le k \le N-1,\ 0 \le n \le N-1 \end{cases}$$

$c(0,0) = c(0,1) = \frac{1}{\sqrt{N}} = \frac{1}{\sqrt{2}}$

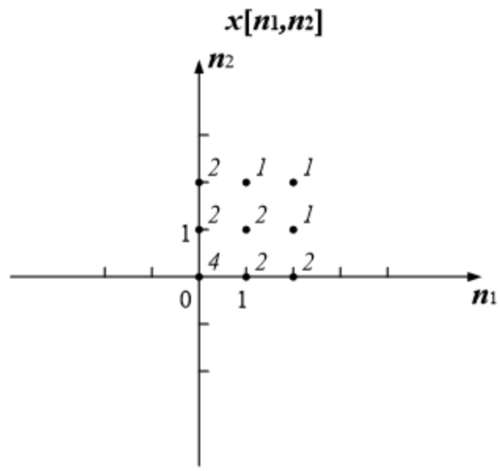
$c(1,0) = \sqrt{\frac{2}{2}} cos[\frac{-(2(0)+1)\pi(1)}{2(2)}] = \frac{1}{\sqrt{2}}$

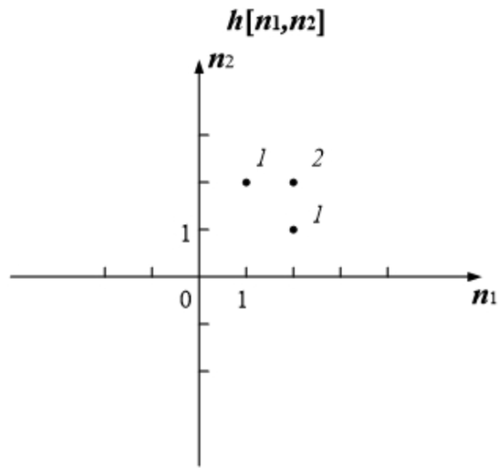
$c(1,1) = \sqrt{\frac{2}{2}} cos[\frac{(2(1)+1)\pi(1)}{2(2)}] = \frac{-1}{\sqrt{2}}$

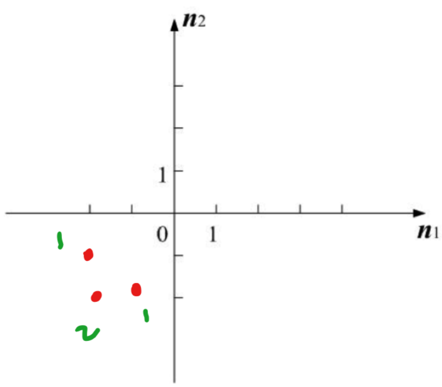
$$dct = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 1 & a \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$$

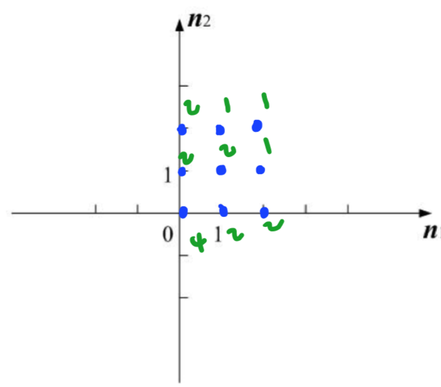
$$dct = \begin{bmatrix} \frac{a}{2} & \frac{-a+4}{2} \\ \frac{-a-2}{2} & -\frac{-a-2}{2} \end{bmatrix}$$

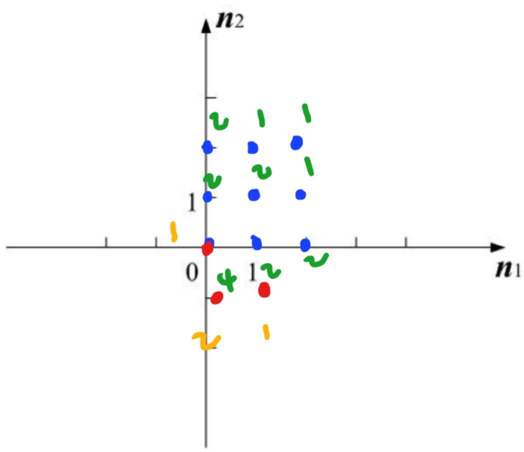
Problem 5 (20 points) Given a 2-D signal $\boldsymbol{x[n_1, n_2]}$ and a 2-D system $\boldsymbol{h[n_1, n_2]}$ as shown, calculate the 2-D convolution of $\boldsymbol{y[n_1, n_2] = x[n_1, n_2] ** h[n_1, n_2]}$. (Show some steps)

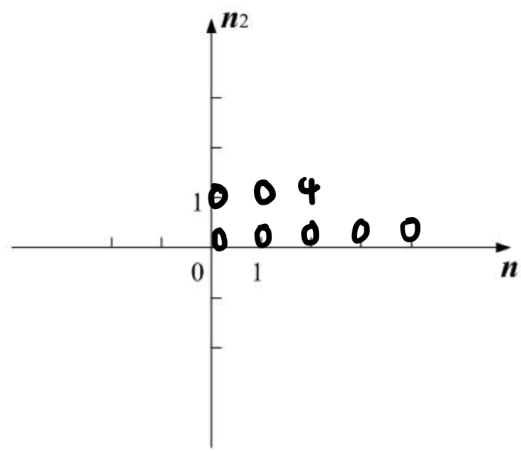


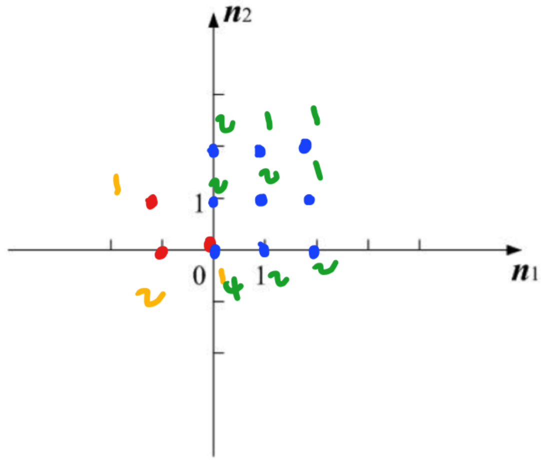




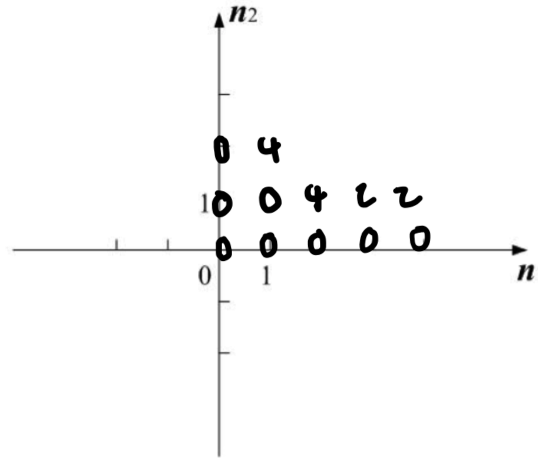




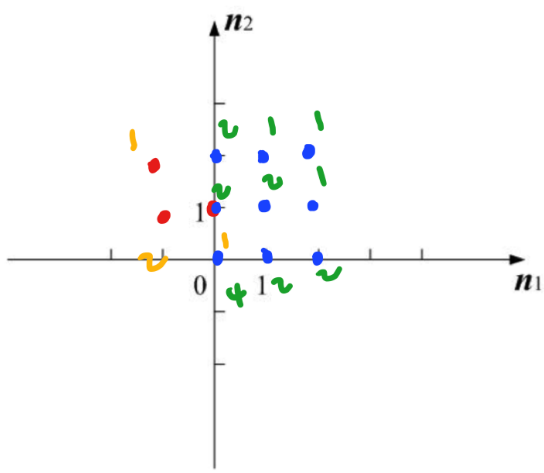




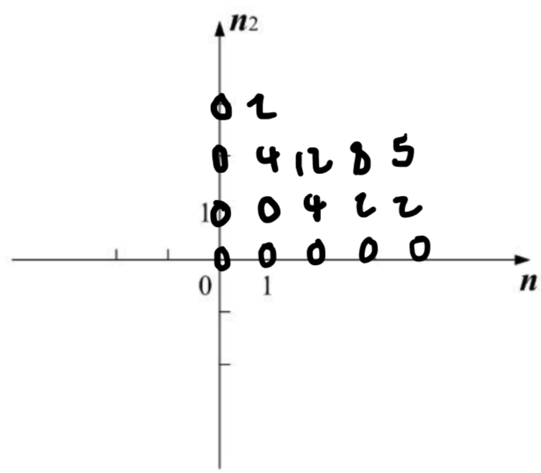
n_2
n_1
0
1



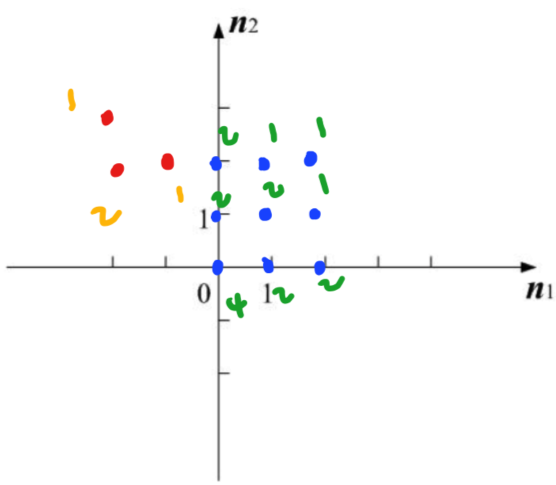
n_2
n_1
0 4
0 0 4 2 2
0 0 0 0 0
0
1



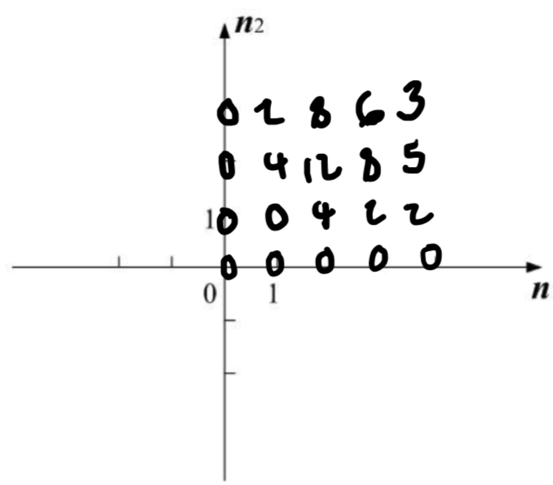
n_2
n_1
0
1



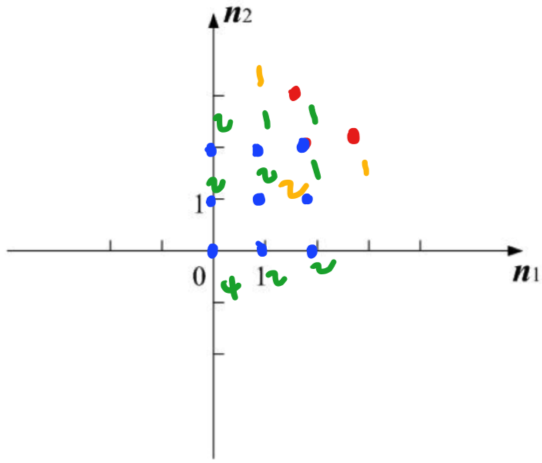
n_2
n_1
0 2
0 4 12 8 5
0 0 4 2 2
0 0 0 0 0
0
1

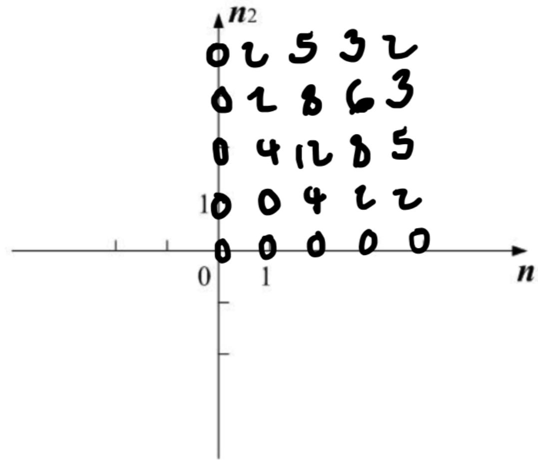


n_2
n_1
0
1



n_2
n_1
0 2 8 6 3
0 4 12 8 5
0 0 4 2 2
0 0 0 0 0
0
1





$$y[n,n] = \{0, 0, 0, 0, 0, 0, 0, 4, 2, 2, 4, 12, 8, 5, 0, 2, 8, 6, 3, 0, 2, 5, 3, 2\}$$

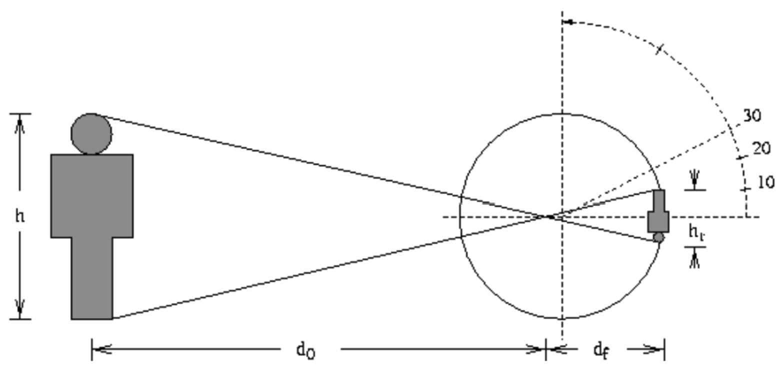
Problem 6 (10 points) Consider the human visual system

a) Discuss if our eyes are more sensitive to small amplitude noise in dark regions or bright regions of an image. Please provide explanation.

Brightness is closely related to intensity, but it is not proportional to intensity, but rather, a logarithmic function of intensity. According to Weber's ratio, $\frac{\Delta I}{I} \approx const \approx 0.02$ for a large range of values I, it is suggested that our brightness perception is closely related to contrast. Thus, it is implied that we are better able to notice differences in intensity when the image is **dark** than when it is light.

b) Describe the procedure to calculate the optimal viewing distance of an image. (You answers should be no longer than 5 short sentences.)

When viewing an object with a specific height (h), the distance (d_f) between the focal center of the lens and the retina is typically between 14 and 17 mm. Assuming these values are known, the next step is to recall that the viewing distance (d_0) is inversely proportional to the perceived height in the retina (h_r). We can use the expression $d_0 = \frac{d_f h}{h_r}$, where h_r increases with viewing angle. With a known viewing angle θ, h_r can be calculated as $d_f[2tan\theta]$, thus resulting in the optimal viewing distance being equal to $\frac{1}{2}[\frac{h}{d_f tan\theta}]$. It must be noted that there is a cutoff point where the distance is too big/small for proper clarity.



- The distance between the focal center of the lens and the retina d_f=14 ~ 17 mm
- The size of the object at retina is $h_r = \frac{d_f h}{d_0}$