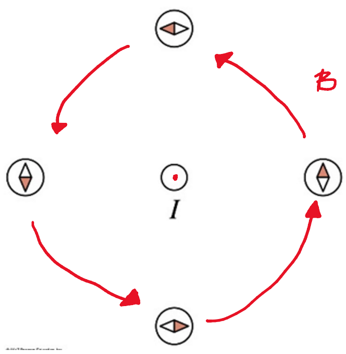
Final Exam

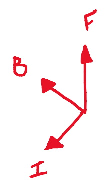
Wednesday, December 9, 2020   1:04 PM

"I pledge my honor I have abided by the Stevens Honor system."

-Alex Gaskins

1. (1 pt) What is the current direction in the wire in the figure? Explain. The brown end of the compass is the north pole. (write a sentence)

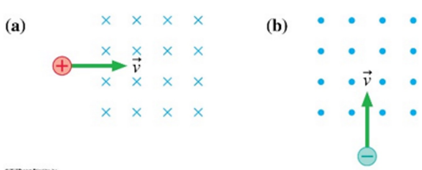




(Direction of B is determined by the orientation of the north pole)

According to the right-hand rule, the current is orthogonal to B and F, and thus current travels out of the page.

2. (2 pts) What is the initial direction of deflection for the charged particles entering the magnetic fields shown in the figure? Explain.

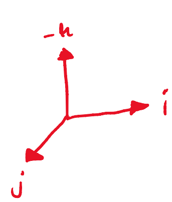


A.)

B direction $= -\hat{k}$

Proton travels to the right $= +\hat{i}$

Direction of deflection is determined by the magnetic force where



$F = q(v \times B)$

$F = i \times (-k) = -(-j)$

$F = +j$ (upwards)

$F = 1 \times (-k) = (-)$

$F = +j$ (upwards)

$\boxed{\text{Direction is in the positive } \hat{j} \text{ direction.}}$

B.)

B is out of the page $= \hat{k}$

V travels upwards $= \hat{j}$

$F = -q(v \times B) = -1(i)$

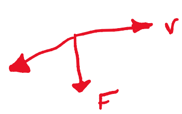
$F = -i$ (to the left)

$\boxed{\text{Direction is in the negative } \hat{i} \text{ direction.}}$

3. (2pts) Determine the magnetic field direction that causes the charged particles shown in the figure to experience the magnetic force. Explain.

$F = q(v \times B)$

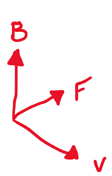
A.)



Positive charge, so apply right hand rule

$\boxed{\text{Magnetic field points out the page}}$

B.)

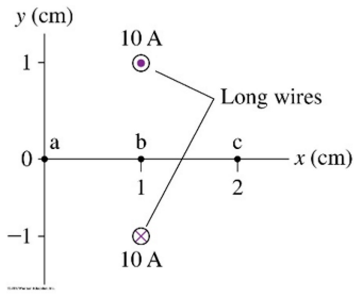


but charge is negative (left-hand rule)

$F = -q(v \times B)$

Magnetic field points in the upward direction

4. (2pts) What are the magnetic fields at points a to c in the figure? Give your answers as vectors.

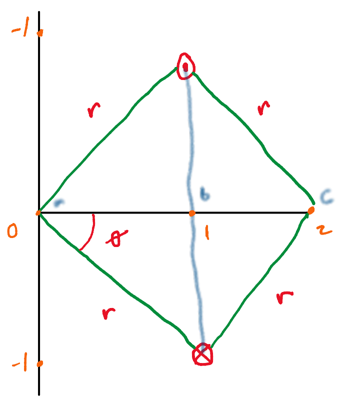


$$B = \frac{\mu_0 I}{2\pi r}$$

$$r = \sqrt{(1\times10^{-2})^2 + (1\times10^{-2})^2}$$

$$r = \sqrt{.0002}$$

$$r = .0141 \text{ m.}$$



$$B_2 = \frac{(4\pi\times10^{-7})(10)}{2\pi(.0141)}$$

$$B_2 = 1.418\times10^{-4}\ T = B_1$$

$$\theta = \tan^{-1}\left(\frac{.01}{.01}\right)$$

$$\theta = 45^\circ$$

$$B_1' = B_1\cos(\theta\hat{i}) - B_1\sin(\theta\hat{j})$$

$$B_2' = B_2\cos(\theta\hat{i}) - B_2\sin(\theta\hat{j})$$

$$B_a = B_1' + B_2'$$

$$B_a = (B_1 + B_2)\cos(\theta\hat{i}) + \cancel{(B_2 - B_1)}\sin(\theta\hat{j})$$

$$B_a = (2(1.418\times10^{-4}))\cos(45\hat{i})$$

$$\boxed{B_a = 2\times10^{-4}\ T\ \hat{i} \quad \text{at point } a}$$

$$B_1 = B_2 = \frac{\mu_0 I}{} = \frac{(4\pi\times10^{-7})(10)}{} = 2\times10^{-4}\ T$$

$$B_1 = B_2 = \frac{\mu_0 I}{2\pi r} = \frac{(4\pi \times 10^{-7})(10)}{2\pi(.01)} = 2 \times 10^{-4}\ T$$

$$B_1' = B_1 \hat{i}$$

$$B_2' = B_2 \hat{i}$$

$$B_b = B_1' + B_2'$$

$$B_b = (2 \times 10^{-4})\hat{i} + (2 \times 10^{-4})\hat{i}$$

$$\boxed{B_b = 4 \times 10^{-4}\ T\ \hat{i} \text{ at point } b}$$

$$B_2 = \frac{(4\pi \times 10^{-7})(10)}{2\pi(.0141)} \qquad \theta = \tan^{-1}\left(\frac{.01}{.01}\right)$$

$$B_2 = 1.418 \times 10^{-4}\ T = B_1 \qquad \theta = 45°$$

$$B_1' = B_1 \cos(\theta \hat{i}) - B_1 \sin(\theta \hat{j})$$

$$B_2' = B_2 \cos(\theta \hat{i}) - B_2 \sin(\theta \hat{j})$$

$$B_c = B_1' + B_2'$$

$$B_c = (B_1 + B_2)\cos(\theta \hat{i}) + \cancel{(B_2 - B_1)\sin(\theta \hat{j})}$$

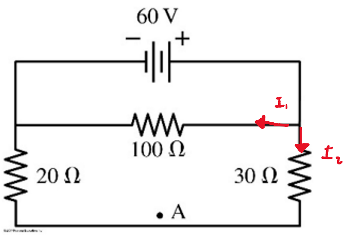
$$B_c = (2(1.418 \times 10^{-4}))\cos(45 \hat{i})$$

$$\boxed{B_c = 2 \times 10^{-4}\ T\ \hat{i} \text{ at point } c}$$

5. (2 pts) Point A in the figure is 2.0 mm from the wire. You can assume that the wire is very long and that all the other wires are too far away to contribute to the magnetic field.

a. what is the current in the circuit?

b. what is the magnetic field at point A.

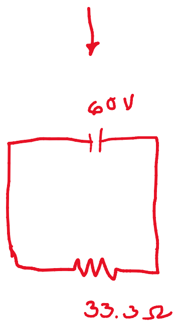


$R_{eq_1} = 30 + 20$

$R_{eq_1} = 50\,\Omega$

$R_{eq_2} = \frac{50(100)}{150}$

$R_{eq_2} = 33.3\,\Omega$



A.)

$V = IR$

$I = \frac{60}{33.3}$

$\boxed{I = 1.8\text{ A}}$

$I_1 = \frac{60}{100}$

$I_1 = .6\text{ A}$

$I = I_1 + I_2$

$1.8 = .6 + I_2$

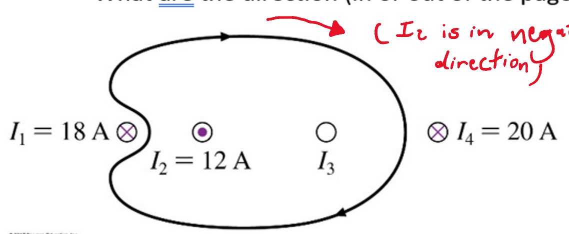
$I_2 = 1.2\text{ A.}$

B.)

$$B_A = \frac{\mu_0 I_2}{2\pi r} = \frac{(4\pi \times 10^{-7})(1.2)}{2\pi(2\times 10^{-3})}$$

$$\boxed{B_A = 1.2 \times 10^{-4}\ T}$$

5. (1 pt) The value of the line integral of $\vec{B}$ around the closed path in the figure is 1.38x10^-5 Tm. What are the direction (in or out of the page) and magnitude of I3?



(I_2 is in negative direction)

$\int B\,ds = \mu_0 I_{through}$

$I_{through} = I_2 + I_3$

$I_{through} = -I_2 + I_3$

$I_{through} = -12 + I_3$

$I_{through} = -I_2 + I_3$

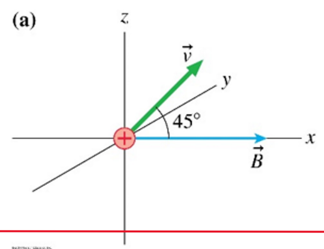
$I_{through} = -12 + I_3$

$1.38 \times 10^{-5} = \mu_0 (-12 + I_3)$

$10.98 = -12 + I_3$

$\boxed{I_3 = 22.98\ A \text{ into the page}}$

6. (4 pts) A proton moves in the magnetic field $\vec{B} = 0.50\hat{\imath}T$ with a speed of 1.0x10^7 m/s in the directions shown in the figure. For each, what is the magnetic force $\vec{F}$ on the proton? Give your answers in component form.

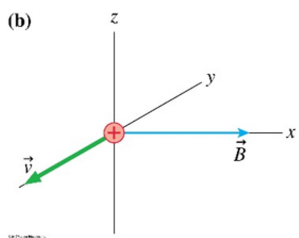


$F = q(v \times B)$

$F = (1.6 \times 10^{-19})(1.0 \times 10^7)(.5)\sin(45)$

$F = 5.66 \times 10^{-13}$ N.

$\boxed{F = \langle 0\hat{\imath},\ 5.66 \times 10^{-13}\hat{\jmath},\ 0\hat{k} \rangle \text{ N.}}$

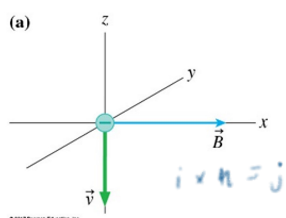


$F = q(v \times B)$

$F = (1.6 \times 10^{-19})(1.0 \times 10^7)(.5)\sin(90)$

$F = 8 \times 10^{-13}$ N.

$\boxed{F = \langle 0\hat{\imath},\ 0\hat{\jmath},\ 8 \times 10^{-13}\hat{k} \rangle \text{ N.}}$



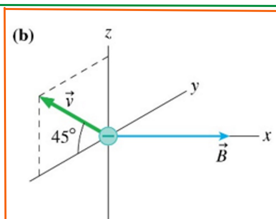
$i \times k = j$

$F = -q(v \times B)$

$F = -(1.6 \times 10^{-19})(1.0 \times 10^7)(.5)\sin(90)$

$F = -8 \times 10^{-13}$ N.

$\boxed{F = \langle 0\hat{\imath},\ -8 \times 10^{-13}\hat{\jmath},\ 0\hat{k} \rangle \text{ N.}}$



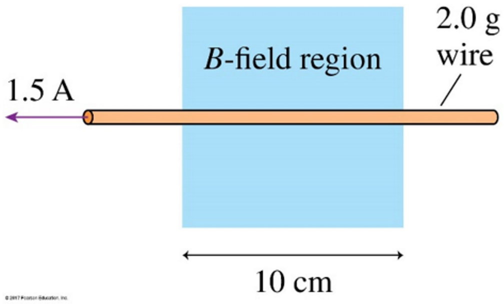
$F = -q(v \times B)$

$F = -(1.6 \times 10^{-19})(1.0 \times 10^7)(.5)\left(-\frac{1}{\sqrt{2}}\hat{\jmath} + \frac{1}{\sqrt{2}}\hat{k}\right)$

$F = 5.66 \times 10^{-13}(-\hat{k} - \hat{\jmath})$ N.

$\boxed{F = \langle 0\hat{\imath},\ -5.66 \times 10^{-13}\hat{\jmath},\ -5.66 \times 10^{-13}\hat{k} \rangle \text{ N.}}$

7. (1 pt) What magnetic field strength and direction will levitate the 2.0 g wire in the figure?



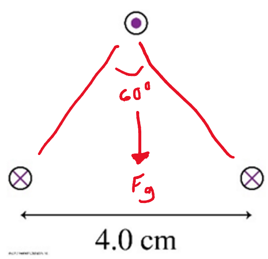
$F = BIL = mg$

$B = \frac{mg}{IL} = \frac{(2\times10^{-3})(9.8)}{(1.5)(.1)}$

$\boxed{B = .131\ T}$

Into the page

8. (2 pts) the figure is a cross section through three long wires with linear density 50 g/m. they each carry equal currents in the directions shown. The lower two wires are 4.0 cm apart and are attached to a table. What current I will allow the upper wire to "float" so as to form an equilateral triangle with the lower wires?



$F_1 = F_2 = \frac{\mu_0 I^2}{2\pi d} L$

$\mu = .05\ kg/m$

$\mu = \frac{m}{L}$

$m = \mu L$

$F_g = mg$

$F_g = \mu L g$

$F_g = 2F_1 \cos(\theta)$

$F_g = 2\left(\frac{\mu_0 I^2}{2\pi d} L\right)\cos(30)$

$\mu L g = \left(\frac{\mu_0 I^2}{\pi d}\right)\cos(30)$

$I = \sqrt{\frac{\pi \mu g d}{\mu_0 \cos(30)}}$

$I = \sqrt{\frac{(\pi)(.05)(9.8)(.04)}{(4\pi\times10^{-7})\cos(30)}}$

$\boxed{I = 237.86\ A.}$

9. (1 pt) a square current loop 5.0 cm on each side carriers a 500 mA current. The loop is in a 1.2 T uniform magnetic field. The axis of the loop, perpendicular to the plane of the loop, is 30 degrees away from the field direction. What is the magnitude of the torque on the current loop?

$A = (5\times10^{-2})^2$

$A = 25\times10^{-4}\ m.$

$$\tau = IAB\sin(30)$$

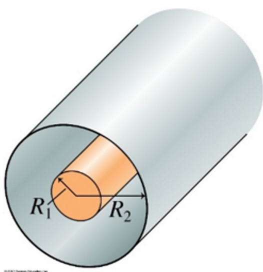
$$\tau = (500\times10^{-3})(25\times10^{-4})(1.2)\sin(30)$$

$$\boxed{\tau = 7.5\times10^{-4}\ \text{N}\cdot\text{m}}$$

10. (2pts) the coaxial cable shown in the figure consists of a solid inner conductor of radius R1 surrounded by a hollow, very thin outer conductor of radius R2. The two carry equal currents I, but in opposite directions. The current density is uniformly distributed over each conductor.

a. find expressions for three magnetic fields: within the inner conductor, in the space between the conductors, and outside the outer conductor.

b. draw a graph of B versus r from r=0 to r=2R2 if R1=1/3R2.



A.) $J_1 = \frac{I}{A_1} = \frac{I}{\pi R_1^2}$

$I_{enclosed} = J_1(\pi r^2) = \frac{I}{\pi R_1^2}(\pi r^2)$

$I_{enclosed} = \frac{I r^2}{R_1^2}$

$2\pi r = \mu_0 I$

$\int B_{in}\, dL = \mu_0 I_{enclosed} = \mu_0\left(\frac{I r^2}{R_1^2}\right)$

$B_{in}(2\pi r) = \mu_0\left(\frac{I r}{R_1^2}\right)$

$$\boxed{B_{in} = \frac{\mu_0 I r}{2\pi R_1^2}}$$

$\int B_{between}\, dL = \mu_0 I$

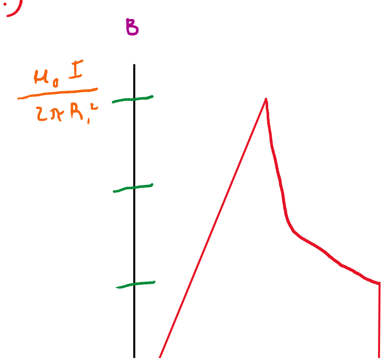
$B_{between}(2\pi r) = \mu_0 I$

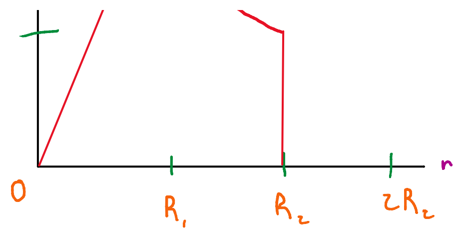
$$\boxed{B_{between} = \frac{\mu_0 I}{2\pi r}}$$

$\int B_{out}\, dL = \mu_0(0)$

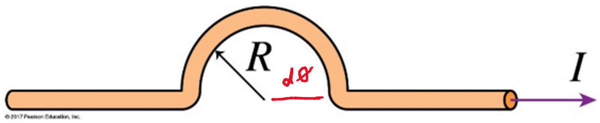
$$\boxed{B_{out} = 0}$$

B.)





11. (1 pt) What is the magnetic field strength at the center of the semicircle in the figure?



$$B = \frac{\mu_0 I\, ds \times R}{4\pi R^2} = \frac{\mu_0}{4\pi}\left(\frac{I R\, d\theta}{R^2}\right) = \frac{\mu_0}{4\pi}\left(\frac{I}{R}\, d\theta\right)$$

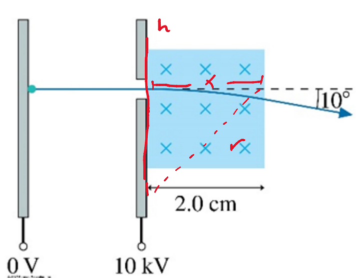
$$B_{net} = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\frac{\mu_0}{4\pi}\right)\left(\frac{I}{R}\right) d\theta$$

$$B_{net} = \left(\frac{\mu_0 I}{4\pi R}\right)\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\theta = \left[\left(\frac{\mu_0 I}{4\pi R}\right)(\theta)\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$B_{net} = \left(\frac{\mu_0 I}{4\pi R}\right)\left(\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right) = \left(\frac{\mu_0 I}{4\pi R}\right)(\pi) = \frac{\mu_0 I}{4R}$$

$$\boxed{\text{Magnetic field strength at the center of the semicircle is } \frac{\mu_0 I}{4R}}$$

12. (1pt) an electron in a cathode ray tube is accelerated through a potential difference of 10 kV, then passes through the 2.0 cm wide region of uniform magnetic field as shown in the figure. What field strength will deflect the electron by 10 degrees?



$$h = r - r\cos(\theta)$$

$$r - h = r\cos(\theta)$$

$$(r-h)^2 + x^2 = r^2(\cos^2(\theta) + \sin^2(\theta))$$

$$\approx r^2 \quad \text{where small } |\theta| \text{ case: } \sin(\theta) = \frac{x}{r}$$

$$\sin(10) = \frac{(2\times10^{-2})}{r}$$

$$r = \frac{(2\times10^{-2})}{\sin(10)}$$

$$r = .1152\, m.$$

$$K = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2K}{m}} \;;\quad K = qV$$

$$v = \sqrt{\frac{2(1.6\times10^{-19})(10\times10^3)}{}}$$

$$v = \sqrt{\frac{2(1.6\times10^{-19})(10\times10^{3})}{(9\times10^{-31})}}$$

$$v = 5.963\times10^{7}\ m/s$$

$$r = \frac{mv}{qB} \ ; \quad B = \frac{mv}{qr}$$

$$B = \frac{(9\times10^{-31})(5.963\times10^{7})}{(1.6\times10^{-19})(.1152)}$$

$$\boxed{B = 2.912\times10^{-3}\ T}$$

13. (3 pts) the figure shows a mass spectrometer, an instrument to identify different molecules and elements by measuring the charge to mass ratio, q/m. the sample is ionized and accelerated through the potential ΔV, and then they enter a region of uniform magnetic field. The field bends the ions into circular trajectories, but after just half a circle they either strike the wall or pass through a small opening to a detector. As the accelerating voltage is slowly increased, different ions reach the detector and are measured. Consider a mass spectrometer with a 200.0 mT magnetic field and an 8.0000 cm spacing between the entrance and exit holes. What accelerating voltages ΔV are required to detect the ions

a. O_2^+

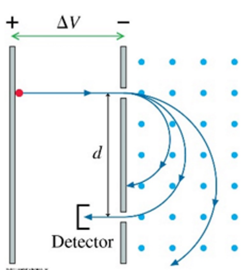
b. N_2^+

c. CO^+

The atomic masses you will need are

$^{12}_{6}C$	$12.000\ u$
$^{14}_{7}N$	$14.003\ u$
$^{16}_{8}O$	$15.995\ u$

1 u =1.6605x10^-27 kg   e=1.6022x10^-19 C



$$K_1 + U_1 = K_2 + U_2 \qquad r = \frac{mv}{qr}$$

$$0 + e\Delta V = \frac{1}{2}mv^2 + 0 \qquad v = \frac{eBd}{2m}$$

$$\Delta V = \frac{mv^2}{2e}$$

$$\Delta V = \frac{m}{2e}\left(\frac{e^2B^2d^2}{4m^2}\right) = \frac{eB^2d^2}{8m}$$

$$\Delta V = qB^2d^2$$

$$\Delta V = \frac{qB^2 d^2}{8m}$$

A.) $$\Delta V_{O_2} = \frac{(1.6\times10^{-19})(.2)^2(.08)^2}{8(5.3119\times10^{-26})}$$

$$\boxed{\Delta V_{O_2} = 96.39\ V}$$

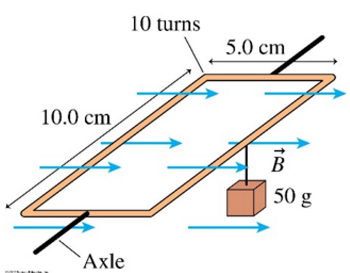
B.) $$\Delta V_{N_2} = \frac{(1.6\times10^{-19})(.2)^2(.08)^2}{8(4.6504\times10^{-26})}$$

$$\boxed{\Delta V_{N_2} = 110.1\ V}$$

C.) $$\Delta V_{CO} = \frac{(1.6\times10^{-19})(.2)^2(.08)^2}{(12+15.995)(1.6605\times10^{-27})}$$

$$\boxed{\Delta V_{CO} = 110.14\ V}$$

14. (1 pt) the 10 turn loop of wire shown in the figure lies in a horizontal plane, parallel to a uniform horizontal magnetic field, and carries a 2.0 A current. The loop is free to rotate about a nonmagnetic axle through center. A 50 g mass hangs from one edge of the loop. What magnetic field strength will prevent the loop from rotating about the axle?



$$\tau = r\times F = r(mg) = \left(\frac{5.0\times10^{-2}}{2}\right)(50\times10^{-3})(9.8)$$

$$\tau = .01225\ N\cdot m$$

$$.01225 = .1B$$

$$\boxed{B = .1225\ T}$$

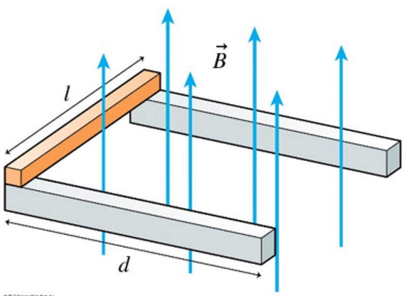
$$\mu = nIA = nI(l(b))$$
$$\mu = (10)(2.0)(10\times10^{-2})(5\times10^{-2})$$
$$\mu = .1\ Am^2$$

$$\tau = \mu\times B$$
$$\tau = \mu B\sin(\theta)$$
$$\tau = (.1)B\sin(90)$$

15. (2 pts) a conducting bar of length l and mass m rests at the left end of the two frictionless rails of length d as shown in the figure. A uniform magnetic field of strength B points upward.

a. in which direction, into or out of the page, will a current through the conducting bar cause the bar to experience a force to the right?

b. find an expression for the bar's speed as it leaves the rails at the right end



A.) $F = B i L \hat{i}$

$\vec{B} = B\hat{j}$

$\vec{F} = i(B \times L)$

$B i L \hat{i} = i(B \times L)$ $\qquad \hat{i} = -\hat{j} \times \hat{k}$

$\boxed{\text{Into the page}}$

B.) $\vec{F}\vec{d} = \frac{1}{2}mv^2$

$B i L \hat{i}\, d\hat{i} = \frac{1}{2}mv^2$

$$\boxed{v = \sqrt{\frac{2BiLd}{m}}}$$