Reg. No. ______________

Indian Institute of Information Technology Ranchi

Department of Electronics & Communication Engineering

B. Tech End Semester Examination - Spring Semester 2022-23

Semester: IV

Course Instructor: Dr. Priyabrat Garanayak, Dr. Rajiv Kumar

Course Code: EC-2006/EI-2006

Course Name: Signal and Systems

QUESTION PAPER

Duration: 3 Hrs. **Max Marks:** 100

Instructions:

(1). Number in [ ] indicates marks.
(2). Any missing data can be assumed suitably.
(3). Symbols have their usual meaning.
(4). Non-Programmable Scientific Calculator are allowed

Section A: Answer any ten questions.

1 (a) Consider an aperiodic signal $x(t) = e^{-at}$, $a > 0$, for $t \geq 0$ and zero otherwise. Find the energy and the power of this signal and determine whether the signal is finite energy, finite power, or both [4]

?

(b) Plot the following signal: [4]

$$y(t) = 3r(t+3) - 6r(t+1) + 3r(t) - 3u(t-3)$$

(c) Find the even and odd components of [4]

$$x(t) = e^{-2t}\cos t$$

?

(d) Evaluate the following integral [4]

$$f(t) = \int_0^{10} ((t-1)^3 - 3(t-6) + 1)\delta(t-3)dt$$

(e) Show that the system described by the differential equation [4]

$$\frac{dy(t)}{dt} + 4y(t) + 1 = x(t)$$

is nonlinear.

(f) Check whether the following system is time-invariant: [4]

$(t-1)^3$

$= (t^2+1-2t)(t-1)$

$= t^3 - t^2 + t - 1 - 2t^2 + 2t$

$= t^3 - 3t^2 - 1 + 3t$

$$y(t) = tx(t) + 5$$

(g) Consider the Rectangular Pulse or gate function $x(t) = u(t) - u(t-1)$. Obtain $y(t) = x(t) * x(t)$, that is, the convolution of rectangular pulse with itself. [4]

(h) Find the Fourier transform of following function [4]

$$x(t) = \sin \omega t$$

(i) Find the Fourier transform of $y(t) = u(t) - u(t-1)$ [4]

(j) Discuss the properties of Fourier Transform. [4]

(k) Obtain the Laplace transform of $x(t) = e^{-t}\cos 4t\, u(t)$ [4]

(l) Obtain the inverse Laplace transform of $X(S) = \frac{4}{(s^2+1)(s+3)}$ [4]

Section B: Answer any three questions

?

3 (a) A contineous-time signal is shown in figure 1. Sketch each of the following signal: [10]

$$y_1(t) = x\left(\frac{t}{2} + 1\right)$$

$$y_2(t) = -1 + 2x(t)$$

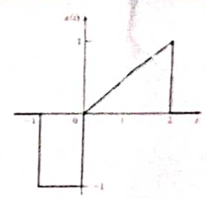

Figure 1: Signal $x(t)$

(b) Express the signals shown in figure 2 in terms of unit step functions. [6]

Calculate its derivative and sketch it.

$\delta[n-1] + \cancel{2\delta[n-2]} + \cancel{3\delta[n-3]} + \delta[n-2] +$
$2\delta[n-3] + 3\delta[n-4] + \delta[n-3]$
$+ 2\delta[n-4] + 3\delta[n-5]$

$= \delta[n-1] + 3\delta[n-2] +$
$6\delta[n-3] + 5\delta[n-4] + 3\delta[n-5]$.

Figure 2: The input signal $x(t)$.

(c) Write down the difference between the following **[4]**
- Continuous-time and Discrete-time Signals
- Causal and Non-Causal Systems

4 **(a)** What is convolution? Explain LTI systems **[2]**

(b) If the durations of $x(t)$ and $h(t)$ are T_1 and T_2, respectively, then the duration of $y(t) = x(t) * h(t)$ is __________ **[2]**

If the areas under $x(t)$ and $h(t)$ are A_1 and A_2, respectively, then the area under $y(t) = x(t) * h(t)$ is __________.

(c) Obtain the convolution of these two signals **[8]**

$$x(t) = 2u(t)$$

$$h(t) = e^{-2t}u(t)$$

(d) Consider $x[n]$ and $h[n]$ as shown in figure 3, respectively. (The signals are all zero outside the ranges indicated.) Evaluate $y[n] = x[n] * h[n]$ **[8]**

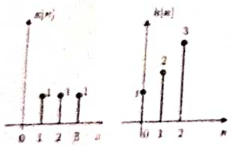

Figure 3: The input signal $x[n]$ and impulse response h[n]

5 **(a)** What is the difference between Fourier series and Fourier transform? **[4]**

(b) Given the periodic train of pulses shown in figure 4, obtain Fourier coefficients a_0, a_n, and b_n. Find the Fourier Series Expansion for the waveform. **[8]**

Figure 4: Periodic train of pulses $x(t)$.

? **(c)** Determine the Fourier transform of the signum function, that is **[8]**

$$x(t) = sgn(t) = \begin{cases} 1 & t > 0 \\ -1 & t < 0 \end{cases}$$

(a) Obtain the Laplace Transform of $x(t) = e^{-10t}u(t)$ and establish the ROC **[4]**

(b) Determine the Laplace transform of $x(t) = t\,u(t-2)$ **[4]**

(c) Find the initial and final values of $X(S) = \frac{4}{S} + \frac{S+8}{S^2+4S+10}$ **[4]**

(d) Consider the system characterized by **[8]**

$$\frac{d^2y}{dt^2} + 3\frac{dy}{dt} - 4y = e^{-t}u(t)$$

Use the Laplace transform to solve the differential equation subject to

$$y(0) = 1, \frac{dy(0)}{dt} = 0$$

End