Final

Tuesday, May 11, 2021 9:52 PM

"I pledge my honor I have abided by the Stevens Honor system."

\- Alex Gaskins

1. A sensor has a Thevenin equivalent circuit with V_T and R_T. The output of this sensor is connected to a two-resistor voltage attenuator shown in the circuit drawing. (R_1 and R_2 is the two-resistor attenuator). The output of the attenuator will be connected to a measuring equipment which has an input resistance R_{in}.

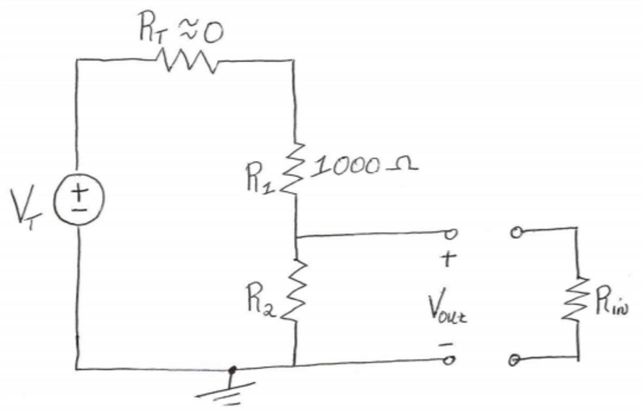

Figure One

For this problem, we can consider that R_T is approximately equal to zero and can be neglected.

a. You are given R_1 equals 1000 Ω. Determine the numerical value of R_2 which will result in the value of V_{out} to equal $(0.2)\cdot V_T$ before connecting to R_{in}. (Before connecting to R_{in}, the value of V_{out} should be equal $(0.2)\cdot V_T$. Determine the required value of R_2). **(2 points)**

Before Connected:

$V_{out} = .2V_T \qquad V_{in} = V_T$

$$V_{out} = \frac{R_2}{R_1 + R_2}(V_{in})$$

$$.2\,V_T = \frac{R_2}{1000 + R_2}(V_T)$$

$$.2(1000 + R_2) = R_2$$

$$200 = .8R_2$$

$$\boxed{R_2 = 250\,\Omega}$$

b. Connecting the output of the attenuator to R_{in} will result in a decrease in the value of V_{out}. (After connecting to R_{in}, V_{out} will no longer be equal to $(0.2)\cdot V_T$). We require that the voltage V_{out} be no lower than $(0.17)\cdot V_T$ after connecting to R_{in}. Determine the minimum value of R_{in} which will result in a value of V_{out} no lower than $(0.17)\cdot V_T$ after connecting to R_{in}. **(15 points)**

$$R_{eq} = \frac{R_2 R_{in}}{R_2 + R_{in}}$$

$$V_{out} = .17\, V_T$$

$$.17\, V_T = \frac{R_{eq}}{R_{eq} + R_1} V_T$$

$$.17\,(R_1) + .17\,(R_{eq})$$

$$.17\, R_1 = .83\, R_{eq}$$

$$\frac{.17\,(1000)}{.83} = R_{eq} = 204.8$$

$$204.8 = \frac{250\, R_{in}}{250 + R_{in}}$$

$$R_{in} = .819\,(250 + R_{in})$$

$$204.8 = .1807\, R_{in}$$

$$\boxed{R_{in} = 1133\,\Omega}$$

2. This is a two-part problem on estimation.

a. You are asked to determine the average transmit power for a new wireless sensor. Twenty-seven sensors are tested to determine the average transmit power and you obtain the following results: the sample mean is 9.8 and the sample standard deviation is 2.7 (all measurements are in units of mW).

Estimate the population mean and the 90% confidence interval on the mean. Enter your numerical answer in the online answer box and show your work in your exam worksheet. **(7 points)**

$$\bar{x} \pm t_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right)$$

$$\frac{\alpha}{2} = .05$$

$$9.8 \pm 1.706\left[\frac{2.7}{\sqrt{27}}\right]$$

90%
confidence

$9.8 \pm .886$

$8.914 \leq \mu \leq 10.686$

b. Please repeat this problem without using the Student-t table; please use the z-table instead. (Everything, including n, remains the same except you are now using a value from the z table instead of the Student-t table. The sample mean, the sample standard deviation, the number of samples n, and the confidence interval remain the same. I thought it would be interesting to see the two results!) **(3 points)**

$\frac{.90}{2} = .45$ $\quad z = 1.645$

$= 9.8 \pm 1.645 \left(\frac{2.7}{\sqrt{27}}\right)$

$8.95 \leq \mu \leq 10.65$

3. (Note; there are no units in this problem; units are not required to solve this problem). You are given a sensor which can be modeled as a first-order system. The input to this sensor is x(t) and the output of the sensor is y(t). You are given the following:

$x(t) = 100$	for $t > 0$ and
$x(t) = 0$	for $t < 0$

The initial condition of the output y(t) is given as:

$$y(t = 0^+) = 5$$

a. The time constant of the system, τ, is 3.4 msec. Determine the time it will take the sensor output to reach 85% of the final output value of the sensor. **(5 points)**

$h = y_i - y_f$ $\quad A = 100$

$h = 5 - 100 = -95$ $\quad \tau = 3.4\,ms$

$y(t) = he^{-t/\tau} + A = -95e^{-t/3.4} + 100 = .85(y(t))$

$85 = -95e^{-t/3.4} + 100$

$$85 = -95e^{-t/3.4} + 100$$

$$\boxed{t = 6.27 \text{ ms.}}$$

b. The measurement error, e(t), is defined as:

e(t) = Measured Value – True Value

The time constant of the system, τ, is 3.4 msec. Determine the numerical value of the measurement error, e(t), at time, t, equal to 5 msec. **(5 points)**

$$e(t) = -95e^{-(5/3.4)}$$

$$\boxed{e(t) = -21.83}$$

c. In this part of the problem, you are requested to determine a new value of the time constant, τ. Determine the maximum value of the time constant of the system, τ, such that the system will satisfy both of the following requirements: **(10 points)**

- The time for the sensor output to reach 85% of the final output value is reduced to one-half of the correct answer to part (a) of this problem. (If the correct answer to part (a) is given by the variable T, we want to select a new value of τ which will result in the sensor output reaching 85% of the final output value in time T/2).
- The measurement error, e(t), at time, t, equal to 5 msec is reduced to one-half the measurement error in a system with a time constant, τ, of 3.4 msec at time, t, equal to 5 msec. (If the correct answer to part (b) is given by the variable E at 5 msec, we want to select a new value of τ which will result in the sensor error being E/2 at 5 msec).

$$\left.\begin{aligned} \frac{-21.83}{2} &= -10.915 \\ -10.915 &= -95e^{-(5/\tau)} \\ -2.164 &= -\frac{5}{\tau} \end{aligned}\right\} \quad \frac{E}{2} = 2.31 \text{ ms.}$$

$$\frac{T}{2} = \frac{6.27}{2} = 3.135 \text{ ms} = t \quad \Big\} \quad T = \ldots$$

$$\frac{1}{2} = \frac{6.}{2} = 3.135\ ms = t$$

$$.85(95)+5 = -95e^{-t/\tau} + 100$$

$$.15 = e^{-t/\tau}$$

$$\boxed{\frac{T}{2} = 1.7\ ms.}$$

d. The input to this first-order sensor is now changed to:

x(t) = (4.8)·t for t > 0

Using the correct value of the time constant, τ, determined in part (c) of this problem, determine the steady-state output error and the steady-state delay. **(2 points each, 4 points total)**

$$y(t) = (5 - 4.8t)e^{-t/\tau} + 4.8t$$

$$\tau = 1.7\ ms.$$

$$e(t) = \left((5-4.8t)e^{-t/1.7} + 4.8t\right) - 4.8t$$

$$0 = (5-4.8t)e^{-t/1.7}$$

$$t = 1.042\ ms.$$

4. This is a two-part question on parameter estimation.

a. A colleague has used parameter estimation to estimate the value of the population mean. They report their results to you as follows:

506.5360 ≤ μ ≤ 518.8640

The standard deviation of the sample is 43.4 and the sample size is 75. Determine the confidence level of this estimate. Enter your numerical answer in the online answer box and show your work in your exam worksheet. **(7 points)**

$$\bar{x} = \frac{518.8640 + 506.5360}{2} = 512.7$$

$$518.864 - 512.7 = z\left(\frac{43}{\sqrt{75}}\right) \qquad z = 1.23$$

For $z = 1.23$, $\frac{\alpha}{2} = .3907$ $\qquad 2(.3907) = .7814\,(100)$

$= \boxed{78.14\%}$

b. Another colleague has used parameter estimation to estimate the value of the population mean. They report their results to you as follows:

$$506.5360 \leq \mu \leq 518.8640$$

The standard deviation of the sample is 43.4 and the confidence level of this estimate is 97%. (Note: 97% is not the answer to Part A of the problem). Determine the same size used by this different colleague in their estimate. Enter your numerical answer in the online answer box and show your work in your exam worksheet. **(8 points)**

$$512.7 \pm \frac{43.4}{\sqrt{n}} = 518.864$$

$$6.164 = 2.17\left(\frac{43.4}{\sqrt{n}}\right)$$

$$2.84 = \frac{43.4}{\sqrt{n}}$$

$$n = \left(\frac{43.4}{2.84}\right)^2 = 233.4$$

$$\boxed{n = 234}$$

5. This is a multi-part question.

a. Consider a sinusoidal input to a first-order sensor. Under what specific conditions are we guaranteed that the dynamic measurement error of this system in steady state is small? Enter your answer in the online answer box. (Be specific; define and name your variable(s)). **(4 points)**

The input frequency and time constant (Omega and Tau) of the sensor determine the error of the sensor. As the product of the ratio of omega to tau approaches 0, so does the error.

b. Consider a sinusoidal input to a second-order sensor. Under what specific conditions are we guaranteed that the dynamic measurement error of this system in steady state is small? Enter your answer in the online answer box. (Be specific; define and name your variable(s)). **(4 points)**

Omega must be significantly larger than the natural undamped frequency (omega n) to minimize error.

6. In this problem you are asked to determine the order of an n^{th}-order unity dc gain low-pass Butterworth filter to meet two design requirements. (Note; "unity dc gain" means the gain at f = 0 is one). A sensor provides an output signal with a frequency of 30 Hz. This signal is applied to the input of the filter. (We shall refer to this signal as the "good" signal). A noise signal of 50 Hz is also present at the input to the filter. The design requirements of the filter are:

- The voltage gain of the filter at the noise frequency should be less than or equal to 0.05
- The voltage gain of the filter at frequency equal to 100 Hz (double the noise frequency) should be less than or equal to 0.01

a. Using a corner (or cut-off) frequency of 35 Hz, determine the minimum required order of the filter to satisfy each of the design requirements. (Please solve for two values of "n" and please clearly label each answer. After you solve for two values of "n", please clearly indicate which value of "n"' will satisfy both requirements). **(12 points)**

$f_c = 35\ Hz$

$$.05 = \frac{1}{\sqrt{1+(F/F_c)^{2n}}} \qquad .01 = \frac{1}{\sqrt{1+\left(\frac{100}{35}\right)^{2n}}}$$

$$.05\sqrt{\left(1+\left(\frac{50}{35}\right)^{2n}\right)} = 1 \qquad .01\left(\sqrt{1+\left(\frac{100}{35}\right)^{2n}}\right) = 1$$

$$399 = \left(\frac{50}{35}\right)^{2n} \qquad 999 = \left(\frac{100}{35}\right)^{2n}$$

$$n = 8.4 = 9 \qquad n = 4.4 = 5$$

Satisfies

b. Next, consider the following. The corner (or cut-off) frequency of this filter is kept constant at 35 Hz and the order of the filter, n, determined in the first part of this problem is decreased. Which of the following statements are true? (Please enter the number of all which are true into the online answer box): **(4 points)**

1. The gain of the "good" signal decreases
2. The gain of the "noise" signal decreases
3. The gain of the "good" signal increases
4. The gain of the "noise" signal increases

$$\frac{1}{\sqrt{1+\frac{50}{35}^{2n}}} \quad n=9 \quad G=.97 \qquad \frac{1}{\sqrt{1+\frac{50}{35}^{2n}}} \quad n=8 \quad G=.96$$

1, 4

c. Next, consider the following. The corner (or cut-off) frequency of this filter is increased from 35 Hz to 40 Hz and the order of the filter, n, determined in the first part of this problem remains the same. Which of the following statements are true? (Please enter the number of all which are true into the online answer box): **(4 points)**

1. The gain of the “good” signal decreases
2. The gain of the “noise” signal decreases
3. The gain of the “good” signal increases
4. The gain of the “noise” signal increases

$$\frac{1}{\sqrt{1+\frac{50}{40}^{2n}}}$$

$n=9$

$G = .997$

3, 4

7. This is a multi-part question on parameter estimation. **Note: single word answers only**; no partial credit; take your time. **(6 points total)**

a. Consider Gaussian parameter estimation; as the sample size is decreased, does the confidence interval increase or decrease? (Note; as the sample size is decreased, the confidence level and the standard deviation of the population remain fixed). **(2 points)**

Increase

b. As the confidence level in Gaussian parameter estimation is decreased, does the confidence interval increase or decrease? (Note; as the confidence level is decreased, the standard deviation of the population and the sample size remain fixed). **(2 points)**

Decrease

c. Consider Gaussian parameter estimation; as the standard deviation of the population is decreased, does the confidence interval increase or decrease? (Note; as the standard deviation of the population is decreased, the confidence level and the sample size remain fixed). **(2 points)**

Decrease