

Theorem: Zero Derivative Implies Constant

Let f be a real function that is continuous on the interval $[a, b]$ and differentiable on (a, b) and that $\forall x \in (a, b)$, we have

$$f'(x) = 0$$

Then f is constant on $[a, b]$

Proof

Let $m \in (a, b]$ then f satisfies the MVT on $[a, m]$ so we get $k \in (a, m)$ such that

$$f'(k) = \frac{f(m) - f(a)}{m - a}$$

But also we know that $f'(k) = 0$ therefore $f(m) - f(a) = 0$ and we have:

$$f(m) = f(a)$$

