

# Sinusoidal Steady State Analysis

## Drill Exercises

DE 7.1 [a]  $\omega = 2\pi f = 3769.91 \text{ rad/s}, \quad f = 600 \text{ Hz}$

[b]  $T = 1/f = 1.67 \text{ ms}$

[c]  $V_m = 10 \text{ V}$

[d]  $v(0) = 10(0.6) = 6 \text{ V}$

[e]  $\phi = -53.13^\circ; \quad \phi = \frac{-53.13^\circ(2\pi)}{360^\circ} = -0.9273 \text{ rad}$

[f]  $3769.91t = 143.13/57.3 = 2.498 \text{ rad}, \quad t = 662.62 \mu\text{s}$

[g]  $(dv/dt) = (-10)3769.91 \sin(3769.91t - 53.13^\circ)$

$(dv/dt) = 0 \quad \text{when} \quad 3769.91t - 53.13^\circ = 0^\circ$

or  $3769.91t = 0.9273 \text{ rad}$

Therefore  $t = 245.97 \mu\text{s}$

DE 7.2  $V_{\text{rms}} = \sqrt{\frac{1}{T} \int_0^{T/2} V_m^2 \sin^2 \frac{2\pi}{T} t dt}$

$$\int_0^{T/2} V_m^2 \sin^2 \left( \frac{2\pi}{T} t \right) dt = \frac{V_m^2}{2} \int_0^{T/2} \left( 1 - \cos \frac{4\pi}{T} t \right) dt = \frac{V_m^2 T}{4}$$

Therefore  $V_{\text{rms}} = \sqrt{\frac{1}{T} \frac{V_m^2 T}{4}} = \frac{V_m}{2}$

DE 7.3 [a] The numerical values of the terms in Eq. 7.9 are

$V_m = 20, \quad R/L = 1066.67, \quad \omega L = 60$

$\sqrt{R^2 + \omega^2 L^2} = 100$

$\phi = 25^\circ, \quad \theta = \tan^{-1} 60/80, \quad \theta = 36.87^\circ$

$i = \left[ -195.72e^{-1066.67t} + 200 \cos(800t - 11.87^\circ) \right] \text{ mA}, \quad t \geq 0$

[b] Transient component  $= -195.72e^{-1066.67t}$  mA  
 Steady-state component  $= 200 \cos(800t - 11.87^\circ)$  mA

[c] By direct substitution into Eq 7.9,  $i(1.875 \text{ ms}) = 28.39 \text{ mA}$

[d]  $0.2 \text{ A}, \quad 800 \text{ rad/s}, \quad -11.87^\circ$

[e] The current lags the voltage by  $36.87^\circ$ .

DE 7.4 [a]  $\mathbf{V} = 170/\underline{-40^\circ} \text{ V}$

[b]  $\mathbf{I} = 10/\underline{-70^\circ} \text{ A}$

[c]  $\mathbf{I} = 5/\underline{36.87^\circ} + 10/\underline{-53.13^\circ}$   
 $= 4 + j3 + 6 - j8 = 10 - j5 = 11.18/\underline{-26.57^\circ} \text{ A}$

[d]  $\mathbf{V} = 300/\underline{45^\circ} - 100/\underline{-60^\circ} = 212.13 + j212.13 - (50 - j86.60)$   
 $= 162.13 + j298.73 = 339.90/\underline{61.51^\circ} \text{ mV}$

DE 7.5 [a]  $v = 18.6 \cos(\omega t - 54^\circ) \text{ V}$

[b]  $\mathbf{I} = 20/\underline{45^\circ} - 50/\underline{-30^\circ} = 14.14 + j14.14 - 43.3 + j25$   
 $= -29.16 + j39.14 = 48.81/\underline{126.68^\circ}$

Therefore  $i = 48.81 \cos(\omega t + 126.68^\circ) \text{ mA}$

[c]  $\mathbf{V} = 20 + j80 - 30/\underline{15^\circ} = 20 + j80 - 28.98 - j7.76$   
 $= -8.98 + j72.24 = 72.79/\underline{97.08^\circ}$

$v = 72.79 \cos(\omega t + 97.08^\circ) \text{ V}$

DE 7.6 [a]  $\omega L = (10^4)(20 \times 10^{-3}) = 200 \Omega$

[b]  $Z_L = j200 \Omega$

[c]  $\mathbf{V}_L = \mathbf{I}Z_L = (10/\underline{30^\circ})(200/\underline{90^\circ}) \times 10^{-3} = 2/\underline{120^\circ} \text{ V}$

[d]  $v_L = 2 \cos(10,000t + 120^\circ) \text{ V}$

DE 7.7 [a]  $X_C = \frac{-1}{\omega C} = -\frac{10^6}{4000(5)} = -50 \Omega$

[b]  $Z_C = jX_C = -j50 \Omega$

[c]  $\mathbf{I} = \frac{30/\underline{25^\circ}}{50/\underline{-90^\circ}} = 0.6/\underline{115^\circ} \text{ A}$

[d]  $i = 0.6 \cos(4000t + 115^\circ) \text{ A}$

DE 7.8  $\mathbf{I}_1 = 100/\underline{25^\circ} = 90.63 + j42.26$

$$\mathbf{I}_2 = 100/\underline{145^\circ} = -81.92 + j57.36$$

$$\mathbf{I}_3 = 100/\underline{-95^\circ} = -8.71 - j99.62$$

$$\mathbf{I}_4 = -(\mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3) = (0 + j0) \text{ A}, \quad \text{therefore } i_4 = 0 \text{ A}$$

DE 7.9 [a]  $\mathbf{I} = \frac{125/\underline{-60^\circ}}{|Z|/\underline{\theta_z}} = \frac{125}{|Z|}/\underline{(-60 - \theta_z)^\circ}$

$$\text{But } -60 - \theta_z = -105^\circ \quad \therefore \theta_z = 45^\circ$$

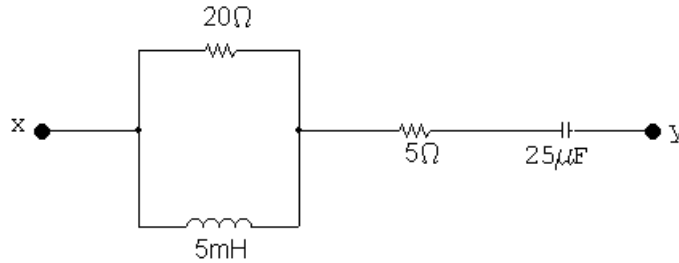
$$Z = 90 + j160 + jX_C$$

$$\therefore X_C = -70 \Omega; \quad -\frac{1}{\omega C} = -70$$

$$\therefore C = \frac{1}{(70)(5000)} = 2.86 \mu\text{F}$$

[b]  $\mathbf{I} = \frac{125/\underline{-60^\circ}}{(90 + j90)} = 0.982/\underline{-105^\circ} \text{ A}; \quad \therefore |\mathbf{I}| = 0.982 \text{ A}$

DE 7.10 [a]



$$\omega = 2000 \text{ rad/s}$$

$$\omega L = 10 \Omega, \quad \frac{-1}{\omega C} = -20 \Omega$$

$$Z_{xy} = \frac{20(j10)}{(20 + j10)} + 5 - j20 = 4 + j8 + 5 - j20 = (9 - j12) \Omega$$

[b]  $\omega L = 40 \Omega, \quad \frac{-1}{\omega C} = -5 \Omega$

$$Z_{xy} = 5 - j5 + \left[ \frac{(20)(j40)}{20 + j40} \right] = 5 - j5 + 16 + j8 = (21 + j3) \Omega$$

$$\begin{aligned}
 \text{[c]} \quad Z_{xy} &= \left[ \frac{20(j\omega L)}{20 + j\omega L} \right] + \left( 5 - \frac{j10^6}{25\omega} \right) \\
 &= \frac{20\omega^2 L^2}{400 + \omega^2 L^2} + \frac{j400\omega L}{400 + \omega^2 L^2} + 5 - \frac{j10^6}{25\omega}
 \end{aligned}$$

The impedance will be purely resistive when the  $j$  terms cancel, i.e.,

$$\frac{400\omega L}{400 + \omega^2 L^2} = \frac{10^6}{25\omega}$$

Solving for  $\omega$  yields  $\omega = 4000 \text{ rad/s}$ .

$$\text{[d]} \quad Z_{xy} = \frac{20\omega^2 L^2}{400 + \omega^2 L^2} + 5 = 10 + 5 = 15 \Omega$$

DE 7.11

$$\mathbf{V} = 150 \angle 0^\circ, \quad \mathbf{I}_s = \frac{150 \angle 0^\circ}{15} = 10 \angle 0^\circ \text{ A}$$

$$\mathbf{I}_L = \frac{10(20)}{20 + j20} = 5 - j5 = 7.07 \angle -45^\circ \text{ A}$$

$$i_L = 7.07 \cos(4000t - 45^\circ) \text{ A}, \quad I_m = 7.07 \text{ A}$$

$$\text{DE 7.12 [a]} \quad Y = \frac{1}{3 + j4} + \frac{1}{16 - j12} + \frac{1}{-j4}$$

$$= 0.12 - j0.16 + 0.04 + j0.03 + j0.25$$

$$= 0.16 + j0.12 = 200 \angle 36.87^\circ \text{ mS}$$

$$\text{[b]} \quad G = 160 \text{ mS}$$

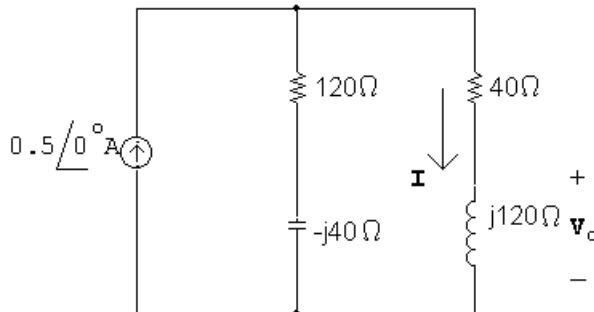
$$\text{[c]} \quad B = 120 \text{ mS}$$

$$\text{[d]} \quad \mathbf{I} = 8 \angle 0^\circ \text{ A}, \quad \mathbf{V} = \frac{\mathbf{I}}{Y} = \frac{8}{0.2 \angle 36.87^\circ} = 40 \angle -36.87^\circ \text{ V}$$

$$\mathbf{I}_C = \frac{\mathbf{V}}{Z_C} = \frac{40 \angle -36.87^\circ}{4 \angle -90^\circ} = 10 \angle 53.13^\circ \text{ A}$$

$$i_C = 10 \cos(\omega t + 53.13^\circ) \text{ A}, \quad I_m = 10 \text{ A}$$

DE 7.13 Construct the phasor domain equivalent circuit:



$$\mathbf{I} = \frac{0.5(120 - j40)}{160 + j80} = 0.25 - j0.25 \text{ A}$$

$$\mathbf{V}_o = j120\mathbf{I} = 30 + j30 = 42.43/\underline{45^\circ}$$

$$v_o = 42.43 \cos(2000t + 45^\circ) \text{ V}$$

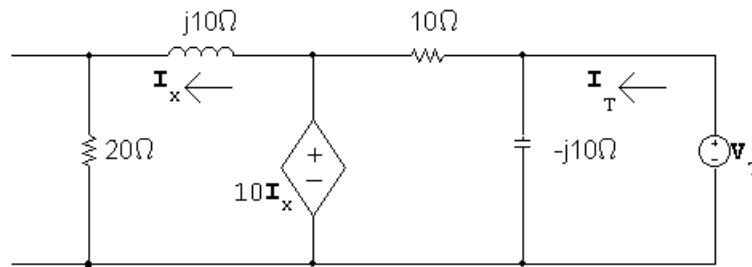
DE 7.14 Use the lower node as the reference node. Let  $\mathbf{V}_1$  = node voltage across the  $20\ \Omega$  resistor and  $\mathbf{V}_{\text{Th}}$  = node voltage across the capacitor. Writing the node voltage equations gives us

$$\frac{\mathbf{V}_1}{20} - 2/\underline{45^\circ} + \frac{\mathbf{V}_1 - 10\mathbf{I}_x}{j10} = 0 \quad \text{and} \quad \mathbf{V}_{\text{Th}} = \frac{-j10}{10 - j10}(10\mathbf{I}_x)$$

We also have

$$\mathbf{I}_x = \frac{\mathbf{V}_1}{20}$$

Solving these equations for  $\mathbf{V}_{\text{Th}}$  gives  $\mathbf{V}_{\text{Th}} = 10/\underline{45^\circ}\text{V}$ . To find the Thévenin impedance, we remove the independent current source and apply a test voltage source at the terminals a, b. Thus



It follows from the circuit that

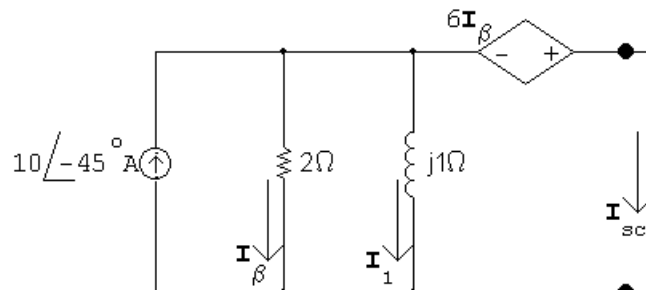
$$10\mathbf{I}_x = (20 + j10)\mathbf{I}_x$$

Therefore

$$\mathbf{I}_x = 0 \quad \text{and} \quad \mathbf{I}_T = \frac{\mathbf{V}_T}{-j10} + \frac{\mathbf{V}_T}{10}$$

$$Z_{\text{Th}} = \frac{\mathbf{V}_T}{\mathbf{I}_T}, \quad \text{therefore} \quad Z_{\text{Th}} = (5 - j5)\ \Omega$$

DE 7.15 Short circuit current



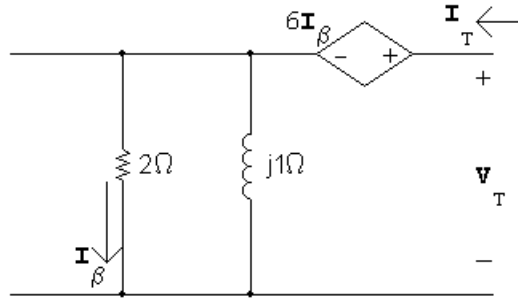
With the short circuit

$$\mathbf{I}_\beta = \frac{-6\mathbf{I}_\beta}{2}$$

$$2\mathbf{I}_\beta = -6\mathbf{I}_\beta; \quad \therefore \quad \mathbf{I}_\beta = 0$$

$$\mathbf{I}_1 = 0; \quad \therefore \quad \mathbf{I}_{sc} = 10\angle -45^\circ \text{ A} = \mathbf{I}_N$$

The Norton impedance is the same as the Thévenin impedance. Thus



$$\mathbf{V}_T = 6\mathbf{I}_\beta + 2\mathbf{I}_\beta = 8\mathbf{I}_\beta, \quad \mathbf{I}_\beta = \frac{j1}{2 + j1}\mathbf{I}_T$$

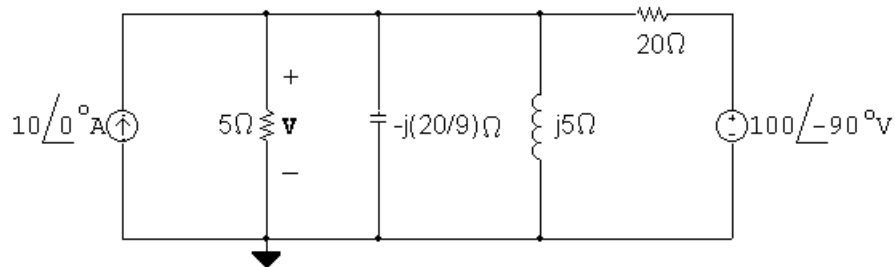
$$Z_{Th} = \frac{\mathbf{V}_T}{\mathbf{I}_T} = \frac{8\mathbf{I}_\beta}{[(2 + j1)/j1]\mathbf{I}_\beta} = \frac{j8}{2 + j1} = 1.6 + j3.2\Omega$$

DE 7.16 The phasor domain circuit is as shown in the following diagram. The node voltage equation is

$$-10 + \frac{\mathbf{V}}{5} + \frac{9\mathbf{V}}{-j20} + \frac{\mathbf{V}}{j5} + \frac{\mathbf{V} - 100\angle -90^\circ}{20} = 0$$

$$\text{Therefore } \mathbf{V} = 10 - j30 = 31.62\angle -71.57^\circ$$

$$\text{Therefore } v = 31.62 \cos(50,000t - 71.57^\circ) \text{ V}$$



DE 7.17 Let  $\mathbf{I}_a$ ,  $\mathbf{I}_b$ , and  $\mathbf{I}_c$  be the three clockwise mesh currents going from left to right. Summing the voltages around meshes a and b gives

$$33.8 = (1 + j2)\mathbf{I}_a + (3 - j5)(\mathbf{I}_a - \mathbf{I}_b)$$

and

$$0 = (3 - j5)(\mathbf{I}_b - \mathbf{I}_a) + 2(\mathbf{I}_b - \mathbf{I}_c).$$

But

$$\mathbf{V}_x = -j5(\mathbf{I}_a - \mathbf{I}_b),$$

therefore

$$\mathbf{I}_c = -0.75[-j5(\mathbf{I}_a - \mathbf{I}_b)].$$

Solving for  $\mathbf{I} = \mathbf{I}_a = 29 + j2 = 29.07/3.95^\circ \text{ A}$ .

DE 7.18 [a]  $\mathbf{V} = 100/\underline{-45^\circ} \text{ V}$ ,  $\mathbf{I} = 20/\underline{15^\circ} \text{ A}$

Therefore

$$P = \frac{1}{2}(100)(20) \cos[-45 - (15)] = 500 \text{ W}, \quad \text{A} \rightarrow \text{B}$$

$$Q = 1000 \sin -60^\circ = -866.03 \text{ VAR}, \quad \text{B} \rightarrow \text{A}$$

[b]  $\mathbf{V} = 100/\underline{-45^\circ}$ ,  $\mathbf{I} = 20/\underline{165^\circ}$

$$P = 1000 \cos(-210^\circ) = -866.03 \text{ W}, \quad \text{B} \rightarrow \text{A}$$

$$Q = 1000 \sin(-210^\circ) = 500 \text{ VAR}, \quad \text{A} \rightarrow \text{B}$$

[c]  $\mathbf{V} = 106/\underline{-45^\circ}$ ,  $\mathbf{I} = 20/\underline{-105^\circ}$

$$P = 1000 \cos(60^\circ) = 500 \text{ W}, \quad \text{A} \rightarrow \text{B}$$

$$Q = 1000 \sin(60^\circ) = 866.03 \text{ VAR}, \quad \text{A} \rightarrow \text{B}$$

[d]  $P = 1000 \cos(-120^\circ) = -500 \text{ W}$ ,  $\text{B} \rightarrow \text{A}$

$$Q = 1000 \sin(-120^\circ) = -866.03 \text{ VAR}, \quad \text{B} \rightarrow \text{A}$$

DE 7.19

$$p_f = \cos(\theta_v - \theta_i) = \cos[15 - (75)] = \cos -60^\circ = 0.5 \text{ leading}$$

$$r_f = \sin(\theta_v - \theta_i) = \sin -60^\circ = -0.866$$

DE 7.20 From Example 7.4,

$$I_{\text{eff}} = \frac{0.18}{\sqrt{3}}$$

$$\begin{aligned} P &= I_{\text{eff}}^2 R \\ &= \left( \frac{0.0324}{3} \right) (5000) \\ &= 54 \text{ W} \end{aligned}$$

DE 7.21 [a]  $Z = (39 + j26) \parallel (-j52) = 48 - j20 = 52 \angle -22.62^\circ \Omega$

$$\text{Therefore } \mathbf{I}_\ell = \frac{250 \angle 0^\circ}{48 - j20 + 1 + j4} = 4.85 \angle 18.08^\circ \text{ A(rms)}$$

$$\mathbf{V}_L = Z \mathbf{I}_\ell = (52 \angle -22.62^\circ)(4.85 \angle 18.08^\circ) = 252.20 \angle -4.54^\circ \text{ V(rms)}$$

$$\mathbf{I}_L = \frac{\mathbf{V}_L}{39 + j26} = 5.38 \angle -38.23^\circ \text{ A(rms)}$$

$$\begin{aligned} \text{[b]} \quad S_L &= (252.20 \angle -4.54^\circ)(5.38 \angle +38.23^\circ) = 1357 \angle 33.69^\circ \\ &= (1129.09 + j752.73) \text{ VA} \end{aligned}$$

$$P_L = 1129.09 \text{ W}; \quad Q_L = 752.73 \text{ VAR}$$

$$\text{[c]} \quad P_\ell = |\mathbf{I}_\ell|^2 1 = (4.85)^2 \cdot 1 = 23.52 \text{ W}; \quad Q_\ell = |\mathbf{I}_\ell|^2 4 = 94.09 \text{ VAR}$$

$$\text{[d]} \quad S_g(\text{delivering}) = 250 \mathbf{I}_\ell^* = (1152.62 - j376.36) \text{ VA}$$

Therefore the source is delivering 1152.62 W and absorbing 376.36 magnetizing VAR.

$$\text{[e]} \quad Q_{\text{cap}} = \frac{|\mathbf{V}_L|^2}{-52} = \frac{(252.20)^2}{-52} = -1223.18 \text{ VAR}$$

Therefore the capacitor is delivering 1223.18 magnetizing VAR.

$$\text{Check: } 94.09 + 752.73 + 376.36 = 1223.18 \text{ VAR} \quad \text{and}$$

$$1129.09 + 23.52 = 1152.62 \text{ W}$$

DE 7.22 Series circuit derivation:

$$250 \mathbf{I}^* = (40,000 - j30,000)$$

$$\text{Therefore } \mathbf{I}^* = 160 - j120 = 200 \angle -36.87^\circ \text{ A(rms)}$$

$$\mathbf{I} = 200 \angle 36.87^\circ \text{ A(rms)}$$



$$Z = \frac{250}{200/\underline{36.87^\circ}} = 1.25/\underline{-36.87^\circ} = (1 - j0.75) \Omega$$

Therefore  $R = 1 \Omega$ ,  $X_C = -0.75 \Omega$

Parallel circuit derivation:

$$P = \frac{(250)^2}{R}; \quad \text{therefore} \quad R = \frac{(250)^2}{40,000} = 1.5625 \Omega$$

$$Q = \frac{(250)^2}{X_C}; \quad \text{therefore} \quad X_C = \frac{(250)^2}{-30,000} = -2.083 \Omega$$

DE 7.23

$$S_1 = 15,000(0.6) + j15,000(0.8) = 9000 + j12,000 \text{ VA}$$

$$S_2 = 6000(0.8) + j6000(0.6) = 4800 - j3600 \text{ VA}$$

$$S_T = S_1 + S_2 = 13,800 + j8400 \text{ VA}$$

$$S_T = 200\mathbf{I}^*; \quad \text{therefore} \quad \mathbf{I}^* = 69 + j42 \quad \mathbf{I} = 69 - j42 \text{ A}$$

$$\mathbf{V}_s = 200 + j\mathbf{I} = 200 + j69 + 42 = 242 + j69 = 251.64/\underline{15.91^\circ} \text{ V(rms)}$$

## Problems

P 7.1 [a] By hypothesis

$$i = 10 \cos(\omega t + \theta)$$

$$\frac{di}{dt} = -10\omega \sin(\omega t + \theta)$$

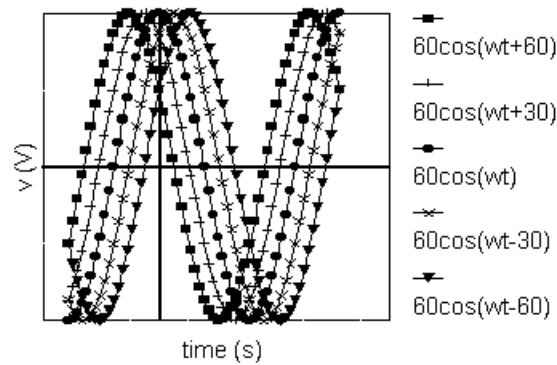
$$\therefore 10\omega = 20,000\pi; \quad \omega = 2000\pi \text{ rad/s}$$

[b]  $f = \frac{\omega}{2\pi} = 1000 \text{ Hz}; \quad T = \frac{1}{f} = 1 \text{ ms} = 1000 \mu\text{s}$

$$\frac{150}{1000} = \frac{3}{20}, \quad \therefore \theta = -90 - \frac{3}{20}(360) = -144^\circ$$

$$\therefore i = 10 \cos(2000\pi t - 144^\circ) \text{ A}$$

P 7.2



[a] Left as  $\phi$  becomes more positive

[b] Right

P 7.3 [a] 170 V

[b]  $2\pi f = 120\pi; \quad f = 60\text{Hz}$

[c]  $\omega = 120\pi = 376.99 \text{ rad/s}$

[d]  $\theta(\text{rad}) = \frac{-\pi}{180}(60) = \frac{-\pi}{3} = -1.05 \text{ rad}$

[e]  $\theta = -60^\circ$

[f]  $T = \frac{1}{f} = \frac{1}{60} = 16.67 \text{ ms}$

[g]  $120\pi t - \frac{\pi}{3} = 0; \quad \therefore t = \frac{1}{360} = 2.78 \text{ ms}$

$$\begin{aligned}
[\mathbf{h}] \quad v &= 170 \cos \left[ 120\pi \left( t + \frac{0.125}{18} \right) - \frac{\pi}{3} \right] \\
&= 170 \cos[120\pi t + (15\pi/18) - (\pi/3)] \\
&= 170 \cos[120\pi t + (\pi/2)] \\
&= -170 \sin 120\pi t \text{ V}
\end{aligned}$$

$$[\mathbf{i}] \quad 120\pi(t - t_o) - (\pi/3) = 120\pi t - (\pi/2)$$

$$\therefore 120\pi t_o = \frac{\pi}{6}; \quad t_o = \frac{25}{18} \text{ ms}$$

$$[\mathbf{j}] \quad 120\pi(t - t_o) - (\pi/3) = 120\pi t$$

$$\therefore 120\pi t_o = \frac{\pi}{3}; \quad t_o = \frac{25}{9} \text{ ms}$$

$$\text{P 7.4} \quad [\mathbf{a}] \quad \frac{T}{2} = \frac{1250}{6} + \frac{250}{6} = 250 \mu\text{s}; \quad T = 500 \mu\text{s}$$

$$f = \frac{1}{T} = \frac{10^6}{500} = 2000 \text{ Hz}$$

$$[\mathbf{b}] \quad v = V_m \sin(\omega t + \theta)$$

$$\omega = 2\pi f = 4000\pi \text{ rad/s}$$

$$4000\pi \left( \frac{-250}{6} \times 10^{-6} \right) + \theta = 0; \quad \therefore \theta = \frac{\pi}{6} \text{ rad} = 30^\circ$$

$$v = V_m \sin[4000\pi t + 30^\circ]$$

$$75 = V_m \sin 30^\circ; \quad V_m = 150 \text{ V}$$

$$v = 150 \sin[4000\pi t + 30^\circ] = 150 \cos[4000\pi t - 60^\circ] \text{ V}$$

$$\text{P 7.5} \quad [\mathbf{a}] \quad \text{From Eq. 7.9 we have}$$

$$\begin{aligned}
L \frac{di}{dt} &= \frac{V_m R \cos(\phi - \theta)}{\sqrt{R^2 + \omega^2 L^2}} e^{-(R/L)t} - \frac{\omega L V_m \sin(\omega t + \phi - \theta)}{\sqrt{R^2 + \omega^2 L^2}} \\
Ri &= \frac{-V_m R \cos(\phi - \theta) e^{-(R/L)t}}{\sqrt{R^2 + \omega^2 L^2}} + \frac{V_m R \cos(\omega t + \phi - \theta)}{\sqrt{R^2 + \omega^2 L^2}} \\
L \frac{di}{dt} + Ri &= V_m \left[ \frac{R \cos(\omega t + \phi - \theta) - \omega L \sin(\omega t + \phi - \theta)}{\sqrt{R^2 + \omega^2 L^2}} \right]
\end{aligned}$$

But

$$\frac{R}{\sqrt{R^2 + \omega^2 L^2}} = \cos \theta \quad \text{and} \quad \frac{\omega L}{\sqrt{R^2 + \omega^2 L^2}} = \sin \theta$$

Therefore the right-hand side reduces to

$$V_m \cos(\omega t + \phi)$$

At  $t = 0$ , Eq. 7.9 reduces to

$$i(0) = \frac{-V_m \cos(\phi - \theta)}{\sqrt{R^2 - \omega^2 L^2}} + \frac{V_m \cos(\phi - \theta)}{\sqrt{R^2 + \omega^2 L^2}} = 0$$

$$[b] \quad i_{ss} = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \cos(\omega t + \phi - \theta)$$

Therefore

$$L \frac{di_{ss}}{dt} = \frac{-\omega L V_m}{\sqrt{R^2 + \omega^2 L^2}} \sin(\omega t + \phi - \theta)$$

and

$$R i_{ss} = \frac{V_m R}{\sqrt{R^2 + \omega^2 L^2}} \cos(\omega t + \phi - \theta)$$

$$\begin{aligned} L \frac{di_{ss}}{dt} + R i_{ss} &= V_m \left[ \frac{R \cos(\omega t + \phi - \theta) - \omega L \sin(\omega t + \phi - \theta)}{\sqrt{R^2 + \omega^2 L^2}} \right] \\ &= V_m \cos(\omega t + \phi) \end{aligned}$$

$$P \ 7.6 \quad [a] \quad \mathbf{Y} = 100/\underline{45^\circ} + 500/\underline{-60^\circ} = 483.86/\underline{-48.48^\circ}$$

$$y = 483.86 \cos(300t - 48.48^\circ)$$

$$[b] \quad \mathbf{Y} = 250/\underline{30^\circ} - 150/\underline{50^\circ} = 120.51/\underline{4.8^\circ}$$

$$y = 120.51 \cos(377t + 4.8^\circ)$$

$$[c] \quad \mathbf{Y} = 60/\underline{60^\circ} - 120/\underline{-215^\circ} + 100/\underline{90^\circ} = 152.88/\underline{32.94^\circ}$$

$$y = 152.88 \cos(100t + 32.94^\circ)$$

$$[d] \quad \mathbf{Y} = 100/\underline{40^\circ} + 100/\underline{160^\circ} + 100/\underline{-80^\circ} = 0$$

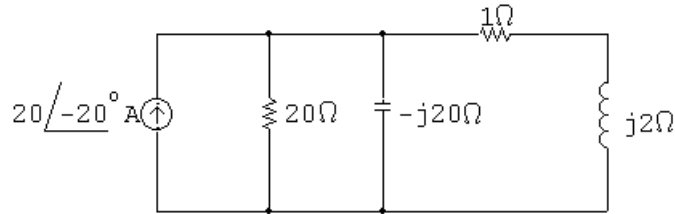
$$y = 0$$

$$\begin{aligned} P \ 7.7 \quad u &= \int_{t_o}^{t_o+T} V_m^2 \cos^2(\omega t + \phi) dt \\ &= V_m^2 \int_{t_o}^{t_o+T} \frac{1}{2} + \frac{1}{2} \cos(2\omega t + 2\phi) dt \\ &= \frac{V_m^2}{2} \left\{ \int_{t_o}^{t_o+T} dt + \int_{t_o}^{t_o+T} \cos(2\omega t + 2\phi) dt \right\} \\ &= \frac{V_m^2}{2} \left\{ T + \frac{1}{2\omega} [\sin(2\omega t + 2\phi)]_{t_o}^{t_o+T} \right\} \\ &= \frac{V_m^2}{2} \left\{ T + \frac{1}{2\omega} [\sin(2\omega t_o + 4\pi + 2\phi) - \sin(2\omega t_o + 2\phi)] \right\} \\ &= V_m^2 \left( \frac{T}{2} \right) + \frac{1}{2\omega} (0) = V_m^2 \left( \frac{T}{2} \right) \end{aligned}$$

P 7.8  $V_m = \sqrt{2}V_{\text{rms}} = \sqrt{2}(120) = 169.71 \text{ V}$

P 7.9 [a]  $j\omega L = j(5 \times 10^4)(40 \times 10^{-6}) = j2 \Omega$

$$\frac{1}{j\omega C} = -j\frac{10^6}{5 \times 10^4} = -j20 \Omega; \quad \mathbf{I}_g = 20 \angle -20^\circ \text{ A}$$



[b]  $\mathbf{V}_o = 20 \angle -20^\circ Z_e$

$$Z_e = \frac{1}{Y_e}; \quad Y_e = \frac{1}{20} + j\frac{1}{20} + \frac{1}{1 + j2}$$

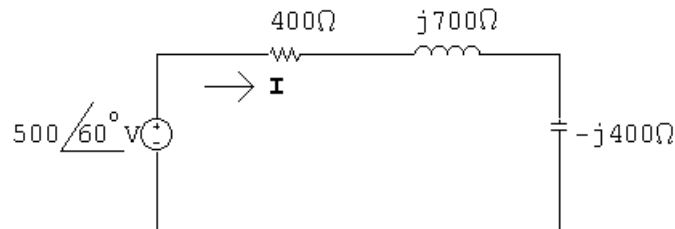
$$Y_e = 0.05 + j0.05 + 0.20 - j0.40 = 0.25 - j0.35 \text{ S}$$

$$Z_e = \frac{1}{0.25 - j0.35} = 2.32 \angle 54.46^\circ \Omega$$

$$\mathbf{V}_o = (20 \angle -20^\circ)(2.32 \angle 54.46^\circ) = 46.4 \angle 34.46^\circ \text{ V}$$

[c]  $v_o = 46.4 \cos(5 \times 10^4 t + 34.46^\circ) \text{ V}$

P 7.10 [a]



[b]  $\mathbf{I} = \frac{500 \angle 60^\circ}{400 + j700 - j400} = 1 \angle 23.13^\circ \text{ A}$

[c]  $i = 1 \cos(8000t + 23.13^\circ) \text{ A}$

P 7.11 [a] 50 Hz

[b]  $\theta_v = 0^\circ$

[c]  $\mathbf{I} = \frac{340 \angle 0^\circ}{j\omega L} = \frac{340}{\omega L} \angle -90^\circ = 8.5 \angle -90^\circ; \quad \theta_i = -90^\circ$

[d]  $\frac{340}{\omega L} = 8.5; \quad \omega L = 40 \Omega$

[e]  $L = \frac{40}{100\pi} = \frac{400}{\pi} \text{ mH} = 127.32 \text{ mH}$

[f]  $Z_L = j\omega L = j40 \Omega$

P 7.12 [a]  $\omega = 2\pi f = 80\pi \times 10^3 = 251.33 \text{ krad/s} = 251,327.41 \text{ rad/s}$

[b]  $\mathbf{I} = \frac{2.5 \times 10^{-3} \angle 0^\circ}{1/j\omega C} = j\omega C(2.5 \times 10^{-3}) \angle 0^\circ = 2.5 \times 10^{-3} \omega C \angle 90^\circ$

$\therefore \theta_i = 90^\circ$

[c]  $125.66 \times 10^{-6} = 2.5 \times 10^{-3} \omega C$

$\frac{1}{\omega C} = \frac{2.5 \times 10^{-3}}{125.66 \times 10^{-6}} = 19.89 \Omega, \quad \therefore X_C = -19.89 \Omega$

[d]  $C = \frac{1}{19.89(\omega)} = \frac{1}{(19.89)(80\pi \times 10^3)}$

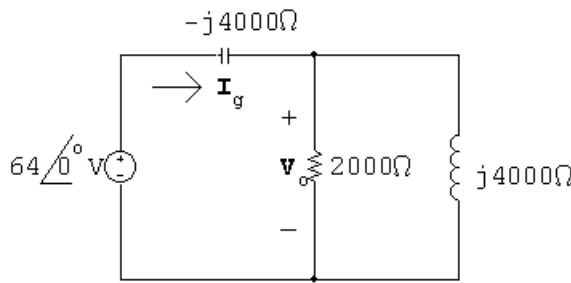
$C = 0.2 \times 10^{-6} = 0.2 \mu\text{F}$

[e]  $Z_c = j \left( \frac{-1}{\omega C} \right) = -j19.89 \Omega$

P 7.13  $\frac{1}{j\omega C} = \frac{10^9}{(31.25)(8000)} = -j4000 \Omega$

$j\omega L = j8000(500)10^{-3} = j4000 \Omega$

$\mathbf{V}_g = 64 \angle 0^\circ \text{ V}$



$Z_e = \frac{(2000)(j4000)}{2000 + j4000} = 1600 + j800 \Omega$

$Z_T = 1600 + j800 - j4000 = 1600 - j3200 \Omega$

$\mathbf{I}_g = \frac{64 \angle 0^\circ}{1600 - j3200} = 8 + j16 \text{ mA}$

$\mathbf{V}_o = Z_e \mathbf{I}_g = (1600 + j800)(0.008 + j0.016) = j32 = 32 \angle 90^\circ \text{ V}$

$v_o = 32 \cos(8000t + 90^\circ) \text{ V}$

$$\text{P 7.14} \quad Z = 400 + j(5)(40) - j\frac{1000}{(5)(0.4)} = 500 \angle -36.87^\circ \Omega$$

$$\mathbf{I}_o = \frac{750 \angle 0^\circ \times 10^{-3}}{500 \angle -36.87^\circ} = 1.5 \angle 36.87^\circ \text{ mA}$$

$$i_o(t) = 1.5 \cos(5000t + 36.87^\circ) \text{ mA}$$

$$\begin{aligned} \text{P 7.15} \quad [\mathbf{a}] \quad Z_p &= \frac{\frac{R}{j\omega C}}{R + (1/j\omega C)} = \frac{R}{1 + j\omega RC} \\ &= \frac{12,500}{1 + j(1000)(12,500)C} = \frac{12,500}{1 + j12.5 \times 10^6 C} \\ &= \frac{12,500(1 - j12.5 \times 10^6 C)}{1 + 156.25 \times 10^{12} C^2} \\ &= \frac{12,500}{1 + 156.25 \times 10^{12} C^2} - j\frac{156.25 \times 10^9 C}{1 + 156.25 \times 10^{12} C^2} \end{aligned}$$

$$j\omega L = j1000(5) = j5000$$

$$\therefore 5000 = \frac{156.25 \times 10^9 C}{1 + 156.25 \times 10^{12} C^2}$$

$$\therefore 781.25 \times 10^{15} C^2 - 156.25 \times 10^9 C + 5000 = 0$$

$$\therefore C^2 - 20 \times 10^{-8} C + 64 \times 10^{-16} = 0$$

$$\therefore C_{1,2} = 10 \times 10^{-8} \pm \sqrt{100 \times 10^{-16} - 64 \times 10^{-16}}$$

$$C_1 = 10 \times 10^{-8} + 6 \times 10^{-8} = 16 \times 10^{-8} = 160 \text{ nF} = 0.16 \mu\text{F}$$

$$C_2 = 10 \times 10^{-8} - 6 \times 10^{-8} = 4 \times 10^{-8} = 40 \text{ nF} = 0.04 \mu\text{F}$$

$$[\mathbf{b}] \quad R_e = \frac{12,500}{1 + 156.25 \times 10^{12} C^2}$$

$$\text{When } C = 160 \text{ nF} \quad R_e = 2500 \Omega;$$

$$\mathbf{I}_g = \frac{250 \angle 0^\circ}{2500} = 0.1 \angle 0^\circ \text{ A}; \quad i_g = 100 \cos 1000t \text{ mA}$$

$$\text{When } C = 40 \text{ nF} \quad R_e = 10,000 \Omega;$$

$$\mathbf{I}_g = \frac{250 \angle 0^\circ}{10,000} = 0.025 \angle 0^\circ \text{ A}; \quad i_g = 25 \cos 1000t \text{ mA}$$

P 7.16 [a]  $Y_p = \frac{1}{10 + j2\omega} + j4 \times 10^{-3}\omega$

$$= \frac{10 - j2\omega}{100 + 4\omega^2} + j4 \times 10^{-3}\omega$$

$$= \frac{10}{100 + 4\omega^2} - \frac{j2\omega}{100 + 4\omega^2} + j4 \times 10^{-3}\omega$$

$Y_p$  is real when

$$4 \times 10^{-3}\omega = \frac{2\omega}{100 + 4\omega^2}$$

or  $\omega^2 = 100$ ;  $\omega = 10 \text{ rad/s}$ ;  $f = 5/\pi = 1.59\text{Hz}$

[b]  $Y_p(10 \text{ rad/s}) = \frac{10}{500} = 20 \text{ mS}$

$$Z_p(10 \text{ rad/s}) = \frac{10^3}{20} = 50 \Omega$$

$$Z(10 \text{ rad/s}) = 50 + 150 = 200 \Omega$$

$$\mathbf{I}_o = \frac{\mathbf{V}_g}{200} \text{ A} = \frac{10\angle 0^\circ}{200} = 50\angle 0^\circ \text{ mA}$$

$$i_o = 50 \cos 10t \text{ mA}$$

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P 7.17  $\mathbf{V}_g = 50\angle -45^\circ \text{ V}$ ;  $\mathbf{I}_g = 100\angle -8.13^\circ \text{ mA}$

$$Z = \frac{\mathbf{V}_g}{\mathbf{I}_g} = 500\angle -36.87^\circ \Omega = 400 - j300 \Omega$$

$$Z = 400 + j\left(0.04\omega - \frac{2.5 \times 10^6}{\omega}\right)$$

$$\therefore 0.04\omega - \frac{2.5 \times 10^6}{\omega} = -300$$

$$\therefore \omega^2 + 7500\omega - 62.5 \times 10^6 = 0$$

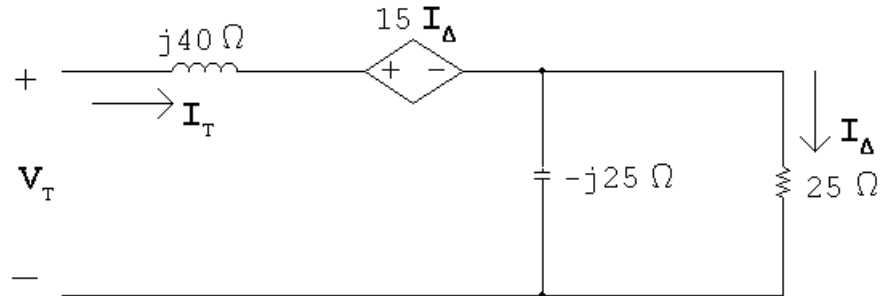
$$\therefore \omega = -3750 \pm \sqrt{(3750)^2 + 62.5 \times 10^6} = -3750 \pm 8750$$

$$\omega > 0, \quad \therefore \omega = 5000 \text{ rad/s}$$



P 7.18  $j\omega L = j1.6 \times 10^6(25 \times 10^{-6}) = j40 \Omega$

$$\frac{1}{j\omega C} = \frac{10^{-6} \times 10^9}{j1.6(25)} = -j25 \Omega$$



$$V_T = j40I_T + 15I_\Delta + 25I_\Delta$$

$$I_\Delta = \frac{I_T(-j25)}{25 - j25} = \frac{-jI_T}{1 - j1}$$

$$V_T = j40I_T + 40 \frac{(-jI_T)}{1 - j1}$$

$$\frac{V_T}{I_T} = Z_{ab} = j40 + 20(-j)(1 + j) = 20 + j20 \Omega = 28.28/\underline{45^\circ} \Omega$$

P 7.19 First find the admittance of the parallel branches

$$\frac{1}{Z_p} = Y_p = \frac{1}{6 - j2} + \frac{1}{4 + j12} + \frac{1}{5} + \frac{1}{j10} = 0.375 - j0.125 \text{ S}$$

$$Z_p = \frac{1}{Y_p} = \frac{1}{0.375 - j0.125} = 2.4 + j0.8 \Omega$$

$$Z_{ab} = -j12.8 + 2.4 + j0.8 + 13.6 = 16 - j12 \Omega$$

$$Y_{ab} = \frac{1}{Z_{ab}} = \frac{1}{16 - j12} = 0.04 + j0.03 \text{ S}$$

$$= 40 + j30 \text{ mS} = 50/\underline{36.87^\circ} \text{ mS}$$

$$\begin{aligned}
\text{P 7.20 [a]} \quad Z_g &= 4000 - j\frac{10^9}{25\omega} + \frac{10^4(j2\omega)}{10^4 + j2\omega} \\
&= 4000 - j\frac{10^9}{25\omega} + \frac{2 \times 10^4 j\omega(10^4 - j2\omega)}{10^8 + 4\omega^2} \\
&= 4000 - j\frac{10^9}{25\omega} + \frac{4 \times 10^4 \omega^2}{10^8 + 4\omega^2} + j\frac{2 \times 10^8 \omega}{10^8 + 4\omega^2} \\
\therefore \frac{10^9}{25\omega} &= \frac{0.2 \times 10^9 \omega}{10^8 + 4\omega^2} \\
10^8 + 4\omega^2 &= 5\omega^2 \\
\omega^2 &= 10^8; \quad \omega = 10,000 \text{ rad/s}
\end{aligned}$$

[b] When  $\omega = 10,000 \text{ rad/s}$

$$\begin{aligned}
Z_g &= 4000 + \frac{4 \times 10^4 (10^4)^2}{10^8 + 4(10^4)^2} = 12,000 \Omega \\
\therefore \mathbf{I}_g &= \frac{45/\underline{0^\circ}}{12,000} = 3.75/\underline{0^\circ} \text{ mA} \\
\mathbf{V}_o &= \mathbf{V}_g - \mathbf{I}_g Z_1 \\
Z_1 &= 4000 - j\frac{10^9}{25 \times 10^4} = 4000 - j4000 \Omega \\
\mathbf{V}_o &= 45/\underline{0^\circ} - (3.75 \times 10^{-3})(4000 - j4000) = 45 - (15 - j15) \\
&= 30 + j15 = 33.54/\underline{26.57^\circ} \text{ V} \\
v_o &= 33.54 \cos(10,000t + 26.57^\circ) \text{ V}
\end{aligned}$$

$$\begin{aligned}
\text{P 7.21 [a]} \quad Z_1 &= 1600 - j\frac{10^9}{10^4(62.5)} = 1600 - j1600 \Omega \\
Z_1 &= \frac{4000(j10^4 L)}{4000 + j10^4 L} = \frac{4 \times 10^5 L^2 + j16 \times 10^4 L}{16 + 100L^2} \\
Z_T = Z_1 + Z_2 &= 1600 + \frac{4 \times 10^5 L^2}{16 + 100L^2} - j1600 + j\frac{16 \times 10^4 L}{16 + 100L^2} \\
Z_T \text{ is resistive when} \\
\frac{16 \times 10^4 L}{16 + 100L^2} &= 1600 \quad \text{or} \\
L^2 - L + 0.16 &= 0
\end{aligned}$$

Solving,  $L_1 = 0.8 \text{ H}$  and  $L_2 = 0.2 \text{ H}$ .

[b] When  $L = 0.8$  H:

$$Z_T = 1600 + \frac{4 \times 10^5(0.64)}{16 + 64} = 4800 \Omega$$

$$\mathbf{I}_g = \frac{96/\underline{0^\circ}}{4.8} \times 10^{-3} = 20/\underline{0^\circ} \text{ mA}$$

$$i_g = 20 \cos 10,000t \text{ mA}$$

When  $L = 0.2$  H:

$$Z_T = 1600 + \frac{4 \times 10^5(0.04)}{16 + 4} = 2400 \Omega$$

$$i_g = 40 \cos 10,000t \text{ mA}$$

P 7.22 [a] 
$$\begin{aligned} Z_{ab} &= j5\omega + \frac{(4000)(10^9/j\omega 625)}{4000 + (10^9/j\omega 625)} \\ &= j5\omega + \frac{4 \times 10^{12}}{2500 \times 10^3 j\omega + 10^9} \\ &= j5\omega + \frac{4 \times 10^7}{10^4 + j25\omega} \\ &= j5\omega + \frac{4 \times 10^{11}}{10^8 + 625\omega^2} - j\frac{100 \times 10^7\omega}{10^8 + 625\omega^2} \\ \therefore 5 &= \frac{10^9}{10^8 + 625\omega^2} \\ 5 \times 10^8 + 3125\omega^2 &= 10^9 \\ \omega &= 4 \times 10^2 = 400 \text{ rad/s} \end{aligned}$$

[b] 
$$Z_{ab}(400) = j2000 + \frac{(4000)(-j4000)}{4000 - j4000} = 2 \text{ k}\Omega$$

P 7.23 
$$Z_1 = 10 - j40 \Omega$$

$$Z_2 = \frac{(5 - j10)(10 + j30)}{15 + j20} = 10 - j10 \Omega$$

$$Z_3 = \frac{20(j20)}{20 + j20} = 10 + j10 \Omega$$

$$\therefore Z_{ab} = Z_1 + Z_2 + Z_3 = 30 - j40 \Omega = 50/\underline{-53.13^\circ} \Omega$$

P 7.24 [a]  $Y_1 = \frac{1}{5000} = 0.2 \times 10^{-3} \text{ S}$

$$Y_2 = \frac{1}{1200 + j0.2\omega}$$

$$= \frac{1200}{1.44 \times 10^6 + 0.04\omega^2} - j \frac{0.2\omega}{1.44 \times 10^6 + 0.04\omega^2}$$

$$Y_3 = j\omega 50 \times 10^{-9}$$

$$Y_T = Y_1 + Y_2 + Y_3$$

For  $i_g$  and  $v_o$  to be in phase the  $j$  component of  $Y_T$  must be zero; thus,

$$\omega 50 \times 10^{-9} = \frac{0.2\omega}{1.44 \times 10^6 + 0.04\omega^2}$$

or

$$0.04\omega^2 + 1.44 \times 10^6 = \frac{0.2 \times 10^9}{50} = 4 \times 10^6$$

$$\therefore 0.04\omega^2 = 2.56 \times 10^6 \quad \therefore \omega = 8000 \text{ rad/s} = 8 \text{ krad/s}$$

[b]  $Y_T = 0.2 \times 10^{-3} + \frac{1200}{1.44 \times 10^6 + 0.04(64) \times 10^6} = 0.5 \times 10^{-3} \text{ S}$

$$\therefore Z_T = 2000 \Omega$$

$$\mathbf{V}_o = (2.5 \times 10^{-3} \angle 0^\circ)(2000) = 5 \angle 0^\circ$$

$$v_o = 5 \cos 8000t \text{ V}$$

P 7.25 [a]  $Z_1 = R_1 + j\omega L_1$

$$Z_2 = \frac{R_2(j\omega L_2)}{R_2 + j\omega L_2} = \frac{\omega^2 L_2^2 R_2 + j\omega L_2 R_2^2}{R_2^2 + \omega^2 L_2^2}$$

$$Z_1 = Z_2 \quad \text{when} \quad R_1 = \frac{\omega^2 L_2^2 R_2}{R_2^2 + \omega^2 L_2^2} \quad \text{and} \quad L_1 = \frac{R_2^2 L_2}{R_2^2 + \omega^2 L_2^2}$$

[b]  $R_1 = \frac{(4 \times 10^8)(6.25)(5 \times 10^4)}{25 \times 10^8 + (4 \times 10^8)(6.25)} = 2.5 \times 10^4$

$$\therefore R_1 = 25 \text{ k}\Omega$$

$$L_1 = \frac{(25 \times 10^8)2.5}{50 \times 10^8} = 1.25 \text{ H}$$

P 7.26 [a]  $Y_2 = \frac{1}{R_2} - \frac{j}{\omega L_2}$

$$Y_1 = \frac{1}{R_1 + j\omega L_1} = \frac{R_1 - j\omega L_1}{R_1^2 + \omega^2 L_1^2}$$

Therefore  $Y_2 = Y_1$  when

$$R_2 = \frac{R_1^2 + \omega^2 L_1^2}{R_1} \quad \text{and} \quad L_2 = \frac{R_1^2 + \omega^2 L_1^2}{\omega^2 L_1}$$

[b]  $R_2 = \frac{25 \times 10^6 + 10^8(0.25)}{5 \times 10^3} = 10 \times 10^3$

$$\therefore R_2 = 10 \text{ k}\Omega$$

$$L_2 = \frac{50 \times 10^6}{10^8(0.5)} = 1 \text{ H}$$

P 7.27 [a]  $Z_1 = R_1 - j\frac{1}{\omega C_1}$

$$Z_2 = \frac{R_2/j\omega C_2}{R_2 + (1/j\omega C_2)} = \frac{R_2}{1 + j\omega R_2 C_2} = \frac{R_2 - j\omega R_2^2 C_2}{1 + \omega^2 R_2^2 C_2^2}$$

$$Z_1 = Z_2 \quad \text{when} \quad R_1 = \frac{R_2}{1 + \omega^2 R_2^2 C_2^2} \quad \text{and}$$

$$\frac{1}{\omega C_1} = \frac{\omega R_2^2 C_2}{1 + \omega^2 R_2^2 C_2^2} \quad \text{or} \quad C_1 = \frac{1 + \omega^2 R_2^2 C_2^2}{\omega^2 R_2^2 C_2}$$

[b]  $R_1 = \frac{500}{1 + (64 \times 10^8)(25 \times 10^4)(625 \times 10^{-18})} = 250 \Omega$

$$C_1 = \frac{2}{(64 \times 10^8)(25 \times 10^4)(25 \times 10^{-9})} = 50 \text{ nF}$$

P 7.28 [a]  $Y_2 = \frac{1}{R_2} + j\omega C_2$

$$Y_1 = \frac{1}{R_1 + (1/j\omega C_1)} = \frac{j\omega C_1}{1 + j\omega R_1 C_1} = \frac{\omega^2 R_1 C_1^2 + j\omega C_1}{1 + \omega^2 R_1^2 C_1^2}$$

Therefore  $Y_1 = Y_2$  when

$$R_2 = \frac{1 + \omega^2 R_1^2 C_1^2}{\omega^2 R_1 C_1^2} \quad \text{and} \quad C_2 = \frac{C_1}{1 + \omega^2 R_1^2 C_1^2}$$

[b]  $R_2 = \frac{1 + (4 \times 10^8)(4 \times 10^6)(2500 \times 10^{-18})}{(4 \times 10^8)(2 \times 10^3)(2500 \times 10^{-18})} = 2500 = 2.5 \text{ k}\Omega$

$$C_2 = \frac{50 \times 10^{-9}}{5} = 10 \text{ nF}$$

P 7.29 [a]  $\mathbf{V}_g = 150/\underline{20^\circ}$ ;  $\mathbf{I}_g = 30/\underline{-52^\circ}$

$$\therefore Z = \frac{\mathbf{V}_g}{\mathbf{I}_g} = 5/\underline{72^\circ} \Omega$$

[b]  $i_g$  lags  $v_g$  by  $72^\circ$ : ← 相位滞后

$$2\pi f = 8000\pi; \quad f = 4000 \text{ Hz}; \quad T = 1/f = 250 \mu\text{s}$$

$$\therefore i_g \text{ lags } v_g \text{ by } \frac{72}{360}(250) = 50 \mu\text{s}$$

P 7.30  $\frac{1}{j\omega C} = -j\frac{10^6}{10^4} = -j100 \Omega$

$$j\omega L = j(500)(1) = j500 \Omega$$

$$\text{Let } Z_1 = 50 - j100 \Omega; \quad Z_2 = 250 + j500 \Omega$$

$$\mathbf{I}_g = 125/\underline{0^\circ} \text{ mA}$$

$$\begin{aligned} \mathbf{I}_o &= \frac{\mathbf{I}_g Z_1}{Z_1 + Z_2} = \frac{125/\underline{0^\circ}(50 - j100)}{(300 + j400)} \\ &= -12.5 - j25 \text{ mA} = 27.95/\underline{-116.57^\circ} \text{ mA} \end{aligned}$$

$$i_o = 27.95 \cos(500t - 116.57^\circ) \text{ mA}$$

P 7.31  $Z_o = 600 - j\frac{10^6}{(5000)(0.25)} = 600 - j800 \Omega$

$$Z_T = 300 + j2000 + 600 - j800 = 900 + j1200 \Omega = 1500/\underline{53.13^\circ} \Omega$$

$$\mathbf{V}_o = \mathbf{V}_g \frac{Z_o}{Z_T} = \frac{(75/\underline{0^\circ})(1000/\underline{-53.13^\circ})}{1500/\underline{53.13^\circ}} = 50/\underline{-106.26^\circ} \text{ V}$$

$$v_o = 50 \cos(5000t - 106.26^\circ) \text{ V}$$

P 7.32  $\mathbf{V}_1 = 240/\underline{53.13^\circ} = 144 + j192 \text{ V}$

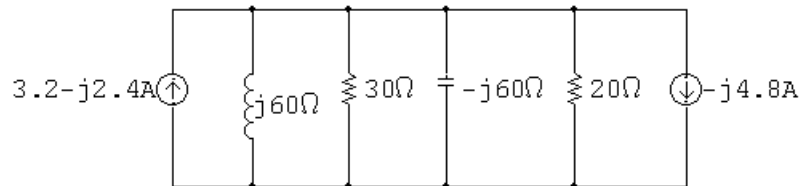
$$\mathbf{V}_2 = 96/\underline{-90^\circ} = -j96 \text{ V}$$

$$j\omega L = j(4000)(15 \times 10^{-3}) = j60 \Omega$$

$$\frac{1}{j\omega C} = -j \frac{6 \times 10^6}{(4000)(25)} = -j60 \Omega$$

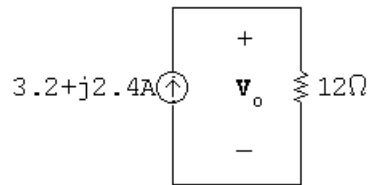
$$\frac{\mathbf{V}_1}{j60} = \frac{144 + j192}{j60} = 3.2 - j2.4 \text{ A}$$

$$\frac{\mathbf{V}_2}{20} = -j \frac{96}{20} = -j4.8 \text{ A}$$



$$Y = \frac{1}{j60} + \frac{1}{30} + \frac{1}{-j60} + \frac{1}{20} = \frac{j5}{j60} = \frac{1}{12}$$

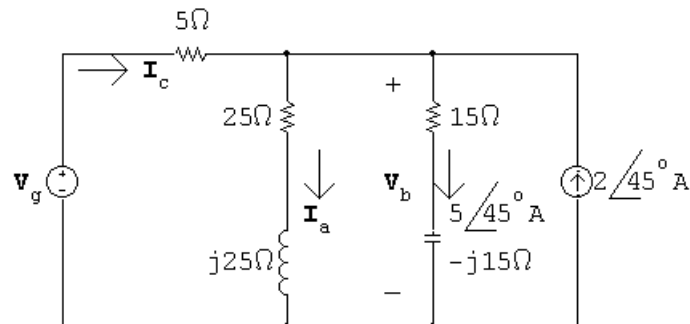
$$Z = \frac{1}{Y} = 12 \Omega$$



$$\mathbf{V}_o = 12(3.2 + j2.4) = 38.4 + j28.8 \text{ V} = 48/\underline{36.87^\circ} \text{ V}$$

$$v_o = 48 \cos(4000t + 36.87^\circ) \text{ V}$$

P 7.33 [a]



$$\mathbf{V}_b = (15 - j15)5/\underline{45^\circ} = 75\sqrt{2}/\underline{0^\circ} \text{ V}$$

$$\mathbf{I}_a = \frac{75\sqrt{2}}{25 + j25} = 3\angle -45^\circ \text{ A}$$

$$\mathbf{I}_c = \mathbf{I}_a + 5\angle 45^\circ - 2\angle 45^\circ = 3\sqrt{2} \text{ A}$$

$$\mathbf{V}_g = 5\mathbf{I}_c + \mathbf{V}_b = 15\sqrt{2} + 75\sqrt{2} = 90\sqrt{2} \text{ V} = 127.28\angle 0^\circ \text{ V}$$

[b]  $i_a = 3 \cos(800t - 45^\circ) \text{ A}$

$$i_c = 4.24 \cos 800t \text{ A}$$

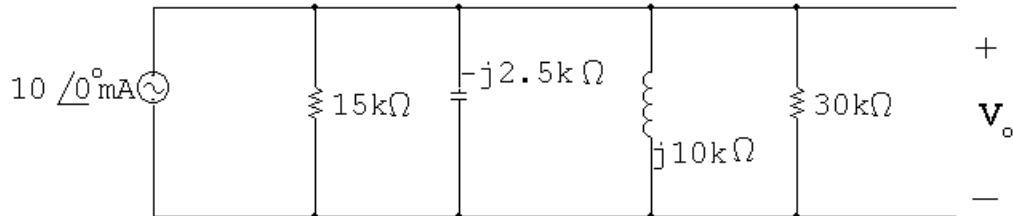
$$v_g = 127.28 \cos 800t \text{ V}$$

P 7.34  $\mathbf{I}_s = 15\angle 0^\circ \text{ mA}$

$$\frac{1}{j\omega C} = \frac{10^6}{j0.05(8000)} = -j2500 \Omega$$

$$j\omega L = j8000(1.25) = j10,000 \Omega$$

After two source transformations we have



$$15 \text{ k}\Omega \parallel 30 \text{ k}\Omega = 10 \text{ k}\Omega$$

$$Y_o = \frac{10^{-3}}{10} + \frac{1}{-j2500} + \frac{1}{j10^4} = 10^{-4}(1 + j3)$$

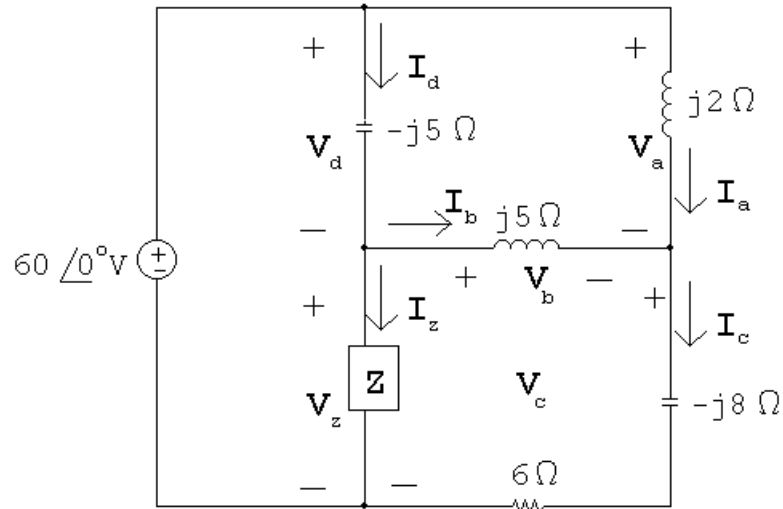
$$Z_o = \frac{10^4}{1 + j3} = (1 - j3) \text{ k}\Omega$$

$$\mathbf{V}_o = \mathbf{I}_g Z_o = (10)(1 - j3) = 10 - j30 = 31.62\angle -71.57^\circ \text{ V}$$

$$v_o = 31.62 \cos(8000t - 71.57^\circ) \text{ V}$$



P 7.35



$$\mathbf{V}_a = j2\mathbf{I}_a = j2(-j5) = 10\angle 0^\circ \text{ V}$$

$$\mathbf{V}_c = 60\angle 0^\circ - \mathbf{V}_a = 50\angle 0^\circ \text{ V}$$

$$\mathbf{I}_c = \frac{\mathbf{V}_c}{6 - j8} = \frac{50\angle 0^\circ}{10\angle -53.13^\circ} = 5\angle 53.13^\circ = 3 + j4 \text{ A}$$

$$\mathbf{I}_b = \mathbf{I}_c - \mathbf{I}_a = 3 + j4 - (-j5) = 3 + j9 \text{ A} = 9.49\angle 71.57^\circ \text{ A}$$

$$\mathbf{V}_b = \mathbf{I}_b(j5) = (3 + j9)(j5) = -45 + j15 \text{ V}$$

$$\mathbf{V}_z = \mathbf{V}_b + \mathbf{V}_c = -45 + j15 + 50 + j0 = 5 + j15 \text{ V}$$

$$\mathbf{V}_d + \mathbf{V}_z = 60\angle 0^\circ; \quad \therefore \mathbf{V}_d = 60 - 5 - j15 = 55 - j15 \text{ V}$$

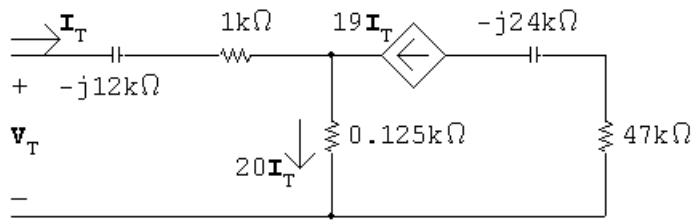
$$\mathbf{I}_d = \frac{\mathbf{V}_d}{-j5} = 3 + j11 \text{ A}$$

$$\mathbf{I}_z = \mathbf{I}_d - \mathbf{I}_b = 3 + j11 - 3 - j9 = j2 \text{ A}$$

$$\mathbf{Z} = \frac{\mathbf{V}_z}{\mathbf{I}_z} = \frac{5 + j15}{j2} = 7.5 - j2.5 \Omega$$

P 7.36  $\frac{1}{\omega C_1} = \frac{(10^{-3})(10^9)}{25(10/3)} = 12 \text{ k}\Omega$

$$\frac{1}{\omega C_2} = \frac{(10^{-3})(10^9)}{25(5/3)} = 24 \text{ k}\Omega$$

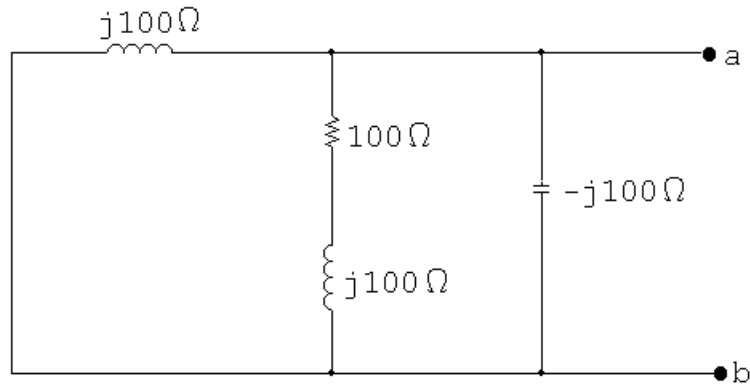


$$\mathbf{V}_T = (1 - j12)\mathbf{I}_T + 20\mathbf{I}_T(0.125)$$

$$Z_{\text{Th}} = \frac{\mathbf{V}_T}{\mathbf{I}_T} = 3.5 - j12 \text{ k}\Omega$$

P 7.37 [a]  $j\omega L = j(1000)(100) \times 10^{-3} = j100 \Omega$

$$\frac{1}{j\omega C} = -j \frac{10^6}{(1000)(10)} = -j100 \Omega$$



$$Y_{\text{ab}} = \frac{1}{j100} + \frac{1}{100 + j100} + \frac{1}{-j100}$$

$$= \frac{1}{100} \left[ \frac{1}{j} + \frac{1}{1 + j1} + \frac{j}{1} \right]$$

$$Y_{\text{ab}} = \frac{1}{100} \left[ -j + \frac{1 - j1}{2} + j \right]$$

$$= \frac{1 - j1}{200}; \quad Z_{\text{ab}} = \frac{200}{1 - j1} = 100(1 + j1)$$

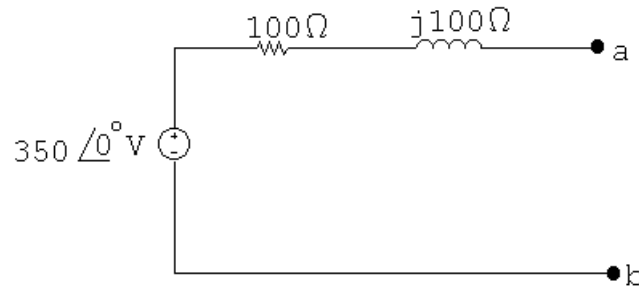
$$\therefore \mathbf{V}_{\text{ab}} = 100(1 + j1) \left[ \frac{247.49/\underline{45^\circ}}{j100} \right]$$

$$= \sqrt{2}/\underline{45^\circ} \cdot 1/\underline{-90^\circ} \cdot 247.49/\underline{45^\circ}$$

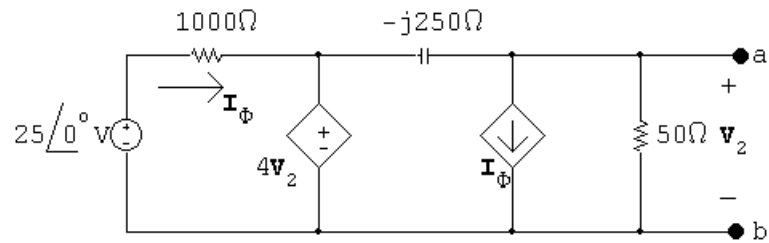
$$\mathbf{V}_{\text{Th}} = 350/\underline{0^\circ} \text{ V}$$

[b]  $Z_{\text{Th}} = Z_{\text{ab}} = 100 + j100 \Omega$

[c]



P 7.38



$$\frac{V_2}{50} + \frac{25 - 4V_2}{1000} + \frac{V_2 - 4V_2}{-j250} = 0$$

Solving,

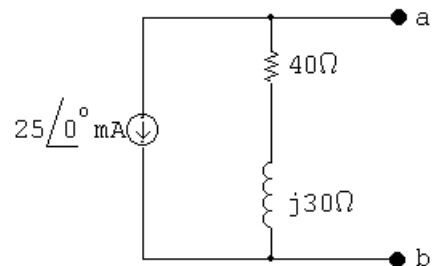
$$V_2 = -10 - j0.75 \text{ V} = 1.25 \angle 216.87^\circ \text{ V}$$

$$\mathbf{I}_{sc} = -\mathbf{I}_\phi = \frac{-25 \angle 0^\circ}{1000} = -25 \angle 0^\circ \text{ mA}$$

$$Z_{\text{Th}} = \frac{1.25 \angle 216.87^\circ}{-25 \times 10^{-3} \angle 0^\circ} = 50 \angle 36.87^\circ \Omega = 40 + j30 \Omega$$

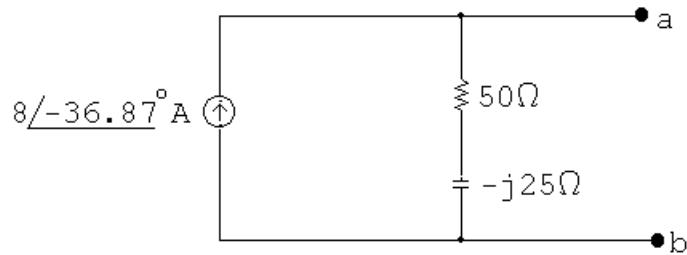
$$\mathbf{I}_N = \mathbf{I}_{sc} = -25 \angle 0^\circ \text{ mA}$$

$$Z_N = Z_{\text{Th}} = 50 \angle 36.87^\circ = 40 + j30 \Omega$$

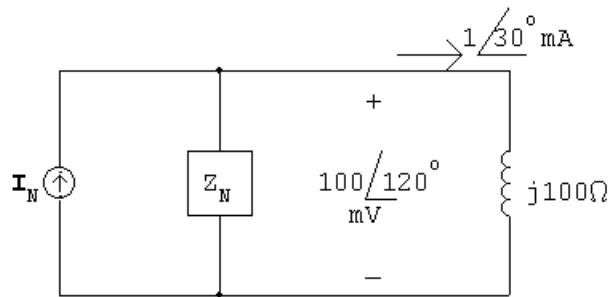


P 7.39  $\mathbf{I}_N = \mathbf{I}_{sc} = \frac{(16/\underline{0^\circ})(25)}{25 + 15 + j30} = 6.4 - j4.8 \text{ A} = 8/\underline{-36.87^\circ} \text{ A}$

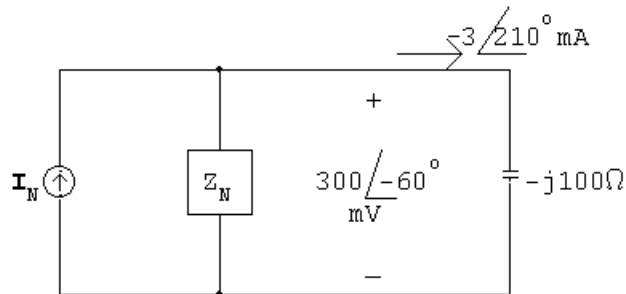
$$Z_N = \frac{(-j50)(40 + j30)}{40 + j30 - j50} = 50 - j25 \Omega$$



P 7.40



$$\mathbf{I}_N = \frac{0.1/\underline{120^\circ}}{Z_N} + 1/\underline{30^\circ} \text{ mA}, \quad Z_N \text{ in k}\Omega$$



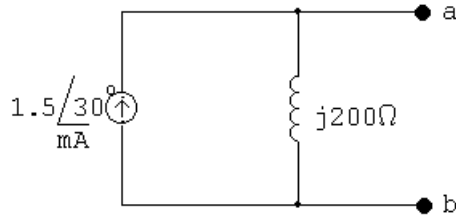
$$\mathbf{I}_N = \frac{0.3/\underline{-60^\circ}}{Z_N} + (-3/\underline{210^\circ}) \text{ mA}, \quad Z_N \text{ in k}\Omega$$

$$\frac{0.1/\underline{120^\circ}}{Z_N} + 1/\underline{30^\circ} = \frac{0.3/\underline{-60^\circ}}{Z_N} + (-3/\underline{210^\circ})$$

$$\frac{0.3/\underline{-60^\circ} - 0.1/\underline{120^\circ}}{Z_N} = 1/\underline{30^\circ} + 3/\underline{210^\circ}$$

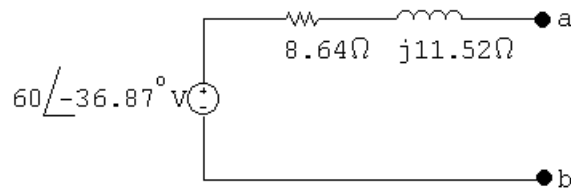
$$Z_N = \frac{0.3 \angle -60^\circ - 0.1 \angle 120^\circ}{1 \angle 30^\circ + 3 \angle 210^\circ} = 0.2 \angle 90^\circ = j0.2 \text{ k}\Omega$$

$$\mathbf{I}_N = \frac{0.1 \angle 120^\circ}{0.2 \angle 90^\circ} + 1 \angle 30^\circ = 1.5 \angle 30^\circ \text{ mA}$$

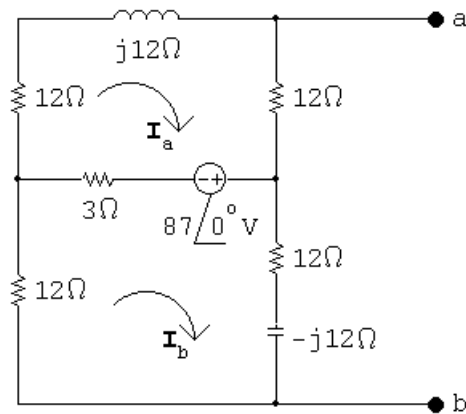


P 7.41  $\mathbf{V}_{Th} = \frac{75(24)}{24 + j18} = 60 \angle -36.87^\circ \text{ V}$

$$Z_{Th} = \frac{(24)(j18)}{24 + j18} = 8.64 + j11.52 \Omega$$



P 7.42



$$(27 + j12)\mathbf{I}_a - 3\mathbf{I}_b = -87 \angle 0^\circ$$

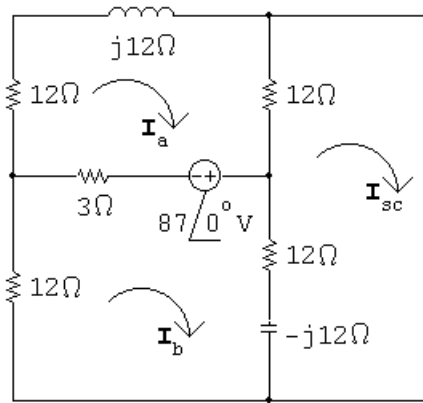
$$-3\mathbf{I}_a + (27 - j12)\mathbf{I}_b = 87 \angle 0^\circ$$

Solving,

$$\mathbf{I}_a = -2.4167 + j1.21; \quad \mathbf{I}_b = 2.4167 + j1.21$$

$$\mathbf{V}_{\text{Th}} = 12\mathbf{I}_a + (12 - j12)\mathbf{I}_b = 14.5/\underline{0^\circ} \text{ V}$$

Short Circuit Test:



$$(27 + j12)\mathbf{I}_a - 3\mathbf{I}_b - 12\mathbf{I}_{\text{sc}} = -87$$

$$-3\mathbf{I}_a + (27 - j12)\mathbf{I}_b - (12 - j12)\mathbf{I}_{\text{sc}} = 87$$

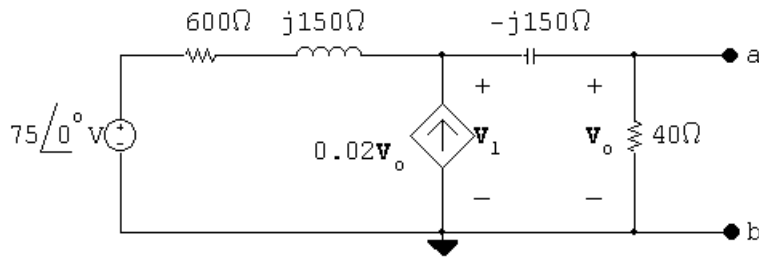
$$-12\mathbf{I}_a - (12 - j12)\mathbf{I}_b + (24 - j12)\mathbf{I}_{\text{sc}} = 0$$

Solving,

$$\mathbf{I}_{\text{sc}} = 1/\underline{0^\circ}$$

$$Z_{\text{Th}} = \frac{\mathbf{V}_{\text{Th}}}{\mathbf{I}_{\text{sc}}} = \frac{14.5/\underline{0^\circ}}{1/\underline{0^\circ}} = 14.5 \Omega$$

P 7.43



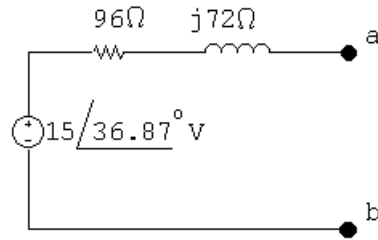
$$\frac{\mathbf{V}_1 - 75}{150(4 + j1)} - \frac{0.02\mathbf{V}_1(40)}{40 - j150} + \frac{\mathbf{V}_1}{40 - j150} = 0$$

$$\therefore \mathbf{V}_1 = \frac{75(4 - j15)}{16 - j12}$$

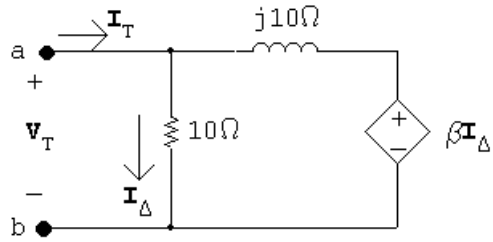
$$\begin{aligned} \mathbf{V}_{\text{Th}} &= \frac{40\mathbf{V}_1}{40 - j150} = \frac{4}{4 - j15} \cdot \frac{75(4 - j15)}{16 - j12} \\ &= \frac{75}{4 - j3} = 15/\underline{36.87^\circ} \text{ V} \end{aligned}$$

$$\mathbf{I}_{\text{sc}} = \frac{75}{600} = \frac{1}{8} \text{ A}$$

$$\mathbf{Z}_{\text{Th}} = \frac{\mathbf{V}_{\text{Th}}}{\mathbf{I}_{\text{sc}}} = 120/\underline{36.87^\circ} = 96 + j72 \Omega$$



P 7.44 [a]



$$\mathbf{I}_T = \frac{\mathbf{V}_T}{10} + \frac{\mathbf{V}_T + \beta\mathbf{V}_T/10}{j10}$$

$$\frac{\mathbf{I}_T}{\mathbf{V}_T} = \frac{1}{10} + \frac{(1 - \beta/10)}{j10} = \frac{(10 - \beta) + j10}{j100}$$

$$\therefore \mathbf{Z}_{\text{Th}} = \frac{\mathbf{V}_T}{\mathbf{I}_T} = \frac{1000 + j100(10 - \beta)}{(10 - \beta)^2 + 100}$$

$\mathbf{Z}_{\text{Th}}$  is real when  $\beta = 10$ .

$$[\mathbf{b}] \mathbf{Z}_{\text{Th}} = \frac{1000}{100} = 10 \Omega$$

$$[\mathbf{c}] \mathbf{Z}_{\text{Th}} = 5 + j5$$

$$\frac{1000}{(10 - \beta)^2 + 100} = 5; \quad (10 - \beta)^2 = 100$$

$$\therefore 10 - \beta = \pm 10; \quad \beta = 10 \mp 10$$

$$\beta = 0; \quad \beta = 20$$

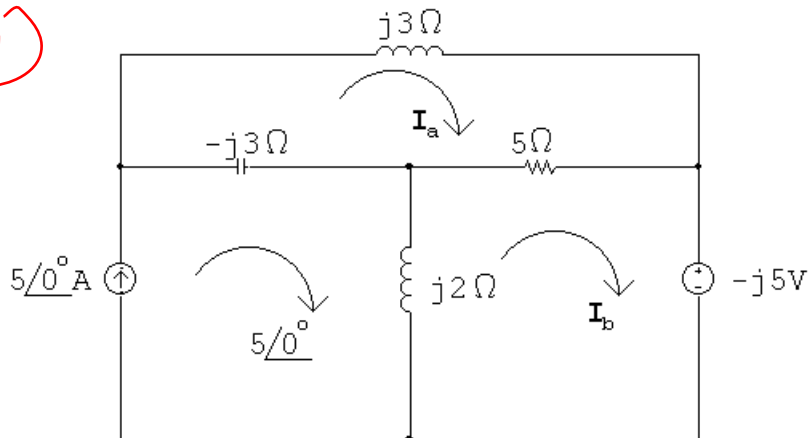
But the  $j$  term can only equal the real term with  $\beta = 0$ . Thus,  $\beta = 0$ .

[d]  $Z_{Th}$  will be capacitive when  $\beta > 10$ :

$$\therefore 10 < \beta \leq 50$$

P 7.45

①



$$j3\mathbf{I}_a + 5(\mathbf{I}_a - \mathbf{I}_b) - j3(\mathbf{I}_a - 5) = 0$$

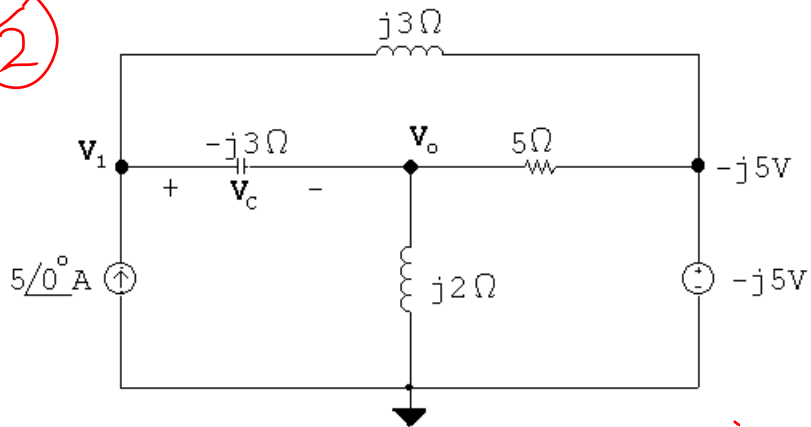
$$j2(\mathbf{I}_b - 5) + 5(\mathbf{I}_b - \mathbf{I}_a) - j5 = 0$$

Solving,

$$\mathbf{I}_a = -j3; \quad \mathbf{I}_g = -j3 = 3\angle -90^\circ \text{ A}$$

P 7.46

②



③ 2 步 法 求 电 压

$$\frac{\mathbf{V}_o}{j2} + \frac{\mathbf{V}_o + j5}{5} + \frac{\mathbf{V}_o - \mathbf{V}_1}{-j3} = 0$$

$$(5 + j6)\mathbf{V}_o + 10\mathbf{V}_1 = 30$$

$$-5 + \frac{\mathbf{V}_1 - \mathbf{V}_o}{-j3} + \frac{\mathbf{V}_1 + j5}{j3} = 0$$

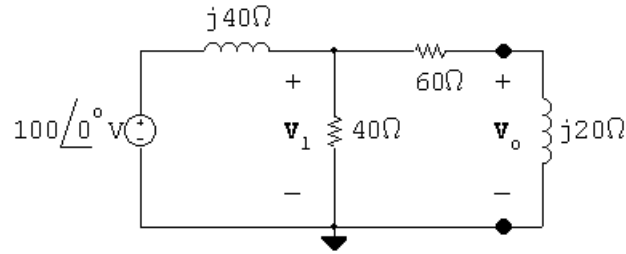
$$\mathbf{V}_o = j10; \quad \mathbf{V}_1 = 9 - j5$$

$$\mathbf{V}_c = \mathbf{V}_1 - \mathbf{V}_o = 9 - j5 - j10 = 9 - j15 = 17.49\angle -59.04^\circ \text{ V}$$

$$\begin{aligned} I_g &= \frac{V_1 - (-5j)}{3j} = \frac{9}{3j} \\ &= -3j = 3\angle -90^\circ \end{aligned}$$



P 7.47



$$\frac{\mathbf{V}_1 - 100}{j40} + \frac{\mathbf{V}_1}{40} + \frac{\mathbf{V}_1}{60 + j20} = 0$$

Solving for  $\mathbf{V}_1$  yields

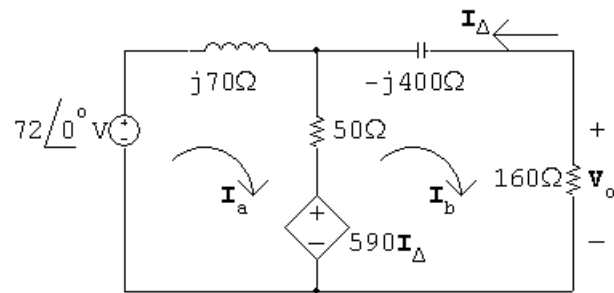
$$\mathbf{V}_1 = 30 - j40 \text{ V}$$

$$\mathbf{V}_o = \frac{\mathbf{V}_1}{60 + j20}(j20) = \left( \frac{j}{3 + j} \right) \mathbf{V}_1$$

$$\mathbf{V}_o = 15 + j5 \text{ V} = 15.81 \angle 18.43^\circ \text{ V}$$

P 7.48  $j\omega L = j5000(14 \times 10^{-3}) = j70 \Omega$ 

$$\frac{1}{j\omega C} = \frac{-j}{(5000)(0.5 \times 10^{-6})} = -j400 \Omega$$



$$72 \angle 0^\circ = (50 + j70)\mathbf{I}_a - 50\mathbf{I}_b + 590(-\mathbf{I}_b)$$

$$0 = -50\mathbf{I}_a - 590(-\mathbf{I}_b) + (210 - j400)\mathbf{I}_b$$

Solving,

$$\mathbf{I}_b = (50 - j50) \text{ mA}$$

$$\mathbf{V}_o = 160\mathbf{I}_b = 8 - j8 = 11.31 \angle -45^\circ$$

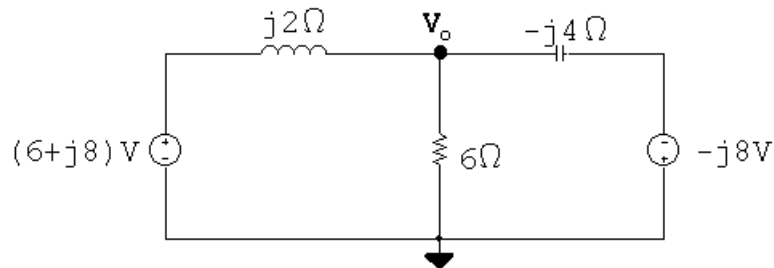
$$v_o = 11.31 \cos(5000t - 45^\circ) \text{ V}$$

P 7.49  $j\omega L = j(5000)(0.4 \times 10^{-3}) = j2\ \Omega$

$$\frac{1}{j\omega C} = -j \frac{10^6}{(5000)(50)} = -j4\ \Omega$$

$$\mathbf{V}_{g1} = 10/\underline{53.13^\circ} = 6 + j8\ \text{V}$$

$$\mathbf{V}_{g2} = 8/\underline{-90^\circ} = -j8\ \text{V}$$



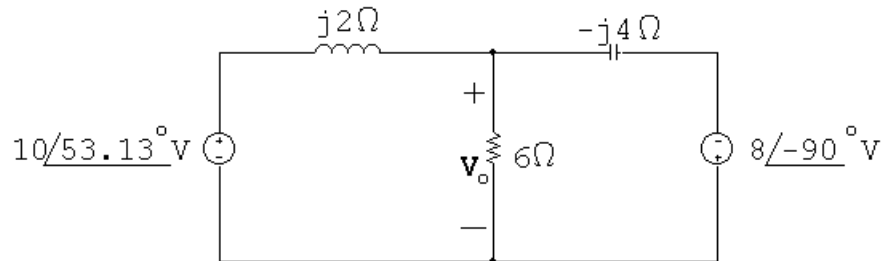
$$\frac{\mathbf{V}_o - 6 - j8}{j2} + \frac{\mathbf{V}_o}{6} + \frac{\mathbf{V}_o + (-j8)}{-j4} = 0$$

Solving,

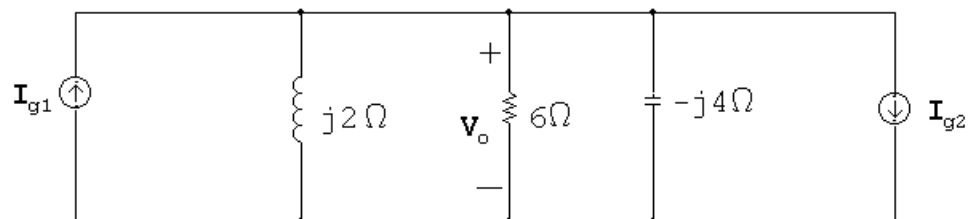
$$\mathbf{V}_o = 12/\underline{0^\circ}$$

$$v_o(t) = 12 \cos 5000t\ \text{V}$$

P 7.50 From the solution to Problem 7.49 the phasor-domain circuit is



Making two source transformations yields



$$\mathbf{I}_{g1} = \frac{10/\underline{53.13^\circ}}{j2} = 5/\underline{-36.87^\circ} = 4 - j3\ \text{A}$$

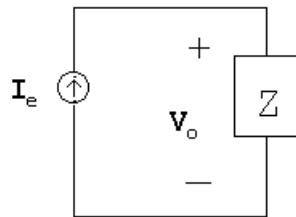
$$\mathbf{I}_{g2} = \frac{8\angle -90^\circ}{-j4} = 2\angle 0^\circ = 2 \text{ A}$$

$$Y = \frac{1}{j2} + \frac{1}{6} + \frac{1}{-j4} \text{ S}$$

$$Z = \frac{1}{Y} = \frac{1}{(1/6) - j(1/4)} = 1.85 + j2.77 \Omega$$

$$\mathbf{I}_e = \mathbf{I}_{g1} - \mathbf{I}_{g2} = 4 - j3 - 2 = 2 - j3 \text{ A}$$

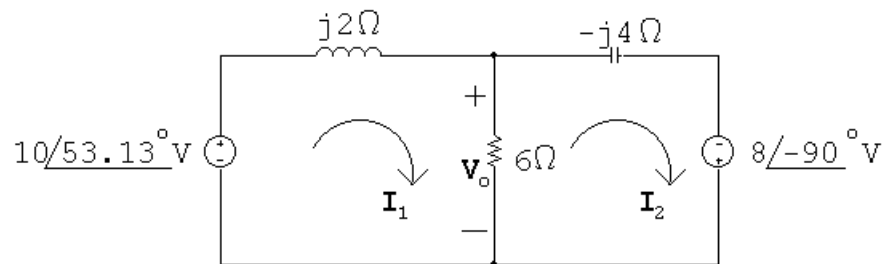
Hence the circuit reduces to



$$\mathbf{V}_o = Z\mathbf{I}_e = (1.85 + j2.77)(2 - j3) = 12\angle 0^\circ$$

$$\therefore v_o(t) = 12 \cos 5000t \text{ V}$$

P 7.51 From the solution to Problem 7.49 the phasor-domain circuit is



$$10\angle 53.13^\circ = (6 + j2)\mathbf{I}_1 - 6\mathbf{I}_2$$

$$8\angle -90^\circ = -6\mathbf{I}_1 + (6 - j4)\mathbf{I}_2$$

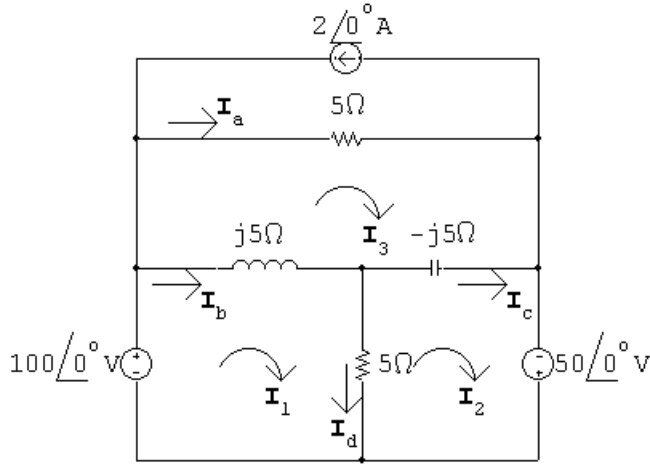
$$\mathbf{V}_o = (\mathbf{I}_1 - \mathbf{I}_2)6$$

Solving,

$$\mathbf{V}_o = 12\angle 0^\circ \text{ V}$$

$$v_o(t) = 12 \cos 5000t \text{ V}$$

P 7.52



$$100\angle 0^\circ = (5 + j5)\mathbf{I}_1 - 5\mathbf{I}_2 - j5\mathbf{I}_3$$

$$50\angle 0^\circ = -5\mathbf{I}_1 + (5 - j5)\mathbf{I}_2 + j5\mathbf{I}_3$$

$$-10\angle 0^\circ = -j5\mathbf{I}_1 + j5\mathbf{I}_2 + 5\mathbf{I}_3$$

Solving,

$$\mathbf{I}_1 = 58 - j20 \text{ A}; \quad \mathbf{I}_2 = 58 + j10 \text{ A}; \quad \mathbf{I}_3 = 28 + j0 \text{ A}$$

$$\mathbf{I}_a = \mathbf{I}_3 + 2 = 30 + j0 \text{ A}$$

$$\mathbf{I}_b = \mathbf{I}_1 - \mathbf{I}_3 = 58 - j20 - 28 = 30 - j20 \text{ A}$$

$$\mathbf{I}_c = \mathbf{I}_2 - \mathbf{I}_3 = 58 + j10 - 28 = 30 + j10 \text{ A}$$

$$\mathbf{I}_d = \mathbf{I}_1 - \mathbf{I}_2 = 58 - j20 - 58 - j10 = -j30 \text{ A}$$

P 7.53  $\mathbf{V}_2$  is the voltage across the  $-j10\Omega$  impedance.

$$\frac{\mathbf{V}_1 - \mathbf{V}_g}{20} + \frac{\mathbf{V}_1}{j5} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{Z} = 0$$

$$\frac{(40 + j30) - (100 - j50)}{20} + \frac{40 + j30}{j5} + \frac{(40 + j30) - \mathbf{V}_2}{Z} = 0$$

$$\therefore \mathbf{V}_2 = 40 + j30 + (3 - j4)Z$$

$$\frac{\mathbf{V}_2 - \mathbf{V}_1}{Z} + \frac{\mathbf{V}_L}{-j10} - \mathbf{I}_g + \frac{\mathbf{V}_2 - \mathbf{V}_g}{3 + j1} = 0$$

$$\frac{\mathbf{V}_2 - (40 + j30)}{Z} + \frac{\mathbf{V}_2}{-j10} - (20 + j30) + \frac{\mathbf{V}_2 - (100 - j50)}{3 + j1} = 0$$

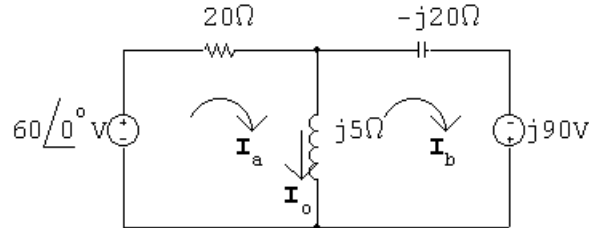
Substituting the expression for  $\mathbf{V}_2$  found at the start and simplifying yields

$$Z = 12 + j16\Omega$$

P 7.54  $\mathbf{V}_a = 60/\underline{0^\circ} \text{ V}; \quad \mathbf{V}_b = 90/\underline{90^\circ} \text{ V}$

$$j\omega L = j(4 \times 10^4)(125 \times 10^{-6}) = j5\Omega$$

$$\frac{-j}{\omega C} = \frac{-j10^6}{40,000(1.25)} = -j20\Omega$$



$$60 = (20 + j5)\mathbf{I}_a - j5\mathbf{I}_b$$

$$j90 = -j5\mathbf{I}_a - j15\mathbf{I}_b$$

Solving,

$$\mathbf{I}_a = 2.25 - j2.25 \text{ A}; \quad \mathbf{I}_b = -6.75 + j0.75 \text{ A}$$

$$\mathbf{I}_o = \mathbf{I}_a - \mathbf{I}_b = 9 - j3 = 9.49/\underline{-18.43^\circ} \text{ A}$$

$$i_o(t) = 9.49 \cos(40,000t - 18.43^\circ) \text{ A}$$

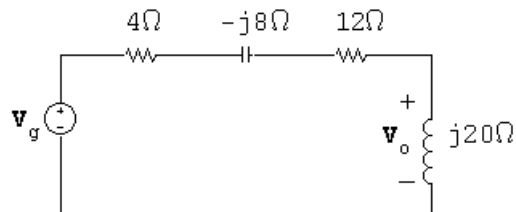
P 7.55 [a]  $\frac{1}{j\omega C} = \frac{10^9}{j8 \times 10^5(125)} = -j10\Omega$

$$j\omega L = j8 \times 10^5(25 \times 10^{-6}) = j20\Omega$$

$$Z_e = \frac{(-j10)(20)}{20 - j10} = 4 - j8\Omega$$

$$\mathbf{I}_g = 5/\underline{0^\circ}$$

$$\mathbf{V}_g = \mathbf{I}_g Z_e = 5(4 - j8) = 20 - j40 \text{ V}$$



$$\mathbf{V}_o = \frac{(20 - j40)(j20)}{(16 + j12)} = 44 - j8 = 44.72/\underline{-10.30^\circ} \text{ V}$$

$$v_o = 44.72 \cos(8 \times 10^5 t - 10.30^\circ) \text{ V}$$

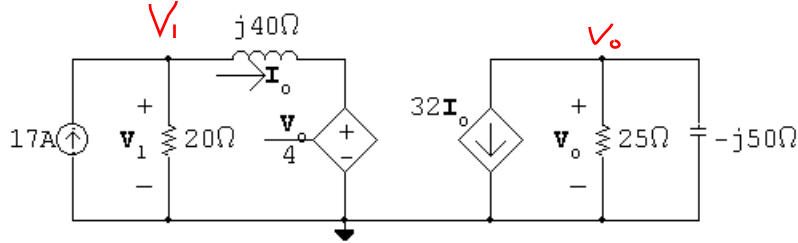
$$[b] \quad \omega = 2\pi f = 8 \times 10^5; \quad f = \frac{4 \times 10^5}{\pi}$$

$$T = \frac{1}{f} = \frac{\pi}{4 \times 10^5} = 2.5\pi \mu\text{s}$$

$$\therefore \frac{10.30}{360}(2.5\pi) = 224.82 \text{ ns}$$

$$\therefore v_o \text{ lags } i_g \text{ by } 224.82 \text{ ns}$$

P 7.56



$$\frac{\mathbf{V}_o}{25} + \frac{\mathbf{V}_o}{-j50} + 32\mathbf{I}_o = 0$$

$$(2 + j)\mathbf{V}_o = -1600\mathbf{I}_o$$

$$\mathbf{V}_o = (-640 + j320)\mathbf{I}_o$$

$$\mathbf{I}_o = \frac{\mathbf{V}_1 - (\mathbf{V}_o/4)}{j40}$$

$$\therefore \mathbf{V}_1 = (-160 + j120)\mathbf{I}_o$$

$$17 = \frac{\mathbf{V}_1}{20} + \mathbf{I}_o = (-8 + j6)\mathbf{I}_o + \mathbf{I}_o = (-7 + j6)\mathbf{I}_o$$

$$\therefore \mathbf{I}_o = \frac{17}{(-7 + j6)} = -1.4 - j1.2 \text{ A} = 1.84 \angle -139.40^\circ \text{ A}$$

$$\mathbf{V}_o = (-640 + j320)\mathbf{I}_o = 1280 + j320 = 1319.39 \angle 14.04^\circ \text{ V}$$

P 7.57  $-15 \angle 0^\circ + \frac{\mathbf{V}_o}{8} + \frac{\mathbf{V}_o - 2.5\mathbf{I}_\Delta}{j5} + \frac{\mathbf{V}_o}{-j10} = 0$

$$\mathbf{I}_\Delta = \frac{\mathbf{V}_o}{-j10}$$

Solving,

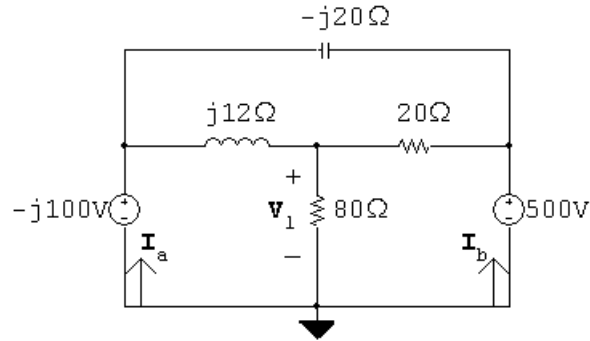
$$\mathbf{V}_o = 72 + j96 = 120 \angle 53.13^\circ \text{ V}$$

P 7.58  $j\omega L = j10^4(1.2 \times 10^{-3}) = j12\ \Omega$

$$\frac{1}{j\omega C} = \frac{-j10^6}{5 \times 10^4} = -j20\ \Omega$$

$$\mathbf{V}_a = 100\angle -90^\circ = -j100\ \text{V}$$

$$\mathbf{V}_b = 500\angle 0^\circ = 500\ \text{V}$$



$$\frac{\mathbf{V}_1}{80} + \frac{\mathbf{V}_1 - 500}{20} + \frac{\mathbf{V}_1 + j100}{j12} = 0$$

Solving,

$$\mathbf{V}_1 = 160\angle 53.13^\circ\ \text{V} = 96 + j128\ \text{V}$$

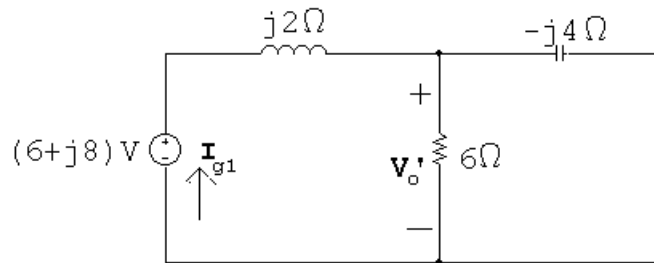
$$\begin{aligned}\mathbf{I}_a &= \frac{-j100 - 96 - j128}{j12} + \frac{-j100 - 500}{-j20} \\ &= 14 - j17 = 22.02\angle -129.47^\circ\ \text{A}\end{aligned}$$

$$i_a = 22.02 \cos(10,000t - 129.47^\circ)\ \text{A}$$

$$\begin{aligned}\mathbf{I}_b &= \frac{500 - 96 - j128}{20} + \frac{500 + j100}{-j20} \\ &= 15.2 + j18.6 = 24.02\angle 50.74^\circ\ \text{A}\end{aligned}$$

$$i_b = 24.02 \cos(10,000t + 50.74^\circ)\ \text{A}$$

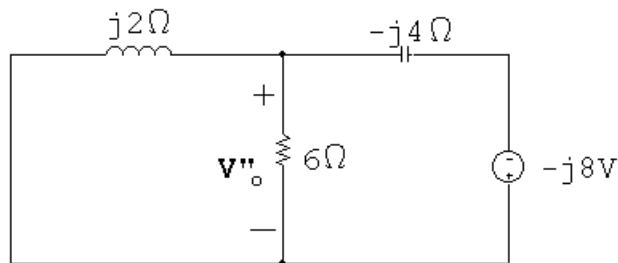
P 7.59 From the solution to Problem 7.49 the phasor-domain circuit with the right-hand source removed is



$$Z_{e1} = \frac{6(-j4)}{(6 - j4)} = \frac{-j24}{6 - j4} \Omega$$

$$\mathbf{V}'_o = \frac{Z_{e1}}{Z_{e1} + j2} (6 + j8) = \frac{192 - j144}{8 - j12} \text{ V}$$

With the left hand source removed



$$Z_{e2} = \frac{6(j2)}{6 + j2} = \frac{j12}{6 + j2} \Omega$$

$$\mathbf{V}''_o = \frac{Z_{e2}}{-j4 + Z_{e2}} (j8) = \frac{-96}{8 - j12} \text{ V}$$

$$\mathbf{V}_o = \mathbf{V}'_o + \mathbf{V}''_o = \frac{192 - j144 - 96}{8 - j12} = 12 + j0 \text{ V}$$

$$v_o(t) = 12 \cos 5000t \text{ V}$$

P 7.60 [a]  $P = \frac{1}{2} (340)(20) \cos(60 - 15) = 2400 \cos 45^\circ = 2404.16 \text{ W (abs)}$

$$Q = 2400 \sin 45^\circ = 2404.16 \text{ VAR (abs)}$$



$$[\mathbf{b}] \quad P = \frac{1}{2}(16)(75) \cos(-15 - 60) = 600 \cos(-75^\circ) = 155.29 \text{ W} \quad (\text{abs})$$

$$Q = 600 \sin(-75^\circ) = -579.56 \text{ VAR} \quad (\text{del})$$

$$[\mathbf{c}] \quad P = \frac{1}{2}(625)(4) \cos(40 - 150) = 1250 \cos(-110^\circ) = -427.53 \text{ W} \quad (\text{del})$$

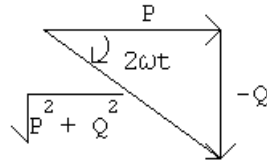
$$Q = 1250 \sin(-110^\circ) = -1174.62 \text{ VAR} \quad (\text{del})$$

$$[\mathbf{d}] \quad P = \frac{1}{2}(180)(10) \cos(130 - 20) = 900 \cos(110^\circ) = -307.82 \text{ W} \quad (\text{del})$$

$$Q = 900 \sin(110^\circ) = 845.72 \text{ VAR} \quad (\text{abs})$$

$$\text{P 7.61} \quad p = P + P \cos 2\omega t - Q \sin 2\omega t; \quad \frac{dp}{dt} = -2\omega P \sin 2\omega t - 2\omega Q \cos 2\omega t$$

$$\frac{dp}{dt} = 0 \quad \text{when} \quad -2\omega P \sin 2\omega t = 2\omega Q \cos 2\omega t \quad \text{or} \quad \tan 2\omega t = -\frac{Q}{P}$$



$$\cos 2\omega t = \frac{P}{\sqrt{P^2 + Q^2}}; \quad \sin 2\omega t = -\frac{Q}{\sqrt{P^2 + Q^2}}$$

Let  $\theta = \tan^{-1}(-Q/P)$ , then  $p$  is maximum when  $2\omega t = \theta$  and  $p$  is minimum when  $2\omega t = (\theta + \pi)$ .

$$\text{Therefore} \quad p_{\max} = P + P \cdot \frac{P}{\sqrt{P^2 + Q^2}} - \frac{Q(-Q)}{\sqrt{P^2 + Q^2}} = P + \sqrt{P^2 + Q^2}$$

$$\text{and} \quad p_{\min} = P - P \cdot \frac{P}{\sqrt{P^2 + Q^2}} - Q \cdot \frac{Q}{\sqrt{P^2 + Q^2}} = P - \sqrt{P^2 + Q^2}$$

$$\text{P 7.62} \quad W_{\text{dc}} = \frac{V_{\text{dc}}^2}{R} T; \quad W_s = \int_{t_o}^{t_o+T} \frac{v_s^2}{R} dt$$

$$\therefore \quad \frac{V_{\text{dc}}^2}{R} T = \int_{t_o}^{t_o+T} \frac{v_s^2}{R} dt$$

$$V_{\text{dc}}^2 = \frac{1}{T} \int_{t_o}^{t_o+T} v_s^2 dt$$

$$V_{\text{dc}} = \sqrt{\frac{1}{T} \int_{t_o}^{t_o+T} v_s^2 dt} = V_{\text{rms}} = V_{\text{eff}}$$

P 7.63 [a] Area under one cycle of  $v_g^2$ :

$$\begin{aligned} A &= (400)(4)(20 \times 10^{-6}) + 10,000(2)(20 \times 10^{-6}) \\ &= 21,600(20 \times 10^{-6}) \end{aligned}$$

Mean value of  $v_g^2$ :

$$\text{M.V.} = \frac{A}{120 \times 10^{-6}} = \frac{21,600(20 \times 10^{-6})}{120 \times 10^{-6}} = 3600$$

$$\therefore V_{\text{rms}} = \sqrt{3600} = 60 \text{ V(rms)}$$

$$\text{[b]} P = \frac{V_{\text{rms}}^2}{R} = \frac{3600}{12} = 300 \text{ W}$$

~~P 7.64~~ 63  $i(t) = \frac{30}{40} \times 10^3 t = 750t \quad 0 \leq t \leq 40 \text{ ms}$

$$i(t) = M - \frac{30}{10} \times 10^3 t \quad 40 \text{ ms} \leq t \leq 50 \text{ ms}$$

$$i(t) = 0 \text{ when } t = 50 \text{ ms}$$

$$\therefore M = 3000(50 \times 10^{-3}) = 150$$

$$i(t) = 150 - 3000t \quad 40 \text{ ms} \leq t \leq 50 \text{ ms}$$

$$\therefore I_{\text{rms}} = \sqrt{\frac{1000}{50} \left\{ \int_0^{0.04} (750)^2 t^2 dt + \int_{0.04}^{0.05} (150 - 3000t)^2 dt \right\}}$$

$$\int_0^{0.04} (750)^2 t^2 dt = (750)^2 \frac{t^3}{3} \Big|_0^{0.04} = 12$$

$$(150 - 3000t)^2 = 22,500 - 9 \times 10^5 t + 9 \times 10^6 t^2$$

$$\int_{0.04}^{0.05} 22,500 dt = 225$$

$$\int_{0.04}^{0.05} 9 \times 10^5 t dt = 45 \times 10^4 t^2 \Big|_{0.04}^{0.05} = 405$$

$$9 \times 10^6 \int_{0.04}^{0.05} t^2 dt = 3 \times 10^6 t^3 \Big|_{0.04}^{0.05} = 183$$

$$\therefore I_{\text{rms}} = \sqrt{20\{12 + (225 - 405 + 183)\}} = \sqrt{300} = 17.32 \text{ A}$$

$$\text{P 7.65} \quad P = I_{\text{rms}}^2 R \quad \therefore \quad R = \frac{24 \times 10^3}{300} = 80 \, \Omega$$

$$\text{P 7.66} \quad \frac{1}{\omega C} = \frac{10^9}{(5000)(80)} = 2500 \, \Omega$$

$$Z_f = \frac{-j2500(7500)}{7500 - j2500} = 750 - j2250 \, \Omega$$

$$Z_i = 1500 \, \Omega$$

$$\therefore \quad \frac{Z_f}{Z_i} = \frac{750 - j2250}{1500} = 0.5 - j1.5$$

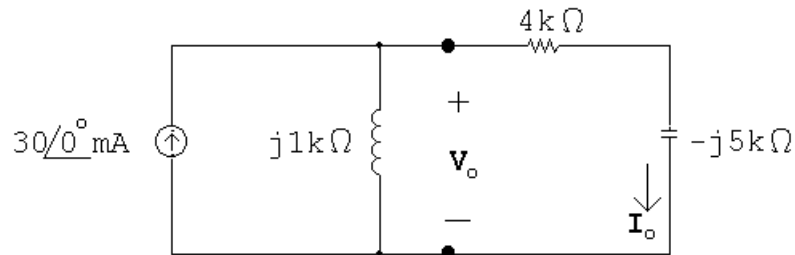
$$\mathbf{V}_o = -\frac{Z_f}{Z_i} \mathbf{V}_g; \quad \mathbf{V}_g = 4\angle 0^\circ \text{ V}$$

$$\mathbf{V}_o = (-0.5 + j1.5)(4) = -2 + j6 = 6.32\angle 108.43^\circ \text{ V}$$

$$P = \frac{1}{2} \frac{V_m^2}{R} = \frac{1}{2} \frac{(4)(10)}{1000} = 20 \times 10^{-3} = 20 \text{ mW}$$

~~P 7.67~~ 65  $\mathbf{I}_g = 30\angle 0^\circ \text{ mA}$

$$j\omega L = j(100)(10) = j1000 \, \Omega; \quad \frac{1}{j\omega C} = \frac{10^6}{j(100)(2)} = -j5000 \, \Omega$$



$$\mathbf{I}_o = \frac{30\angle 0^\circ (j1000)}{4000 - j4000} = 3.75\sqrt{2}\angle 135^\circ \text{ mA}$$

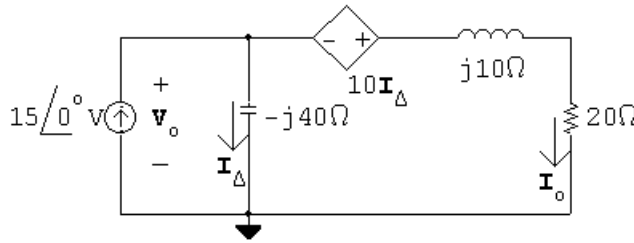
$$P = |\mathbf{I}_o|_{\text{rms}}^2 (4000) = (3.75)^2 (4000) = 56.25 \text{ mW}$$

$$Q = |\mathbf{I}_o|_{\text{rms}}^2 (-5000) = -70.3125 \text{ mVAR}$$

$$S = P + jQ = 56.25 - j70.3125 \text{ mVA}$$

$$|S| = 90.044 \text{ mVA}$$

P 7.68 <sup>66</sup>  $j\omega L = j10,000(10^{-3}) = j10\ \Omega$ ;  $\frac{1}{j\omega C} = \frac{10^6}{j10,000(2.5)} = -j40\ \Omega$



$$-15 + \frac{\mathbf{V}_o}{-j40} + \frac{\mathbf{V}_o + 10(\mathbf{V}_o / -j40)}{20 + j10} = 0$$

$$\therefore \mathbf{V}_o \left[ \frac{1}{-j40} + \frac{1 + j0.25}{20 + j10} \right] = 15$$

$$\therefore \mathbf{V}_o = 300 - j100\ \text{V}$$

$$\therefore \mathbf{I}_\Delta = \frac{\mathbf{V}_o}{-j40} = 2.5 + j7.5\ \text{A}$$

$$\mathbf{I}_o = 15\angle 0^\circ - \mathbf{I}_\Delta = 15 - 2.5 - j7.5 = 12.5 - j7.5 = 14.58\angle -30.9^\circ\ \text{A}$$

$$P_{20\Omega} = \frac{1}{2} |\mathbf{I}_o|^2 20 = 2125\ \text{W}$$

P 7.69 [a]  $Z_1 = 240 + j70 = 250\angle 16.26^\circ\ \Omega$

$$\text{pf} = \cos(16.26^\circ) = 0.96\ \text{lagging}$$

$$\text{rf} = \sin(16.26^\circ) = 0.28$$

$$Z_2 = 160 - j120 = 200\angle -36.87^\circ\ \Omega$$

$$\text{pf} = \cos(-36.87^\circ) = 0.80\ \text{leading}$$

$$\text{rf} = \sin(-36.87^\circ) = -0.60$$

$$Z_3 = 30 - j40 = 50\angle -53.13^\circ\ \Omega$$

$$\text{pf} = \cos(-53.13^\circ) = 0.6\ \text{leading}$$

$$\text{rf} = \sin(-53.13^\circ) = -0.8$$

[b]  $Y = Y_1 + Y_2 + Y_3$

$$Y_1 = \frac{1}{250 \angle 16.26^\circ}; \quad Y_2 = \frac{1}{200 \angle -36.87^\circ}; \quad Y_3 = \frac{1}{50 \angle -53.13^\circ}$$

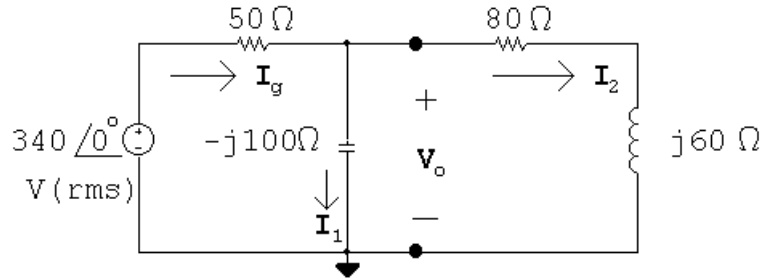
$$Y = 19.84 + j17.88 \text{ mS}$$

$$Z = \frac{1}{Y} = 37.44 \angle -42.03^\circ \Omega$$

$$\text{pf} = \cos(-42.03^\circ) = 0.74 \text{ leading}$$

$$\text{rf} = \sin(-42.03^\circ) = -0.67$$

P 7.70 [a]



$$\frac{\mathbf{V}_o}{-j100} + \frac{\mathbf{V}_o - 340}{50} + \frac{\mathbf{V}_o}{80 + j60} = 0$$

$$\therefore \mathbf{V}_o = 238 - j34 \text{ V}$$

$$\mathbf{I}_g = \frac{340 - 238 + j34}{50} = 2.04 + j0.68 \text{ A}$$

$$\begin{aligned} S_g &= \mathbf{V}_g \mathbf{I}_g^* = (340)(2.04 - j0.68) \\ &= 693.6 - j231.2 \text{ VA} \end{aligned}$$

[b] Source is delivering 693.6 W.

[c] Source is absorbing 231.2 magnetizing VAR.

[d]  $\mathbf{I}_1 = \frac{\mathbf{V}_o}{-j100} = 0.34 + j2.38 \text{ A}$

$$\begin{aligned} S_1 &= \mathbf{V}_o \mathbf{I}_1^* = (238 - j34)(0.34 - j2.38) \\ &= 0 - j578 \text{ VA} \end{aligned}$$

$$\mathbf{I}_2 = \frac{\mathbf{V}_o}{80 + j60} = \frac{238 - j34}{80 + j60} = 1.7 - j1.7 \text{ A}$$

$$\begin{aligned} S_2 &= \mathbf{V}_o \mathbf{I}_2^* = (238 - j34)(1.7 + j1.7) \\ &= 462.4 + j346.8 \text{ VA} \end{aligned}$$

$$S_{50\Omega} = |\mathbf{I}_g|^2(50) + j0 = (2.15)^2(50) = 231.2 \text{ W}$$

$$[\text{e}] \quad \sum P_{\text{del}} = 693.6 \text{ W}$$

$$\sum P_{\text{diss}} = 462.4 + 231.2 = 693.6 \text{ W}$$

$$\therefore \sum P_{\text{del}} = \sum P_{\text{diss}} = 693.6 \text{ W}$$

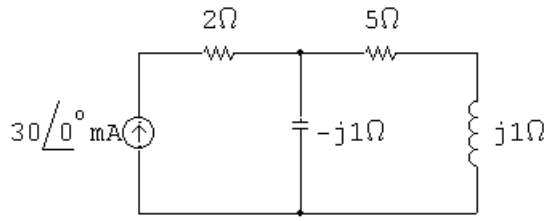
$$[\text{f}] \quad \sum Q_{\text{abs}} = 231.2 + 346.8 = 578 \text{ VAR}$$

$$\sum Q_{\text{dev}} = 578 \text{ VAR}$$

$$\therefore \sum \text{mag VAR dev} = \sum \text{mag VAR abs} = 578$$

$$\text{P 7.71} \quad \mathbf{I}_g = 30 \angle 0^\circ \text{ mA}; \quad \frac{1}{j\omega C} = \frac{10^6}{j(25 \times 10^3)(40)} = -j1 \Omega$$

$$j\omega L = j(25 \times 10^3)(40) \times 10^{-6} = j1 \Omega$$



$$Z_1 = j1 \parallel (5 + j1) = 0.2 - j1 \Omega$$

$$Z_{\text{eq}} = 2 + Z_1 = 2.2 - j1 \Omega$$

$$P_g = |I_{\text{rms}}|^2 \text{Re}\{Z_{\text{eq}}\} = \left(\frac{30}{\sqrt{2}} \times 10^{-3}\right)^2 (2.2) = 990 \mu\text{W}$$

$$\text{P 7.72} \quad [\text{a}] \quad P = \frac{1}{2} \frac{(240)^2}{480} = 60 \text{ W}$$

$$-\frac{1}{\omega C} = \frac{-9 \times 10^6}{(5000)(5)} = -360 \Omega$$

$$Q = \frac{1}{2} \frac{(240)^2}{(-360)} = -80 \text{ VAR}$$

$$p_{\text{max}} = P + \sqrt{P^2 + Q^2} = 60 + \sqrt{(60)^2 + (80)^2} = 160 \text{ W (del)}$$

$$[\text{b}] \quad p_{\text{min}} = 60 - 100 = -40 \text{ W (abs)}$$

$$[\text{c}] \quad P = 60 \text{ W}$$

$$[\text{d}] \quad Q = -80 \text{ VAR}$$

[e] generate

[f]  $\text{pf} = \cos(\theta_v - \theta_i)$

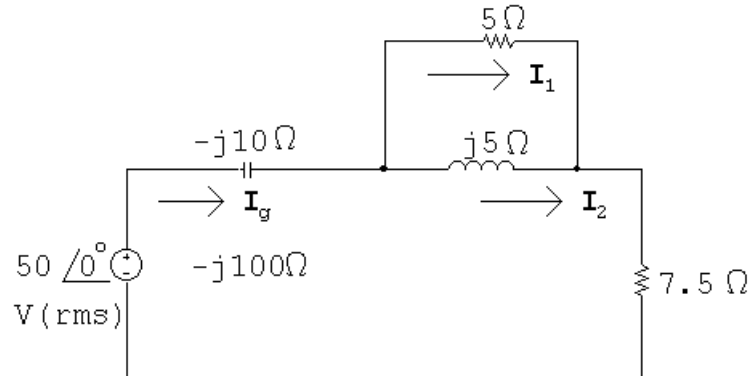
$$\mathbf{I} = \frac{240}{480} + \frac{240}{-j360} = 0.5 + j0.67 = 0.83/\underline{53.13^\circ} \text{ A}$$

$$\therefore \text{pf} = \cos(0 - 53.13^\circ) = 0.6 \text{ leading}$$

[g]  $\text{rf} = \sin(-53.13^\circ) = -0.8$

P 7.73 [a]  $\frac{1}{j\omega C} = \frac{10^6}{j10^5} = -j10 \Omega$

$$j\omega L = j10^5(50 \times 10^{-6}) = j5 \Omega$$



$$Z = -j10 + \frac{(5)(j5)}{5 + j5} + 7.5 = 10 - j7.5 \Omega$$

$$\mathbf{I}_g = \frac{50/\underline{0^\circ}}{10 - j7.5} = 3.2 + j2.4 \text{ A}$$

$$S_g = \frac{1}{2} \mathbf{V}_g \mathbf{I}_g^* = 25(3.2 - j2.4) = 80 - j60 \text{ VA}$$

$$P = 80 \text{ W (del)}; \quad Q = 60 \text{ VAR (abs)}$$

$$|S| = |S_g| = 100 \text{ VA}$$

[b]  $\mathbf{I}_1 = \frac{\mathbf{I}_g(j5)}{5 + j5} = \frac{1}{2}(3.2 + j2.4)(1 + j1) = 0.4 + j2.8 \text{ A}$

$$P_{5\Omega} = \frac{1}{2} |\mathbf{I}_1|^2 (5) = 20 \text{ W}$$

$$P_{7.5\Omega} = \frac{1}{2} |\mathbf{I}_g|^2 (7.5) = 60 \text{ W}$$

$$\sum P_{\text{diss}} = 20 + 60 = 80 \text{ W} = \sum P_{\text{dev}}$$

$$[\text{c}] \quad \mathbf{I}_{j5} = \frac{\mathbf{I}_g 5}{5 + j5} = \frac{1}{2}(3.2 + j2.4)(1 - j1) = 2.8 - j0.4 \text{ A}$$

$$Q_{j5\Omega} = \frac{1}{2}|\mathbf{I}_{j5}|^2(5) = 20 \text{ VAR( abs)}$$

$$Q_{-j10\Omega} = \frac{1}{2}|\mathbf{I}_g|^2(-10) = -80 \text{ VAR(dev)}$$

$$\sum Q_{\text{abs}} = 20 + 60 = 80 \text{ VAR} = \sum Q_{\text{dev}}$$

P 7.74 [a]  $S_1 = 24,960 + j47,040 \text{ VA}$

$$S_2 = \frac{|\mathbf{V}_L|^2}{Z_2^*} = \frac{(480)^2}{5 + j5} = 23,040 - j23,040 \text{ VA}$$

$$S_1 + S_2 = 48,000 + j24,000 \text{ VA}$$

$$480\mathbf{I}_L^* = 48,000 + j24,000; \quad \therefore \quad \mathbf{I}_L = 100 - j50 \text{ A(rms)}$$

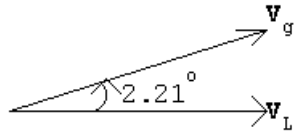
$$\begin{aligned} \mathbf{V}_g &= \mathbf{V}_L + \mathbf{I}_L(0.02 + j0.20) = 480 + (100 - j50)(0.02 + j0.20) \\ &= 492 + j19 = 492.37 \angle 2.21^\circ \text{ Vrms} \end{aligned}$$

$$|\mathbf{V}_g| = 492.37 \text{ Vrms}$$

[b]  $T = \frac{1}{f} = \frac{1}{60} = 16.67 \text{ ms}$

$$\frac{2.21^\circ}{360^\circ} = \frac{t}{16.67 \text{ ms}}; \quad \therefore \quad t = 102.39 \mu\text{s}$$

[c]  $\mathbf{V}_L$  lags  $\mathbf{V}_g$  by  $2.21^\circ$  or  $102.31 \mu\text{s}$



P 7.75 [a]  $S_1 = 18 + j24 \text{ kVA}; \quad S_2 = 36 - j48 \text{ kVA}; \quad S_3 = 18 + j0 \text{ kVA}$

$$S_T = S_1 + S_2 + S_3 = 72 - j24 \text{ kVA}$$

$$2400\mathbf{I}^* = (72 - j24) \times 10^3; \quad \therefore \quad \mathbf{I} = 30 + j10 \text{ A}$$

$$Z = \frac{2400}{30 + j10} = 72 - j24 \Omega = 75.89 \angle -18.43^\circ \Omega$$

[b]  $\text{pf} = \cos(-18.43^\circ) = 0.9487 \text{ leading}$



P 7.76 [a] From the solution to Problem 7.75 we have

$$\mathbf{I}_L = 30 + j10 \text{ A(rms)}$$

$$\begin{aligned}\therefore \mathbf{V}_s &= 2400\angle 0^\circ + (30 + j10)(0.2 + j1.6) = 2390 + j50 \\ &= 2390.52\angle 1.20^\circ \text{ V(rms)}\end{aligned}$$

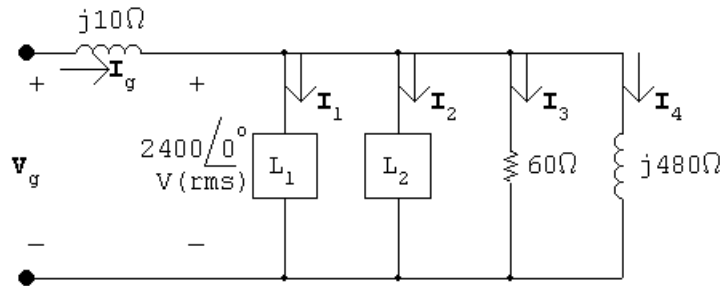
[b]  $|\mathbf{I}_L| = \sqrt{1000}$

$$P_\ell = (1000)(0.2) = 200 \text{ W} \quad Q_\ell = (1000)(1.6) = 1600 \text{ VAR}$$

[c]  $P_s = 72,000 + 200 = 72.2 \text{ kW} \quad Q_s = -24,000 + 1600 = -22.4 \text{ kVAR}$

[d]  $\eta = \frac{72}{72.2}(100) = 99.72\%$

P 7.77



$$2400\mathbf{I}_1^* = 24,000 + j18,000$$

$$\mathbf{I}_1^* = 10 + j7.5; \quad \therefore \mathbf{I}_1 = 10 - j7.5 \text{ A(rms)}$$

$$2400\mathbf{I}_2^* = 48,000 - j30,000$$

$$\mathbf{I}_2^* = 20 - j12.5; \quad \therefore \mathbf{I}_2 = 20 + j12.5 \text{ A(rms)}$$

$$\mathbf{I}_3 = \frac{2400\angle 0^\circ}{60} = 40 + j0 \text{ A}; \quad \mathbf{I}_4 = \frac{2400\angle 0^\circ}{j480} = 0 - j5 \text{ A}$$

$$\mathbf{I}_g = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3 + \mathbf{I}_4 = 70 \text{ A}$$

$$\mathbf{V}_g = 2400 + (70)(j10) = 2400 + j700 = 2500\angle 16.26^\circ \text{ V(rms)}$$

P 7.78  $S_T = 52,800 - j\frac{52,800}{0.8}(0.6) = 52,800 - j39,600 \text{ VA}$

$$S_1 = 40,000(0.96 + j0.28) = 38,400 + j11,200 \text{ VA}$$

$$S_2 = S_T - S_1 = 14,400 - j50,800 = 52,801.52\angle -74.17^\circ \text{ VA}$$

$$\text{pf} = \cos(-74.17^\circ) = 0.2727 \text{ leading}$$

P 7.79 [a]  $\mathbf{I} = \frac{7200 \angle 0^\circ}{140 + j480} = 14.4 \angle -73.74^\circ \text{ A (rms)}$

$$P = (14.4)^2(2) = 414.72 \text{ W}$$

[b]  $Y_L = \frac{1}{138 + j460} = \frac{138 - j460}{230,644}$

$$\therefore -j\omega C = -j \frac{460}{230,644} \quad \therefore X_C = \frac{-230,644}{460} = -501.40 \Omega$$

[c]  $Z_L = \frac{230,644}{138} = 1671.33 \Omega$

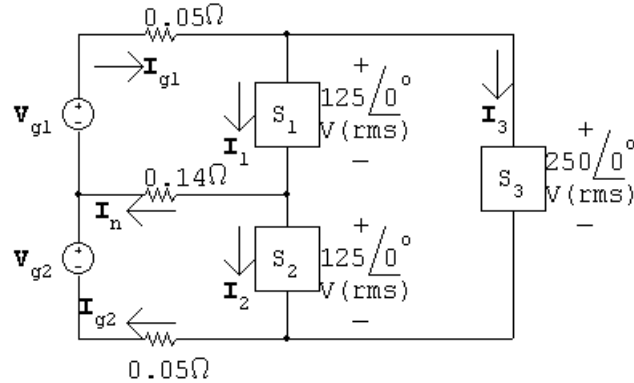
[d]  $\mathbf{I} = \frac{7200}{1673.33 + j20} = 4.30 \angle -0.68^\circ \text{ A}$

$$P = (4.30)^2(2) = 37.02 \text{ W}$$

[e]  $\% = \frac{37.02}{414.72}(100) = 8.93\%$

Thus the power loss after the capacitor is added is 8.93% of the power loss before the capacitor is added.

P 7.80 [a]



$$\mathbf{I}_1 = \frac{5000 - j2000}{125} = 40 - j16 \text{ A (rms)}$$

$$\mathbf{I}_2 = \frac{3750 - j1500}{125} = 30 - j12 \text{ A (rms)}$$

$$\mathbf{I}_3 = \frac{8000 + j0}{250} = 32 + j0 \text{ A (rms)}$$

$$\therefore \mathbf{I}_{g1} = 72 - j16 \text{ A (rms)}$$

$$\mathbf{I}_n = \mathbf{I}_1 - \mathbf{I}_2 = 10 - j4 \text{ A (rms)}$$

$$\mathbf{I}_{g2} = 62 - j12 \text{ A}$$

$$\mathbf{V}_{g1} = 0.05\mathbf{I}_{g1} + 125 + j0 + 0.14\mathbf{I}_n = 130 - j1.36 \text{ V (rms)}$$

$$\mathbf{V}_{g2} = -0.14\mathbf{I}_n + 125 + j0 + 0.05\mathbf{I}_{g2} = 126.7 - j0.04 \text{ V(rms)}$$

$$S_{g1} = [(130 - j1.36)(72 + j16)] = [9381.76 + j1982.08] \text{ VA}$$

$$S_{g2} = [(126.7 - j0.04)(62 + j12)] = [7855.88 + j1517.92] \text{ VA}$$

Note: Both sources are delivering average power and magnetizing VAR to the circuit.

[b]  $P_{0.05} = |\mathbf{I}_{g1}|^2(0.05) = 272 \text{ W}$

$$P_{0.14} = |\mathbf{I}_n|^2(0.14) = 16.24 \text{ W}$$

$$P_{0.05} = |\mathbf{I}_{g2}|^2(0.05) = 199.4 \text{ W}$$

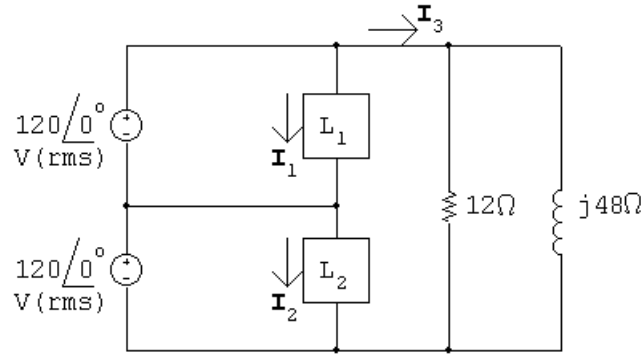
$$\sum P_{\text{dis}} = 272 + 16.24 + 199.4 + 5000 + 3750 + 8000 = 17,237.64 \text{ W}$$

$$\sum P_{\text{dev}} = 9381.76 + 7855.88 = 17,237.64 \text{ W} = \sum P_{\text{dis}}$$

$$\sum Q_{\text{abs}} = 2000 + 1500 = 2500 \text{ VAR}$$

$$\sum Q_{\text{del}} = 1982.08 + 1517.92 = 3500 \text{ VAR} = \sum Q_{\text{abs}}$$

P 7.81 [a]



$$120\mathbf{I}_1^* = 1800 + j600; \quad \therefore \mathbf{I}_1 = 15 - j5 \text{ A(rms)}$$

$$120\mathbf{I}_2^* = 1200 - j900; \quad \therefore \mathbf{I}_2 = 10 + j7.5 \text{ A(rms)}$$

$$\mathbf{I}_3 = \frac{240}{12} + \frac{240}{j48} = 20 - j5 \text{ A(rms)}$$

$$\mathbf{I}_{g1} = \mathbf{I}_1 + \mathbf{I}_3 = 35 - j10 \text{ A}$$

$$S_{g1} = 120(35 + j10) = 4200 + j1200 \text{ VA}$$

Thus the  $\mathbf{V}_{g1}$  source is delivering 4200 W and 1200 magnetizing vars.

$$\mathbf{I}_{g2} = \mathbf{I}_2 + \mathbf{I}_3 = 30 + j2.5 \text{ A(rms)}$$

$$S_{g2} = 120(30 - j2.5) = 3600 - j300 \text{ VA}$$

Thus the  $\mathbf{V}_{g2}$  source is delivering 3600 W and absorbing 300 magnetizing vars.

$$[b] \sum P_{\text{gen}} = 4200 + 3600 = 7800 \text{ W}$$

$$\sum P_{\text{abs}} = 1800 + 1200 + \frac{(240)^2}{12} = 7800 \text{ W} = \sum P_{\text{gen}}$$

$$\sum Q_{\text{del}} = 1200 + 900 = 2100 \text{ VAR}$$

$$\sum Q_{\text{abs}} = 300 + 600 + \frac{(240)^2}{48} = 2100 \text{ VAR} = \sum Q_{\text{del}}$$

P 7.82 [a]  $S_L = 24 + j7 \text{ kVA}$

$$125\mathbf{I}_L^* = (24 + j7) \times 10^3; \quad \mathbf{I}_L^* = 192 + j56 \text{ A(rms)}$$

$$\therefore \mathbf{I}_L = 192 - j56 \text{ A(rms)}$$

$$\begin{aligned} \mathbf{V}_s &= 125 + (192 - j56)(0.006 + j0.048) = 128.84 + j8.88 \\ &= 129.15 \angle 3.94^\circ \text{ V(rms)} \end{aligned}$$

$$|\mathbf{V}_s| = 129.15 \text{ V(rms)}$$

$$[b] P_\ell = |\mathbf{I}_\ell|^2(0.006) = (200)^2(0.006) = 240 \text{ W}$$

$$[c] \frac{(125)^2}{X_C} = -7000; \quad X_C = -2.23 \Omega$$

$$-\frac{1}{\omega C} = -2.23; \quad C = \frac{1}{(2.23)(120\pi)} = 1188.36 \mu\text{F}$$

$$[d] \mathbf{I}_\ell = 192 + j0 \text{ A(rms)}$$

$$\begin{aligned} \mathbf{V}_s &= 125 + 192(0.006 + j0.048) = 126.152 + j9.216 \\ &= 126.49 \angle 4.18^\circ \text{ V(rms)} \end{aligned}$$

$$|\mathbf{V}_s| = 126.49 \text{ V(rms)}$$

$$[e] P_\ell = (192)^2(0.006) = 221.184 \text{ W}$$

P 7.83 [a]  $\Delta = R_a R_b R_c - R_1^2 R_b - R_2^2 R_a - R_n(2R_1 R_2 + R_n R_c)$

$$R_a = R_1 + R_n + R_l = 30 + 1 + 0.5 = 31.5 \Omega$$

$$R_b = R_2 + R_n + R_l = 300 + 1 + 0.5 = 301.5 \Omega$$

$$R_c = R_1 + R_2 + R_3 = 30 + 300 + 15 = 345 \Omega$$

$$\begin{aligned} \Delta &= (31.5)(301.5)(345) - 900(301.5) - 9 \times 10^4(31.5) \\ &\quad - 1[2(30)(300) + 1(345)] \\ &= 151,856.25 \end{aligned}$$

$$\begin{aligned}
N_a &= \mathbf{V}_{g1}[(R_b R_c - R_2^2) + R_n R_c + R_1 R_2] \\
&= 120[(301.5)(345) - 9 \times 10^4 + 345 + 30(300)] \\
&= 2,803,500
\end{aligned}$$

$$\begin{aligned}
N_b &= \mathbf{V}_{g1}[R_n R_c + R_1 R_2 + R_a R_c - R_1^2] \\
&= 120[345 + (30)(300) + 31.5(345) - 900] \\
&= 2,317,500
\end{aligned}$$

$$\mathbf{I}_a = \frac{N_a}{\Delta}; \quad \mathbf{I}_b = \frac{N_b}{\Delta}$$

$$\mathbf{I}_n = \mathbf{I}_a - \mathbf{I}_b = \frac{N_a - N_b}{\Delta} = 3.2/\underline{0^\circ} \text{A(rms)}$$

[b]

$$\begin{aligned}
N_c &= \mathbf{V}_{g1}[R_2 R_n + R_1 R_b + R_2 R_a + R_1 R_n] \\
&= 120[300 + 30(301.5) + 31.5(300) + 30] \\
&= 2,259,000
\end{aligned}$$

$$\mathbf{I}_{L1} = \frac{N_a - N_c}{\Delta}$$

$$\mathbf{V}_1 = 30\mathbf{I}_{L1} = \frac{30(N_a - N_c)}{\Delta} = 107.57/\underline{0^\circ} \text{V(rms)}$$

[c]  $\mathbf{I}_{L2} = \frac{N_b - N_c}{\Delta}$

$$\mathbf{V}_2 = 300\mathbf{I}_{L2} = \frac{300(N_b - N_c)}{\Delta} = 115.57/\underline{0^\circ} \text{V(rms)}$$

[d]  $\mathbf{V}_3 = 15\mathbf{I}_c = 15\frac{N_c}{\Delta} = 233.14/\underline{0^\circ} \text{V(rms)}$

CHECK:

$$\mathbf{V}_3 = \mathbf{V}_1 + \mathbf{V}_2 = 107.57/\underline{0^\circ} + 115.57/\underline{0^\circ} = 233.14/\underline{0^\circ} \text{V(rms)}$$

[e]  $P_1 = \frac{|\mathbf{V}_1|^2}{R_1} = \frac{(107.57)^2}{30} = 385.70 \text{ W}$

$$P_2 = \frac{|\mathbf{V}_2|^2}{R_2} = \frac{(115.57)^2}{300} = 44.52 \text{ W}$$

$$P_3 = \frac{|\mathbf{V}_3|^2}{R_3} = \frac{(233.14)^2}{15} = 3319.39 \text{ W}$$

$$[\text{f}] \quad \mathbf{I}_a = \frac{N_a}{\Delta} = 18.46 \angle 0^\circ \text{ A (rms)}$$

$$\mathbf{I}_b = \frac{N_b}{\Delta} = 15.26 \angle 0^\circ \text{ A (rms)}$$

$$P_a = (120)(18.46) \cos 0^\circ$$

$$P_b = (120)(15.26) \cos 0^\circ$$

$$\sum P_{\text{gen}} = 120(18.46 + 15.26) = 4046.72 \text{ W}$$

$$[\text{g}] \quad P_a = |\mathbf{I}_a|^2(0.5) = 170.41 \text{ W}$$

$$P_n = |\mathbf{I}_n|^2(1) = 10.24 \text{ W}$$

$$P_b = |\mathbf{I}_b|^2(0.5) = 116.45 \text{ W}$$

$$\sum P_{\text{diss}} = 170.41 + 10.24 + 116.45 + 385.70$$

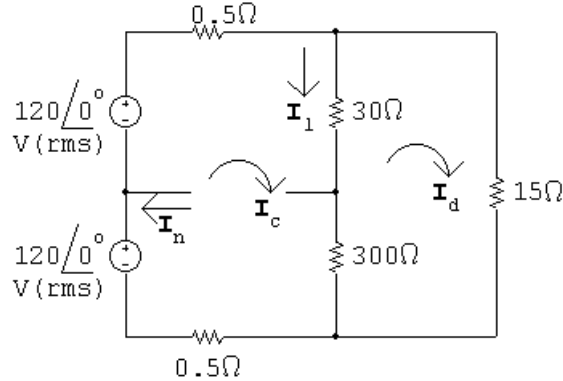
$$+ 44.52 + 3319.39$$

$$= 4046.72 \text{ W}$$

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P 7.84 [a]  $\mathbf{I}_n = 0$  by hypothesis.

[b] With the neutral conductor open the circuit becomes:



The two mesh current equations are

$$240 \angle 0^\circ = 331 \mathbf{I}_c - 330 \mathbf{I}_d$$

$$0 = -330 \mathbf{I}_c + 345 \mathbf{I}_d$$

$$\therefore \mathbf{I}_c = \frac{82,800}{5295} + 15.64 \angle 0^\circ \text{ A (rms)}$$

$$\mathbf{I}_d = \frac{79,200}{5295} + 14.96 \angle 0^\circ \text{ A (rms)}$$

$$\mathbf{I}_1 = \mathbf{I}_c - \mathbf{I}_d = 0.68 \angle 0^\circ \text{ A (rms)}$$

$$\mathbf{V}_1 = 30 \mathbf{I}_1 = 20.40 \angle 0^\circ \text{ V (rms)}$$

$$[\text{c}] \quad \mathbf{V}_2 = 300\mathbf{I}_1 = 203.97\angle 0^\circ \text{V(rms)}$$

$$[\text{d}] \quad \mathbf{V}_3 = 15\mathbf{I}_d = 224.36\angle 0^\circ \text{V(rms)}$$

$$[\text{e}] \quad P_{R_1} = (20.40)^2/30 = 13.87 \text{ W}$$

$$P_{R_2} = (203.97)^2/300 = 138.67 \text{ W}$$

$$P_{R_3} = (224.36)^2/15 = 3355.91 \text{ W}$$

$$[\text{f}] \quad \sum P_{\text{gen}} = 240|\mathbf{I}_c| \cos 0^\circ = (240)(15.64)(1) = 3752.97 \text{ W}$$

$$[\text{g}] \quad \sum P_{\text{diss}} = (15.64)^2(1) + 13.87 + 138.67 + 3355.91 = 3752.97 \text{ W}$$

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