Prove (3.12) **Double Negation**: $\neg\neg p \equiv p$
by showing equivalence to a previous theorem

<u>Proof</u>

$$\begin{array}{ll}
 & \neg\neg p \equiv p \\
= & \qquad \langle\ \text{(3.11) } \neg p \equiv q \equiv p \equiv \neg q \text{, with } p, q := \neg p, p\ \rangle \\
 & \neg p \equiv \neg p \\
= & \qquad \langle\ \text{Identity of } \equiv\ \rangle \\
 & true \quad //
\end{array}$$

Prove (3.13) **Negation of *false***: $\neg$*false* $\equiv$ *true*
by showing equivalence to a previous theorem

<u>Proof</u>

$$
\begin{array}{ll}
 & \neg false \\
= & \qquad \langle \text{ (3.8) Definition of false } \rangle \\
 & \neg\neg true \\
= & \qquad \langle \text{ Double negation } \rangle \\
 & true \quad //
\end{array}
$$

Prove (3.14) $(p \not\equiv q) \;\equiv\; \neg p \equiv q$
by showing equivalence to a previous theorem

<u>Proof</u>

$$
\begin{array}{ll}
 & p \not\equiv q \\
= & \qquad \langle \text{ (3.10) Definition of } \not\equiv \ \rangle \\
 & \neg(p \equiv q) \\
= & \qquad \langle \text{ (3.10) Distributivity of } \neg \text{ over } \equiv \ \rangle \\
 & \neg p \equiv q \quad //
\end{array}
$$

Prove (3.19) **Mutual interchangeability**: $p \not\equiv q \equiv r \quad \equiv \quad p \equiv q \not\equiv r$
by showing equivalence to a previous theorem

<u>Proof</u>

$$
\begin{array}{ll}
 & p \not\equiv q \equiv r \\
= & \qquad \langle \text{ (3.10) Definition of } \not\equiv \text{ with } p, q := p, q \equiv \text{r} \rangle \\
 & \neg(p \equiv q \equiv r) \\
= & \qquad \langle \text{ (3.10) Definition of } \not\equiv \text{ with } p, q := p \equiv q, r \ \rangle \\
 & p \equiv q \not\equiv r \quad //
\end{array}
$$