Homework 5

Wednesday, October 13, 2021 12:14 PM

"I pledge my honor I have abided by the Stevens Honor system."

- Alex [signature]

7.2 Calculate the built-in potential barrier, V_{bi}, for Si, Ge, and GaAs pn junctions if they each have the following dopant concentrations at $T = 300$ K:

(a) $N_d = 10^{14}\ \text{cm}^{-3}$ $N_a = 10^{17}\ \text{cm}^{-3}$

(b) $N_d = 5 \times 10^{16}$ $N_a = 5 \times 10^{16}$

(c) $N_d = 10^{17}$ $N_a = 10^{17}$

$$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right)$$

a.)

Si:

$$V_{bi} = (0.0259) \ln\left(\frac{10^{14}(10^{17})}{(1.5\times10^{10})^2}\right) = \boxed{0.635\ \text{V.}}$$

Ge:

$$V_{bi} = (0.0259) \ln\left(\frac{10^{14}(10^{17})}{(2.4\times10^{13})^2}\right) = \boxed{0.253\ \text{V.}}$$

GaAs:

$$V_{bi} = (0.0259) \ln\left(\frac{10^{14}(10^{17})}{(1.8\times10^{6})^2}\right) = \boxed{1.10\ \text{V.}}$$

b.)

Si:

$$V_{bi} = (0.0259) \ln\left(\frac{(5\times10^{16})(5\times10^{16})}{(1.5\times10^{10})^2}\right) = \boxed{0.778\ \text{V.}}$$

Ge:

$$V_{bi} = (0.0259) \ln\left(\frac{(5\times10^{16})(5\times10^{16})}{(2.4\times10^{13})^2}\right) = \boxed{0.396\ \text{V.}}$$

GaAs:

$$V_{bi} = (0.0259) \ln\left(\frac{(5\times10^{16})(5\times10^{16})}{(1.8\times10^{6})^2}\right) = \boxed{1.25\ \text{V.}}$$

c.)

Si:

$$V_{bi} = (0.0259) \ln\left(\frac{10^{17}(10^{17})}{(1.5\times10^{10})^2}\right) = \boxed{0.814 \text{ V.}}$$

Ge:

$$V_{bi} = (0.0259) \ln\left(\frac{10^{17}(10^{17})}{(2.4\times10^{13})^2}\right) = \boxed{0.432 \text{ V.}}$$

GaAs:

$$V_{bi} = (0.0259) \ln\left(\frac{10^{17}(10^{17})}{(1.8\times10^{6})^2}\right) = \boxed{1.28 \text{ V.}}$$

7.10 Consider a uniformly doped silicon pn junction with doping concentrations $N_a = 2 \times 10^{17}$ cm^{-3} and $N_d = 4 \times 10^{16}$ cm^{-3}. (*a*) Determine V_{bi} at $T = 300$ K. (*b*) Determine the temperature at which V_{bi} increases by 2 percent. (Trial and error may have to be used.)

a.) $$V_{bi} = \frac{kT}{q} \ln\left(\frac{N_a N_d}{n_i^2}\right) = (0.0259) \ln\left(\frac{(2\times10^{17})(4\times10^{16})}{(1.5\times10^{10})^2}\right) = \boxed{0.807 \text{ V.}}$$

b.)

$V_{bi,} = 0.807$ increases by 2% $\quad 1.02(0.807) = 0.823$

$$31.202\left(\frac{kT}{q}\right) = 0.823$$

$$\boxed{T = 305.81 \text{ K}}$$

7.16 An abrupt silicon pn junction at $T = 300$ K has impurity doping concentrations of $N_a = 5 \times 10^{16}$ cm^{-3} and $N_d = 10^{15}$ cm^{-3}. Calculate (*a*) V_{bi}, (*b*) W at (*i*) $V_R = 0$ and (*ii*) $V_R = 5$ V, and (*c*) $|\text{E}_{\max}|$ at (*i*) $V_R = 0$ and (*ii*) $V_R = 5$.

a.) $$V_{bi} = \frac{kT}{q} \ln\left(\frac{N_a N_d}{n_i^2}\right) = (0.0259) \ln\left(\frac{(5\times10^{16})(10^{15})}{(1.5\times10^{10})^2}\right) = \boxed{0.676 \text{ V.}}$$

b.) [i] $$W = \left(\frac{2\varepsilon_r\varepsilon_0(V_{bi}+V_R)}{e}\left(\frac{N_a+N_d}{N_a N_d}\right)\right)^{\frac{1}{2}}$$

b.) [i] $W = \left(\frac{2 \varepsilon_r \varepsilon_0 (V_{bi} + V_R)}{e} \left(\frac{N_a + N_d}{N_a N_d} \right) \right)^{\frac{1}{2}}$

$= \left(\frac{2(11.7)(8.85 \times 10^{-12})(0.676 + 0)}{1.6 \times 10^{-19}} \left(\frac{(5 \times 10^{16}) + 10^{15}}{(5 \times 10^{16})(10^{15})} \right) \right)^{\frac{1}{2}}$

$= 9.45 \times 10^{-7} = \boxed{0.945\ \mu m}$

[ii] $W = \left(\frac{2 \varepsilon_r \varepsilon_0 (V_{bi} + V_R)}{e} \left(\frac{N_a + N_d}{N_a N_d} \right) \right)^{\frac{1}{2}}$

$= \left(\frac{2(11.7)(8.85 \times 10^{-12})(0.676 + 5)}{1.6 \times 10^{-19}} \left(\frac{(5 \times 10^{16}) + 10^{15}}{(5 \times 10^{16})(10^{15})} \right) \right)^{\frac{1}{2}}$

$= 2.738 \times 10^{-6} = \boxed{2.738\ \mu m}$

c.) [i] $|E_{max}| = \frac{2(V_{bi} + V_R)}{W}$

$|E_{max}| = \frac{2(0.676 + 0)}{9.45 \times 10^{-7}}$

$|E_{max}| = \boxed{1.43 \times 10^{6}\ V/m}$

[ii] $|E_{max}| = \frac{2(V_{bi} + V_R)}{W}$

$|E_{max}| = \frac{2(0.676 + 5)}{2.738 \times 10^{-6}}$

$$|E_{max}| = \boxed{4.15\times10^{6}\ V/m}$$

7.20 (*a*) The peak electric field in a reverse-biased silicon pn junction is $|E_{max}| = 3 \times 10^5$ V/cm. The doping concentrations are $N_d = 4 \times 10^{15}$ cm^{-3} and $N_a = 4 \times 10^{17}$ cm^{-3}. Find the magnitude of the reverse-biased voltage. (*b*) Repeat part (*a*) for $N_d = 4 \times 10^{16}$ cm^{-3} and $N_a = 4 \times 10^{17}$ cm^{-3}. (*c*) Repeat part (*a*) for $N_d = N_a = 4 \times 10^{17}$ cm^{-3}.

a.) $V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right) = (0.0259)\ln\left(\frac{(4\times10^{17})(4\times10^{15})}{(1.5\times10^{10})^2}\right) = \boxed{0.766\ V.}$

$$|E_{max}| = \left(\frac{2e(V_{bi}+V_R)}{\varepsilon_s}\left(\frac{N_a N_d}{N_a+N_d}\right)\right)^{\frac{1}{2}}$$

$$V_R = \frac{\varepsilon_s}{2e}\left(\frac{N_a+N_d}{N_a N_d}\right)E_{max}^2 - V_{bi} = \frac{(11.7(8.85\times10^{-12}))}{2(1.6\times10^{-19})}\left(\frac{(4\times10^{17})+(4\times10^{15})}{(4\times10^{17})(4\times10^{15})}\right)(3\times10^5)^2 - 0.766$$

$V_R = \boxed{72.8\ V.}$

b.) $V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right) = (0.0259)\ln\left(\frac{(4\times10^{17})(4\times10^{16})}{(1.5\times10^{10})^2}\right) = \boxed{0.826\ V.}$

$$|E_{max}| = \left(\frac{2e(V_{bi}+V_R)}{\varepsilon_s}\left(\frac{N_a N_d}{N_a+N_d}\right)\right)^{\frac{1}{2}}$$

$$V_R = \frac{\varepsilon_s}{2e}\left(\frac{N_a+N_d}{N_a N_d}\right)E_{max}^2 - V_{bi} = \frac{(11.7(8.85\times10^{-12}))}{2(1.6\times10^{-19})}\left(\frac{(4\times10^{17})+(4\times10^{16})}{(4\times10^{17})(4\times10^{16})}\right)(3\times10^5)^2 - 0.826$$

$V_R = \boxed{7.18\ V.}$

c.) $V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right) = (0.0259)\ln\left(\frac{(4\times10^{17})(4\times10^{17})}{(1.5\times10^{10})^2}\right) = \boxed{0.886\ V.}$

$$|E_{max}| = \left(\frac{2e(V_{bi}+V_R)}{\varepsilon_s}\left(\frac{N_a N_d}{N_a+N_d}\right)\right)^{\frac{1}{2}}$$

$$V_R = \frac{\varepsilon_s}{2e}\left(\frac{N_a+N_d}{N_a N_d}\right)E_{max}^2 - V_{bi} = \frac{(11.7(8.85\times10^{-12}))}{2(1.6\times10^{-19})}\left(\frac{(4\times10^{17})+(4\times10^{17})}{(4\times10^{17})(4\times10^{17})}\right)(3\times10^5)^2 - 0.886$$

$V_R = \boxed{0.57\ V.}$

7.30 A silicon p⁺n junction has doping concentrations of $N_a = 2 \times 10^{17}$ cm^{-3} and $N_d = 2 \times 10^{15}$ cm^{-3}. The cross-sectional area is 10^{-5} cm^2. Calculate (*a*) V_{bi} and (*b*) the junction capacitance at (*i*) $V_R = 1$ V, (*ii*) $V_R = 3$ V, and (*iii*) $V_R = 5$ V. (*c*) Plot $1/C^2$ versus V_R and show that the slope can be used to find N_d and the intercept at the voltage axis yields V_{bi}.

a.) $V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right) = (0.0259) \ln\left(\frac{(2\times10^{17})(2\times10^{15})}{(1.5\times10^{10})^2}\right)$

$V_{bi} = \boxed{0.73 \text{ V.}}$

b.) $C_{j_0} = A\sqrt{\left(\frac{\varepsilon_s q}{2}\right)\left(\frac{N_a N_d}{N_a + N_d}\right)\frac{1}{V_{bi}}} = (10^{-5})\sqrt{\left(\frac{(1.04\times10^{-12})(1.6\times10^{-19})}{2(0.73)}\right)\left(\frac{(2\times10^{17})(2\times10^{15})}{(2\times10^{17})+(2\times10^{15})}\right)}$

$C_{jo} = 0.1502\,pF$

[i]

$\frac{C_{jo}}{C_j} = \sqrt{\frac{V_{bi}+V_R}{V_{bi}}} = \sqrt{\frac{0.73+1}{0.73}} = 1.538 \qquad C_j = \frac{C_{jo}}{1.538} = \boxed{97.713 \text{ fF}}$

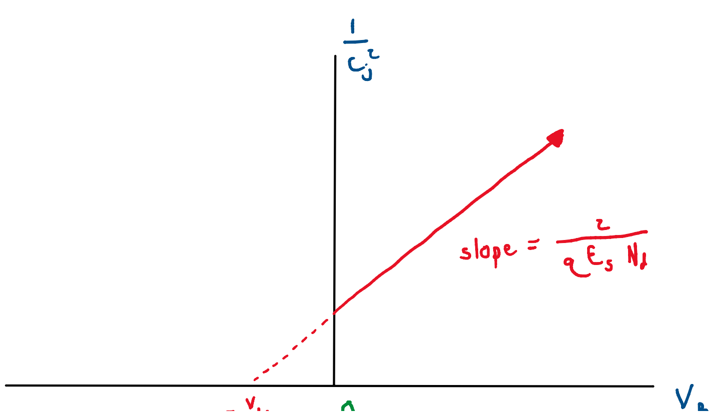
[ii]

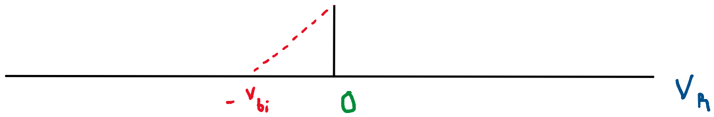
$\frac{C_{jo}}{C_j} = \sqrt{\frac{V_{bi}+V_R}{V_{bi}}} = \sqrt{\frac{0.73+3}{0.73}} = 2.257 \qquad C_j = \frac{C_{jo}}{2.257} = \boxed{66.585 \text{ fF}}$

[iii]

$\frac{C_{jo}}{C_j} = \sqrt{\frac{V_{bi}+V_R}{V_{bi}}} = \sqrt{\frac{0.73+5}{0.73}} = 2.797 \qquad C_j = \frac{C_{jo}}{2.797} = \boxed{53.76 \text{ fF}}$

c.)





7.38 (*a*) A symmetrically doped silicon pn junction diode has doping concentrations of $N_a = N_d = 2 \times 10^{16}$ cm^{-3}. Assuming the critical electric field is $E_{crit} = 4 \times 10^5$ V/cm, determine the breakdown voltage. (*b*) Repeat part (*a*) if the doping concentrations are $N_a = N_d = 5 \times 10^{15}$ cm^{-3}.

a.) $$V_{bi} = \frac{kT}{q} \ln\left(\frac{N_a N_d}{n_i^2}\right) = (0.0259) \ln\left(\frac{(2\times10^{16})(2\times10^{16})}{(1.5\times10^{10})^2}\right) = 0.73 \text{ V.}$$

$$|E_{max}| = \left(\frac{2e(V_{bi}+V_B)}{\epsilon_s}\left(\frac{N_a N_d}{N_a + N_d}\right)\right)^{\frac{1}{2}}$$

$$4\times10^5 = \left(\frac{2(1.6\times10^{-19})(V_{bi}+V_B)}{(11.7)(8.85\times10^{-12})}\left(\frac{(2\times10^{16})(2\times10^{16})}{(2\times10^{16})+(2\times10^{16})}\right)\right)^{\frac{1}{2}}$$

$$V_{bi} + V_B = 51.77$$

$$V_B = 51.77 - 0.73 = \boxed{51.04 \text{ V.}}$$

b.) $$V_{bi} = \frac{kT}{q} \ln\left(\frac{N_a N_d}{n_i^2}\right) = (0.0259) \ln\left(\frac{(5\times10^{15})(5\times10^{15})}{(1.5\times10^{10})^2}\right) = 0.659 \text{ V.}$$

$$|E_{max}| = \left(\frac{2e(V_{bi}+V_B)}{\epsilon_s}\left(\frac{N_a N_d}{N_a + N_d}\right)\right)^{\frac{1}{2}}$$

$$4\times10^5 = \left(\frac{2(1.6\times10^{-19})(V_{bi}+V_B)}{(11.7)(8.85\times10^{-12})}\left(\frac{(5\times10^{15})(5\times10^{15})}{(5\times10^{15})+(5\times10^{15})}\right)\right)^{\frac{1}{2}}$$

$$V_{bi} + V_B = 207.1$$

$$V_B = 207.1 - 0.659 = \boxed{206.44 \text{ V.}}$$