# MAT223 - Linear Algebra

Classnotes for Summer 2019

by

*Callum Cassidy-Nolan*

Computer Science

University of Toronto

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## 1.1 Linear Transformations

**Definition 1 (Linear Transformation)**

*Let $V$ and $W$ be subspaces. A function $\mathcal{T} : V \to W$ is called a linear transformation if for all $\vec{u}, \vec{v} \in V$ and $a \in \mathbb{R}$ it satisfies*

1. $\mathcal{T}(\vec{u} + \vec{v}) = \mathcal{T}(\vec{u}) + \mathcal{T}(\vec{v})$
2. $\mathcal{T}(a\vec{u}) = a\mathcal{T}(\vec{u})$

**Example 1.1.1**

We'll show that $\mathcal{T} : \mathbb{R}^2 \to \mathbb{R}^2$ where $\mathcal{T}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x+2 \\ y \end{bmatrix}$ is not a linear transformation.

Let $\vec{j} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \vec{k} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ we have that

$$\mathcal{T}(\begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix}) = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

But then we can see that

$$\mathcal{T}(\begin{bmatrix} 0 \\ 0 \end{bmatrix}) + \mathcal{T}(\begin{bmatrix} 0 \\ 0 \end{bmatrix}) = \begin{bmatrix} 2 \\ 0 \end{bmatrix} + \begin{bmatrix} 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \end{bmatrix}$$

Then we conclude that $\mathcal{T}(\vec{j}+\vec{k}) \neq \mathcal{T}(\vec{j}) + \mathcal{T}(\vec{k})$

**Example 1.1.2**

We'll show that $\mathcal{P}$ is a linear transformation [1]

[1] We'll show that it is closed under addition and multiplication

$$\mathcal{P}(\begin{bmatrix} x \\ y \end{bmatrix}) = comp_{\vec{u}} \begin{bmatrix} x \\ y \end{bmatrix}$$

Let $\vec{j}, \vec{k} \in \mathbb{R}^2$ we know that

$$comp_{\vec{u}}\vec{j} = \left(\frac{\vec{u} \cdot \vec{j}}{||\vec{u}||^2}\right)\vec{u} \text{ and } comp_{\vec{u}}\vec{k} = \left(\frac{\vec{u} \cdot \vec{k}}{||\vec{u}||^2}\right)\vec{u}$$

And thus their product yields

$$comp_{\vec{u}}\vec{j} + comp_{\vec{u}}\vec{k} = \left(\frac{\vec{u} \cdot \left(\vec{j}+\vec{k}\right)}{||\vec{u}||^2}\right)\vec{u}$$

Which is equal to

$$comp_{\vec{u}}\left(\vec{j}+\vec{k}\right)$$

We must then show that it holds under multiplication let $\alpha \in \mathbb{R}$ and we know that

$$\alpha comp_{\vec{u}}\vec{j} = \alpha \left(\frac{\vec{u} \cdot \vec{j}}{||\vec{u}||^2}\right)\vec{u} = \left(\frac{\vec{u} \cdot \alpha\vec{j}}{||\vec{u}||^2}\right)\vec{u} = comp_{\vec{u}}\alpha\vec{j}$$

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