MAT237 - Multi-variable Calculus

Callum Cassidy-Nolan

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Chapter 1

Lecture 1 - Review

1.1 Sets & tuples

Definition 1 (Tuple)
A n tuple is an ordered list of n elements $(x_1, \ldots, x_n)$ notation

- *couple, a 2-tuple*
- *triple, a 3-tuple*

Fundamental Property

$$(x_1, \ldots, x_m) = (y_1, \ldots, y_m) \Leftrightarrow \forall i \in \{1, \ldots, m\}, x_i = y_i$$

Recall

$$\{1, 2, 3\} = \{3, 2, 1\}$$

But

$$(1, 2, 3) \neq (3, 2, 1)$$

In addition

$$(1, 2, 2, 3) \neq (1, 2, 3)$$

Also the comparison here doesn't even make sense since they are different sizes.

Definition 2 (Cartesian Product)
For sets A, B

$$A \times B = \big\{(a, b) : a \in A, b \in B\big\}$$

Note if we have $A = \emptyset$ or $B = \emptyset$ then $A \times B = \emptyset$

Example 1.1.1

$$A = \{\pi, e\} \text{ and } B = \left\{1, \sqrt{2}, \pi\right\}$$

$$A \times B = \left\{(\pi, 1), \left(\pi, \sqrt{2}\right), (\pi, \pi), \dots\right\}$$

For multiple cartesian products we have

$$A_1 \times A_2, \dots, A_n = \left\{(a_1, a_2, \dots, a_m) : a_i \in A_i\right\}$$

Exercise 1.1.1
Is the following true ?

$$(A \times B) \times C = A \times (B \times C) = A \times B \times C$$

No, observe the tuples of $(A \times B) \times C$ *are of the form*

$$\big((a, b), c\big)$$

In the same way we observe that none of them are equal . Though in a functional type of sense, they are equal as they all still convey the same fundamental idea.

1.2 Functions

Definition 3 (Function)
A function is the data of two sets, A and B together with a "rule" that associates to each $x \in A$ *a unique* $f(x) \in B$.
We define a function like this

$$f : A \to B$$

Where A is the domain and B is the codomain.

Definition 4 (Image)
The image of $E \subseteq A$ *by* f *is*

$$f(E) = \left\{f(x) : x \in E\right\}$$

Definition 5 (Pre-Image)
The pre-image of $F \in B$ by f is

$$f^{-1}(F) = \{x \in A : f(x) \in F\}$$

Definition 6 (Graph)
The graph of f is

$$\Gamma f = \{(x, y) \in A \times B : y = f(x)\}$$

Definition 7 (Injective)
A function $f : A \to B$ is injective or one-to-one

$$\forall x_1, x_2 \in A, f(x_1) = f(x_2) \implies x_1 = x_2$$

We have the contrapositive

$$\forall x_1, x_2 \in A, x_1 \neq x_2 \implies f(x_1) \neq f(x_2)$$

Definition 8 (Onto)
A function is surjective or onto if

$$\forall y \in B, \exists x \in A, y = f(x)$$

Definition 9 (Bijective)
$f : A \to B$ is bijective if it is injective and surjective.

$$\forall y \in B, \exists! x \in A, y = f(x)$$

Definition 10 (Inverse)
$f : A \to B$ has an inverse if and only if there exists a function $g : B \to A$ such that

$$\forall x \in A, g \circ f(x) = x$$

and

$$\forall x \in B, f \circ g(x) = x$$

then we say that g is the inverse of f and $g = f^{-1}$