Group Members: Alex Gaskins, Sejad Kiyam I, Joseph Mistretta

MA 221 **Workshop 2** **Sep 15, 2020**

1. Classify each of the following differential equations as *linear*, *separable*, or *Bernoulli*. Identify all classifications that apply.

	Linear	Separable	Bernoulli
(a) $\frac{dy}{dt} = 3y(1-y^2)$	✗	✓	✓
(b) $\frac{dy}{dx} = \frac{x-y}{x}$	✓	✗	✓
(c) $(x+1)\frac{dy}{dx} = 2y + 10$	✓	✓	✗
(d) $\frac{dy}{dx} = \frac{y^2+y}{x^2+x}$	✗	✓	✓
(e) $xy\frac{dy}{dx} + y^2 = 2x$	✗	✗	✓
(f) $(3y + \ln x)dx + xdy = 0$	✓	✗	✗

2. For each of the equations in (1),

   If ***separable***, verify by setting up the integrals, $\int u(y)\,dy = \int v(x)\,dx$.

   If ***linear***, identify an integrating factor, multiply the DE by the integrating factor, and verify that the modified equation can be expressed in the form, $\frac{d}{dx}\Big(u(x)y\Big) = v(x)$.

   If ***Bernoulli***, determine an appropriate substitution, $w = w(y)$, such that the ODE for $w(x) = w(y(x))$ satisfies a linear ODE. Express the linear ODE for $w(x)$ in *standard* form.

3. Find an explicit solution to the initial value problem (IVP) and determine its existence interval.

$$\sin(x) + y\frac{dy}{dx} = 0, \qquad y(0) = 1$$

$\frac{dy}{dx}\, y = -\sin(x)$

$\int y\,dy = \int -\sin(x)\,dx$

$y^2 = 2\cos(x) - 1$

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$\frac{dy}{dx} y = -\sin(x)$

$\int y\,dy = \int -\sin(x)\,dx$

$\frac{y^2}{2} = \cos(x) + c$

$y^2 = 2\cos(x) + c$

$(1)^2 = 2\cos(0) + c$

$1 = 2 + c$

$c = -1$

$y^2 = 2\cos(x) - 1$

$y = 0$

$\frac{1}{2} = \cos(x)$

$x = \cos^{-1}(1/2)$

$x = \frac{\pi}{3}$

$y = \sqrt{2\cos(x) - 1}$

$(-\frac{\pi}{3}, \frac{\pi}{3})$

4. Solve the IVP and determine the largest interval on which the solution is defined. ☆

$$x(x+1)\frac{dy}{dx} + xy = 1, \qquad y(1) = 0$$

$\frac{dy}{dx} + \underbrace{\frac{1}{x+1}}_{P(x)} y = \underbrace{\frac{1}{x(x+1)}}_{f(x)}$ * Linear

Integrating factor: $\mu(x) = e^{\int P(x)\,dx} = e^{\int \frac{1}{x+1}dx} = |x+1|$

Multiply by $\mu(x)$

$y = \frac{1}{\mu(x)} \cdot \int \mu(x) f(x)\,dx$

$(x+1)\frac{dy}{dx} + y = \frac{1}{x}$

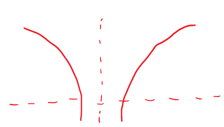
$x = 1 \quad y = 0$

$y = \frac{1}{x+1}(\ln|x| + c)$

$0 = \frac{0}{2} + \frac{c}{2}$ $c = 0$

$\boxed{y = \frac{\ln|x|}{x+1} = \frac{\ln x}{x+1} \quad I: (0, \infty)}$

$\ln|x|$



5. Solve for the explicit solution, $y = \phi(x)$, to the following IVP and determine its existence interval.

$$y^{1/2}\frac{dy}{dx} + y^{3/2} = 1, \quad y(0) = 4$$

$y^{1/2}\left(\frac{dy}{dx} + y\right) = 1$

$\frac{dy}{dx} + y = \frac{1}{\sqrt{y}}$

$\frac{dy}{dx} = \frac{1}{y^{1/2}} - y = \frac{1}{(y^{-1/2} - y)}dy = dx = \ln|y^{-1/2} - y| = x$

$\frac{-\frac{1}{2}\ln|y|}{\ln|y|} = x + c$

$c = \frac{-\frac{1}{2}\ln|4|}{\ln|4|}$

$c = -\frac{1}{2}$

$\ln|y^{-1/2} - y| = x - 1/2$

$-y^{-1/2} = e^{x - 1/2}$

$\ln|y^{-1/2}|$

$$\frac{\ln|y^{-1/2}|}{\ln|y|} = x \cdot \frac{1}{2}$$

$\ln|y| \neq 0$

$y \neq 0$

$(-\infty, \infty)$