Theorem: Vector Space Unique Inverse

Every element in a vector space V has a unique additive inverse.

Proof

Suppose V is a vector space, and let $v \in V$ and supppose we have two additive inverses of v, call them x and y, where $x \neq y$. In that case

$$x = x + 0 = x + (v + y) = (x + v) + y = 0 + y = y$$

Thus $x = y$, a contradiction, so we know that there is a unique additive inverse. ■