Definition: Field

A field is a set $\mathbb{F}$, containing at least two elements on which two operations $+$ (addition) and $\cdot$ (multiplication) are defined so that for each pair of elements $x, y \in \mathbb{F}$ there are unique elements $x + y$ and $x \cdot y$ in $\mathbb{F}$ for which the following conditions hold for all elements $x, y, z \in \mathbb{F}$

- $x + y = y + x$ *(commutativity of addition)*
- $(x + y) + z = x + (y + z)$ *(associativity of addition)*
- *There is an element $0 \in \mathbb{F}$, which is named 0 such that $x + 0 = x$ (existance of additive identity)*
- *For each x, we have an element $-x \in \mathbb{F}$ such that $x + (-x) = 0$ (existance of additive inverse)*
- $xy = yx$ *(commutivity of multiplication)*
- $(x \cdot y) \cdot z = x \cdot z + y \cdot z$ *and* $x \cdot (y + z) = x \cdot y + x \cdot z$ *(distributivity)*
- *There is an element $1 \in \mathbb{F}$, such that $1 \neq 0$ and $x \cdot 1 = x$ (existance of a multiplicative identity)*
- *if $x \neq 0$, then there is an element $x^{-1} \in \mathbb{F}$ such that $x \cdot x^{-1} = 1$ (existance of multiplicative inverses)*