Theorem: Scalar Times Vector is Zero implies one of them Zero

Suppose that $a \in \mathbb{F}, v \in V$, *and that* $av = 0$ *we will prove that*

$$a = 0 \in \mathbb{F} \text{ or } v = 0 \in V$$

Proof

Supposing $a \neq 0$, *then by multiplying both sides by* $\frac{1}{a}$ *we obtain*

$$v = \left(\frac{1}{a}\right) 0 = 0 \in \mathbb{F}$$

by compatibility of scalar multiplication, and the fact that any scalar times the identity element is still the identity. Otherwise $a = 0$ *and the proof is done.* ■