Bayesian Machine Learning

03/05/21 - François HU

Outline

1 Bayesian statistics
- define a probabilistic model
- apply bayesian inference
- conjugate priors

2 Latent variable models
- define latent variable and apply them to simplify probabilistic model
- cluster data with latent models like GMM
- train probabilistic models with EM-algorithm

3 Variational Inference
- apply variational inference for probabilistic models
- understand variational interpretation of LDA
- application of LDA to text mining

4 Markov Chain Monte Carlo
- train / do inference almost any probabilistic model with MCMC
- pros and cons of MCMC / VI

5 Extensions and oral presentations

PREREQUISITE

THEORY

1. Notions of **probability & statistics**
2. **Statistical Learning :** supervised **&** unsupervised learning
3. **Information theory :** Entropy, KL-divergence, …
4. **Monte Carlo & Markov Chain**

APPLICATION

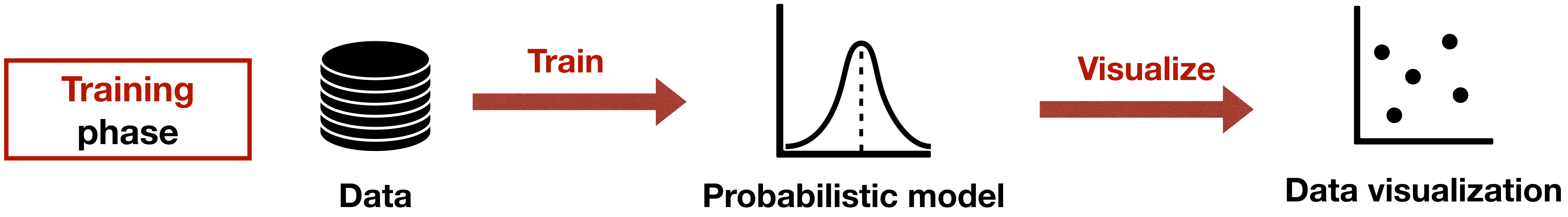
Python (or at least R)

ALGORITHM

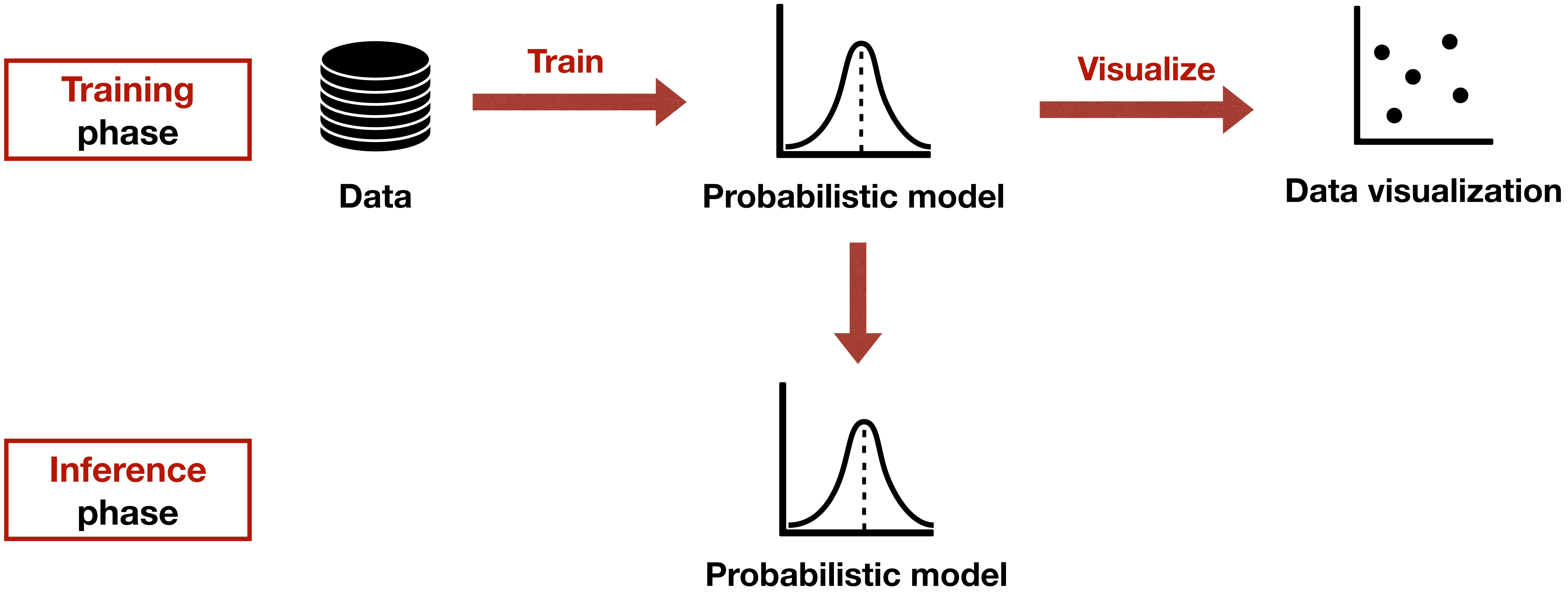
Some « classical » supervised & unsupervised models

0 Gentle introduction to statistical learning

Simplified statistical learning process



Simplified statistical learning process



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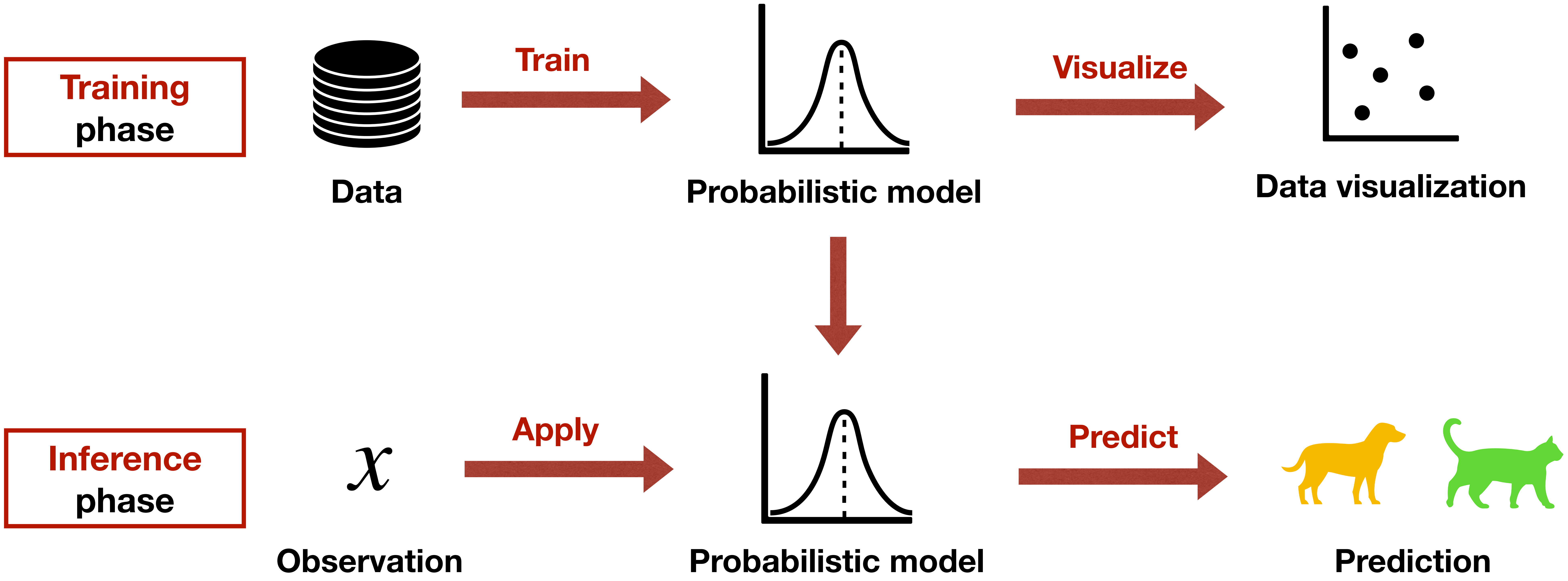
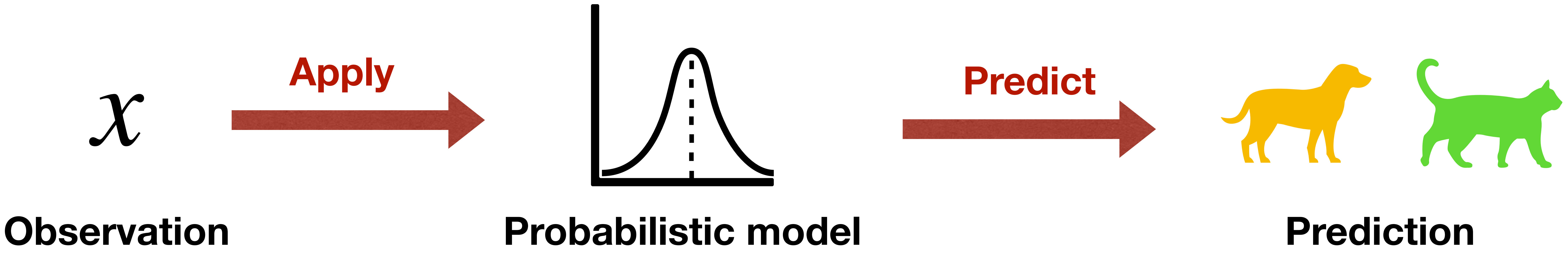


Inference phase



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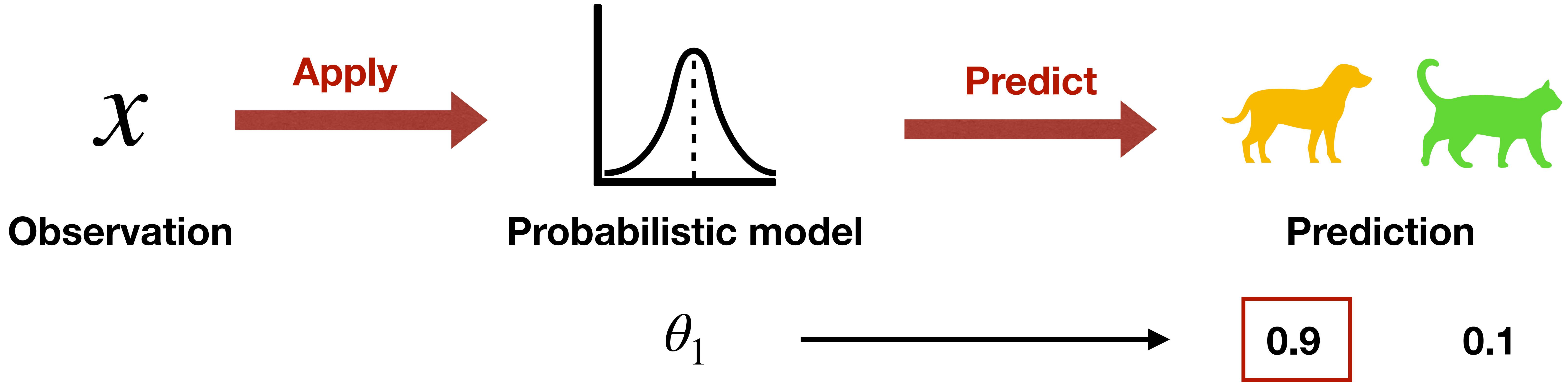

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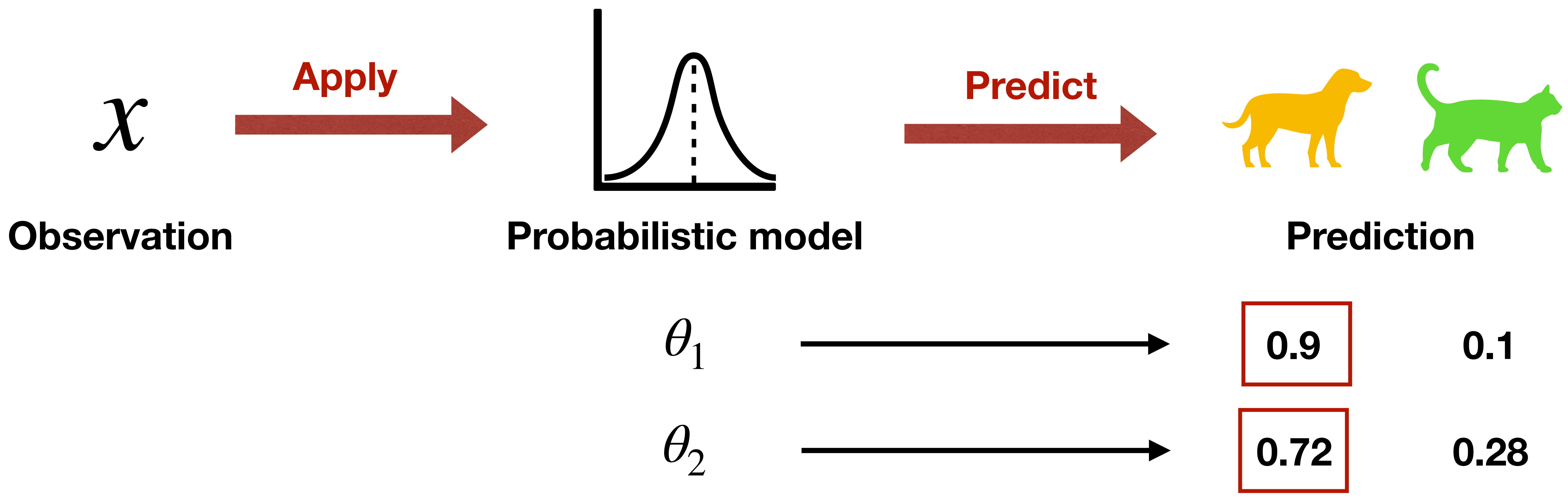

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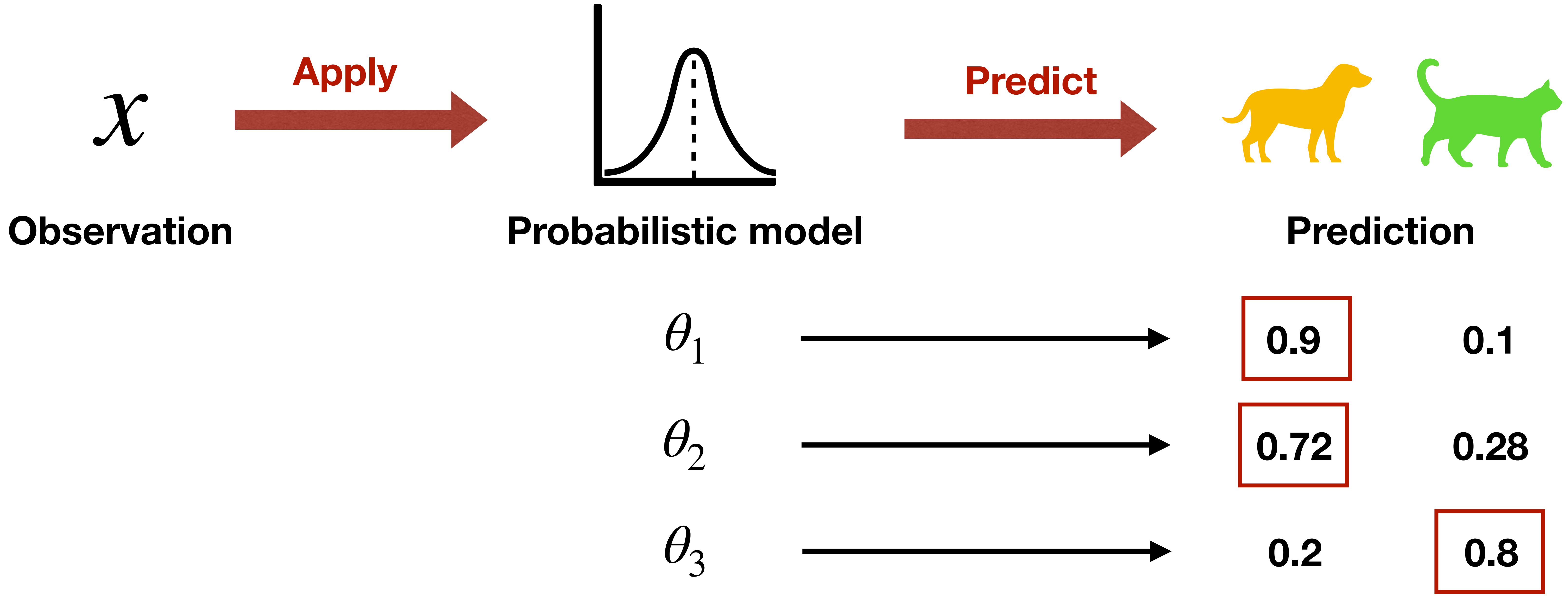
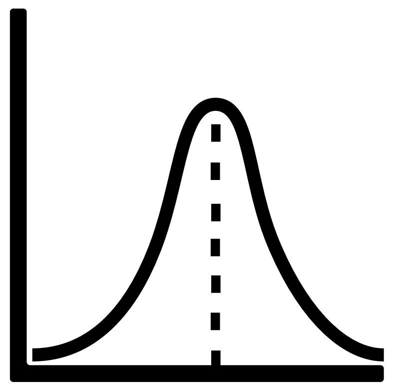
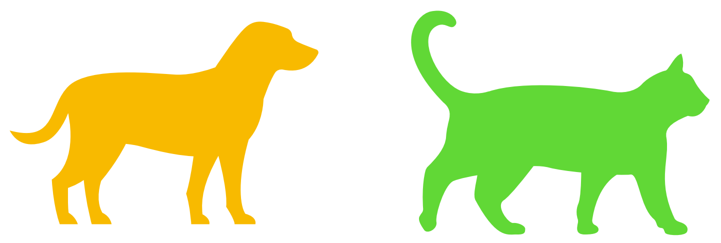


Inference phase

Aim during training phase : find

θ **such that** $\forall (x, y) \in \mathcal{X} \times \mathcal{Y}, h_\theta(x) \approx y$

x $\xrightarrow{\text{Apply}}$  $\xrightarrow{\text{Predict}}$ 

Observation **Probabilistic model** **Prediction**

$\theta_1 \longrightarrow$	$\boxed{0.9}$	0.1
$\theta_2 \longrightarrow$	$\boxed{0.72}$	0.28
$\theta_3 \longrightarrow$	0.2	$\boxed{0.8}$

$\dots$

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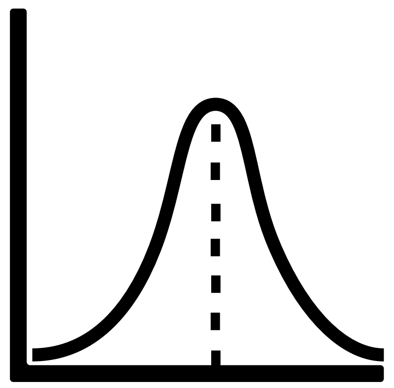
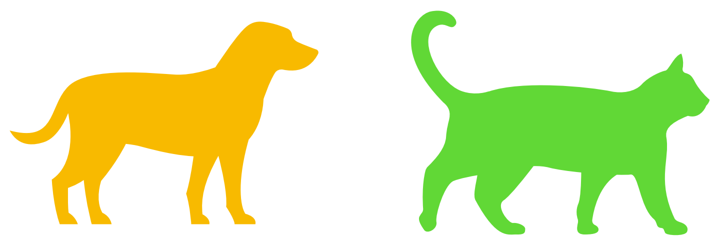
x $\xrightarrow{\text{Apply}}$  $\xrightarrow{\text{Predict}}$ 

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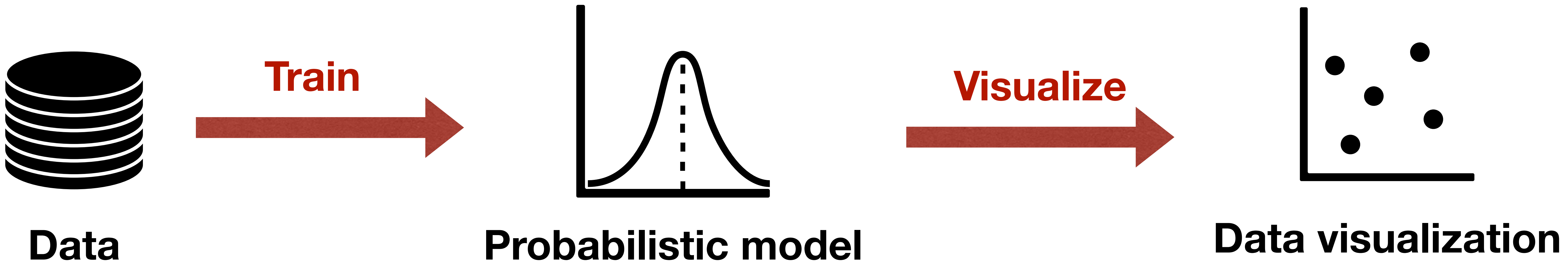
...

- « Classical » frequentist point of view
- Parameter θ is fixed and the data X is random

Training phase



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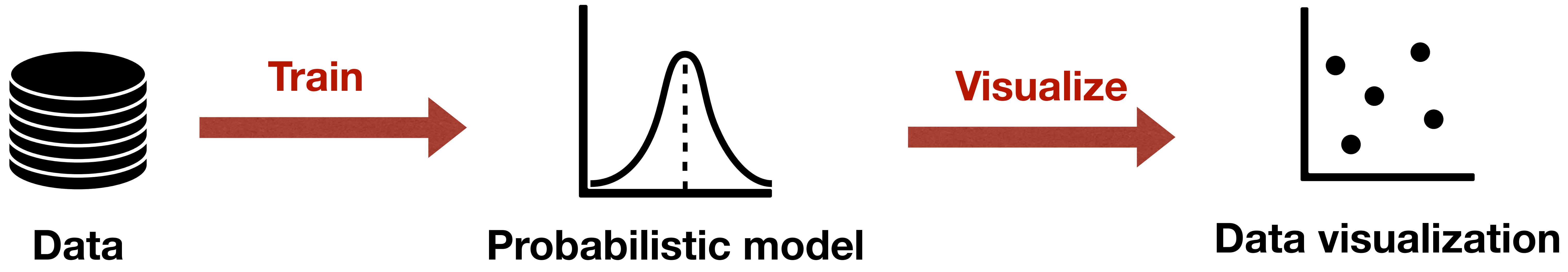

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- Only works well if we have big data : $|X| \gg |\theta|$
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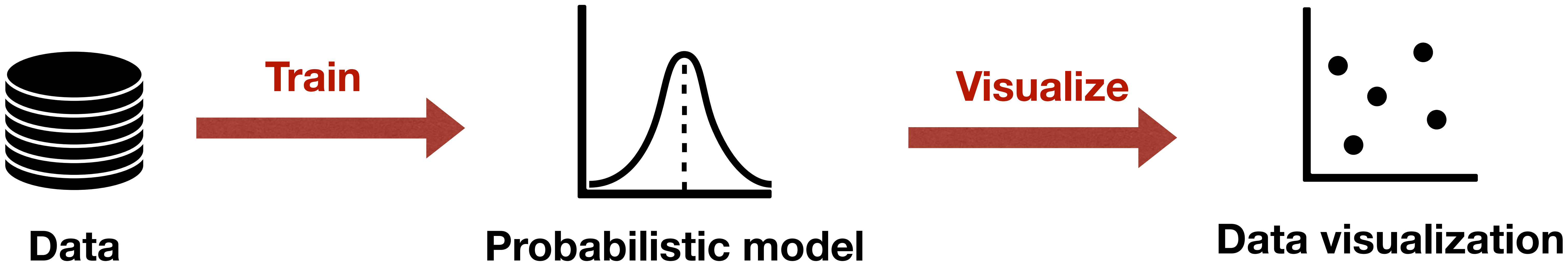

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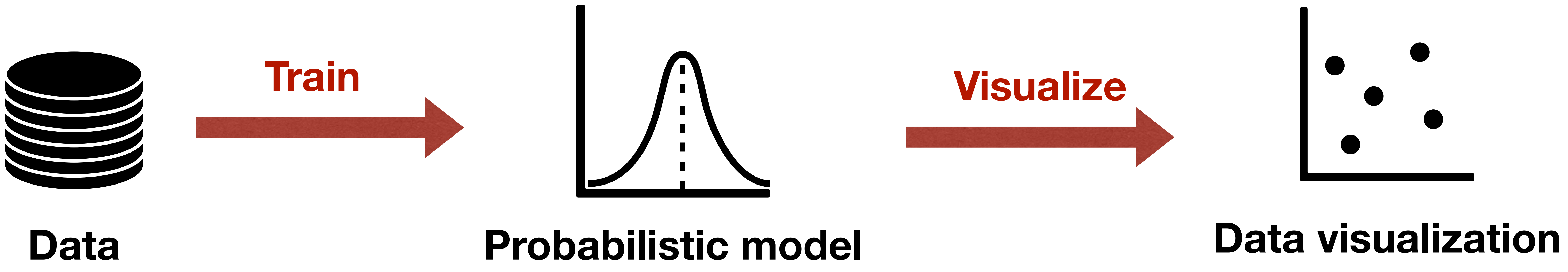

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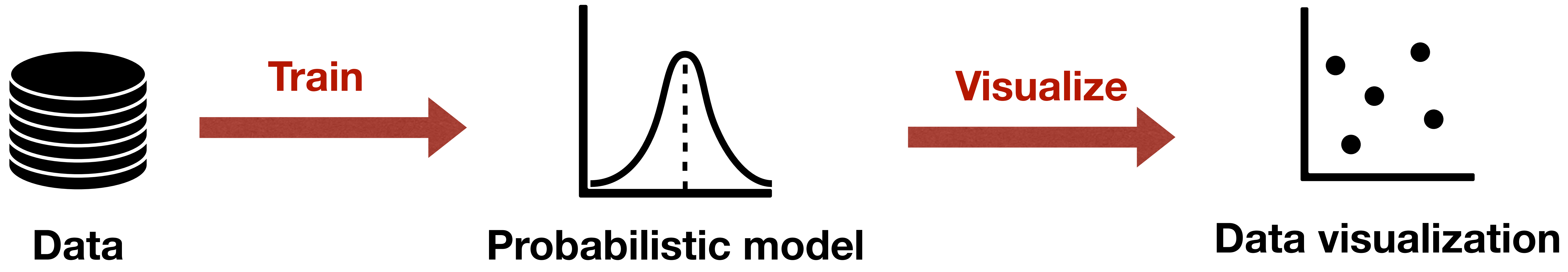

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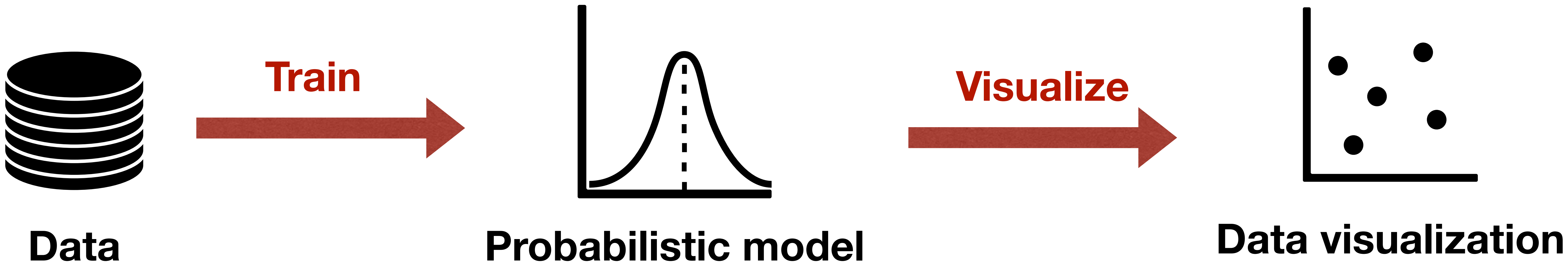

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Bayesian statistics

1 Introduction to bayesian statistics

1. Introduction to bayesian statistics

Probability & statistics : small review

Probability (non axiomatic definition) of an event : relative **frequency** of an event in an infinite trials

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Probability & statistics : small review

Probability (non axiomatic definition) of an event : relative **frequency** of an event in an infinite trials

Example : a deck of 52 playing cards

$P($  $) = \frac{1}{52}$ $\qquad$ $P(\spadesuit) = \frac{1}{4}$

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Random variable

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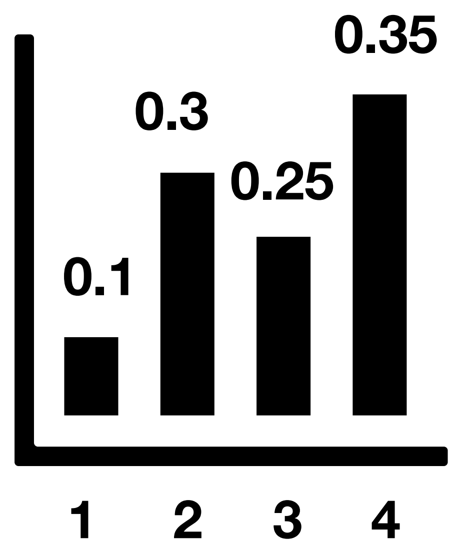
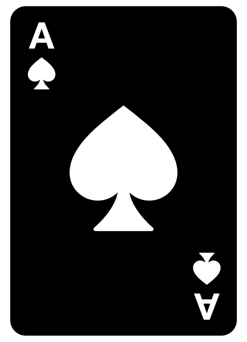
Random variable

- **Discrete** variable



Example



Probability Mass Function (PMF)

$$P(X) = \begin{cases} 0.1 & \text{if } X = 1 \\ 0.3 & \text{if } X = 2 \\ 0.25 & \text{if } X = 3 \\ 0.35 & \text{if } X = 4 \end{cases}$$

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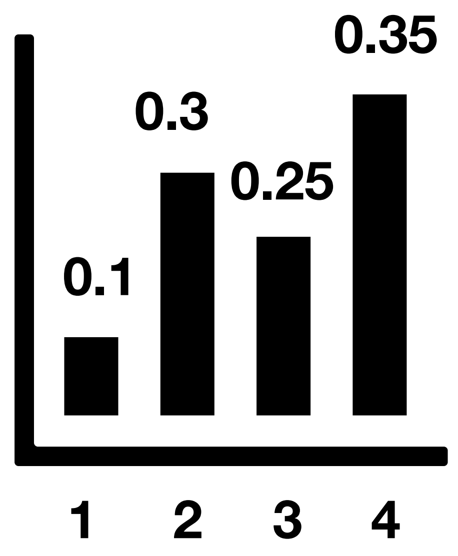

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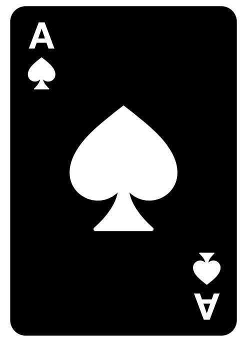

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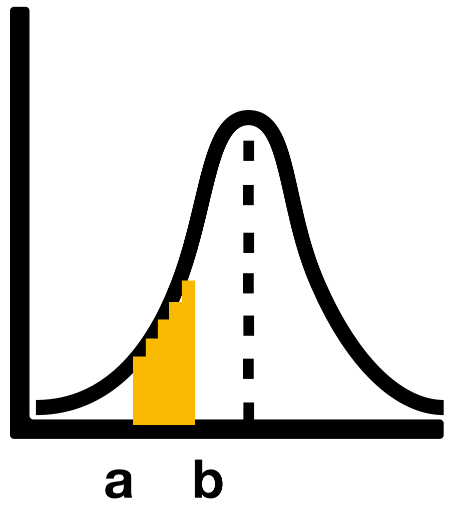
- **Continuous** variable



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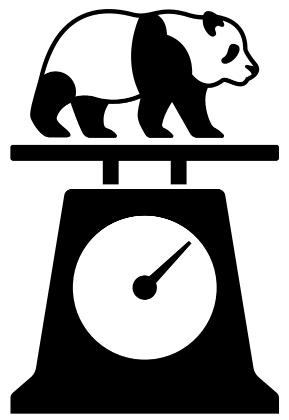

Probability Density function (PDF)

$$P(X \in [a, b]) = \int_a^b p(s)ds$$

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Random variable - **Discrete** variable - **Continuous** variable

Independence

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Independence : two random variables X and Y are **independent** if

$$\underbrace{P(X,Y)}_{\textbf{joint probability}} = \underbrace{P(X)P(Y)}_{\textbf{marginals}}$$

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joint probability **marginals**

Example :

dependency : **one** deck of 52 playing cards

$$P(X_1 = A\heartsuit, X_2 = A\heartsuit) = 0$$

$$P(X_1 = A\heartsuit) \cdot P(X_2 = A\heartsuit) = \frac{1}{52^2}$$

independency : **two** dices

$$P(X_1 = \text{⚃}, X_2 = \text{⚃}) = \frac{1}{6^2}$$

$$P(X_1 = \text{⚃}) \cdot P(X_2 = \text{⚃}) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{6^2}$$

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Conditional probability

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Conditional probability : probability of X **given that Y happened**

joint probability

$$\underbrace{P(X|Y)}_{\textbf{conditional}} = \frac{P(X, Y)}{P(Y)}$$

marginal

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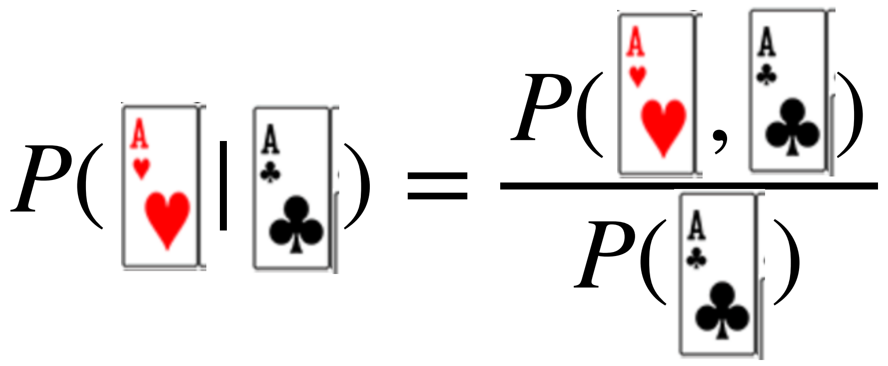

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Rules :

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Rules :

Chain rule $P(X_1, X_2) = P(X_1|X_2) \times P(X_2)$

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$$P(X \mid Y) = \frac{P(X, Y)}{P(Y)}$$

Rules :

Chain rule $P(X_1, X_2, X_3) = P(X_1 \mid X_2, X_3) \times P(X_2 \mid X_3) \times P(X_3)$

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Chain rule

$$P(X_1, \ldots, X_n) = \prod_{k=1,\ldots,n} P(X_k \mid X_1, \ldots, X_{k-1})$$

Sum rule

$$P(X) = \sum_{Y \in \mathcal{Y}} P(X, Y) \qquad P(X) = \int_{Y \in \mathcal{Y}} P(X, Y) \cdot dY$$

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Bayes theorem :

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Rules : **Chain rule** **Sum rule**

Bayes theorem :

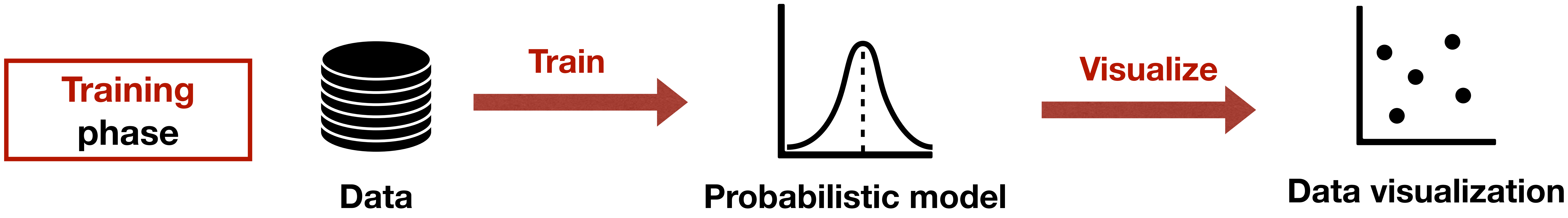
θ Parameters

X Observations (data)

$$\underbrace{P(\theta \mid X)}_{\text{Posterior}} = \frac{P(X, \theta)}{P(X)} = \frac{\overbrace{P(X \mid \theta)}^{\text{Likelihood}} \times \overbrace{P(\theta)}^{\text{Prior}}}{\underbrace{P(X)}_{\text{Evidence}}}$$

1. Introduction to bayesian statistics

Frequentist VS Bayesian point of view



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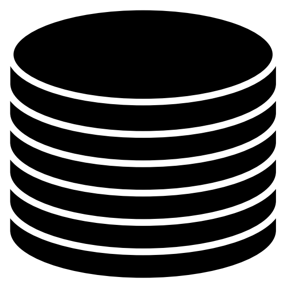
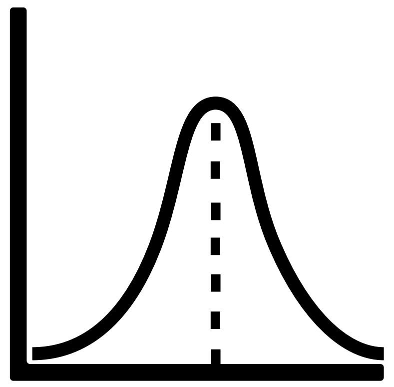
Frequentist VS Bayesian point of view

Training phase



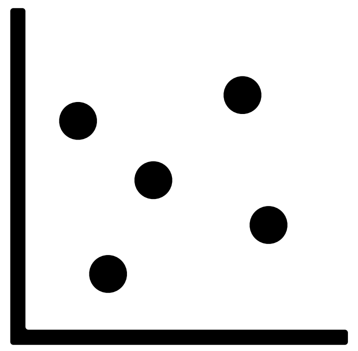
Data

Train



Probabilistic model

Visualize



Data visualization

Frequentist

θ is fixed
X is random

Maximum Likelihood Estimation

$$\hat{\theta} = \arg\max_{\theta} P(X \mid \theta)$$

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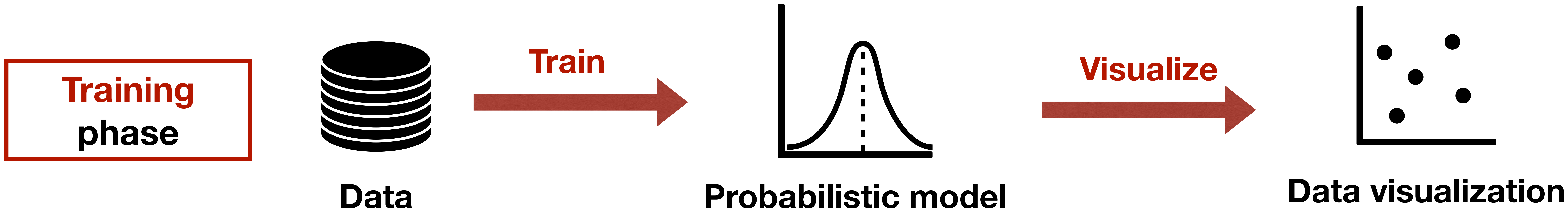

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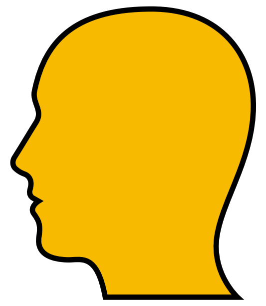

Bayesian

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Frequentist VS Bayesian point of view

Frequentist

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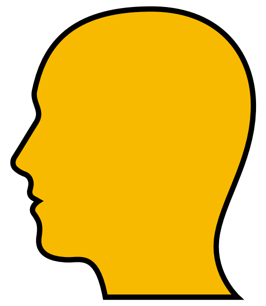

Bayesian

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Frequentist VS Bayesian point of view

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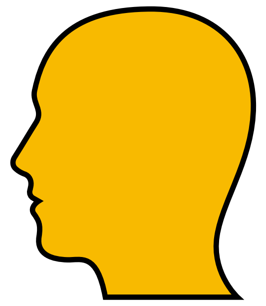

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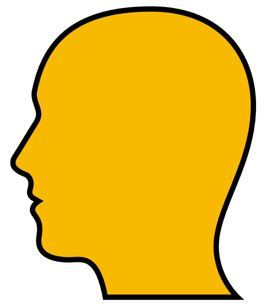


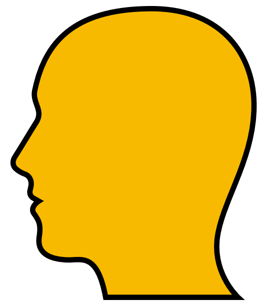
Bayesian

1. Introduction to bayesian statistics

Bayesian point of view : classification

Bayes theorem

$$P(\theta | X) = \frac{P(X | \theta) \times P(\theta)}{P(X)}$$



Bayesian

Training **phase**

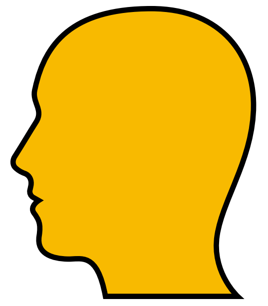
$$P(\theta | X_{train}, y_{train}) = \frac{P(y_{train} | X_{train}, \theta) \times P(\theta)}{P(y_{train} | X_{train})}$$

1. Introduction to bayesian statistics

Bayesian point of view : training

Bayes theorem

$$P(\theta | X) = \frac{P(X | \theta) \times P(\theta)}{P(X)}$$



Bayesian

Training phase

$$P(\theta | X_{train}, y_{train}) = \frac{P(y_{train} | X_{train}, \theta) \times \boxed{P(\theta)}}{P(y_{train} | X_{train})}$$

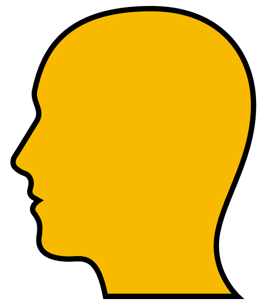
Can regularize your model when training on your data

1. Introduction to bayesian statistics

Bayesian point of view : inference

Bayes theorem

$$P(\theta | X) = \frac{P(X | \theta) \times P(\theta)}{P(X)}$$



Bayesian

Training phase

$$P(\theta | X_{train}, y_{train}) = \frac{P(y_{train} | X_{train}, \theta) \times \boxed{P(\theta)}}{P(y_{train} | X_{train})}$$

Can regularize your model when training on your data

Inference phase

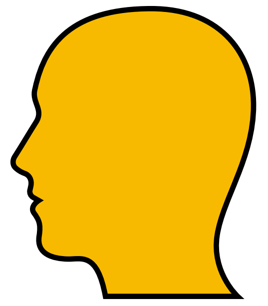
$$P(y_{new} | X_{new}, X_{train}, y_{train}) = \int P(y_{new} | X_{train}, \theta) \times P(\theta | X_{train}, y_{train}) d\theta$$

1. Introduction to bayesian statistics

Bayesian point of view : online learning

Bayes theorem

$$P(\theta | X) = \frac{P(X | \theta) \times P(\theta)}{P(X)}$$



Bayesian

Training phase

$$P(\theta | X_{train}, y_{train}) = \frac{P(y_{train} | X_{train}, \theta) \times \boxed{P(\theta)}}{P(y_{train} | X_{train})}$$

Can regularize your model when training on your data

Inference phase

$$P(y_{new} | X_{new}, X_{train}, y_{train}) = \int P(y_{new} | X_{train}, \theta) \times P(\theta | X_{train}, y_{train}) d\theta$$

Online learning

$$P_{new}(\theta) = P(\theta | x_{new}) = \frac{P(x_{new} | \theta) \times P_{old}(\theta)}{P(x_{new})}$$

New prior

Posterior

2 Probabilistic models

2. Probabilistic model

Probabilistic Graphical Model (PGM)

Probabilistic graphical models : analysis using **diagrammatic representations** of probability distributions

2. Probabilistic model

Probabilistic Graphical Model (PGM)

Probabilistic graphical models : analysis using **diagrammatic representations** of probability distributions

- **Nodes :** random variables
- **Links :** probabilistic relationships

2. Probabilistic model

Probabilistic Graphical Model (PGM)

Probabilistic graphical models : analysis using **diagrammatic representations** of probability distributions

Bayesian networks
(**Directed** graphical models)

Nodes : random variables

Links : probabilistic relationships

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Markov random fields
(**Undirected** graphical models)

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The focus of our course !

Markov random fields
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The focus of our course !

Markov random fields
(**Undirected** graphical models)

Model : joint probability over all variables $P(X_1, \cdots, X_N) = \ldots$

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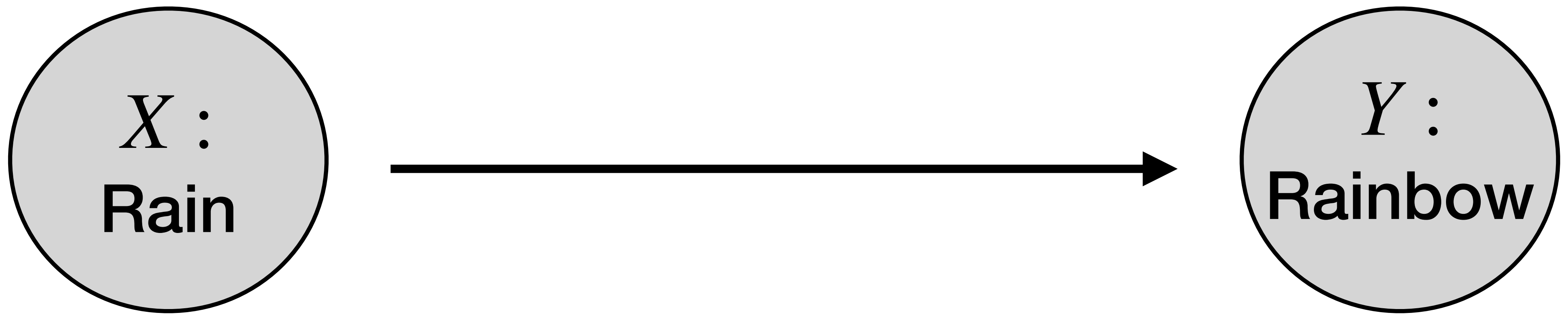
The focus of our course !

Model : joint probability over all variables $P(X_1, \cdots, X_N) = \ldots$

Example :



$$P(X, Y) = \ldots$$

2. Probabilistic model

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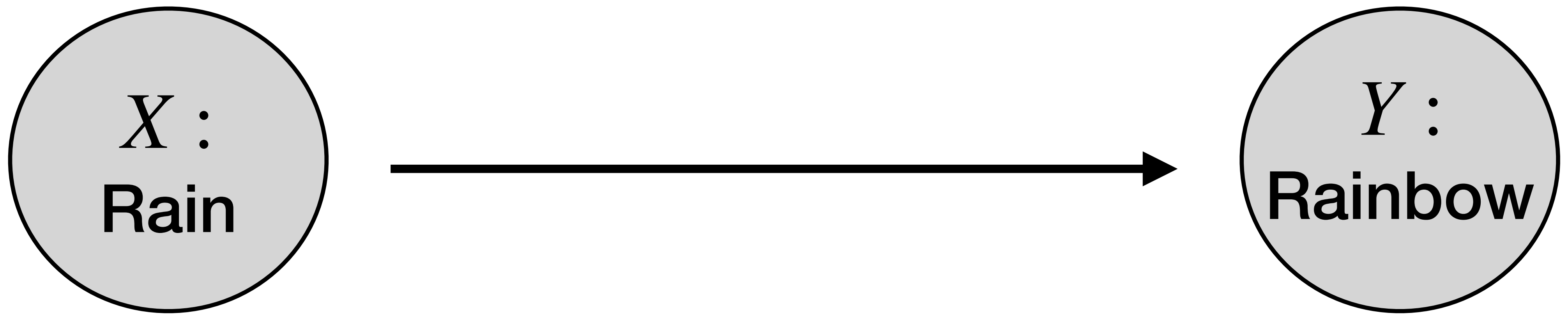

$$P(X, Y) = P(Y|X) \times P(X)$$

2. Probabilistic model

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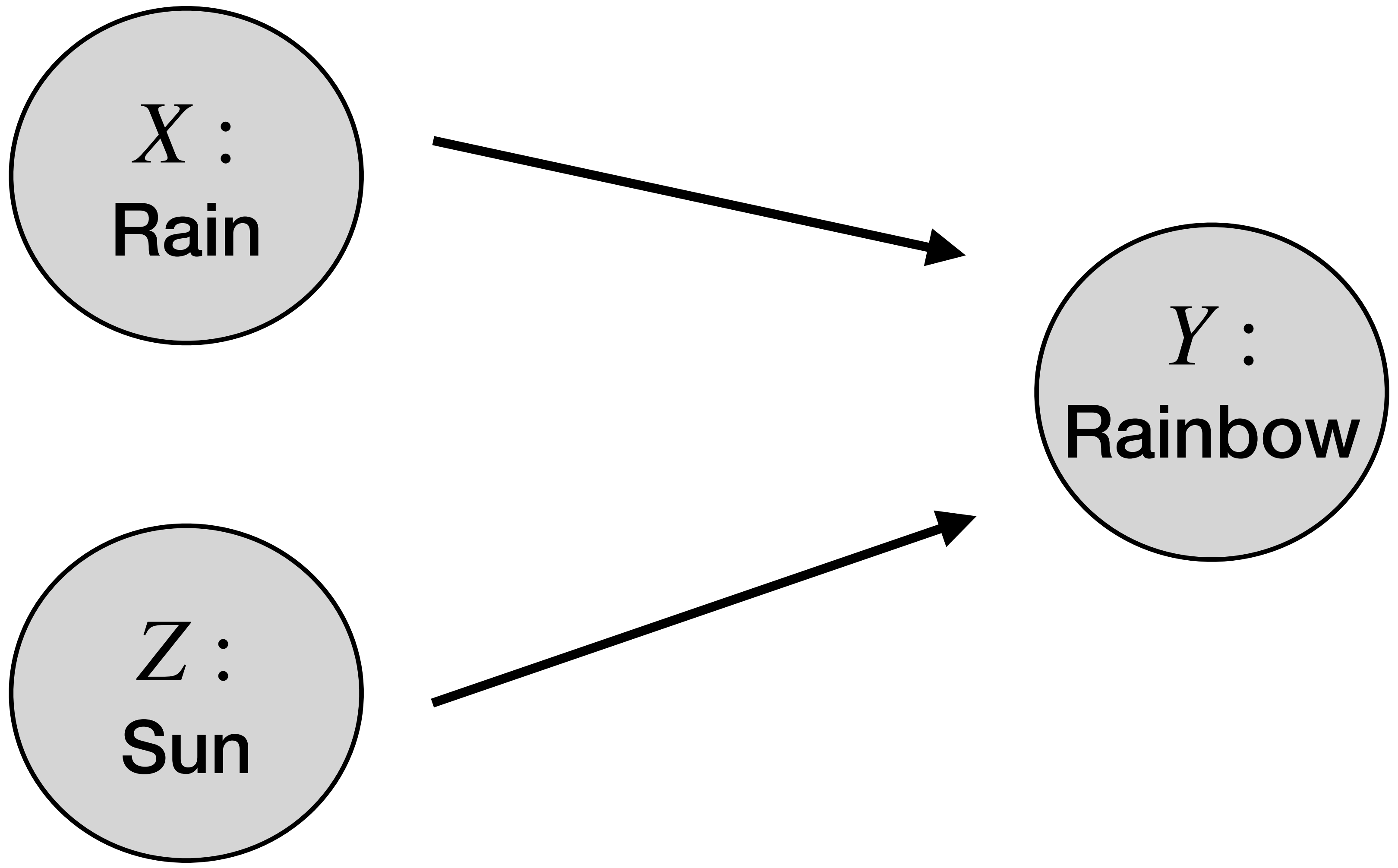
Example :



$$P(X, Y, Z) = \ldots$$

Features

X , Z conditionally independent given Y

2. Probabilistic model

Probabilistic Graphical Model (PGM)

Probabilistic graphical models : analysis using **diagrammatic representations** of probability distributions

Bayesian networks
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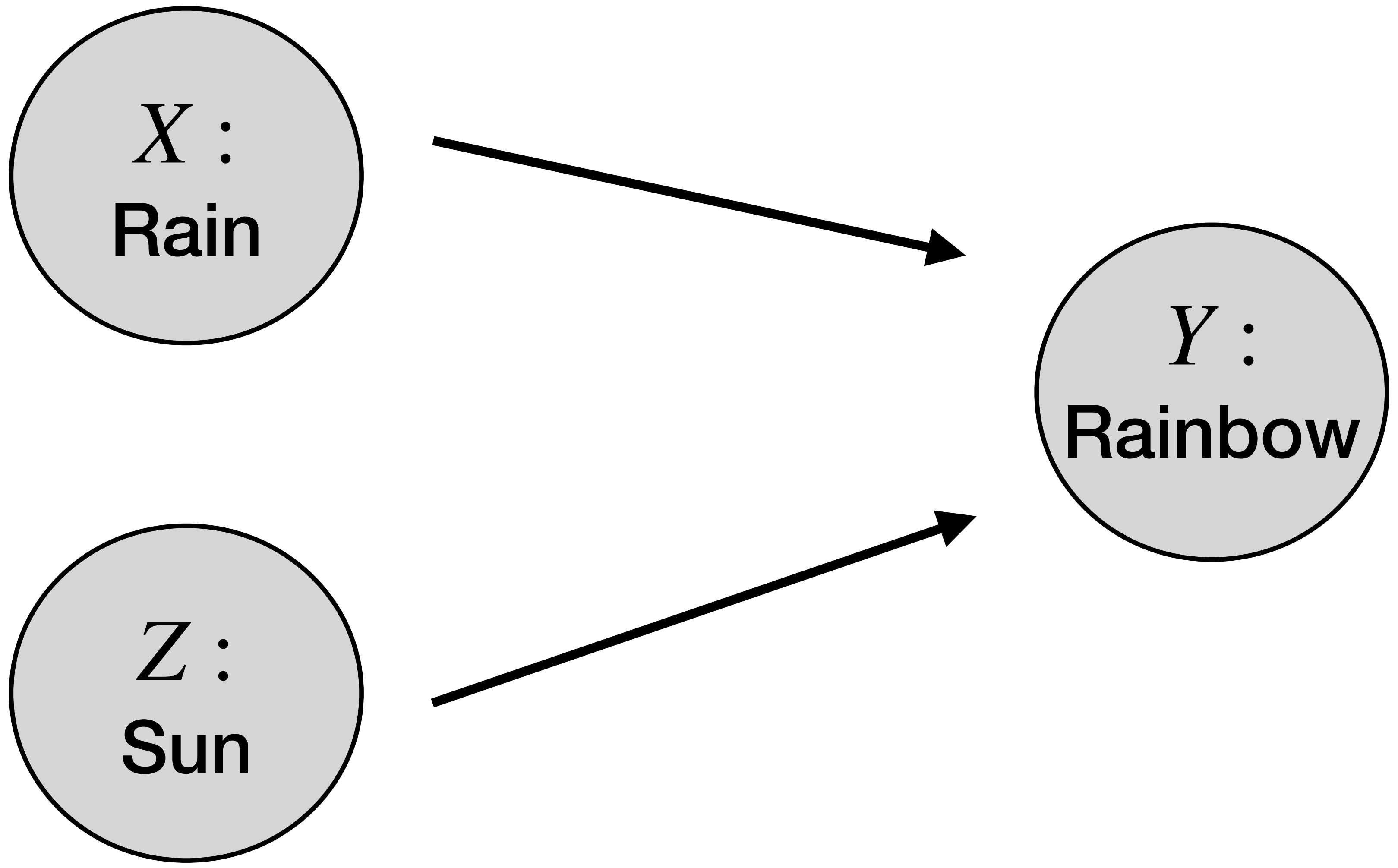
Model : joint probability over all variables $P(X_1, \cdots, X_N) = \dots$

Example :



$$P(X, Y, Z) = P(Y|X, Z) \times P(X, Z)$$
$$= P(Y|X, Z) \times P(X) \times P(Z)$$

Features

X , Z conditionally independent given Y

2. Probabilistic model

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(**Directed** graphical models)

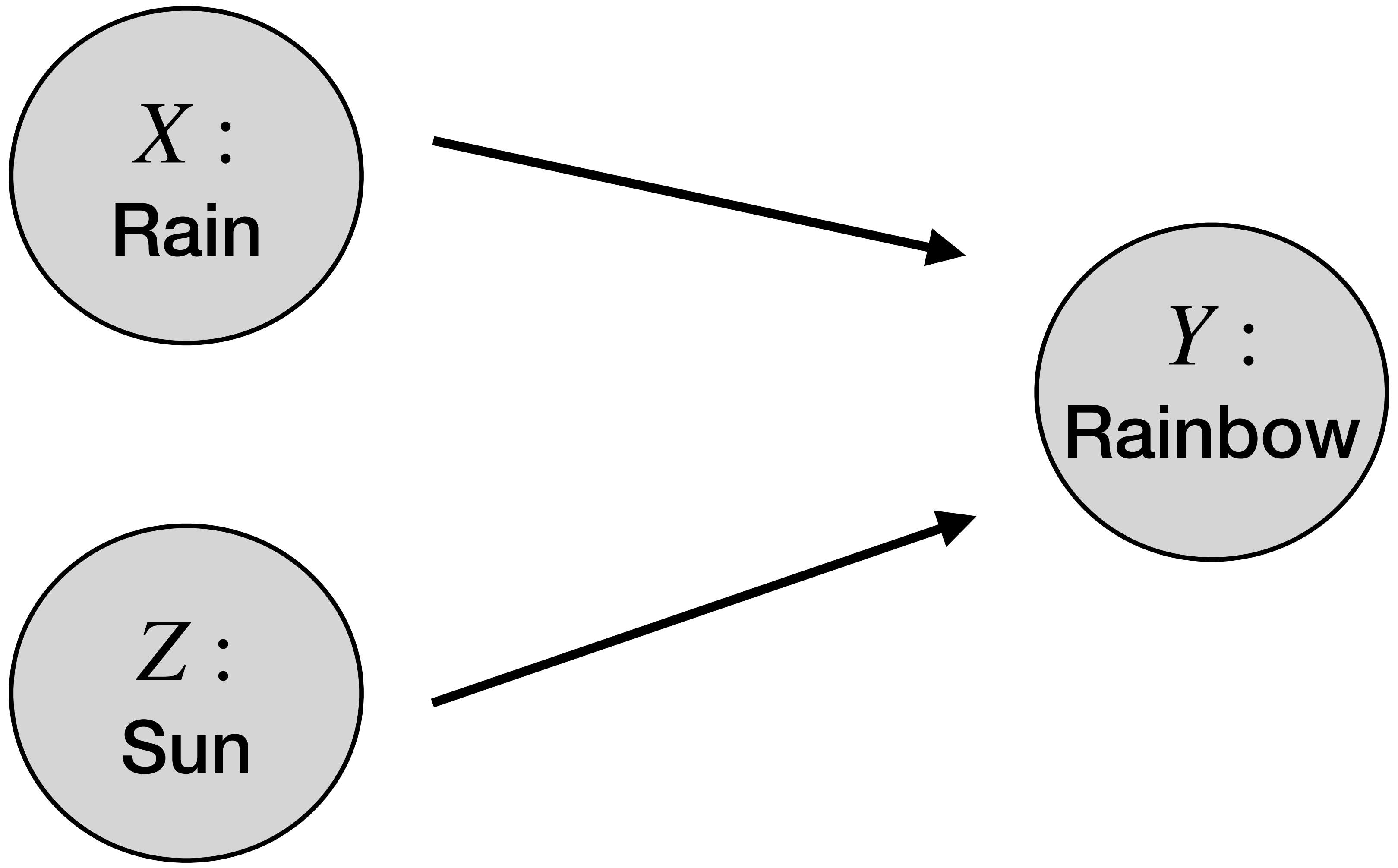
The focus of our course !

Markov random fields
(**Undirected** graphical models)

Model : joint probability over all variables

$$P(X_1, \cdots, X_N) = \prod_{i=1,\cdots,N} P(\ X_i \mid parents(X_i)\ )$$

Example :



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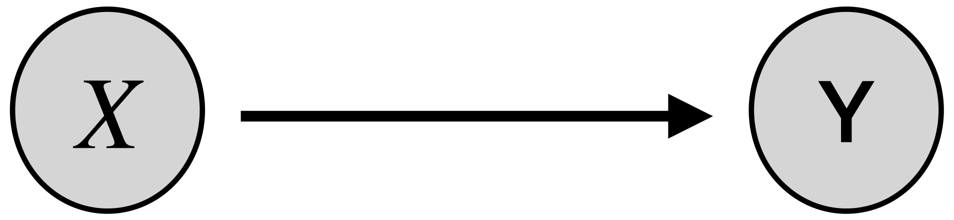
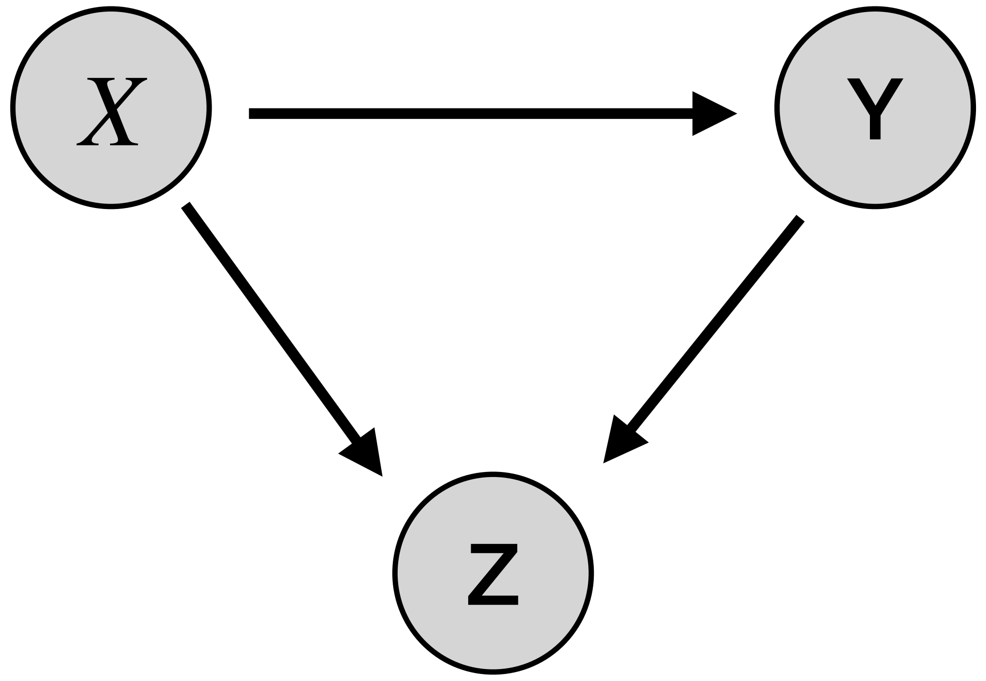
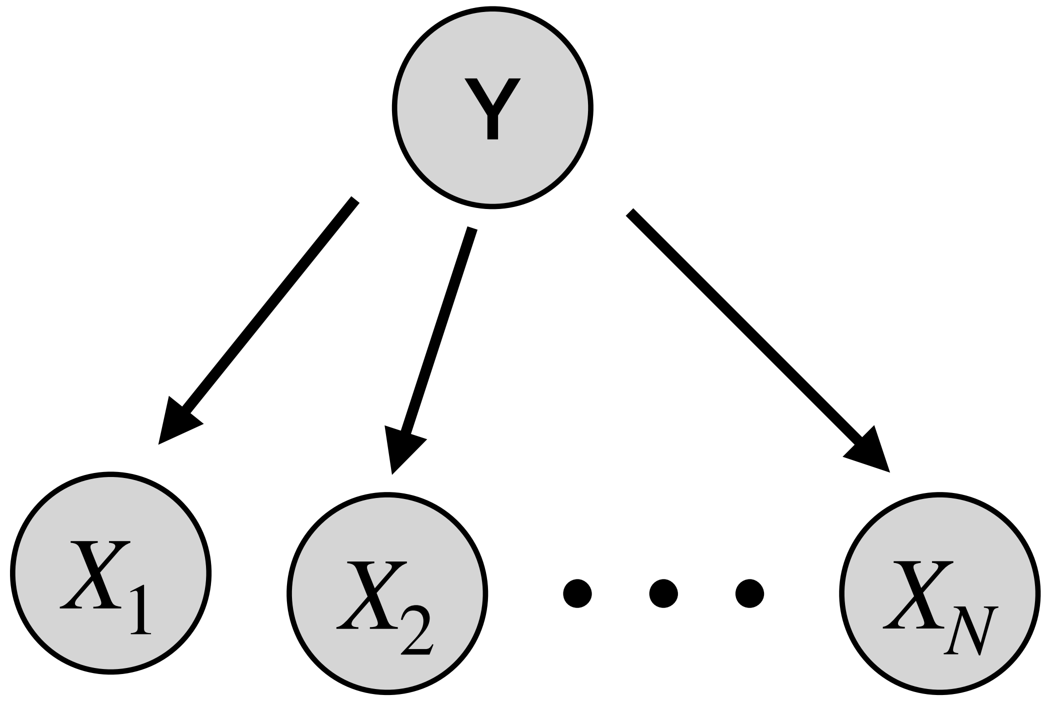


$P(X, Y) = P(Y | X) \times P(X)$



$P(X, Y, Z) = \ldots$



$P(Y, X_1, \cdots, X_N) =$

2. Probabilistic model

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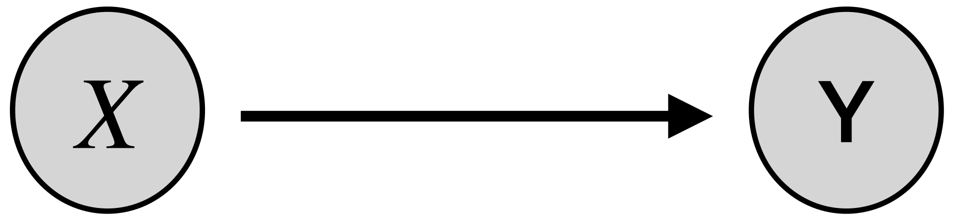
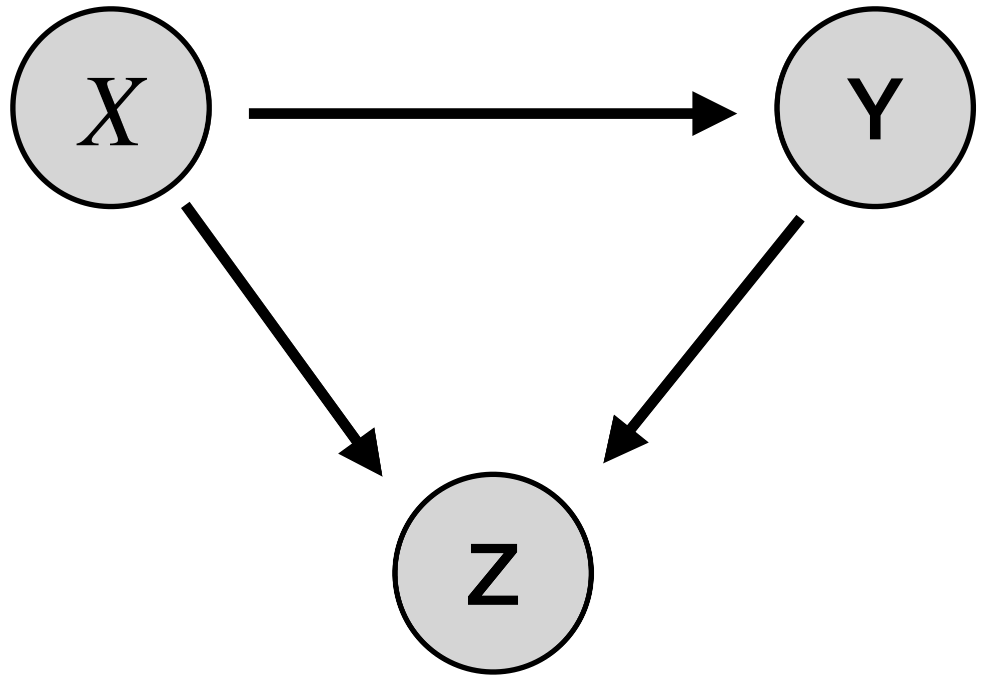
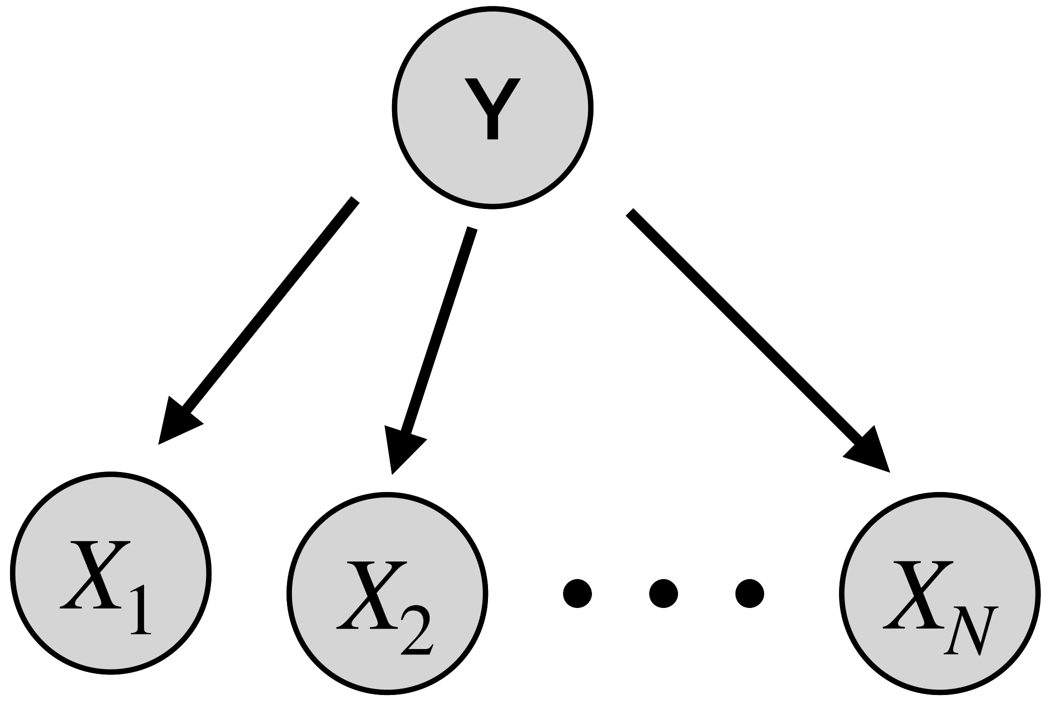


$$P(X, Y) = P(Y|X) \times P(X)$$



$$P(X, Y, Z) = P(Z|X, Y) \times P(X, Y)$$
$$= P(Z|X, Y) \times P(Y|X) \times P(X)$$



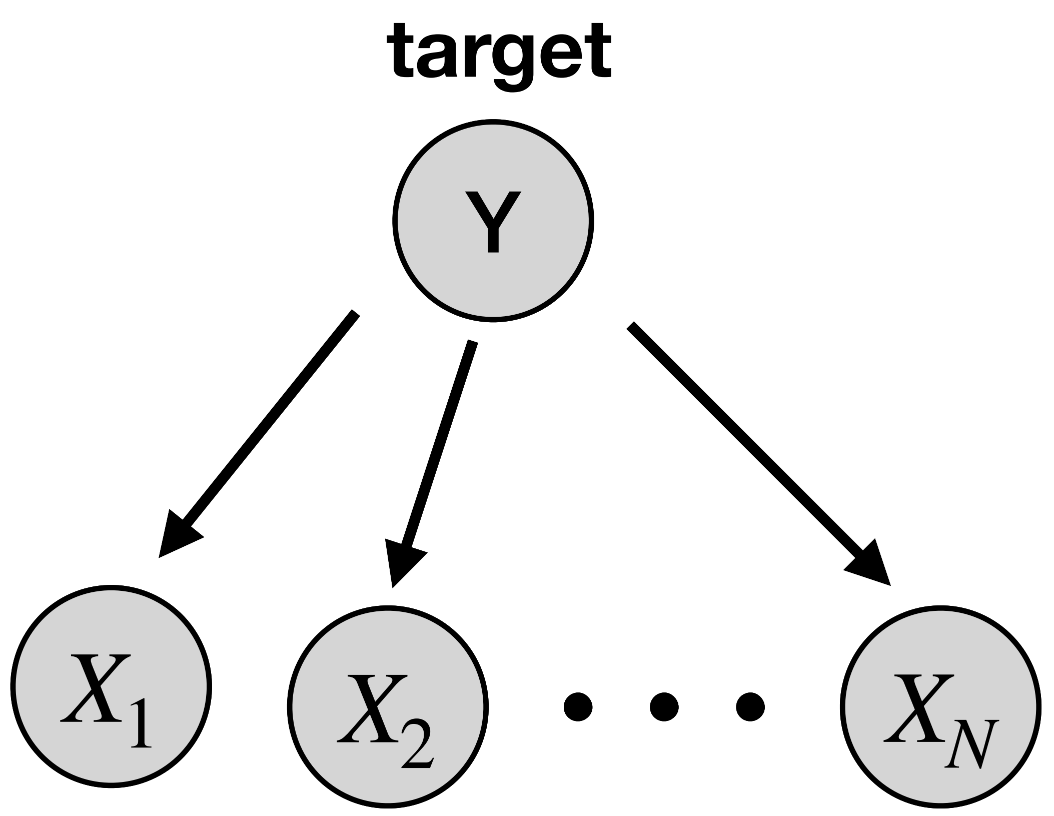
$$P(Y, X_1, \cdots, X_N) = P(Y) \prod_{i=1\ldots,N} P(X_i | Y)$$

2. Probabilistic model

Plates and examples of probabilistic model

Naive Bayes Classifier

$$P(Y, X_1, \cdots, X_N) = P(Y) \prod_{i=1\ldots,N} P(X_i \mid Y)$$



Features
X_i , X_j conditionally independent given Y

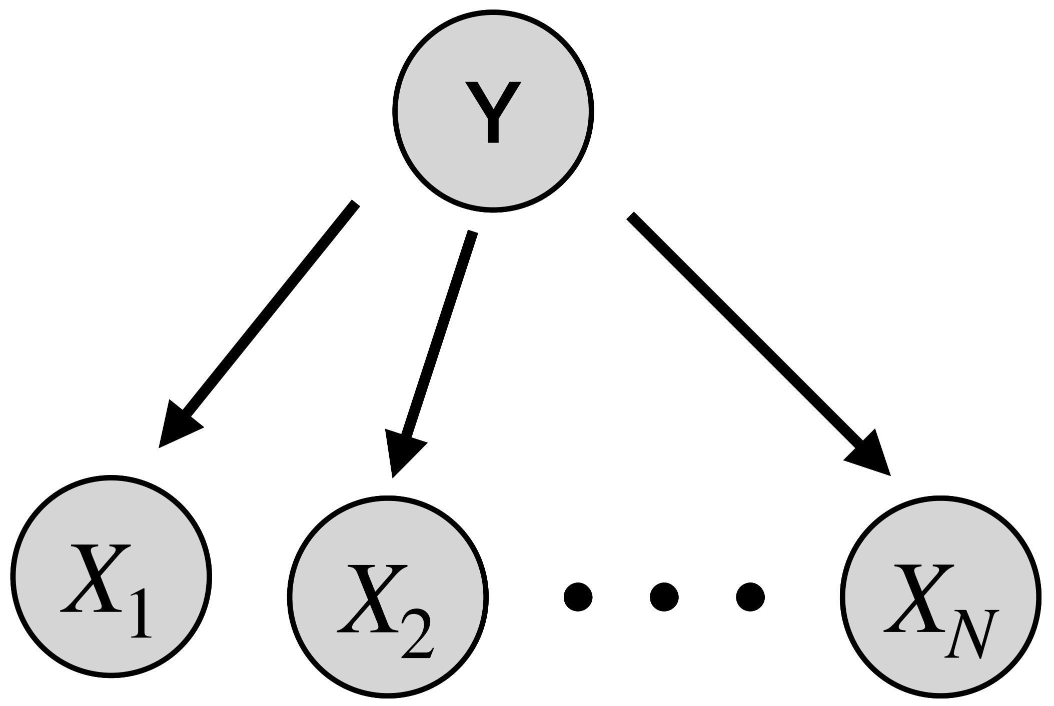
2. Probabilistic model

Plates and examples of probabilistic model

Naive Bayes Classifier

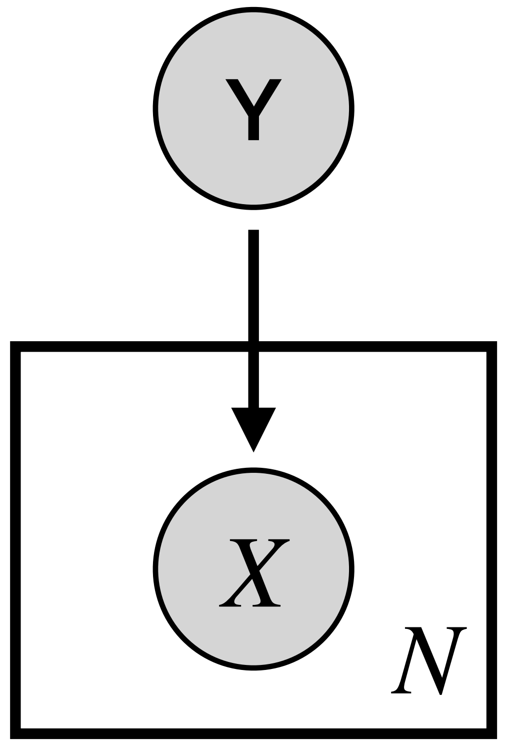
$$P(Y, X_1, \cdots, X_N) = P(Y) \prod_{i=1\ldots,N} P(X_i \mid Y)$$

target



Features
X_i , X_j conditionally independent given Y

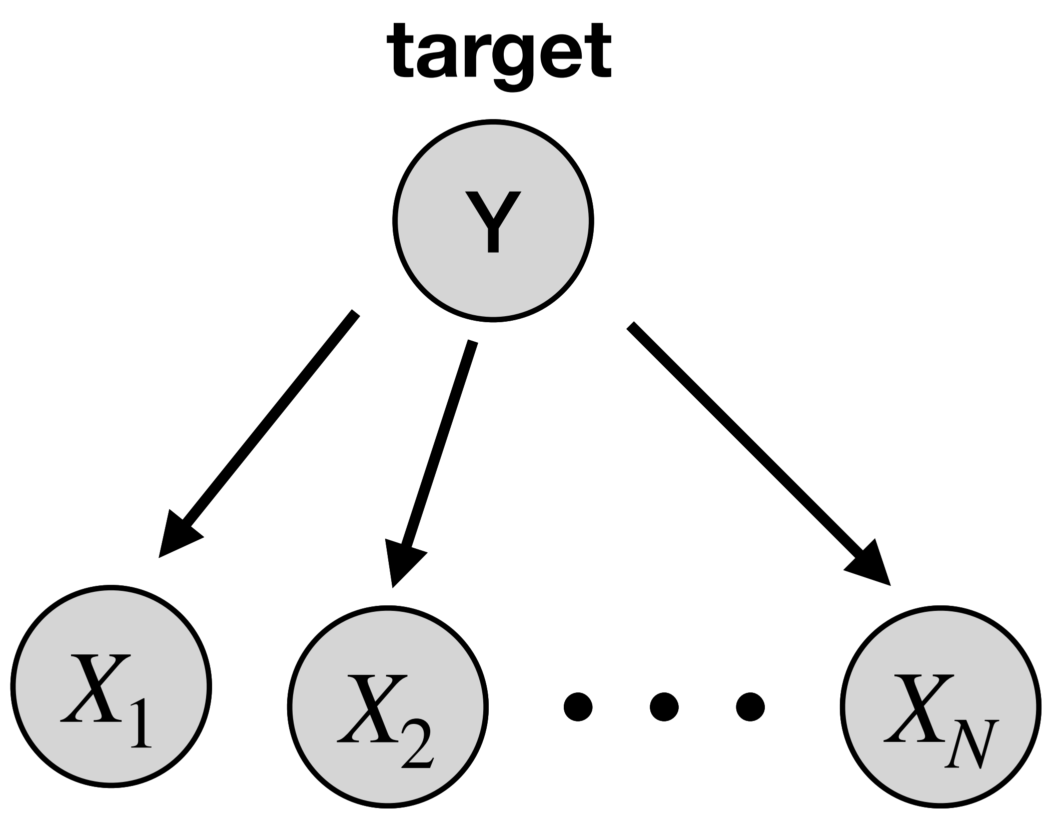
Plate notation $\longrightarrow$



2. Probabilistic model

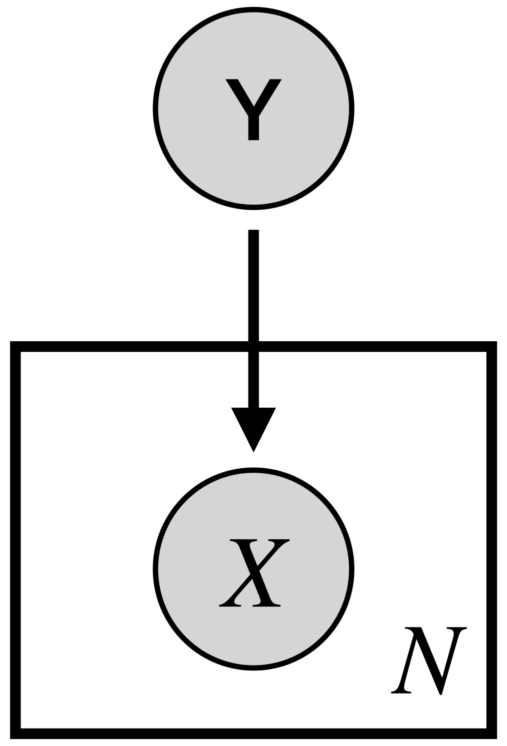
Plates and examples of probabilistic model

Naive Bayes Classifier $$P(Y, X_1, \cdots, X_N) = P(Y) \prod_{i=1\ldots,N} P(X_i \mid Y)$$



Features
X_i , X_j conditionally independent given Y

Plate notation →



Reminder : Frequentist linear regression $x_i \in \mathbb{R}^d \quad y_1 \in \mathbb{R}$

Scalar notation :

$$D = \{(x_1, y_1), \ldots, (x_n, y_n)\}$$

$$y_i = x_i^T \theta + \epsilon_i$$

Matrix notation :

$\mathbf{X} = (x_1, \ldots, x_n)$ and $\boldsymbol{y} = (y_1, \ldots, y_n)$

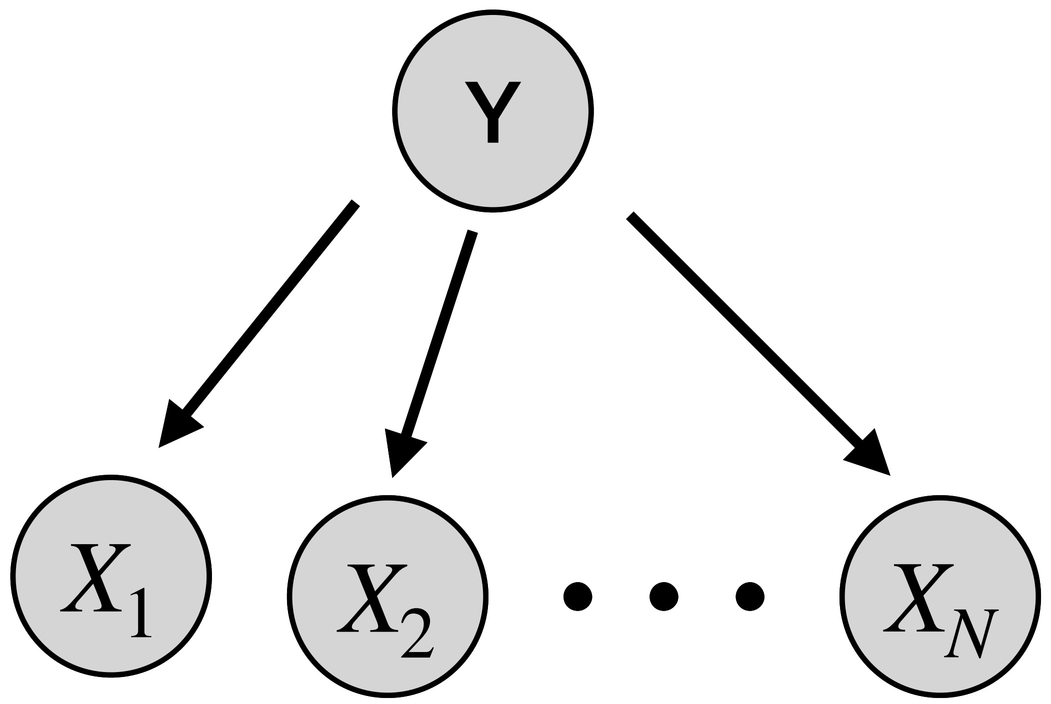
$$y = \mathbf{X}^T \theta + \epsilon$$

2. Probabilistic model

Plates and examples of probabilistic model

Naive Bayes Classifier $P(Y, X_1, \cdots, X_N) = P(Y) \prod_{i=1\ldots,N} P(X_i \mid Y)$

target



Features
X_i , X_j conditionally
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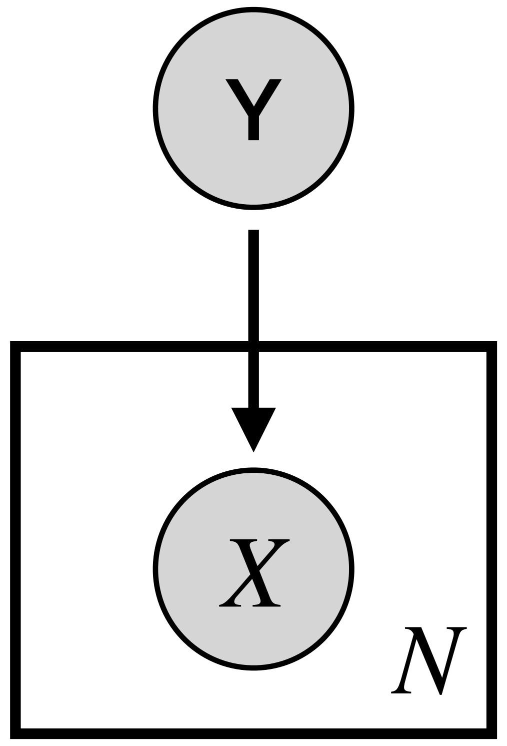
Plate notation



Reminder : Frequentist linear regression $x_i \in \mathbb{R}^d$ $y_1 \in \mathbb{R}$

Scalar notation :

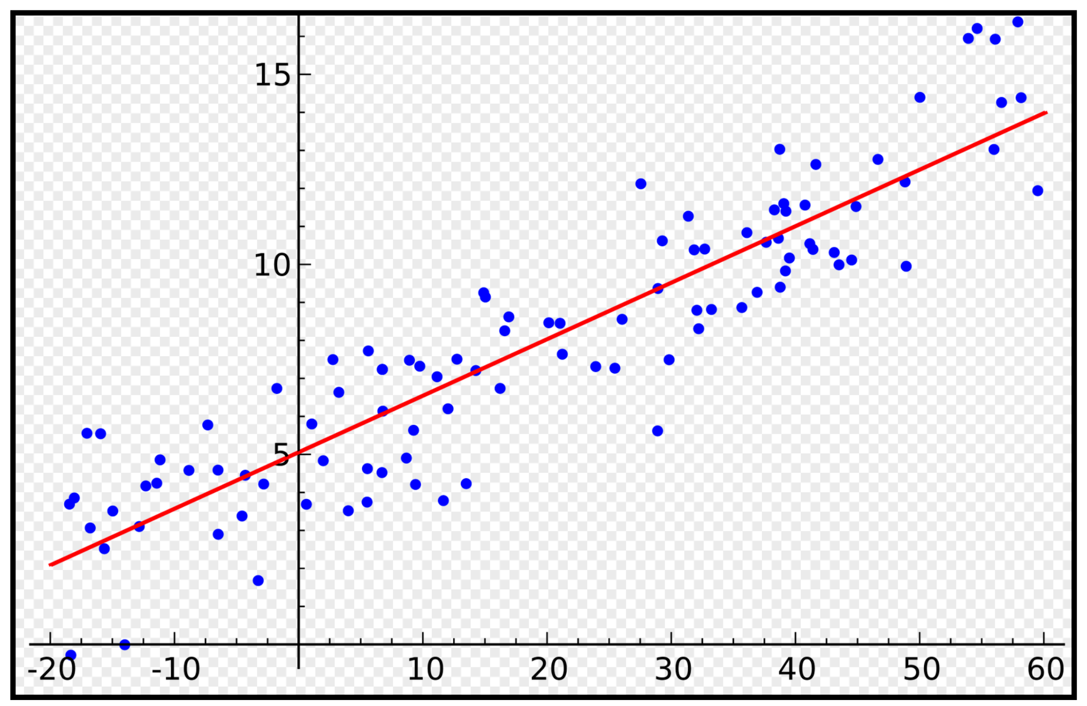
$D = \{(x_1, y_1), \ldots, (x_n, y_n)\}$

$$y_i = x_i^T \theta + \epsilon_i$$

Matrix notation :

$\mathbf{X} = (x_1, \ldots, x_n)$ and $\boldsymbol{y} = (y_1, \ldots, y_n)$

$$y = \mathbf{X}^T \theta + \epsilon$$



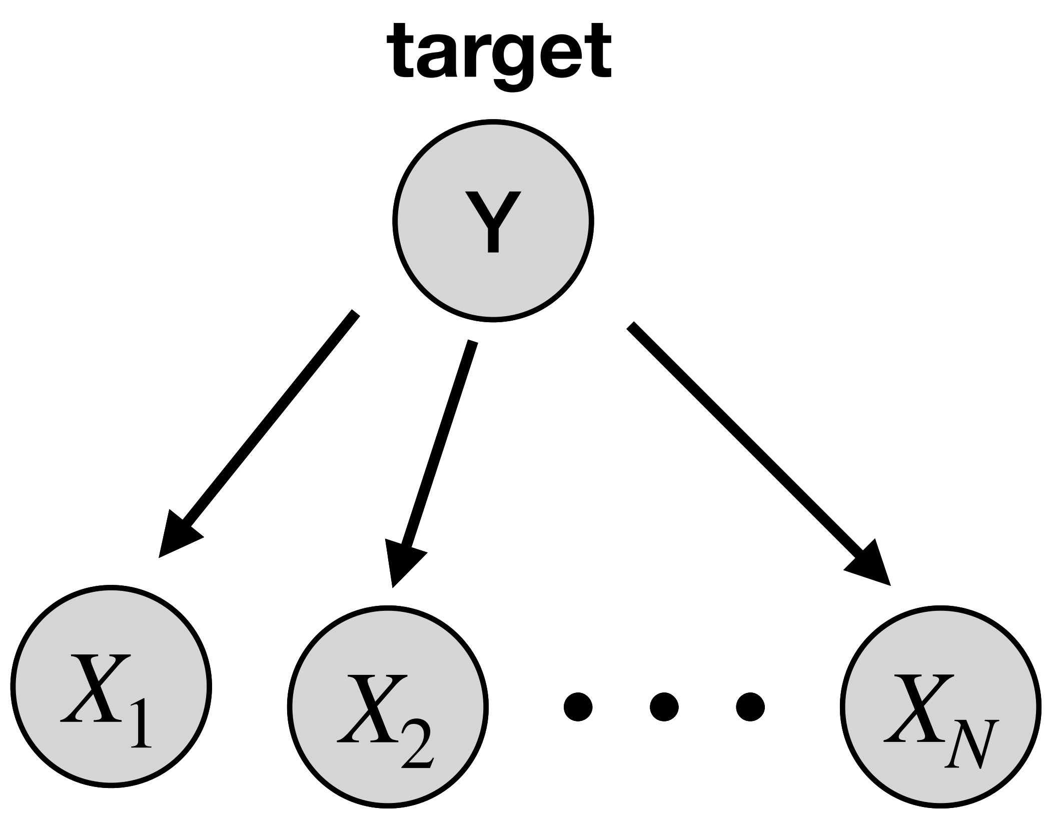
$$\min_{\theta} \|\theta^t X - y\|^2$$

2. Probabilistic model

Plates and examples of probabilistic model

Naive Bayes Classifier

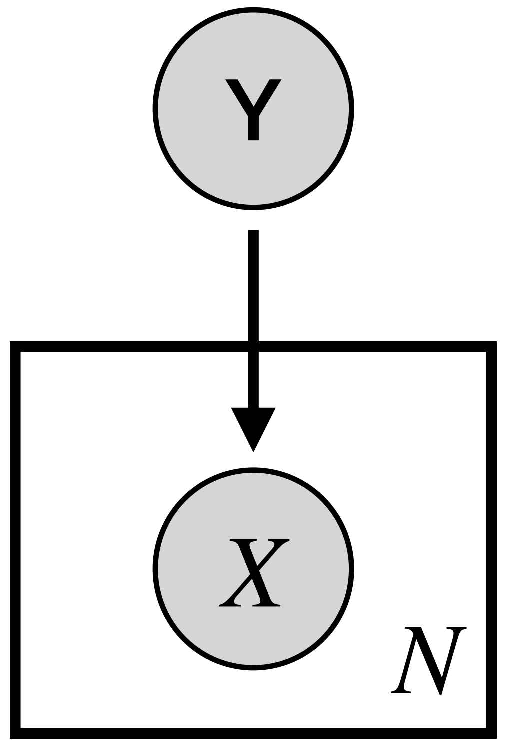
$$P(Y, X_1, \cdots, X_N) = P(Y) \prod_{i=1\ldots,N} P(X_i \mid Y)$$



Features

X_i , X_j conditionally independent given Y

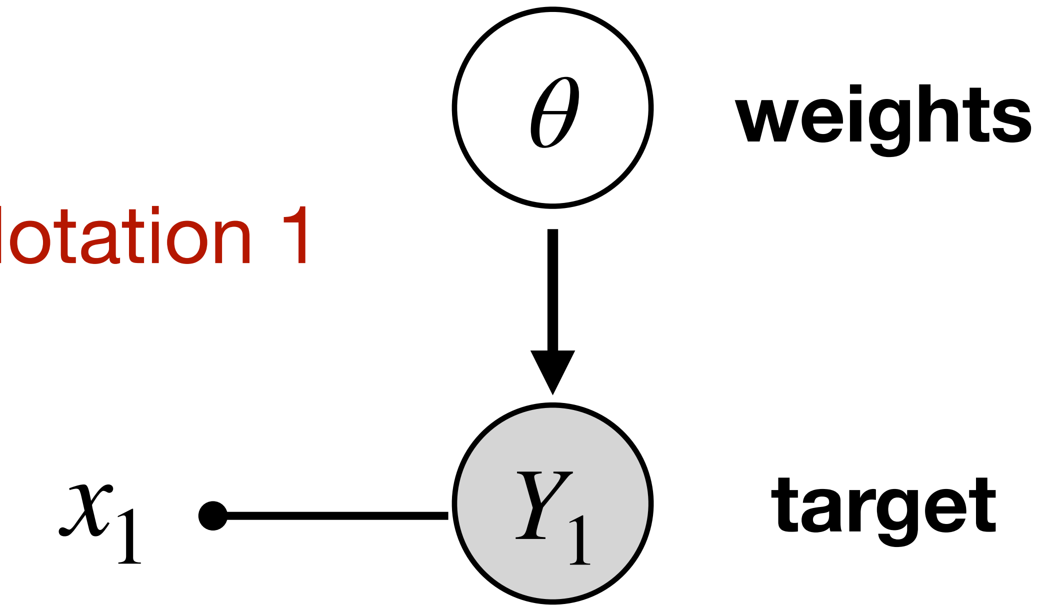
Plate notation



Bayesian Linear regression

(x_1, y_1) with $x_1 \in \mathbb{R}^d$, $y_1 \in \mathbb{R}$

Notation 1



$$P(\theta, y_1 \mid x_1) = P(y_1 \mid \theta, x_1) \times P(\theta)$$

$$P(y_1 \mid \theta, x_1) = \ldots$$

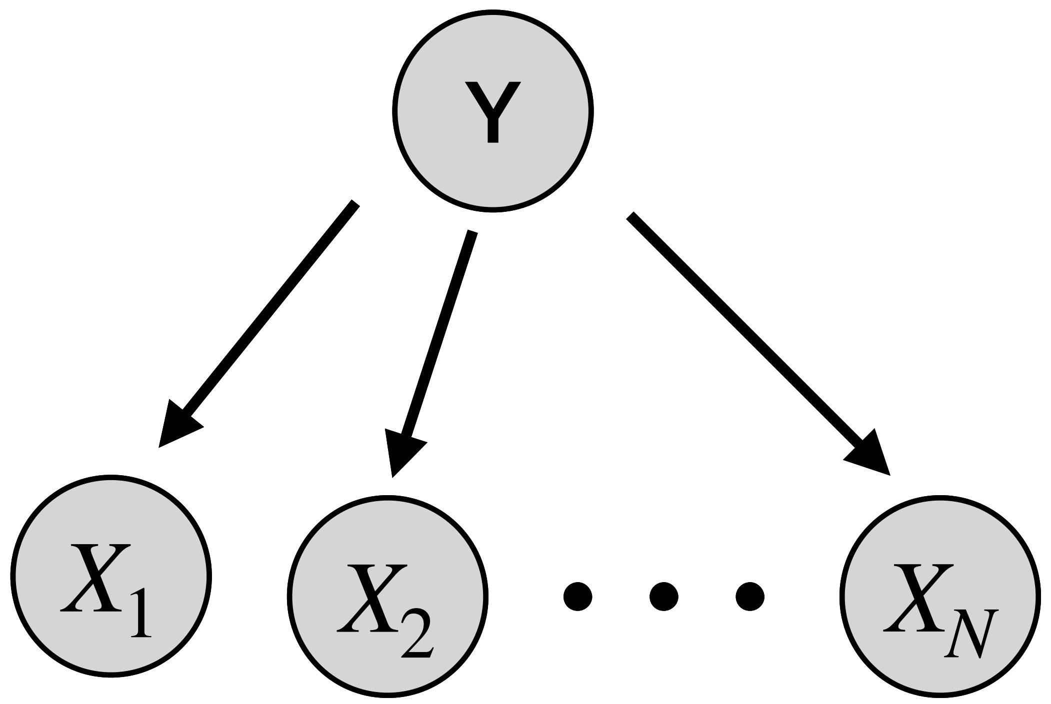
$$P(\theta) = \ldots$$

2. Probabilistic model

Plates and examples of probabilistic model

Naive Bayes Classifier $P(Y, X_1, \cdots, X_N) = P(Y) \prod_{i=1\ldots,N} P(X_i \mid Y)$

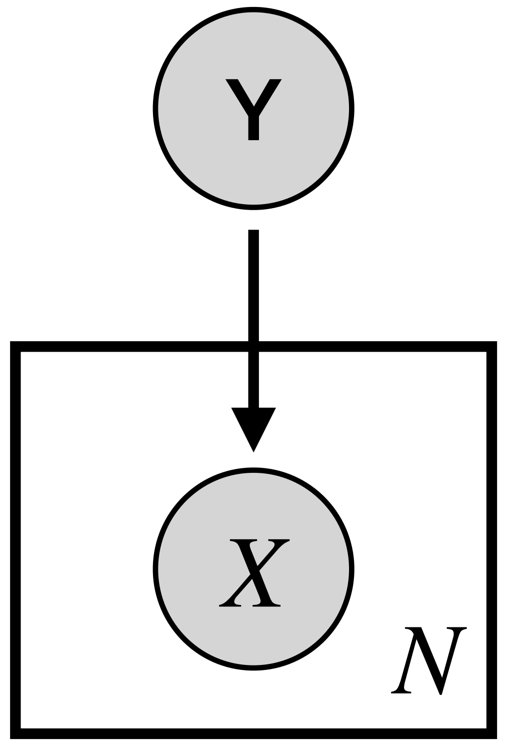
target



Features

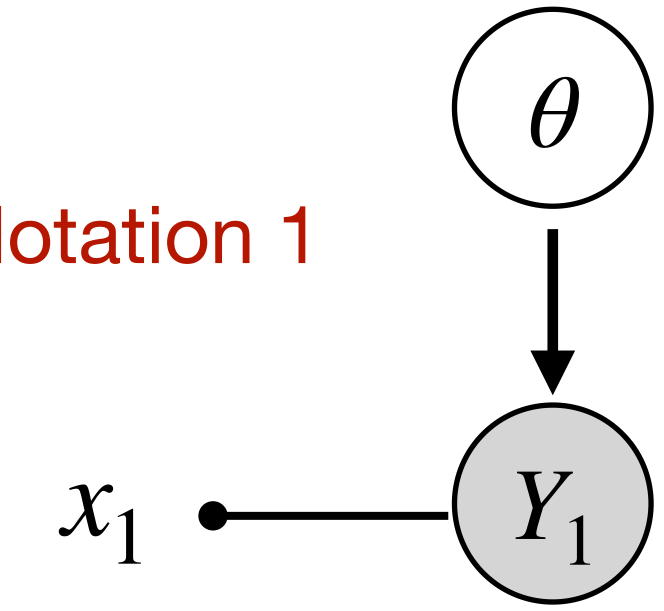
X_i , X_j conditionally independent given Y

Plate notation



Bayesian Linear regression (x_1, y_1) with $x_1 \in \mathbb{R}^d$, $y_1 \in \mathbb{R}$

Notation 1



weights

target

$$P(\theta, y_1 \mid x_1) = P(y_1 \mid \theta, x_1) \times P(\theta)$$

$$P(y_1 \mid \theta, x_1) = \mathcal{N}(y_1 \mid \theta^T x_1,\ \sigma^2)$$

$$P(\theta) = \mathcal{N}(\theta \mid 0,\ \gamma^2)$$

Legend :

Fixed variable

Hidden (latent) r.v.

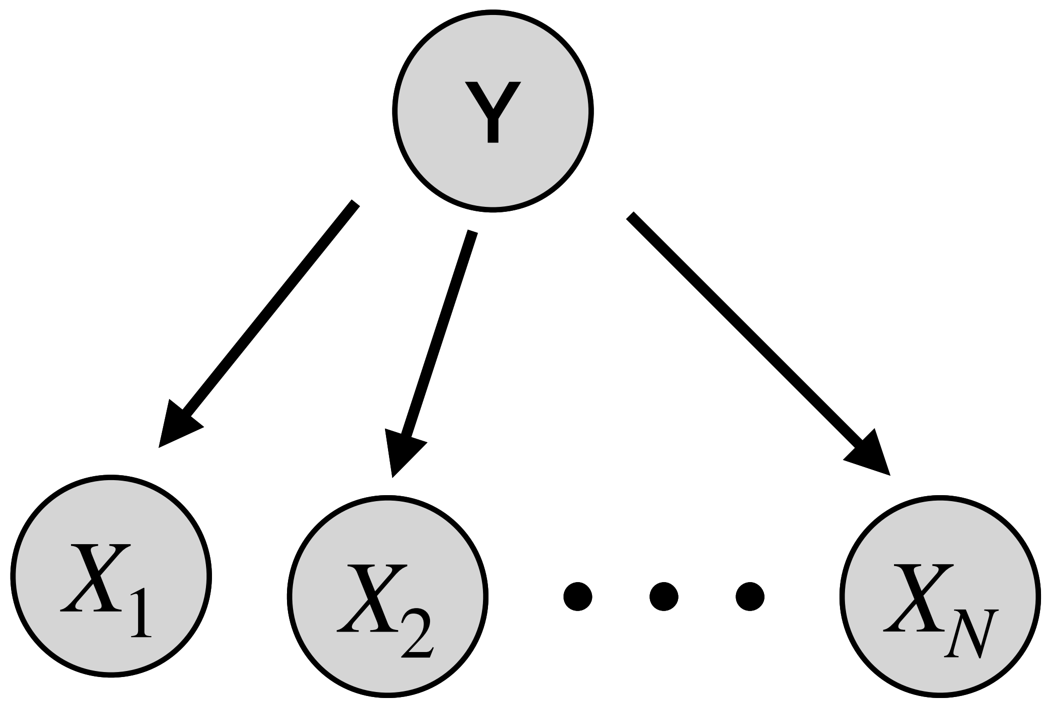
Observed r.v.

2. Probabilistic model

Plates and examples of probabilistic model

Naive Bayes Classifier $P(Y, X_1, \cdots, X_N) = P(Y) \prod_{i=1\ldots,N} P(X_i \mid Y)$

target



Features
X_i , X_j conditionally
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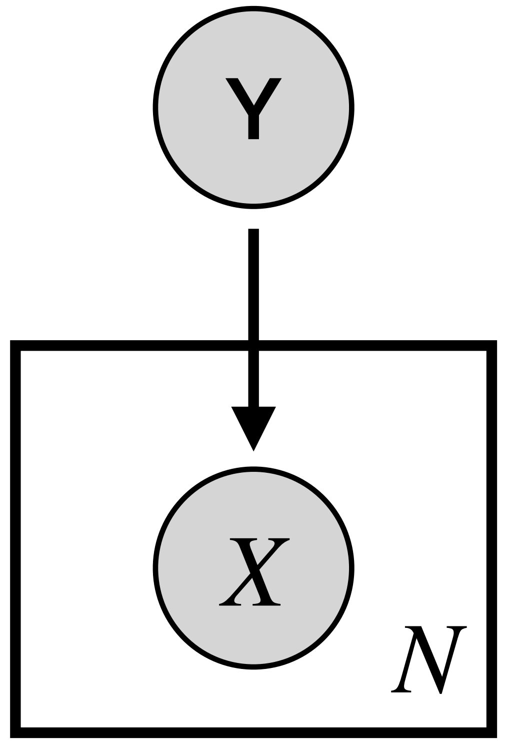
Plate notation



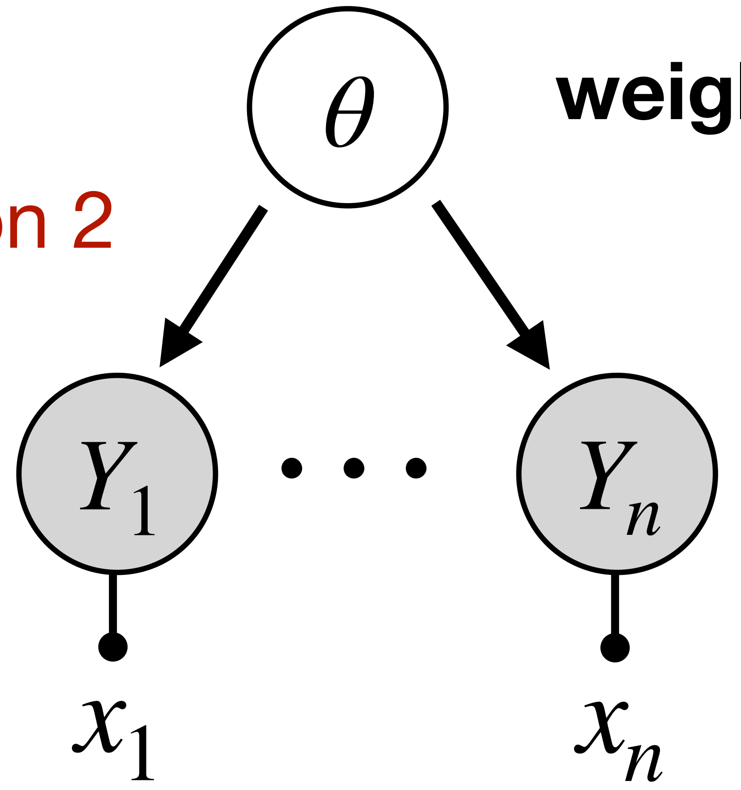
Bayesian Linear regression $D = \{(x_1, y_1), \ldots, (x_n, y_n)\}$ with $x_i \in \mathbb{R}^d$, $y_i \in \mathbb{R}$

$$P(\theta, y_i \mid x_i) = P(y_i \mid \theta, x_i) \times P(\theta)$$

$$P(y_i \mid \theta, x_i) = \mathcal{N}(y_i \mid \theta^T x_i, \sigma^2)$$

$$P(\theta) = \mathcal{N}(\theta \mid 0, \gamma^2)$$

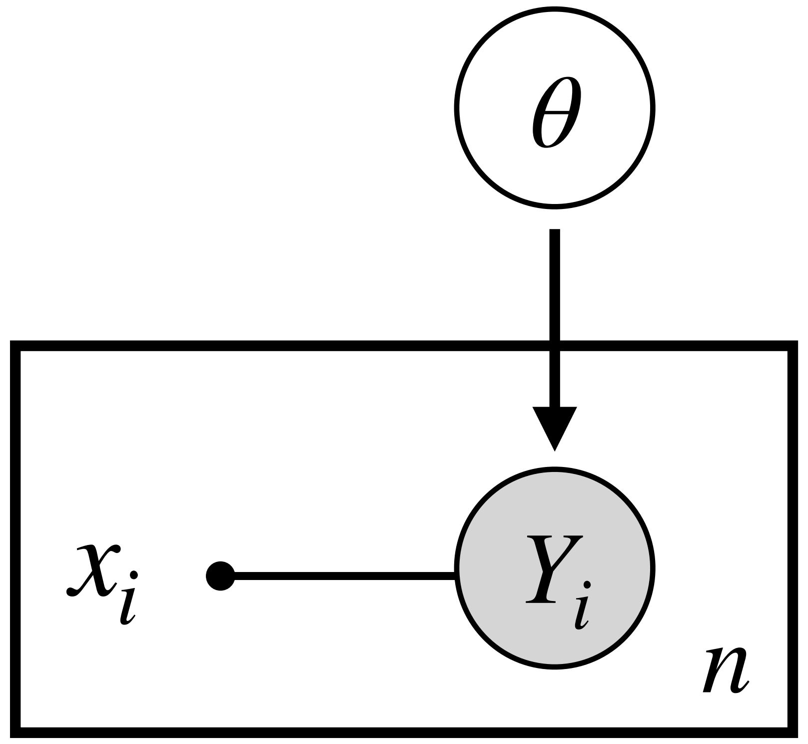
Notation 2



weights

target

Plate notation



Legend :

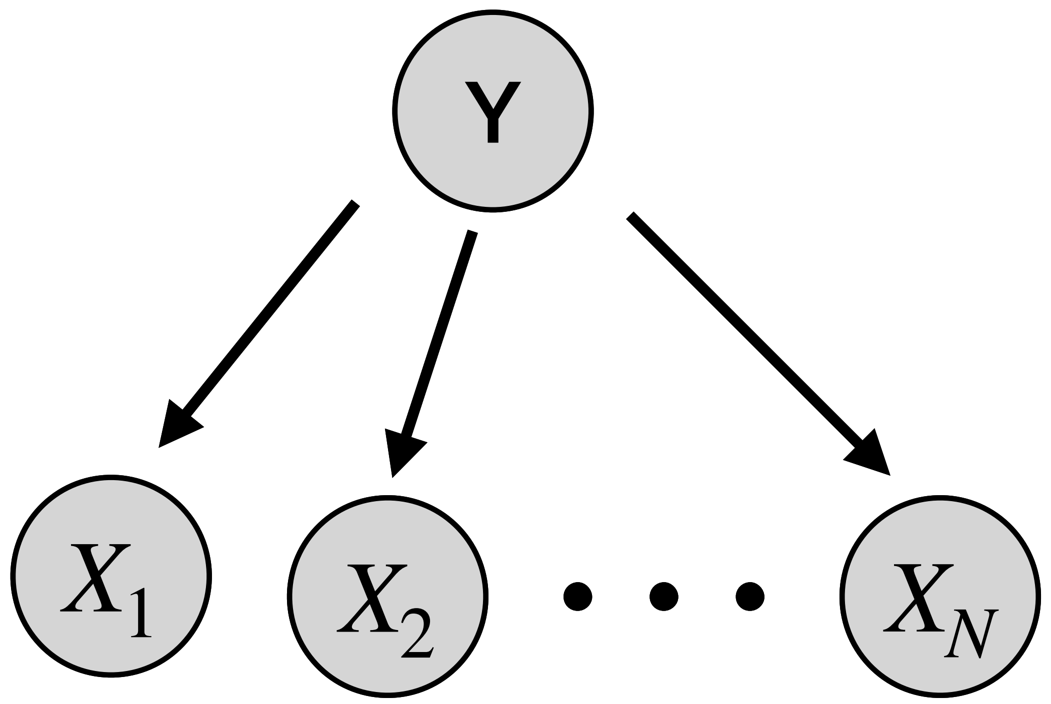
2. Probabilistic model

Plates and examples of probabilistic model

Naive Bayes Classifier

$$P(Y, X_1, \cdots, X_N) = P(Y) \prod_{i=1\ldots,N} P(X_i \mid Y)$$

target



Features
X_i , X_j conditionally
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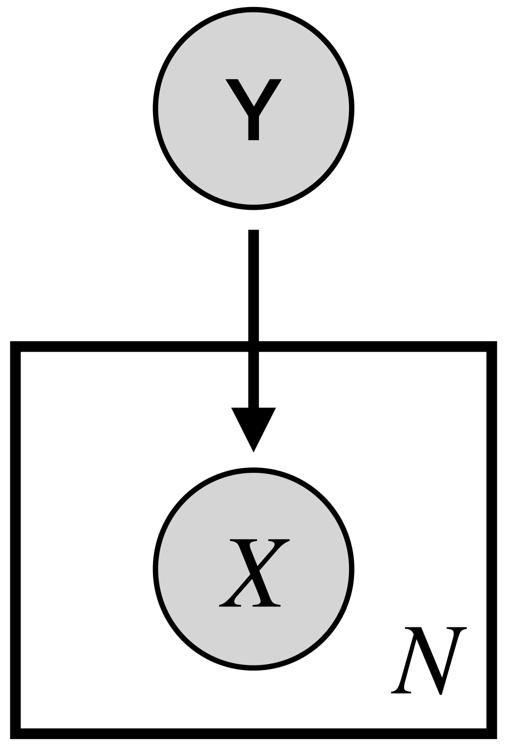
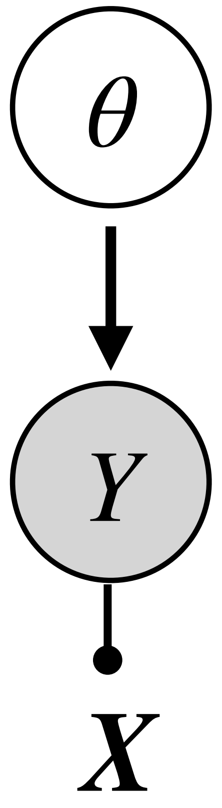
Plate notation



Bayesian Linear regression $\mathbf{X} = (x_1, \ldots, x_n)$ and $\boldsymbol{y} = (y_1, \ldots, y_n)$ with $x_i \in \mathbb{R}^d$, $y_i \in \mathbb{R}$

Notation 3



weights

target

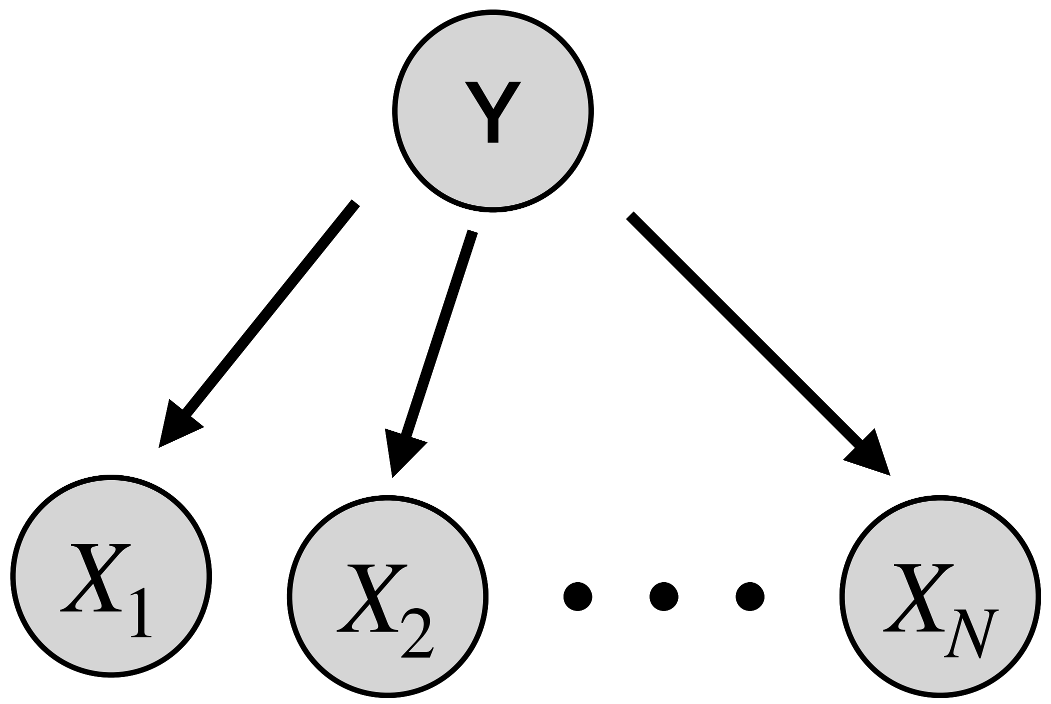
$$P(\theta, y \mid x) = P(y \mid \theta, x) \times P(\theta)$$
$$P(y \mid \theta, x) = \ldots$$
$$P(\theta) = \ldots$$

2. Probabilistic model

Plates and examples of probabilistic model

Naive Bayes Classifier $P(Y, X_1, \cdots, X_N) = P(Y) \prod_{i=1\ldots,N} P(X_i \mid Y)$

target



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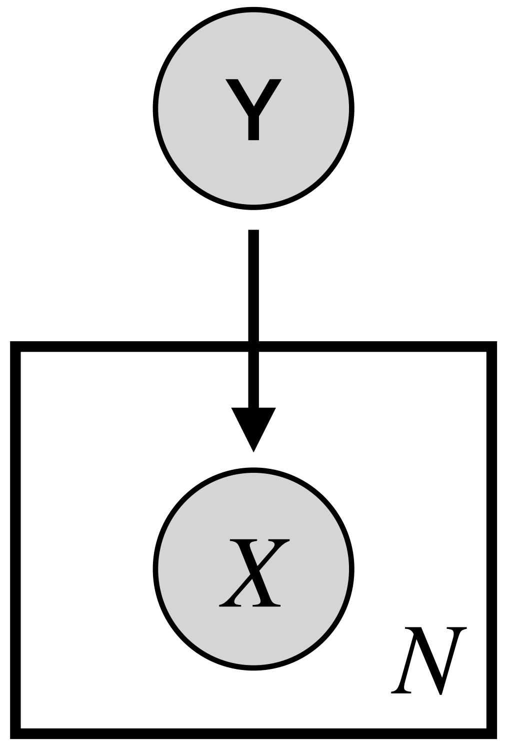
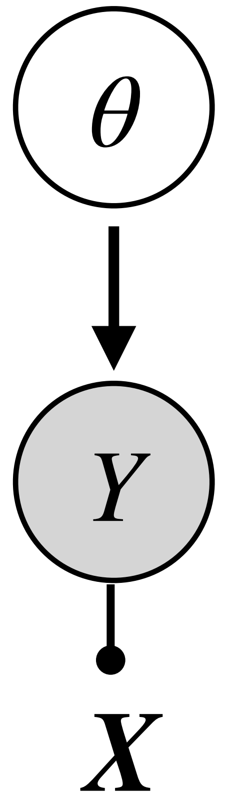


Bayesian Linear regression $\mathbf{X} = (x_1, \ldots, x_n)$ and $\boldsymbol{y} = (y_1, \ldots, y_n)$ with $x_i \in \mathbb{R}^d$, $y_i \in \mathbb{R}$

Notation 3



$$P(\theta, y \mid x) = P(y \mid \theta, x) \times P(\theta)$$

$$P(y \mid \theta, x) = \mathcal{N}(y \mid \theta^T X, \sigma^2 I_n)$$

$$P(\theta) = \mathcal{N}(\theta \mid 0, \gamma^2 I_n)$$

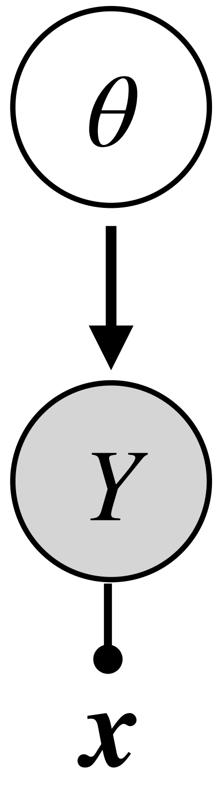
Legend :

Fixed variable
Hidden (latent) r.v.
Observed r.v.

2. Probabilistic model

Linear regression

Bayesian Linear regression



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Objective :

Frequentist linear regression

Objective : $\min\limits_{\theta} \mathcal{L}(\theta) = \min\limits_{\theta} \|\theta^t X - y\|^2$

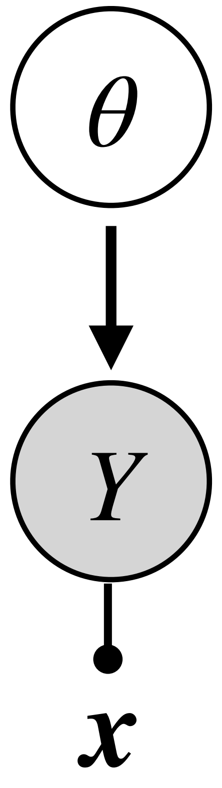
$$\hat{\theta} = \arg\min_{\theta} \mathcal{L}(\theta)$$

$$\hat{\theta} = (X^T X)^{-1} X^T y$$

2. Probabilistic model

Linear regression

Bayesian Linear regression



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Objective : $\arg\max_{\theta} P(\theta \mid X, y) = \arg\max_{\theta} P(\theta, y \mid X)$

Frequentist linear regression

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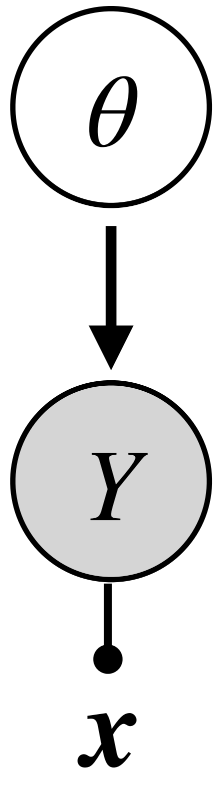
Linear regression

Bayesian Linear regression



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Frequentist linear regression

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Theorem :

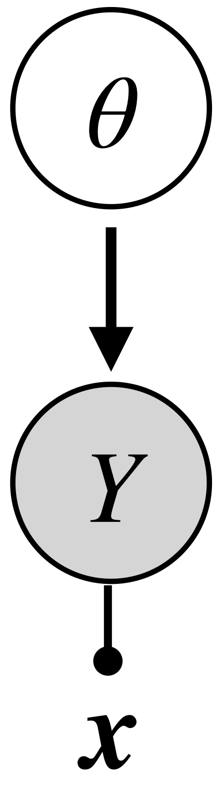
There exists $\lambda \in \mathbb{R}$ such that : $\arg\max_{\theta} P(\theta \mid X, y) = \arg\min_{\theta} \left\{ \|\theta^t X - y\|^2 + \lambda \|\theta\|^2 \right\}$

So by adding a normal prior on the weight we turned this problem into a L_2 **regularised problem**

2. Probabilistic model

Linear regression

Bayesian Linear regression



$$P(\theta, y \,|\, X) = P(y \,|\, \theta, X) \times P(\theta)$$

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Objective : $\arg\max_\theta P(\theta \,|\, X, y) = \arg\max_\theta P(\theta, y \,|\, X)$

Frequentist linear regression

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So by adding a normal prior on the weight we turned this problem into a L_2 **regularised problem**

Proof : see the whiteboard in class <u>**or**</u> left as an exercice

3 Analytical Inference

3. Analytical Inference

Reminder of posterior distribution

Posterior distribution

Likelihood **Prior**

$$P(\theta \mid X) = \frac{P(X, \theta)}{P(X)} = \frac{P(X \mid \theta) \times P(\theta)}{P(X)}$$

Posterior

Evidence

3. Analytical Inference

Reminder of posterior distribution

Posterior distribution

Fixed by **model**

Fixed by **us**

Likelihood   Prior

$$P(\theta|X) = \frac{P(X,\theta)}{P(X)} = \frac{P(X|\theta) \times P(\theta)}{P(X)}$$

Posterior

Evidence

Fixed by **data**

3. Analytical Inference

Reminder of posterior distribution

Posterior distribution

Fixed by model — **Likelihood**

Prior — **Fixed by us**

$$P(\theta|X) = \frac{P(X,\theta)}{P(X)} = \frac{P(X|\theta) \times P(\theta)}{P(X)}$$

Posterior

Evidence

HARD TO COMPUTE

Fixed by data

$$P(X) = \int_\theta P(X|\theta) \cdot P(\theta) \cdot d\theta$$

3. Analytical Inference

Maximum a posteriori (MAP) : definition & remarks

Posterior distribution

Fixed by model

Fixed by us

Likelihood Prior

$$\underbrace{P(\theta|X)}_{\text{Posterior}} = \frac{P(X,\theta)}{P(X)} = \frac{P(X|\theta) \times P(\theta)}{P(X)}$$

Evidence

HARD TO COMPUTE

Fixed by data

$$P(X) = \int_\theta P(X|\theta) \cdot P(\theta) \cdot d\theta$$

Remarks

- We have to avoid computing the evidence
- Naive approach : maximum a posteriori ,

$$\hat{\theta}_{MAP} = \arg\max_\theta \left\{ \frac{P(\theta|X) \cdot P(\theta)}{P(X)} \right\}$$

$$= \arg\max_\theta P(X|\theta) \cdot P(\theta)$$

- This maximization can be done with numerical optimization problem

3. Analytical Inference

Maximum a posteriori (MAP) : limitations

Posterior distribution

Fixed by model

Fixed by us

Likelihood Prior

$$P(\theta|X) = \frac{P(X,\theta)}{P(X)} = \frac{P(X|\theta) \times P(\theta)}{P(X)}$$

Posterior

Evidence

HARD TO COMPUTE

Fixed by data

$$P(X) = \int_\theta P(X|\theta) \cdot P(\theta) \cdot d\theta$$

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- We have to avoid computing the evidence
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$$\hat{\theta}_{MAP} = \arg\max_\theta \left\{ \frac{P(\theta|X) \cdot P(\theta)}{P(X)} \right\}$$

$$= \arg\max_\theta P(X|\theta) \cdot P(\theta)$$

- This maximization can be done with numerical optimization problem

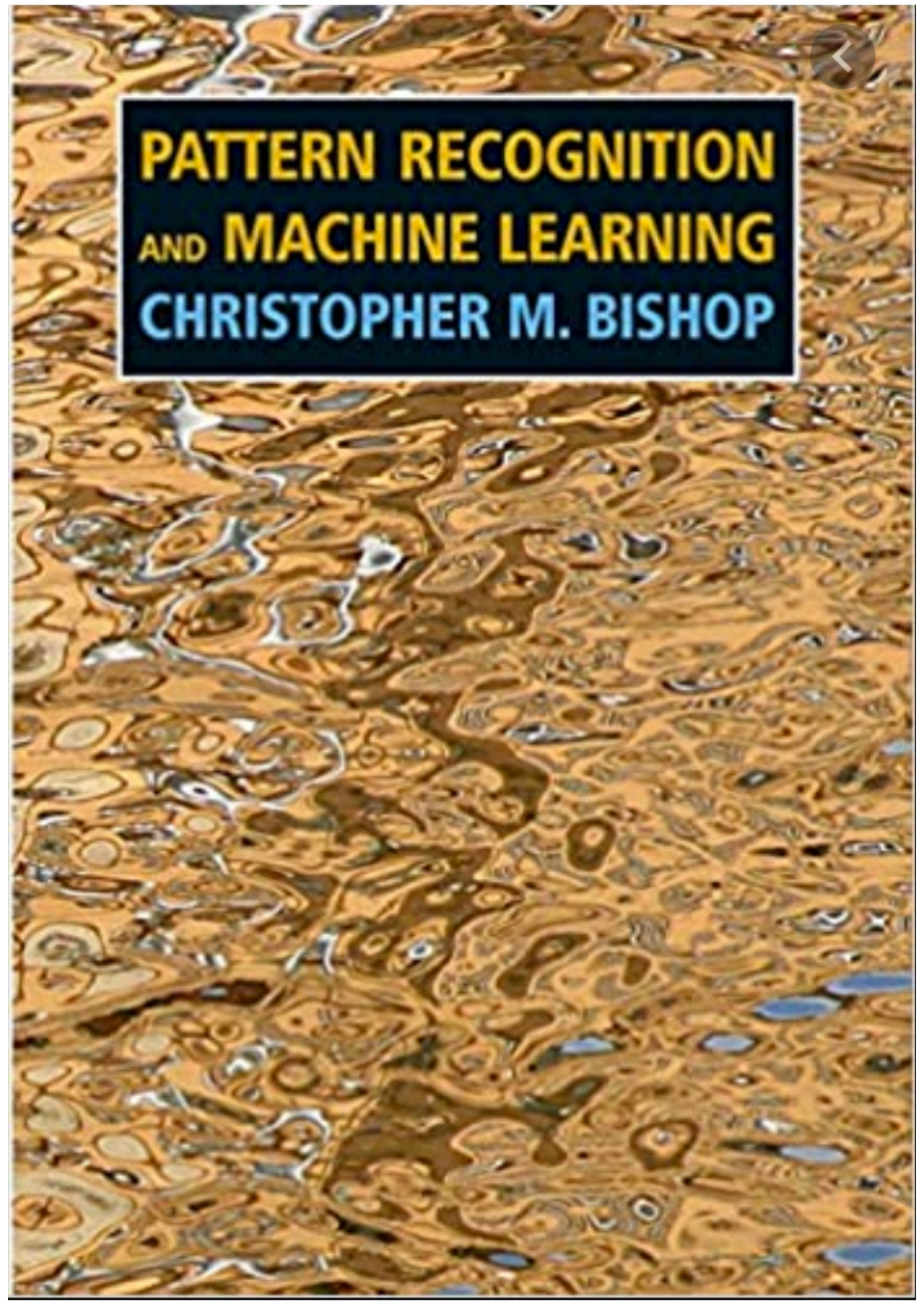
Limitations (among many others)

1. in general, not representative of bayesian methods : $\hat{\theta}_{MAP}$ is a point estimate like $\hat{\theta}_{MLE}$
   - can't compute credible intervals because it doesn't return a pdf/pmf (not a bayesian inference)
2. can't use online learning : the prior is not well updated

3. Analytical Inference

Maximum a posteriori (MAP) : book

For more theoretical details (and example on analytical inference) :



4 Conjugate distributions

4. Conjugate distributions

Conjugate distributions : avoid computing evidence

Posterior distribution

Fixed by model

Fixed by us

Likelihood

Prior

$$P(\theta|X) = \frac{P(X,\theta)}{P(X)} = \frac{P(X|\theta) \times P(\theta)}{P(X)}$$

Posterior

Evidence

Fixed by data

4. Conjugate distributions

Conjugate distributions : avoid computing evidence

Posterior distribution

Fixed by **model**

Fixed by **us**

Likelihood **Prior**

$$P(\theta|X) = \frac{P(X,\theta)}{P(X)} = \frac{P(X|\theta) \times P(\theta)}{P(X)}$$

Posterior

Evidence

Fixed by **data**

Remarks

- We have to avoid computing the evidence
- We can choose a convenient prior which enable us to compute the **posterior :**

  Conjugate prior

4. Conjugate distributions

Conjugate distributions : avoid computing evidence

Posterior distribution

Fixed by model — **Likelihood**

Prior — **Fixed by us**

$$P(\theta|X) = \frac{P(X,\theta)}{P(X)} = \frac{P(X|\theta) \times P(\theta)}{P(X)}$$

Posterior

Evidence — **Fixed by data**

Remarks

- We have to avoid computing the evidence
- We can choose a convenient prior which enable us to compute the **posterior :**

  Conjugate prior

Conjugate prior

$P(\theta)$ is **conjugate** to $P(X|\theta)$ if the $P(\theta)$ and $P(X|\theta)$ lie in the same family of distributions (gaussian for example)

4. Conjugate distributions

Conjugate distributions : avoid computing evidence

Posterior distribution

Fixed by **model**

Fixed by **us**

Likelihood Prior

$$P(\theta|X) = \frac{P(X,\theta)}{P(X)} = \frac{P(X|\theta) \times P(\theta)}{P(X)}$$

Posterior

Evidence

Fixed by **data**

Remarks

- We have to avoid computing the evidence
- We can choose a convenient prior which enable us to compute the **posterior :**

  Conjugate prior

Conjugate prior

$P(\theta)$ is **conjugate** to $P(X|\theta)$ if the $P(\theta)$ and $P(X|\theta)$ lie in the same family of distributions (gaussian for example)

Example

$$P(\theta|X) = \frac{\mathcal{N}(X|\theta,\sigma^2) \times P(\theta)}{P(X)}$$

$\mathcal{N}(\theta|\mu_{prior}, \sigma^2_{prior})$

$\mathcal{N}(\theta|\mu_{posterior}, \sigma^2_{posterior})$

In the context of a gaussian, the prior for the **mean** is a gaussian !

4. Conjugate distributions

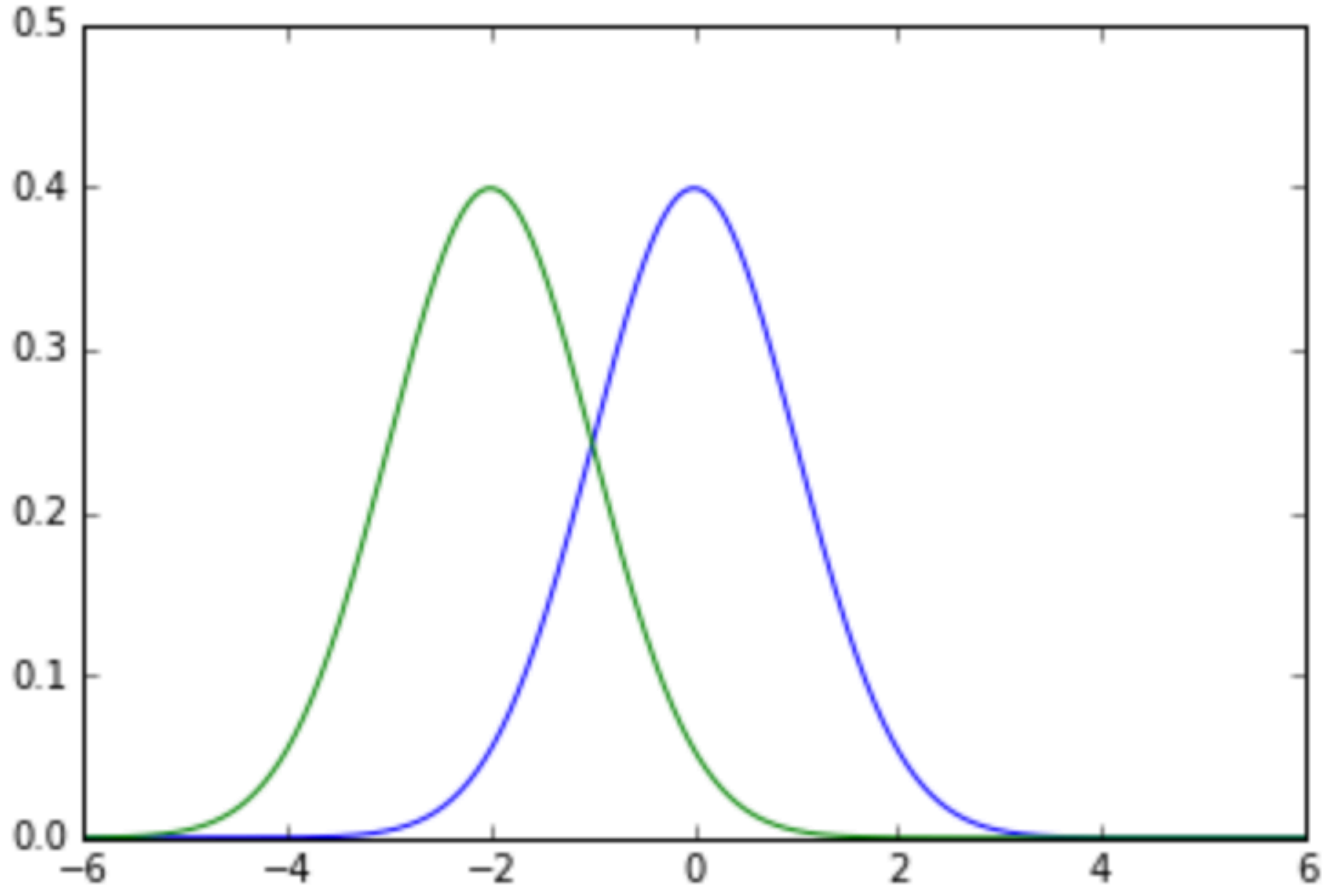
Conjugate distributions : example

Example

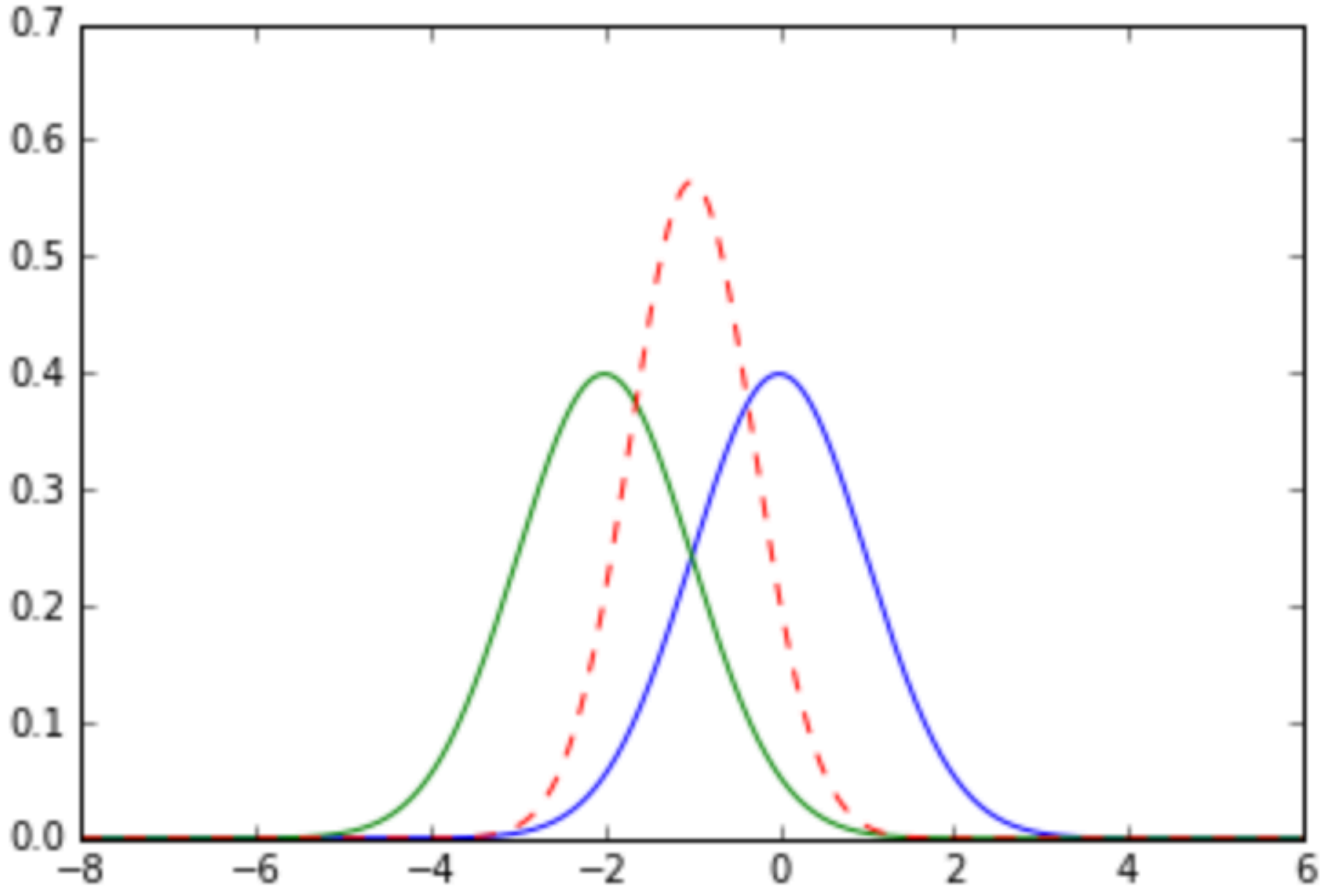
$$P(\theta|X) = \frac{\mathcal{N}(X|\theta,\sigma^2) \times P(\theta)}{P(X)}$$

$\mathcal{N}(\theta|\mu_{posterior}, \sigma^2_{posterior})$

$\mathcal{N}(\theta|\mu_{prior}, \sigma^2_{prior})$



pointwise product



Exercice (left as an exercice, correction in the next lecture)

Show that $\mathcal{N}(\theta|x/2,1/2) = \dfrac{\mathcal{N}(x|\theta,1) \times \mathcal{N}(\theta|0,1)}{P(x)}$

4. Conjugate distributions

Usual distributions : Gamma distribution

Gamma distribution

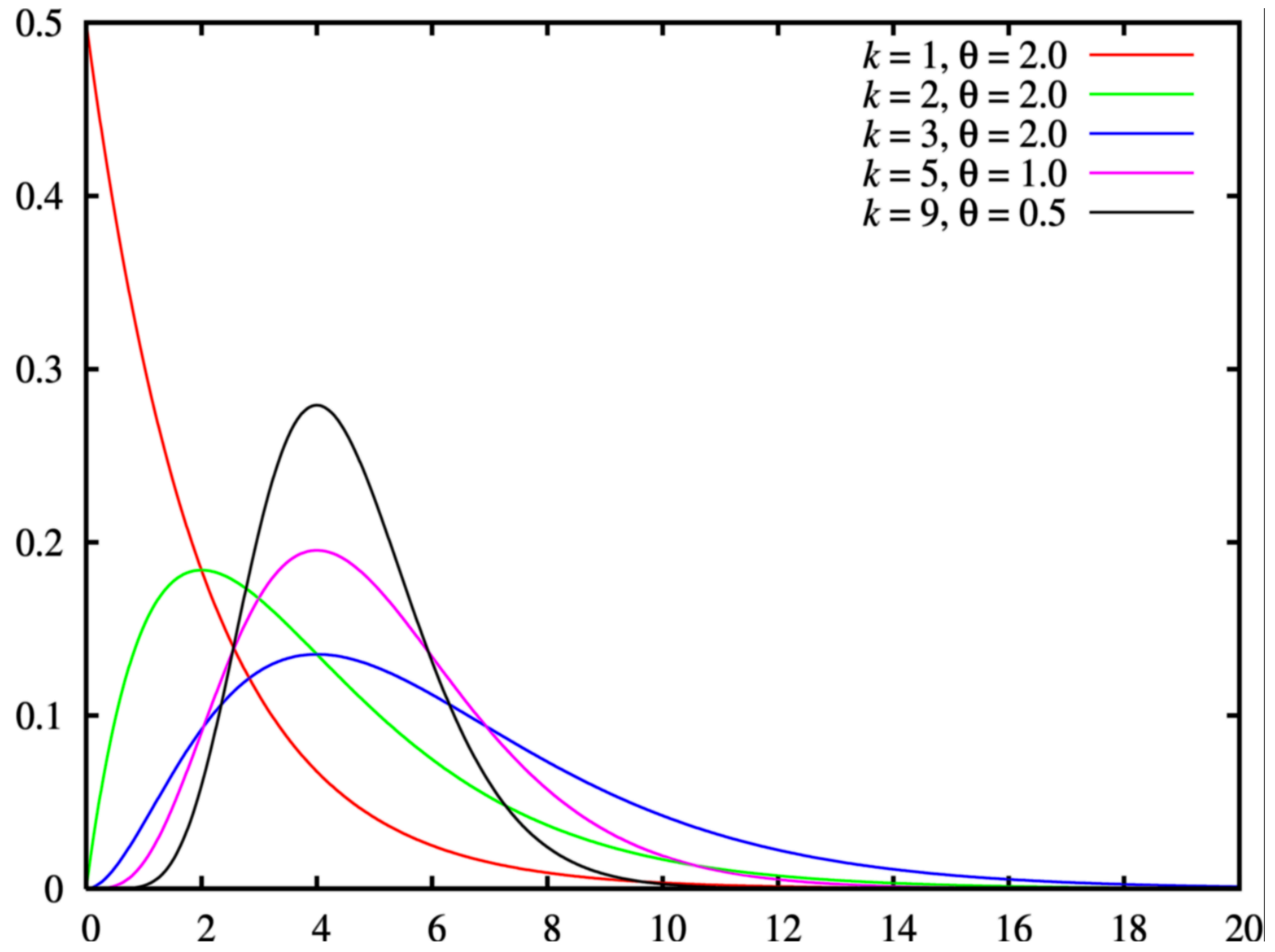
PDF : $\Gamma(x\,|\,\alpha,\beta) = \dfrac{\beta^{\alpha}}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}$ with $x, \alpha, \beta > 0$

$\Gamma(\alpha) = (\alpha - 1)!$

mean : $\mathbb{E}[x] = \dfrac{\alpha}{\beta}$

variance : $V(x) = \dfrac{\alpha}{\beta^2}$

mode : $Mode\,[x] = \dfrac{\alpha - 1}{\beta}$



Example

$$\Gamma(\gamma\,|\,\alpha_{posterior}, \beta_{posterior}) \quad P(\gamma\,|\,x) = \frac{\mathcal{N}(x\,|\,\mu, \gamma^{-1}) \times P(\gamma)}{P(x)} \quad \Gamma(\gamma\,|\,\alpha_{prior}, \beta_{prior})$$

In the context of a gaussian, the prior for the **precision** is a gamma !

4. Conjugate distributions

Usual distributions : Gamma distribution

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PDF : $\Gamma(x \mid \alpha, \beta) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}$ with $x, \alpha, \beta > 0$

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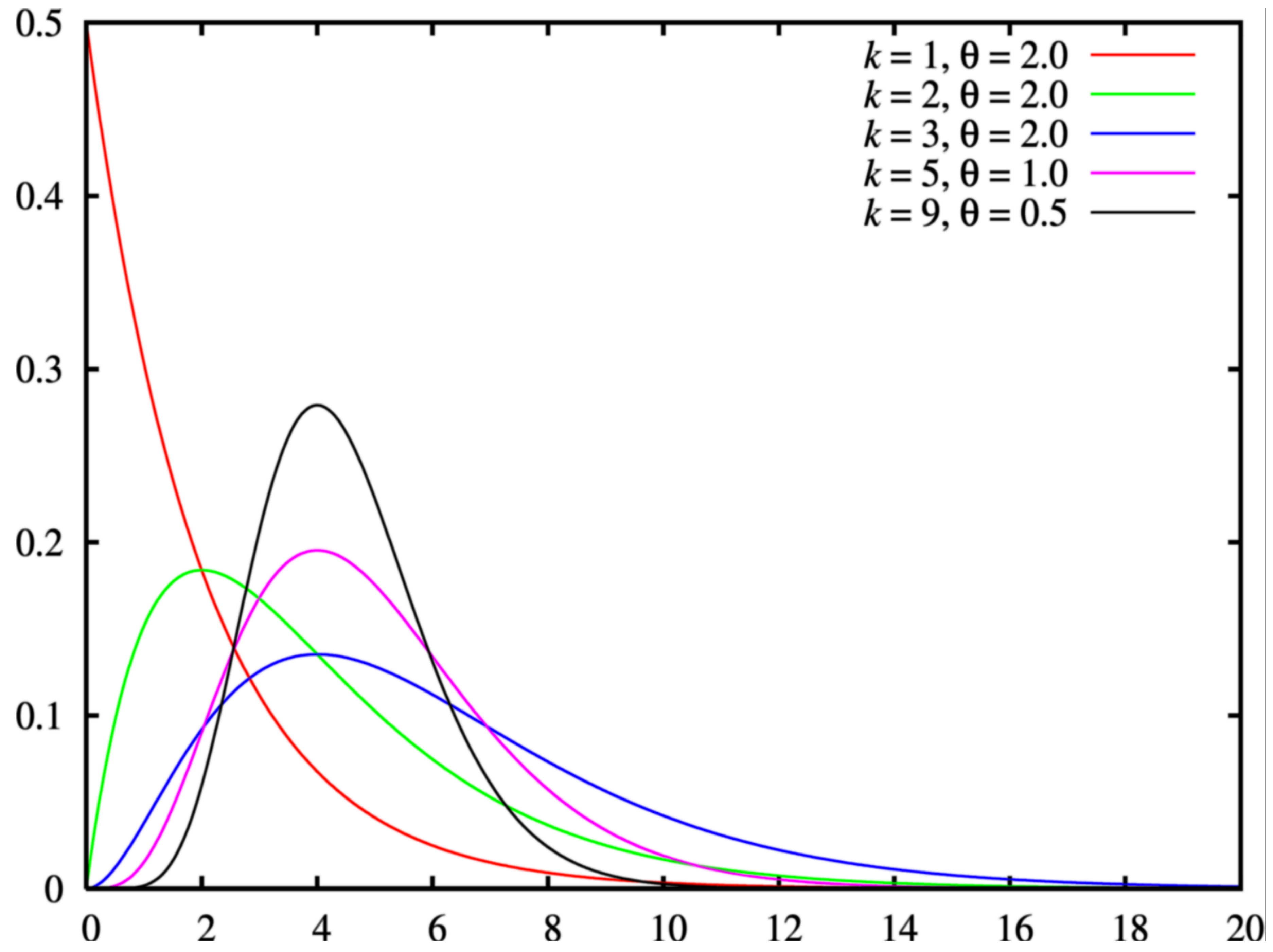


Exercice (left as an exercice, correction in the next lecture)

$$P(\gamma \mid x) = \frac{\mathcal{N}(x \mid \mu, \gamma^{-1}) \times P(\gamma)}{P(x)}$$

$\Gamma(\gamma \mid \alpha_{prior}, \beta_{prior})$

$\Gamma(\gamma \mid \alpha_{posterior}, \beta_{posterior})$

$\Gamma(\gamma \mid \alpha_{prior} + 1/2,\ \beta_{prior} + (x - \mu)^2/2)$

In the context of a gaussian, the prior for the **precision** is a gamma !

4. Conjugate distributions

Usual distributions : Beta distribution

Beta distribution

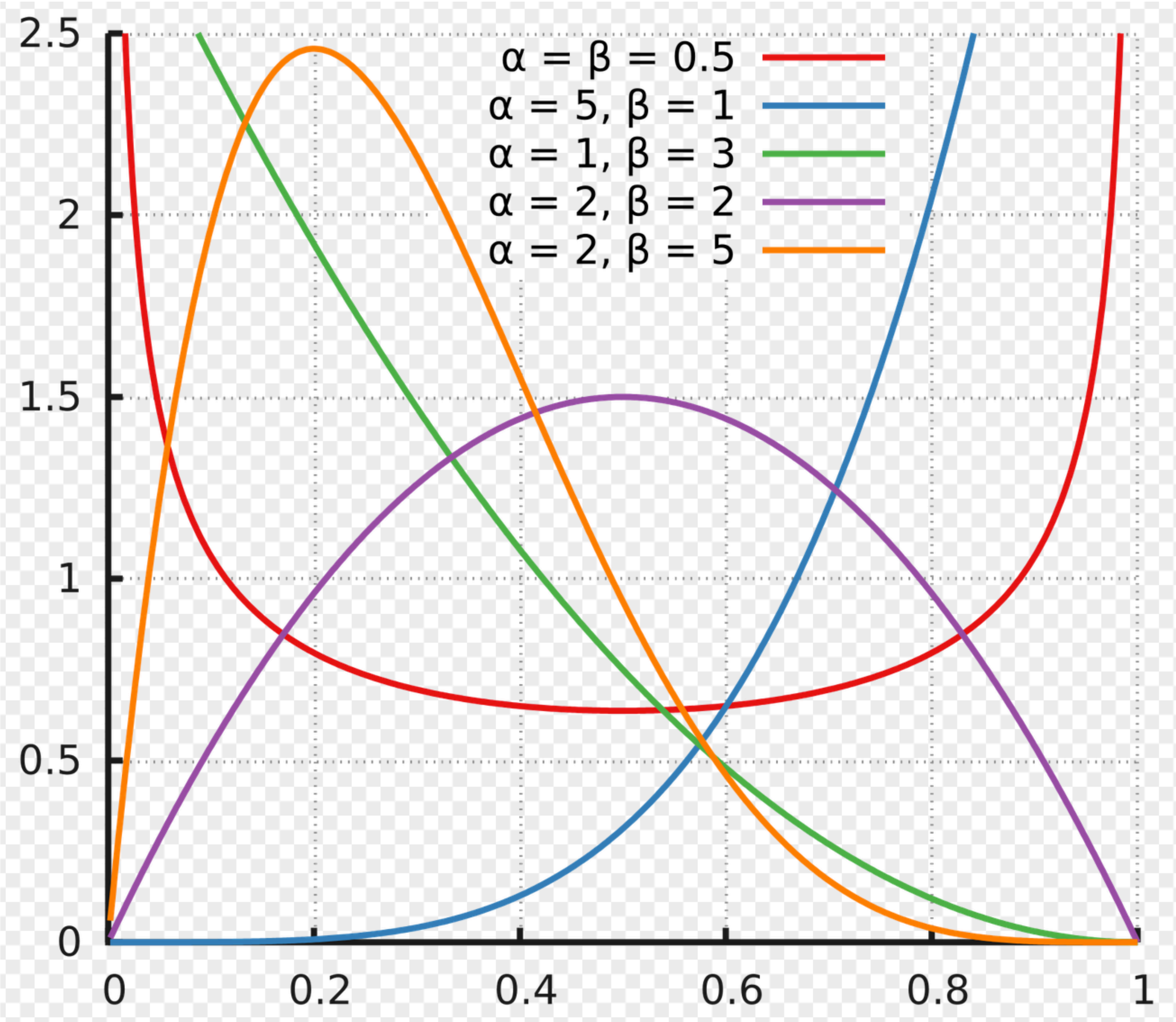
PDF : $$B(x\,|\,\alpha,\beta) = \frac{1}{B(\alpha,\beta)} x^{\alpha-1}(1-x)^{\beta-1} \text{ with } \alpha,\beta > 0 \text{ and } x \in [0,1]$$

$$B(\alpha,\beta) = \frac{\Gamma(\alpha)\cdot\Gamma(\beta)}{\Gamma(\alpha+\beta)}$$

mean : $$\mathbb{E}[x] = \frac{\alpha}{\alpha+\beta}$$

variance : $$V(x) = \frac{\alpha\beta}{(\alpha+\beta)^2\cdot(\alpha+\beta-1)}$$

mode : $$Mode\,[x] = \frac{\alpha-1}{\alpha+\beta-2}$$



Example

$$\underbrace{B(\theta\,|\,\alpha_{posterior},\beta_{posterior})}\ —\ P(\theta\,|\,x) = \frac{Ber(x\,|\,\theta)\times P(\theta)}{P(x)} \ —\ B(\theta\,|\,\alpha_{prior},\beta_{prior})$$

In the context of a Bernoulli distribution, the prior is a beta !

4. Conjugate distributions

Usual distributions : Beta distribution

Beta distribution

PDF : $B(x\,|\,\alpha,\beta) = \dfrac{1}{B(\alpha,\beta)} x^{\alpha-1}(1-x)^{\beta-1}$ with $\alpha,\beta > 0$ and $x \in [0,1]$

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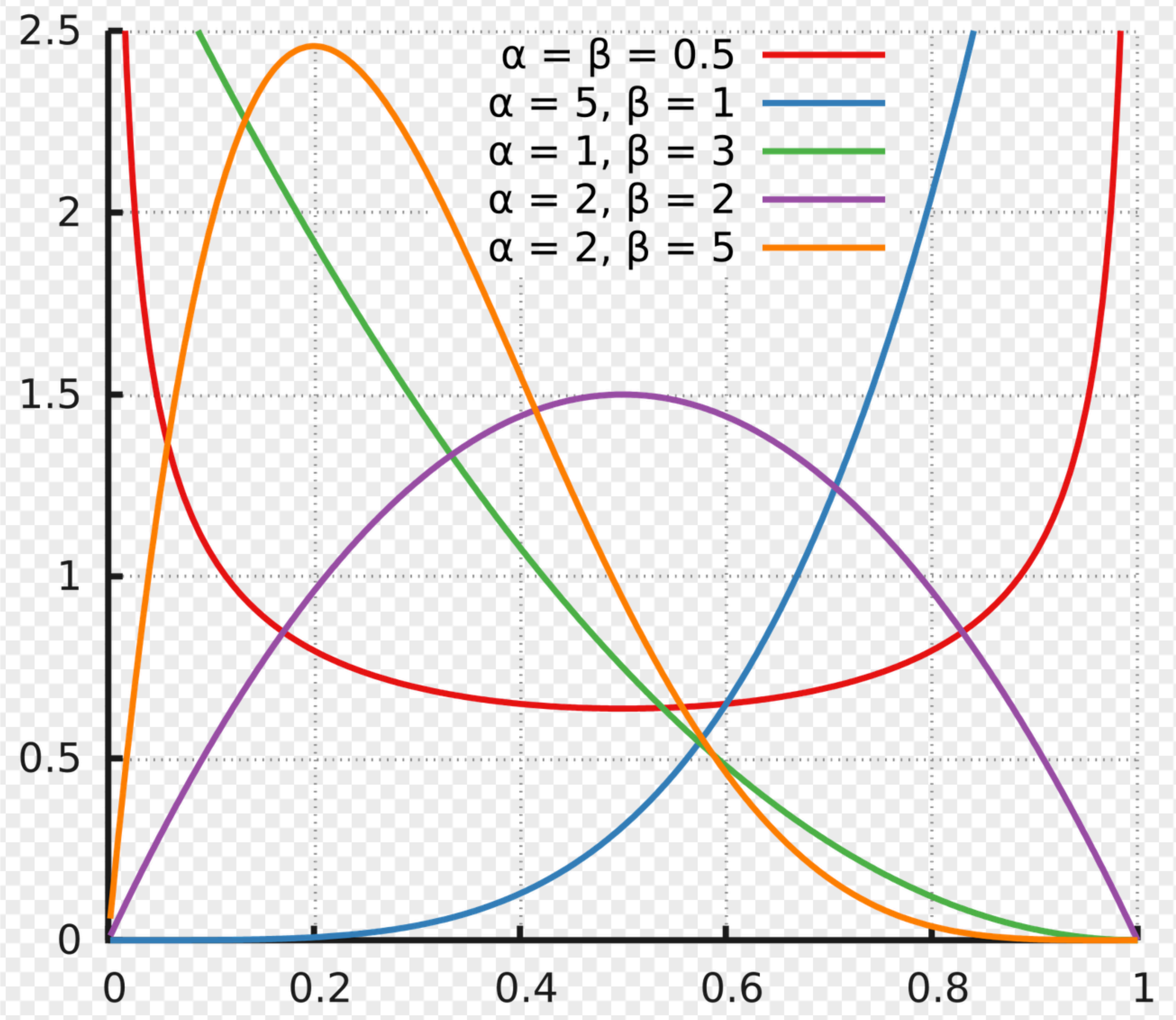

Exercice (left as an exercice, correction in the next lecture)

$$\theta^{n_1}\cdot(1-\theta)^{n_0}$$

$$B(\theta\,|\,\alpha_{prior},\beta_{prior})$$

$$B(\theta\,|\,\alpha_{posterior},\beta_{posterior}) \qquad P(\theta\,|\,x) = \frac{Ber(x\,|\,\theta)\times P(\theta)}{P(x)}$$

$$B(\theta\,|\,n_1+\alpha_{prior}, n_0+\beta_{prior})$$

In the context of a Bernoulli distribution, the prior is a beta !

4. Conjugate distributions

Limitations

Posterior distribution

Fixed by **model**

Fixed by **us**

Likelihood Prior

$$P(\theta|X) = \frac{P(X,\theta)}{P(X)} = \frac{P(X|\theta) \times P(\theta)}{P(X)}$$

Posterior

Evidence

Fixed by **data**

Remarks

- We have to avoid computing the evidence
- We can choose a convenient prior which enable us to compute the **posterior :**

  Conjugate prior

- It computes the exact posterior
- Easy for online learning

- For some (complex) models, the conjugate prior can be inadequate (improper prior)
- Can be unrealistic (non-informative prior)

! Road map

Bayesian statistics (03/05/21)

1

Bayesian perspective :

Likelihood

Prior distribution

$$P(\theta | X) = \frac{P(X, \theta)}{P(X)} = \frac{P(X | \theta) \cdot P(\theta)}{P(X)}$$

Posterior distribution

Evidence

Hard to compute !

θ parameters

X observations

Exemple :
Naive Bayes classifier,
Linear regression, ….

MAP : $\arg\max_{\theta} P(X | \theta) \cdot P(\theta)$

Conjugate distribution

Pros :	Cons :
- exact posterior	- conjugate prior maybe inadequate

Latent variable models (17/05/21)

2

Variational Inference (31/05/21)

3

Markov Chain Monte Carlo (07/06/21)

4

Extensions (14/06/21)

5