- Q1: Read the below paper and write a summary (a minimum of half a page) to describe what you have learned from it. (20 points)

R.T. O'Brien and J.M. Watkins, “A unified approach for teaching root locus and bode compensator design, Proceedings of the 2003 American Control Conference, 2003.

The article discussed how the methods used to teach both root locus and bode design can be perceived as impractical by students due to the heavily theoretical backbone behind its methodology. Unfortunately, the intensity of the algorithms used to present the principles and functionality of these techniques can skew how they are incorporated into practical scenarios. Progress has been made in response to this issue, where labs focused on testing control systems in real world scenarios are becoming more common as this drawback is becoming more familiar. However, in the midst of this response, the issue can still be passively addressed by redirecting attention towards more important design qualification metrics, such as the type of compensator and actuator being applied. In many aspects, root locus may be seen as an impractical concept being taught, in comparison to more recent incorporations like Bode and PID. The article compared the traditional root locus methods to these bode methods to establish a more identifiable connection between these two methods. The conclusion was that the root locus method may not be as practical as bode in general, but it is ideal in the perspective of developing an understanding towards the importance of proper compensator functionality and how this can be more efficiently computed depending on the method applied. As a result, it is still of interest to reinforce the methods used in root locus calculation in spite of more straightforward bode routines.

Q2: Sketch the Bode plot, and use Matlab to check the accuracy of your plot. (40 points)

$$G(s) = \frac{(s+2)}{s^2(s+10)(s^2+6s+25)}$$

$$G(s) = \frac{(s+2)}{s^2(s+10)(s^2+6s+25)} = \frac{(j\omega+2)}{(j\omega)^2(j\omega+10)((j\omega)^2+6(j\omega)+25)} = \frac{j\omega+2}{-\omega^2(10+j\omega)[25-\omega^2+j6\omega]}$$

Magnitude: $\left|\frac{j\omega+2}{-\omega^2(10+j\omega)[25-\omega^2+j6\omega]}\right| = \frac{\sqrt{4+\omega^2}}{\omega^2\sqrt{\omega^2+100}\sqrt{(25-\omega^2)^2+(6\omega)^2}}$

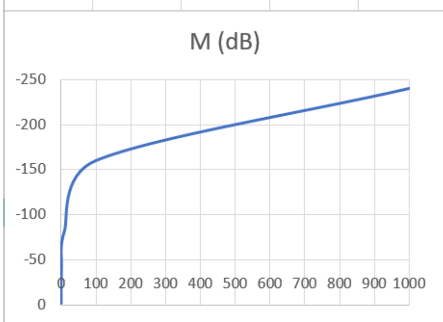
Phase:

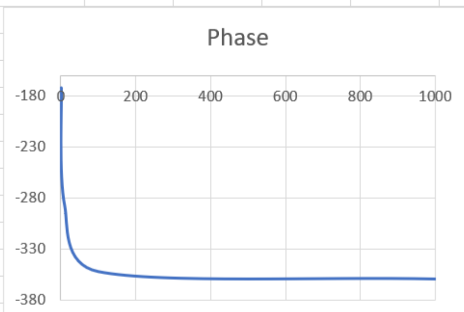
$$\omega \le 5 = -180 + tan^{-1}\frac{\omega}{2} - tan^{-1}\frac{\omega}{10} - tan^{-1}\frac{6\omega}{25-\omega^2}$$

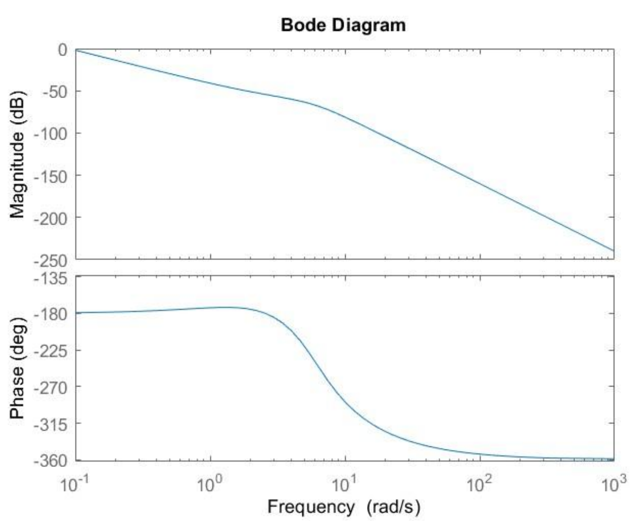
$$\omega \ge 5 = -180 + tan^{-1}\frac{\omega}{2} - tan^{-1}\frac{\omega}{10} - (180 + tan^{-1}\frac{6\omega}{25-\omega^2})$$

ω	M (dB)
0.1	-1.9268
1	-40.92
10	-82.489
100	-160.035
1000	-240

ω	Phase (°)
0.1	-179.08
1	-173.182
10	-287.65
100	-351.99
1000	-359.19







- **Q3: For the system $KG(s) = \frac{K}{(s+2)^2(s+10)}$ (40 points)**
  - a. Approximate the range of K for which the system is stable; use Matlab to check the accuracy of your solution.
  - b. Approximate GM and PM for K=100 via the Bode plot; use Matlab to check the accuracy of your plot.

Transfer function $KG(s) = \frac{K}{(s+2)^2(s+10)} = \frac{K}{s^3+14s^2+44s+40}$

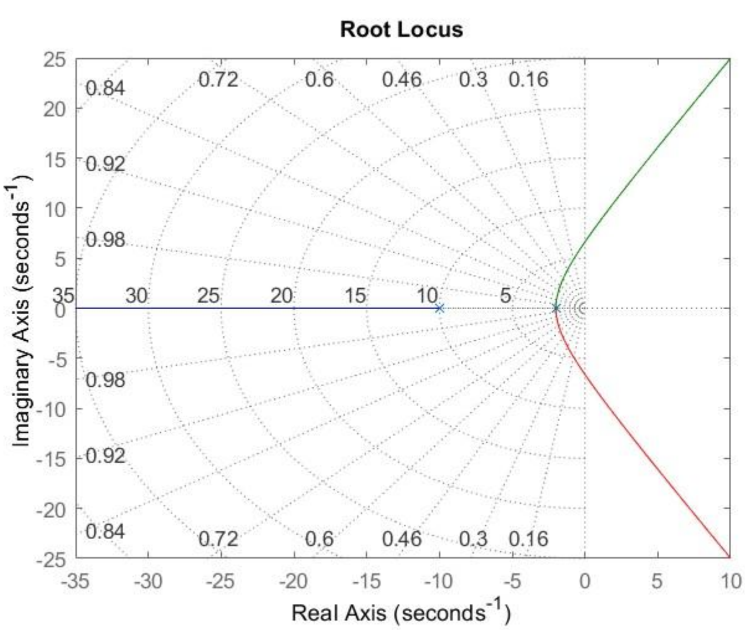
Characteristic equation: $1 + KG(s) = 1 + \frac{K}{s^3+14s^2+44s+40} = s^3 + 14s^2 + 44s + 40 + K$

Find K using Routh-Hurwitz criterion:

s^3	1	44
s^2	14	$40 + K$
s^1	$\frac{44\bullet14-(40+K)}{14}$	
s^0	$40 + K$	

$40 + K > 0$ $\qquad$ $\frac{44\bullet14-(40+K)}{14} > 0$

$K > -40$ $\qquad$ $K < 576$



$KG(j\omega) = \frac{K}{(j\omega+2)^2(j\omega+10)} = \frac{100}{(j\omega+2)^2(j\omega+10)}$

$KG(j\omega)\theta = -\ 2tan^{-1}(\frac{\omega}{2}) - tan^{-1}(\frac{\omega}{10})$

$|KG(j\omega)| = \frac{100}{(\omega^2+4)\sqrt{\omega^2+100}} = 1$

$(\omega^2 + 4)\sqrt{\omega^2 + 100} = 100$

$\omega \approx 2.05$

$KG(j2.05)\theta = -\ 2tan^{-1}(\frac{2.05}{2}) - tan^{-1}(\frac{2.05}{10}) = -\ 91.41 - 11.59 = -\ 103^o$

$PM = 180^o - 103^o = 77^o$

Phase crossover frequency

$KG(j\omega)(-\ 180^o) = -\ 2tan^{-1}(\frac{\omega}{2}) - tan^{-1}(\frac{\omega}{10}) = -\ 180^o$

$\omega_{PC} = 7\ \frac{rad}{sec}$

$|KG(j7)| = \ -\ 20log_{10}[\frac{100}{((7)^2+4)\sqrt{(7)^2+100}}] = 16.21\ dB$