Homework 25-27

Tuesday, April 20, 2021 5:30 PM

"I pledge my honor I have abided by the Stevens Honor system."

\- Alex Gaskins

Problem 25.1. Draw a graph Γ satisfying the following conditions or proof that such a graph does not exist:

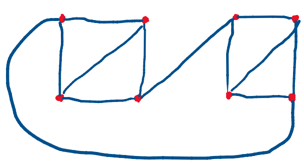
a) *(3 points)* Γ is a simple graph on 7 vertices for which each vertex has degree 3.
b) *(3 points)* Γ is a **simple connected** graph on 8 vertices for which each vertex has degree 3.
c) *(3 points)* Γ is a **simple non-connected** graph on 8 vertices for which each vertex has degree 4.
d) *(5 points)* Γ has 9 vertices, has an Euler cycle but no Hamiltonian cycle.
e) *(5 points)* Γ has 9 vertices, has a Hamiltonian cycle but no Euler cycle.
f) *(3 points)* Γ has 6 vertices, is connected, but has no circuits of length more than 1.

A.) Degree sum = 7(3) = 21

For a simple graph, degree sum must be even.

Thus, this graph does not exist

B.)



C.) Simple non-connected graph.

For degree 4, 5 verticies are in one component with remaining 3 verticies separate subset.

3 verticies are non-connected, and simple graph, but for a simple graph the maximum degree with three or less verticies is 2.

Each vertex cannot have a degree of four.

Thus, this graph is not possible.

D.)

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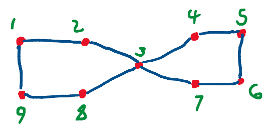
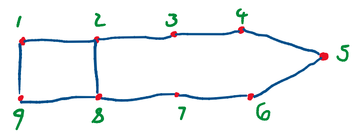


1, 2, 3, 4, 5, 6, 7, 8, 9, 1 form a Euler cycle with no edge overlap.

No hamiltonian cycle because 3 is repeated.

E.)



1, 2, 3, 4, 5, 6, 7, 8, 9, 1 form a hamiltonian cycle with no vertex repetition.

Euler cycle requires each degree to have an even vertex, but 2 and 8 have a degree of three so there is no Euler cycle.

F.)

Tree with 6 verticies makes a connected and acyclic graph.

Problem P26.1. Let Γ and Δ be the graphs below.

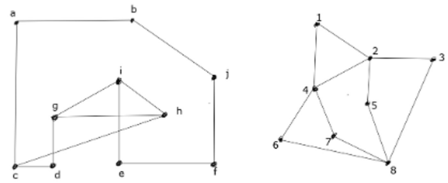
a) *(5 points)* Does Γ have an Euler circuit?

b) *(5 points)* Does Δ have an Euler circuit?

c) *(5 points)* Does Γ have a Hamiltonian circuit?

d) *(5 points)* Does Δ have a Hamiltonian circuit?

Explain your answers: if you answer yes, provide an example of a corresponding circuit; if you answer no, prove that there is no such circuit.



Γ Δ

A.) c, i, g, h have odd degrees, which means an Euler circuit is not possible.

B.) Because all degrees are even, there is an Euler Circuit.

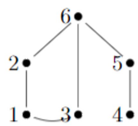
1, 4, 2, 5, 8, 7, 4, 6, 8, 3, 2, 1 forms an Euler Circuit.

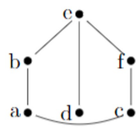
C.) a, b, j, f, e, i, h, g, d, c, a forms a Hamiltonian circuit where no verticies are repeated.

D.) Assume for a graph with a Hamiltonian circuit each vertex has a degree of 2, but degree of 2 = 4 as well as 4 and 8. Assume 2, 4, 8 are removed, there is a conflict because if 2, 5, 4 or 2, 3, 4 or 2, 1, 4 then it makes the degree of 5, 3, or 1 equal to one, which means it is impossible to make the degree of 2 equal to two.

Thus, Δ has no Hamiltonian Circuit.

Problem P27.1. *(10 points)*

Prove that the graphs 

and 

are not isomorphic.

Assuming the graphs are isomorphic, 4 would correspond to d (They are both leap verticies).

But 4 is adjacent to 5 which has a degree of 2, while d is adjacent to e which has a degree of 3.

Thus the map 5 → e is not possible.

Because of this contradiction, the graphs are not isomorphic.