# MAT236 - Intro to the Theory of Computation

Callum Cassidy-Nolan

September 11, 2019

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# List of Definitions

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# *List of Theorems*

# Chapter 1

# Lecture 1

**Definition 1** (Predicate)
*defined over some variable and denoted by a statement about a set of elements.*

**Example**

$$P(n): \text{“}n \text{ is an odd natural number”}, \text{ where } n \in \mathbb{N}$$

## 1.1 Simple Induction

Allows us to prove a predicate P holds for all natural numbers greater than or equal to $b \in \mathbb{N}$ that is

$$\forall n \in \mathbb{N}, n \geq b \implies P(n)$$

The principle bhind it is this

- If $P(b)$

# Chapter 2

# Preliminaries

## 2.1 Sets

- We now have the concept of the cardinality of a set being infinity that is for a set $A$ we have, $|A| = \infty$
- $\infty$: for all integers $k$ we have $k < \infty$
- Describing by listing all elements explicitly is called an extensional description, we can state the property that characterises it's elements, then we have an internal description (set-builder).
- A proper sub/super $A \subset B$ set means that every element of $A$ is also an element of $B$ moreover, there is at least one element of $B$ that is not in $A$.
- If $A \cap B = \emptyset$ , that is $A$ and $B$ have nothing in common, then we say they are disjoint.
- The difference of $A - B$ is the set of elements in $A$, that don't belong to $B$
- The intersection or union of an arbitary number, or infinite number of sets, is written as ($I$ is a set of indicies)

$$\cup_{i \in I} A_i = \{x : \text{ for some } i \in I, x \in A_i\}$$
$$\cap_{i \in I} A_i = \{x : \text{ for each } i \in I, x \in A_i\}$$

- Partition: For a set $A$ a partition of $A$ is a set, that satisifies the following.
  - $\mathcal{X} \subseteq \mathcal{P}(A)$ such that $X \in \mathcal{X}, \mathcal{X} \neq \emptyset$
  - $X, Y \in \mathcal{X}$ such that $X \neq Y, X \cap Y = \emptyset$ and $\cup_{X \in \mathcal{X}} X = A$

### 2.1.1 Ordered Pairs

- Ordered pairs actually can be defined more primitively, we have $(a, b)$ we can define it as the set $\big\{\{a\}, \{a, b\}\big\}$, the element that is of length 1, is viewed as the first element of the ordered set, the element of size two represents the ordered pair, one of which is the first, and the other is the second.

- For example if we have $(j, k) = (l, m)$, then $\big\{\{j\}, \{j, k\}\big\} = \big\{\{l\}, \{l, m\}\big\}$ therefore we must verify that they are subsets of eachother.