- Q1: Read the below paper and write a summary (a minimum of half a page) to describe what you have learned from it. (20 points)

Neculai Andrei, “Modern Control Theory - A historical perspective,”

We have made a lot of progress in the field of control systems, but this was not an overnight process. In fact, the earliest traces of recognition to the contents of control systems date back to the time of ancient Greece, where Aristotle mentioned the effectiveness of a self-controlling system, stating “if every instrument could accomplish its own work, obeying or anticipating the will of others, if the shuttle weaved and the pick touched the lyre without a hand to guide them, chief workmen would not need servants, nor masters slaves.” The necessity to create a system that operates on input feedback has shown to be of importance, but it wasn’t until much later on that it became more than just an idea. This was greatly due to the lack of a proper scientific method and mathematics that extended beyond simple algebra and linear systems. After the invention of Calculus by Issac Newton, as well as physical experimentation becoming more widespread thanks to Galileo, control systems began to become more attainable. This first led to the Frequency-Domain Approach, which emphasized the importance of a transfer function, and representing the input versus output response on a plane that suits its parameters. It was found that the transfer function is the Laplace transform of the system impulse response, and hosts the transfer characteristics of the system. Nyquist and Bode would later become involved with stability, and worked with the root locus method to model the system’s behavior. Optimal system behavior was explored by Bernoulli, but it wasn’t until Rudolf Kalman that nonlinear stability was explored in-depth. While state/space was heavily worked on in these theoretical areas, input/output was given attention again after the polynomial-matrix approach was incorporated, presenting a very general compensator able to achieve any desired closed loop transfer matrix.

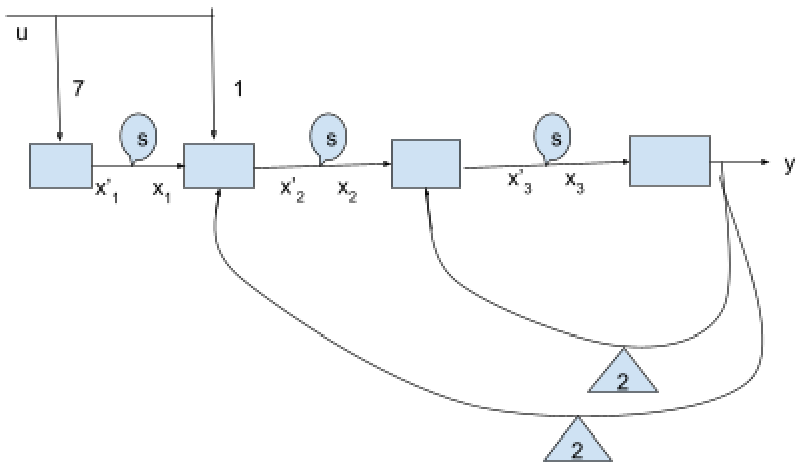
■ Q2: Derive the control, model, and observer canonical forms. (25 points)

$$G(s) = \frac{(s+7)}{s(s^2+2s+2)}$$

$$G(s) = \frac{Y(s)}{U(s)} = \frac{(s+7)}{s(s^2+2s+2)} = \frac{s+7}{s^3+2s^2+2s}$$

$$\frac{d^3y}{dt^3} + 2\frac{d^2y}{dt^2} + 2\frac{dy}{dt} = \frac{du}{dt} + 7u \qquad \frac{d^3t}{dt^3} = \frac{du}{dt} + 7u - 2\frac{d^2y}{dt^2} + 2\frac{dy}{dt}$$

$$\frac{d^3t}{dt^3} = \frac{d}{dt}[u - 2y] - 2\frac{d^2y}{dt^2} + 7u \qquad y = \int\int(u - 2y)dt - 2\int ydt + \int\int\int 7udt$$



$$x'_1 = 7u \qquad x'_2 = u - 2y + x_1 = u - 2(x_3) + x_1 \qquad x'_3 = -2y + x_2 = -2x_3 + x_2$$

<u>Observer Canonical:</u>

$$\begin{vmatrix} x'_1 \\ x'_2 \\ x'_3 \end{vmatrix} \quad \begin{vmatrix} 0 & 0 & 0 \\ 1 & 0 & -2 \\ 0 & 1 & -2 \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix} + \begin{vmatrix} 7 \\ 1 \\ 0 \end{vmatrix} u \qquad y = \begin{vmatrix} 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} x'_1 \\ x'_2 \\ x'_3 \end{vmatrix}$$

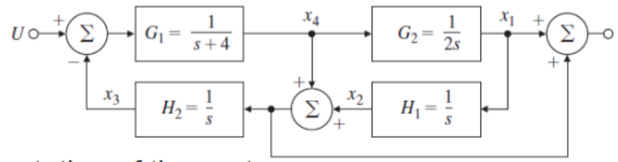
■ Q3: Convert the following state-space system into control canonical form: $\boldsymbol{A} = \begin{bmatrix} -2 & 1 \\ -2 & 0 \end{bmatrix}, \boldsymbol{B} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}, \boldsymbol{C} = [1 \quad 0], D = [0]$ (25 points)

$$Transfer\ function = \frac{b_3(C_3 s^3 + C_2 s^2 + C_1 s + C_0)}{s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0}$$

$$x' = \begin{vmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -a_0 & -a_1 & -a_2 & -a_3 \end{vmatrix} |x| + \begin{vmatrix} 0 \\ 0 \\ 0 \\ b_3 \end{vmatrix} u$$

$$Y = \begin{vmatrix} C_0 & C_1 & C_2 & C_3 \end{vmatrix} |x|$$

- Q4: For the block diagram shown below, (30 points)
  - (a) Find the transfer function from U to Y;
  - (b) Write the state-space equations (in matrix form) for the system using the state variables x1,x2,x3,x4 defined in the figure, i.e., $\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix} = [?]\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_n\end{bmatrix} + [?]U,\ y = [?]\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_n\end{bmatrix} + [?]U$



  - (c) Provide control canonical form presentation of the system.

(a.)

$Transfer\ Function = \frac{P_1D_1+P_2D_2}{D}$ $\quad P_1 = G_1G_2 \quad D_1 = 1 \quad P_2 = G_1 \quad D_2 = 1$

$D = 1 + G_1G_2H_1H_2 + G_1H_2$

$Transfer\ Function = \frac{G_1G_2+G_1}{1+G_1H_2+G_1G_2H_1H_2}$ $\quad G_1 = \frac{1}{s+4} \quad G_2 = \frac{1}{2s} \quad H_1 = \frac{1}{s} \quad H_2 = \frac{1}{s}$

$= \frac{(\frac{1}{s+4})(\frac{1}{2s})+\frac{1}{s+4}}{1+(\frac{1}{s+4})(\frac{1}{s})+\frac{1}{2s^3(s+4)}} = \frac{2s^2(1+2s)}{2(2s^4+8s^3+2s^2+1)}$

(b.)

$x_4 = 2x'_1 \quad x'_1 = \frac{x_4}{2} \quad x'_3 = x_2 + x_4 \quad x'_2 = x_1 \quad x'_4 = \text{-}4x_4 - x_3 + 4$

$x'_3 = x_2 + x_1 \quad x'_4 = \text{-}x_3 - 4x_4 + u \quad x' = Ax + Bu$

$A = |0\ \ 0\ \ 0\ \ \frac{1}{2}| \quad |0|$

$|1\ \ 0\ \ 0\ \ 0|\ |x| + |0|u$

$|0\ \ 1\ \ 0\ \ 1| \quad |0|$

$|0\ \ 0\ \text{-}1\ \ \text{-}4| \quad |1|$

$Y = |1\ \ 0\ \ 0\ \ 0|\ |x_1| \quad |1|$

$|x_2| + |0|u$

$|x_3| \quad |0|$

$|x_4| \quad |0|$

(c.)

<u>Control canonical:</u> $|B: AB: A^2B|$

$|0|\ |0\ \ 0\ \ 0\ \ \frac{1}{2}|\ |0| : |0\ \ 0\ \ 0\ \ \frac{1}{2}|^2\ |0|$

$|0|\ |1\ \ 0\ \ 0\ \ 0|\ |0| : |1\ \ 0\ \ 0\ \ 0|\ \ |0|$

$|0|\ |0\ \ 1\ \ 0\ \ 1|\ |0| : |0\ \ 1\ \ 0\ \ 1|\ \ |0|$

$|1|\ |0\ \ 0\ \text{-}1\ \ \text{-}4|\ |1| : |0\ \ 0\ \text{-}1\ \ \text{-}4|\ \ |1|$