Theorem: Scalar Times Vector is Zero implies one of them Zero

Suppose that $a \in \mathbb{F}, v \in V$, and that $av = 0$ we will prove that

$$a = 0 \in \mathbb{F} \text{ or } v = 0 \in V$$

Proof

Supposing $a \neq 0$, then by multiplying both sides by $\frac{1}{a}$ we obtain

$$v = \left(\frac{1}{a}\right) 0 = 0 \in \mathbb{F}$$

by compatibility of scalar multiplication, and the fact that any scalar times the identity element is still the identity. Otherwise $a = 0$ and the proof is done. ■