Theorem: Shift Equivalence for Intervals

For any $\widehat{x}, \widehat{y} \in \mathbb{W}$ *and* $k \in \mathbb{Z}$

$$I\left(\widehat{x}, \widehat{y}\right) = I\left(\widehat{x} + k, \widehat{y} + k\right)$$

In other words, if we take two notes, the interval between them is the same interval between the notes you get when we shift both by the same number of semitones.

Proof

$$\begin{aligned} I\left(\widehat{x} + k, \widehat{y} + k\right) &\overset{SIN}{=} I\left(\widehat{x+k}, \widehat{y+k}\right) \\ &\overset{\mathtt{D}}{=} (y + k) - (x + k) \\ &= y - x \\ &\overset{\mathtt{D}}{=} I\left(x, y\right) \end{aligned}$$

■