Homework 6

Monday, October 18, 2021 11:52 AM

"I pledge my honor I have abided by the Stevens Honor system."

- Alex [signature]

8.2 A silicon pn junction has impurity doping concentrations of $N_d = 2 \times 10^{15}$ cm^{-3} and $N_a = 8 \times 10^{15}$ cm^{-3}. Determine the minority carrier concentrations at the edges of the space charge region for (*a*) $V_a = 0.45$ V, (*b*) $V_a = 0.55$ V, and (*c*) $V_a = -0.55$ V.

$$p_0 = \frac{n_i^2}{N_d} = \frac{(1.5\times10^{10})^2}{2\times10^{15}} = 1.125\times10^{5}/cm^3$$

$$n_0 = \frac{n_i^2}{N_a} = \frac{(1.5\times10^{10})^2}{8\times10^{15}} = 2.8125\times10^{4}/cm^3$$

a.) $$p_n(x_n) = p_0\left[\exp\left(\frac{eV_a}{kT}\right)\right] = (1.125\times10^5)\exp\left(\frac{0.45}{0.0259}\right)$$

$$\boxed{p_n(x_n) = 3.95\times10^{12}/cm^3}$$

$$n_p(-x_p) = n_0\left[\exp\left(\frac{eV_a}{kT}\right)\right] = (2.8125\times10^4)\exp\left(\frac{0.45}{0.0259}\right)$$

$$\boxed{n_p(-x_p) = 9.88\times10^{11}/cm^3}$$

b.) $$p_n(x_n) = p_0\left[\exp\left(\frac{eV_a}{kT}\right)\right] = (1.125\times10^5)\exp\left(\frac{0.55}{0.0259}\right)$$

$$\boxed{p_n(x_n) = 1.88\times10^{14}/cm^3}$$

$$n_p(-x_p) = n_0\left[\exp\left(\frac{eV_a}{kT}\right)\right] = (2.8125\times10^4)\exp\left(\frac{0.55}{0.0259}\right)$$

$$\boxed{n_p(-x_p) = 4.69 \times 10^{13} / cm^3}$$

c.) $p_n(x_n) = p_o\left[\exp\left(\frac{eV_a}{kT}\right)\right] = (1.125 \times 10^5)\exp\left(\frac{-0.55}{0.0259}\right)$

$$\boxed{p_n(x_n) = 6.74 \times 10^{-6} / cm^3}$$

$n_p(-x_p) = n_o\left[\exp\left(\frac{eV_a}{kT}\right)\right] = (2.8125 \times 10^4)\exp\left(\frac{-0.55}{0.0259}\right)$

$$\boxed{n_p(-x_p) = 0 / cm^3}$$

8.4 (*a*) The doping concentrations in a silicon pn junction are $N_d = 5 \times 10^{15}$ cm^{-3} and $N_a = 5 \times 10^{16}$ cm^{-3}. The minority carrier concentration at either space charge edge is to be no larger than 10 percent of the respective majority carrier concentration. (*i*) Determine the maximum forward-bias voltage that can be applied to the junction and still meet the required specifications. (*ii*) Is the n-region or p-region concentration the factor that limits the forward-bias voltage? (*b*) Repeat part (*a*) if the doping concentrations are $N_d = 3 \times 10^{16}$ cm^{-3} and $N_a = 7 \times 10^{15}$ cm^{-3}.

a.) $n_{p_0} = \frac{n_i^2}{N_a} = \frac{(1.5 \times 10^{10})^2}{5 \times 10^{16}} = 4.5 \times 10^3 / cm^3$

$p_{n_0} = \frac{n_i^2}{N_d} = \frac{(1.5 \times 10^{10})^2}{5 \times 10^{15}} = 4.5 \times 10^4 / cm^3$

[i] $p_n = p_{n_0}\exp\left(\frac{V_a}{V_t}\right)$; $V_a = V_t \ln\left(\frac{p_n}{p_{n0}}\right)$

$p_n = \frac{1}{10}(5 \times 10^{15}) = 0.5 \times 10^{15} / cm^3$

$V_a = (0.0259)\ln\left(\frac{0.5 \times 10^{15}}{4.5 \times 10^4}\right) = \boxed{0.599 \text{ V.}}$

[ii] n-region

b.) $n_{p_0} = \frac{n_i^2}{N_a} = \frac{(1.5 \times 10^{10})^2}{7 \times 10^{15}} = 3.214 \times 10^4 / cm^3$

b.)

$$n_{p_0} = \frac{n_i^2}{N_a} = \frac{(1.5\times10^{10})^2}{7\times10^{15}} = 3.214\times10^{4}\ /cm^3$$

$$p_{n_0} = \frac{n_i^2}{N_d} = \frac{(1.5\times10^{10})^2}{3\times10^{16}} = 7.5\times10^{3}\ /cm^3$$

[i] $p_n = p_{n_0}\exp\left(\frac{V_a}{V_t}\right)$; $V_a = V_t \ln\left(\frac{p_n}{p_{n0}}\right)$

$$n_p = \frac{1}{10}(7\times10^{15}) = 0.7\times10^{15}\ /cm^3$$

$$V_a = (0.0259)\ln\left(\frac{0.7\times10^{15}}{3.214\times10^{4}}\right) = \boxed{0.6165\ V.}$$

[ii] p-region

8.16 Consider an ideal silicon pn junction diode with the geometry shown in Figure P8.16. The doping concentrations are $N_a = 5 \times 10^{16}\ \text{cm}^{-3}$ and $N_d = 1.5 \times 10^{16}\ \text{cm}^{-3}$, and the minority carrier lifetimes are $\tau_{n0} = 2 \times 10^{-7}$ s and $\tau_{p0} = 8 \times 10^{-8}$ s. The cross-sectional area is $A = 5 \times 10^{-4}\ \text{cm}^2$. Calculate (*a*) the ideal reverse-saturation current due to holes, (*b*) the ideal reverse-saturation current due to electrons, (*c*) the hole concentration at $x = x_n$ for $V_a = 0.8V_{bi}$, (*d*) the electron current at $x = x_n$ for $V_a = 0.8\,V_{bi}$, and (*e*) the electron current at $x = x_n + (1/2)L_p$ for $V_a = 0.8\,V_{bi}$.

a.)

$$I_h = eA\sqrt{\frac{D_h}{\tau_{p0}}}\left(\frac{n^2}{N_d}\right) = (1.6\times10^{-19})(5\times10^{-4})\sqrt{\frac{10}{(8\times10^{-8})}}\left(\frac{(1.5\times10^{10})^2}{1.5\times10^{16}}\right)$$

$$\boxed{I_h = 1.342\times10^{-14}\ A.}$$

b.)

$$I_e = eA\sqrt{\frac{D_e}{\tau_{n0}}}\left(\frac{n^2}{N_a}\right) = (1.6\times10^{-19})(5\times10^{-4})\sqrt{\frac{25}{(2\times10^{-7})}}\left(\frac{(1.5\times10^{10})^2}{5\times10^{16}}\right)$$

$$\boxed{I_e = 4.025\times10^{-15}\ A.}$$

c.)

$$V = (0.0259)\ln\left(\frac{N_a N_d}{n^2}\right) = (0.0259)\ln\left(\frac{(5\times10^{16})(1.5\times10^{16})}{(1.5\times10^{10})^2}\right) = 0.7468\ V.$$

$$V' = (0.8)(0.7468) = 0.5975\ V.$$

$$p(x) = \frac{n_i^2}{N_d}\exp\left(\frac{V'}{V_t}\right) = \frac{(1.5\times10^{10})^2}{1.5\times10^{16}}\exp\left(\frac{0.5975}{0.0259}\right) = \boxed{1.5647\times10^{14}\ /cm^3}$$

d.)

$$I_n(-x_p) = I_e\exp\left(\frac{V'}{V}\right) = (4.025\times10^{-15})\exp\left(\frac{0.5975}{0.0259}\right) = \boxed{4.198\times10^{-5}\ A.}$$

e.)

$$I_p = I_{ip}\exp\left(\frac{V'}{V_t}\right) = (1.342\times10^{-14})\exp\left(\frac{0.5975}{0.0259}\right) = 1.399\times10^{-4}\ A.$$

$$I_n = 4.198\times10^{-5} + 1.399\times10^{-4} = 1.82\times10^{-4}\ A.$$

$$I_p\left(x_n + \tfrac{1}{2}L_p\right) = (1.399\times10^{-4})\exp\left(\frac{-\frac{1}{2}L_p}{L_p}\right) = 8.49\times10^{-5}\ A.$$

[illegible] 10^{-4} [illegible] 10^{-5} [illegible] 10^{-5} A

$$I_p\left(x_n+\frac{1}{2}L_p\right) = (1.399\times10^{-4})\exp\left(\frac{-\frac{1}{2}L_p}{L_p}\right) = 8.49\times10^{-5}\text{ A.}$$

$$I = 1.82\times10^{-4} - 8.49\times10^{-5} = \boxed{9.71\times10^{-5}\text{ A.}}$$

8.18 The limit of low injection is normally defined to be when the minority carrier concentration at the edge of the space charge region in the low-doped region becomes equal to one-tenth the majority carrier concentration in this region. Determine the value of the forward-bias voltage at which the limit of low injection is reached for the diode described in (*a*) Problem 8.7 and (*b*) Problem 8.8.

A.)

$$n_p = n_{p_0}\exp\left(\frac{eV_a}{kT}\right); \quad n_{p_0} = \frac{n_i^2}{N_a}; \quad (0.1)N_a = \frac{n_i^2}{N_a}\exp\left(\frac{eV_a}{kT}\right)$$

$$V_a = \frac{kT}{e}\ln\left(\frac{(0.1)N_a^2}{n_i^2}\right) = (0.0259)\ln\left(\frac{(0.1)(4\times10^{15})^2}{(2.4\times10^{13})^2}\right) = \boxed{0.205\text{ V.}}$$

B.)

$$p_n = p_{n_0}\exp\left(\frac{eV_a}{kT}\right); \quad p_{n_0} = \frac{n_i^2}{N_d}; \quad (0.1)N_d = \frac{n_i^2}{N_d}\exp\left(\frac{eV_a}{kT}\right)$$

$$V_a = \frac{kT}{e}\ln\left(\frac{(0.1)N_d^2}{n_i^2}\right) = (0.0259)\ln\left(\frac{(0.1)(8\times10^{15})^2}{(1.5\times10^{10})^2}\right) = \boxed{0.623\text{ V.}}$$

8.24 (*a*) A silicon pn junction diode has the geometry shown in Figure 8.11 in which the n region is "short" with a length $W_n = 0.7\ \mu$m. The doping concentrations are $N_a = 2\times10^{17}$ cm^{-3} and $N_d = 2\times10^{15}$ cm^{-3}. The cross-sectional area is $A = 10^{-3}$ cm^2. Determine (*i*) the maximum forward-bias voltage such that low injection is still valid, and (*ii*) the resulting current at this forward-bias voltage. (*b*) Repeat part (*a*) if the doping concentrations are reversed such that $N_a = 2\times10^{15}$ cm^{-3} and $N_d = 2\times10^{17}$ cm^{-3}.

A.)

[i] $$p_n(x_n) = p_{n_0}\exp\left(\frac{V_a}{V_t}\right); \quad p_{n_0} = \frac{n_i^2}{N_d}; \quad (0.1)N_d = \left(\frac{n_i^2}{N_d}\right)\exp\left(\frac{V_a}{V_t}\right)$$

$$V_a = V_t\left(\ln\left(\frac{(0.1)N_d^2}{n_i^2}\right)\right) = (0.0259)\ln\left(\frac{(0.1)(2\times10^{15})^2}{(1.5\times10^{10})^2}\right) = \boxed{0.5516\text{ V.}}$$

[ii] $$I_p = \frac{AeD_p}{W_n}\left(\frac{n_i^2}{N_d}\right)\exp\left(\frac{V_a}{V_t}\right) = \frac{(10^{-3})(1.6\times10^{-19})(10)(1.5\times10^{10})^2}{(0.7\times10^{-6})(2\times10^{15})}\exp\left(\frac{0.5516}{0.0259}\right)$$

$$\boxed{I_p = 4.565\text{ mA}}$$

B.)

[i] $V_a = V_t \left(\ln\left(\frac{(0.1)N_a^2}{n_i^2}\right)\right) = (0.0259)\ln\left(\frac{(0.1)(2\times10^{15})^2}{(1.5\times10^{10})^2}\right) = \boxed{0.5516\text{ V.}}$

[ii] $I_n = eA\sqrt{\frac{D_e}{\tau_{n0}}}\left(\frac{n_i^2}{N_a}\right)\exp\left(\frac{V_a}{V_t}\right) = (1.6\times10^{-19})(5\times10^{-4})\sqrt{\frac{25}{(5\times10^{-7})}}\left(\frac{(1.5\times10^{10})^2}{2\times10^{15}}\right)\exp\left(\frac{0.5516}{0.0259}\right)$

$I_n = 2.2597\times10^{-4}\text{ A}$

$I = I_n + I_p = \boxed{0.2716\text{ mA}}$

8.42 A one-sided p^+n silicon diode has doping concentrations of $N_a = 4\times10^{17}$ cm^{-3} and $N_d = 8\times10^{15}$ cm^{-3}. The diode cross-sectional area is $A = 5\times10^{-4}$ cm^2. (*a*) The maximum diffusion capacitance is to be limited to 1 nF. Determine (*i*) the maximum current through the diode, (*ii*) the maximum forward-bias voltage, and (*iii*) the diffusion resistance. (*b*) Repeat part (*a*) if the maximum diffusion capacitance is limited to 0.25 nF.

A.)

[i] $I_d = \frac{2V(C_d)}{\tau_0} = \frac{2(0.0259)(10^{-9})}{(10^{-7})} = \boxed{0.518\text{ mA}}$

[ii] $I_p = eA\sqrt{\frac{D_p}{\tau_{p0}}}\left(\frac{n_i^2}{N_d}\right)\exp\left(\frac{V_a}{V_t}\right)$

$0.518\times10^{-3} = (5\times10^{-4})(1.6\times10^{-19})\sqrt{\frac{10}{10^{-7}}}\left(\frac{(1.5\times10^{10})^2}{8\times10^{15}}\right)\exp\left(\frac{V_a}{V_t}\right)$

$V = (0.0259)\ln\left(\frac{0.518\times10^{-3}}{2.25\times10^{-14}}\right) = \boxed{0.618\text{ V.}}$

[iii] $R_d = \frac{V_t}{I_d} = \frac{(0.0259)}{(0.518\times10^{-3})} = \boxed{50\ \Omega}$

B.)

[i] $I_p = \frac{2V(C_d)}{} = \frac{2(0.0259)(0.25\times10^{-9})}{} = \boxed{0.1295\text{ mA}}$

[i] $I_p = \dfrac{2V(C_d)}{\tau_0} = \dfrac{2(0.0259)(0.25\times10^{-9})}{(10^{-7})} = \boxed{0.1295 \text{ mA}}$

[ii] $V = (0.0259)\ln\left(\dfrac{0.1295\times10^{-3}}{2.25\times10^{-14}}\right) = \boxed{0.5821 \text{ V.}}$

[iii] $R_d = \dfrac{V_t}{I_d} = \dfrac{(0.0259)}{(0.1295\times10^{-3})} = \boxed{200\ \Omega}$