Exam 3 "I pledge my honor I have abided by the Stevens Honor system."

Saturday, November 21, 2020 8:59 AM

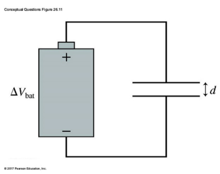
- Alex Gaskins

1. The parallel plate capacitor in the figure is connected to a battery having a potential difference ΔV_{bat}. Without breaking any of the connections, insulating handles are used to increase the plate separation to 2d.

a. (1 pt) Does the potential difference ΔV_C change as the separation increases? If so, by what factor? If not, why?

b. (1 pt) Does the capacitance change? If so, by what factor? If not, why?

c. (1 pt) Does the capacitor charge Q change? If so, by what factor? If not, why?



A.) Potential between the plates doesn't change if plate separation is increased and the battery remains connected.

B.) $C_1 = \frac{\kappa \varepsilon_0 A}{d}$

$C_2 = \frac{\kappa \varepsilon_0 A}{2d}$

$$\frac{C_2}{C_1} = \frac{\left(\frac{\kappa \varepsilon_0 A}{2d}\right)}{\left(\frac{\kappa \varepsilon_0 A}{d}\right)}$$

Changes by a factor of 2.

$$\frac{C_2}{C_1} = \frac{d}{2d} = \boxed{\frac{1}{2}}$$

C.) If separation is increased, charge decreases.

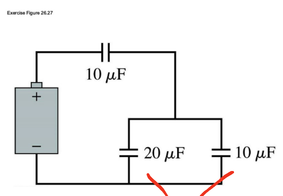
$Q = CV$

$$\frac{Q_2}{Q_1} = \frac{C_2 V}{C_1 V} = \frac{1}{2}$$

$$\boxed{Q_2 = \frac{Q_1}{2}}$$

Decreases by a factor of 2.

2. What is the
   a. (1 pt) The equivalent capacitance of the three capacitors shown in the figure?
   b. (1 pt) If there is 5x10^-5 C of charge in the lower 10 µF that capacitor what is the charge in the 20 µF capacitor
   c. (1 pt) The charge in the upper 10uF capacitor
   d. (1 pt) The potential of the battery



30 µF

A.) $C_{eq} = \frac{30(10)}{40} = \frac{300}{40} = \frac{30}{4} = \boxed{7.5\,\mu F}$

B.) $Q = CV$

$V = \frac{Q}{C}$

$\frac{Q_1}{C_1} = \frac{Q_2}{C_2}$ $\qquad \frac{(5\times10^{-5})}{(10\times10^{-6})} = \frac{Q_2}{(20\times10^{-6})}$

$Q_2 = \frac{(20\times10^{-6})(5\times10^{-5})}{(10\times10^{-6})}$

$\boxed{Q_2 = 1\times10^{-4}\ C.}$

C.)

$Q = CV$

$(5\times10^{-5}) = (10\times10^{-6})\,V$

$V = \frac{(5\times10^{-5})}{(10\times10^{-6})}$

$V = 5\ V.$

$Q_{eq} = C_{eq}\,V$

$Q_{eq} = (30\times10^{-6})(5)$

$Q_{eq} = 1.5\times10^{-4}\ C.$

In series $q_1 = q_2$

Q for top $10\mu F$ capacitor = $\boxed{1.5\times10^{-4}\ C.}$

D.)

$$(1.5\times10^{-4}) = (10\times10^{-6})V$$

$$V = 15\ V.$$

$$V_{battery} = 15 + 5$$

$$\boxed{V_{battery} = 20V.}$$

3. Two 5.0 mm x 5.0 mm electrodes are held 0.10 mm apart and are attached to a 9.0 V battery. Without disconnecting a battery, 0.10 mm thick sheet of Mylar is inserted between the plates. What are the capacitors potential difference, electric field and charge:
   a. (1 pt) Before
   b. (1 pt) After the mylar is inserted

A.)

$$A = bl$$

$$A = (5.0\times10^{-3})(5.0\times10^{-3})$$

$$A = 2.5\times10^{-5}\ m.^2$$

$$C = \frac{\varepsilon_0 A}{d} = \frac{(8.85\times10^{-12})(2.5\times10^{-5})}{(.1\times10^{-3})}$$

$$C = 2.21\times10^{-12}\ F$$

$$E = \frac{V}{d} = \frac{9}{(.1\times10^{-3})}$$

$$E = 90000\ \frac{V}{m}$$

$$Q = CV = 2.21\times10^{-12}(9)$$

$Q = 1.99 \times 10^{-11}$ C.

B.) Potential remains the same

$V = 9V.$

$$E = \frac{9}{(.1 \times 10^{-3})} = 90000 \, V/m$$

$\kappa = 3.1$

$$C = 3.1 \left(\frac{(8.85 \times 10^{-12})(2.5 \times 10^{-5})}{(.1 \times 10^{-3})} \right)$$

$C = 6.85 \times 10^{-12}$ F

$Q = CV = 6.85 \times 10^{-12} (9)$

$Q = 6.16 \times 10^{-11}$ C.

4. Two 2.0 cm x 2.0 cm metal electrodes are space 1.0 mm apart and connected to the terminals of a 9.0 V battery.

a. (1 pt) What are the charge on each electrode and the potential difference between them

The wires are then disconnected, and insulated handles are used to pull the plates apart to a new spacing of 2.0 mm.

b. (1 pt) What are the charge on each electrode and the potential difference between them.

A.) Potential difference = 9 V. because they are connected to 9 V. battery

$$C = \frac{\varepsilon_0 A}{d} = \frac{(8.85 \times 10^{-12})(4 \times 10^{-4})}{(1 \times 10^{-3})}$$

$Q = CV$

$$Q = \left(\frac{(8.85 \times 10^{-12})(4 \times 10^{-4})}{(1 \times 10^{-3})} \right) (9)$$

$$Q = \left(\frac{(8.85\times10^{-12})(4\times10^{-4})}{(1\times10^{-3})}\right)(9)$$

$$Q = 3.2\times10^{-11}\ C.$$

B.) Q is constant if wires are disconnected.

C and V change

$$C \propto \frac{1}{d}$$

so if d is doubled, c is halved.

$$C = \frac{(8.85\times10^{-12})(4\times10^{-4})}{2(1\times10^{-3})}$$

$$C = 1.77\times10^{-12}\ F$$

$$Q = CV$$

$$V = \frac{Q}{C} = \frac{(3.2\times10^{-11})}{(1.77\times10^{-12})}$$

$$V = 18.079\ V.$$

$$\approx 18\ V.$$

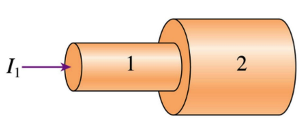
5. The wire in consists of two segments of different diameters but made from the same metal. The current in segment 1 is I1.

a. (1 pt) Compare the currents in the two segments. That is, is I2 greater than, less than, or equal to I1? Explain.

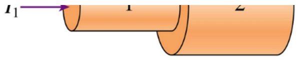
b. (1 pt) Compare the current densities J1 and J2 in the two segments.

c. (1 pt) Compare the electric field strengths E1 and E2 in the two segments.

d. (1 pt) Compare the drift speeds vd1 and vd2 in the two segments.



in a wire is



A.) Current in a wire is equal to its output current.

$I_1 = I_2$

output of I_1 is equal to input of I_2.

B.) current density is equal to current divided by area.

current stays the same, but A_2 is greater than A_1.

$J_1 > J_2$

C.) $E \propto J$

Electric field is directly proportional to current.

$E_1 > E_2$

D.) Drift velocity is directly proportional to electric field.

$V_{d1} > V_{d2}$

6. An engineer cuts a 1.0 m long, 0.33 mm diameter wire, connects it across a 1.5 V battery, and finds that the current in the wire is 8.0 A?

a. (1 pt) What is the resistance of the wire?
b. (1 pt) What is the current density in the wire?
c. (1 pt) What is the electric field in the wire?
d. (1 pt) What material is the wire made of?

A.) $V = IR$

$R = \frac{1.5}{8}$

$$R = \frac{1.5}{8}$$

$$\boxed{R = .1875\ \Omega}$$

B.)

$$J = \frac{I}{A} = \frac{I}{\frac{1}{4}\pi(d)^2} = \frac{8}{\frac{1}{4}\pi(.33\times10^{-3})^2}$$

$$\boxed{J = 9.35\times10^{7}\ \frac{A}{m^2}}$$

C.)

$$E = \frac{V}{d} = \frac{1.5}{1}$$

$$\boxed{E = 1.5\ \frac{V}{m}}$$

D.)

$$V = IR$$

$$R = \rho\frac{L}{A}$$

$$V = I\rho\frac{L}{A}$$

$$\rho = \frac{VA}{IL} = \frac{1.5\left(\frac{1}{4}\pi(.33\times10^{-3})^2\right)}{8(1)}$$

$\rho = 1.6\times10^{-8}\ \Omega\cdot m$

Thus the wire is made of silver.

7. The starter motor of a car engine draws a current of 150 A from the battery. The copper wire to the motor is 5.0 mm in diameter and 1.2 m long. The starter motor runs for 0.80 s until the car engine starts.

a. (1 pt) How much charge passes through the starter motor?

b. (1 pt) How far does an electron travel along the wire while the starter motor is on.

c. (1 pt) How long would it take an electron to travel from the beginning of the wire to the end of the wire?

A.) $Q = I\Delta t = 150(.80)$

$$\boxed{Q = 120\,C.}$$

C.) $t = \frac{d}{v}$

$$t = \frac{1.2}{5.61\times10^{-4}}$$

$$\boxed{t = 2139.04\,s.}$$

B.) $v_d = \frac{J}{ne}$ $\quad J = \frac{I}{A}$

$$v_d = \frac{I}{\frac{1}{4}ne\pi d^2}$$

$$v_d = \frac{150}{\frac{1}{4}(8.5\times10^{28})(1.602\times10^{-19})\pi(5\times10^{-3})^2}$$

$$v_d = 5.61\times10^{-4}\ m/s$$

$$d = v\Delta t$$

$$d = (5.61\times10^{-4})(.8)$$

$$\boxed{d = 4.49\times10^{-4}\,m.}$$

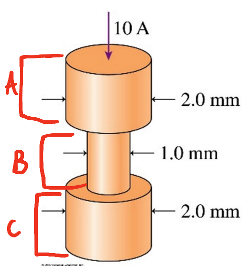
8. An aluminum wire consists of the three segments shown in the figure. The current in the top segment is 10 A. For each of these three segments, find the

a. (1 pt) Current I

b. (1 pt) Current density J

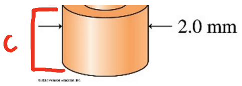
c. (1 pt) Electric field E.

d. (1 pt) Drift velocity vd



A.) Current is 10 A. throughout the wire.

B.) A: $J = \frac{I}{A} = \frac{4I}{\pi d^2}$



C.) $E = \dfrac{J}{\sigma}$

$\sigma = 3.5 \times 10^{-8}$ for aluminum

A:

$E = .091 \, \frac{V}{m}$

B:

$E = .371 \, \frac{V}{m}$

C:

$E = .091 \, V/m$

B.)

A: $J = \dfrac{I}{A} = \dfrac{4I}{\pi d^2}$

$J = \dfrac{40}{\pi(.002)^2}$

$J = 3.2 \times 10^6 \, \frac{A}{m^2}$

B:

$J = \dfrac{40}{\pi(.001)^2}$

$J = 1.3 \times 10^7 \, \frac{A}{m^2}$

C:

$J = 3.2 \times 10^6 \, \frac{A}{m^2}$

D.)

A: $J = n_e e V_d$

$V_d = \dfrac{J}{n_e e}$

$V_d = \dfrac{(3.2 \times 10^6)}{(6 \times 10^{28})(1.6 \times 10^{-19})}$

$V_d = 3.32 \times 10^{-4} \, m/s$

B:

$V_d = \dfrac{(1.3 \times 10^7)}{(6 \times 10^{28})(1.6 \times 10^{-19})}$

$V_d = 1.35 \times 10^{-3} \, m/s$

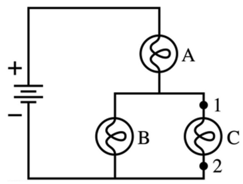
C:

$V_d = 3.32 \times 10^{-4} \, m/s$

9. Bulbs A, B, and C in the figure are identical, and all are glowing.

a. (1 pt) Rank in order, from most to least, the brightnesses of the three bulbs. Explain.

b. (1 pt) Suppose a wire is connected between points 1 and 2. What happens to each bulb? Does it get brighter, stay the same , get dimmer or go out? Explain.

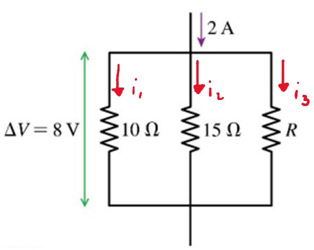


A.) $A > B = C$

Using KCL, the current through A is equal to the sum of the currents through both B and C. Less current means less power.

B.) Bulbs B and C would go out since the current would travel through the path of least resistance (the wire). Since A would be the only source of resistance in the circuit, it would have more current going through it. Thus, A would get brighter.

10. (1 pt) What is the value of the resistor in R in the figure?



$V = IR$

$I = \frac{V}{R}$

$i_1 = \frac{8}{10} = .8$ A.

$i_2 = \frac{8}{15} = .53$ A.

$.8 + .53 + i_3 = 2$

$i_3 = .66$ A.

$R = \frac{V}{I}$

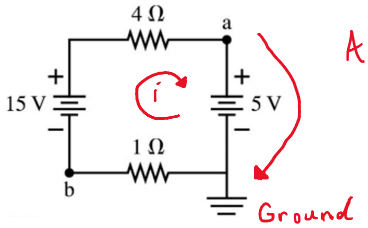
$R = \frac{8}{.66}$

$\boxed{R = 12.12\ \Omega}$

11. In the figure, what is

a. (1 pt) The current in each resistor?

b. (1 pt) The value of the potential at points a and b?



A.) $-15 + 4i + 5 + i = 0$

$5i = 10$

$\boxed{i = 2\,A. \text{ through each resistor}}$

B.) $V_a - 5 = 0$

$\boxed{V_a = 5\,V.}$

$V = IR = 2(1)$

$V_b + 2(1) = 0$

$\boxed{V_b = -2\,V.}$

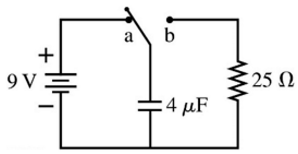
12. The switch in the figure has been in position a for a long time. It is changed to position b at t=0 s. What are the charge Q on the capacitor and the current I through the resistor

a. (1 pt) Immediately after the switch is closed?

b. (1 pt) T=50e-6 s

c. (1 pt) T=200e-6 s

d. (1 pt) At what time is the current 15% of the initial value



A.) $Q = CV$

$Q = (4 \times 10^{-6})(9)$

$Q = 3.6 \times 10^{-5}$ C.

$\tau = RC$

$\tau = (25)(4 \times 10^{-6})$

$\tau = 1 \times 10^{-4}$

$I = \frac{Q}{\tau}$

$I = .36$ A.

B.) $Q = Q_0 e^{-t/\tau}$

$Q = (3.6 \times 10^{-5})\, e^{-\frac{(50 \times 10^{-6})}{(1 \times 10^{-4})}}$

$Q = 2.18 \times 10^{-5}$ C.

$I = I_0 e^{-t/\tau}$

$I = (.36)\, e^{-\frac{(50 \times 10^{-6})}{(1 \times 10^{-4})}}$

$I = .22$ A.

C.) $Q = Q e^{-t/\tau}$

$I = I e^{-t/\tau}$

C.)

$$Q = Q_0 e^{-t/\tau}$$

$$Q = (3.6 \times 10^{-5})\, e^{-\frac{(200 \times 10^{-6})}{(1 \times 10^{-4})}}$$

$$Q = 4.87 \times 10^{-6} \text{ C.}$$

$$I = I_0 e^{-t/\tau}$$

$$I = (.36)\, e^{-\frac{(200 \times 10^{-6})}{(1 \times 10^{-4})}}$$

$$I = .049 \text{ A.}$$

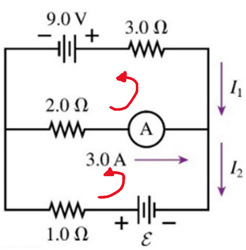
D.)

$$.36(.15) = .054 \text{ A.}$$

$$.054 = (.36)\, e^{-t/(1 \times 10^{-4})}$$

$$t = 1.89 \times 10^{-4} \text{ s.}$$

13. (3 pt) The ammeter in the figure reads 3.0 A. Find I1, I2 and ε.



$$i_1 = i_2 - 3$$

$$\varepsilon - 2(3) - i_2 = 0$$

$$9 + 2(3) - 3i_1 = 0$$

$$-3i_1 = -15$$

$$i_1 = 5 \text{ A.}$$

$$i_2 = 8 \text{ A.}$$

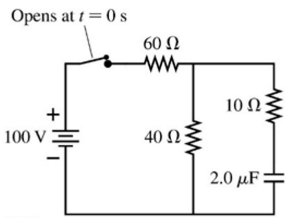
$$\varepsilon = 6 + i_2$$

$$\varepsilon = 14 \text{ V.}$$

Bonus

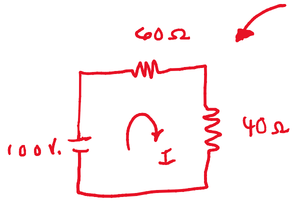
The switch in the figure has been closed for a very long time.

a. (1 pt) What is the charge on the capacitor?
b. (1 pt) What is the current through the 40 ohm resistor
c. (1 pt) The switch is opened at t=0 s. At what time has the charge on the capacitor decreased to 10% of its initial value.



A.)

Closed for a very long time



$R_{eq} = R_1 + R_2$

$R_{eq} = 60 + 40$

$R_{eq} = 100\ \Omega$

$I = \frac{V}{R_1} = \frac{100}{100}$

$I = 1$ A.

$V = IR_2 = 1(40)$

$V = 40$ V.

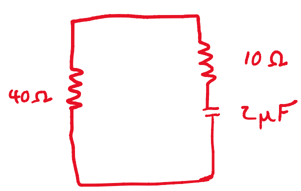
$Q = CV$

$Q = 2(40)$

$\boxed{Q = 80\ \mu C}$

B.) From Part A, $\boxed{I = 1\text{ A.}}$

C.)



$R_{eq} = R_2 + R_2$

$R_{eq} = 10 + 40$

$R_{eq} = 50\ \Omega$

$\tau = RC$

$\tau = (50)(2 \times 10^{-6})$

$\tau = 1 \times 10^{-4}$

$Q = Q_0 e^{-t/\tau}$

$8 \times 10^{-6} = 80 \times 10^{-6} e^{-\frac{t}{1 \times 10^{-4}}}$

$\boxed{t = 2.30 \times 10^{-4}\text{ s.}}$

$Q = .1 Q_0$

$Q = .1(80 \times 10^{-6})$

$Q = 8 \times 10^{-6}$ F