

Circuit Variables and Circuit Elements

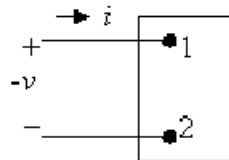
Drill Exercises

$$\text{DE 1.1} \quad q = \int_0^{\infty} 20e^{-5000t} dt = 4000 \mu\text{C}$$

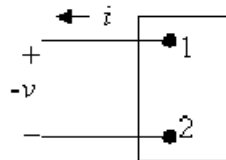
$$\text{DE 1.2} \quad i = \frac{dq}{dt} = te^{-\alpha t}, \quad \frac{di}{dt} = (1 - \alpha t)e^{-\alpha t}, \quad \frac{di}{dt} = 0 \quad \text{when } t = \frac{1}{\alpha};$$

$$\text{Therefore } i_{\max} = \frac{1}{\alpha e} = \frac{1}{0.03679e} \cong 10 \text{ A}$$

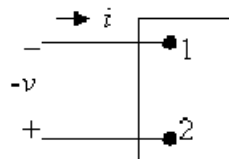
DE 1.3 [a]



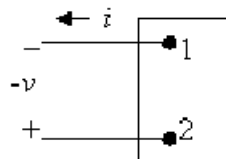
(a)



(b)



(c)



(d)

Therefore

$$\text{(a) } v = -20 \text{ V}, \quad i = -4 \text{ A}; \quad \text{(b) } v = -20 \text{ V}, \quad i = 4 \text{ A}$$

$$\text{(c) } v = 20 \text{ V}, \quad i = -4 \text{ A}; \quad \text{(d) } v = 20 \text{ V}, \quad i = 4 \text{ A}$$

[b] Using the reference system in Fig. 1.3(a), $p = vi = (-20)(-4) = 80 \text{ W}$, so the box is absorbing power.

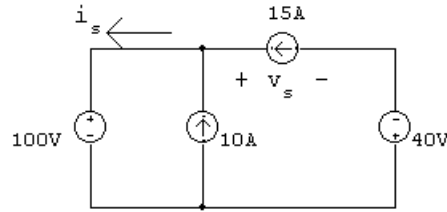
[c] The box is absorbing 80 W.

$$\text{DE 1.4} \quad p = vi = 20 \times 10^4 e^{-10,000t} \text{ W}; \quad w = \int_0^\infty 20 \times 10^4 e^{-10,000t} dt = 20 \text{ J}$$

$$\text{DE 1.5} \quad p = 800 \times 10^3 \times 1.8 \times 10^3 = 1440 \times 10^6 = 1440 \text{ MW}$$

from Oregon to California

DE 1.6



The interconnection is valid:

$$i_s = 10 + 15 = 25 \text{ A}$$

$$p_{100\text{V}} = 100i_s = 2500 \text{ W (absorbing)}$$

$$p_{10\text{A}} = -100(10) = -1000 \text{ W (generating)}$$

$$-100 + v_s - 40 = 0 \quad \text{so } v_s = 140 \text{ V}$$

$$p_{15\text{A}} = -15(140) = -2100 \text{ W (generating)}$$

$$p_{40\text{V}} = 15(40) = 600 \text{ W (absorbing)}$$

$$\sum p_{\text{dev}} = p_{10\text{A}} + p_{15\text{A}} = 3100 \text{ W}$$

$$\sum p_{\text{abs}} = p_{100\text{V}} + p_{40\text{V}} = 3100 \text{ W}$$

$$\sum p_{\text{dev}} = \sum p_{\text{abs}} = 3100 \text{ W}$$

$$\text{DE 1.7} \quad [\mathbf{a}] \quad v_l - v_c + v_1 - v_s = 0, \quad i_l R_l - i_c R_c + i_1 R_1 - v_s = 0$$

$$i_s R_l + i_s R_c + i_s R_1 - v_s = 0$$

$$[\mathbf{b}] \quad i_s = v_s / (R_l + R_c + R_1)$$

$$\text{DE 1.8} \quad [\mathbf{a}] \quad 24 = v_2 + v_5 - v_1 = 3i_5 + 7i_5 - (-2i_5) = 12i_5$$

$$\text{Therefore } i_5 = 24/12 = 2 \text{ A}$$

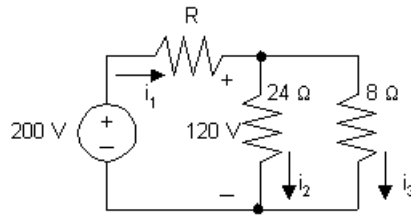
$$[\mathbf{b}] \quad v_1 = -2i_5 = -4 \text{ V}$$

$$[\mathbf{c}] \quad v_2 = 3i_5 = 6 \text{ V}$$

$$[\mathbf{d}] \quad v_5 = 7i_5 = 14 \text{ V}$$

[e] $p_{24} = -(24)(2) = -48$ W; therefore 24 V source is delivering 48 W.

DE 1.9



$$i_2 = 120/24 = 5 \text{ A}$$

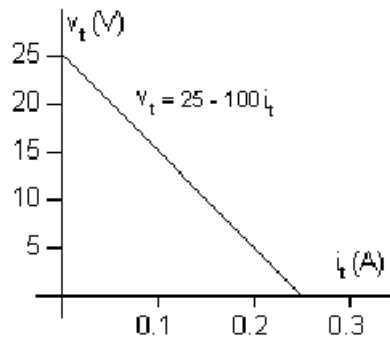
$$i_3 = 120/8 = 15 \text{ A}$$

$$i_1 = i_2 + i_3 = 20 \text{ A}$$

$$-200 + 20R + 120 = 0$$

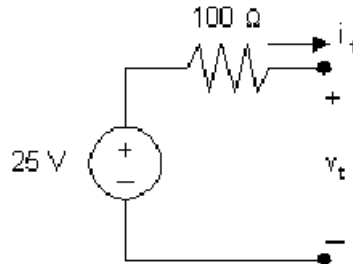
$$R = 80/20 = 4 \Omega$$

DE 1.10 [a] Plotting a graph of v_t versus i_t gives

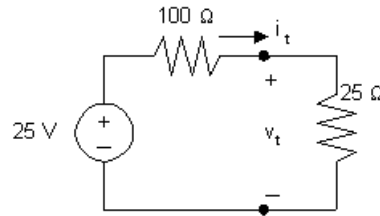


Note that when $i_t = 0$, $v_t = 25$ V; therefore the voltage source must be 25 V. When v_t is zero, $i_t = 0.25$ A, hence the resistor must be $25/0.25$ or 100Ω .

A circuit model having the same $v - i$ characteristic is a 25 V source in series with a 100Ω resistor.

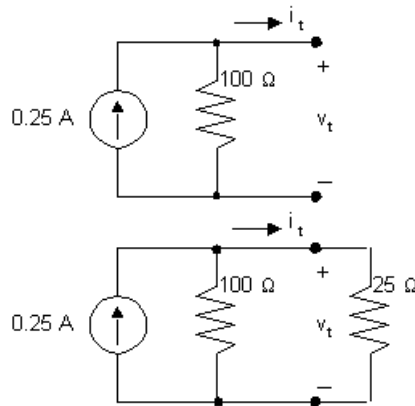


[b]



$$i_t = \frac{25}{125} = 0.2 \text{ A}; \quad p = (0.2)^2(25) = 1 \text{ W}.$$

DE 1.11 [a] Since we are constructing the model from two elements, we have two choices on interconnecting them—series or parallel. From the $v - i$ characteristic we require $v_t = 25 \text{ V}$ when $i_t = 0$. The only way we can satisfy this requirement is with a parallel connection. The constraint that $v_t = 0$ when $i_t = 0.25 \text{ A}$ tells us the ideal current source must produce 0.25 A . Therefore the parallel resistor must be $25/0.25$ or 100Ω .



[b]

$$-0.25 + \frac{v_t}{100} + \frac{v_t}{25} = 0, \quad 5v_t = 25, \quad v_t = 5 \text{ V}$$

$$p = \frac{v_t^2}{25} = 1 \text{ W}.$$

Problems

P 1.1 $i = \frac{dq}{dt} = 24 \cos 4000t$

Therefore, $dq = 24 \cos 4000t \, dt$

$$\int_{q(0)}^{q(t)} dx = 24 \int_0^t \cos 4000y \, dy$$

$$q(t) - q(0) = 24 \left. \frac{\sin 4000y}{4000} \right|_0^t$$

But $q(0) = 0$ by hypothesis, i.e., the current passes through its maximum value at $t = 0$, so $q(t) = 6 \times 10^{-3} \sin 4000t \text{ C} = 6 \sin 4000t \text{ mC}$

P 1.2 $p = (6)(100) \times 10^{-3} = 0.6 \text{ W}; \quad w = (0.6)(3)(60)(60) = 6480 \text{ J}$

P 1.3 Assume we are standing at box A looking toward box B, then $p = vi$.

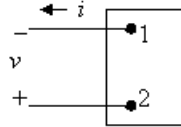
[a] $p = (120)(5) = 600 \text{ W}$ from A to B

[b] $p = (250)(-8) = -2000 \text{ W}$ from B to A

[c] $p = (-150)(16) = -2400 \text{ W}$ from B to A

[d] $p = (-480)(-10) = 4800 \text{ W}$ from A to B

P 1.4 [a]



$$p = vi = (40)(-10) = -400 \text{ W}$$

Power is being delivered by the box.

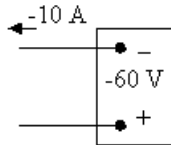
[b] Entering

[c] Gain

P 1.5 [a] $p = vi = (-60)(-10) = 600 \text{ W}$, so power is being absorbed by the box.

[b] Entering

[c] Lose



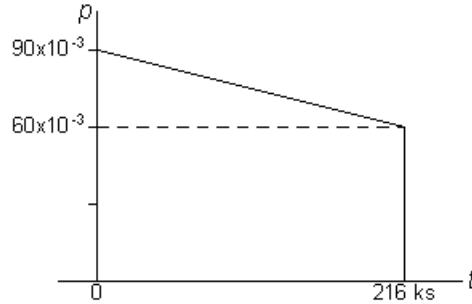
P 1.6 [a] Looking from A to B the current i is in the direction of the voltage rise across the 12 V battery, therefore $p = vi = -12(30) = -360 \text{ W}$. Thus the power flow is from B to A, and Car A has the “dead” battery.

[b] $w = \int_0^t p \, dx = \int_0^t 360 \, dx$

$$w = 360t = 360(1 \times 60) = 21.6 \text{ kJ}$$

P 1.7 $p = vi; \quad w = \int_0^t p \, dx$

Since the energy is the area under the power vs. time plot, let us plot p vs. t .



$$p(0) = (6)(15 \times 10^{-3}) = 90 \times 10^{-3} \text{ W}$$

$$p(216 \text{ ks}) = (4)(15 \times 10^{-3}) = 60 \times 10^{-3} \text{ W}$$

$$w = (60 \times 10^{-3})(216 \times 10^3) + \frac{1}{2}(216)(30) = 16.2 \text{ kJ}$$

Note: $60 \text{ hr} \equiv 216,000 \text{ s} = 216 \text{ ks}$

P 1.8 [a] $p = vi = 30e^{-500t} - 30e^{-1500t} - 40e^{-1000t} + 50e^{-2000t} - 10e^{-3000t}$
 $p(1 \text{ ms}) = 3.1 \text{ mW}$

[b]
$$w(t) = \int_0^t (30e^{-500x} - 30e^{-1500x} - 40e^{-1000x} + 50e^{-2000x} - 10e^{-3000x}) dx$$

$$= 21.67 - 60e^{-500t} + 20e^{-1500t} + 40e^{-1000t} - 25e^{-2000t} + 3.33e^{-3000t} \mu\text{J}$$

$$w(1 \text{ ms}) = 1.24 \mu\text{J}$$

[c] $w_{\text{total}} = 21.67 \mu\text{J}$

P 1.9 [a] $v(20 \text{ ms}) = 100e^{-1} \sin 3 = 5.19 \text{ V}$
 $i(20 \text{ ms}) = 20e^{-1} \sin 3 = 1.04 \text{ A}$
 $p(20 \text{ ms}) = vi = 5.39 \text{ W}$

[b]

$$\begin{aligned}
p &= vi = 2000e^{-100t} \sin^2 150t \\
&= 2000e^{-100t} \left[\frac{1}{2} - \frac{1}{2} \cos 300t \right] \\
&= 1000e^{-100t} - 1000e^{-100t} \cos 300t \\
w &= \int_0^\infty 1000e^{-100t} dt - \int_0^\infty 1000e^{-100t} \cos 300t dt \\
&= 1000 \left. \frac{e^{-100t}}{-100} \right|_0^\infty - 1000 \left\{ \frac{e^{-100t}}{(100)^2 + (300)^2} [-100 \cos 300t + 300 \sin 300t] \right\} \bigg|_0^\infty \\
&= 10 - 1000 \left[\frac{100}{1 \times 10^4 + 9 \times 10^4} \right] = 10 - 1 \\
w &= 9 \text{ J}
\end{aligned}$$

P 1.10 [a] $0 \leq t \leq 10 \text{ ms}$:

$$v = 1000t \text{ V}; \quad i = 0.6 \text{ mA}; \quad p = 0.6t \text{ mW}$$

 $10 \leq t \leq 25 \text{ ms}$:

$$v = 10 \text{ V}; \quad i = 0.6 \text{ mA}; \quad p = 6 \text{ mW}$$

 $25 \leq t \leq 35 \text{ ms}$:

$$v = 75 - 2500t \text{ V}; \quad i = 0 \text{ mA}; \quad p = 0 \text{ mW}$$

 $35 \leq t \leq 60 \text{ ms}$:

$$v = -50 + 1000t \text{ V}; \quad i = -0.4 \text{ mA}; \quad p = 20 - 400t \text{ mW}$$

 $60 \leq t \leq 70 \text{ ms}$:

$$v = -50 + 1000t \text{ V}; \quad i = 0 \text{ mA}; \quad p = 0 \text{ mW}$$

 $70 \leq t \leq 80 \text{ ms}$:

$$v = 20 \text{ V}; \quad i = -0.5 \text{ mA}; \quad p = -10 \text{ mW}$$

 $80 \leq t \leq 90 \text{ ms}$:

$$v = 180 - 2000t \text{ V}; \quad i = 0 \text{ mA}; \quad p = 0 \text{ mW}$$

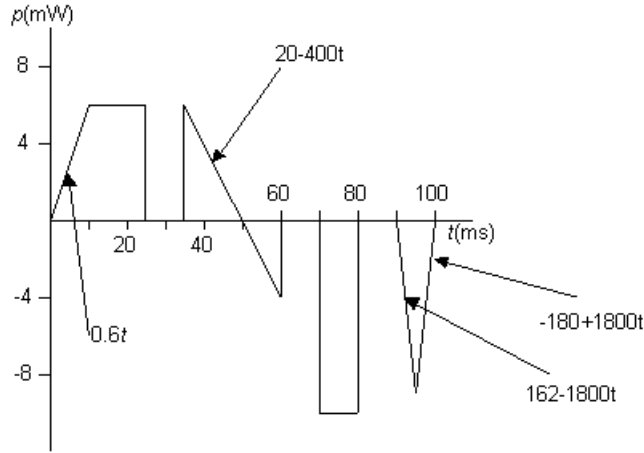
 $90 \leq t \leq 95 \text{ ms}$:

$$v = 180 - 2000t \text{ V}; \quad i = 0.9 \text{ mA}; \quad p = 162 - 1800t \text{ mW}$$

 $95 \leq t \leq 100 \text{ ms}$:

$$v = -200 + 2000t \text{ V}; \quad i = 0.9 \text{ mA}; \quad p = -180 + 1800t \text{ mW}$$

$$\begin{aligned}
\text{[b]} \quad w(25) &= \frac{1}{2}(6)(10) + (6)(15) = 120 \mu\text{J} \\
w(60) &= 120 + \frac{1}{2}(15)(6) - \frac{1}{2}(10)(4) = 145 \mu\text{J} \\
w(90) &= 145 - (10)(10) = 45 \mu\text{J} \\
w(100) &= 45 - \frac{1}{2}(10)(9) = 0 \mu\text{J}
\end{aligned}$$



P 1.11 [a] $p = vi = (2e^{-500t} - 2e^{-1000t}) \text{ W}$

$$\frac{dp}{dt} = -1000e^{-500t} + 2000e^{-1000t} = 0 \text{ at } t = 1.4 \text{ ms}$$

$$p_{\max} = p(1.4 \text{ ms}) = 0.5 \text{ W}$$

$$\begin{aligned}
\text{[b]} \quad w &= \int_0^{\infty} [2e^{-500t} - 2e^{-1000t}] dt = \left[\frac{2}{-500}e^{-500t} - \frac{2}{-1000}e^{-1000t} \right]_0^{\infty} \\
&= 2 \text{ mJ}
\end{aligned}$$

P 1.12 [a] $p = vi = 900 \sin(200\pi t) \cos(200\pi t) = 450 \sin(400\pi t) \text{ W}$

Therefore, $p_{\max} = 450 \text{ W}$

[b] $p_{\max}(\text{extracting}) = 450 \text{ W}$

$$\begin{aligned}
\text{[c]} \quad p_{\text{avg}} &= 200 \int_0^{5 \times 10^{-3}} 450 \sin(400\pi t) dt \\
&= 9 \times 10^4 \left[\frac{-\cos 400\pi t}{400\pi} \right]_0^{2.5 \times 10^{-3}} = \frac{225}{\pi} [1 - \cos 2\pi] = 0
\end{aligned}$$

$$\text{[d]} \quad p_{\text{avg}} = \frac{180}{\pi} [1 - \cos 2.5\pi] = \frac{180}{\pi} = 57.3 \text{ W}$$

P 1.13 [a] $q = \text{area under } i \text{ vs. } t \text{ plot}$

$$\begin{aligned}
&= \left[\frac{1}{2}(5)(4) + (10)(4) + \frac{1}{2}(8)(4) + (8)(6) + \frac{1}{2}(3)(6) \right] \times 10^3 \\
&= [10 + 40 + 16 + 48 + 9]10^3 = 123,000 \text{ C}
\end{aligned}$$

$$\begin{aligned}
\text{[b]} \quad w &= \int p dt = \int v i dt \\
v &= 0.2 \times 10^{-3}t + 9 \quad 0 \leq t \leq 15 \text{ ks} \\
0 \leq t &\leq 4000s \\
i &= 15 - 1.25 \times 10^{-3}t \\
p &= 135 - 8.25 \times 10^{-3}t - 0.25 \times 10^{-6}t^2 \\
w_1 &= \int_0^{4000} (135 - 8.25 \times 10^{-3}t - 0.25 \times 10^{-6}t^2) dt \\
&= (540 - 66 - 5.3333)10^3 = 468.667 \text{ kJ} \\
4000 \leq t &\leq 12,000 \\
i &= 12 - 0.5 \times 10^{-3}t \\
p &= 108 - 2.1 \times 10^{-3}t - 0.1 \times 10^{-6}t^2 \\
w_2 &= \int_{4000}^{12,000} (108 - 2.1 \times 10^{-3}t - 0.1 \times 10^{-6}t^2) dt \\
&= (864 - 134.4 - 55.467)10^3 = 674.133 \text{ kJ} \\
12,000 \leq t &\leq 15,000 \\
i &= 30 - 2 \times 10^{-3}t \\
p &= 270 - 12 \times 10^{-3}t - 0.4 \times 10^{-6}t^2 \\
w_3 &= \int_{12,000}^{15,000} (270 - 12 \times 10^{-3}t - 0.4 \times 10^{-6}t^2) dt \\
&= (810 - 486 - 219.6)10^3 = 104.4 \text{ kJ} \\
w_T &= w_1 + w_2 + w_3 = 468.667 + 674.133 + 104.4 = 1247.2 \text{ kJ}
\end{aligned}$$

$$\begin{aligned}
\text{P 1.14 [a]} \quad p &= v i \\
&= 400 \times 10^3 t^2 e^{-800t} + 700t e^{-800t} + 0.25 e^{-800t} \\
&= e^{-800t} [400,000t^2 + 700t + 0.25] \\
\frac{dp}{dt} &= \{e^{-800t} [800 \times 10^3 t + 700] - 800e^{-800t} [400,000t^2 + 700t + 0.25]\} \\
&= [-3,200,000t^2 + 2400t + 5]100e^{-800t} \\
\text{Therefore, } \frac{dp}{dt} &= 0 \text{ when } 3,200,000t^2 - 2400t - 5 = 0
\end{aligned}$$

so p_{\max} occurs at $t = 1.68 \text{ ms}$.

$$\begin{aligned}
\text{[b]} \quad p_{\max} &= [400,000(.00168)^2 + 700(.00168) + 0.25]e^{-800(.00168)} \\
&= 666 \text{ mW}
\end{aligned}$$

$$\begin{aligned}
[\text{c}] \quad w &= \int_0^t p dx \\
w &= \int_0^t 400,000x^2 e^{-800x} dx + \int_0^t 700x e^{-800x} dx + \int_0^t 0.25 e^{-800x} dx \\
&= \frac{400,000 e^{-800x}}{-512 \times 10^6} [64 \times 10^4 x^2 + 1600x + 2] \Big|_0^t + \\
&\quad \frac{700 e^{-800x}}{64 \times 10^4} (-800x - 1) \Big|_0^t + 0.25 \frac{e^{-800x}}{-800} \Big|_0^t
\end{aligned}$$

When $t = \infty$ all the upper limits evaluate to zero, hence

$$w = \frac{(400,000)(2)}{512 \times 10^6} + \frac{700}{64 \times 10^4} + \frac{0.25}{800} = 2.97 \text{ mJ.}$$

P 1.15 [a] $p = 0 \quad t < 0, \quad p = 0 \quad t > 3 \text{ s}$

$$\begin{aligned}
p &= vi = t(3-t)(6-4t) = 18t - 18t^2 + 4t^3 \text{ mW} \quad 0 \leq t \leq 3 \text{ s} \\
\frac{dp}{dt} &= 18 - 36t + 12t^2 = 12(t^2 - 3t + 1.5) \\
\frac{dp}{dt} &= 0 \quad \text{when } t^2 - 3t + 1.5 = 0 \\
t &= \frac{3 \pm \sqrt{9-6}}{2} = \frac{3 \pm \sqrt{3}}{2} \\
t_1 &= 3/2 - \sqrt{3}/2 = 0.634 \text{ s}; \quad t_2 = 3/2 + \sqrt{3}/2 = 2.366 \text{ s} \\
p(t_1) &= 18(0.634) - 18(0.634)^2 + 4(0.634)^3 = 5.196 \text{ mW} \\
p(t_2) &= 18(2.366) - 18(2.366)^2 + 4(2.366)^3 = -5.196 \text{ mW}
\end{aligned}$$

Therefore, maximum power is being delivered at $t = 0.634 \text{ s}$.

[b] $p_{\max} = 5.196 \text{ mW}$ (delivered)

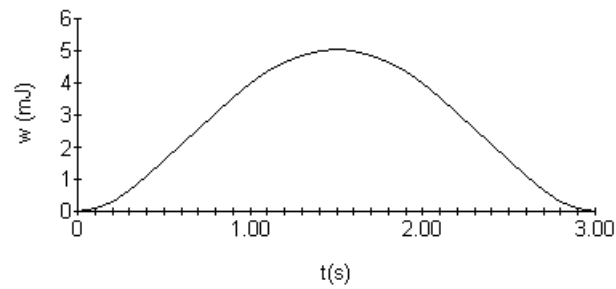
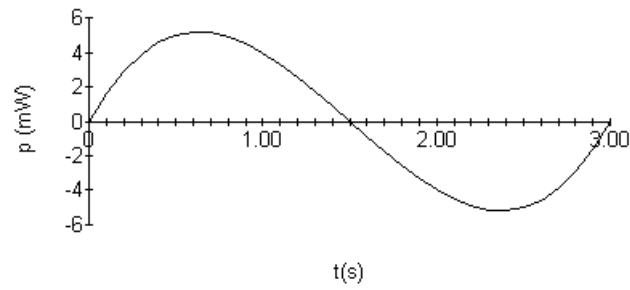
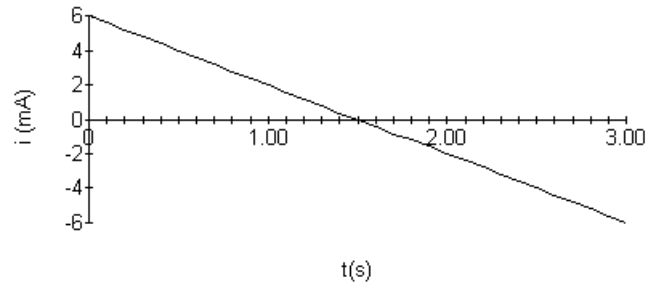
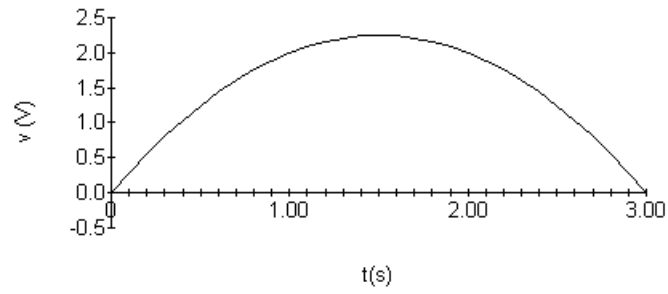
[c] Maximum power is being extracted at $t = 2.366 \text{ s}$.

[d] $p_{\max} = 5.196 \text{ mW}$ (extracted)

[e] $w = \int_0^t p dx = \int_0^t (18x - 18x^2 + 4x^3) dx = 9t^2 - 6t^3 + t^4$

$$w(0) = 0 \text{ mJ} \quad w(2) = 4 \text{ mJ}$$

$$w(1) = 4 \text{ mJ} \quad w(3) = 0 \text{ mJ}$$



$$\begin{aligned}
 \text{P 1.16 [a]} \quad p &= vi = 12 \times 10^5 t^2 e^{-1000t} \text{ W} \\
 \frac{dp}{dt} &= 12 \times 10^5 [t^2(-1000)e^{-1000t} + e^{-1000t}(2t)] \\
 &= 12 \times 10^5 t e^{-1000t} [t(2 - 1000t)]
 \end{aligned}$$

$$\frac{dp}{dt} = 0 \text{ at } t = 0, \quad t = 2 \text{ ms}$$

We know p is a minimum at $t = 0$ since v and i are zero at $t = 0$.

$$\text{[b]} \quad p_{\max} = 12 \times 10^5 (2 \times 10^{-3})^2 e^{-2} = 649.61 \text{ mW}$$

$$\begin{aligned}
[\text{c}] \quad w &= 12 \times 10^5 \int_0^\infty t^2 e^{-1000t} dt \\
&= 12 \times 10^5 \left\{ \frac{e^{-1000t}}{(-1000)^3} [10^6 t^2 + 2,000t + 2] \right\} \Big|_0^\infty = 2.4 \text{ mJ}
\end{aligned}$$

P 1.17 [a] From the diagram and the table we have

$$\begin{aligned}
p_a &= -v_a i_a = -(46.16)(6) = -276.96 \text{ W} & (\text{del}) \\
p_b &= v_b i_b = (14.16)(4.72) = 66.8352 \text{ W} & (\text{abs}) \\
p_c &= v_c i_c = (-32)(-6.4) = 204.80 \text{ W} & (\text{abs}) \\
p_d &= -v_d i_d = -(22)(1.28) = -28.16 \text{ W} & (\text{del}) \\
p_e &= -v_e i_e = -(33.60)(1.68) = -56.448 \text{ W} & (\text{del}) \\
p_f &= v_f i_f = (66)(-0.4) = -26.40 \text{ W} & (\text{del}) \\
p_g &= v_g i_g = (2.56)(1.28) = 3.2768 \text{ W} & (\text{abs}) \\
p_h &= -v_h i_h = -(-0.4)(0.4) = 0.16 \text{ W} & (\text{abs})
\end{aligned}$$

$$\sum P_{\text{del}} = 276.96 + 28.16 + 56.448 + 26.40 = 387.9680 \text{ W}$$

$$\sum P_{\text{abs}} = 66.8352 + 204.80 + 3.2768 + 0.16 = 275.072 \text{ W}$$

Therefore, $\sum P_{\text{del}} \neq \sum P_{\text{abs}}$ and the subordinate engineer is correct.

[b] We can also check the data using Kirchhoff's laws.

From Fig. P1.17 the following equations should be satisfied:

$$\begin{aligned}
i_a - i_b - i_d &= 0 & (\text{ok}) \\
i_b + i_c - i_e &= 0 & (\text{no}) \\
i_f - i_a - i_c &= 0 & (\text{ok}) \\
i_d &= i_g & (\text{ok}) \\
i_g + i_e + i_h &= 0 & (\text{no}) \\
i_h &= -i_f & (\text{ok})
\end{aligned}$$

Using Kirchhoff's current law, it appears i_e is in error.

From Kirchhoff's voltage law we have

$$\begin{aligned}
v_b - v_a - v_c &= 0 & (\text{ok}) \\
-v_d - v_b + v_e + v_g &= 0 & (\text{ok}) \\
-v_e + v_c + v_f + v_h &= 0 & (\text{ok})
\end{aligned}$$

Therefore all the voltages are consistent with Kirchhoff's voltage law.

Assume i_e is in error. Therefore,

$$i_e = i_b + i_c = -i_g - i_h = 4.72 - 6.40 = -1.28 - 0.4 = -1.68 \text{ A}$$

So the error is in the sign of i_e ; i_e equals minus 1.68 A.

Correcting i_e leads to

$$\sum P_{\text{del}} = \sum P_{\text{abs}} = 331.52 \text{ W}$$

P 1.18

$$\begin{aligned}
 p_a &= v_a i_a = (48)(12) = 576 \text{ W} && (\text{abs}) \\
 p_b &= v_b i_b = (18)(-4) = -72 \text{ W} && (\text{del}) \\
 p_c &= -v_c i_c = -(30)(-10) = 300 \text{ W} && (\text{abs}) \\
 p_d &= v_d i_d = (36)(16) = 576 \text{ W} && (\text{abs}) \\
 p_e &= -v_e i_e = -(36)(8) = -288 \text{ W} && (\text{del}) \\
 p_f &= -v_f i_f = -(-54)(14) = 756 \text{ W} && (\text{abs}) \\
 p_g &= -v_g i_g = -(84)(22) = -1848 \text{ W} && (\text{del}) \\
 \sum P_{\text{del}} &= 72 + 288 + 1848 = 2208 \text{ W} \\
 \sum P_{\text{abs}} &= 576 + 300 + 576 + 756 = 2208 \text{ W} \\
 \text{Therefore, } \sum P_{\text{del}} &= \sum P_{\text{abs}} = 2208 \text{ W}
 \end{aligned}$$

P 1.19 [a] From an examination of reference polarities, the following elements employ the passive convention: a, c, e , and f .

[b]

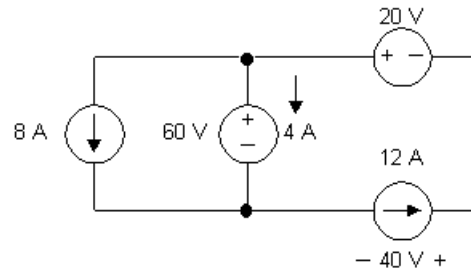
$$\begin{aligned}
 p_a &= -56 \text{ W} && (\text{del}) \\
 p_b &= -14 \text{ W} && (\text{del}) \\
 p_c &= 150 \text{ W} && (\text{abs}) \\
 p_d &= -50 \text{ W} && (\text{del}) \\
 p_e &= -18 \text{ W} && (\text{del}) \\
 p_f &= -12 \text{ W} && (\text{del}) \\
 \sum P_{\text{abs}} &= 150 \text{ W}; && \sum P_{\text{del}} = 56 + 14 + 50 + 18 + 12 = 150 \text{ W}.
 \end{aligned}$$

P 1.20 (a) 9 (b) 7 (c) 4 (d) $v_a - R_a, v_b - R_b, v_c - R_c$ (e) 6

(f)

- (1) $v_a - R_a - R_d - R_b - v_b$
- (2) $R_d - R_f - R_e$
- (3) $v_b - R_b - R_d - R_f - R_c - v_c$
- (4) $v_c - R_c - R_f - R_a - v_a$
- (5) $v_a - R_a - R_f - R_e - R_b - v_b$
- (6) $v_a - R_a - R_d - R_e - R_c - v_c$
- (7) $v_b - R_b - R_e - R_c - v_c$

P 1.21 The interconnect is valid since it does not violate Kirchhoff's laws.



$$-60 + 20 + 40 = 0 \quad (\text{KVL})$$

$$8 + 4 - 12 = 0 \quad (\text{KCL})$$

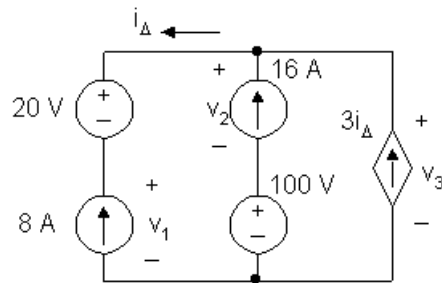
$$\sum P_{\text{dev}} = 4(60) + 8(60) = 720 \text{ W}$$

$$\sum P_{\text{abs}} = 12(20) + 12(40) = 720 \text{ W}$$

$$\sum P_{\text{dev}} = \sum P_{\text{abs}} = 720 \text{ W}$$

P 1.22 [a] Yes, Kirchhoff's laws are not violated.

[b] No, because the voltages across the independent and dependent current sources are indeterminate. For example, define v_1 , v_2 , and v_3 as shown:



Kirchhoff's voltage law requires

$$v_1 + 20 = v_3$$

$$v_2 + 100 = v_3$$

Conservation of energy requires

$$8(20) + 8v_1 + 16v_2 + 1600 - 24v_3 = 0$$

or

$$v_1 + 2v_2 - 3v_3 = -220$$

Now arbitrarily select a value of v_3 and show the conservation of energy will be satisfied. Examples:

If $v_3 = 200$ V then $v_1 = 180$ V and $v_2 = 100$ V. Then

$$180 + 200 - 600 = -220 \text{ (CHECKS)}$$

If $v_3 = -100$ V, then $v_1 = -120$ V and $v_2 = -200$ V. Then

$$-120 - 400 + 300 = -220 \text{ (CHECKS)}$$

P 1.23 [a] Yes, independent voltage sources can carry whatever current is required by the connection; independent current source can support any voltage required by the connection.

[b] 30 V source: absorbing

10 V source: delivering

8 A source: delivering

[c] $P_{30V} = (30)(8) = 240 \text{ W (abs)}$

$$P_{10V} = -10(8) = -80 \text{ W (del)}$$

$$P_{8A} = -20(8) = -160 \text{ W (del)}$$

$$\sum P_{\text{abs}} = \sum P_{\text{del}} = 240 \text{ W}$$

[d] Yes, 30 V source is delivering, the 10 V source is delivering, and the 8 A source is absorbing

$$P_{30V} = -30(8) = -240 \text{ W (del)}$$

$$P_{10V} = -10(8) = -80 \text{ W (del)}$$

$$P_{8A} = +40(8) = 320 \text{ W (abs)}$$

P 1.24 The interconnection is valid because it does not violate Kirchhoff's laws.

$$i_{\Delta} = -25 \text{ A}; \quad 6i_{\Delta} = -150 \text{ V}$$

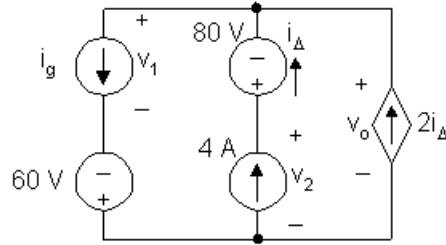
$$-200 + 50 - (-150) = 0$$

But the power developed in the circuit cannot be determined, as the currents in the 200 V, 50 V, and $6i_{\Delta}$ sources are unspecified.

P 1.25 The interconnection is not valid because it violates Kirchhoff's current law:

$$3 \text{ A} + (-5 \text{ A}) \neq 8 \text{ A}.$$

P 1.26



$$i_{\Delta} = 4 \text{ A so } i_g = 12 \text{ A}$$

$$v_o = 100 \text{ V}$$

$$-60 + v_1 = 100, \text{ so } v_1 = 160 \text{ V}$$

$$v_2 - 80 = 100, \text{ so } v_2 = 180 \text{ V}$$

$$\sum P_{\text{dev}} = 180(4) + 100(8) + 60(12) = 2240 \text{ W}$$

$$\begin{aligned} \text{CHECK: } \sum P_{\text{diss}} &= 160(12) + 80(4) = 1920 + 320 \\ &= 2240 \text{ W} \text{ — CHECKS} \end{aligned}$$

P 1.27 The interconnection is valid because it does not violate Kirchhoff's laws:

$$p_{V\text{-sources}} = -(100 - 60)(5) = -200 \text{ W}.$$

P 1.28 First there is no violation of Kirchhoff's laws, hence the interconnection is valid.

Kirchhoff's voltage law requires

$$v_1 + v_2 = 150 - 50 = 100 \text{ V}$$

The conservation of energy law requires

$$20v_1 - 10v_1 + 10v_2 + 500 - 1500 = 0$$

or

$$v_1 + v_2 = 100$$

Hence any combination of v_1 and v_2 that adds to 100 is a valid solution. For example if $v_1 = 80 \text{ V}$ and $v_2 = 20 \text{ V}$

$$P_{\text{abs}} = 80(20) + 10(20) + 50(10) = 2300 \text{ W}$$

$$P_{\text{dev}} = 1500 + 80(10) = 2300 \text{ W}$$

If $v_1 = 60 \text{ V}$ and $v_2 = 40 \text{ V}$

$$P_{\text{abs}} = 60(20) + 10(40) + 500 = 2100 \text{ W}$$

$$P_{\text{dev}} = 60(10) + 1500 = 2100 \text{ W}$$

If $v_1 = -100 \text{ V}$ and $v_2 = 200 \text{ V}$

$$P_{\text{abs}} = 10(100) + 10(200) + 10(50) = 3500 \text{ W}$$

$$P_{\text{dev}} = 20(100) + 10(150) = 3500 \text{ W}$$

P 1.29 [a] $1.6 = i_g - i_a$
 $80i_a = 1.6(30 + 90) = 192$ therefore, $i_a = 2.4 \text{ A}$
 $i_g = i_a + 1.6 = 2.4 + 1.6 = 4 \text{ A}$

[b] $v_g = 90(1.6) = 144 \text{ V}$

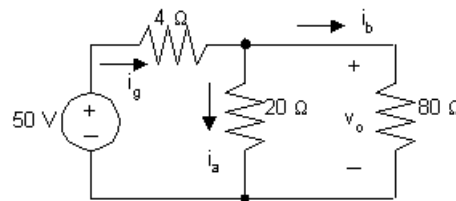
[c] $\sum P_{\text{dis}} = 2.4^2(80) + 1.6^2(120) = 768 \text{ W}$
 $\sum P_{\text{dev}} = (4)(192) = 768 \text{ W}$
 Therefore, $\sum P_{\text{dis}} = \sum P_{\text{dev}} = 768 \text{ W}$

P 1.30 [a] $v_o = 8i_a + 14i_a + 18i_a = 40(20) = 800 \text{ V}$
 $800 = 10i_o$
 $i_o = 800/10 = 80 \text{ A}$

[b] $i_g = i_a + i_o = 20 + 80 = 100 \text{ A}$

[c] $p_g(\text{delivered}) = (100)(800) = 80,000 \text{ W} = 80 \text{ kW}$

P 1.31 [a]



$$20i_a = 80i_b \quad i_g = i_a + i_b = 5i_b$$

$$i_a = 4i_b$$

$$50 = 4i_g + 80i_b = 20i_b + 80i_b = 100i_b$$

$$i_b = 0.5 \text{ A, therefore, } i_a = 2 \text{ A} \quad \text{and} \quad i_g = 2.5 \text{ A}$$

[b] $i_b = 0.5 \text{ A}$

[c] $v_o = 80i_b = 40 \text{ V}$

[d] $p_{4\Omega} = i_g^2(4) = 6.25(4) = 25 \text{ W}$

$$p_{20\Omega} = i_a^2(20) = (4)(20) = 80 \text{ W}$$

$$p_{80\Omega} = i_b^2(80) = 0.25(80) = 20 \text{ W}$$

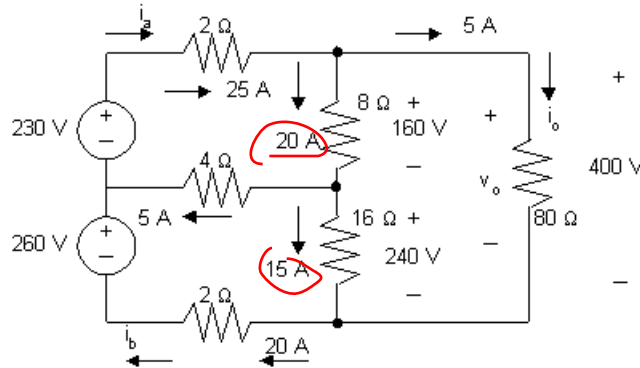
[e] $p_{5V} \text{ (delivered)} = 5i_g = 125 \text{ W}$

Check:

$$\sum P_{\text{dis}} = 25 + 80 + 20 = 125 \text{ W}$$

$$\sum P_{\text{del}} = 125 \text{ W}$$

P 1.32 [a]



$$v_o = 20(8) + 16(15) = 400 \text{ V}$$

$$i_o = 400/80 = 5 \text{ A}$$

$$i_a = 25 \text{ A}$$

$$P_{230} \text{ (supplied)} = (230)(25) = 5750 \text{ W}$$

$$i_b = 5 + 15 = 20 \text{ A}$$

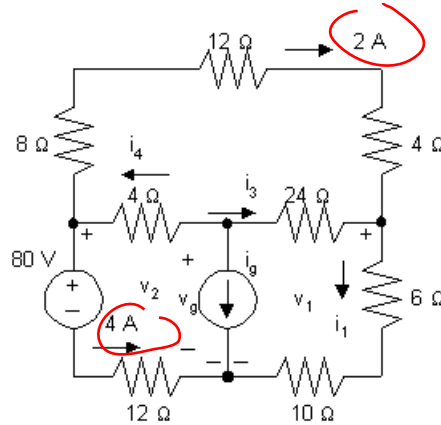
$$P_{260} \text{ (supplied)} = (260)(20) = 5200 \text{ W}$$

$$\begin{aligned} \text{[b]} \quad \sum P_{\text{dis}} &= (25)^2(2) + (20)^2(8) + (5)^2(4) + (15)^2(16) + (20)^2(2) + (5)^2(80) \\ &= 1250 + 3200 + 100 + 3600 + 800 + 2000 = 10,950 \text{ W} \end{aligned}$$

$$\sum P_{\text{sup}} = 5750 + 5200 = 10,950 \text{ W}$$

$$\text{Therefore, } \sum P_{\text{dis}} = \sum P_{\text{sup}} = 10,950 \text{ W}$$

P 1.33 [a]



$$v_2 = 80 + 4(12) = 128 \text{ V}$$

$$v_1 = 128 - 24(2) = 80 \text{ V}$$

$$i_1 = \frac{v_1}{16} = \frac{80}{16} = 5 \text{ A}$$

$$i_3 = i_1 - 2 = 5 - 2 = 3 \text{ A}$$

$$v_g = v_1 + 24i_3 = 80 + 72 = 152 \text{ V}$$

$$v_g - 4i_4 = v_2$$

$$4i_4 = v_g - v_2 = 152 - 128 = 24 \text{ V}$$

$$i_4 = 24/4 = 6 \text{ A}$$

$$i_g = -(i_3 + i_4) = -(3 + 6) = -9 \text{ A}$$

$$[b] \quad p_{8\Omega} = (2)^2(8) = 32 \text{ W} \quad p_{4\Omega} = (6)^2(4) = 144 \text{ W}$$

$$p_{12\Omega} = (2)^2(12) = 48 \text{ W} \quad p_{6\Omega} = (5)^2(6) = 150 \text{ W}$$

$$p_{4\Omega} = (2)^2(4) = 16 \text{ W} \quad p_{10\Omega} = (5)^2(10) = 250 \text{ W}$$

$$p_{24\Omega} = (3)^2(24) = 216 \text{ W} \quad p_{12\Omega} = (4)^2(12) = 192 \text{ W}$$

$$[c] \quad v_g = 152 \text{ V}$$

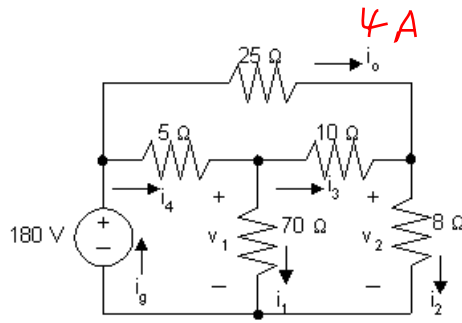
[d]

$$\sum P_{\text{dis}} = 32 + 48 + 16 + 216 + 144 + 150 + 250 + 192 + 80(4) = 1368 \text{ W}$$

$$\sum P_{\text{del}} = (152)(9) = 1368 \text{ W}$$

$$\text{Therefore, } \sum P_{\text{dis}} = \sum P_{\text{del}}$$

P 1.34 [a]



$$v_2 = 180 - 100 = 80 \text{ V}$$

$$i_2 = \frac{v_2}{8} = 10 \text{ A}$$

$$i_3 + 4 = i_2, \quad i_3 = 10 - 4 = 6 \text{ A}$$

$$v_1 = v_2 + v_3 = 80 + 6(10) = 140 \text{ V}$$

$$i_1 = \frac{v_1}{70} = \frac{140}{70} = 2 \text{ A}$$

$$[\mathbf{b}] \quad p_{5\Omega} = 8^2(5) = 320 \text{ W}$$

$$p_{25\Omega} = (4)^2(25) = 400 \text{ W}$$

$$p_{70\Omega} = 2^2(70) = 280 \text{ W}$$

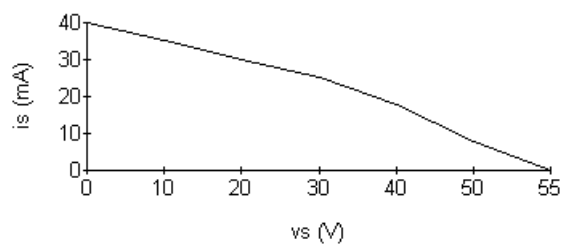
$$p_{10\Omega} = 6^2(10) = 360 \text{ W}$$

$$p_{8\Omega} = 10^2(8) = 800 \text{ W}$$

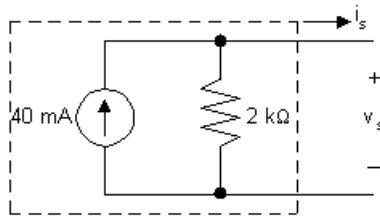
$$[\mathbf{c}] \quad \sum P_{\text{dis}} = 320 + 400 + 280 + 360 + 800 = 2160 \text{ W}$$

$$P_{\text{dev}} = 180i_g = 180(12) = 2160 \text{ W}$$

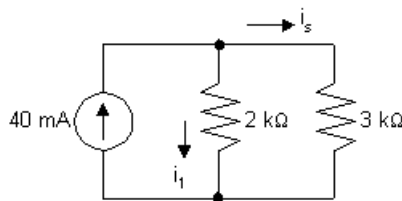
P 1.35 [a]



[b] $\Delta v = 20 \text{ V}; \quad \Delta i = 10 \text{ mA}; \quad R = \frac{\Delta v}{\Delta i} = 2 \text{ k}\Omega$



[c] $2i_1 = 3i_s, \quad i_1 = 1.5i_s$
 $40 = i_1 + i_s = 2.5i_s, \quad i_s = 16 \text{ mA}$

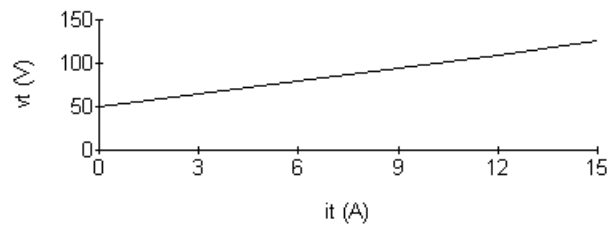


[d] $v_s(\text{open circuit}) = (40 \times 10^{-3})(2 \times 10^3) = 80 \text{ V}$

[e] $v_s(\text{open circuit}) = 55 \text{ V}$

[f] Linear model cannot predict the nonlinear behavior of the practical current source.

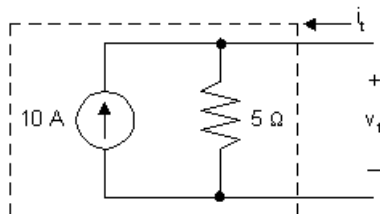
P 1.36 [a] Plot the $v - i$ characteristic



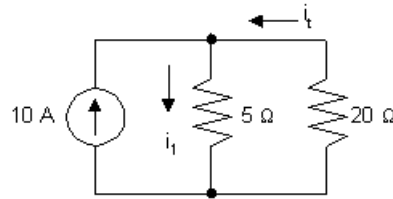
From the plot:

$$R = \frac{\Delta v}{\Delta i} = \frac{(125 - 50)}{(15 - 0)} = 5 \Omega$$

When $i_t = 0$, $v_t = 50 \text{ V}$; therefore the ideal current source has a current of 10 A



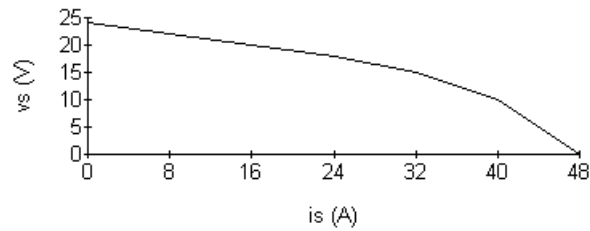
[b]



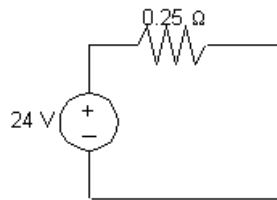
$$10 + i_t = i_1 \quad \text{and} \quad 5i_1 = -20i_t$$

Therefore, $10 + i_t = -4i_t$ so $i_t = -2$ A

P 1.37 [a]



[b] $R = \frac{24 - 18}{24 - 0} = \frac{6}{24} = 0.25 \Omega$



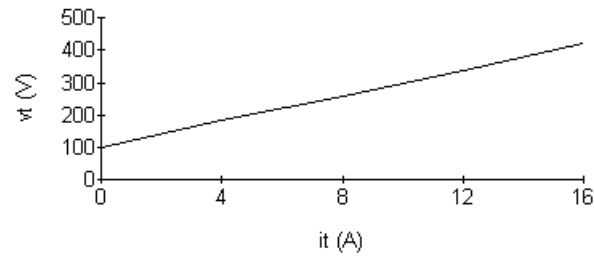
[c] $i = \frac{24}{1.25} = 19.2$ A, $v = 24 - 19.2(0.25) = 19.2$ V

[d] $i_{sc} = \frac{24}{0.25} = 96$ A

[e] $i_{sc} = 48$ A (from graph)

[f] Linear model cannot predict nonlinear behavior of voltage source.

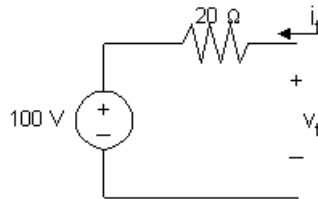
P 1.38 [a] Plot the $v-i$ characteristic:



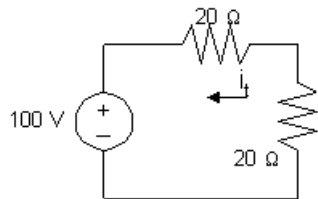
From the plot:

$$R = \frac{\Delta v}{\Delta i} = \frac{(420 - 100)}{(16 - 0)} = 20 \, \Omega$$

When $i_t = 0$, $v_t = 100$ V; therefore the ideal voltage source has a voltage of 100 V

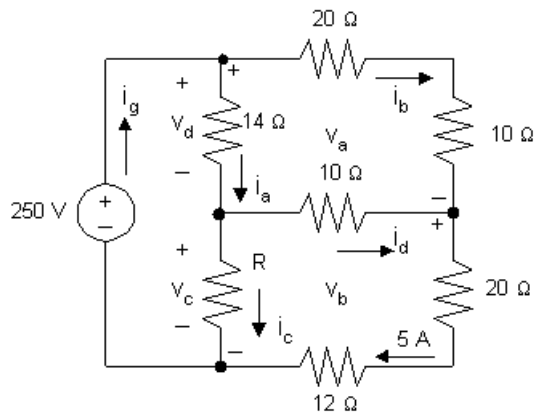


[b]



$$i_t = -100/(20 + 20) = -2.5 \text{ A}; \text{ Therefore, } p_{20\Omega} = (-2.5)^2(20) = 125 \text{ W}$$

P 1.39 [a]



$$v_b = 5(20 + 12) = 160 \text{ V}$$

$$v_b + v_a = 250 \text{ V, so } v_a = 90 \text{ V}$$

$$i_b = 90/(20 + 10) = 3 \text{ A}$$

$$i_d = 5 - i_b = 2 \text{ A}$$

$$v_c = v_b + 10(i_d) = 180 \text{ V}$$

$$v_d = 250 - v_c = 70 \text{ V} = 14(i_a); \text{ therefore, } i_a = 5 \text{ A}$$

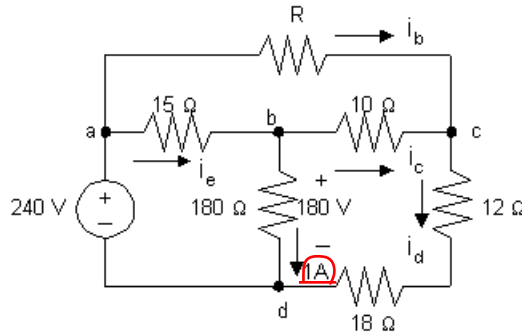
$$i_c = i_a - i_d = 5 - 2 = 3 \text{ A}$$

$$R = v_c/i_c = 180/3 = 60 \Omega$$

[b] $i_g = 5 + 3 = 8 \text{ A}$

$$p_g (\text{supplied}) = (250)(8) = 2000 \text{ W}$$

P 1.40



$$v_{ab} = 240 - 180 = 60 \text{ V}; \quad \text{therefore, } i_e = 60/15 = 4 \text{ A}$$

$$i_c = i_e - 1 = 4 - 1 = 3 \text{ A}; \quad \text{therefore, } v_{bc} = 10i_c = 30 \text{ V}$$

$$v_{cd} = 180 - v_{bc} = 180 - 30 = 150 \text{ V};$$

$$\text{therefore, } i_d = v_{cd}/(12 + 18) = 150/30 = 5 \text{ A}$$

$$i_b = i_d - i_c = 5 - 3 = 2 \text{ A}$$

$$v_{ac} = v_{ab} + v_{bc} = 60 + 30 = 90 \text{ V}$$

$$R = v_{ac}/i_b = 90/2 = 45 \Omega$$

CHECK: $i_g = i_b + i_e = 2 + 4 = 6 \text{ A}$

$$p_{\text{dev}} = (240)(6) = 1440 \text{ W}$$

$$\begin{aligned} \sum P_{\text{dis}} &= 1(180) + 4(45) + 9(10) + 25(12) \\ &\quad + 25(18) + 16(15) = 1440 \text{ W (CHECKS)} \end{aligned}$$

P 1.41 [a] $15.2 = 10,000i_\beta - 0.80 + (200)30i_\beta$

$$16 = (16,000)i_\beta$$

$$i_\beta = 1 \text{ mA}$$

$$200(30i_\beta) + v_y + 500(29i_\beta) - 25 = 0$$

$$v_y = 25 - 6000i_\beta - 14,500i_\beta$$

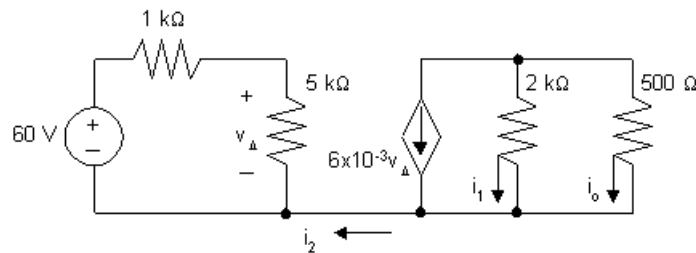
$$\text{Therefore, } v_y = 4.5 \text{ V}$$

[b] $\sum P_{\text{gen}} = 15.2i_\beta + 25(29)i_\beta + 0.8i_\beta = 741i_\beta = 741 \text{ mW}$

$$\begin{aligned} \sum P_{\text{dis}} &= 10^4(i_\beta)^2 + 200(30i_\beta)^2 + 29i_\beta(4.5) + 500(29i_\beta)^2 \\ &= 741 \text{ mW.} \end{aligned}$$

P 1.42 [a] $i_2 = 0$ because no current can exist in a single conductor connecting two parts of a circuit.

[b]



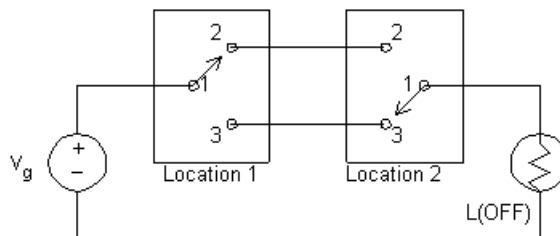
$$60 = 6000i_g \quad i_g = 10 \text{ mA}$$

$$v_\Delta = 5000i_g = 50 \text{ V} \quad 6 \times 10^{-3}v_\Delta = 300 \text{ mA}$$

$$2000i_1 = 500i_o, \text{ so } i_1 + 4i_1 = -300 \text{ mA; therefore, } i_1 = -60 \text{ mA}$$

[c] $300 - 60 + i_2 = 0$, so $i_o = -240 \text{ mA}$.

P 1.43 [a]



[b]

