Definition: Note Integer Notation

- *Is a notational system which maps the letter names for notes in the western system of music to an integer:*

$$\begin{array}{cccccccccccc} C & \cdot & D & \cdot & E & F & \cdot & G & \cdot & A & \cdot & B \\ \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow \\ \widehat{0} & \widehat{1} & \widehat{2} & \widehat{3} & \widehat{4} & \widehat{5} & \widehat{6} & \widehat{7} & \widehat{8} & \widehat{9} & \widehat{10} & \widehat{11} \end{array}$$

- *The hat is added to denote that we are talking about the pitch produced by playing this note on a device which creates sound.*
- *We may also denote which octave band we are within by writing*

$$\widehat{9}_4$$

Which represents an A4, the sound generated with a frequency of 440Hz

- *We may consider elements such as $\widehat{12}, \widehat{-1}$ by moving circularly, so that $\widehat{12} \leftrightarrow C$ and $\widehat{-1} \leftrightarrow B$. But you can refer to any note using the elements in the initial mapping, so it is standard to use those numbers instead.*
  - *In other words, without considering which octave a note is in, we have the following equivalence for any $k \in \mathbb{Z}$ and $x \in \{0, \ldots, 12\}$*

$$\widehat{x} = \widehat{x + 12 \cdot k}$$

Which says if you add 12 semitones to any note, it will be the same note differing by an integer number of octaves