Homework 1

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"I pledge my honor I have abided by the Stevens Honor system."

- Alex Gaskins

3.26 (*a*) Determine the total number (#/cm³) of energy states in silicon between E_c and $E_c + 2kT$ at (*i*) $T = 300$ K and (*ii*) $T = 400$ K. (*b*) Repeat part (*a*) for GaAs.

$$g(E)\,dE = \left[\frac{4\pi a^3}{h^3}(2m)^{3/2}(\sqrt{E})\right]$$

$$= \left[\frac{4\pi(2m)^{3/2}}{h^3}\left(\frac{2}{3}\right)\left(E^{3/2}\right)\right]$$

a.)

i]

$$\left[\frac{4\pi\left(2(1.08)(9.11\times10^{-31})\right)^{3/2}}{(6.26\times10^{-34})^3}\left(\frac{2}{3}\right)\left(2(1.38\times10^{-23})(300)\right)^{3/2}\right]$$

$$= 5.97\times10^{25}\ m^{-3} \times [100\ cm]^{-3}$$

$$= \boxed{5.97\times10^{19}\ cm^{-3}}$$

ii]

$$\left[\frac{4\pi\left(2(1.08)(9.11\times10^{-31})\right)^{3/2}}{(6.26\times10^{-34})^3}\left(\frac{2}{3}\right)\left(2(1.38\times10^{-23})(400)\right)^{3/2}\right]$$

$$= 9.23\times10^{25}\ m^{-3} \times [100\ cm]^{-3}$$

$$= \boxed{9.23\times10^{19}\ cm^{-3}}$$

b.)

i.)
$$\left[\frac{4\pi\left(2(0.067)(9.11\times10^{-31})\right)^{3/2}}{(6.26\times10^{-34})^3}\left(\frac{2}{3}\right)\left(2(1.38\times10^{-23})(300)\right)^{3/2}\right]$$

$$= 9.26\times10^{23}\ m^{-3} \times [100\ cm]^{-3}$$

$$= \boxed{9.26\times10^{17}\ cm^{-3}}$$

ii.)
$$\left[\frac{4\pi\left(2(0.067)(9.11\times10^{-31})\right)^{3/2}}{(6.26\times10^{-34})^3}\left(\frac{2}{3}\right)\left(2(1.38\times10^{-23})(400)\right)^{3/2}\right]$$

$$= 1.43\times10^{24}\ m^{-3} \times [100\ cm]^{-3}$$

$$= \boxed{1.43\times10^{18}\ cm^{-3}}$$

3.27 (*a*) Determine the total number (#/cm^3) of energy states in silicon between E_v and $E_v - 3\,kT$ at (*i*) $T = 300$ K and (*ii*) $T = 400$ K. (*b*) Repeat part (*a*) for GaAs.

a.)

i.)
$$\left[\frac{4\pi(2m_p^*)^{3/2}}{h^3}\left(-\frac{2}{3}\right)(-3kT)^{3/2}\right]$$

$$= \frac{4\pi\left(2(0.56)(9.11\times10^{-31})\right)^{3/2}}{(6.26\times10^{-34})^3}\left(\frac{2}{3}\right)(3kT)^{3/2}$$

$$= 2.969\times10^{55}\left(3\left(4.144\times10^{-21}\right)\right)^{3/2}$$

$$= 4.12\times10^{25}\ m^{-3}\ \left[100\right]^{-3}_{cm}$$

$$= \boxed{4.12\times10^{19}\ cm^{-3}}$$

ii]

$$\left[\frac{4\pi\left(2m_p^*\right)^{3/2}}{h^3}\left(-\frac{2}{3}\right)\left(-3kT\right)^{3/2}\right]$$

$$= \frac{4\pi\left(2\left(0.56\right)\left(9.11\times10^{-31}\right)\right)^{3/2}}{\left(6.26\times10^{-34}\right)^3}\left(\frac{2}{3}\right)\left(3kT\right)^{3/2}$$

$$= 2.969\times10^{55}\left(3\left(5.53\times10^{-21}\right)\right)^{3/2}$$

$$= 6.34\times10^{25}\ m^{-3}\ \left[100\right]^{-3}_{cm}$$

$$= \boxed{6.34\times10^{19}\ cm^{-3}}$$

b.)

i]

$$\left[\frac{4\pi\left(2m_p^*\right)^{3/2}}{h^3}\left(-\frac{2}{3}\right)\left(-3kT\right)^{3/2}\right]$$

$$= \frac{4\pi\left(2\left(0.48\right)\left(9.11\times10^{-31}\right)\right)^{3/2}}{\left(6.26\times10^{-34}\right)^3}\left(\frac{2}{3}\right)\left(3kT\right)^{3/2}$$

$$= 2.3564\times10^{55}\left(3\left(4.144\times10^{-21}\right)\right)^{3/2}$$

$$= 3.27\times10^{25}\ m^{-3}\ \left[100\right]^{-3}_{cm}$$

$$= \boxed{3.27\times10^{19}\ cm^{-3}}$$

ii] $\left[\frac{4\pi(2m_p^*)^{3/2}}{h^3}\left(-\frac{2}{3}\right)(-3kT)^{3/2}\right]$

$$= \frac{4\pi(2(0.48)(9.11\times10^{-31}))^{3/2}}{(6.26\times10^{-34})^3}\left(\frac{2}{3}\right)(3kT)^{3/2}$$

$$= 2.3564\times10^{55}\,(3(5.53\times10^{-21}))^{3/2}$$

$$= 5.03\times10^{25}\ m^{-3}\ [100]^{-3}_{cm}$$

$$= \boxed{5.03\times10^{19}\ cm^{-3}}$$

3.32 Determine the probability that an energy level is occupied by an electron if the state is above the Fermi level by (*a*) *kT*, (*b*) 5*kT*, and (*c*) 10*kT*.

$$f(E) = \frac{1}{1+\exp\left(\frac{E-E_F}{kT}\right)}$$

a.) $E-E_F = kT$

$$f(E) = \frac{1}{1+\exp(1)} = \boxed{0.269}$$

b.) $E-E_F = 5kT$

$$f(E) = \frac{1}{1+\exp(5)} = \boxed{0.0067}$$

c.) $E-E_F = 10kT$

$$f(E) = \frac{1}{1+\exp(10)} = \boxed{4.54\times10^{-5}}$$

3.35 The probability that a state at $E_c + kT$ is occupied by an electron is equal to the probability that a state at $E_v - kT$ is empty. Determine the position of the Fermi energy level as a function of E_c and E_v.

$$\exp\left(\frac{E_F - E}{kT}\right) = \exp\left(\frac{E_F - kT - E_c}{kT}\right) = f_F$$

$$1 - f_F = \exp\left(\frac{E - E_F}{kT}\right) = \exp\left(\frac{-[E_F - (E_v - kT)]}{kT}\right)$$

$$= \exp\left(\frac{-(E_c + kT - E_F)}{kT}\right) = \exp\left(\frac{-(E_F - E_v + kT)}{kT}\right)$$

$$E_c + \cancel{kT} - E_F = E_F - E_v + \cancel{kT}$$

$$= \boxed{E_F = \frac{E_c + E_v}{2}}$$

3.41 The Fermi energy for copper at $T = 300$ K is 7.0 eV. The electrons in copper follow the Fermi–Dirac distribution function. (*a*) Find the probability of an energy level at 7.15 eV being occupied by an electron. (*b*) Repeat part (*a*) for $T = 1000$ K. (Assume that E_F is a constant.) (*c*) Repeat part (*a*) for $E = 6.85$ eV and $T = 300$ K. (*d*) Determine the probability of the energy state at $E = E_F$ being occupied at $T = 300$ K and at $T = 1000$ K.

$kT = 0.0259$

a.) $$f(E) = \frac{1}{1 + \exp\left(\frac{7.15 - 7.0}{0.0259}\right)} = 0.00304$$

$f(E) = 0.304\%$

b.) $kT = 0.08633$

$$f(E) = \frac{1}{1 + \exp\left(\frac{7.15 - 7.0}{0.08633}\right)} = 0.1496$$

$f(E) = 14.96\%$

c.) $$f(E) = \frac{1}{1 + \exp\left(\frac{6.85 - 7.0}{0.0259}\right)} = 0.997$$

$f(E) = 99.7\%$

d.) $E = E_F$

$f(E) = \frac{1}{2}$ @ all temps.