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- 6.1** Consider silicon at  $T = 300$  K that is doped with donor impurity atoms to a concentration of  $N_d = 5 \times 10^{15} \text{ cm}^{-3}$ . The excess carrier lifetime is  $2 \times 10^{-7} \text{ s}$ . (a) Determine the thermal equilibrium recombination rate of holes. (b) Excess carriers are generated such that  $\delta n = \delta p = 10^{14} \text{ cm}^{-3}$ . What is the recombination rate of holes for this condition?

$$n_i = 1.5 \times 10^{10} / \text{cm}^3 \quad N_d = 5 \times 10^{15} / \text{cm}^3 \quad R_{p0} = \frac{p_0}{\tau} = \frac{45000}{2 \times 10^{-7}}$$

$$a.) \quad n = \frac{(1.5 \times 10^{10})^2}{5 \times 10^{15}} = 45000$$

$$R_{p0} = 2.25 \times 10^{11} / \text{cm}^3 \cdot \text{s}$$

$$b.) \quad R = \frac{(N_d + p_0)}{\tau_{p0} N_d} a_p = \frac{(5 \times 10^{15} + 45000)}{(2 \times 10^{-7}) (5 \times 10^{15})} (10^{14})$$

$$R = 5 \times 10^{20} / \text{cm}^3 \cdot \text{s}$$

- 6.10** Germanium at  $T = 300$  K is uniformly doped with donor impurity atoms to a concentration of  $4 \times 10^{13} \text{ cm}^{-3}$ . The excess carrier lifetime is found to be  $\tau_{p0} = 2 \times 10^{-6} \text{ s}$ . (a) Determine the ambipolar (i) diffusion coefficient and (ii) mobility. (b) Find the electron and hole lifetimes.

$$n = \frac{N_d}{2} + \sqrt{\left(\frac{N_d}{2}\right)^2 + n_i^2} = \frac{4 \times 10^{13}}{2} + \sqrt{\left(\frac{4 \times 10^{13}}{2}\right)^2 + (2.4 \times 10^{13})^2} = 5.124 \times 10^{13} / \text{cm}^3$$

$$p = \frac{n_i^2}{n} = \frac{(2.4 \times 10^{13})^2}{5.124 \times 10^{13}} = 1.124 \times 10^{13} / \text{cm}^3$$

$$a.) \quad [i] \quad \frac{\mu_n}{D_n} = \frac{e}{kT} \quad D_n = (0.0259)(3900) = 101.01 \text{ cm}^2/\text{s} \quad \frac{\mu_p}{D_p} = \frac{e}{kT} \quad D_p = (0.0259)(1900) = 49.21 \text{ cm}^2/\text{s}$$

$$D' = \frac{D_n D_p (n + p)}{D_n n + D_p p} = \frac{(101.01)(49.21)(5.124 \times 10^{13} + 1.124 \times 10^{13})}{(101.01)(5.124 \times 10^{13}) + (49.21)(1.124 \times 10^{13})} = 54.21 \text{ cm}^2/\text{s}$$

$$[ii] \quad \mu' = \frac{\mu_n \mu_p (p - n)}{\mu_n n + \mu_p p} = \frac{(3900)(1900)(1.124 \times 10^{13} - 5.124 \times 10^{13})}{(3900)(5.124 \times 10^{13}) + (1900)(1.124 \times 10^{13})} = -1340.01 \text{ cm}^2/\text{V} \cdot \text{s}$$

$$b.) \quad \text{Excess carrier lifetime} = \text{hole lifetime} = \tau_{p0} = 2 \times 10^{-6} \text{ s}$$

$$\frac{n}{\tau_{n0}} = \frac{p}{\tau_{p0}}$$

$$\tau_{n0} = 9.12 \times 10^{-6} \text{ s}$$

$$\frac{5.124 \times 10^{13}}{\tau_{n0}} = \frac{1.124 \times 10^{13}}{2 \times 10^{-6}}$$

↑  
electron lifetime

- 6.13** An n-type GaAs semiconductor at  $T = 300$  K is uniformly doped at  $N_d = 5 \times 10^{15} \text{ cm}^{-3}$ . The minority carrier lifetime is  $\tau_{p0} = 5 \times 10^{-8} \text{ s}$ . Assume mobility values of  $\mu_n = 7500 \text{ cm}^2/\text{V}\cdot\text{s}$  and  $\mu_p = 310 \text{ cm}^2/\text{V}\cdot\text{s}$ . A light source is turned on at  $t = 0$  generating excess carriers uniformly at a rate of  $g' = 4 \times 10^{21} \text{ cm}^{-3} \text{ s}^{-1}$  and turns off at  $t = 10^{-6} \text{ s}$ .
- (a) Determine the excess carrier concentrations versus time over the range  $0 \leq t \leq \infty$ .
- (b) Calculate the conductivity of the semiconductor versus time over the same time period as part (a).

a.) For  $t: 0 \leq t \leq 10^{-6}$

$$\Delta p = g' \tau_{p0} \left[ 1 - \exp\left(-\frac{t}{\tau_{p0}}\right) \right] = (4 \times 10^{21})(5 \times 10^{-8}) \left( 1 - e^{-\left(\frac{t}{5 \times 10^{-8}}\right)} \right)$$

$$\Delta p = (2 \times 10^{14}) \left( 1 - e^{-\left(\frac{t}{5 \times 10^{-8}}\right)} \right)$$

For  $t > 10^{-6}$ :

$$\Delta p = g' \tau_{p0} \left[ \exp\left(-\frac{t}{\tau_{p0}}\right) \right] = (4 \times 10^{21})(5 \times 10^{-8}) \left( e^{-\left(\frac{t}{5 \times 10^{-8}}\right)} \right)$$

$$\Delta p = (2 \times 10^{14}) \left( e^{-\left(\frac{t}{5 \times 10^{-8}}\right)} \right)$$

$$\Delta p = \begin{cases} (2 \times 10^{14}) \left( 1 - e^{-\left(\frac{t}{5 \times 10^{-8}}\right)} \right), & 0 \leq t \leq 10^{-6} \\ (2 \times 10^{14}) \left( e^{-\left(\frac{t}{5 \times 10^{-8}}\right)} \right), & t > 10^{-6} \end{cases}$$

b.)  $\sigma_i = e \mu_n N_d$      $\sigma_r = e (\Delta n \mu_n + \Delta p \mu_p)$      $\sigma = \sigma_i + \sigma_r$

$$\sigma = 1.6 \times 10^{-19} \left( 7500 (5 \times 10^{15}) + (7500 + 310) (2 \times 10^{14}) \left( 1 - e^{-\left(\frac{t}{5 \times 10^{-8}}\right)} \right) \right)$$

$$\sigma = 6 + 0.25 \left( 1 - e^{-\left(\frac{t}{5 \times 10^{-8}}\right)} \right) / \Omega \cdot \text{cm}$$

**6.16** In a GaAs material at  $T = 300$  K, the doping concentrations are  $N_d = 8 \times 10^{15} \text{ cm}^{-3}$  and  $N_a = 2 \times 10^{15} \text{ cm}^{-3}$ . The thermal equilibrium recombination rate is  $R_o = 4 \times 10^4 \text{ cm}^{-3} \text{ s}^{-1}$ . (a) What is the minority carrier lifetime? (b) A uniform generation rate for excess carriers results in an excess carrier recombination rate of  $R' = 2 \times 10^{21} \text{ cm}^{-3} \text{ s}^{-1}$ . What is the steady-state excess carrier concentration? (c) What is the excess carrier lifetime?

$$n_o = N_d - N_a = 8 \times 10^{15} - 2 \times 10^{15} = 6 \times 10^{15} / \text{cm}^3$$

$$p_o = \frac{n_i^2}{n_o} = \frac{(1.8 \times 10^6)^2}{6 \times 10^{15}} = 5.4 \times 10^{-4} / \text{cm}^3$$

$$a.) R_o = \frac{p_o}{\tau_{p0}} \quad \tau_{p0} = \frac{p_o}{R_o} = \frac{5.4 \times 10^{-4}}{4 \times 10^4} = 1.35 \times 10^{-8} \text{ s}$$

$$b.) \Delta p = g' \tau_{p0} = (2 \times 10^{21}) (1.35 \times 10^{-8})$$

$$\Delta p = 2.7 \times 10^{13} / \text{cm}^3$$

c.) Minority carrier lifetime = Excess carrier lifetime

$$1.35 \times 10^{-8} \text{ s}$$

**6.18** A semiconductor is uniformly doped with  $10^{17} \text{ cm}^{-3}$  acceptor atoms and has the following properties:  $D_n = 27 \text{ cm}^2/\text{s}$ ,  $D_p = 12 \text{ cm}^2/\text{s}$ ,  $\tau_{n0} = 5 \times 10^{-7} \text{ s}$ , and  $\tau_{p0} = 10^{-7} \text{ s}$ . An external source has been turned on for  $t < 0$  producing a uniform concentration of excess carriers at a generation rate of  $g' = 10^{21} \text{ cm}^{-3} \text{ s}^{-1}$ . The source turns off at time  $t = 0$  and back on at time  $t = 2 \times 10^{-6} \text{ s}$ . (a) Derive the expressions for the excess carrier concentration as a function of time for  $0 \leq t \leq \infty$ . (b) Determine the value of excess carrier concentration at (i)  $t = 0$ , (ii)  $t = 2 \times 10^{-6} \text{ s}$ , and (iii)  $t = \infty$ . (c) Plot the excess carrier concentration as a function of time.

$$\begin{aligned} a.) \quad \Delta n &= g' \tau_{n0} \exp\left(-\frac{t}{\tau_{n0}}\right) = (10^{21}) (5 \times 10^{-7}) e^{-\frac{t}{\tau_{n0}}} \\ &= (5 \times 10^{14}) e^{-\frac{t}{\tau_{n0}}} \end{aligned}$$

$$\textcircled{a} \quad t = 2 \times 10^{-6} \text{ s}$$

$$\Delta n_1 = (5 \times 10^{14}) e^{-\frac{2 \times 10^{-6}}{5 \times 10^{-7}}} = 9.16 \times 10^{12} / \text{cm}^3$$

$$\Delta n = (5 \times 10^{14} - 9.16 \times 10^{12}) \left[1 - \exp\left(-\frac{t}{\tau_{n0}}\right)\right] + 9.16 \times 10^{12}$$

$$\Delta n = (4.908 \times 10^{14}) \left[1 - \exp\left(-\frac{t}{\tau_{n0}}\right)\right] + 9.16 \times 10^{12} / \text{cm}^3$$

b.)

$$[i] \quad t = 0 \text{ s}$$

$$\Delta n = (5 \times 10^{14}) e^{-\frac{0}{\tau_{n0}}} = (5 \times 10^{14}) e^0 = (5 \times 10^{14}) / \text{cm}^3$$

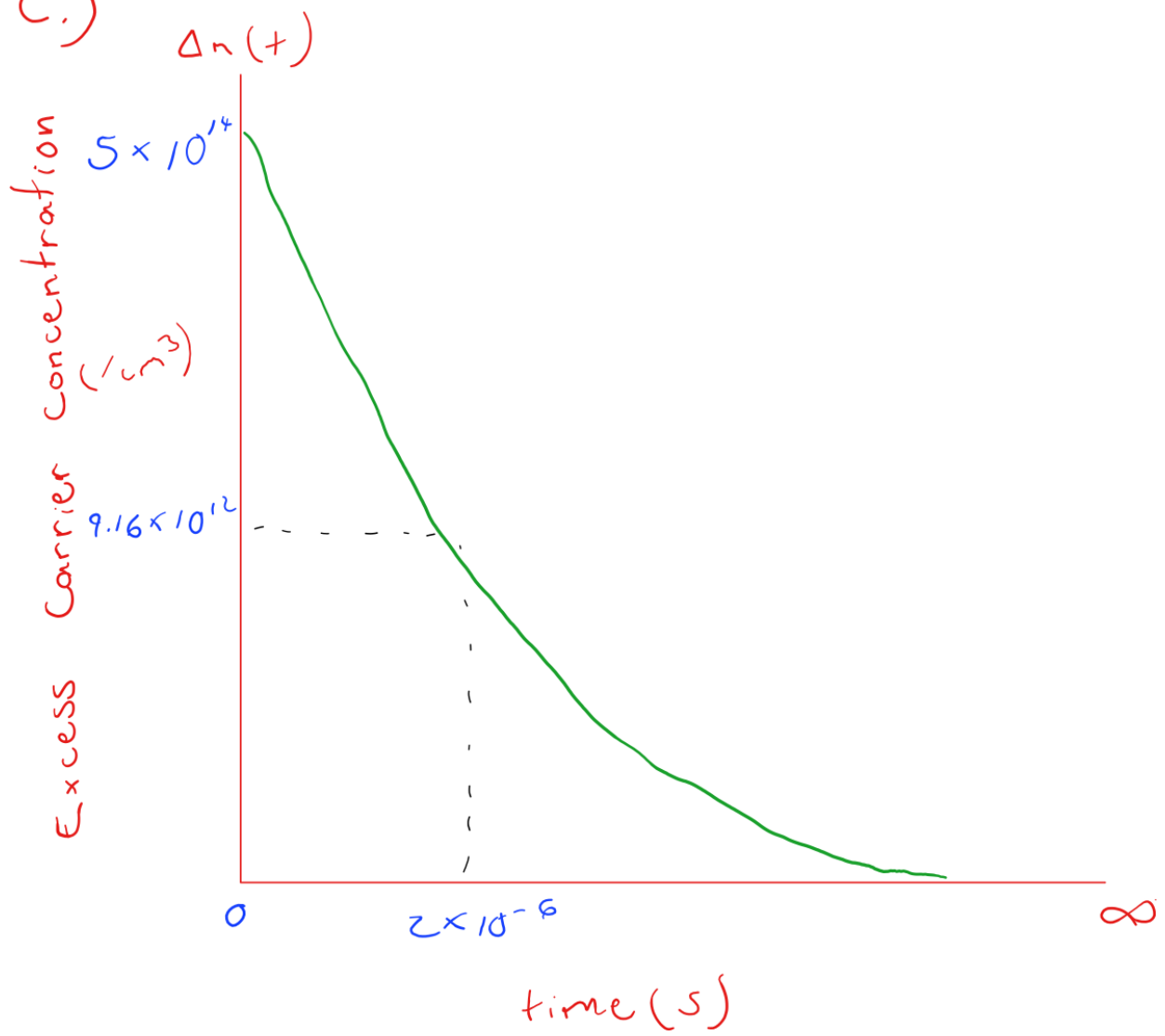
$$[ii] \quad t = 2 \times 10^{-6} \text{ s}$$

$$\Delta n = (5 \times 10^{14}) e^{-\frac{2 \times 10^{-6}}{5 \times 10^{-7}}} = 9.16 \times 10^{12} / \text{cm}^3$$

$$[iii] \quad t = \infty$$

$$\Delta n = (5 \times 10^{14}) e^{-\frac{\infty}{5 \times 10^{-7}}} = (5 \times 10^{14}) / \text{cm}^3$$

c.)



**6.30**

An n-type silicon semiconductor, doped at  $N_d = 4 \times 10^{16} \text{ cm}^{-3}$ , is steadily illuminated such that  $g' = 2 \times 10^{21} \text{ cm}^{-3} \text{ s}^{-1}$ . Assume  $\tau_{n0} = 10^{-6} \text{ s}$  and  $\tau_{p0} = 5 \times 10^{-7} \text{ s}$ .

(a) Determine the thermal-equilibrium value of  $E_F - E_{Fi}$ . (b) Calculate the quasi-Fermi levels for electrons and holes with respect to  $E_{Fi}$ . (c) What is the difference (in eV) between  $E_{Fn}$  and  $E_F$ ?

a.)  $n_i = 1.5 \times 10^{10}$

$$E_F - E_{Fi} = kT \left[ \ln \left( \frac{n_0}{n_i} \right) \right] = 0.259 \ln \left( \frac{4 \times 10^{16}}{1.5 \times 10^{10}} \right)$$

$$= \boxed{0.383225 \text{ eV}}$$

b.)  $\Delta n = g' \tau_{p0} = (2 \times 10^{21}) (5 \times 10^{-7}) = 10^{15} / \text{cm}^3 \cdot \text{s}$

Holes:

$$E_{Fi} - E_{Fp} = kT \left[ \ln \left( \frac{p_0}{p_i} \right) \right] = 0.259 \ln \left( \frac{5.625 \times 10^3 + 10^{15}}{1.5 \times 10^{10}} \right)$$

$$= \boxed{0.287680 \text{ eV}}$$

Electrons:

$$E_{Fn} - E_{Fi} = kT \left[ \ln \left( \frac{n_0}{n_i} \right) \right] = 0.259 \ln \left( \frac{4 \times 10^{16} + 10^{15}}{1.5 \times 10^{10}} \right)$$

$$= \boxed{0.383865 \text{ eV}}$$

c.)

$$E_{Fn} - E_F = (E_{Fn} - E_{Fi}) - (E_F - E_{Fi})$$

$$= 0.3838 - 0.3832 = \boxed{0.000640 \text{ eV}}$$