

# MAT247 - Probability with Computer Applications

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## *List of Theorems*



# Chapter 1

## Lecture 1

### 1.1 Stats vs Probability

#### Statistics

- Reverse Probability
- Making observations and then based on probability we describe the original model
- Critique whether a model is correct

#### Probability

- How likely based on a theoretical situation, for example a coin toss has a 50/50 chance of one of the outcomes.
- We know all the parameters.

### 1.2 To do Well

- Understand the formulas
- Make sure to the hard questions
- Office hours in HS 386

### 1.3 Useful Terminology

**Definition 1** (Random Experiment)

*A process of gathering data or observations. We can perform the experiment multiple times so long as the conditions aren't changed and the outcome of each experiment is random, we don't know what the result will be, though we know the set of possible outcomes.*

Examples

- Rolling a die, and observing the number on the top face.
- Rolling  $n$  dice and observing the resulting pair of numbers.
- Drawing 3 cards from a deck of cards
- Asking a professor how old they are

**Definition 2** (Sample Space)

*This is the set of all possible outcomes/results from a random experiment , we denote this set as*

$$\Omega \text{ or } S$$

*The sample space depends on the outcome of interest.*

**Note 1**

*If our outcome of interest is say, whether after flipping a coin it is heads or tails, we don't care how many times it turned in the air before landing on the ground.*

Examples

1.  $S = \{n \in \mathbb{N} : 1 \leq n \leq 20\}$ . All the faces of the dice.
2.  $\Omega_1 = \{\text{ True , False }\}$
3.  $\Omega_2 = \mathbb{R}^{\geq 0}$ . If they are precise in their measurement, though more likely to any hour between 0 and 10.

**Definition 3** (Event)

*Any subset of our sample space of interest.*

- *Simple Event: One with exactly one outcome*
- *Compound Event: One with multiple outcomes*

**Simple Event**

- flipping a coin, and observing whether it has landed on its edge (that's possible!)

**Compound Event**

- flipping 3 coins, and observing whether at least one has landed on its edge.

**Definition 4** (Complement)

*The complement of an event A is the event consisting of outcomes that are not in A. We denote this as  $A^c$ .*

**For example:** the complement of our previous example, that would be if no, coin has landed on its edge.

**Definition 5** (Union)

*The union of two events A and B is the event consisting of outcomes in A or B or both. Denoted as  $A \cup B$*

- *The union of events is usually said as A or B*
- *Observe  $A \cup A^c = \Omega$*

**Definition 6** (Intersection)

*The intersection of two events A and B is the event consisting of outcomes in both A and B, written as  $A \cap B$*

- *The intersection of events is described by A and B*
- *We see  $A \cap A^c = \emptyset$*

**Definition 7** (Disjoint)

*events A and B are disjoint, if they have no overlapping events , that is  $A \cap B = \emptyset$*

**1.4 Event Laws**

- Commutative Law:  $A \cup B = B \cup A$
- Associative Law:

$$(A \cup B) \cup C = A \cup (B \cup C)$$

$$(A \cap B) \cap C = A \cap (B \cap C)$$

- Distributive Law:  $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$  (This one will be useful for proving  $P(A \cup B \cup C) = P(A) + P(B) + \dots + P(A \cap B \cap C)$  )

**1.5 Mutually Exclusive****Definition 8** (Mutually Exclusive)

*Two events are mutually exclusive if the events cannot both occur. That is if event A occurs, event B cannot occur. In a Venn Diagram A and B being disjoint means that  $A \cap B = \emptyset$ .*

**Example**

- The events  $A = \{ \text{Roll and even } \# \}$  and  $B = \{ \text{Roll and odd } \# \}$  from rolling a die are mutually exclusive, since a number is either even or odd, not both.

**1.6 Independence****Definition 9** (Independent)

*Two events A and B are independent if the occurrence of one event does not affect the occurrence of the other in any way.*

**Example**

- The events
  - $C = \{ \text{Roll an even } \# \text{ on the first toss} \}$  and

- $D = \{ \text{Roll an odd } \# \text{ on the second toss}\}$

are independent of each other.

mutually exclusive events must be dependent recall that two mutually exclusive events cannot both occur, therefore if one of the events is true, it forces the other to be false, and so the occurrence of one does affect the occurrence of the other.

### Example 1.6.1

*Are all dependent events mutually exclusive ? recall, if A is mutually exclusive of B, that is*

$$A \text{ occurs} \implies P(B) = 0$$

*and if A is dependent on B*

Should this be swapped?

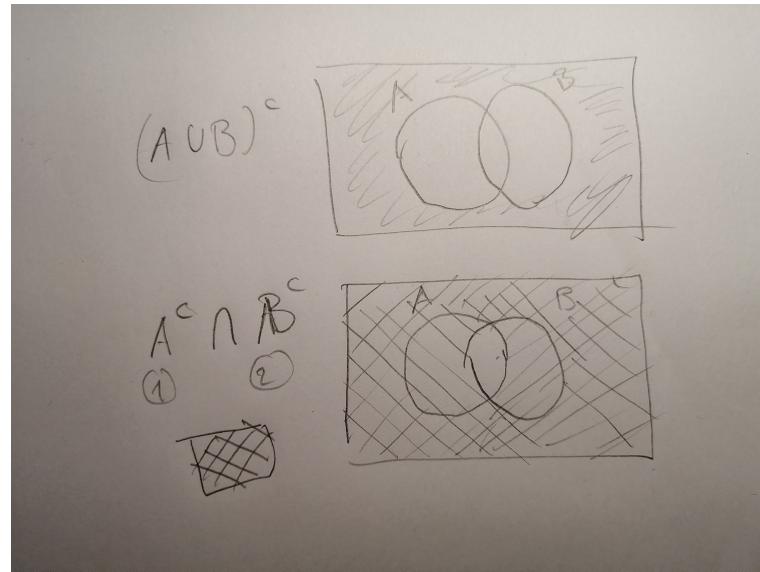
$$A \text{ occurs} \implies P(B \text{ given } A \text{ occurred}) \neq P(B)$$

**Example :** Let A be the event that the person next to me is sick, and B the event that I am sick, note the outcome of interest is sickness. We can see that if indeed the person next to me is sick then the probability of me being sick shall increase. Though they are not mutually exclusive of eachother, if I am sick, it is possible that the person next to me is as well.

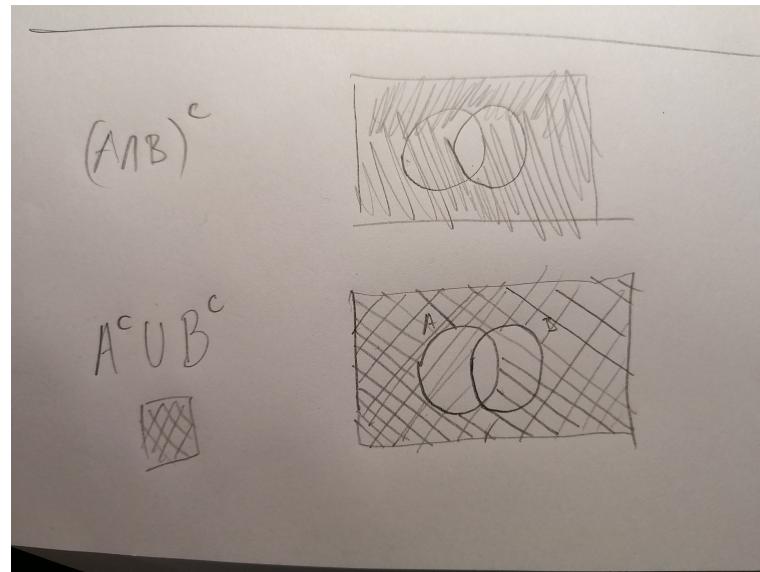
## 1.7 DeMorgan's Laws

From Venn Diagrams, some results follow, between union and intersection of events.

- $(A \cup B)^c = A^c \cap B^c$ , saying Not in either A nor B is the same as not in A and not in B



- $(A \cap B)^c = A^c \cup B^c$ , that is not both and and B, is not in A, or not in B or not in either A or B



The general version we have for set events  $\{A_1, A_2, \dots, A_n\}$

- $(\bigcup_{i=1}^n A_i)^c = \bigcap_{i=1}^n A_i^c$
- $(\bigcap_{i=1}^n A_i)^c = \bigcup_{i=1}^n A_i^c$

## 1.8 Sample Space Examples

1. Flipping a coin 3 times in a row

$$\Omega = \{HHH, HHT, HTH, HTT, TTT, TTH, THT, THH\}$$

observe that getting two heads, means you also got one tail, therefore  $P(\text{ Two H }) = P(\text{ One T }) = \frac{3}{8}$

2. Flipping a fair coin 3 times and measuring the number of tails

$$\Omega = \{0, 1, 2, 3\}$$

Not all of these are equally as likely, as there are multiple ways of getting just 1 tail, ex,  $HHT$  and  $HTH$

3. Number Selection of an individual's Lotto649 ticket

$$\Omega = \{(a, b, c, d, e, f) : 1 \leq a, b, c, d, e, f \leq 49 \in \mathbb{Z}\}$$

note  $n(\Omega) = 6^{49}$  since there are 49 options for each position

## 1.9 Event Examples

1. Tossing two different dice. The event of tossing a double

$$D = \{(1, 1), (2, 2), \dots, (6, 6)\}$$

2. Tossing such that the sum of the two dice is 8

$$S = \{(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)\}$$

observe that the first index of the tuple is the first dice and the second position is for the second dice, therefore  $(2, 6) \neq (6, 2)$

3. Tossing doubles or tossing a sum of 8

$$C = A \cup B$$

4. Tossing doubles and tossing a sum of 8

$$D = A \cap B = \{(4, 4)\}$$

## 1.10 Exclusive & independence Examples

- a) If we know that the course subject is comp sci, then we can assume that the percentage of males will be higher, if we do not know the course subject, then we have no idea, then we don't know if the percentage of males will be higher. Therefore Dependent.
- b) The fact that the coin is baised doesn't matter, every time the coin is tossed, it is still the same probability, therefore getting a head on the first toss tells us nothing about what will happen on the second toss. Independent
- c) If we have someone who doesn't like hiking, then we know that the probability of them climbing mount everest, is lower than just a normal random individual climbing mount everest, therefore by definition it is dependent, though note it is not mutually exclusive since even if they don't like hiking, maybe they want to conquer their fear, so they climb the mountain.
- d) dependent ,  $P(\text{Red Card}) = \frac{1}{2}$  and  $P(\text{Red Card if QoH}) = 1$  therefore by definition we know it is dependent . Note if  $G$  occurs then ,  $P(\text{H given G}) > \frac{1}{52}$  since one card has been removed

## 1.11 Intro to Probability

**Definition 10** (Probability)

*In a random experiment with sample space  $\Omega$ , the probability of an event  $A$ , denoted  $P(A)$  is a function that assigns to event  $A$  a numerical value that measures the chance that event  $A$  will occur.*

### 1.11.1 Axioms of Probability

1.  $P(A) \geq 0$  , negative probability doesn't make sense
2.  $P(\Omega) = 1$ , that is the probability for anything to happen must be 100
3. For a set of disjoint events (that is they are mutually exclusive )  $A_1, A_2, \dots, A_{n-1}, A_n$  in  $\Omega$

$$P\left(\bigcup_{i=1}^n A_i\right) = \sum_{i=1}^n P(A_i)$$

this says that probability of any of them happening, is the sum of each probability individually. note this one is very useful for proving things

### 1.11.2 probability proof

We'll show the complement relationship

*Proof.*

Recall  $A \cup A^c = \Omega$ , and  $A \cap A^c = \emptyset$ , it follows that

$$\begin{aligned}P(\Omega) &= P(A \cup A^c) \\1 &= P(A) + P(A^c) \\P(A) &= 1 - P(A^c)\end{aligned}$$

■

**Example** If there is a 30% chance it will rain tomorrow, we can conclude that there is a 70% chance that it will not.