Roll No. 2022UG3004

Indian Institute of Information Technology Ranchi

Department of Mathematics

B. Tech Mid Semester Examination – Autumn Semester 2022-23

Semester: I **Branch: CE/CSE/ECE(ES&IoT)/CSE(DS&AI)**

Course Code: MA-1001 **Course Name: Mathematics-I**

QUESTION PAPER

Duration: 2 hrs. **Max Marks: 60**

Instructions:

(1) Answer all the questions. Number in [ ] indicates marks.
(2) Scientific calculator is allowed in the examination.
(3) Any missing data can be assumed suitably.
(4) All symbol have there usual meaning.

1 **(a)** Evaluate $\int_0^1 \frac{x^{m-1}+x^{n-1}}{(1+x)^{m+n}}\,dx$ [5]

(b) Prove that [5]

$$\Gamma\left(\tfrac{1}{4}\right)\Gamma\left(\tfrac{3}{4}\right) = \pi\sqrt{2}$$

(c) Evaluate the integral: $\int_0^{log2}\int_0^{x}\int_0^{x+\log y} e^{x+y+z}dz\,dy\,dx.$ [5]

(d) If $f(x,y) = \tan^{-1}(xy)$, compute an approximate value of $f(0.9,-1.2)$. [5]

2 **(a)** Examine the convergence of the series: $\sum\left(\sqrt[3]{n^3+1}-n\right)$. [10]

(b) If $xyz = 8$, find the value of x,y,z for which $u = \frac{5xyz}{x+2y+4z}$ is maximum. [10]

3 **(a)** Test the convergence of the series $\sum_1^{\infty} \frac{(n+1)^n x^n}{n^{n+1}}$. [10]

(b) If $x = \sqrt{vw}$, $y = \sqrt{wu}$, $z = \sqrt{uv}$ and $u = rsin\theta\cos\phi$, $v = rsin\theta sin\phi$, $w = rcos\theta$. Calculate $\frac{\partial(x,y,z)}{\partial(r,\theta,\phi)}$. [10]

End