

Definition: Field

A field is a set \mathbb{F} , containing at least two elements on which two operations $+$ (addition) and \cdot (multiplication) are defined so that for each pair of elements $x, y \in \mathbb{F}$ there are unique elements $x + y$ and $x \cdot y$ in \mathbb{F} for which the following conditions hold for all elements $x, y, z \in \mathbb{F}$

$x + y = y + x$ (commutativity of addition) $(x + y) + z = x + (y + z)$ (associativity of addition) There is an element $0 \in \mathbb{F}$, which is named 0 such that $x + 0 = x$ (existence of additive identity) For each x , we have an element $-x \in \mathbb{F}$ such that $x + (-x) = 0$ (existence of additive inverse) $xy = yx$ (commutivity of multiplication) $(x \cdot y) \cdot z = x \cdot z + y \cdot z$ and $x \cdot (y + z) = x \cdot y + x \cdot z$ (distributivity) There is an element $1 \in \mathbb{F}$, such that $1 \neq 0$ and $x \cdot 1 = x$ (existence of a multiplicative identity) if $x \neq 0$, then there is an element $x^{-1} \in \mathbb{F}$ such that $x \cdot x^{-1} = 1$ (existence of multiplicative inverses)