- Q1: Read the below paper and write a summary (a minimum of half a page) to describe what you have learned from it. (20 points)

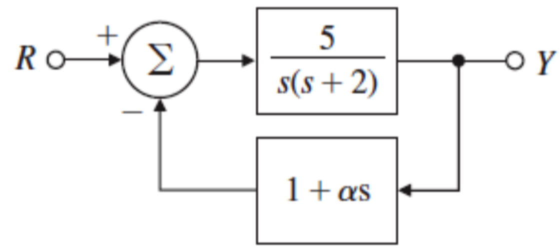
Marcel Carvalho and Minhoto Teixeira, “Direct Expressions for Ogata's Lead-Lag Design Method Using Root Locus,” IEEE Transactions on Education, Vol. 37, No. I, February 1994.

The paper presents the useful nature of incorporating the root locus into both written as well as computer-based approaches. The efficient nature of applying a simple compensator like PID or lead-lag is convenient in many cases, as it is a much faster alternative to an in-depth analysis using full fledged control systems. For example, in the control of dc motors and in magnetic suspension systems, PID and lead-lag controllers are sufficient. With this, the avoidance of delving into the geometrical work that accompanies these determination methods is important in ensuring the simplicity and speed in using this alternative. Computer softwares like MATLAB and Program CC were credited for providing the ability to quickly perform these tasks, as they are designed and optimized to avoid these complicated geometric calculations by incorporating root locus and Bode plot methods instead. However, in some designs, the determination of poles and zeros of the compensator are made using a graphic mean or a computer solution. This impedes the process that can be simplified through the direct expression method that is presented in the paper. Essentially, the direct expression method discussed for the design of a lead-lag handler, which functions using the root locus method and the corresponding procedure presented, allows for the avoidance of geometrical determination or computer solutions for the obtainment of poles and zeros, which is made in and, enabling a computer-based design to be made faster and with greater accuracy. An example of the procedure in action is provided, and shows how poles and zeros can be found without the need to incorporate an entire graph, and even allow for a simple written analysis using the formulas demonstrated by Ogata.

Q2: Put the characteristic equation of the system shown in below figure in root-locus form, with respect to parameter α, and identify the corresponding L(s), a

Sketch the root locus with respect to α. Use Matlab to check the accuracy of your root locus.

(40 points)



$Transfer\ function:\ G(s)\ =\frac{5}{s(s+2)}$

$Feedback:\ H(s)\ =\ 1\ +\ as$

$Characteristic\ equation:\ 1\ +\ G(s)H(s)\ =\ 0$

$1\ +\ [\frac{5}{s(s+2)}][1\ +\ as]\ =\ 0$

$s(s\ +\ 2)\ +\ 5(1\ +\ as)\ =\ s^2\ +\ 2s\ +\ 5\ +\ 5as\ =\ 0$

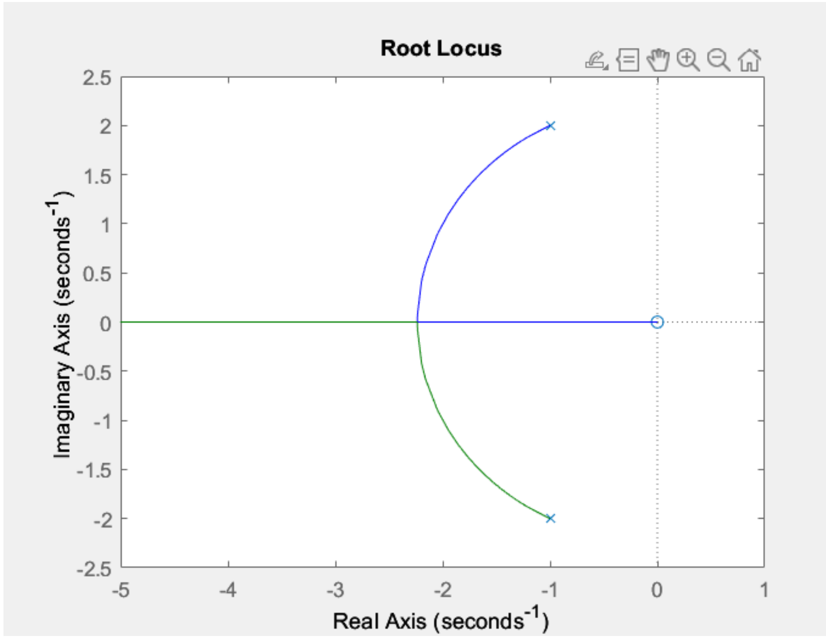
$Write\ in\ format\ 1\ +\ K\frac{b(s)}{a(s)}:$

$\frac{s^2+2s+5+5as}{s^2+2s+5} = \frac{0}{s^2+2s+5}$

$1\ +\ \frac{5as}{s^2+2s+5}\ =\ 0$

$Open\ loop\ transfer\ function:\ L(s)\ =\ \frac{b(s)}{a(s)}\ =\ \frac{5s}{s^2+2s+5}$

```
% Plot the root locus
num=[5 0];
den=[1 2 5];
sys=tf(num,den);
rlocus(sys)
```



Closed loop transfer function: $T(s) = \frac{\frac{5}{s(s+2)}}{1+(1+as)(\frac{5}{s(s+2)})} = \frac{5}{s^2+2s+5+5as} = \frac{5}{s^2+(2+5a)s+5}$

Closed loop pole locations for $a = 0$:

$s^2 + (2 + 5(0))s + 5 = 0$

$s == 1 \pm 2i$

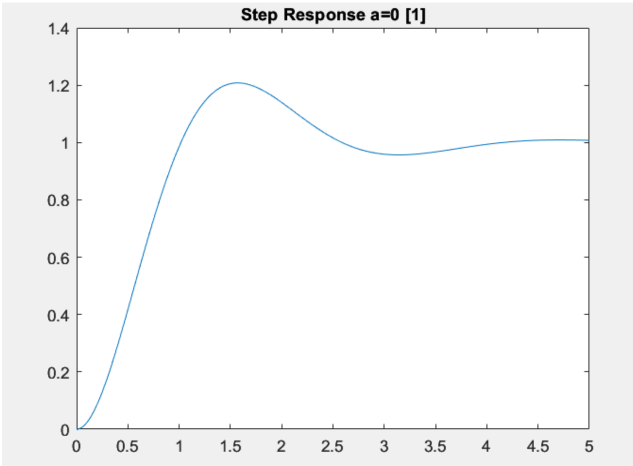
$\frac{Y(s)}{R(s)} = \frac{5}{s^2+2s+5}$; $R(s) = \frac{1}{s}$; $Y(s) = \frac{5}{s(s^2+2s+5)}$

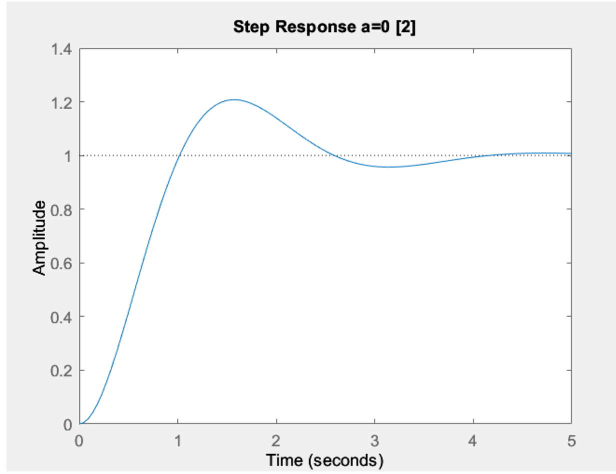
Step response for $a = 0$:

$Y(s) = \frac{5}{s(s^2+2s+5)} = \frac{1}{s} - \frac{s+2}{(s+1)^2+2^2} = \frac{1}{s} - \frac{s+1}{(s+1)^2+2^2} - \frac{1}{(s+1)^2+2^2}$

Apply inverse Laplace transform: $y(t) = u(t) - e^{-t}cos(2t)u(t) - 0.5e^{-t}sin(2t)u(t)$

```
% Step response
t=0:0.01:5;
y=1-exp(-t).*cos(2.*t)-0.5.*exp(-t).*sin(2.*t);
plot(t,y);
title('Step Response a=0 [1]');

%================================================

a=0;
num=[5];
den=[1 2+5*a 5];
sys=tf(num,den);
step(sys);
title('Step Response a=0 [2]');
```





// Same result

$Closed\ loop\ pole\ locations\ for\ a\ =\ 0.5$:

$s^2 + (2 + 5(0.5))s + 5 = 0$

$s = -2,\ -2.5$

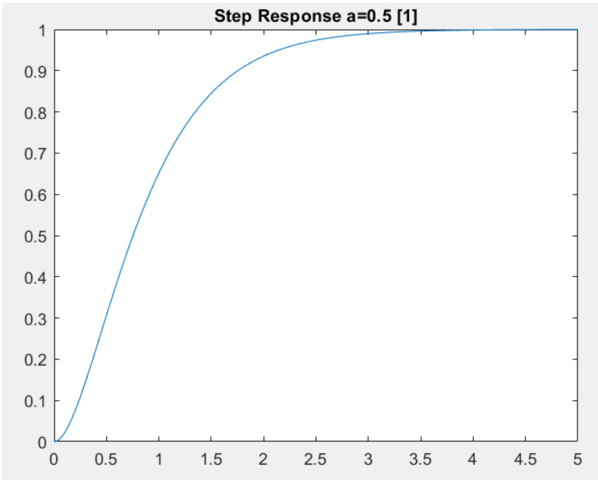
$\frac{Y(s)}{R(s)} = \frac{5}{s^2+4.5s+5};\ R(s) = \frac{1}{s};\ Y(s) = \frac{5}{s(s^2+4.5s+5)}$

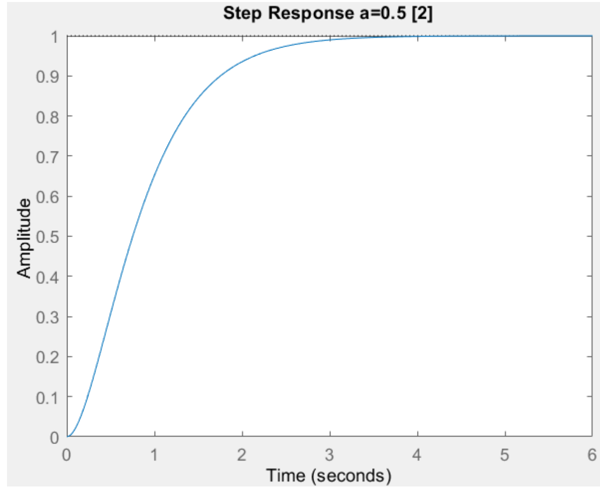
$Step\ response$:

$Y(s) = \frac{5}{s(s+2)(s+2.5)} = \frac{1}{s} - \frac{5}{s+2} + \frac{4}{s+2.5}$

$Apply\ inverse\ Laplace\ transform$: $y(t) = u(t) - 5e^{-2t}u(t) + 4e^{-2.5t}u(t) = [1 - 5e^{-2t} + 4e^{-2.5t}]u(t)$

```
% Step response
t=0:0.01:5;
y=1-5.*exp(-2.*t)+4.*exp(-2.5.*t);
plot(t,y);
title('Step Response a=0.5 [1]');

%============================================

a=0.5;
num=[5];
den=[1 2+5*a 5];
sys=tf(num,den);
step(sys);
title('Step Response a=0.5 [2]');
```






```
// Same result
```

$Closed\ loop\ pole\ locations\ for\ a\ =\ 2:$

$$s^2\ +\ (2\ +\ 5(2))s\ +\ 5\ =\ 0$$

$$s\ =\ -\ 0.432,\ \ -\ 11.5567$$

$$\frac{Y(s)}{R(s)}\ =\ \frac{5}{s^2+12s+5};\ R(s)\ =\ \frac{1}{s};\ Y(s)\ =\ \frac{5}{s(s^2+12s+5)}$$

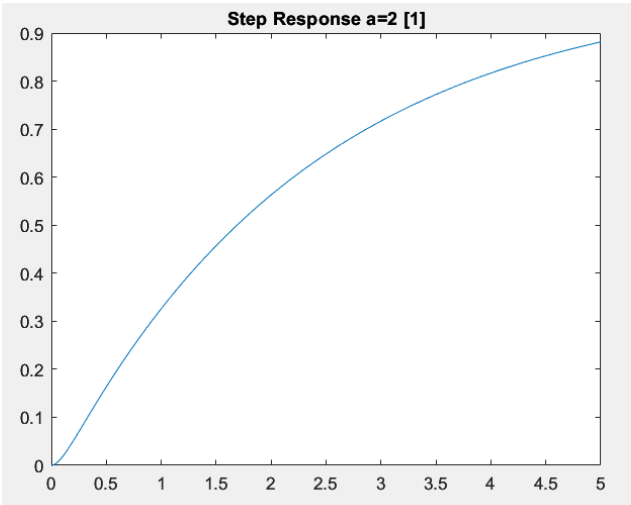
$Step\ response:$

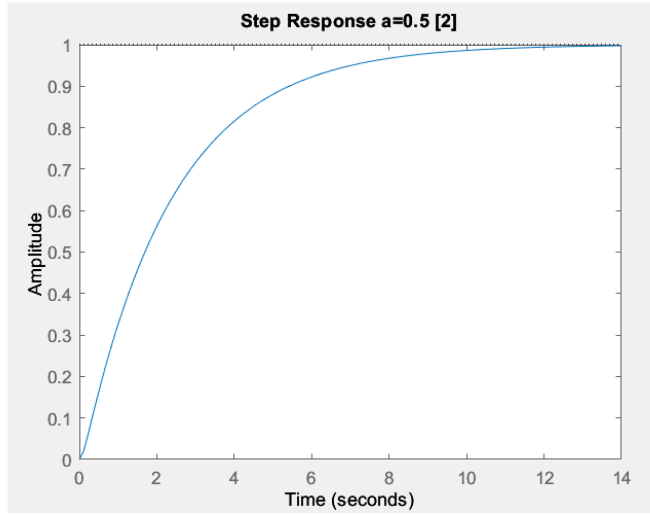
$$Y(s)\ =\ \frac{5}{s(s+0.432)(s+11.5567)}\ =\ \frac{1.0015}{s}\ -\ \frac{1.0404}{s+0.432}\ +\ \frac{0.039}{s+11.5567}$$

$Apply\ inverse\ Laplace\ transform:$

$$y(t)\ =\ 1.0015u(t)\ -\ 1.0404e^{-0.432t}u(t)\ +\ 0.039e^{-11.5567t}u(t)\ =\ [1.0015\ -\ 1.0404e^{-0.432t}\ +\ 0.039e^{-11.5567t}]u(t)$$

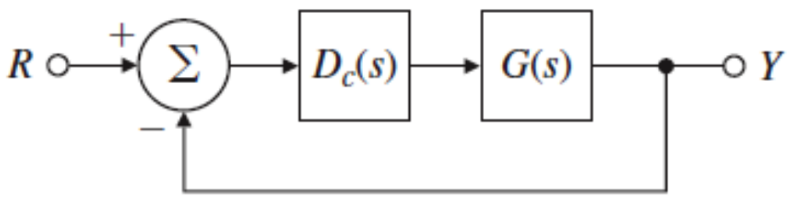
```
% Step response
t=0:0.01:5;
y=1.0015-1.0404.*exp(-0.432.*t)+0.039.*exp(-11.5567.*t);
plot(t,y);
title('Step Response a=2 [1]');

%===============================================

a=2;
num=[5];
den=[1 2+5*a 5];
sys=tf(num,den);
step(sys);
title('Step Response a=0.5 [2]');
```


Step Response a=2 [1]
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5


Step Response a=0.5 [2]
Amplitude
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
2
4
6
8
10
12
14
Time (seconds)

```
// Same result
```

- Q3: Consider the below system with $G(s) = \frac{1}{(s+2)(s+3)}$ and $D_c(s) = K\frac{s+a}{s+b}$. Using the root-locus, find values of K, a, and b that will produce closed-loop poles at $s = -1 \pm j$.
  Is this a lead or lag compensator? (40 points)



$L(s) = G(s) = \frac{1}{(s+2)(s+3)}$ *before compensation*

$L(s) = G(s)D_C(s) = [\frac{1}{(s+2)(s+3)}][K\frac{s+a}{s+b}]$ *after compensation*

Poles at $s = -2$ *and* $s = -3$

Find transfer function for poles at $s = -1 \pm i$

$(s + (1 + i))(s + (1 - i)) = (s + 1)^2 + 1 = s^2 + 2s + 2 = 0$

$$\frac{Y(s)}{R(s)} = \frac{G(s)D(s)}{1+G(s)D(s)H(s)} = \frac{[\frac{1}{(s+2)(s+3)}][K\frac{s+3}{s+b}]}{1+[\frac{1}{(s+2)(s+3)}][K\frac{s+3}{s+b}][1]} = \frac{\frac{K}{(s+2)(s+b)}}{1+\frac{K}{(s+2)(s+b)}} = \frac{K}{s^2+(2+b)s+2b+K}$$

Characteristic equation:

$s^2 + (2 + b)s + 2b + K = 0$

[1]

$2 + b = 2$

$b = 0$

[2]

$2b + K = 2$

$2(0) + K = 2$

$K = 2$

$a = 3,\ b = 0,\ K = 2$