Definition: Vector Space

A vector space over a field $\mathbb{F}$ *is a set* V*, with two operations: (where* $\vec{v}, \vec{u}, \vec{w} \in V$*)*

- *A binary operation named vector addition:* $v + w$
- *Scalar multiplication:* $x \mapsto \alpha x$*, which is an operation from* $F \times V$ *to* V

That satisfy the following axioms:

- *Associativity of addition:* $\vec{u} + (\vec{v} + \vec{w}) = (\vec{u} + \vec{v}) + \vec{w}$
- *Commutativity of addition:* $\vec{u} + \vec{v} = \vec{v} + \vec{u}$
- *Identity element of addition: there exists an element* $\vec{0} \in V$*, called the [[Zero n-Tuple—zero vector]], such that* $\vec{v} + \vec{0} = \vec{v}$ *for all* $\vec{v} \in V$
- *Inverse elements of addition: For every* $\vec{v} \in V$ *there exists an element* $-\vec{v} \in V$*, called the additive inverse of* $\vec{v}$ *such that* $\vec{v} + (-\vec{v}) = \vec{0}$
- *Compatibility of scalar multiplication with field multiplication* $\alpha(\beta\vec{v}) = (\alpha\beta)\vec{v}$
- *Identity element of scalar multiplication:* $1\vec{v} = \vec{v}$ *where 1 is the multiplicative identity in* $\mathbb{F}$
- *Distributivity of scalar multiplication with respect to vector addition:* $\alpha(\vec{u} + \vec{v}) = \alpha\vec{u} + \alpha\vec{v}$
- *Distributivity of scalar multiplication with respect to field addition* $(\alpha + \beta)\vec{v} = \alpha\vec{v} + \beta\vec{v}$