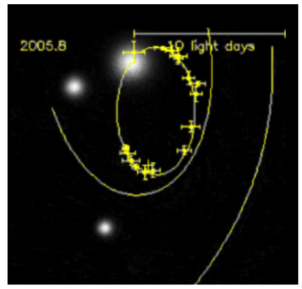
Problem 1 (4 pts).

As we discussed in class, half of this year's Nobel Prize in Physics is awarded for the work discovering the super massive black hole at the center of our galaxy. As shown in the snapshot below, by monitoring the stars near the galactic center and tracing their orbits, the mass of the central object can be estimated from Newton's version of Kepler's third law $p^2 = \frac{4\pi^2}{G(M1+M2)} a^3$ as we have seen before.



One of the stars in the center of our galaxy called S2 has a relatively short orbital period, 15.2 years and thus its entire orbit was mapped out. The semi-major of its orbit is determined to be 5.5 light days (952.3 AU). Based on this, what is the mass of the object at the very center of our galaxy (in solar masses)?

You can directly calculate the mass using the equation above – you have to use SI units and the mass you get would be in kg. Then you'll need to convert it to solar masses.

However, for this specific problem, there is an easier way. Remember we can apply $p^2 = \frac{4\pi^2}{G(M1+M2)} a^3$ to the orbital motion of Earth around the Sun as well. So we have two equations:

$p_{S2}^2 = \frac{4\pi^2}{GM_{BH}} a_{S2}^3$ and $p_{Earth}^2 = \frac{4\pi^2}{GM_{Sun}} a_{Earth}^3$, where we have made use of the fact that the mass of the satellite object is much smaller than that of the central object and is thus ignored. Taking the ratio of the two equations, we have: $\frac{p_{S2}^2}{P_{Earth}^2} = \frac{M_{Sun}}{M_{BH}} \frac{a_{S2}^3}{a_{Earth}^3}$. The beauty of this is that we don't have to bother with SI units – it's a ratio so we can use whatever units that are convenient, especially knowing p_{Earth} = 1 year, and a_{Earth} = 1 AU.

$$\frac{p_{S2}^2}{p_{Earth}^2} = \frac{M_{Sun} \times a_{S2}^3}{M_{BH} \times a_{Earth}^3} = \frac{M_{Sun} \times a_{S2}^3}{M_{BH}}$$

$$p_{S2}^2 = \frac{M_{Sun} \times a_{S2}^3}{M_{BH}}$$

$$M_{BH} = \frac{M_{Sun} \times a_{S2}^3}{p_{S2}^2} = \frac{M_{Sun} \times (952.3)^3}{(15.2)^2} = [3737955.9326] M_{Sun}$$

$$\frac{M_{BH}}{M_{Sun}} = 3.73 \times 10^6$$

Problem 2.

a) (2 pts) Our Sun is about 27000 light years away from the center of our galaxy, and it is orbiting around the center at a speed of about 220 km/s on average. Assuming the orbit is perfectly circular, what is the total mass enclosed within the Sun's orbit (in solar masses)? Feel free to review the orbital motion part of the lecture slides.

$$v = \sqrt{\frac{GM}{R}}$$

$$M = \frac{v^2 R}{G}$$

$$t = 60 \times 60 \times 24 \times 365 = 3.15 \times 10^7\, s$$

$$R = 27000 \times (3 \times 10^8) \times (3.15 \times 10^7) = 2.55 \times 10^{20}\, m$$

$$M = \frac{v^2 R}{G} = \frac{(2.2\times10^5)^2 (2.55\times10^{20})}{(6.67\times10^{-11})} = 1.85 \times 10^{41}\, kg$$

<u>In Solar Masses:</u>

$$M_{Sun} = 1.99 \times 10^{30}\, kg$$

$$M = \frac{1.85\times10^{41}}{1.99\times10^{30}} = [9.29 \times 10^{10}] M_{Sun}$$

b) If we observe the distribution of stars in our galaxy, we will find that it tapers off as you go from the center of the galaxy outwards. Beyond where the Sun is, there isn't much visible matter (stars or gas) out there. However, if you observe the orbital speed out there, it is actually roughly the same as the Sun's orbital speed.
i) (1 pts) Does this mean the total mass enclosed at larger and larger radius increases, decreases or stays the same?
ii) (1 pt) Does your answer in i) contradict with the fact that visible matter in our galaxy is concentrated only in the central region within the solar radius? How would you reconcile this "discrepancy"?

[i] This means that the total mass enclosed at a larger radius stays the same, as the orbital speed remains almost constant at a larger radius, despite the increase in distance.

[ii] Yes, this contradicts the observation that visible matter is concentrated at the center of the galaxy. To reconcile this "discrepancy," one can assumes that some sort of invisible or dark matter is present even at the larger radii, resulting in the orbital speed to be constant.

Problem 3 (2 pts).

In class we discussed how to measure distance on different scales. On very large scales, there is a Hubble flow due to the expansion of the Universe governed by Hubble's law: cz = v = H_0 d, where c is speed of light in vacuum, z is redshift, v is the recession speed, Ho is the Hubble constant and d is distance. (This linear relation is only valid for low redshift though.)

A group of galaxies called Robert's quartet has a receding speed of 3400 km/s. The current accepted value for Ho is 70 km/s/Mpc. How far away is this group of galaxies (in Mpc)?

$$v = H_0 d$$

$$d = \frac{v}{H_0} = \frac{3400}{70} = 48.57\ Mpc$$