Theorem: Chord wrt Different Root

Let $\widehat{r} \mid x_1, x_2, \ldots, x_{n-1}, x_n$ *be a chord, and* $\widehat{R}$ *be a different root. Then* $\widehat{r} \mid x_1, x_2, \ldots, x_{n-1}, x_n$ *with respect to* $\widehat{R}$ *is a new chord*

$$\widehat{R} \mid x_1 - I\left(\widehat{r}, \widehat{R}\right), x_2 - I\left(\widehat{r}, \widehat{R}\right), \ldots, x_{n-1} - I\left(\widehat{r}, \widehat{R}\right), x_n - I\left(\widehat{r}, \widehat{R}\right)$$

Proof

- *We know that there exists* $k \in \mathbb{Z}$ *such that* $\widehat{r} + k = \widehat{R}$*, namely* $k = I\left(\widehat{r}, \widehat{R}\right)$
- *We want to find out what the interval is from* $\widehat{R}$ *to* x_i *, written as* $I\left(\widehat{R}, \widehat{r + x_i}\right)$ *which is defined to be equal to:* $r + x_i - R$
  - *But we already know that* $R = r + k$*, therefore that equation becomes*

$$r + x_i - (r + k) = x_i - k$$

- *Since* $k = I\left(\widehat{r}, \widehat{R}\right)$*, then we can conclude that*

$$I\left(\widehat{R}, \widehat{r + x_i}\right) = x_i - I\left(\widehat{r}, \widehat{R}\right)$$

- *That is, each of the intervals from the first chord get transformed to intervals in the second chord by subtracting the interval from* $\widehat{r}$ *to* $\widehat{R}$

$\blacksquare$

Examples

- *Say there is a change from* $\widehat{8} \mid 0, 3, 7, 9,$ *(Ab 6) to* $\widehat{1} \mid 0, 4, 7, 10$ *(Db 7)*
  - *Using the above Idea we can see that* $I\left(\widehat{8}, \widehat{1}\right) = 5$ *and so we we can subtract 5 from each interval and take it modulo 12 to get our new intervals with respect to the new root:*

$$0 - 5, 3 - 5, 7 - 5, 9 - 5 = 7, 10, 2, 4$$

  Therefore $\widehat{8} \mid 0, 3, 7, 9$ *with respect to* $\widehat{1}$ *is:*

$$\widehat{1} \mid 7, 10, 2, 4$$

- *The upshot is that now we can easily see how the chord we are playing now can be tweaked to fit the next chord's context.*