Theorem: Number of Permutations on N Elements

The number of permutations on $[n]$ is $n!$

Proof

- *Firstly we know that $f(1) = x_1$ where $x_1 \in [n]$, so $f(1)$ has n different options to map to*
- *Now say $f(2) = x_2$, then it could be one of $[n] \setminus \{x_1\}$, so there are $n-2$ options for what $f(2)$ could be*
  - *x_2 must be one of $[n] \setminus \{x_1\}$ because we cannot have $f(2) = f(1)$, because a permutation is a bijective self map.*
- *Then $n-3$ options for $f(3)$ and in general there will be $n-i+1$ options for $f(i)$*
- *Following this, an enumeration of all the possible permutations would be:*

$$n \cdot (n-1) \cdot (n-2) \cdots (3) \cdot 2 \cdot 1$$

- *That is the definition of $n!$ so more consicely, there are $n!$ such permutations*

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