

The Operational Amplifier

Drill Exercises

DE 4.1 [a] $v_o = (-80/16)v_s, \quad v_o = -5v_s$

$$v_s(\text{ V}) \quad 0.4 \quad 2.0 \quad 3.5 \quad -0.6 \quad -1.6 \quad -2.4$$

$$v_o(\text{ V}) \quad -2.0 \quad -10.0 \quad -15.0 \quad 3.0 \quad 8.0 \quad 10.0$$

[b] $-15 = -5v_s, \quad v_s = 3 \text{ V}; \quad 10 = -5v_s, \quad v_s = -2 \text{ V}$

Therefore $-2 \leq v_s \leq 3 \text{ V}$

DE 4.2 $v_o = (-R_x/16)v_s = (0.64R_x/16) = 10 \text{ V}$

Therefore $R_x = \frac{160}{0.64} = 250 \text{ k}\Omega, \quad 0 \leq R_x \leq 250 \text{ k}\Omega$

DE 4.3 [a] $v_o = -\frac{250}{5}v_a - \frac{250}{25}v_b = -50(0.1) - 10(0.25) = -5 - 2.5 = -7.5 \text{ V}$

[b] $v_o = -50v_a - 2.5 = -10 \text{ V}; \quad \text{therefore } 50v_a = 7.5, \quad v_a = 0.15 \text{ V}$

[c] $v_o = -5 - 10v_b = -10 \text{ V}; \quad 10v_b = 5, \quad v_b = 0.5 \text{ V}$

[d] $v_o = -50v_a + 10v_b = -5 + 2.5 = -2.5 \text{ V}$

$$v_o = -50v_a + 2.5 = -10 \text{ V};$$

$$50v_a = 12.5, \quad v_a = 0.25 \text{ V}$$

$$v_o = -5 + 10v_b = 15 \text{ V}; \quad 10v_b = 20; \quad v_b = 2.0 \text{ V}$$

DE 4.4 [a] $\frac{v_n}{4500} + \frac{v_n - v_o}{63,000} = 0, \quad \text{therefore } v_o = 15v_n, \quad v_n = v_p$

Thus $v_o = 15v_p, \quad v_p = \frac{0.4R_x}{15,000 + R_x}$

So when $R_x = 60 \text{ k}\Omega, \quad v_p = 0.32 \text{ V}, \quad v_o = 4.8 \text{ V}$

$$[\mathbf{b}] \quad \frac{15(0.4R_x)}{15,000 + R_x} = 5, \quad R_x = 75 \text{ k}\Omega$$

DE 4.5 [a] Assume v_a is acting alone. Replacing v_b with a short circuit yields $v_p = 0$, therefore $v_n = 0$ and we have

$$\frac{0 - v_a}{R_a} + \frac{0 - v'_o}{R_b} + i_n = 0, \quad i_n = 0$$

Therefore

$$\frac{v'_o}{R_b} = -\frac{v_a}{R_a}, \quad v'_o = \frac{R_b}{R_a}v_a$$

Assume v_b is acting alone. Replace v_a with a short circuit. Now

$$v_p = v_n = \frac{v_b R_d}{R_c + R_d}$$

$$\frac{v_n}{R_a} + \frac{v_n - v''_o}{R_b} + i_n = 0, \quad i_n = 0$$

$$\left(\frac{1}{R_a} + \frac{1}{R_b}\right) \left(\frac{R_d}{R_c + R_d}\right) v_b - \frac{v''_o}{R_b} = 0$$

$$v''_o = \left(\frac{R_b}{R_a} + 1\right) \left(\frac{R_d}{R_c + R_d}\right) v_b = \frac{R_d}{R_a} \left(\frac{R_a + R_b}{R_c + R_d}\right) v_b$$

$$v_o = v'_o + v''_o = \frac{R_d}{R_a} \left(\frac{R_a + R_b}{R_c + R_d}\right) v_b - \frac{R_b}{R_a} v_a$$

$$[\mathbf{b}] \quad \frac{R_d}{R_a} \left(\frac{R_a + R_b}{R_c + R_d}\right) = \frac{R_b}{R_a}, \quad \text{therefore} \quad R_d(R_a + R_b) = R_b(R_c + R_d)$$

$$R_d R_a = R_b R_c, \quad \text{therefore} \quad \frac{R_a}{R_b} = \frac{R_c}{R_d}$$

$$\text{When } \frac{R_d}{R_a} \left(\frac{R_a + R_b}{R_c + R_d}\right) = \frac{R_b}{R_a}$$

$$\text{Eq. (4.22) reduces to } v_o = \frac{R_b}{R_a} v_b - \frac{R_b}{R_a} v_a = \frac{R_b}{R_a} (v_b - v_a).$$

$$\text{DE 4.6 [a]} \quad v_o = \frac{20(60)}{10(24)} v_b - \frac{50}{10} v_a = 5(v_b - v_a) = 20 - 5v_a$$

$$20 - 5v_a = \pm 10 \text{ V}$$

$$5v_a = 20 \mp 10, \quad v_a = 2 \text{ V}, \quad v_a = 6 \text{ V}$$

$$\text{Therefore } -2 \leq v_a \leq 6 \text{ V}$$

$$[\mathbf{b}] \quad v_o = \frac{8(60)}{10(12)}v_b - 5v_a = 4v_b - 5v_a$$

$$4v_b - 5v_a = 16 - 5v_a = \pm 10 \text{ V}$$

$$16 \mp 10 = 5v_a, \quad v_a = 1.2 \text{ V}, \quad v_a = 5.2 \text{ V}$$

$$\text{Therefore } 1.2 \leq v_a \leq 5.2 \text{ V}$$

$$\text{DE 4.7 } [\mathbf{a}] \quad A_{\text{dm}} = \frac{(24)(26) + (25)(25)}{(2)(1)(25)} = 24.98$$

$$[\mathbf{b}] \quad A_{\text{cm}} = \frac{(1)(24) - 25(1)}{1(25)} = -0.04$$

$$[\mathbf{c}] \quad \text{CMRR} = \left| \frac{24.98}{0.04} \right| = 624.50$$

$$\text{DE 4.8 } A_{\text{cm}} = \frac{(20)(50) - (50)R_x}{20(50 + R_x)}$$

$$A_{\text{dm}} = \frac{50(20 + 50) + 50(50 + R_x)}{2(20)(50 + R_x)}$$

$$\frac{A_{\text{dm}}}{A_{\text{cm}}} = \frac{R_x + 120}{2(20 - R_x)}$$

$$\therefore \frac{R_x + 120}{2(20 - R_x)} = \pm 1000$$

$$\text{If we use } +1000 \quad R_x = 19.93 \text{ k}\Omega$$

$$\text{If we use } -1000 \quad R_x = 20.07 \text{ k}\Omega$$

Problems

P 4.1 Since the current into the inverting input terminal of an ideal op-amp is zero, the voltage across the $3.3\text{ M}\Omega$ resistor is $(2.5)(3.3)$ or 8.25 V . Therefore the voltmeter reads 8.25 V .

P 4.2
$$v_p = \frac{18}{24}(12) = 9\text{ V} = v_n$$

$$\frac{v_n - 24}{30} + \frac{v_n - v_o}{20} = 0$$

$$v_o = (45 - 48)/3 = -1.0\text{ V}$$

$$i_L = \frac{v_o}{5} \times 10^{-3} = -\frac{1}{5} \times 10^{-3} = -200 \times 10^{-6}$$

$$i_L = -200\text{ }\mu\text{A}$$

P 4.3
$$\frac{v_b - v_a}{20} + \frac{v_b - v_o}{160} = 0, \quad \text{therefore } v_o = 9v_b - 8v_a$$

[a] $v_a = 1.5\text{ V}, \quad v_b = 0\text{ V}, \quad v_o = -12\text{ V}$

[b] $v_a = 3.0\text{ V}, \quad v_b = 0\text{ V}, \quad v_o = -18\text{ V (sat)}$

[c] $v_a = 1.0\text{ V}, \quad v_b = 2\text{ V}, \quad v_o = 10\text{ V}$

[d] $v_a = 4.0\text{ V}, \quad v_b = 2\text{ V}, \quad v_o = -14\text{ V}$

[e] $v_a = 6.0\text{ V}, \quad v_b = 8\text{ V}, \quad v_o = 18\text{ V (sat)}$

[f] If $v_b = 4.5\text{ V}, \quad v_o = 40.5 - 8v_a = \pm 18$

$$\therefore 2.8125 \leq v_a \leq 7.3125\text{ V}$$

P 4.4 [a]
$$i_a = \frac{120}{6} \times 10^{-6} = 20\text{ }\mu\text{A}$$

$$v_a = -20 \times 10^3 i_a = -400\text{ mV}$$

[b]
$$\frac{v_a}{60,000} + \frac{v_a}{20,000} + \frac{v_a - v_o}{240,000} = 0$$

$$\therefore v_o = 17v_a = -6.8\text{ V}$$

[c] $i_a = 20\text{ }\mu\text{A}$

[d]
$$i_o = \frac{-v_o}{80,000} + \frac{v_a - v_o}{240,000} = 111.67\text{ }\mu\text{ A}$$

P 4.5 $v_o = (1)(9) = 9 \text{ V}; \quad i_{15k\Omega} = \frac{9}{15,000} = 0.6 \text{ mA};$

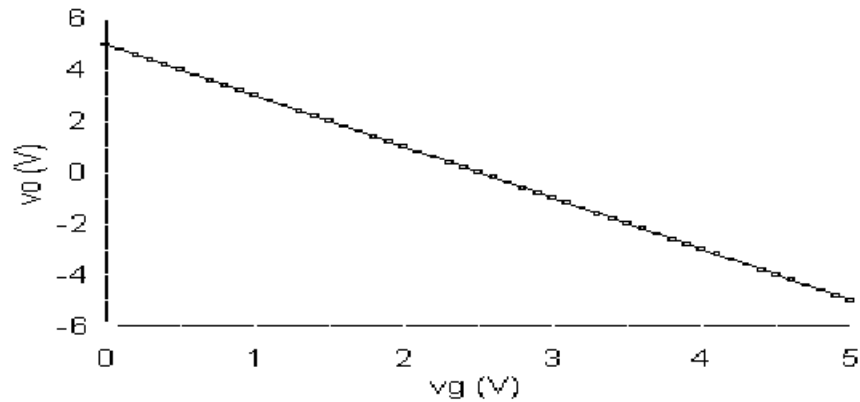
$$i_{6k\Omega} = \frac{9}{6000} = 1.5 \text{ mA}; \quad i_{9k\Omega} = \frac{9}{9000} = 1 \text{ mA}$$

$$\therefore i_o = -0.6 - 1.5 - 1 = -3.1 \text{ mA}$$

P 4.6 [a] First, note that $v_n = v_p = 2.5 \text{ V}$
Let v_{o1} equal the voltage output of the op-amp. Then

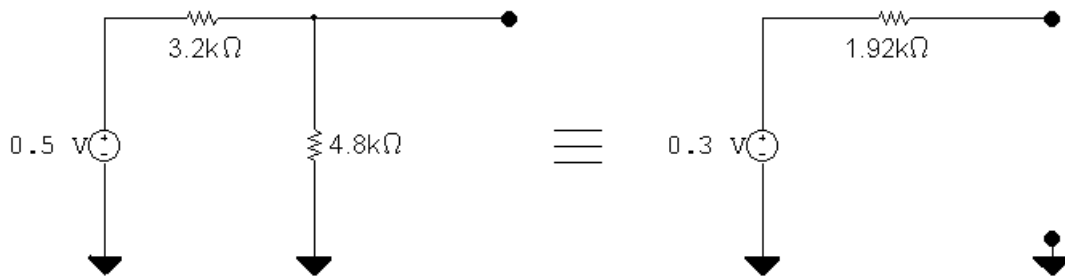
$$\frac{2.5 - v_g}{5000} + \frac{2.5 - v_{o1}}{10,000} = 0, \quad \therefore v_{o1} = 7.5 - 2v_g$$

$$\text{Also note that } v_{o1} - 2.5 = v_o, \quad \therefore v_o = 5 - 2v_g$$



[b] Yes, the circuit designer is correct!

P 4.7 [a] Replace the combination of v_g , $3.2 \text{ k}\Omega$, and the $4.8 \text{ k}\Omega$ resistors with its Thévenin equivalent.



$$\text{Then } v_o = \frac{-[30 + \sigma 170]}{1.92} (0.30)$$

At saturation $v_o = -10 \text{ V};$ therefore

$$-\left(\frac{30 + \sigma 170}{1.92}\right) (0.3) = -10, \quad \text{or } \sigma = 0.2$$

Thus for $0 \leq \sigma < 0.20$ the operational amplifier will not saturate.

[b] When $\sigma = 0.12$, $v_o = \frac{-(30 + 20.4)}{1.92}(0.30) = -7.875 \text{ V}$

Also $\frac{v_o}{180} + \frac{v_o}{50.4} + i_o = 0$

$\therefore i_o = -\frac{v_o}{180} - \frac{v_o}{50.4} = \frac{7.875}{180} + \frac{7.875}{50.4} \text{ mA} = 200 \mu\text{A}$

P 4.8 [a] Let v_Δ be the voltage from the potentiometer contact to ground. Then

$$\frac{0 - v_g}{5} + \frac{0 - v_\Delta}{15} = 0$$

$$-3v_g - v_\Delta = 0, \quad \therefore v_\Delta = -150 \text{ mV}$$

$\checkmark \therefore \frac{v_\Delta}{\alpha R_\Delta} + \frac{v_\Delta - 0}{15,000} + \frac{v_\Delta - v_o}{(1 - \alpha)R_\Delta} = 0$

$$\frac{v_\Delta}{\alpha} + 10v_\Delta + \frac{v_\Delta - v_o}{1 - \alpha} = 0$$

$$v_\Delta \left(\frac{1}{\alpha} + 10 + \frac{1}{1 - \alpha} \right) = \frac{v_o}{1 - \alpha}$$

$$\therefore v_o = -0.15 \left[1 + 10(1 - \alpha) + \frac{(1 - \alpha)}{\alpha} \right]$$

When $\alpha = 0.3$, $v_o = -0.15(1 + 7 + 7/3) = -1.55 \text{ V}$

When $\alpha = 0.75$, $v_o = -0.15(1 + 2.5 + 1/3) = -0.575 \text{ V}$

$$\therefore -1.55 \text{ V} \leq v_o \leq -0.575 \text{ V}$$

[b] $-0.15 \left[1 + 10(1 - \alpha) + \frac{(1 - \alpha)}{\alpha} \right] = -6$

$$\alpha + 10\alpha(1 - \alpha) + (1 - \alpha) = 40\alpha$$

$$\alpha + 10\alpha - 10\alpha^2 + 1 - \alpha = 40\alpha$$

$$\therefore 10\alpha^2 + 30\alpha - 1 = 0 \quad \text{so} \quad \alpha \cong 0.033$$

P 4.9 [a] $\frac{v_d - v_a}{72} + \frac{v_d - v_b}{120} + \frac{v_d - v_c}{450} + \frac{v_d}{600} + \frac{v_d - v_o}{180} = 0$

$$v_o = 180 \left(-\frac{10}{72,000} + \frac{2}{120,000} + \frac{23}{450,000} + \frac{8}{600,000} + \frac{8}{180,000} \right) = -2.4 \text{ V}$$

$$\begin{aligned}
\text{[b]} \quad v_o &= -8.4 - 0.4v_c \\
-8.4 - 0.4v_c &= -16; \quad v_c = 19 \text{ V} \\
-8.4 - 0.4v_c &= 16; \quad v_c = -61 \text{ V} \\
-61 \text{ V} &\leq v_c \leq 19 \text{ V}
\end{aligned}$$

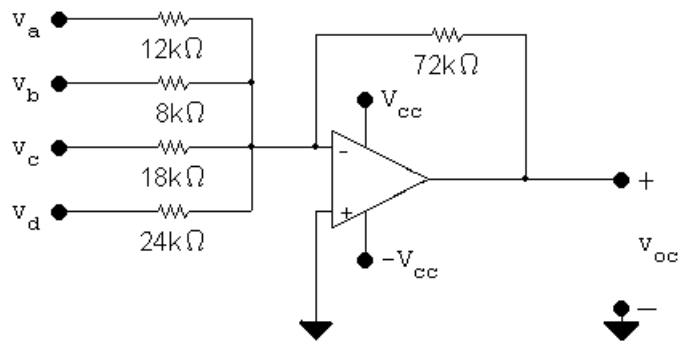
$$\begin{aligned}
\text{P 4.10 [a]} \quad \frac{v_d - v_a}{72,000} + \frac{v_d - v_b}{120,000} + \frac{v_d - v_c}{450,000} + \frac{v_d}{600,000} + \frac{v_d - v_o}{R_f} &= 0 \\
(25/3)v_d - (25/3)v_a + 5v_d - 5v_b + (4/3)v_d - (4/3)v_c + v_d + & \\
\frac{600}{R_f}v_d &= \frac{600}{R_f}v_o \\
(47/3)v_d + \frac{600}{R_f}v_d - (25/3)v_a - 5v_b - (4/3)v_c &= \frac{600}{R_f}v_o \\
(376/3) + \frac{4800}{R_f} - 150 - 30 + 20 &= \frac{600}{R_f}v_o \\
14400 - 104R_f &= 1800v_o \quad \text{or} \quad 104R_f = 14400 - 1800v_o \\
v_o &= \pm 16 \text{ V}, \quad \text{but} \quad R_f > 0 \\
\therefore 104R_f &= 14400 - 1800(-16) \quad \text{or} \quad R_f = 415.38 \text{ k}\Omega
\end{aligned}$$

$$\begin{aligned}
\text{[b]} \quad i_f &= \frac{8 - (-16)}{415.38 \times 10^3} = 57.78 \mu\text{A} \\
i_{27\text{k}\Omega} &= \frac{v_o}{0.027 \times 10^6} = -592.59 \mu\text{A} \\
i_o - i_f + i_{27\text{k}\Omega} &= 0 \\
i_o &= 57.78 - (-592.59) = 650.37 \mu\text{A}
\end{aligned}$$

$$\begin{aligned}
\text{P 4.11 [a]} \quad v_o &= -\frac{220}{33}v_a - \frac{220}{22}v_b - \frac{220}{80}v_c = -8 + 15 - 11 = -4 \text{ V} \\
\text{[b]} \quad v_o &= -19 - 10v_b = \pm 6 \\
\therefore v_b &= -1.3 \text{ V} \quad \text{when} \quad v_o = -6 \text{ V}; \\
v_b &= -2.5 \text{ V} \quad \text{when} \quad v_o = 6 \text{ V} \\
\therefore -2.5 \text{ V} &\leq v_b \leq -1.3 \text{ V}
\end{aligned}$$

$$\begin{aligned}
\text{P 4.12} \quad v_o &= -\left[\frac{R_f}{3000}(0.15) + \frac{R_f}{5000}(0.1) + \frac{R_f}{25,000}(0.25) \right] \\
-6 &= -8 \times 10^{-5}R_f; \quad R_f = 75 \text{ k}\Omega; \quad \therefore 0 \leq R_f \leq 75 \text{ k}\Omega
\end{aligned}$$

P 4.13



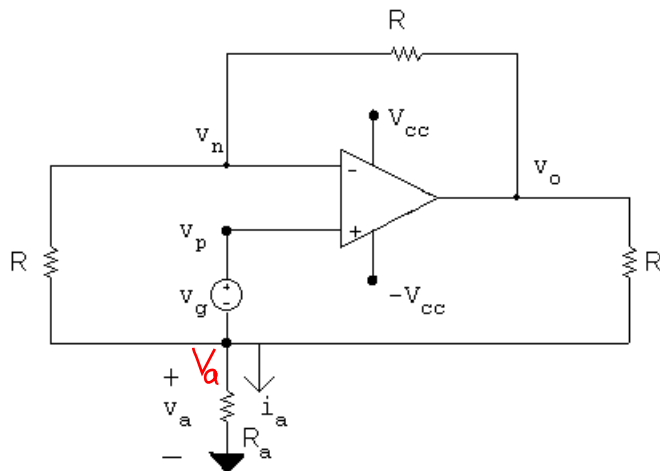
$$v_o = -(6v_a + 9v_b + 4v_c + 3v_d)$$

$$v_o = -\left[\frac{72}{R_a}v_a + \frac{72}{R_b}v_b + \frac{72}{R_c}v_c + \frac{72}{R_d}v_d\right]$$

$$\therefore R_a = 72,000/6 = 12 \text{ k}\Omega \quad R_c = 72,000/4 = 18 \text{ k}\Omega$$

$$R_b = 72,000/9 = 8 \text{ k}\Omega \quad R_d = 72,000/3 = 24 \text{ k}\Omega$$

P 4.14 [a]



$$\text{KCL } v_n: \frac{v_n - v_a}{R} + \frac{v_n - v_o}{R} = 0$$

$$2v_n - v_a = v_o$$

$$V_a: \frac{v_a}{R_a} + \frac{v_a - v_n}{R} + \frac{v_a - v_o}{R} = 0$$

$$v_a \left[\frac{1}{R_a} + \frac{2}{R} \right] - \frac{v_n}{R} = \frac{v_o}{R}$$

$$v_a \left(2 + \frac{R}{R_a} \right) - v_n = v_o$$

$$v_n = v_p = v_a + v_g$$

$$\therefore 2v_n - v_a = 2v_a + 2v_g - v_a = v_a + 2v_g$$

$$\therefore v_a - v_o = -2v_g \quad (1)$$

$$2v_a + v_a \left(\frac{R}{R_a} \right) - v_a - v_g = v_o$$

$$\therefore v_a \left(1 + \frac{R}{R_a} \right) - v_o = v_g \quad (2)$$

Now combining equations (1) and (2) yields

$$-v_a \frac{R}{R_a} = -3v_g$$

$$\text{or } v_a = 3v_g \frac{R_a}{R}$$

$$\text{Hence } i_a = \frac{v_a}{R_a} = \frac{3v_g}{R} \quad \text{Q.E.D.}$$

[b] At saturation $V_o = \pm V_{cc}$

$$\therefore v_a = \pm V_{cc} - 2v_g \quad (3)$$

and

$$\therefore v_a \left(1 + \frac{R}{R_a} \right) = \pm V_{cc} + v_g \quad (4)$$

Dividing Eq (4) by Eq (3) gives

$$1 + \frac{R}{R_a} = \frac{\pm V_{cc} + v_g}{\pm V_{cc} - 2v_g}$$

$$\therefore \frac{R}{R_a} = \frac{\pm V_{cc} + v_g}{\pm V_{cc} - 2v_g} - 1 = \frac{3v_g}{\pm V_{cc} - 2v_g}$$

$$\text{or } R_a = \frac{(\pm V_{cc} - 2v_g)}{3v_g} R \quad \text{Q.E.D.}$$

P 4.15 [a] Assume the op-amp is operating within its linear range, then

$$i_L = \frac{3}{1.5} = 2 \text{ mA}$$

$$\text{For } R_L = 2.5 \text{ k}\Omega \quad v_o = (2.5 + 1.5)(2) = 8 \text{ V}$$

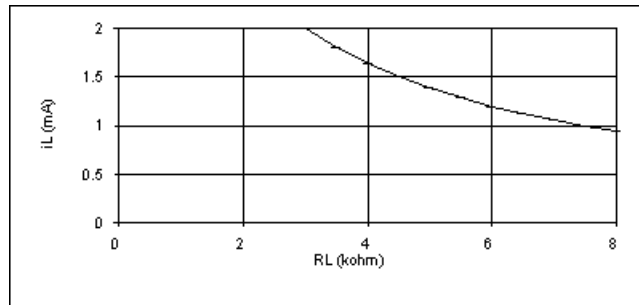
Now since $v_o < 9 \text{ V}$ our assumption of linear operation is correct, therefore

$$i_L = 2 \text{ mA}$$

$$[b] 9 = 2(1.5 + R_L); \quad R_L = 3 \text{ k}\Omega$$

- [c] As long as the op-amp is operating in its linear region i_L is independent of R_L . From (b) we found the op-amp is operating in its linear region as long as $R_L \leq 3 \text{ k}\Omega$. Therefore when $R_L = 6.5 \text{ k}\Omega$ the op-amp is saturated. We can estimate the value of i_L by assuming $i_p = i_n \ll i_L$. Then $i_L = 9/(1.5 + 6.5) = 1.125 \text{ mA}$. To justify neglecting the current into the op-amp assume the drop across the $47 \text{ k}\Omega$ resistor is negligible, and the input resistance to the op-amp is at least $500 \text{ k}\Omega$. Then $i_p = i_n = (3 - 1.5)/500 \times 10^{-3} = 3 \mu\text{A}$. But $3 \mu\text{A} \ll 1.125 \text{ mA}$, hence our assumption is reasonable.

[d]



- P 4.16 [a] The output voltage of the first op-amp is $v_{o1} = -(80/20)v_g = -4v_g$. The output voltage of the second op-amp is $v_{o2} = -1.6v_{o1} = 6.4v_g$. When v_g has its largest value, i.e., 1.2 V ,

$$v_{o1} = -4.8 \text{ V} \quad \text{and} \quad v_{o2} = 7.68 \text{ V}$$

Therefore neither op-amp saturates. The expression for i_g is

$$i_g = \frac{v_g}{20,000} + \frac{v_g - 6.4v_g}{R_o} = v_g \left[\frac{1}{20,000} - \frac{5.4}{R_o} \right]$$

$$i_g = 0 \quad \text{when} \quad \left(\frac{1}{20,000} - \frac{5.4}{R_o} \right) = 0, \quad \text{or} \quad R_o = 108 \text{ k}\Omega$$

$$\text{[b]} \quad i_{R_o} = \frac{6.4v_g - v_g}{R_o} = \frac{5.4v_g}{R_o} = 50v_g \mu\text{A} = 50 \mu\text{A}$$

$$p_{R_o} = (50 \times 10^{-6})^2 (108 \times 10^3) = 270 \mu\text{W}$$

- P 4.17 Let v_{o1} be the output voltage of the first operational amplifier and v_{o2} the output voltage of the second operational amplifier. Then

$$\frac{0 - 1}{12,000} + \frac{0 - v_{o1}}{48,000} + \frac{0 - v_{o2}}{100,000} = 0$$

$$-50 - 12.5v_{o1} - 6v_{o2} = 0$$

$$\frac{v_{o1}}{30,000} + \frac{v_{o1} - v_{o2}}{6000} = 0$$

$$\therefore 6v_{o1} = 5v_{o2}$$

$$\therefore -50 - 12.5[(5/6)v_{o2}] - 6v_{o2} = 0 \quad \text{so} \quad v_{o2} = -3.05 \text{ V}$$

$$i_a = \frac{v_{o2}}{36,000} = -0.0846 \text{ mA}$$

$$i_a = -84.6 \mu\text{A}$$

P 4.18 [a] Let v_{o1} = output voltage of the amplifier on the left. Let v_{o2} = output voltage of the amplifier on the right. Then

$$\frac{0 - V_{o1}}{47\text{k}} = \frac{1 - 0}{10\text{k}} \quad \rightarrow \quad v_{o1} = \frac{-47}{10}(1) = -4.7 \text{ V}; \quad v_{o2} = \frac{-220}{33}(-0.15) = 1.0 \text{ V} \quad \frac{0 - V_{o2}}{22\text{k}} = \frac{-0.15 - 0}{33\text{k}}$$

$$i_a = \frac{v_{o2} - v_{o1}}{1000} = 5.7 \text{ mA}$$

[b] $i_a = 0$ when $v_{o1} = v_{o2}$ so from (a) $v_{o2} = 1 \text{ V}$

Thus

$$\frac{-47}{10}(v_L) = 1$$

$$v_L = -\frac{10}{47} = -212.77 \text{ mV}$$

P 4.19 [a] $p_{16\text{k}\Omega} = \frac{(320 \times 10^{-3})^2}{(16 \times 10^3)} = 6.4 \mu\text{W}$

[b] $v_{16\text{k}\Omega} = \left(\frac{16}{64}\right)(320) = 80 \text{ mV}$

$$p_{16\text{k}\Omega} = \frac{(80 \times 10^{-3})^2}{(16 \times 10^3)} = 0.4 \mu\text{W}$$

[c] $\frac{p_a}{p_b} = \frac{6.4}{0.4} = 16$

[d] Yes, the operational amplifier serves several useful purposes:

- First, it enables the source to control 16 times as much power delivered to the load resistor. When a small amount of power controls a larger amount of power, we refer to it as *power amplification*.
- Second, it allows the full source voltage to appear across the load resistor, no matter what the source resistance. This is the *voltage follower* function of the operational amplifier.

- Third, it allows the load resistor voltage (and thus its current) to be set without drawing any current from the input voltage source. This is the *current amplification* function of the circuit.

P 4.20 [a] $v_p = v_s, \quad v_n = \frac{R_1 v_o}{R_1 + R_2}, \quad v_n = v_p$

Therefore $v_o = \left(\frac{R_1 + R_2}{R_1} \right) v_s = \left(1 + \frac{R_2}{R_1} \right) v_s$

[b] $v_o = v_s$

[c] Because $v_o = v_s$, thus the output voltage follows the signal voltage.

P 4.21 [a] $v_p = v_n = \frac{45}{75} v_g = 0.6 v_g$

$$\therefore \frac{0.6 v_g}{15} + \frac{0.6 v_g - v_o}{48} = 0$$

$$\therefore v_o = 2.52 v_g = 2.52(3), \quad v_o = 7.56 \text{ V}$$

[b] $v_o = 2.52 v_g = \pm 10$

$$v_g = \pm 3.97 \text{ V}, \quad -3.97 \leq v_g \leq 3.97 \text{ V}$$

[c] $\frac{0.6 v_g}{15} + \frac{0.6 v_g - v_o}{R_f} = 0$

$$\left(\frac{0.6 R_f}{15} + 0.6 \right) v_g = v_o = \pm 10$$

$$\therefore 3R_f + 45 = \pm 150; \quad 3R_f = 150 - 45; \quad R_f = 35 \text{ k}\Omega$$

P 4.22 [a] $\frac{v_p - v_a}{R_a} + \frac{v_p - v_b}{R_b} + \frac{v_p - v_c}{R_c} + \frac{v_p}{R_g} = 0$

$$\therefore v_p = \frac{R_b R_c R_g}{D} v_a + \frac{R_a R_c R_g}{D} v_b + \frac{R_a R_b R_g}{D} v_c$$

where $D = R_b R_c R_g + R_a R_c R_g + R_a R_b R_g + R_a R_b R_c$

$$\frac{v_n}{R_s} + \frac{v_n - v_o}{R_f} = 0$$

$$v_n \left(\frac{1}{R_s} + \frac{1}{R_f} \right) = \frac{v_o}{R_f}$$

$$\therefore v_o = \left(1 + \frac{R_f}{R_s} \right) v_n = k v_n$$

where $k = \left(1 + \frac{R_f}{R_s} \right)$

$$v_p = v_n$$

$$\therefore v_o = kv_p$$

or

$$v_o = \frac{kR_g R_b R_c}{D} v_a + \frac{kR_g R_a R_c}{D} v_b + \frac{kR_g R_a R_b}{D} v_c$$

$$\frac{kR_g R_b R_c}{D} = 3 \quad \therefore \frac{R_b}{R_a} = 1.5$$

$$\frac{kR_g R_a R_c}{D} = 2 \quad \therefore \frac{R_c}{R_b} = 2$$

$$\frac{kR_g R_a R_b}{D} = 1 \quad \therefore \frac{R_c}{R_a} = 3$$

$$\text{Since } R_a = 2 \text{ k}\Omega \quad R_b = 3 \text{ k}\Omega \quad R_c = 6 \text{ k}\Omega$$

$$\therefore D = [(3)(6)(4) + (2)(6)(4) + (2)(3)(4) + (2)(3)(6)] \times 10^9 = 180 \times 10^9$$

$$\frac{k(4)(3)(6) \times 10^9}{180 \times 10^9} = 3$$

$$k = \frac{540 \times 10^9}{72 \times 10^9} = 7.5$$

$$\therefore 7.5 = 1 + \frac{R_f}{R_s}$$

$$\frac{R_f}{R_s} = 6.5$$

$$R_f = (6.5)(12,000) = 78 \text{ k}\Omega$$

$$[\mathbf{b}] \quad v_o = 3(0.8) + 2(1.5) + 2.10 = 7.5 \text{ V}$$

$$v_n = v_p = \frac{7.5}{7.5} = 1.0 \text{ V}$$

$$i_a = \frac{0.8 - 1}{2000} = \frac{-0.2}{2000} = -0.1 \text{ mA} = -100 \mu\text{A}$$

$$i_b = \frac{1.5 - 1.0}{3000} = \frac{0.5}{3000} = 166.67 \mu\text{A}$$

$$i_c = \frac{2.10 - 1.0}{6000} = \frac{1.1}{6000} = 183.33 \mu\text{A}$$

$$i_g = \frac{1}{4000} = 250 \mu\text{A}$$

$$i_s = \frac{v_n}{12,000} = \frac{1}{12,000} = 83.33 \mu\text{A}$$

P 4.23 [a] $\frac{v_p - v_a}{80 \times 10^3} + \frac{v_p - v_b}{64 \times 10^3} = 0$

$$\therefore 9v_p = 4v_a + 5v_b$$

$$\frac{v_n}{18,000} + \frac{v_n - v_o}{72,000} = 0$$

$$\therefore v_o = 5v_n = 5v_p = (20/9)v_a + (25/9)v_b = 4.44 \text{ V}$$

[b] $v_p = v_n = \frac{v_o}{5} = 0.889 \text{ V}$

$$i_a = \frac{v_a - v_p}{80 \times 10^3} = -4.86 \mu\text{A}$$

$$i_b = \frac{v_b - v_p}{64 \times 10^3} = 4.86 \mu\text{A}$$

[c] $(20/9)$ for v_a

$(25/9)$ for v_b

P 4.24 [a] $\frac{v_p - v_a}{R_a} + \frac{v_p - v_b}{R_b} + \frac{v_p - v_c}{R_c} = 0$

$$\therefore v_p = \frac{R_b R_c}{D} v_a + \frac{R_a R_c}{D} v_b + \frac{R_a R_b}{D} v_c$$

where $D = R_b R_c + R_a R_c + R_a R_b$

$$\frac{v_n}{10,000} + \frac{v_n - v_o}{R_f} = 0$$

$$\left(\frac{R_f}{10,000} + 1 \right) v_n = v_o$$

Let $\frac{R_f}{10,000} + 1 = k$

$$v_o = kv_n = kv_p$$

$$\therefore v_o = \frac{kR_b R_c}{D} v_a + \frac{kR_a R_c}{D} v_b + \frac{kR_a R_b}{D} v_c$$

$$\therefore \frac{kR_b R_c}{D} = 5 \quad \therefore \frac{R_c}{R_a} = 5$$

$$\frac{kR_a R_c}{D} = 4$$

$$\frac{kR_a R_b}{D} = 1 \quad \therefore \frac{R_c}{R_b} = 4$$

$$\therefore R_c = 5R_a = 5 \text{ k}\Omega$$

$$R_b = R_c/4 = 1.25 \text{ k}\Omega$$

$$\therefore D = (1.25)(5) + (1)(5) + (1.25)(1) = 12.5 \times 10^6$$

$$\therefore k = \frac{5D}{R_b R_c} = \frac{(5)(12.5) \times 10^6}{(1.25)(5) \times 10^6} = 10$$

$$\therefore \frac{R_f}{10,000} + 1 = 10, \quad R_f = 90 \text{ k}\Omega$$

$$[\mathbf{b}] \quad v_o = 5(0.5) + 4(1) + 1.5 = 8 \text{ V}$$

$$v_n = v_o/10 = 0.8 \text{ V} = v_p$$

$$i_a = \frac{v_a - v_p}{1000} = \frac{0.5 - 0.8}{1000} = -300 \mu\text{A}$$

$$i_b = \frac{v_b - v_p}{1250} = \frac{1 - 0.8}{1250} = 160 \mu\text{A}$$

$$i_c = \frac{v_c - v_p}{5000} = \frac{1.5 - 0.8}{5000} = 140 \mu\text{A}$$

$$\text{P 4.25} \quad [\mathbf{a}] \quad v_o = \frac{R_d(R_a + R_b)}{R_a(R_c + R_d)}v_b - \frac{R_b}{R_a}v_a = \frac{33(100)}{20(80)}(0.90) - 4(0.45)$$

$$v_o = 1.8563 - 1.8 = 56.25 \text{ mV}$$

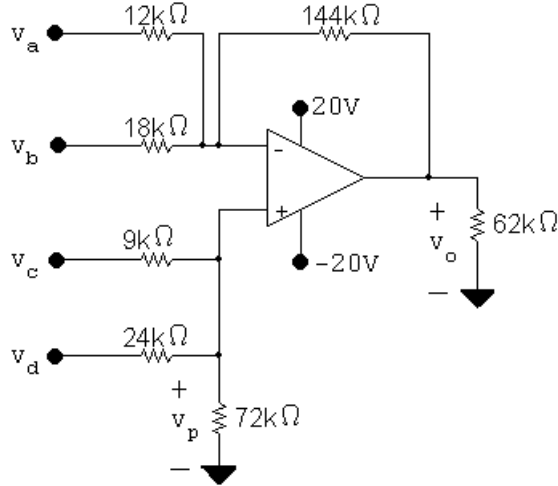
$$[\mathbf{b}] \quad v_n = v_p = \frac{(0.90)(33)}{80} = 371.25 \text{ mV}$$

$$i_a = \frac{(450 - 371.25)10^{-3}}{20 \times 10^3} = 3.9375 \mu\text{A}$$

$$R_a = \frac{v_a}{i_a} = \frac{450 \times 10^{-3}}{3.9375 \times 10^{-6}} = 114.3 \text{ k}\Omega$$

$$[\mathbf{c}] \quad R_{\text{in } b} = R_c + R_d = 80 \text{ k}\Omega$$

P 4.26 [a]



$$\frac{v_p}{72,000} + \frac{v_p - v_c}{9,000} + \frac{v_p - v_d}{24,000} = 0$$

$$\therefore v_p = (2/3)v_c + 0.25v_d = v_n$$

$$\frac{v_n - v_a}{12,000} + \frac{v_n - v_b}{18,000} + \frac{v_n - v_o}{144,000} = 0$$

$$\begin{aligned} \therefore v_o &= 21v_n - 12v_a - 8v_b \\ &= 21[(2/3)v_c + 0.25v_d] - 12v_a - 8v_b \\ &= 21(0.4 + 0.2) - 12(0.5) - 8(0.3) = 4.2 \text{ V} \end{aligned}$$

[b] $v_o = 14v_c + 4.2 - 6 - 2.4$

$$\pm 15 = 14v_c - 4.2$$

$$\therefore 14v_c = \pm 15 + 4.2$$

$$\therefore v_c = 1.371 \text{ V} \quad \text{and} \quad v_c = -0.771 \text{ V}$$

$$\therefore -771 \leq v_c \leq 1371 \text{ mV}$$

P 4.27 [a]

$$v_n = v_p = \alpha v_g$$

$$\frac{v_n - v_g}{R_1} + \frac{v_n - v_o}{R_f} = 0$$

$$(v_n - v_g) \frac{R_f}{R_1} + v_n - v_o = 0$$

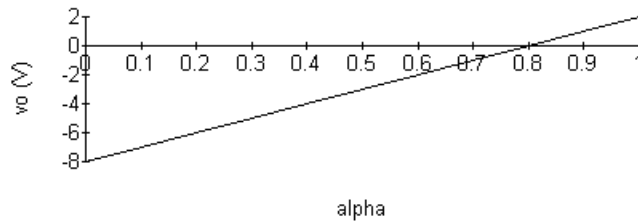
$$v_o = (\alpha v_g - v_g)4 + \alpha v_g$$

$$= [(\alpha - 1)4 + \alpha]v_g$$

$$= (5\alpha - 4)v_g$$

$$= (5\alpha - 4)(2) = 10\alpha - 8$$

α	v_o	α	v_o	α	v_o
0.0	-8 V	0.4	-4 V	0.8	0 V
0.1	-7 V	0.5	-3 V	0.9	1 V
0.2	-6 V	0.6	-2 V	1.0	2 V
0.3	-5 V	0.7	-1 V		



[b] Rearranging the equation for v_o from (a) gives

$$v_o = \left(\frac{R_f}{R_1} + 1 \right) v_g \alpha - \left(\frac{R_f}{R_1} \right) v_g$$

Therefore,

$$\text{slope} = \left(\frac{R_f}{R_1} + 1 \right) v_g; \quad \text{intercept} = - \left(\frac{R_f}{R_1} \right) v_g$$

[c] Using the equations from (b),

$$-6 = \left(\frac{R_f}{R_1} + 1 \right) v_g; \quad 4 = - \left(\frac{R_f}{R_1} \right) v_g$$

Solving,

$$v_g = -2 \text{ V}; \quad \frac{R_f}{R_1} = 2$$

P 4.28 $v_p = v_n = R_b i_b$

$$\frac{R_b i_b - 3000 i_a}{3000} + \frac{R_b i_b - v_o}{R_f} = 0$$

$$\left(\frac{R_b}{3000} + \frac{R_b}{R_f} \right) i_b - i_a = \frac{v_o}{R_f}$$

$$v_o = \left[\frac{R_b R_f}{3000} + R_b \right] i_b - R_f i_a$$

$$\therefore R_f = 2000 \Omega$$

$$(2/3)R_b + R_b = 2000$$

$$\therefore R_b = 1200 \Omega$$

$$\text{P 4.29} \quad v_p = \frac{v_b R_b}{R_a + R_b} = v_n$$

$$\frac{v_n - v_a}{4700} + \frac{v_n - v_o}{R_f} = 0$$

$$v_n \left(\frac{R_f}{4700} + 1 \right) - \frac{v_a R_f}{4700} = v_o$$

$$\therefore \left(\frac{R_f}{4700} + 1 \right) \frac{R_b}{R_a + R_b} v_b - \frac{R_f}{4700} v_a = v_o$$

$$\therefore \frac{R_f}{4700} = 10; \quad R_f = 47 \text{ k}\Omega$$

$$\therefore \frac{R_f}{4700} + 1 = 11$$

$$\therefore 11 \left(\frac{R_b}{R_a + R_b} \right) = 10$$

$$11R_b = 10R_b + 10R_a \quad R_b = 10R_a$$

$$R_a + R_b = 220 \text{ k}\Omega$$

$$11R_a = 220 \text{ k}\Omega$$

$$R_a = 20 \text{ k}\Omega$$

$$R_b = 220 - 20 = 200 \text{ k}\Omega$$

$$\text{P 4.30} \quad v_o = \frac{R_d(R_a + R_b)}{R_a(R_c + R_d)} v_b - \frac{R_b}{R_a} v_a$$

$$\text{By hypothesis: } R_b/R_a = 5; \quad R_c + R_d = 600 \text{ k}\Omega; \quad \frac{R_d(R_a + R_b)}{R_a(R_c + R_d)} = 2$$

$$\therefore \frac{R_d(R_a + 5R_a)}{R_a \cdot 600,000} = 2 \quad \text{so} \quad R_d = 200 \text{ k}\Omega; \quad R_c = 400 \text{ k}\Omega$$

Also, when $v_o = 0$ we have

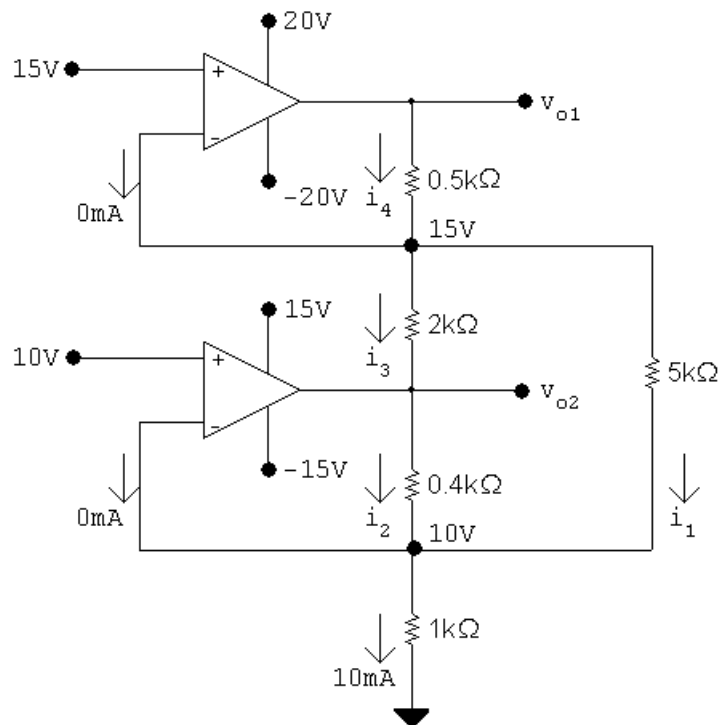
$$\frac{v_n - v_a}{R_a} + \frac{v_n}{R_b} = 0$$

$$\therefore v_n \left(1 + \frac{R_a}{R_b}\right) = v_a; \quad v_n = (5/6)v_a$$

$$i_a = \frac{v_a - (5/6)v_a}{R_a} = \frac{1}{6} \frac{v_a}{R_a}; \quad R_{in} = \frac{v_a}{i_a} = 6R_a = 18 \text{ k}\Omega$$

$$\therefore R_a = 3 \text{ k}\Omega; \quad R_b = 15 \text{ k}\Omega$$

P 4.31



$$i_1 = \frac{15 - 10}{5000} = 1 \text{ mA}$$

$$i_2 + i_1 + 0 = 10 \text{ mA}; \quad i_2 = 9 \text{ mA}$$

$$v_{o2} = 10 + (400)(9) \times 10^{-3} = 13.6 \text{ V}$$

$$i_3 = \frac{15 - 13.6}{2000} = 0.7 \text{ mA}$$

$$i_4 = i_3 + i_1 = 1.7 \text{ mA}$$

$$v_{o1} = 15 + 1.7(0.5) = 15.85 \text{ V}$$

P 4.32 Let v_{o1} be the output voltage of the first op-amp. Then

$$\frac{0 - 1.1}{3000} + \frac{0 - v_{o1}}{18,000} + \frac{0 - v_o}{24,000} = 0$$

$$-26.4 - 4v_{o1} - 3v_o = 0$$

$$\text{But } v_{o1} = \frac{v_o}{30}(27) = 0.9v_o$$

$$\therefore -3.6v_o - 3v_o = 26.4 \quad \text{or} \quad v_o = -4 \text{ V}$$

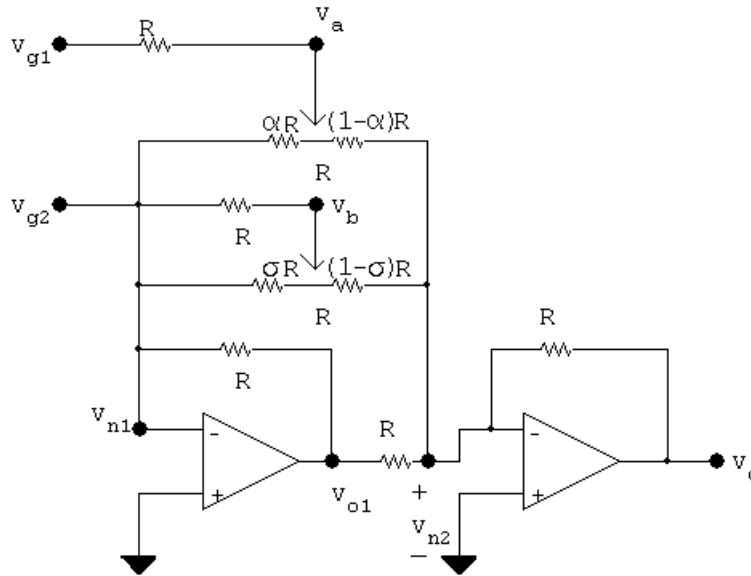
$$i_{24\text{k}\Omega} = \frac{0 - (-4)}{24} = (1/6) \text{ mA}$$

$$i_{3\text{k}\Omega} = \frac{-4}{30} = (2/15) \text{ mA}$$

$$i_{4.5\text{k}\Omega} = \frac{-4}{4.5} = (8/9) \text{ mA}$$

$$\frac{1}{6000} = i_o - \frac{2}{15,000} - \frac{8}{9000}; \quad i_o = 1.1889 \text{ mA}$$

P 4.33 [a] The circuit of Fig. P4.33 is redrawn with intermediate voltages defined to facilitate the analysis.



$$v_{n1} = v_{n2} = 0$$

$$\frac{0 - v_{o1}}{R} + \frac{0 - v_b}{\sigma R} + \frac{0 - v_a}{\alpha R} = 0$$

$$\text{therefore } v_{o1} = -\frac{v_a}{\alpha} - \frac{v_b}{\sigma}$$

$$\frac{0 - v_b}{(1 - \sigma)R} + \frac{0 - v_a}{(1 - \alpha)R} + \frac{0 - v_{o1}}{R} + \frac{0 - v_o}{R} = 0$$

$$\text{therefore } v_o = -v_{o1} - \frac{v_a}{1 - \alpha} - \frac{v_b}{1 - \sigma}$$

$$v_o = \frac{v_a}{\alpha} + \frac{v_b}{\sigma} - \frac{v_a}{1 - \alpha} - \frac{v_b}{1 - \sigma} = \frac{v_a(1 - 2\alpha)}{\alpha(1 - \alpha)} + v_b \frac{(1 - 2\sigma)}{\sigma(1 - \sigma)}$$

$$\frac{v_a - v_{g1}}{R} + \frac{v_a - 0}{\alpha R} + \frac{v_a - 0}{(1 - \alpha)R} = 0$$

$$v_a + \frac{v_a}{\alpha} + \frac{v_a}{1 - \alpha} = v_{g1}$$

$$v_a \left(\frac{\alpha(1 - \alpha) + (1 - \alpha) + \alpha}{\alpha(1 - \alpha)} \right) = v_{g1}$$

$$v_a = \frac{v_{g1}\alpha(1 - \alpha)}{(\alpha - \alpha^2 + 1)}$$

$$\text{By symmetry } v_b = \frac{v_{g2}\sigma(1 - \sigma)}{\sigma - \sigma^2 + 1}$$

$$\text{therefore } v_o = \frac{(1 - 2\alpha)}{(\alpha - \alpha^2 + 1)}v_{g1} + \frac{(1 - 2\sigma)}{(\sigma - \sigma^2 + 1)}v_{g2}$$

[b] $\alpha = \sigma = 1 :$

$$v_o = -v_{g1} - v_{g2} = -(v_{g1} + v_{g2}); \quad \text{inverted summing amplifier}$$

[c] $\alpha = \sigma = 0 :$

$$v_o = v_{g1} + v_{g2}; \quad \text{noninverted summing amplifier}$$

P 4.34 $v_p = \frac{5.6}{8.0}v_g = 0.7v_g = 7 \sin(\pi/3)t \text{ V}$

$$\frac{v_n}{15,000} + \frac{v_n - v_o}{75,000} = 0$$

$$6v_n = v_o; \quad v_n = v_p$$

$$\therefore v_o = 42 \sin(\pi/3)t \text{ V} \quad 0 \leq t \leq \infty$$

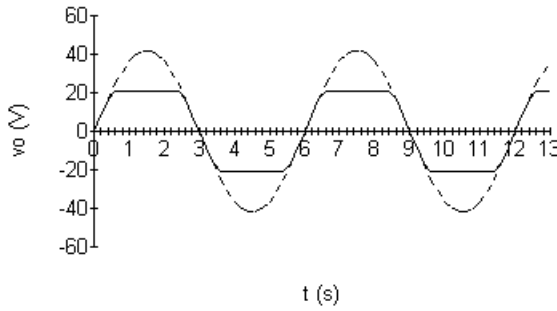
$$v_o = 0 \quad t \leq 0$$

At saturation

$$42 \sin\left(\frac{\pi}{3}\right)t = \pm 21; \quad \sin \frac{\pi}{3}t = \pm 0.5$$

$$\therefore \frac{\pi}{3}t = \frac{\pi}{6}, \quad \frac{5\pi}{6}, \quad \frac{7\pi}{6}, \quad \frac{11\pi}{6}, \quad \text{etc.}$$

$$t = 0.50 \text{ s}, \quad 2.50 \text{ s}, \quad 3.50 \text{ s}, \quad 5.50 \text{ s}, \quad \text{etc.}$$



P 4.35 It follows directly from the circuit that $v_o = -16v_g$
From the plot of v_g we have $v_g = 0, \quad t < 0$

$$v_g = (1/4)t \quad 0 \leq t \leq 2$$

$$v_g = -(1/4)t + 1 \quad 2 \leq t \leq 6$$

$$v_g = (1/4)t - 2 \quad 6 \leq t \leq 10$$

$$v_g = -(1/4)t + 3 \quad 10 \leq t \leq 14$$

$$v_g = (1/4)t - 4 \quad 14 \leq t \leq 18, \quad \text{etc.}$$

Therefore

$$v_o = -4t \quad 0 \leq t \leq 2$$

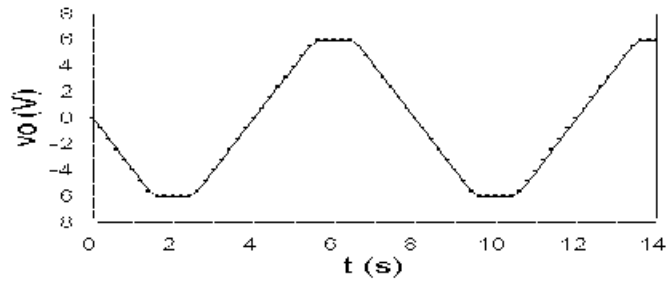
$$v_o = 4t - 16 \quad 2 \leq t \leq 6$$

$$v_o = -4t + 32 \quad 6 \leq t \leq 10$$

$$v_o = 4t - 48 \quad 10 \leq t \leq 14$$

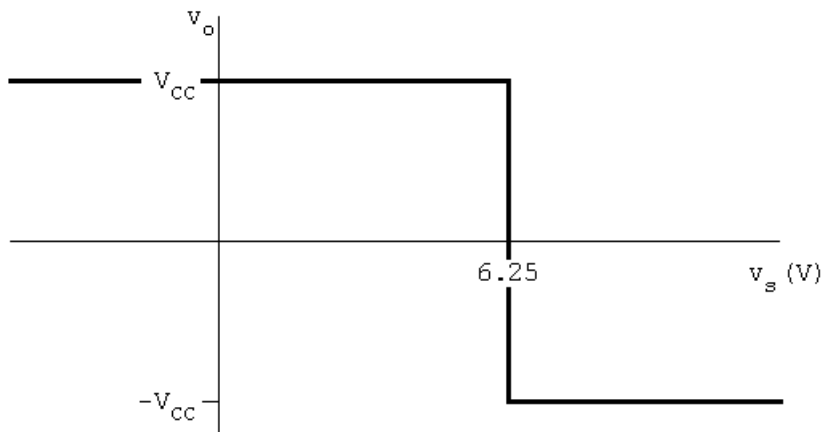
$$v_o = -4t + 64 \quad 14 \leq t \leq 18, \quad \text{etc.}$$

These expressions for v_o are valid as long as the op amp is not saturated. Since the peak values of v_o are ± 6 , the output is clipped at ± 6 . The plot is shown below.



P 4.36 [a] v_o will equal V_{CC} when $v_n < v_{ref}$. Thus

$$v_s \left(\frac{40}{50} \right) < v_{ref} \quad \text{or} \quad v_s < 6.25 \text{ V}$$

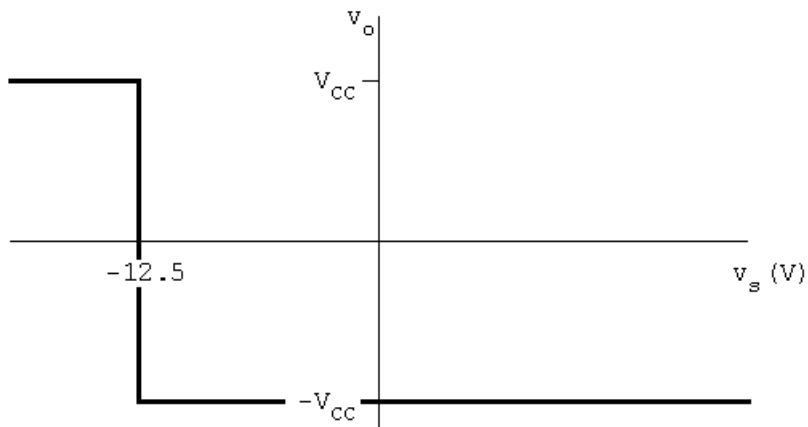


[b] v_o will equal V_{CC} when $v_n < v_{ref}$. Hence

$$v_s < -12.5 \text{ V}$$

v_o will equal $-V_{CC}$ when $v_n > v_{ref}$. Thus

$$v_s < -12.5 \text{ V}$$



- [c] Observe that in Problem 4.36 the inputs to the comparator are interchanged with those in Example 4.2. Hence the v_o versus v_s plots in Problem 4.36 are interchanged with those in Example 4.2. For example, when $v_{\text{ref}} = 5 \text{ V}$

$$v_o = V_{\text{CC}} \quad \text{when } v_s > 6.25 \text{ (Example 4.2)}$$

$$v_s < 6.25 \text{ (Problem 4.36)}$$

$$v_o = -V_{\text{CC}} \quad \text{when } v_s < 6.25 \text{ (Example 4.2)}$$

$$v_s > 6.25 \text{ (Problem 4.36)}$$

when $v_{\text{ref}} = -10 \text{ V}$

$$v_o = V_{\text{CC}} \quad \text{when } v_s > -12.5 \text{ (Example 4.2)}$$

$$v_s < -12.5 \text{ (Problem 4.36)}$$

$$v_o = -V_{\text{CC}} \quad \text{when } v_s < -12.5 \text{ (Example 4.2)}$$

$$v_s > -12.5 \text{ (Problem 4.36)}$$

- P 4.37 [a] The output of the comparator will be zero when $v_n = 0$. Summing the currents away from the inverting input terminal yields

$$\frac{0 - v_s}{R_1} + \frac{0 - v_{\text{ref}}}{R_2} = 0$$

Solving for v_s gives

$$v_s = -\frac{R_1}{R_2} v_{\text{ref}}$$

- [b] The threshold value of v_s is

$$v_s = -\frac{10}{20}(-10) = 5 \text{ V}$$

Assume v_s is slightly less than 5 V, say

$$v_s = (5 - \epsilon) \text{ V}$$

Then

$$\frac{v_n - (5 - \epsilon)}{10} - \frac{v_n + 10}{20} = 0$$

or

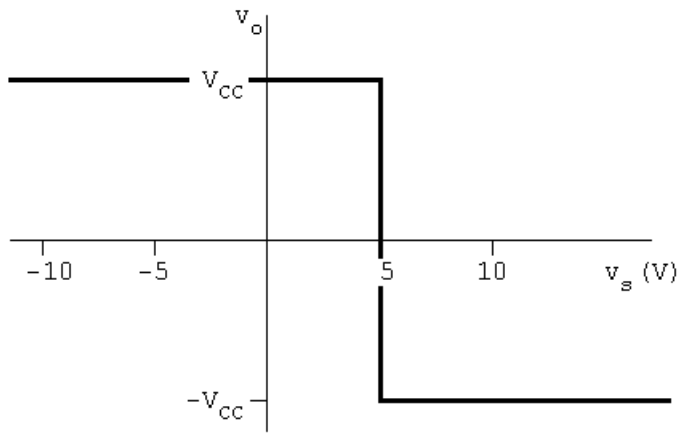
$$v_n = -\frac{2}{3}\epsilon$$

With v_n slightly negative $v_o = V_{\text{CC}}$

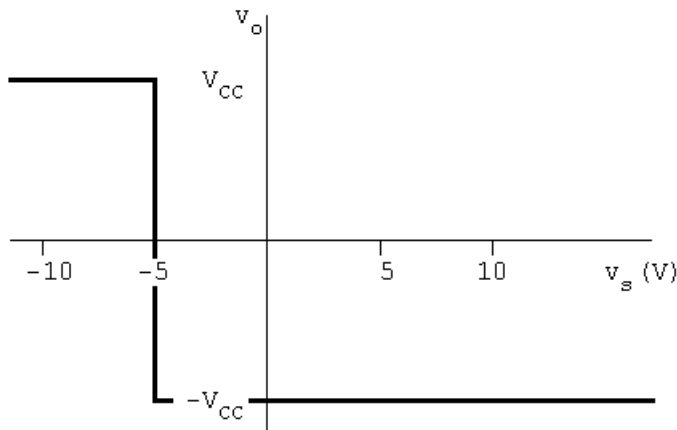
If $v_s = (5 + \epsilon)$ then

$$v_n = \frac{2}{3}\epsilon$$

Therefore, when v_s is slightly larger than the threshold value v_n goes positive and $v_o = -V_{CC}$. Thus, the v_o versus v_s sketch is



[c] When $v_{ref} = 10$ V, the threshold value of v_s is -5 V and the sketch of v_o versus v_s is



P 4.38 The voltages at the inverting input terminal of the comparators, starting with the lower comparator, are: 0.875 V, 1.75 V, 2.625 V, 3.5 V, 4.375 V, 5.25 V, and 6.125 V. When $v_s = 1$ V, all comparator output voltages are low except the lowest one. Therefore, the thermometer code is 0 0 0 0 0 0 1.

When $v_s = 3$ V, the comparator output voltages of the three lowest comparators are high, hence the code is 0 0 0 0 1 1 1.

For $v_s = 5$ V the code is 0 0 1 1 1 1 1 and for $v_s = 7$ V the code is 1 1 1 1 1 1 1.

The results are summarized in the following table:

v_s (V)	Thermometer Code						
1	0	0	0	0	0	0	1
3	0	0	0	0	1	1	1
5	0	0	1	1	1	1	1
7	1	1	1	1	1	1	1

P 4.39 Since the current into the terminals of the ideal comparators is zero the current oriented down through the string of resistors is

$$i = \frac{v_{\text{ref}} - (-v_{\text{ref}})}{8R} = \frac{v_{\text{ref}}}{4R}$$

It follows that

$$v_1 = -v_{\text{ref}} + \frac{v_{\text{ref}}}{4R}(R) = -\frac{3}{4}v_{\text{ref}}$$

$$v_2 = -v_{\text{ref}} + \frac{v_{\text{ref}}}{4R}(2R) = -\frac{1}{2}v_{\text{ref}}$$

$$v_3 = -v_{\text{ref}} + \frac{v_{\text{ref}}}{4R}(3R) = -\frac{1}{4}v_{\text{ref}}$$

$$v_4 = -v_{\text{ref}} + \frac{v_{\text{ref}}}{4R}(4R) = 0$$

$$v_5 = -v_{\text{ref}} + \frac{v_{\text{ref}}}{4R}(5R) = \frac{1}{4}v_{\text{ref}}$$

$$v_6 = -v_{\text{ref}} + \frac{v_{\text{ref}}}{4R}(6R) = \frac{1}{2}v_{\text{ref}}$$

$$v_7 = -v_{\text{ref}} + \frac{v_{\text{ref}}}{4R}(7R) = \frac{3}{4}v_{\text{ref}}$$

P 4.40 From the solution to Problem 4.39 we have

$$v_1 = (-3/4)(7) = -5.25 \text{ V}$$

$$v_2 = (-1/2)(7) = -3.5 \text{ V}$$

$$v_3 = (-1/4)(7) = -1.75 \text{ V}$$

$$v_4 = 0$$

$$v_5 = (1/4)(7) = 1.75 \text{ V}$$

$$v_6 = (1/2)(7) = 3.5 \text{ V}$$

$$v_7 = (3/4)(7) = 5.25 \text{ V}$$

When $v_s = -7 \text{ V}$ all the comparator output voltages will be low, thus the thermometer code is 0 0 0 0 0 0 0.

When $v_s = -5 \text{ V}$, all except the first comparator (counting from the bottom up) output voltage will be low, thus the code is 0 0 0 0 0 0 1.

When $v_s = -3 \text{ V}$, all except the first two comparator output voltages will be low, hence the code is 0 0 0 0 0 1 1.

When $v_s = -1 \text{ V}$, the output voltages of the first three comparators will be high, thus the thermometer code is 0 0 0 0 1 1 1.

Then $v_s = 1 \text{ V}$ the output voltages of the first four comparators will be high (0 0 0 1 1 1 1); when $v_s = 3 \text{ V}$ the first five comparators will be high (0 0 1 1 1 1 1); when $v_s = 5 \text{ V}$ the first six comparators will be high (0 1 1 1 1 1 1); and when $v_s = 7 \text{ V}$ the output voltages of all seven comparators will be high (1 1 1 1 1 1 1). Our results are summarized in the following table:

$v_s \text{ (V)}$	Thermometer Code						
-7	0	0	0	0	0	0	0
-5	0	0	0	0	0	0	1
-3	0	0	0	0	0	1	1
-1	0	0	0	0	1	1	1
1	0	0	0	1	1	1	1
3	0	0	1	1	1	1	1
5	0	1	1	1	1	1	1
7	1	1	1	1	1	1	1

This page intentionally left blank