Property: Constant in Integral

For any function f and constant $c \in \mathbb{R}$

$$\int cf(x)\ dx = c\int f(x)\ dx$$

Proof

- *Suppose that $F(x)$ is an anti-derivative of $f(x)$, that is $F'(x) = f(x)$ since we can pull constants out of derivatives we have*

$$(cF(x))' = cF'(x) = cf(x)$$

- *Therefore $cF(x)$ is an anti-derivative of $cf(x)$, therefore*

$$\int cf(x)\ dx = cF(x) + m = c\left(F(x) + \frac{m}{c}\right) = c\int f(x)\ dx$$

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