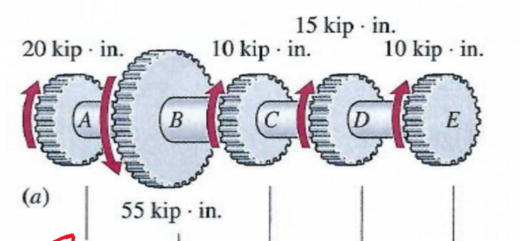
Homework 13

Monday, December 7, 2020 12:00 PM

*"I pledge my honor I have abided by the Stevens Honor system."*

- Alex Gaskins

**Problem 1:** A steel shaft is used to transmit torque from a motor to operating units at different points. The torque is input at gear *B* and is removed at gears *A*, *C*, *D*, and *E*. Determine the torques transmitted by shaft cross sections (resisting torques) in the intervals *AB*, *BC*, *CD*, and *DE* of the shaft.




A-B:

$\Sigma M_{AB} = 0$

$T_{AB} - 20 = 0$

$\boxed{T_{AB} = 20 \text{ kip}\cdot\text{in}}$

A-C:

$\Sigma M_{AC} = 0$

$T_{BC} - 20 + 50 = 0$

$\boxed{T_{BC} = -35 \text{ kip}\cdot\text{in}}$

A-D:

$\Sigma M_{AD} = 0$

$T_{CD} - 20 + 55 - 10 = 0$

$\boxed{T_{CD} = -25 \text{ kip}\cdot\text{in}}$

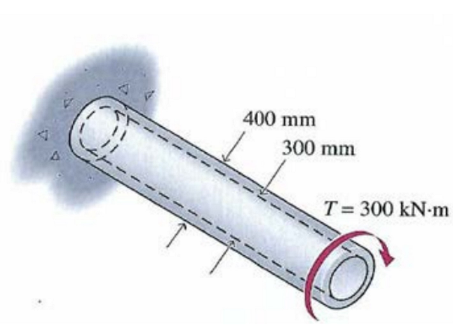
A-E:

$\Sigma M_{AE} = 0$

$T_{DE} - 20 + 55 - 10 - 15 = 0$

$\boxed{T_{DE} = -10 \text{ kip}\cdot\text{in}}$

**Problem 2:** A hollow steel shaft with an outside diameter of 400 mm and an inside diameter of 300 mm is subjected to a torque of 300 kN-m as shown. The modulus of rigidity (G) of steel is 80 GPa. Determine: (a) The maximum shearing stress in the shaft. (b) The shearing stress on a transverse cross section at the inside surface of the shaft. (c) The magnitude of the angle of twist in a 2-meter length.




$J = \frac{\pi}{32}(400^4 - 300^4)$

$J = 1718058482$

a.)

$\tau_{max} = \frac{T r_{outter}}{J}$

$\tau_{max} = (300 \times 10^3)\left(\frac{400}{2} \times 10^3\right)$

$$\tau_{max} = \frac{(300\times10^3)\left(\frac{400}{2}\times10^3\right)}{1718058482}$$

$$\boxed{\tau_{max} = 34.923 \ \frac{N}{mm^2}}$$

c.) $\theta = \frac{TL}{GJ}$

$$\theta = \frac{(300\times10^6)(2(1000)(1000)^2)}{(80\times10^9)(1718058482)}$$

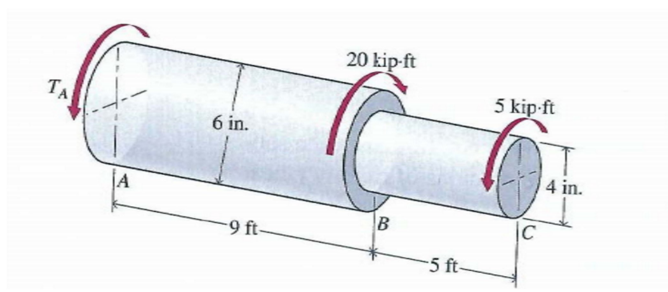
$$\boxed{\theta = 4.36\times10^{-3} \ \text{rads.}}$$

b.) $\tau = \frac{T r_{inside}}{GJ}$

$$\tau = \frac{(300\times10^3)\left(\frac{300}{2}\times10^3\right)}{(80\times10^9)(1718058482)}$$

$$\boxed{\tau = 26.192 \ \frac{N}{mm^2}}$$

**Problem 3:** A solid steel shaft 14 feet long has a diameter of 6 in. for 9 feet of its length and diameter of 4 in. inch for the remaining 5 feet. The shaft is in equilibrium when subjected to three torques as shown. The modulus of rigidity (G) is 12,000 ksi. Determine: (a) The maximum shearing stress in the shaft. (b) The rotation of end *B* of the 6-in segment with respect to end *A*. (c) The rotation of end *C* of the 4-in segment with respect to end *B*. The rotation of end *C* with respect to end *A*.




a.) $J_1 = \frac{\pi}{32}(6)^4$

$J_1 = 127.234 \ in.^4$

$\tau_1 = \frac{Tr}{J_1}$

$$\tau_1 = \frac{(15(12))(3)}{127.234}$$

$\tau_1 = 4.24 \ ksi$

$\frac{\pi}{\ }(4)^4$

b.) $\phi_{AB} = \frac{T_1 L_1}{G_1 J_1}$

$$\phi_{AB} = \frac{(-15)(9)(12)(12)}{(12000)(127.23)}$$

$\phi_{AB} = -.0127 \ rad.$

$$(12000)(127.23)$$

$$J_2 = \frac{\pi}{32}(4)^4$$

$$\boxed{\phi_{AB} = -.0127 \text{ rad.}}$$

Clockwise

$$J_2 = 25.133 \text{ in.}^4$$

$$\text{c.)} \quad \phi_{BC} = \frac{(5)(12)(5)(12)}{(12000)(25.133)}$$

$$\tau_2 = \frac{Tr}{J_2}$$

$$\boxed{\phi_{BC} = .01134 \text{ rad.}}$$

$$\tau_2 = \frac{(5(12))(2)}{25.133}$$

$$\boxed{\tau_2 = 4.775 \text{ ksi}}$$

$\tau_{max}$

$$\text{d.)} \quad \phi_{AC} = \phi_{AB} + \phi_{BC}$$

$$\phi_{AC} = (-.0127) + (.01134)$$

$$\boxed{\phi_{AC} = .00076 \text{ rad.}}$$

Counter clockwise

**Problem 4:** A solid circular shaft with diameters of 2.5 inch and 1.75 inch is subjected to a torque $T$ as shown. The shearing stress is limited to 8000 psi and the angle of twist in the 7-ft length cannot exceed 0.04 radians. Determine the maximum permissible value of $T$. Given $G$ = 4000 ksi.

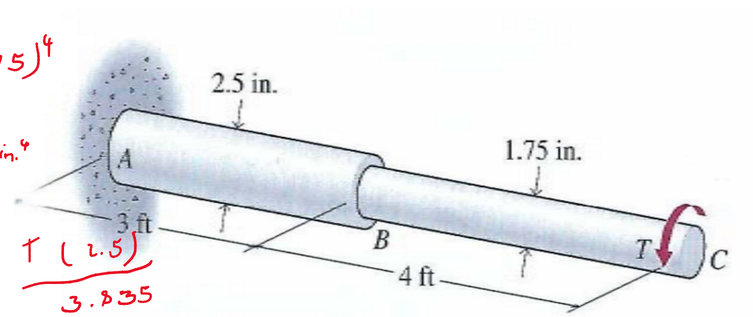
$$J_1 = \frac{\pi}{32} d^4 \qquad J_2 = \frac{\pi}{32} d^4$$

$$J_2 = \frac{\pi}{32}(1.75)^4$$

$$J_1 = \frac{\pi}{32}(2.5)^4$$

$$J_2 = .9208 \text{ in.}^4$$

$$J_1 = 3.835 \text{ in.}^4$$




$$8000 = \frac{T(2.5)}{3.835}$$

$$T = 24544 \text{ lb}\cdot\text{in}$$

$$\tau_{max} = \tau_{BC} = \frac{Tr}{J}$$

$$\tau_{max} = \frac{Tr}{J}$$

$$\theta = \sum \frac{TL}{GJ}$$

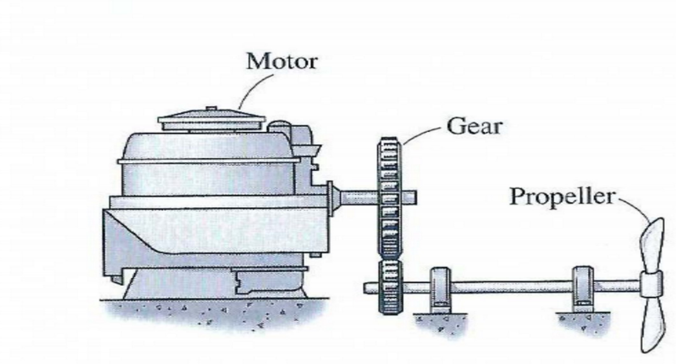
$$.04 = \frac{T}{4000 \times 10^3}\left(\frac{3(12)}{3.835} + \frac{4(12)}{.9208}\right)$$

$$8000 = \frac{T\left(\frac{1.75}{2}\right)}{.9208}$$

$$\boxed{T = 2601.44 \text{ lb}\cdot\text{in}}$$

$$T = 8419 \text{ lb}\cdot\text{in}$$

**Problem 5:** A diesel engine for a small boat is to operate at 200 rpm and deliver 800 horsepower through a gearbox with the ratio of 4:1 to the propeller shaft as shown. Both the shaft from the engine to the gearbox and the proper shaft are to be solid and made of steel. Find the minimum permissible diameter for the two shafts. Given: Maximum allowable Shearing Stress is 20 ksi, and the angle of twist for the 10-feet length should not exceed 4 degrees. Neglect power loss in the gearbox. Hint: Use P = Tω, and one hp equals 550 ft-lb per second.




$P = T\omega$

$800(33{,}000) = T_1(200)(2\pi)$

$T_1 = \frac{66000}{\pi}$ lb·ft

$J = \frac{\pi}{2} r_1^4 \; ; \quad \frac{J}{r_1} = \frac{T}{r'}$

$\frac{\pi}{2} r_1^3 = \frac{T}{r'}$

$r_1^3 = \frac{2}{\pi} \frac{(66000(12))}{(20 \times 10^3)}$

$r_1 = \sqrt[3]{8.024}$

$r_1 = 2.002$ in.

$\frac{r_2^3}{r_1^3} = \frac{4}{1}$

$r_2^3 = r_1^3(4)$

$r_2 = \sqrt[3]{(2.002)^3(4)}$

$r_2 = 3.1779$ in.

$\phi = \frac{TL}{JG}$

$4\left(\frac{\pi}{180}\right) = \frac{\left(66000\left(\frac{12}{4\pi}\right)\right)(10)(12)}{(12 \times 10^6)\left(\frac{\pi r_3^4}{2}\right)}$

$r_3^4 = \frac{(66000(12))(10)(12)}{(12 \times 10^6)(4\pi)\left(4\left(\frac{\pi}{180}\right)\right)\left(\frac{\pi}{2}\right)}$

$r_3 = \sqrt[4]{5.7422}$

$r_3 = 1.348$

$1.348 < 3.1779$

$d = 2(r)$

$d = 2(3.1779)$

$\boxed{d = 6.3538 \text{ in.}}$