Theorem: Interior Subset Closure Superset

$$\mathring{S} \subseteq S \subseteq \overline{S}$$

Proof

- $\mathring{S} \subseteq S$
  - *Let* $x \in \mathring{S}$, *so we have* $\varepsilon \in \mathbb{R}^{>0}$ *such that* $B\left(x, \varepsilon\right) \subseteq S$, *therefore* $x \in B\left(x, \varepsilon\right) \subseteq S$ *so* $x \in S$
- $S \subseteq \overline{S}$
  - *Let* $x \in S$, *then* $x \in S \cap B\left(x, \varepsilon\right)$ *so*
    $$\{x\} \subseteq S \cap B\left(x, \varepsilon\right) \Rightarrow S \cap B\left(x, \varepsilon\right) \neq \varnothing$$

$\blacksquare$