

Definition: Vector Space

A vector space over a field \mathbb{F} is a set V , with two operations: (where $\vec{v}, \vec{u}, \vec{w} \in V$)

- A binary operation named vector addition: $v + w$
- Scalar multiplication: $x \mapsto \alpha x$, which is an operation from $F \times V$ to V

That satisfy the following axioms:

- Associativity of addition: $\vec{u} + (\vec{v} + \vec{w}) = (\vec{u} + \vec{v}) + \vec{w}$
- Commutativity of addition: $\vec{u} + \vec{v} = \vec{v} + \vec{u}$
- Identity element of addition: there exists an element $\vec{0} \in V$, called the [[Zero n-Tuple—zero vector]], such that $\vec{v} + \vec{0} = \vec{v}$ for all $\vec{v} \in V$
- Inverse elements of addition: For every $\vec{v} \in V$ there exists an element $-\vec{v} \in V$, called the additive inverse of \vec{v} such that $\vec{v} + (-\vec{v}) = \vec{0}$
- Compatibility of scalar multiplication with field multiplication $\alpha(\beta\vec{v}) = (\alpha\beta)\vec{v}$
- Identity element of scalar multiplication: $1\vec{v} = \vec{v}$ where 1 is the multiplicative identity in \mathbb{F}
- Distributivity of scalar multiplication with respect to vector addition: $\alpha(\vec{u} + \vec{v}) = \alpha\vec{u} + \alpha\vec{v}$
- Distributivity of scalar multiplication with respect to field addition $(\alpha + \beta)\vec{v} = \alpha\vec{v} + \beta\vec{v}$