Do NOT write your name on this test.

Author code: ◯◯◯◯◯

Part 0. Instructions

0.1. This test is in-class on Monday February 13, 2016. It is intended to take 55 minutes. You must hand it in after 60 minutes.

0.2. Calculators and computers are not allowed.

0.3. Network-connected devices are not allowed.

0.4. You may use one sheet (8.5×11 US Letter, both sides) of notes.

0.5. Space provided should be adequate for complete answers with average handwriting. But, if you don't have enough space, feel free to write on the blank pages. If you do so, please mark the work clearly, so that the grader can find it without difficulty.

0.6. Other students may be taking the test at other times. For the sake of fairness and integrity, do **not** communicate with other students about the contents, difficulty, emphasis, or style of the test. This embargo is in place until 5pm on Tuesday Feb 14.

0.7. This test is worth 20 points, which are evenly-distributed across the pages.

0.8. For full credit, show your thought process clearly, but also keep your explanations as concise as possible. A number in a box with no explanation yields no credit!

0.9. Don't worry too much about simplification. In some cases, "$4 \cdot 3 \cdot 2 \cdot 1$" is more clear than "24" if it helps the reader follow your work.

0.10. Within each Part, symbols (like A or $f(x)$) mean the same thing. However, unless explicitly noted, symbols mean something completely different in different Parts.

0.11. Be polite to your referee — Make your work clear and easy to understand.

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/5	/5	/5	/5	/20

Referee: ☐☐☐☐

Part 1. Perform row-reduction to solve this system. (Hint! The final solution is $(x, y, z) = (4, 6, 8)$. You still need to do the work, though.)

$$\begin{cases} 4x + 24y + 0z = 160 \\ 13x + 47y - 6z = 286 \\ 12x + 78y + 1z = 524 \end{cases}$$

Part 2. Write down all of the 3×4 reduced row-echelon form matrices that have rank 2. (Use $*$ to indicate entries that could be any number.)

Part 3. Suppose that A is a 4×4 matrix for which the following procedure yields I. First, swap ρ_2 and ρ_4. Second, replace ρ_1 with $\rho_1 + 5\rho_3$. Third, multiply ρ_3 by 6, and divide ρ_1 by 5. Finally, replace ρ_4 with $\rho_4 - 3\rho_1$. Recall that these row-operations can be un-done in the correct order, to change I back to A. Find the matrix A.

Part 4. Suppose that A is a 3×3 matrix and that $E_1, E_2, \ldots, E_k$ are elementary matrices such that

$$E_k \cdots E_2 E_1 A = R = \begin{pmatrix} 1 & 0 & \frac{1}{5} \\ 0 & 1 & -3 \\ 0 & 0 & 0 \end{pmatrix}, \quad \text{and} \quad E_k \cdots E_2 E_1 = \begin{pmatrix} 4 & 2 & 8 \\ 8 & 1 & 3 \\ 0 & 1 & 1 \end{pmatrix}.$$

4.1. Find the solution of the system $A\vec{x} = \vec{0}$ in parametric form.

4.2. Write the solution to the inhomogeneous system

$$A \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ -3 \\ 3 \end{pmatrix},$$

or explain precisely why no solution exists.

4.3. Write the solution to the inhomogeneous system

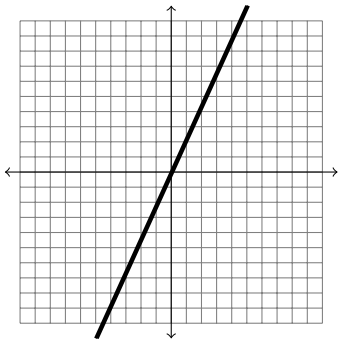
$$A \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 39 \\ -9 \end{pmatrix},$$

or explain precisely why no solution exists.

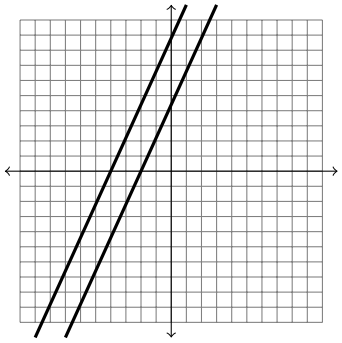
Part 5. The following pictures show two lines, corresponding to a system of two linear equations in two variables, $[A|b]$. For each picture, answer these questions:

- Is the system homogeneous or inhomogeneous?
- How many solutions does the system have?
- What is the rank of the 2×2 matrix A?
- What is the rank of the 2×3 augmented matrix $[A|b]$?

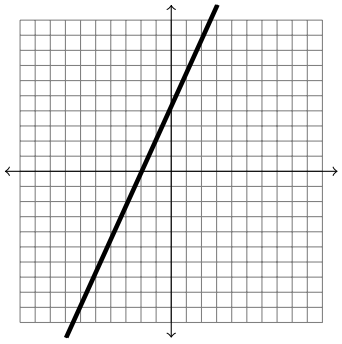
5.1.



5.2.



5.3.



5.4.