Theorem: Boundary of Complement is the Same

$$\partial S = \partial\left(S^{\complement}\right)$$

Proof

- *Recall that* $x \in \partial S$ *if and only if* $\forall \varepsilon \in \mathbb{R}^{>0}$
  - $B(x,\varepsilon) \cap S \neq \varnothing$
  - $B(x,\varepsilon) \cap S^{\complement} \neq \varnothing$
- *Since* $\left(S^{\complement}\right)^{\complement} = S$, *then, we can re-write the above in opposite order*
  - $B(x,\varepsilon) \cap S^{\complement} \neq \varnothing$
  - $B(x,\varepsilon) \cap \left(S^{\complement}\right)^{\complement} \neq \varnothing$
  - *But that's if and only if* $x \in \partial\left(S^{\complement}\right)$, *as required.*

$\blacksquare$