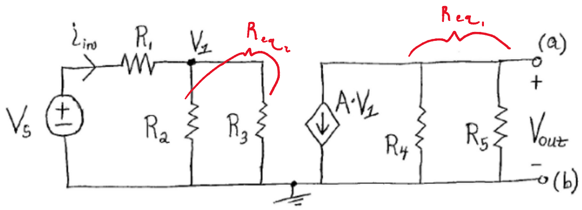
Final Exam

Tuesday, December 15, 2020 7:50 AM

"I pledge my honor I have abided by the Stevens Honor system."

- Alex Gaskins

Problem One: Please refer to circuit drawing below.



Note; there are no numerical values given for parts (a), (b), and (c); for these three parts of the problem, your answer should be in terms of the network elements. **(15 points total)**

Note; the units of the constant "A" are Amps/Volts

Do not use any "Req" term in your solutions.

a) Determine an expression for V_{out}/V_1 Do not enter your answer in the online answer box; please enter your answer in your worksheet. Please clearly label your answer. **(3 points)**

$$R_{eq_1} = \frac{R_4 R_5}{R_4 + R_5}$$

$$R_{eq_2} = \frac{R_2 R_3}{R_2 + R_3}$$

$$V_{out} = R_{eq_1}(-A \cdot V_1)$$

$$\frac{V_{out}}{V_1} = R_{eq_1}(-A)$$

$$\boxed{\frac{V_{out}}{V_1} = -\frac{R_4 R_5}{R_4 + R_5}(A)}$$

b) Determine an expression for V_{out}/V_S Do not enter your answer in the online answer box; please enter your answer in your worksheet. Please clearly label your answer. **(4 points)**

$$V_1 = \frac{R_{eq_2}}{R_1 + R_{eq_2}} V_s$$

$$\frac{V_1}{V_s} = \frac{R_{eq_2}}{R_1 + R_{eq_2}}$$

$\cancel{A}$ V_{out} R (,)

$$\frac{\cancel{V_1}}{V_s} \times \frac{V_{out}}{\cancel{V_1}} = \frac{R_{eq_2}}{R_1 + R_{eq_2}} \left(R_{eq_1}(A)\right)$$

$$\boxed{\frac{V_{out}}{V_s} = -\frac{R_2 R_3}{R_1(R_2+R_3) + R_2 R_3} \left(\frac{R_4 R_5}{R_4 + R_5}\right) A}$$

c) A load resistance is placed between terminals “a” and “b”. We want to select the load resistance which will result in the maximum power being delivered to this load resistor. Determine an expression for this load resistance for maximum power transfer. Your answer should be in terms of the network elements. Do not enter your answer in the online answer box; please enter your answer in your worksheet. Please clearly label your answer. **(5 points)**

From Part A:

$$V_{out} = V_{oc} = -\frac{R_4 R_5}{R_4 + R_5}(A \cdot V_1)$$

$$I_{sc} = -A \cdot V_1$$

$$R_{th} = \frac{V_{oc}}{I_{sc}} = \frac{\left(-\frac{R_4 R_5}{R_4 + R_5}\right)(A \cdot V_1)}{-(A \cdot V_1)}$$

$$R_{th} = \frac{R_4 R_5}{R_4 + R_5}$$

$$\boxed{R_{Load} = R_{th} = \frac{R_4 R_5}{R_4 + R_5}}$$

d) Determine the maximum power that can be delivered to a resistive load for the given circuit. Enter your numerical answer in the online answer box. **(3 points)**

Note; for part (d) you are now given the following values;

- $R_1 = 10\ \Omega$
- $R_2 = R_3 = 100\ \Omega$
- $R_4 = R_5 = 15\ \Omega$
- $V_S = 3$ Volts
- $A = 1.5$ Amps/Volt

$$R_L = \frac{(15)(15)}{15+15} = 7.5\ \Omega \qquad R_{eq_2} = \frac{(100)(100)}{100+100} = 50\ \Omega$$

$$P = \frac{(V_T)^2}{4R_L} = \frac{(-AV_1)^2 R_{eq_1}}{4(R_{eq_1})}$$

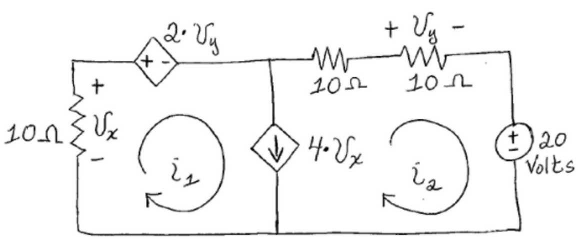
$$V_1 = \frac{R_{eq_2}}{R_{eq_2}+R_1} A = \frac{\frac{R_2R_3}{R_2+R_3}}{\frac{R_2R_3}{R_2+R_3}+R_1} A$$

$$V_1 = \frac{50}{50+10}(3) = 2.5\ V$$

$$P = \frac{(-1.5(2.5))^2(7.5)}{4}$$

$$\boxed{P = 26.37\ \text{Watts}}$$

Problem Two: Consider the circuit shown below. Use mesh analysis for this problem.



Using mesh analysis, determine the values of i_1 and i_2. Enter your numerical answer in the online answer box and clearly label which answer is i_1 and which answer is i_2. **(15 points)**

$$-10i_1 + 2V_y + 10i_2 + 10i_2 + 20 = 0$$

$$20i_2 + 2V_y - 10i_1 = -20$$

$$20i_2 + 2(10i_2) - 10i_1 = -20$$

$$40i_2 - 10i_1 = -20$$

$$40(41i_1) - 10i_1 = -20$$

$$1630i_1 = -20$$

$$\boxed{i_1 = -.0122699\ A.}$$

$V_y = 10i_2$

$V_x = -10i_1$

$4V_x = i_1 - i_2$

$4(-10i_1) = i_1 - i_2$

$i_2 = 41i_1$

$$i_2 = 41\, i_1$$

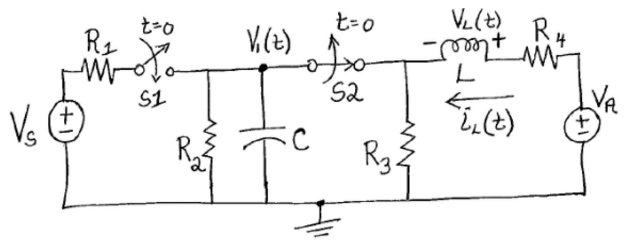
$$= 41(-.0122699)$$

$$i_2 = 41\, i_1$$

$$i_2 = 41(-.0122699)$$

$$\boxed{i_2 = -.503\text{ A.}}$$

Problem Three: Consider the circuit shown below. There are no numerical values given in this problem.



The two voltage sources, V_S and V_A, are each a constant dc source. The circuit has been operating for a long time with Switch One (S1) open prior to t = 0 and Switch Two (S2) closed prior to t = 0. **(25 points total)**

a) Write the first order differential equation for the voltage, $v_1(t)$ for t > 0, in standard form. Do not enter your answer in the online answer box; please enter your answer in your worksheet. Please clearly label your answer. **(10 points)**

$$\frac{V_1 - V_s}{R_1} + \frac{V_1}{R_2} + C\frac{dV_1}{dt} = 0$$

$$\frac{V_1}{R_1} + \frac{V_1}{R_2} + C\frac{dV_1}{dt} = \frac{V_s}{R_1}$$

$$\frac{V_1\left(\frac{1}{R_1} + \frac{1}{R_2}\right) + C\frac{dV_1}{dt} = \frac{V_s}{R_1}}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{V_1\left(\frac{R_1 + R_2}{R_1 R_2}\right) + C\frac{dV_1}{dt} = \frac{V_s}{R_1}}{\frac{R_1 + R_2}{R_1 R_2}}$$

$$\boxed{\frac{R_1 R_2}{R_1 + R_2} C\frac{dV_1}{dt} + V_1 = \frac{V_s}{R_1}\left(\frac{R_1 R_2}{R_1 + R_2}\right)}$$

b) Determine the expression for the time constant in this system. Do not enter your answer in the online answer box; please enter your answer in your worksheet. Please clearly label your answer. **(2 points)**

$$\tau = RC$$

$$\boxed{\tau = \frac{R_1 R_2}{R_1 + R_2} C}$$

$$\boxed{\tau = \frac{R_1 R_2}{R_1 + R_2} C}$$

c) Determine the homogeneous solution. Do not enter your answer in the online answer box; please enter your answer in your worksheet. Please clearly label your answer. (**1 point**)

$$Y_{c_h}(t) = k e^{-t/\tau}$$

$$Y_{c_h}(t) = k e^{-\frac{t}{\frac{R_1 R_2}{R_1+R_2} C}}$$

$$\boxed{Y_{c_h}(t) = k e^{-\frac{R_1+R_2}{R_1 R_2 C}}}$$

d) Determine the particular solution. Do not enter your answer in the online answer box; please enter your answer in your worksheet. Please clearly label your answer. (**2 points**)

$Y_p =$ Right side of equation

$$Y_p = \frac{V_S}{R_1}\left(\frac{R_1 R_2}{R_1+R_2}\right)$$

$$\boxed{Y_p = \frac{R_2}{R_1+R_2} V_S}$$

e) Determine the complete solution and please solve for all constant(s). Clearly identify the expression for "K" and if you have any "Req" in your solution, please clearly define Req. Do not enter your answer in the online answer box; please enter your answer in your worksheet. Please clearly label your answer. (**10 points**)

$$V_1(t=0^+) = V_1(t=0^-)$$

$$[V_1(t=0^-)]$$

$$R_{eq} = \frac{R_2 R_3}{R_2+R_3}$$

$$V_1(t=0^-) = \frac{R_{eq}}{R_{eq}+R_4} V_A = \frac{\frac{R_2 R_3}{R_2+R_3}}{\frac{R_2 R_3}{R_2+R_3} + R_4} V_A = \frac{R_2 R_3}{R_2 R_3 + R_4(R_2+R_3)} V_A$$

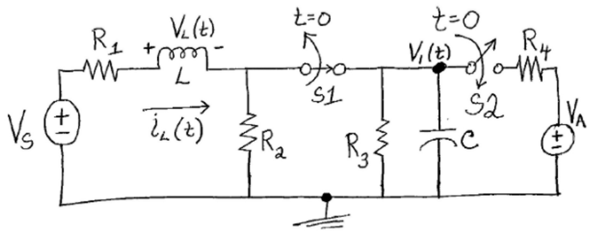
$$\frac{R_2 R_3}{R_2 R_3 + R_4(R_2+R_3)} V_A = k e^0 + \frac{R_2}{R_1+R_2} V_S$$

$$k = \frac{R_2 R_3}{R_2 R_3 + R_4(R_2+R_3)} V_A - \frac{R_2}{R_1+R_2} V_S$$

$$v_1 = \frac{R_2 R_3}{R_2 R_3 + R_4(R_2+R_3)} V_A - \frac{R_2}{R_1+R_2} V_s$$

$$\boxed{v_1 = \left(\frac{R_2 R_3}{R_2 R_3 + R_4(R_2+R_3)} V_A - \frac{R_2}{R_1+R_2} V_s\right) e^{-\frac{R_1+R_2}{R_1 R_2 C}} + \frac{R_2}{R_1+R_2} V_s}$$

Problem Four: Consider the circuit shown below. There are no numerical values given in this problem.



The two voltage sources, V_S and V_A, are each a constant dc source. The circuit has been operating for a long time with Switch One (S1) closed prior to t = 0 and Switch Two (S2) open prior to t = 0. **(15 points total)**

a) Write the first order differential equation for the current, $i_L(t)$ for t > 0, in standard form. Do not enter your answer in the online answer box; please enter your answer in your worksheet. Please clearly label your answer. **(10 points)**

$$V_L = L\frac{di}{dt}$$

$$-V_s + R_1 i_L + R_2 i_L + V_L = 0$$

$$(R_1+R_2) i_L + L\frac{di_L}{dt} = V_s$$

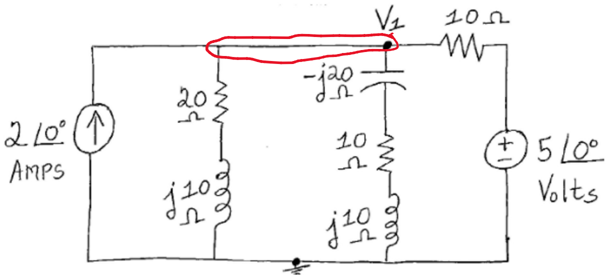
$$\boxed{\frac{L}{R_1+R_2}\frac{di_L}{dt} = \frac{V_s}{R_1+R_2}}$$

b) Please determine an expression for $i_L(t = 0^+)$. Do not enter your answer in the online answer box; please enter your answer in your worksheet. Please clearly label your answer. **(5 points)**

$$i_L(t=0^+) = i_L(t=0^-)$$

$$\boxed{i_L(t=0^+) = \frac{V_s}{R_1}}$$

Problem Five: Consider the circuit shown below.



$-j20 + j10 = -j10$

Determine the steady-state voltage V_1. Note; the voltage V_1 is shown in the given circuit. Please write your answer in phasor form (polar format). Enter your numerical answer in the online answer box. **(20 points)**

$$-2\angle 0° + \frac{V_1}{20+j10} + \frac{V_1}{10-j10} + \frac{V_1 - 5\angle 0°}{10} = 0$$

$$\frac{V_1}{20+j10} + \frac{V_1}{10-j10} + \frac{V_1 - 5\angle 0°}{10} = 2\angle 0°$$

$$V_1\left(\frac{1}{20+j10} + \frac{1}{10-j10} + \frac{1}{10}\right) = 2\angle 0° + \frac{1}{2}\angle 0°$$

$$V_1(.04 - j.02 + .05 + j.05 + .1) = 2.5$$

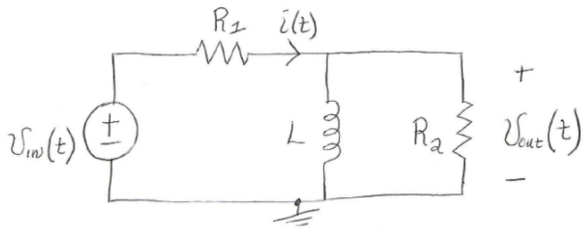
$$V_1(.192 + j.03) = 2.5$$

$$V_1 = \frac{2.5}{.192 + j.03}$$

$$V_1 = \frac{2.5}{.192 + j.03}$$

$$\boxed{\begin{aligned} V_1 &= 13.16\angle -8.97° \\ V_1 &= 13.16\cos(\omega t - 8.97) \end{aligned}}$$

Problem Six: Consider the given circuit.



Note; $v_{in}(t)$, $v_{out}(t)$, and i(t) are shown in the circuit.

The input voltage source is a sinusoidal voltage source with frequency ω radians/second. **(10 points total)**

a) For input frequencies, ω, very (very) close to zero, determine the value or the expression for the output voltage $v_{out}(t)$ in steady state. Enter your answer in the online answer box. **(2 points)**

When inductor impedance is equal to 0, it acts as a short circuit.

so $\boxed{V_{out} = 0\ V.}$

b) For input frequencies, ω, very (very) close to zero, determine the value or the expression for the current i(t) in steady state. Enter your answer in the online answer box. **(3 points)**

i(t) only flows through R_1

so

$$\boxed{i(t) = \frac{V_{in}(t)}{R_1}}$$

c) For input frequencies, ω, approaching infinity, determine the value or the expression for the output voltage $v_{out}(t)$ in steady state. Enter your answer in the online answer box. **(3 points)**

Using voltage division

$$\boxed{V_{out} = \frac{R_2}{R_1 + R_2} V_{in}(t)}$$

d) For input frequencies, ω, approaching infinity, determine the value or the expression for the current i(t) in steady state. Enter your answer in the online answer box. **(2 points)**

Using Ohm's Law

$$\boxed{i(t) = \frac{V_{in}(t)}{R_1 + R_2}}$$