

Theorem: Vector Space Unique Identity

Let V be a vector space and suppose that 0 and $0'$ are two different ($0 \neq 0'$) identities, then

$$0' = 0' + 0 = 0 + 0' = 0$$

from commutivity and the definition of identity element, therefore $0 = 0'$ which is a contradiction.