Exam 1

Monday, March 8, 2021 10:54 AM

"I pledge my honor I have abided by the Stevens Honor system."

- Alex Jaskins

Problem 1. Consider Boolean function $f(a, b) = ((a \to b) \land (b \to a)) \lor a$.

a) *(5 points)* Construct the truth table of f;

a	b	$a \to b$	$b \to a$	$((a \to b) \land (b \to a)) \lor a$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

b) *(3 points)* Construct the truth table of $\neg f$ (you can use the truth table of f).

$\neg(((a \to b) \land (b \to a)) \lor a)$

$= \neg((\neg a \lor b) \land (\neg b \lor a) \lor a)$ (Using DeMorgan's laws and negation laws)

$= (a\bar{b}) \lor (b\bar{a}) \land \neg a$

a	b	$a\bar{b}$	$b\bar{a}$	$\bar{a}$	$(a\bar{b}) \lor (b\bar{a}) \land \neg a$
T	T	F	F	F	F
T	F	T	F	F	F
F	T	F	T	T	T
F	F	F	F	T	F

c) *(4 points)* Is f a contradiction or a tautology? Explain why.

Based on the truth table for f, it is neither a contradiction nor a taughtology, for it is true sometimes and false other times.

d) *(4 points)* Find the DNF of f;

For wherever f is true:

$\underline{\text{DNF:}}$ $(a \land b) \lor (a \land \neg b) \lor (\neg a \land \neg b)$

$f(a,b)$

e) *(2 points)* Is f logically equivalent to $\neg(b \lor \neg a)$? Explain why.

$= \neg b \land a$

a	b	$\neg b$	$\neg b \wedge a$
T	T	F	F
T	F	T	T
F	T	F	F
F	F	T	F

The truth table does not compare to f, so it is not logically equivalent.

f) *(2 points)* Is $\neg f$ logically equivalent to $\neg(b \vee \neg a)$? Explain why.

No, for $\neg(b \vee \neg a)$ to be true, $a = T$ and $b = F$. For $\neg f$ to be true, $a = F$ and $b = T$.

Problem 2. Consider the set X of all people and the following predicates on X:

- $M(x)$ that is true if and only if x has been to the moon.
- $S(x)$ that is true if and only if x has been to space.

Translate the following expressions from English to predicate logic statements with quantifiers:

a) *(5 points)* P_1: "There exists a person who has been to the moon"

$\exists x \in X, M(x)$

b) *(5 points)* P_2: "All people who has been to the moon has also been to space"

$\forall x \in X, (M(x) \rightarrow S(x))$

c) *(5 points)* P_3: "There exists a person who has been to the moon but haven't been to space"

$\exists x \in X, (M(x) \wedge \neg(S(x)))$

d) *(3 points)* P_4: "There is a person who has been to space and to the moon that flew with another person who has been to space but not to the moon". You can use predicates $E(x, y)$ that is true if x and y are the same person, and $F(x, y)$ that is true if x and y flew together.

$\exists x \in X, (F(x,y) \wedge (M(x) \wedge (S(x)) \wedge (S(y) \wedge \neg M(y))))$

bonus (2 points): Prove that, in symbolic form, P_3 is logically equivalent to the negation of P_2

$\neg P_2 = \neg(\forall x \in X, (M(x) \rightarrow S(x))) = \exists x \in X, \neg(M(x) \rightarrow S(x))$

$M(x) \rightarrow S(x) = \neg M(x) \vee S(x)$

$\neg(\neg M(x) \vee S(x)) = M(x) \wedge \neg S(x)$

$= \exists x \in X, (M(x) \wedge \neg S(x)) = P_3$

$= \exists x \in X, (M(x) \wedge \neg S(x))$ - 1 3

Problem 3. Consider the set of integers $\mathbb{Z}$ and the relation R on $\mathbb{Z}$ defined as: aRb if the number of $0's$ at the end of a and the number of 0's at the end of b are equal. E.g. $10R2020$ and $2021R2022$, but $100 \not R\ 1010$.

a) *(10 points)* Prove that R is an equivalence relation.

Reflexive:

✓

a R a ; (i.e. a = 10)

10 R 10 is valid (# of 0's are equal)

Symmetric:

✓

a R b and b R a

if say a = 100 and b = 21,300

They have the same # of 0's in either case

(100 R 21,300 ≡ 21,300 R 100)

Transitive:

✓

a R b and b R c ∴ a R c is true because in every case they have the same # of zeros

b) *(5 points)* Give 3 (different) examples of integers a that are in the equivalence class of 134.

- 134 R 35 b R a
- 134 R 138 b R a
- 134 R 5112 b R a

c) *(3 points)* How many integers are there in the equivalence class of 134? Explain your answer.

Any number without a zero at the end of it

d) *(2 points)* How many different equivalence classes of R are there? Explain your answer.

Infinitely many, for more zeros can be added (Approaches infinity)