Let $a, b, c, d \in \mathbb{Z}$ such that $\chi : ad + bc \neq 0$. Prove that if $ad + bc$ divides each of a, b, c, d then one has

$$|ad + bc| = 1$$

Proof By our assumption of divisibility we get $k_a, k_b, k_c, k_d \in \mathbb{Z}$ such that for any $x \in \{aa, b, c, d\}$ we have

$$\phi : \quad (ad + bc) \cdot k_x = x$$

Therefore

$$\gamma : \quad ad + bc = (ad + bc) \cdot (k_a k_d + k_b k_c) \Leftrightarrow k_a k_d + k_b k_c = 1$$

And by using ϕ we see that $ad = (ad + bc)^2 \, k_a k_d$ and $ad = (ad + bc)^2 \, k_b k_c$, therefore

$$ad + bc = (ad + bc)^2 \, (k_a k_d + k_b k_c) \stackrel{\gamma}{=} (ad + bc)^2 \, (1)$$

Therefore $ad + bc = 1$