

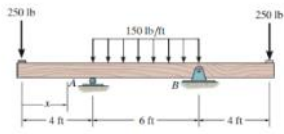
Homework 10

Tuesday, November 10, 2020 5:07 PM

"I pledge my honor I have abided by the Stevens Honor system."

- Alex J. Adams

11-2. Draw the shear and moment diagrams for the beam, and determine the shear and moment in the beam as functions of x for $0 \leq x < 4$ ft, $4 \text{ ft} < x < 10$ ft, and $10 \text{ ft} < x < 14$ ft.



$$\sum M_B = 0$$

$$-A_y(6) + 250(10) + (6(150)\left(\frac{6}{2}\right)) - 250(4) = 0$$

$$6A_y = 4200$$

$$A_y = 700 \text{ lbs.}$$

$$\sum F_y = 0$$

$$A_y + B_y - (6(150)) - 250 - 250 = 0$$

$$A_y + B_y = 1400$$

$$(700) + B_y = 1400$$

$$B_y = 700 \text{ lbs.}$$

$$0 \leq x < 4 \text{ ft}$$

$$V(x) = -250 \text{ lbs. (shear force)}$$

$$M(x) = -150 \text{ lb}\cdot\text{ft (bending moment)}$$

$$4 \text{ ft} < x < 10 \text{ ft}$$

$$V(x) = -250 + A_y - (150)(x-4)$$

$$V(x) = -250 + 700 - (150)(x-4)$$

$$V(x) = 450 - 150x + 600$$

$$V(x) = [1050 - 150x] \text{ lbs.}$$

$$M(x) = -250x + A_y(x-4) - (150)\left(\frac{x-4}{2}\right)\left(\frac{x-4}{2}\right)$$

$$M(x) = -250x + 700x - 7500 - 75\left(x^2 - 8x + 16\right)$$

$$M(x) = [-75x^2 + 1050x - 9000] \text{ lb}\cdot\text{ft}$$

$$10 \text{ ft} < x \leq 14 \text{ ft}$$

$$V(x) = -250 + A_y - 150(6) + B_y$$

$$V(x) = -250 + 700 - (150(6)) + 700$$

$$V(x) = 250 \text{ lbs.}$$

$$M(x) = -250x + A_y(x-4) - (150(6))(x-7) + B_y(x-10)$$

$$M(x) = -250x + (700)(x-4) - (150(6))(x-7) + (700)(x-10)$$

$$M(x) = [250x - 3500] \text{ lb}\cdot\text{ft}$$

Shear force test points:

$$V_x = 0 = -250 \text{ lbs.}$$

$$V_x = 4^- = -250 \text{ lbs.}$$

$$V_x = 4^+ = [1050 - 150(4)] = 450 \text{ lbs.}$$

$$V_x = 10^- = [1050 - 150(10)] = -450 \text{ lbs.}$$

$$V_x = 10^+ = 250 \text{ lbs.}$$

$$V_x = 14 = 250 \text{ lbs.}$$

Bending moment test points:

$$M_x = 0 = -250x = 0$$

$$M_x = 4^- = -250x = -1000 \text{ lb}\cdot\text{ft}$$

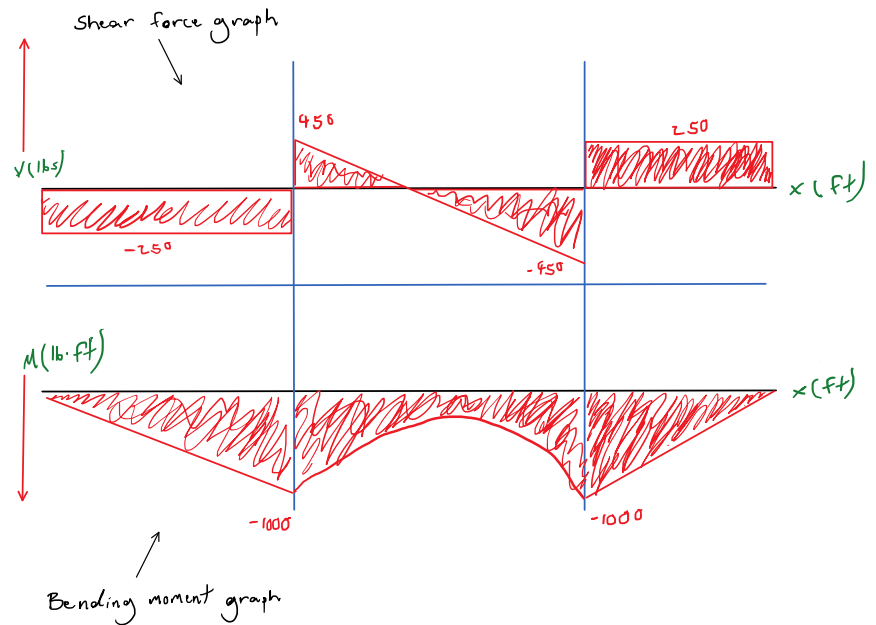
$$M_x = 4^+ = [-75x^2 + 1050x - 9000] = -1000 \text{ lb}\cdot\text{ft}$$

$$M_x = 7 = [-75x^2 + 1050x - 9000] = -3250 \text{ lb}\cdot\text{ft}$$

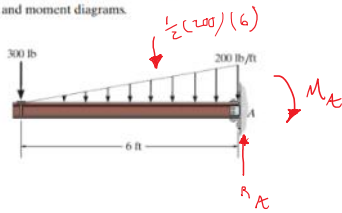
$$M_x = 10^- = [-75x^2 + 1050x - 9000] = -1000 \text{ lb}\cdot\text{ft}$$

$$M_x = 10^+ = [250x - 3500] = -1000 \text{ lb}\cdot\text{ft}$$

$$M_x = 14 = [250x - 3500] = 0$$



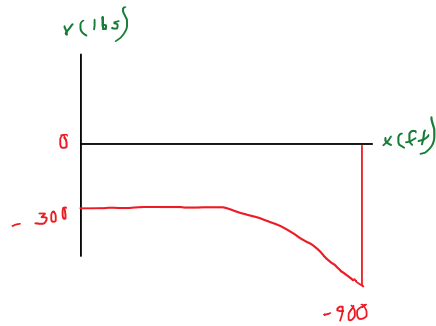
11-5. Express the internal shear and moment in the cantilevered beam as a function of x and then draw the shear and moment diagrams.



$$\begin{aligned} \sum F_y &= 0 \\ R_A - 300 - \frac{1}{2}(200)(6) &= 0 \\ R_A &= 900 \text{ lb.} \\ M_A - R_A(6) + \frac{1}{2}(200)(6)(4) &= 0 \\ M_A - 900(6) + \frac{1}{2}(200)(6)(4) &= 0 \\ M_A &= 3000 \text{ lb}\cdot\text{ft} \\ W &= 200\left(\frac{x}{6}\right) \\ W &= 33.33x \end{aligned}$$

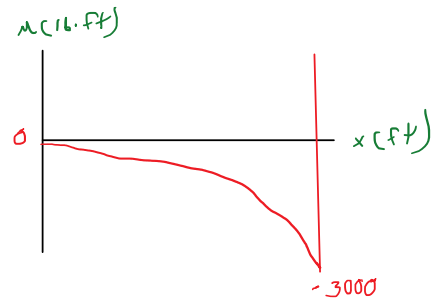
Shear force test points:

$$\begin{aligned} \sum F_y &= 0 \\ -300 - \frac{1}{2}(33.33x)(x) - V &= 0 \\ V &= (-300 - 16.67x^2) \text{ lb} \\ x &= 0 \\ V &= -300 \text{ lbs.} \\ x &= 6 \\ V &= -900 \text{ lbs.} \end{aligned}$$

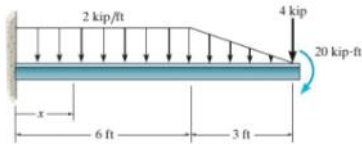


Bending moment test points:

$$\begin{aligned} \sum M_A &= 0 \\ M + \frac{1}{2}(33.33x)(x)\left(\frac{x}{3}\right) + 300x &= 0 \\ M &= (-300x - 5.556x^3) \text{ lb}\cdot\text{ft} \\ x &= 0 \\ M &= 0 \\ x &= 6 \\ M &= -3000 \text{ lb}\cdot\text{ft} \end{aligned}$$



*11-8. Draw the shear and moment diagrams for the beam, and determine the shear and moment throughout the beam as functions of x for $0 \leq x \leq 6$ ft and $6 \text{ ft} \leq x \leq 9$ ft.



$$\begin{aligned} \sum F_y &= 0 \\ R_A - (2(9)) - 4 + \left(\frac{6}{2}\right) &= 0 \\ R_A &= 19 \text{ kip.} \\ \sum M_A &= 0 \\ M_A - (18(9)) - 4(9-20) + 3(8) &= 0 \\ M_A &= 113 \text{ kip}\cdot\text{ft} \end{aligned}$$

$$\begin{aligned} \sum F_y &= 0 \\ R_A - 7x - V_x &= 0 \\ V_x &= 19 - 7x \end{aligned}$$

$$\begin{aligned} \sum M_x &= 0 \\ M_A - R_A(x) + 2x\left(\frac{x}{2}\right) + M_x &= 0 \\ M_x &= 19x - x^2 - 113 \end{aligned}$$

$$\begin{aligned} \sum F_y &= 0 \\ R_A - 2x + \frac{1}{2}(x-6)^2 - V_x &= 0 \\ V_x &= 19 - 2x + \frac{(x-6)^2}{3} \end{aligned}$$

$$6 \text{ ft} \leq x \leq 9 \text{ ft}$$

$$\sum M_x = 0$$

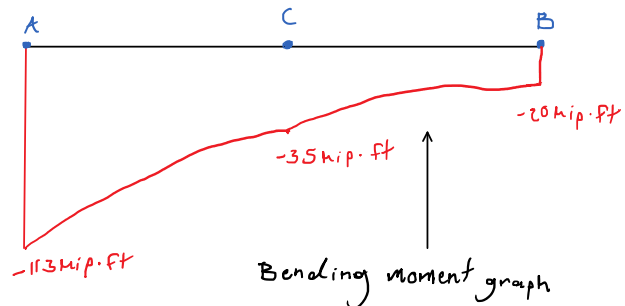
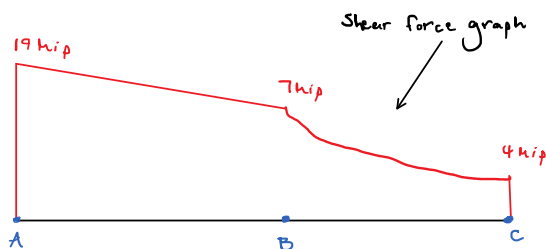
$$\begin{aligned} M_A - R_A(x) + 2x\left(\frac{x}{2}\right) - \left[\frac{1}{2}(x-6)x\right]\frac{1}{3}(x+6) + M_x &= 0 \\ M_x &= \frac{(x-6)^3}{9} - x^2 + 19x - 113 \end{aligned}$$

Shear force:

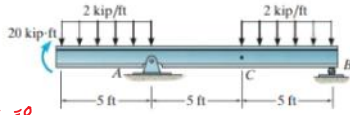
$$\begin{aligned} A &= 19 \text{ kip} \\ B &= 4 \text{ kip} \\ C &= 7 \text{ kip} \end{aligned}$$

Bending moment:

$$\begin{aligned} A &= -113 \text{ kip}\cdot\text{ft} \\ B &= -20 \text{ kip}\cdot\text{ft} \\ C &= -35 \text{ kip}\cdot\text{ft} \end{aligned}$$



11-21. Draw the shear and moment diagrams for the beam.



$$\begin{aligned} \sum F_x &= 0 \\ A_x &= 0 \\ \sum M_B &= 0 \\ A_y(10) + 20 - (2(5))(10 + \frac{5}{2}) - 10(\frac{5}{2}) &= 0 \\ A_y &= 13 \text{ kip} \\ \sum F_y &= 0 \\ A_y + B_y - 2(5) - 2(5) &= 0 \\ A_y + B_y &= 20 \\ 13 + B_y &= 20 \\ B_y &= 7 \text{ kip} \\ \sum F_x &= 0 \\ -(Lx) \cdot V &= 0 \\ V &= -2x \\ x=0 \quad V &= 0 \\ x=5 \quad V &= -10 \text{ kip} \end{aligned}$$

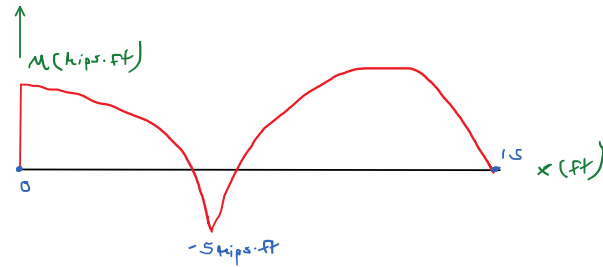
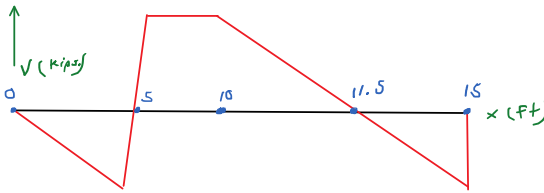
Bending moment:

$$\begin{aligned} \sum M_x &= 0 \\ -20 + 2x(\frac{x}{2}) + M_x &= 0 \\ M_x &= 20 - x^2 \\ x=0 \quad M_x &= 20 \\ x=5 \quad M_x &= 5 \text{ kip}\cdot\text{ft} \\ \sum M_x &= 0 \\ 20 - x^2 &= 0 \\ x &= \sqrt{20} \\ x &= 4.472 \text{ ft} \end{aligned}$$

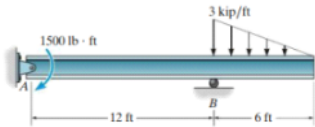
$$\begin{aligned} \sum F_y &= 0 \\ -10 + A_y - V &= 0 \\ V &= A_y - 10 \\ V &= 3 \text{ kips} \\ x=5 \quad V &= 3 \text{ kips} \\ x=10 \quad V &= 3 \text{ kips} \end{aligned}$$

$$\begin{aligned} \sum M_x &= 0 \\ -20 + 10(x - \frac{5}{2}) - A_y(x - 5) + M &= 0 \\ M &= 20 - 10(x - \frac{5}{2}) + 13(x - 5) \\ x=5 \quad M &= -5 \text{ kip}\cdot\text{ft} \\ x=10 \quad M &= 10 \text{ kip}\cdot\text{ft} \\ \sum M_x &= 0 \\ 20 - 10(x - \frac{5}{2}) + 13(x - 5) &= 0 \\ x &= 6.667 \text{ ft} \end{aligned}$$

$$\begin{aligned} \sum F_y &= 0 \\ -10 + A_y - 2(x - 10) - V &= 0 \\ V &= 3 - 2(x - 10) \\ x=10 \quad V &= 15 \\ x=15 \quad V &= -7 \text{ kips} \\ \sum M_x &= 0 \\ -10 + A_y - 2(x - 10) - V &= 0 \\ V &= 3 - 2(x - 10) \\ x=10 \quad V &= 15 \\ x=11.5 \text{ ft} \quad V &= 0 \end{aligned}$$



11-26. Draw the shear and moment diagrams for the beam.



$$\begin{aligned} \sum M_A &= 0 \\ -1.5 - (9(1.5)) + B_y(12) &= 0 \\ B_y &= 10.625 \text{ kip} \\ \sum F_y &= 0 \\ A_y + B_y - 9 &= 0 \\ A_y + B_y &= 9 \\ A_y &= -1.625 \text{ kip} \end{aligned}$$

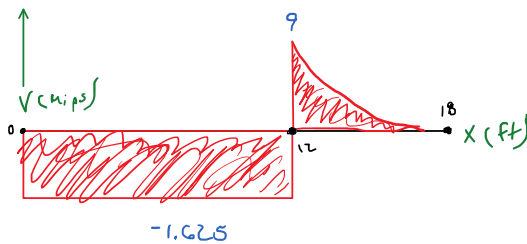
$$\begin{aligned} \frac{w'}{18-x} &= \frac{3}{6} \\ w' &= \frac{3}{6}(18-x) \\ w &= 3 - \frac{3}{6}(18-x) \end{aligned}$$

Shear $0 \leq x < 12 \text{ ft}$

$$\begin{aligned} \sum F_y &= 0 \\ V &= A_y \\ V &= -1.625 \text{ kip} \end{aligned}$$

Bending Moment $0 \leq x < 12 \text{ ft}$

$$\begin{aligned} \sum M &= 0 \\ M &= A_y(x) + 1.5 \\ M &= [-1.625x + 1.5] \text{ kip}\cdot\text{ft} \end{aligned}$$



Shear $12 \text{ ft} < x \leq 18 \text{ ft}$

$$\begin{aligned} \sum F_y &= 0 \\ V &= A_y + B_y - \frac{1}{2}(3 - \frac{3(18-x)}{6})(x - 12) \\ V &= -1.625 + 10.625 - \frac{1}{2}(3 - \frac{3(18-x)}{6})(x - 12) \\ x=12 \quad V &= 9 \text{ kips} \\ x=18 \quad V &= 0 \end{aligned}$$

Bending Moment $12 \text{ ft} < x \leq 18 \text{ ft}$

$$\begin{aligned} M &= -1.625 + 10.625(x - 12) - \frac{1}{2}(3 - \frac{3(18-x)}{6})(x - 12)(\frac{x - 12}{3}) \\ x=12 \quad M &= -15 \text{ kip}\cdot\text{ft} \\ x=18 \quad M &= 0 \end{aligned}$$

