

Definition: Factorial

Direct

Let $n \in \mathbb{N}$, then

$$n! \stackrel{\text{D}}{=} n \cdot (n-1) \cdot (n-2) \cdots 3 \cdot 2 \cdot 1$$

Recursive

$$n! = \begin{cases} 1 & \text{if } n = 0 \\ n \cdot (n-1)! & \text{otherwise} \end{cases}$$

$$0! = 1$$

- The definition of $n!$ as a product of no numbers at all is an example of the convention that the product of no factors is equal to the multiplicative identity
- You can see that $\frac{x!}{x} = (x-1)!$, so $\frac{1!}{1} = 0!$ but $\frac{1!}{1} = 1$ so it makes sense for $0! = 1$