

### **Deduction: Scalar Times Vector is Zero implies one of them Zero**

*Suppose that  $a \in \mathbb{F}$ ,  $v \in V$ , and that  $av = 0$  we will prove that*

$$a = 0 \in \mathbb{F} \text{ or } v = 0 \in V$$

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### **Proof**

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*Supposing  $a \neq 0$ , then by multiplying both sides by  $\frac{1}{a}$  we obtain*

$$v = \left(\frac{1}{a}\right) 0 = 0 \in V$$

*by compatibility of scalar multiplication, and the fact that any scalar times the identity element is still the identity. Otherwise  $a = 0$  and the proof is done. ■*