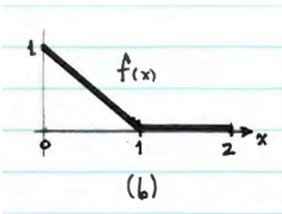
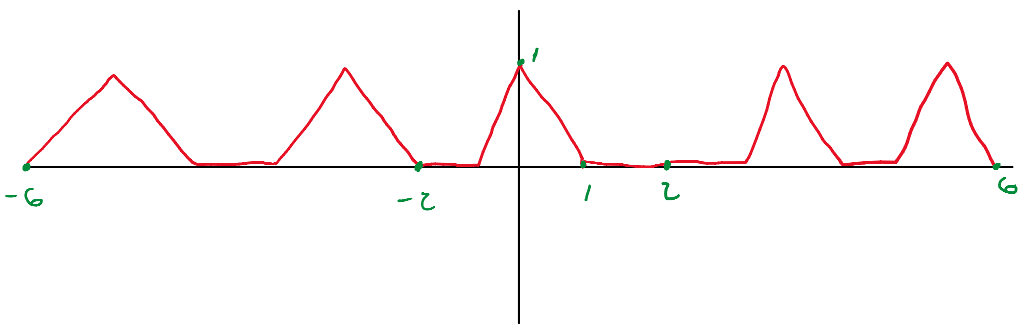
Workshop 12 Group Members: Alex Gaskins, Sydney Mellage, Stephanie Long

Tuesday, November 24, 2020 11:04 AM

1. For each of the following functions defined on $[0, 2]$, sketch the periodic extension to which (i) the Fourier cosine series converges and (ii) the Fourier sine series converges. Plot the graphs on the interval $[-6, 6]$ and identify the convergence at jump discontinuities.

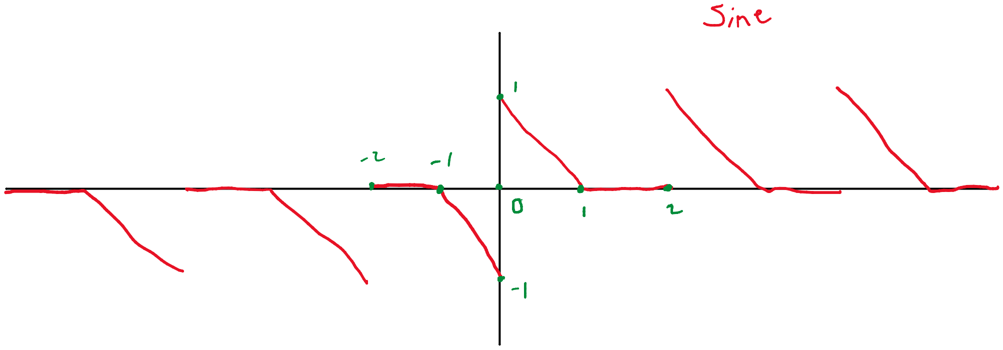


Cosine



Converges at 1

Sine



Converges at 0

2. Find the (*half-range*) Fourier Cosine series for $f(x) = \sin^2(\pi x)$ on the interval $(0, 1)$. Sketch the periodic extension, $f_p(x)$, that represents the pointwise convergence of the cosine series and identify the *fundamental period*.

#2

$F(x) = \sin^2(\pi x)$ on $(0,1)$ $\rightarrow$ $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(n\pi x)$

$a_0 = 2\int_0^1 f(x)\,dx$

$\int_0^1 2\sin^2(\pi x)\,dx$

$\Rightarrow \int_0^1 (1-\cos 2\pi x)\,dx$

$= \left[x - \frac{1}{2\pi}\cdot \sin 2\pi x\right]_0^1$

$= 1 - 0 = 1$

$a_0 = 1$

$a_n = 2\int_0^1 f(x)\cos(n\pi x)\,dx$

$\Rightarrow 2\int_0^1 \sin^2(\pi x)\cos(\pi n x)\,dx$

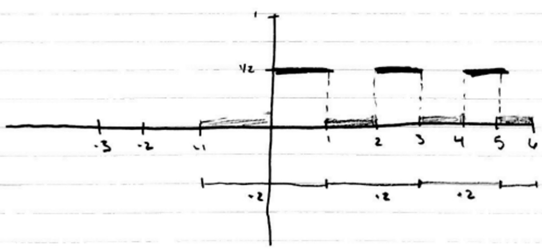
$\Rightarrow \int_0^1 2\cos n\pi x - 2\cos 2\pi x - \cos \pi n x\,dx$

$\Rightarrow \int_0^1 2\cos n\pi x - \cos(2+n)\pi x - \cos(2-n)\pi x\,dx$

$= \frac{2\sin n\pi x}{n\pi} - \frac{\sin(2+n)\pi x}{(2+n)\pi} - \frac{\sin(2-n)\pi x}{(2-n)\pi}\Big|_0^1 = 0$

$f_p(x) = \frac{1}{2}$

period $= 2$



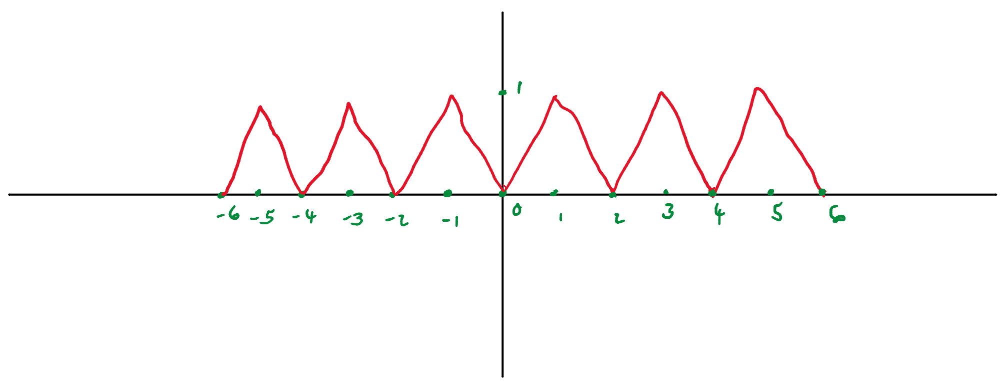
3. Consider the function $f(x)$ defined on $(0,2)$, $f(x) = \begin{cases} x, & 0 < x < 1 \\ -x+2, & 1 \le x < 2 \end{cases}$.

(a) On the interval $[-6,6]$, graph the periodic function $f_p(x)$ representing the pointwise convergence of the Fourier Sine series for $f(x)$ on $(0,2)$.
Identify the *fundamental period* of $f_p(x)$.

(b) Determine the coefficients in the Sine series for $f(x)$ on $(0,2)$. Your result should show $b_2 = b_4 = \cdots = 0$. Try to use symmetry about $x = 1$ on the half interval $0 < x < 2$ to explain why the these coefficients are zero.

$\begin{bmatrix} 1, & [0,1] \\ -1, & [1,2] \end{bmatrix}$

A.) Always converges to 1



B.) $b_n = \int_0^2 f(x)\sin\left(\frac{n\pi x}{2}\right) = \int_0^1 x\sin\left(\frac{n\pi x}{2}\right)dx + \int_1^2 -x+2\sin\left(\frac{n\pi x}{2}\right)$

$b_n = \frac{8\sin\left(\frac{\pi n}{2}\right)}{\pi^2 n^2}$

$b_2 = 0$

$b_4 = 0$

(Odd function)

$f(x) = \sum^{\infty} \frac{8\sin\left(\frac{\pi n}{2}\right)}{\pi^2 n^2}\sin\left(\frac{n\pi x}{2}\right)$

$$f(x) = \sum_{n=1}^{\infty} \frac{8\sin\left(\frac{\pi n}{2}\right)}{\pi^2 n^2} \sin\left(\frac{n\pi x}{2}\right)$$