Definition: Probability Axioms

Let S be a sample space, $E \subseteq S$ be any event and a function P satisfying the following properties

- $P(E) \in [0,1]$
- $P(S) = 1$
- *For disjoint events* $E_1, E_2, E_3, \ldots$

$$P\left(\bigcup_{i=1}^{\infty} E_i\right) = \sum_{i=1}^{\infty} P\left(E_i\right)$$

Then we denote $P(E)$ as the probability of the event E

Remarks

- *Set $E_1 = S$ and for every $i \in \mathbb{N}^{\geq 2}, E_i = \varnothing$, and note that any set and the empty set are disjoint, therefore by the third axiom one sees that* $P\left(S\right) = \sum_{i=1}^{\infty} P\left(E_i\right) = P\left(S\right) + \sum_{i=2}^{\infty} P\left(\varnothing\right)$ *Thus* $P\left(\varnothing\right) = 0$
- *Following that, the third axiom allows us to talk about a finite number of disjoint sets $E_1, E_2, \ldots, E_{n-1}, E_n$ as well, to do so, set each each $E_i = \varnothing$ for $i > n$ to get* $P\left(\bigcup_{i=1}^{n} E_i\right) = P\left(\bigcup_{i=1}^{\infty} E_i\right) = \sum_{i=1}^{n} P\left(E_i\right) + \sum_{i=n+1}^{\infty} P\left(\varnothing\right) = \sum_{i=1}^{n} P\left(E_i\right)$