

Theorem: S Closed iff Boundary is Subset

$$S \text{ is closed} \Leftrightarrow \partial S \subseteq S$$

Proof

- \Rightarrow

- $\partial S \stackrel{\text{D}}{=} \overline{S} \setminus \overset{\circ}{S}$ since $S = \overline{S}$ by assumption, that is $\partial S = S \setminus \overset{\circ}{S} \subseteq S$

- \Leftarrow

- $\overline{S} \stackrel{\text{T}}{=} S \cup \partial S$ now since we've assumed that $\partial S \subseteq S$, $S \cup \partial S = S$ and we get $\overline{S} = S$

