

Theorem: S Closed iff Boundary is Subset

$$S \text{ is closed} \Leftrightarrow \partial S \subseteq S$$

Proof

- \Rightarrow
 - $\partial S \stackrel{\text{D}}{=} \overline{S} \setminus \overset{\circ}{S}$ since $S = \overline{S}$ by assumption, that is $\partial S = S \setminus \overset{\circ}{S} \subseteq S$
- \Leftarrow
 - \overline{S}

