Theorem: Complement Cancels

Let U be a set and let $X \subseteq U$ then

$$(X^{\complement})^{\complement} = X$$

Proof

- $\subseteq$
  - *Let $x \in (X^{\complement})^{\complement}$ then $x \notin X^{\complement}$, then suppose $x \notin X$ then $x \in X^{\complement}$, that's a contradiction, so $x \in X$.*
- $\supseteq$
  - *Let $x \in X$ we want to show that $x \in (X^{\complement})^{\complement}$*
    - *If $x \in X$ then $x \notin X^{\complement} \overset{\mathtt{D}}{=} \{u \in U : u \notin X\}$ therefore $x \in (X^{\complement})^{\complement}$*

$\blacksquare$