Proposition: Finite Geometric Sum

Finite Geometric sum, for any $x \geq 0$:

$$\sum_{i=0}^{k} x^i = \frac{x^{k+1} - 1}{x - 1}$$

Proof

Let $S = \sum_{i=0}^{k} x^i$, then we have

$$xS - S = \sum_{i=0}^{k} x^{i+1} - \sum_{i=0}^{k} x^i = x^{k+1} - x^0 = x^{k+1} - 1$$

therefore

$$S = \frac{x^{k+1} - 1}{x - 1}$$

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