4.1 2-D DFT and DCT are separable, and can be implemented through 1-D DFT or DCT along horizontal and vertical directional separately. For the following 2×2 image block

$$\begin{bmatrix} \mathbf{1} & \mathbf{2} \\ \mathbf{3} & \mathbf{4} \end{bmatrix}$$

4.1.1 Calculate its 2×2 DFT

$$\omega_r^{kn} = e^{-\frac{j2\pi}{N}kn} = e^{-j\pi kn}$$

$$n \in [0,1] \qquad k \in [0,1]$$

$$\begin{matrix} & 0 & 1 \\ 0 & \omega_2^{0} & \omega_2^{0} \\ 1 & \omega_2^{0} & \omega_2^{1} \end{matrix} = \begin{bmatrix} e^{0} & e^{0} \\ e^{0} & e^{-jn} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$dft = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \times \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 4 & 6 \\ -2 & -2 \end{bmatrix} \times \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 10 & -2 \\ -4 & 0 \end{bmatrix}$$

4.1.2 Calculate its 2×2 DCT

$$c(k,n) = \begin{cases} \frac{1}{\sqrt{N}}, & k = 0,\ 0 \le n \le N-1 \\ \sqrt{\frac{2}{N}}\cos\frac{\pi(2n+1)k}{2N}, & 1 \le k \le N-1,\ 0 \le n \le N-1 \end{cases}$$

$$c(0,0) = c(0,1) = \frac{1}{\sqrt{N}} = \frac{1}{\sqrt{2}}$$

$$c(1,0) = \sqrt{\frac{2}{2}}\cos\left[\frac{-(2(0)+1)n(1)}{2(2)}\right] = \frac{1}{\sqrt{2}}$$

$$c(1,1) = \sqrt{\frac{2}{2}}\cos\left[\frac{(2(1)+1)n(1)}{2(2)}\right] = \frac{-1}{\sqrt{2}}$$

$$dct = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$$

$$dft = \begin{bmatrix} 5 & -1 \\ -2 & 0 \end{bmatrix}$$

4.2 A 512x512 gray scale image contains a sinusoidal pattern as shown below.

4.2.1 Formulate this image, i.e. provide analytical expression as a 2-D discrete time function. (Hint: similar to page 14 in CpE462-5.ppt).

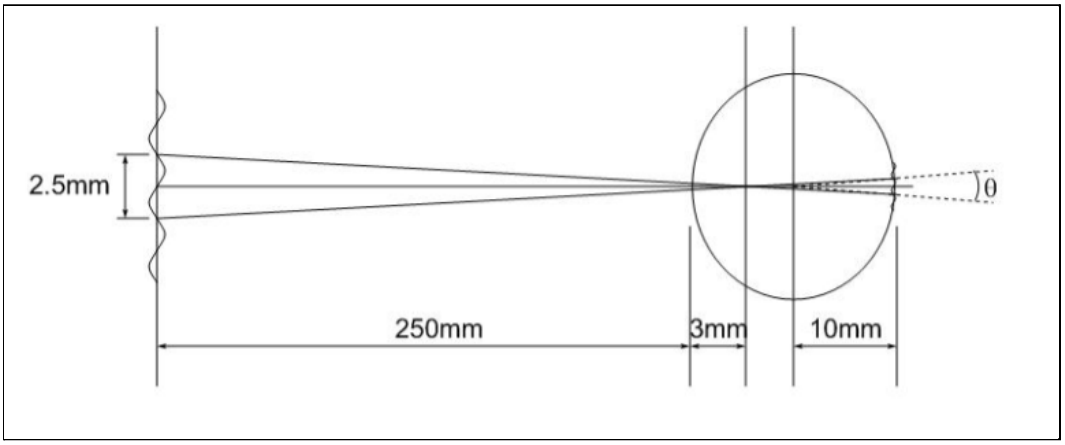
$T = \frac{256}{7} = 36.57$

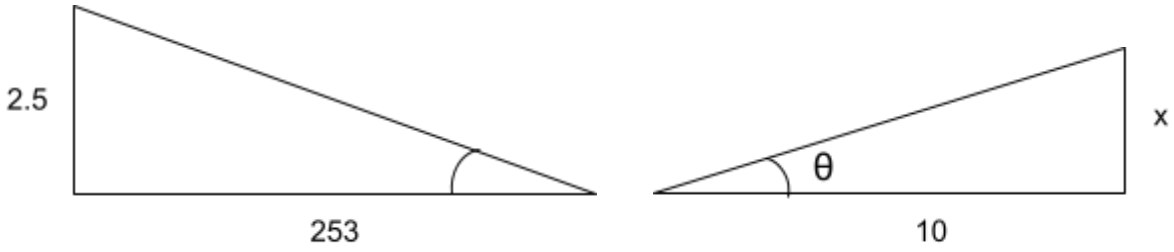
$F = \frac{1}{36.57}$

$f = \frac{2\pi}{T} = \frac{2\pi}{36.57}$

$x[n_1] = Acos(\frac{n_1 2\pi}{36.57})$

4.2.2 Assume the vertical period of the sinusoid is 2.5 mm, i.e. 2.5 mm per cycle, and the image is directly placed in front of the eye at 250 mm away, and the eye is 20 mm in diameter with a focal point of 3 mm from the lens, calculate the period of its projection on retina in distance (mm) and in degree (°).





$tan\theta = \frac{2.5}{253}$ $\theta = 0.566$

$tan\theta = \frac{x}{10}$

$tan(0.566) = \frac{x}{10}$

$10tan(0.566) = 6.3539\,mm$