- Q1: Read the below paper and write a summary (a minimum of half a page) to describe what you have learned from it. (20 points)

A.N. Michel, “Stability: The Common Thread in the Evolution of Feedback Control,” IEEE Control Systems Magazine, Vol. 16, No. 3, June 1996

This article discusses the series of events that built upon one another to enhance stability and feedback control in systems. While the concept of stability in control systems seems like a topic that has been approached through purposeful ventures, it is actually quite the opposite. The advancements made in feedback control are typically only of concern in response to a stunt in a system. For instance, the steam engine governor that was introduced to supply speed control to a steam engine was not a concept that was of great concern until advancements on the steam engine’s performance came to question as industrial initiatives struggled to keep up with increasing demand. Over the years, mathematics became an essential tool to keep up with the underlying drawbacks that may be spontaneously identified in a system, or otherwise calculated through previous observations. This phenomena became more common as systems could be illustrated prior to physical experimentation and simple trial and error. Routh Hurwitz criterion was among the methods introduced to provide insight on observing stability in a system. With technology advancing at an exponential rate, the stability of newly implemented systems soon became prioritized as the methods discussed in the article provide the ability to hypothesize the performance of a system.

Q2: Consider the following system (80 points)

A.) Open-loop poles at $\det(pI - A) = 0$

$$\det \begin{bmatrix} p+0.4 & 0 & 0.01 \\ -1 & p & 0 \\ 1.4 & -9.8 & p+0.02 \end{bmatrix} = 0$$

$$p^3 + \frac{21p^2}{50} - \frac{3p}{500} + \frac{49}{500} = 0$$

Poles: $-0.659,\ 0.118 \pm 0.368i$

B.) Controllable if $\det(C) \neq 0$ $\quad C = [B \ \ AB \ \ A^2B]$

$$C = \begin{bmatrix} 6.3 & -2.618 & 1.137 \\ 0 & 6.3 & -2.618 \\ 9.8 & -9.016 & 65.586 \end{bmatrix}$$

$\det(C) = 2451.3 \neq 0$

$\therefore$ controllable

C.) $s = -2, -1 \pm j$ $\quad$ characteristic equation:

$$(s+2)(s+1-j)(s+1+j) = 0$$

$$s^3 + 4s^2 + 6s + 4 = 0$$

characteristic equation: $\det(sI - (A - Bk)) = 0$

$$A - Bk = \begin{bmatrix} -0.4 & 0 & -0.01 \\ 1 & 0 & 0 \\ -1.4 & 9.8 & -0.02 \end{bmatrix} - \begin{bmatrix} 6.3k_1 & 6.3k_2 & 6.3k_3 \\ 0 & 0 & 0 \\ 9.8k_1 & 9.8k_2 & 9.8k_3 \end{bmatrix}$$

$$sI - (A - Bk) = \begin{bmatrix} s + 0.4 + 6.3k_1 & 6.3k_2 & 0.01 + 6.3k_2 \\ -1 & s & 0 \\ 1.4 + 9.8k_3 & -9.8 + 9.8k_2 & s + 0.02 + 9.8k_3 \end{bmatrix}$$

$\det(sI - (A - Bk)) = s^3 + 4s^2 + 6s + 4$ $\qquad s^3 = s^3$

$$\frac{7k_2}{250} + \frac{3087k_3}{50} + \frac{49}{500} = 4 \qquad \frac{-3s}{500} + \frac{63k_1 s}{500} + \frac{63k_2 s}{10} - \frac{2499\ldots}{500}$$

$$\frac{63k_1 s^2}{10} + \frac{49k_3 s^2}{5} + \frac{21s^2}{50} = 4s^2 \qquad + \frac{3087k_1k_3 s}{50} - \frac{3087k_3^2 s}{50} = 6.$$

$k_1 = 0.471 \quad k_2 = 1.190 \quad k_3 = 0.063$

D.) $s = -8, \ -4 \pm 4\sqrt{3}j$ $\quad \alpha_c(s) = (s+8)(s+4+4\sqrt{3}j)(s+4-4\sqrt{3}j)$

$s^3 + 16s^2 + 128s + 512$

$\alpha_c(A) = A^3 + 16A^2 + 128A + 512I$

$$= \begin{bmatrix} 463.4 & -1.5 & -1.2 \\ 121.8 & 511.9 & -0.2 \\ -17.4 & 1251.4 & 509.6 \end{bmatrix}$$

$$\theta = \begin{bmatrix} C \\ CA \\ CA^2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ -1.4 & 9.8 & -0.02 \\ 10.4 & -0.2 & 0.014 \end{bmatrix} \quad \theta^{-1} = \begin{bmatrix} -0.0014 & 0.0019 & 0.0968 \\ 0.0018 & 0.1023 & 0.0138 \\ 1 & 0 & 0 \end{bmatrix}$$

$$L = \alpha_c(A)\theta^{-1}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 44.7 \\ 18.8 \\ 15.6 \end{bmatrix} \qquad L_1 = 44.7 \quad L_2 = 18.8 \quad L_3 = 15.6$$

E.) Poles at -4 and -4 $\qquad (s+4)^2 = 0$

$s^2 + 8s + 16 = 0$

$A_{aa} = [-0.4] \quad A_{ab} = [0 \ \ -0.01] \quad A_{ba} = \begin{bmatrix} 1 \\ -1.4 \end{bmatrix}$

$A_{bb} = \begin{bmatrix} 0 & 0 \\ 9.8 & -0.02 \end{bmatrix} \quad B_a = [6.3] \quad B_b = \begin{bmatrix} 0 \\ 9.8 \end{bmatrix}$

$L = \alpha_c(A_{bb})\theta^{-1}\begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad \alpha_c(A_{bb}) = A_{bb}^2 + 8A_{bb} + 16I$

$\alpha_c(A_{bb}) = \begin{bmatrix} 16 & 0 \\ 782 & 15.84 \end{bmatrix} \qquad \theta^{-1} = \begin{bmatrix} A_{ab} \\ A_{ab} \cdot A_{bb} \end{bmatrix} = \begin{bmatrix} 0 & -0.01 \\ -0.098 & 0.02 \end{bmatrix}^{-1}$

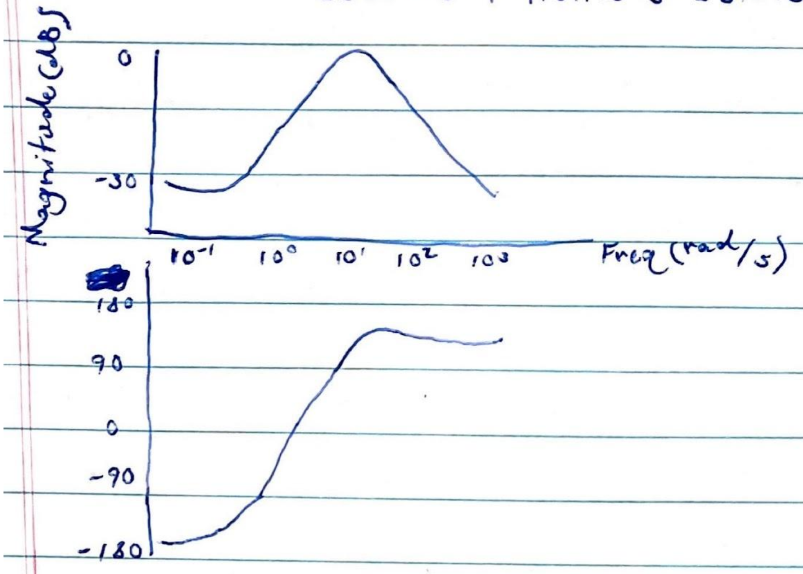
$\theta^{-1} = \begin{bmatrix} -20.41 & -10.20 \\ -100 & 0 \end{bmatrix} \qquad L = \begin{bmatrix} 16 & 0 \\ 78.2 & 15.84 \end{bmatrix}\begin{bmatrix} -20.41 & -10.2 \\ -100 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$L = \begin{bmatrix} -163.27 \\ -798 \end{bmatrix} \qquad L_1 = -163.27 \quad L_2 = -798$

F.) $TF = -K(sI - A + BK + LC)^{-1}L$

$$= -[0.471 \ \ 1.19 \ \ 0.063]\left(\begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{bmatrix} - \begin{bmatrix} -0.4 & 0 & -0.01 \\ 1 & 0 & 0 \\ -1.4 & 9.8 & -0.02 \end{bmatrix} + \begin{bmatrix} 6.3 \\ 0 \\ 9.8 \end{bmatrix}[0.471 \ \ 1.19 \ \ 0.063] + \begin{bmatrix} 44.7 \\ 18.8 \\ 15.6 \end{bmatrix}[0 \ 0 \ 1]\right)^{-1}\begin{bmatrix} 44.7 \\ 18.8 \\ 15.6 \end{bmatrix}$$

$$TF = \frac{-1110212505^2 - 1774863455 - 80242114}{25000005^3 + 490117505^2 - 610466230s + 1919392230}$$



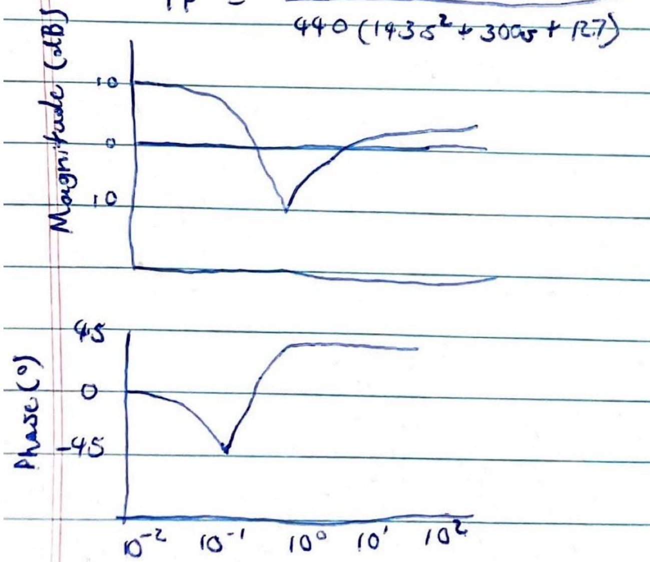
Bode Plot

$$k_a = [0.471] \quad k_b = [1.190 \quad 0.063]$$

G.) $TF = C(sI - A)^{-1}b + D$

$$TF = -k_b\left[sI - (A_{bb} - LA_{ab} - (B_b - LB_a)k_b)\right]^{-1}\left[(A_{bb} - LA_{ab} - (B_b - LB_a)k_b)L + A_{ba} - LA_{aa} - (B_b - LB_a)k_b\right] + (-k_a - k_b L)$$

$$TF = \frac{208s^2 + 110s + 404}{440(143s^2 + 300s + 12.7)}$$



Bode Plot from MatLab: