Roll No. 2022 UG 3008

Indian Institute of Information Technology Ranchi

Department of Mathematics

B. Tech End Semester Examination: Autumn Semester 2022-23

Semester: 1st

Course Instructor: Dr. Shashi Kant and Dr. Rishikesh Dutta Tiwary

Course Code: MA1001

Course Name: Mathematics-I (Calculus and Differential Equations)

QUESTION PAPER

Duration: 3 hrs.

Max Marks: 100

Instructions:

(1). Number in [ ] indicates marks.
(2). Any missing data can be assumed suitably.
(3). Symbols have their usual meaning.

Section A: Answer all the questions.

1. (a) State the Rolle's theorem. Identify the point(s) at which the tangent is parallel to the x-axis for $f(x) = x^2 - 6x + 5$ in the interval $[1,5]$. [6]

(b) Solve the differential equation $xy'' - y' = x^2 e^x$. [6]

(c) If $u = f(r)$, where $r^2 = x^2 + y^2$, show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r} f'(r)$. [4]

(d) Evaluate $\int_0^\infty e^{-x^2} dx$. [4]

2. (a) Evaluate $\int_0^{4a} \int_{\frac{x^2}{4a}}^{2\sqrt{ax}} dydx$ by changing the order of integration. [6]

(b) Evaluate $\iint_S xy\, dxdy$, where S is the region bounded by the x-axis, ordinate $x=2a$ and the curve $x^2 = 4ay$. [6]

(c) Evaluate $J\left(\frac{x,\ y,\ z}{(r,\ \theta,\ \emptyset)}\right)$, where $x = r\sin\emptyset\cos\theta$, $y = r\sin\emptyset\sin\theta$, $z = r\cos\emptyset$. [4]

(d) Find the Laplace transform of $e^{-2t}t^2 + t\sin 3t$. [4]

3. (a) A fluid motion given by $\vec{F} = (x^2 - y^2 + x)\hat{\imath} - (2xy + y)\hat{\jmath}$ is irrotational? If so, find its scalar potential $\emptyset$. [6]

(b) The temperature of the points in space is given by $T(x,y,z) = x^2 + y^2 - z$. A mosquito located at $(1,1,2)$ desires to fly in such a direction that it will get warm as soon as possible. In what direction should it move? [6]

(c) Curl of any gradient field is zero? Justify your answer. [4]

(d) Find the value of n for which the vector field $r^n\vec{r}$ is solenoidal, where $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$. [4]

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4. (a) Find the inverse Laplace transform of (i) $\log \frac{s+5}{s+3}$ (ii) $\frac{e^{-\pi s}}{(s+5)(s+3)}$ [6]

(b) Solve $y'' + 16y = 16 \tan 4x$, using variation of parameters method. [6]

(c) Find the solution of $3e^x \tan y \, dx + (1 - e^x) \sec^2 y \, dy = 0$ [4]

(d) Solve the first order partial differential equation $(y - z)p + (z - x)q = x - y$. [4]

Section B: Answer only one question.

5. (a) Find the Laplace transform of $\frac{\sin t}{t}$. Hence, evaluate $\int_0^\infty \frac{e^{-t} \sin t}{t} dt$. [6]

(b) Use Green's theorem to evaluate $\int_C (2y^2 dx + 3x dy)$, where, C is the boundary of closed region bounded by the curves $y = x$ and $y = x^2$. [6]

(c) Explain ordinary, regular, and irregular points of a 2nd order linear differential equation. Hence, find the power series solution of $y'' + y = 0$, about $x = 0$. [8]

6. (a) Form the partial differential equations for the relations given by: [6]

(i) $\emptyset(x^2 + y^2 + z^2, \; x + y + z) = 0$ (ii) $z = f(x^2 - y^2)$

(b) Show that two Legendre's polynomials of distinct degrees are orthogonal in [-1, 1]. [6]

(c) Solve the following partial differential equations: [8]

(i) $z = px + qy + p^2 + q^2$ (ii) $(D^3 - D'^3)z = x^3 y^3$

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