

Indian Institute of Information Technology Ranchi

Department of Mathematics

B. Tech End Semester Examination – Spring Semester 2022-23

Semester: IV

Course Code: MA-2002

Course Instructor: Dr. Rishikesh Dutta Tiwary

Course Name: Combinatorics and Graph Theory

Duration: 3 Hrs.

QUESTION PAPER

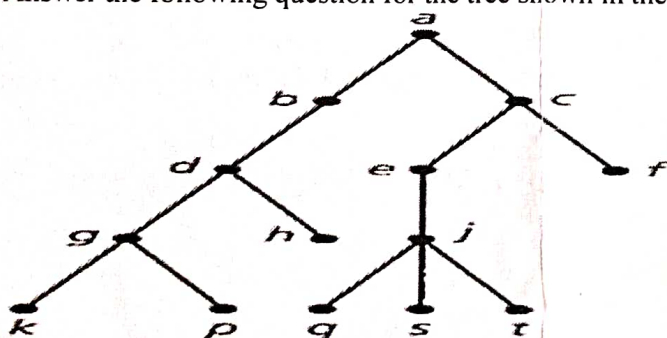
Max Marks: 100

Instructions:

- (1). Number in [] indicates marks.
- (2). Any missing data can be assumed suitably.
- (3). Symbols have their usual meaning.
- (4). Non-programmable scientific calculator is allowed.

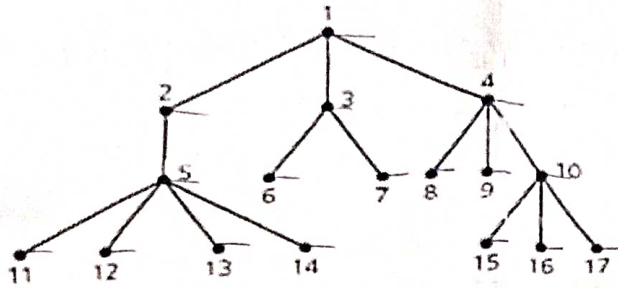
Section A: Answer all the questions.

- 1 (a) Obtain and simplify a generating function for the number of submultisets of $\{\infty . a, \infty . b, \infty . c\}$ in which there are an odd number of a's, an even number of b's, and any number of c's. [5]
- (b) For any undirected graph or multigraph, the number of vertices of odd degree must be even. [5]
- (c) Solve the recurrence relation $2a_n - 7a_{n-1} - 3a_{n-2} = 0$; $a_0 = 2, a_1 = 5$. [5]
- (d) Find a_{12} if $a_{n+1}^2 = 5a_n^2$, where $a_n > 0$ for $n \geq 0$ and $a_0 = 2$. [5]
- 2 (a) If $G = (V, E)$ is an undirected graph, then G is connected if and only if G has a spanning tree. [10]
- (b) In how many ways can a police captain distribute 24 rifles shells to four police officers so that each officer gets atleast three shells but not more than eight? [10]
- 3 (a) Answer the following question for the tree shown in the figure [10]



- i) Which vertices are the leaves?
- ii) Which vertex is the root?
- iii) Which vertex is the parent of g?
- iv) Which vertices are the descendants of c?
- v) Which vertices are the siblings of s?
- vi) What is the level number of vertex f?
- vii) Which vertices have level number 4?

- (b) Explain in detail and list the vertices in the tree shown in the figure when they are visited in a preorder traversal and in a postorder traversal. [10]



Section B: Answer any two questions

- 4 (a) Seven towns A, B, C, D, E, F and G are connected by a system of highways as follows: I-22 goes from A to C, passing through B; I-33 goes from C to D and then passes through B as it continues to F; I-44 goes from D through E to A; I-55 goes from F to B, passing through G; and I-66 goes from G to D. Using vertices for towns and directed edges for segments of highways between towns, draw a directed graph that models this situation. List the paths from G to A. What is the smallest number of highway segments that would have to be closed down in order for travel from B to D to be disrupted? Is it possible to leave town C and return there, visiting each of the other towns only once? [10]

- (b) Determine whether or not the loop-free undirected graphs with the following incidence matrices are isomorphic. [10]

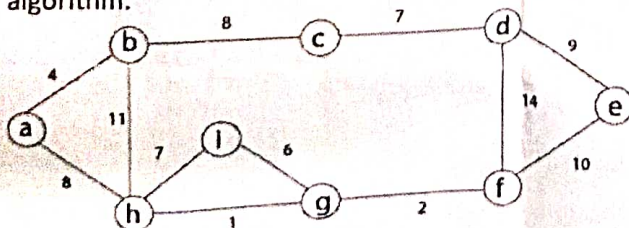
a) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

b) $\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$

- 5 (a) Schedule the final exams for Math 115, Math 116, Math 185, Math 195, CS 101, CS 102, CS 273, and CS 473, using the fewest number of different time slots, if there are no students taking both Math 115 and CS 473, both Math 116 and CS 473, both Math 195 and CS 101, both Math 195 and CS 102, both Math 115 and Math 116, both Math 115 and Math 185, and both Math 185 and Math 195, but there are students in every other pair of courses. Model this problem as a graph-coloring problem, and find the minimum number of time slots. [10]

- (b) Solve the recurrence relation $a_r - 2a_{r-1} - 3a_{r-2} = 0$; $a_0 = 3, a_1 = 1$, by using generating functions. [10]

- 6 (a) Find the Minimum Spanning Tree of the following graph using Kruskal's and Prim's algorithm. [10]



- (b) Prove that if G is a connected graph, then G contains an Euler circuit if and only if every vertex has even degree. [10]

End