- Q1: Read the below paper and write a summary (a minimum of half a page) to describe what you have learned from it. (20 points)

Derek P. Atherton, “Early Developments in Nonlinear Control,” Proceedings of 1994 33rd IEEE Conference on Decision and Control

The early developments in nonlinear control can be traced back to the 1940s and 1950s, when researchers began to explore the use of feedback control systems for nonlinear systems. These early efforts focused on developing control algorithms that could stabilize nonlinear systems and ensure their stability, as well as on understanding the fundamental principles and limitations of nonlinear control.One of the key early developments in nonlinear control was the concept of nonlinear feedback, which involves using feedback signals from the system to adjust the control inputs in a way that is appropriate for the nonlinear system. This approach allowed for the design of control algorithms that could adapt to the specific characteristics of the nonlinear system and ensure its stability. Another important early development was the concept of nonlinear observer design, which involves using measurements of the system's state to estimate its true state and provide information for the control algorithm. This allowed for the design of nonlinear control systems that could operate without perfect knowledge of the system's state, making them more practical and robust in real-world applications. While there remain limitations on the findings in this area, they have been applicable with modern day ventures including microprocessors, and their corresponding algorithms required to properly function. Overall, the early developments in nonlinear control laid the foundation for many of the techniques and methods that are used in modern control engineering, and they continue to be an active area of research and development.

- Q2: Consider the system $\dot{x} = -sinx$, prove that the origin is a stable equilibrium point (35 points).

$(0,0) \rightarrow$ origin

$\dot{x} = -\sin(x)$

$\frac{dx}{dt} = -\sin(x)$; $\frac{dx}{\sin(x)} = -dt = \csc(x)\,dx$

$\ln\left|\tan\left(\frac{x}{2}\right)\right| = -t + c$ $(c=0)$ at $(0,0)$

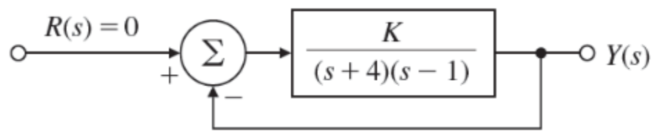
$\ln\left|\tan\left(\frac{x}{2}\right)\right| = -t$

$\tan\left(\frac{x}{2}\right) = e^{-t}$; $x = 2\left[\tan^{-1}\left(e^{-t}\right)\right]$

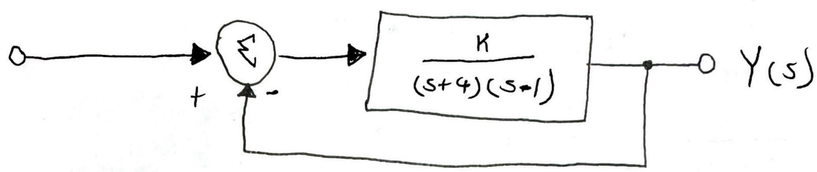
Assess for $t \rightarrow \infty$ (if $x=0$, origin is stable)

$x = 2\left[\tan^{-1}(0)\right] = 2(0) = 0$ $\therefore$ Stable Equilibrium Point.

- Q3: Using the Lyapunov equation $\boldsymbol{A}^T\boldsymbol{P} + \boldsymbol{P}\boldsymbol{A} = -\boldsymbol{Q} = -\boldsymbol{I}$, find the range of K for which the below system will be stable. Compare your results with the stable values for K obtained via Routh's stability criterion. (45 points)



$$A^T P + PA = -Q = -I \qquad P' = \begin{bmatrix} p & q \\ q & n \end{bmatrix}$$



$$\frac{Y(s)}{R(s)} = \frac{\dfrac{K}{(s+4)(s-1)}}{1 + \dfrac{K}{(s+4)(s-1)}} = \frac{K}{s^2 + 3s + (K-4)}$$

$$A = \begin{bmatrix} -a_{n-1} & -a_{n-2} \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} -a_1 & -a_0 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} -3 & (4-K) \\ 1 & 0 \end{bmatrix}$$

<u>Incorporate Lyapunov's Equation:</u>

$$\begin{bmatrix} -3 & (4-K) \\ 1 & 0 \end{bmatrix}^T \begin{bmatrix} p & q \\ q & n \end{bmatrix} + \begin{bmatrix} p & q \\ q & n \end{bmatrix}\begin{bmatrix} -3 & (4-K) \\ 1 & 0 \end{bmatrix} = -\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -3 & 1 \\ (4-K) & 0 \end{bmatrix}\begin{bmatrix} p & q \\ q & n \end{bmatrix} + \begin{bmatrix} p & q \\ q & n \end{bmatrix}\begin{bmatrix} -3 & (4-K) \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 2(q-3p) & p(4-K)+n-3q \\ p(4-K)+n-3q & 2q(4-K) \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$q = \frac{-1}{2(4-K)} \qquad 2\left(\frac{-1}{2(4-K)}\right) - 6p = -1; \quad p = \frac{1}{6}\left[\frac{3-K}{4-K}\right] \quad \therefore K>3 \text{ and } K>4$$

$$\frac{1}{6}\left[\frac{3-K}{4-K}\right](4-K) + n - 3\left[\frac{-1}{2(4-K)}\right] = 0; \quad n = \frac{-K^2+7K-21}{6(4-K)}$$

$$pn - q^2 = \left(\frac{1}{6}\left(\frac{3-K}{4-K}\right)\right)\left[\frac{-K^2+7K-21}{6(4-K)}\right] - \left(\frac{-1}{2(4-K)}\right)^2 > 0$$

$$= \frac{K^3 - 10K^2 + 42K - 72}{36(K-4)^2} > 0; \quad (K-4)(K^2-6K+18) > 0$$

$K>4$ satisfies

<u>Characteristic Equation:</u>

$$1 + G(s) = 1 + \frac{K}{(s+4)(s-1)} = 0; \quad s^2 + 3s - 4 + K = 0$$

s^n		
s^2	1	$(K-4)$
s^1	3	0
s^0	$\frac{3(K-4)}{3}$	0

$K - 4 > 0$

$$\boxed{\therefore \text{ For the system to be stable } K>4}$$