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Moons of Jupiter

Introduction

In our modern society, one can easily perform a simple Google search to find the known value of Jupiter's mass. However, the method by which this value was acquired is often given little to no recognition. Perhaps it is obvious that we cannot simply place Jupiter on a scale to find its mass, so then how is this done?

The purpose of this lab is to demonstrate the physics behind how the mass of Jupiter is acquired. In order to do this, satellite objects that orbit around the planet are needed. This is because the mass of Jupiter plays a role in their orbital strength and gravitational pull. Using factors that can be found through observation, such as the orbital period and semimajor axis of its moons, the mass of Jupiter can be estimated, as will be shown through this experiment.

Data

This lab was performed in Microsoft Excel 2016, using a [data sheet provided by NASA] as a reference for the orbital period and semimajor axis values for each of Jupiter's many moons. From there, R^3 ([Semimajor Axis]^3) and T^2 ([Orbital Period]^2) can be found in order to be incorporated into Newton's Two-Body Equation, derived from Kepler's Third Law: $T^2 = [\frac{4\pi^2}{G(M+m)}]R^3$, where the mass of the satellite (m) can be neglected, since it is so much smaller than Jupiter's mass (the variable of focus). As a result, the equation that is used becomes $T^2 = [\frac{4\pi^2}{GM}]R^3$.

The moons used and their respective data is shown in the figure below:

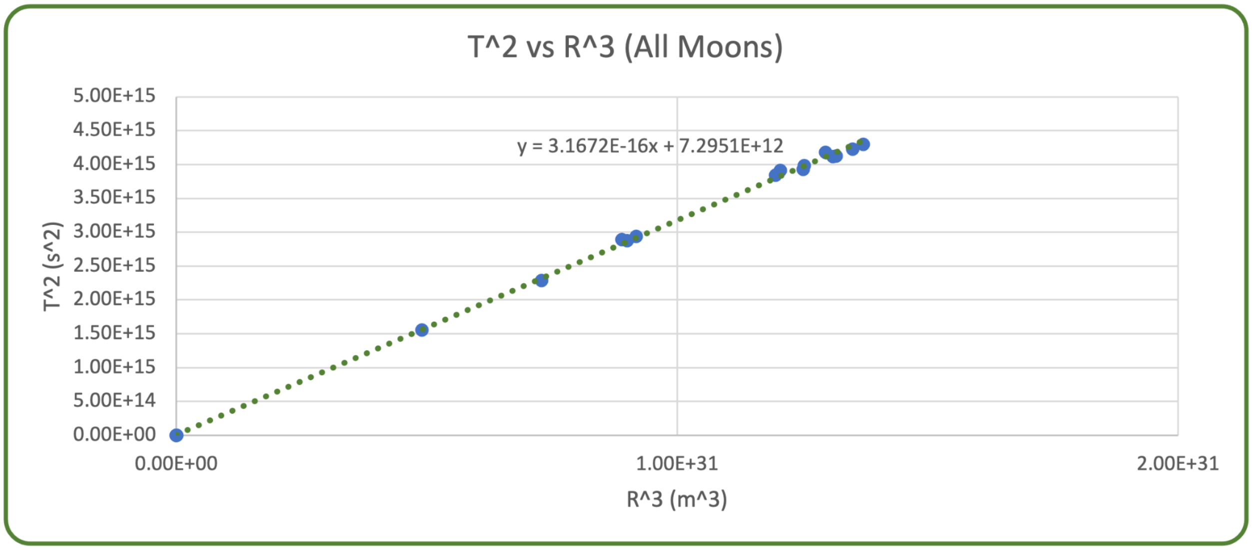
Moons	Semimajor Axis (10^3 km)	Semimajor Axis (m)	Orbital Period (days)	Orbital Period (seconds)	R^3	T^2
Io	421.8	421800000	1.769138	152853.5232	7.50E+25	2.34E+10
Europa	671.1	671100000	3.551181	306822.0384	3.02E+26	9.41E+10
Ganymede	1070.4	1070400000	7.154553	618153.3792	1.23E+27	3.82E+11
Callisto	1882.7	1882700000	16.689017	1441931.0688	6.67E+27	2.08E+12
Carpo	16990	16990000000	456.1	39407040.0000	4.90E+30	1.55E+15
Euporie	19390	19390000000	553.1	47787840.0000	7.29E+30	2.28E+15
Orthosie	20720	20720000000	622.6	53792640.0000	8.90E+30	2.89E+15
Euanthe	20800	20800000000	620.6	53619840.0000	9.00E+30	2.88E+15
Thyone	20940	20940000000	627.3	54198720.0000	9.18E+30	2.94E+15
Eurydome	22870	22870000000	717.3	61974720.0000	1.20E+31	3.84E+15
Arche	22930	22930000000	723.9	62544960.0000	1.21E+31	3.91E+15
Isonoe	23220	23220000000	725.5	62683200.0000	1.25E+31	3.93E+15
Aitne	23230	23230000000	730.2	63089280.0000	1.25E+31	3.98E+15
Sponde	23490	23490000000	748.3	64653120.0000	1.30E+31	4.18E+15
Kalyke	23580	23580000000	743	64195200.0000	1.31E+31	4.12E+15
Pasiphae	23620	23620000000	743.6	64247040.0000	1.32E+31	4.13E+15
Megaclite	23810	23810000000	752.8	65041920.0000	1.35E+31	4.23E+15
Sinope	23940	23940000000	758.9	65568960.0000	1.37E+31	4.30E+15

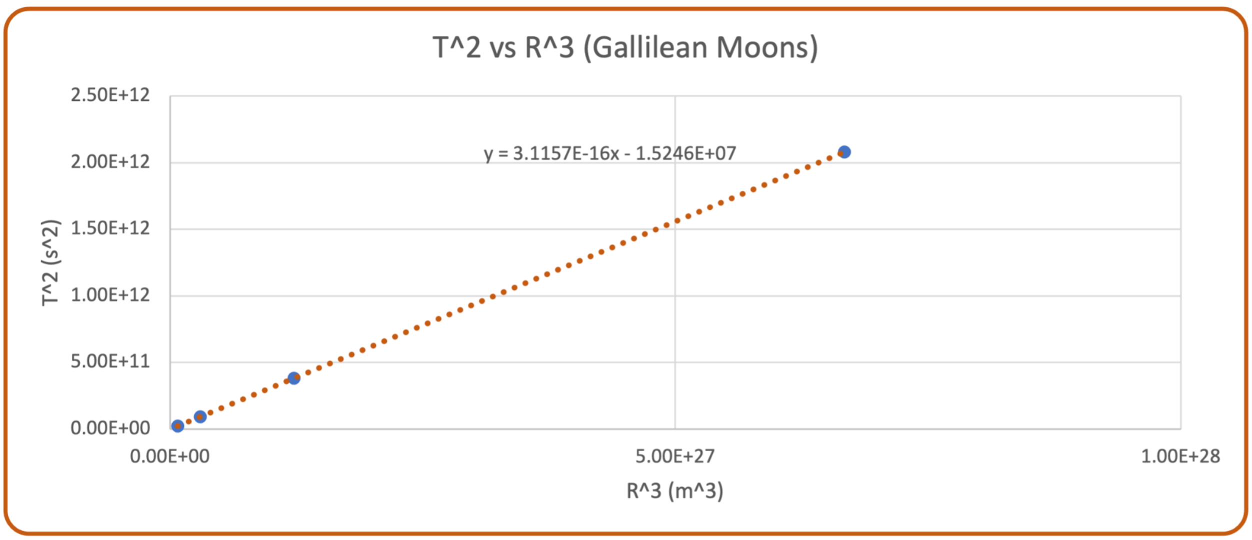
Color Key	
	Gallilean Moons
	Other Moons

As seen from Newton's equation, T^2 is dependent on R^3, with $\frac{4\pi^2}{GM}$ representing the rate of change in the relationship. As a result, the mass of Jupiter (M) can be solved for by simply obtaining the rate of change of the system and equating it to $\frac{4\pi^2}{GM}$ to solve for M.

In this case, two methods were used to find the rate of change of the relationship between T^2 and R^3.

Method 1: Graphical Analysis





Method 2: Least Square Fit

Using the formulas $a = \frac{\sum_{i=1}^{N} x_i y_i - N\overline{xy}}{\sum_{i=1}^{N} x_i^2 - N\overline{x}^2}$ and $b = \frac{\overline{y}\sum_{i=1}^{N} x_i^2 - \overline{x}\sum_{i=1}^{N} x_i y_i}{\sum_{i=1}^{N} x_i^2 - N\overline{x}^2}$ where $y = ax + b$, the rate of change (a) and y-intercept (b) of the data can be accurately acquired for the set of values in the data set, where $y = T^2$ and $x = R^3$. The results are shown below:

Least Square Fit (All Moons)				
x (avg)	8.60E+30			
y (avg)	2.73E+15			
1	-4.23E+47	-1.33E+63	3.18E-16	a
2	-4.23E+47	-1.33E+63	3.18E-16	
3	-4.23E+47	-1.33E+63	3.18E-16	
4	-4.23E+47	-1.33E+63	3.18E-16	
5	-4.15E+47	-1.31E+63	3.18E-16	
6	-4.06E+47	-1.28E+63	3.18E-16	
7	-3.97E+47	-1.25E+63	3.17E-16	
8	-3.97E+47	-1.25E+63	3.17E-16	
9	-3.96E+47	-1.25E+63	3.17E-16	
10	-3.77E+47	-1.19E+63	3.17E-16	
11	-3.76E+47	-1.19E+63	3.17E-16	
12	-3.74E+47	-1.17E+63	3.18E-16	
13	-3.73E+47	-1.17E+63	3.18E-16	
14	-3.69E+47	-1.16E+63	3.17E-16	
15	-3.69E+47	-1.16E+63	3.18E-16	
16	-3.68E+47	-1.16E+63	3.18E-16	
17	-3.66E+47	-1.15E+63	3.18E-16	
18	-3.64E+47	-1.14E+63	3.18E-16	
Total	-7.04E+48	-2.22E+64	3.17585E-16	
1	5.63E+51	1.75E+36	5.63E+51	b
2	9.14E+52	2.85E+37	9.14E+52	
3	1.50E+54	4.69E+38	1.50E+54	
4	4.45E+55	1.39E+40	4.45E+55	
5	2.41E+61	7.62E+45	2.41E+61	
6	5.31E+61	1.66E+46	5.31E+61	
7	7.91E+61	2.57E+46	7.91E+61	
8	8.10E+61	2.59E+46	8.10E+61	
9	8.43E+61	2.70E+46	8.43E+61	
10	1.43E+62	4.59E+46	1.43E+62	
11	1.45E+62	4.72E+46	1.45E+62	
12	1.57E+62	4.92E+46	1.57E+62	
13	1.57E+62	4.99E+46	1.57E+62	
14	1.68E+62	5.42E+46	1.68E+62	
15	1.72E+62	5.40E+46	1.72E+62	
16	1.74E+62	5.44E+46	1.74E+62	
17	1.82E+62	5.71E+46	1.82E+62	
18	1.88E+62	5.90E+46	1.88E+62	
Total	1.81E+63	5.74E+47	1.81E+63	
avg*Total	4.94E+78	4.93E+78	1.92E+12	
Equation	y = 3.17585E-16x + 1.92E+12			

Least Square Fit (Gallilean Moons)				
x (avg)	2.07E+27			
y (avg)	6.45E+11			
1	-5.33E+39	-7.71E+55	6.92E-17	a
2	-5.31E+39	-7.70E+55	6.89E-17	
3	-4.87E+39	-7.56E+55	6.44E-17	
4	8.54E+39	-3.25E+55	-2.62E-16	
Total	-6.97E+39	-2.62E+56	2.66E-17	
1	5.63E+51	1.75E+36	5.63E+51	b
2	9.14E+52	2.85E+37	9.14E+52	
3	1.50E+54	4.69E+38	1.50E+54	
4	4.45E+55	1.39E+40	4.45E+55	
Total	4.61E+55	1.44E+40	4.61E+55	
avg*Total	2.97E+67	2.97E+67	-9.59E+06	
Equation	y = 2.66E-17x - 9.59E+06			

Solving for Jupiter's Mass

As seen from the data obtained, each of the methods provided similar results for the rate of change of the relationship. Using this value, the value of Jupiter's mass can now be solved for. The first step in this process is to set the rate of change equal to $\frac{4\pi^2}{GM}$. From there, the system can be simplified by separating and solving for all of the known constants, where $G = 6.67E\text{-}11\frac{N\bullet m^2}{kg^2}$, yielding $\frac{4\pi^2}{6.67E-11}[\frac{1}{M}] = m$. To solve for M, the expression becomes: $M = \frac{[\frac{4\pi^2}{6.67E-11}]}{m}$. Using Microsoft Excel, the following results were found:

Calculated Mass

Via Graph (All Moons)		Via Graph (Gallilean Moons)	
Mass of Jupiter =	(4pi^2/G)/(a)	Mass of Jupiter =	(4pi^2/G)/(a)
M	1.86878E+27	M	1.89967E+27
%Error	1.5462%	%Error	0.0812%
Via Least Square Fit (All Moons)		**Via Least Square Fit (Gallilean Moons)**	
Mass of Jupiter =	(4pi^2/G)/(a)	Mass of Jupiter =	(4pi^2/G)/(a)
M	1.86369E+27	M	2.23E+28
%Error	1.8145%	%Error	1072.6898%

Actual Mass

M	1.89813E+27

As seen from the data, the results were similar in each case. The exception, in this case, was the mass obtained via the Least Square Fit method for the data pertaining exclusively to the Galilean Moons, which may have been due to the fact that the data set was much smaller, resulting in a greater influence from outliers while solving for the rate of change.

Conclusion

Based on the findings in this experiment, it is evident that Newton's methodology accurately relates orbital motion and position to mass. Using the Two Body Equation, the mass of Jupiter was found using two different methods with two separate data sets (All Moons and Galilean Moons), with notable accuracy overall. Using the known mass of Jupiter, provided by NASA, the experimental values were compared to the known value to find the percent error in each case. The percent error was between 0 and 2 percent for each case, except for with the Least Square Method performed with data from the Galilean Moons, which returned a mass value of 2.23E28 kg, compared to the actual mass of Jupiter at 1.89813E27 kg, yielding a percent error of 1072.69%. As previously mentioned, this may have been due to the fact that a smaller set of data was being used. As you can see, through the laws of physics, the many mysteries that the universe has to offer are all governed by gravity, acting as a key as we continue to explore the cosmos.