

MAT223 - Linear Algebra

CLASSNOTES FOR SUMMER 2019

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List of Theorems

List of Procedures

Linear transformations and such

1.1 Linear Transformations

Definition 1 (Linear Transformation)

Let V and W be subspaces. A function $\mathcal{T} : V \rightarrow W$ is called a linear transformation if for all $\vec{u}, \vec{v} \in V$ and $a \in \mathbb{R}$ it satisfies

1. $\mathcal{T}(\vec{u} + \vec{v}) = \mathcal{T}(\vec{u}) + \mathcal{T}(\vec{v})$
2. $\mathcal{T}(a\vec{u}) = a\mathcal{T}(\vec{u})$

Example 1.1.1

We'll show that \mathcal{R} is a linear transformation where \mathcal{R} is a counter clockwise rotation of $\frac{\pi}{2}$ radians

$$\mathcal{R} \left(\begin{bmatrix} x \\ y \end{bmatrix} \right) = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Let $\vec{u}, \vec{v} \in \mathbb{R}^2$ we know that for some $x_1, y_1, x_2, y_2 \in \mathbb{R}$ that

$$\vec{u} = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} \text{ and } \vec{v} = \begin{bmatrix} x_2 \\ y_2 \end{bmatrix}$$

$$\mathcal{R} \left(\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} \right) + \mathcal{R} \left(\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} \right) = \begin{bmatrix} -y_1 \\ x_1 \end{bmatrix} + \begin{bmatrix} -y_2 \\ x_2 \end{bmatrix}$$


$$= \begin{bmatrix} -(y_1 + y_2) \\ x_1 + x_2 \end{bmatrix}$$

Which is exactly equal to $\mathcal{R}(\vec{u} + \vec{v})$ as required, then let $\alpha \in \mathbb{R}$ and we know that

$$\mathcal{R}(\alpha \vec{u}) = \begin{bmatrix} -\alpha y_1 \\ \alpha x_1 \end{bmatrix}$$

But also that

$$\alpha \mathcal{R}(\vec{u}) = \begin{bmatrix} -\alpha y_1 \\ \alpha x_1 \end{bmatrix}$$

So then we've shown that $\mathcal{R}(\alpha \vec{u}) = \alpha \mathcal{R}(\vec{u})$ but also that $\mathcal{R}(\vec{u} + \vec{v}) = \mathcal{R}(\vec{u}) + \mathcal{R}(\vec{v})$ as req'd 

Example 1.1.2


We'll show that $\mathcal{T} : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ where $\mathcal{T} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x+2 \\ y \end{bmatrix}$ is not a linear transformation.

Let $\vec{j} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, $\vec{k} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ we have that

$$\mathcal{T}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

But then we can see that

$$\mathcal{T}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}\right) + \mathcal{T}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ 0 \end{bmatrix} + \begin{bmatrix} 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \end{bmatrix}$$

Then we conclude that $\mathcal{T}(\vec{j} + \vec{k}) \neq \mathcal{T}(\vec{j}) + \mathcal{T}(\vec{k})$ 

Example 1.1.3

We'll show that \mathcal{P} is a linear transformation ¹

¹ We'll show that it is closed under addition and multiplication

$$\mathcal{P}\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \text{comp}_{\vec{u}} \begin{bmatrix} x \\ y \end{bmatrix}$$

Let $\vec{j}, \vec{k} \in \mathbb{R}^2$ we know that

$$\text{comp}_{\vec{u}}\vec{j} = \left(\frac{\vec{u} \cdot \vec{j}}{\|\vec{u}\|^2} \right) \vec{u} \text{ and } \text{comp}_{\vec{u}}\vec{k} = \left(\frac{\vec{u} \cdot \vec{k}}{\|\vec{u}\|^2} \right) \vec{u}$$

And thus their product yields

$$\text{comp}_{\vec{u}}\vec{j} + \text{comp}_{\vec{u}}\vec{k} = \left(\frac{\vec{u} \cdot (\vec{j} + \vec{k})}{\|\vec{u}\|^2} \right) \vec{u}$$

Which is equal to

$$\text{comp}_{\vec{u}}(\vec{j} + \vec{k})$$

We must then show that it holds under multiplication let $\alpha \in \mathbb{R}$ and we know that

$$\alpha \text{comp}_{\vec{u}}\vec{j} = \alpha \left(\frac{\vec{u} \cdot \vec{j}}{\|\vec{u}\|^2} \right) \vec{u} = \left(\frac{\vec{u} \cdot \alpha \vec{j}}{\|\vec{u}\|^2} \right) \vec{u} = \text{comp}_{\vec{u}}\alpha \vec{j} \quad \rightarrow$$

1.2 Image

Definition 2 (Image)

Let $L : V \rightarrow W$ be a transformation and let $X \subseteq V$ be a set. The **image of the set X under L** , denoted as $L(X)$, is the set

$$L(X) = \{\vec{x} \in W : \vec{x} = L(\vec{y}) \text{ for some } \vec{y} \in X\}$$

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