Ground Target Tracking – a Historical Perspective[1]

Chee-Yee Chong
Booz Allen & Hamilton, Inc.
101 California St., Suite 3300
San Francisco, CA 94111
415-281-4948
chong_chee@bah.com

David Garren
Science Applications International Corp.
4001 North Fairfax Dr, Suite 300
Arlington, VA 22203
703-558-2768
david.a.garren@cpmx.saic.com

Timothy P. Grayson
Defense Advanced Research Projects Agency
3701 North Fairfax Dr.
Arlington, VA 22203
703-248-1519
tgrayson@darpa.mil

Abstract — Airborne sensor platforms such as Joint STARS provide a capability for ground surveillance and monitoring target movements. Because of the high target density and maneuverability, high clutter, low visibility due to terrain masking, etc., ground target tracking presents unique challenges not present in tracking other types of targets. This paper reviews major developments in multi-target tracking over the past four decades and discusses how algorithms developed primarily for tracking air targets can be used for tracking ground targets. The similarities and differences between ground and air target tracking are first compared. We then discuss how simple target state estimation algorithms such as Kalman filtering have evolved into more complicated algorithms for tracking maneuvering targets. Similarly, we discuss how association algorithms have progressed from nearest neighbor to joint probabilistic data association (JPDA), multiple hypothesis tracking (MHT), and multi-dimensional assignment. The adequacy of these techniques for tracking ground targets is discussed.

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1. INTRODUCTION

Over the past four decades, multiple target tracking has developed into a fairly mature technology. Applications now include diverse areas such as air traffic control, air and missile defense, avionics, ocean surveillance, port monitoring, etc. Sensors that provide data include radar, infrared, acoustic, etc. However, until recently, there has not been much emphasis on tracking ground targets due to the lack of sensors.

The introduction of the Ground Moving Target Indicator (GMTI) radar has drastically changed the situation. A GMTI radar can provide detections on moving ground vehicles over a large area. During Operation Desert Storm in 1991, Joint Surveillance Target Attack Radar System (Joint STARS) aircraft proved to be extremely valuable in providing the commander with a ground picture of the battlefield. An example of its capability is the now famous picture that shows "The Mother of All Retreats". In addition to Joint STARS, GMTI have been considered for other platforms such as Unmanned Aerial Vehicles (UAVs). References [31] and [34] trace the development of GMTI for ground surveillance from some early programs to Joint STARS and other potential future platforms.

Besides providing a global picture of vehicle movements, GMTI measurements can be used to track individual targets. Thus, there has been increasing interest in ground target tracking and government programs have been started to develop tracking algorithms for ground targets [47]. However, ground target tracking has characteristics that are quite different from tracking other types of targets, in particular air targets. Specific differences include high target density, high clutter, possible terrain obscuration, etc. Thus, many tracking approaches that have been developed for air targets may perform poorly when used to track ground targets. The objective of this paper is to review the progress of multi-target tracking over the past 40 years and discuss its applicability to tracking ground targets.

We focus primarily on tracking non-cooperative targets over a large area. Thus we have excluded the tracking of vehicles with onboard locators (e.g., a global position system receiver) such as those used in vehicle location and navigation systems [85]. Also excluded from our review are algorithms that track vehicles over a small area using video sensors [19], [35], [44]. This paper is not intended to be a comprehensive survey of multi-target tracking. Good surveys and overviews can be found in the survey paper [3], and the books [4], [5], [9]–[11], [17], [20]. Instead, it is an overview of the major developments in tracking technology and their relevance to tracking ground targets.

The rest of this paper is structured as follows. In Section 2, we discuss differences between tracking ground and air targets such as target dynamics and density, and provide a numerical comparison of the difficulty. Section 3 reviews

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the development of target state estimation which is needed to support association, from Kalman filtering to multiple models for maneuvering targets. Section 4 contains a review of data association approaches including nearest neighbor, joint probabilistic data association, multiple hypothesis tracking and multi-dimensional assignment. This is followed by conclusions in Section 5.

2. TRACKING GROUND VERSUS AIR TARGETS

Even though tracking problems can be found in many applications, e.g., ocean surveillance and submarine tracking, most tracking algorithms have been developed for air targets. Air tracking algorithms are potentially applicable to ground targets because both have relatively high revisit rates. Thus, a comparison of the characteristics of ground and air tracking will be useful in assessing whether current tracking algorithms are adequate for ground targets.

2.1 Qualitative Comparison

A tracking problem is defined by the targets of interest, the sensors that collect the measurements, and the environment in which the targets move and sensors observe the targets.

Environment – Aircraft move in a three dimensional space while ground vehicles move over the surface of the earth. Thus the motion of ground targets is generally two dimensional. For a vehicle that moves on roads, the space becomes one dimensional on a road segment until it reaches an intersection, when it moves to other road segments. The region where the vehicles can move may be further constrained, e.g., rough terrain may limit accessibility. The same constraint may apply to air targets since mountains and ground have to be avoided.

Target Dynamics – Ground targets can accelerate, slow down and stop completely, and remain stationary for an indefinite period of time. Except for helicopters, aircraft dynamics is far more restricted with a minimum speed below which the aircraft will stall. Furthermore, some vehicles can move from off-road to on-road and back. The actual dynamics depend on the local condition, e.g., slope, terrain type, type of road, etc. Thus ground target motion is far more variable than air target motion.

Target Density – Except in very rare cases, air targets maintain reasonable separation from each other for safety reasons. Ground targets, however, can move very close to each other. When stationery, the vehicles can be parked side by side. Thus the density of ground targets is much higher than air targets.

Sensor Detection – Because of the observation geometry, ground targets can be obscured by terrain. Furthermore, MTI radars can only detect targets with radial velocities above the minimum detection velocity (MDV). There is no detection when vehicles stop or move perpendicular to the line of sight of the radar. Thus the detection of ground targets depends on dynamics and may not be continuous. On the other hand, the detection of air targets depends mostly on the radar cross section and less on the target dynamics. Synthetic Aperture Radar (SAR) has been proposed for imaging stopped targets, thus providing continuous tracking capability.

Clutter – Air target operates in a low clutter environment while ground targets move in heavy clutter. The need to remove such clutter also results in lower detection of ground targets.

Table 1 summarizes the differences between air and ground target tracking.

Table 1: Ground versus Air Tracking

	Air Targets	Ground Targets
Environment	3 D	2 D or 1 D
Dynamics	Less mobile, cannot stop	Highly mobile, may stop
Density	Low	High
Detection probability	High	Low due to terrain observation and MDV
Clutter	Low	High

2.2 Quantitative Comparison

Basic Tracking Functions – One approach of characterizing a tracking problem is in terms of the difficulty of performing the tracking functions. The basic functions in multi-target tracking are shown in Figure 1 and consist of prediction, association, and estimation. When measurements are received, the current tracks are predicted to the time of the measurements and associated with the measurements. Then the associated measurements are used to update the state estimates of the tracks. Although these functions are not always performed sequentially, they are present in most tracking algorithms.

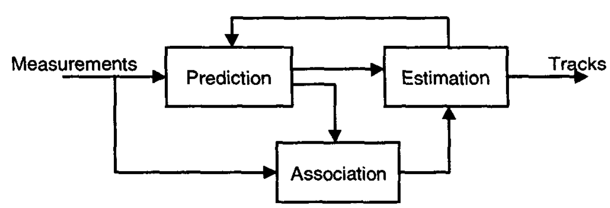

Figure 1: Basic Tracking Functions

Estimation Performance – Prediction and estimation are single target state estimation functions in the absence of measurement uncertainty. Prediction difficulty depends on target dynamics and sensor revisit time. Highly maneuverable targets will result in large prediction uncertainty while fairly static targets with high revisit rates will result in small prediction uncertainty.

Figure 2 compares the prediction and estimation performance between air and ground tracking assuming a four dimensional (2 D position and velocity) linear target motion model. The acceleration noise for air targets and ground targets are assumed to be 1g and 0.1 g respectively. The RMS measurement errors are assumed to be 60 m x 60 m for air targets and 100 m x 10 m (case 1) and 10 m x 10 m (case 2) for ground targets. The investigation of two separate cases for ground targets is intended to model the potential of using a single sensor versus two sensors with orthogonal vantage points. The prediction and estimation errors are computed using steady state Kalman filter equations and thus do not consider maneuvers. However, they represent the typical values in air and ground tracking. Thus, with perfect association, air target tracking is more difficult than ground tracking. However, as we shall see shortly, the main difficulty with ground tracking is association.

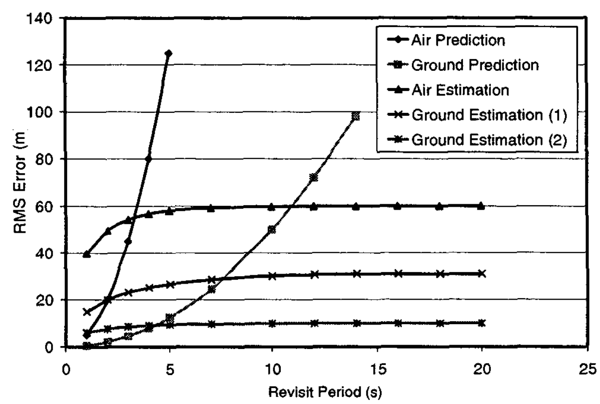

Figure 2: Estimation Performance

Association Performance – Association performance depends on target density, prediction performance and measurement accuracy. However, high target density does not necessarily result in poor association performance if the target state can be predicted accurately or the measurements are accurate. One way of representing association difficulty is in terms of the normalized target density which is the number of targets in the measurement validation gate. These are basically the measurements that can be confused with the true measurement from the target. The normalized target density can be computed as (see [65] for details)

$$\tilde{D} = (D_T + D_{FA}) | \overline{P} + R |^{1/2} \qquad (1)$$

where D_T and D_{FA} are the target and false alarm densities respectively, $\overline{P}$ is the error covariance matrix for the predicted target position, and R is the measurement error covariance matrix. The normalized target density as a function of the revisit period is shown in Figure 3, where the average target separation is assumed to be 2 Km for air targets and 50 m. for ground targets. Note that the normalized target density for ground targets increases rapidly with the revisit period.

The normalized target density represents the intrinsic difficulty in association. The actual performance depends on the specific association algorithm used and is hard to predict analytically. For single scan optimal assignment algorithms, an approximate formula for predicting the probability of correct association is given by [65]:

$$P_{CA} = \exp(-\tilde{D}) \qquad (2)$$

where $\tilde{D}$ is the normalized target density. Figure 4 shows the probability of correct association as a function of normalized target density. It can be seen from Figures 3 and 4 that the association performance for ground target tracking is expected to be much lower than air target tracking. In order to achieve reasonable performance, we need accurate target estimation algorithms and association algorithms that can handle high target density.

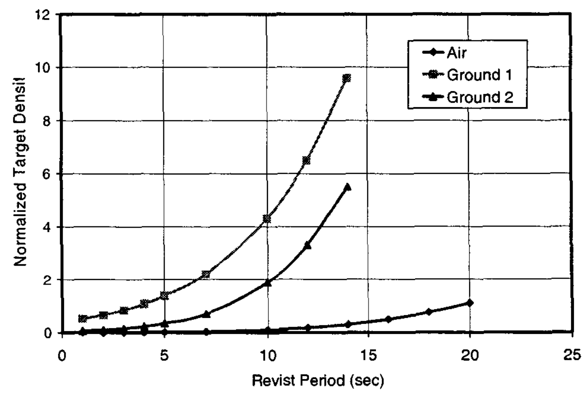

Figure 3: Normalized Target Density

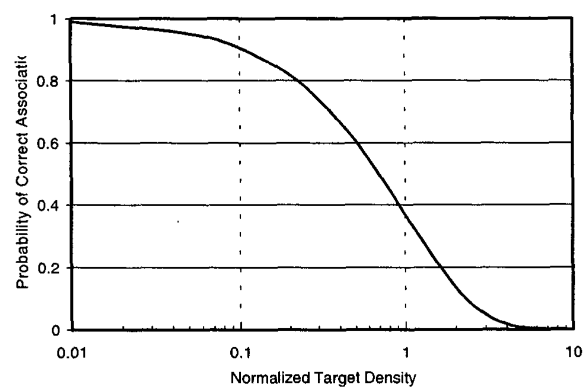

Figure 4: Probability of Correct Association

3. TARGET STATE ESTIMATION

Target state estimation is an important component of any multi-target tracking algorithm. The association of measurements to tracks requires the prediction of the target state of each track to the time of the measurements so that the measurement to track likelihood can be computed. Accurate state prediction is a key to good association performance. Once an association decision has been made, the output of the tracker consists of updated state estimates of the tracks using the associated measurements. In this section, we review the development of target state estimation algorithms and discuss their applicability to ground target tracking. Although the target state can include features, we will focus on the kinematics target state, i.e., the position and velocity.

Target state algorithms can be grouped into linear algorithms that compute the means and covariances of the target state estimate, nonlinear algorithms that compute the conditional probability distributions, and adaptive algorithms that address target maneuvers.

3.1 Linear Estimation Algorithms

These algorithms assume linear target motion and observation models and provide estimates of the target state by means of linear transformations.

Alpha-Beta-Gamma Filters – These constant coefficient filters [14], [76] estimate the target position and velocity from position measurements only. The alpha-beta filter assumes a second order model driven by white noise for the target dynamics while the alpha-beta-gamma filter assumes a third order model. In either case, the fixed coefficients can be shown to be related to the steady state Kalman filter gains.

These filters are simple to implement and were popular in the early days of tracking when computation resources were limited. However, they do not provide accurate estimates and have problems in high clutter or low detection probability situations. Thus, they have been mostly replaced by Kalman filters in recent years.

Kalman Filter – The Kalman filter has been the standard approach to filtering for linear systems since its development in the early sixties [40], [41]. It is based upon the following linear model for the state and measurements:

State equation:

$$x_{k+1} = F_k x_k + G_k w_k \tag{3}$$

Measurement equation:

$$z_k = H_k x_k + v_k \tag{4}$$

where x_k is the target state (position, velocity, etc.) at sampling instant k, z_k is the measurement, w_k is a zero-mean white process noise with covariance Q, and v_k is a zero-mean white measurement noise with covariance R. Kalman filtering consists of two steps: prediction of the current estimate and error covariance to the next time (time update) and combining the prediction with the measurement to update the target state estimate and error covariance (measurement update). In the following $\hat{x}_{k|j}$ and $P_{k|j}$ are the estimate and error covariance at time k given all the observations up to time j.

Time update:

$$\hat{x}_{k|k-1} = F_{k-1}\hat{x}_{k-1|k-1} \tag{5}$$

$$P_{k|k-1} = F_{k-1}P_{k-1|k-1}F_{k-1}' + G_{k-1}QG_{k-1}' \tag{6}$$

Measurement update:

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + W_k v_k \tag{7}$$

$$P_{k|k} = P_{k|k-1} - W_k S_k W_k' \tag{8}$$

where $v_k = z_k - \hat{z}_{k|k-1} = z_k - H_k\hat{x}_{k|k-1}$ is the measurement residual or innovations, $W_k = P_{k|k-1}H_k' S_k^{-1}$ is the Kalman filter gain and $S_k = H_k P_{k|k-1} H_k' + R$ is the covariance of the residual.

Kalman filtering reduces the design of the filter to choosing the appropriate target dynamics and measurement models. For kinematic states, the standard models are

1. Almost constant velocity. The target states are position and velocity, and the state equation is a 2nd order system driven by acceleration noise.
2. Almost constant acceleration. The target states are position, velocity, and acceleration, and the state equation is a 3rd order system driven by white noise.
3. Singer acceleration model [77]. Since the acceleration typically lasts over a certain time interval, a first order Markov model has been used to model this correlation:

$$a(k+1) = e^{-T/\tau}a(k) + \sqrt{1-e^{-2T/\tau}}\,\sigma w(k) \tag{9}$$

where T is the measurement update interval, τ is the acceleration time constant, σ is the standard deviation of the acceleration, and Q is now one.

These linear models are the basic models used for tracking air targets which are not maneuvering. These same models can be used for non-maneuvering ground targets moving on a single road segment (1-D) or in open terrain (2-D). As an example, a second order 1-D model has been used to track vehicles on a road with imaging sensors so as to monitor the traffic density and separation [44]. However, when the targets maneuver or reach an intersection, these simple linear models are no longer adequate.

3.2 Adaptive Filters

The Kalman filter performs well when the target dynamics match the model used in the filter. When a target maneuvers, the model no longer matches the dynamics and performance will degrade. One approach of dealing with target maneuvers is to increase the process noise in the target model. This will usually decrease the tracking accuracy since the state estimate will depend more on the current measurement and not utilize the information accumulated from previous measurements. Increasing the prediction uncertainty will also make association more difficult. Thus several approaches have been developed to detect maneuvers and adapt the filter to the target dynamics in real time.

Parameter adjustment [18], [61] – The structure of the filter (and the underlying model) is fixed. However, the filter will monitor its own performance (such as the size of the residuals) and adapt parameters (such as the process noise covariance or the Kalman filter gain) when a maneuver is detected.

State Augmentation [6] – This approach recognizes that there is a penalty in using a model which tries to work in all situations. Instead, the dimension of the state is changed when a maneuver has been detected. For example, before maneuver, a constant velocity model is used. When a maneuver has been detected, the filter switches to an acceleration model with higher state dimension and switches back to the original model when the maneuver is determined to have ended.

3.3 Multiple Models

The state augmentation approach discussed above makes a hard decision on the target dynamic model that is valid at any particular time. This is similar to making hard association decisions to be discussed later. When the measurement does not contain sufficient information, an incorrect decision may be made, resulting in poor performance. Thus algorithms that maintain multiple target dynamic models have been developed. These algorithms compute the probability of each model being true given the measurements and generate a target state estimate as a weighted sum of the estimates given the individual models.

A description of multiple model estimation can be found in the books [10], [11], [20] , the survey paper [60] and the review [54]. Mathematically, multiple model estimation is the estimation of a hybrid state system [82]. The discrete state m_k is the model in effect at time k and has value in the model set $M = \{m^i\}_{i=1}^r$. The discrete or model state evolves according to the Markov model

$$P(m_{k+1}^j \mid m_k^i) = P(m_{k+1} = m^j \mid m_k = m^i) = p_{ij} \qquad (10)$$

A semi-Markov chain can also be used to include the dependence on the sojourn time. Given the model m^i, the continuous state x_k of the target evolves according to the dynamic system

$$x_k = F_k^i x_{k-1} + G_{k-1}^i w_k^i \qquad (11)$$

with the measurement equation

$$z_k = H_k^i x_k + v_k^i \qquad (12)$$

The basic idea of multiple model estimation is to run a bank of filters corresponding to the number of models, compute the probability of each model being valid, and then obtain the overall estimate using a combination of the estimates from these filters. Let z^k be the measurement sequence up to time k. Then the optimal estimate is given by

$$\hat{x}_{k|k} = E[x_k \mid z^k] = \sum_{(k,l)=1}^{r^k} E[x_k \mid m^{(k,l)}, z^k] P\{m^{(k,l)} \mid z^k\} \qquad (13)$$

where $m^{(k,l)}$ is one of the r^k model sequences up to time k.

Static Multiple Models – The early work [53], [56] on multiple models assumes that the true target motion model is static and contained in a fixed set of models, i.e., $p_{ij} = 0$ for $i \neq j$. Since the model dose not switch, the model sequence consists of only one model, the number of filters is fixed at r, and equation (13) becomes:

$$\hat{x}_{k|k} = E[x_k \mid z^k] = \sum_{i=1}^{r} E[x_k \mid m^i, z^k] P\{m^i \mid z^k\} \qquad (14)$$

Because the target model does not change with time, this approach is not appropriate for maneuvering targets, although it can be used to determine what discrete state the system is in.

Model Sequence Pruning – The optimal multiple model estimator given by (13) requires a filter for each possible model sequence hypothesis. Since the number of model sequences and thus the number of filters increases exponentially with time, the optimal estimator is not practical. An obvious sub-optimal approach is to prune the least likely model sequences according to their probabilities. However, this has been shown to have inferior performance relative to other approaches [84].

Generalized Pseudo Bayesian Estimator – The Generalized Pseudo Bayesian (GPB) method [1], [27], [83] is a suboptimal approach that reduces the number of filters by merging model sequences that end up with the same fixed length sub-sequences. A GPB algorithm of order k, called GPBk requires a bank of r^k filters. Thus the first order GPB estimator, GPB1, will re-initialize the filters with the estimates:

$$\bar{x}_{k-1|k-1}^i = E\{x_{k-1} \mid z^{k-1}\} = \hat{x}_{k-1|k-1} \qquad (15)$$

Interacting Multiple Models – This alternative merging approach was first developed by Blom [21], [22] and has become very popular. It is conceptually similar to the GPB2 algorithm but has only GPB1 complexity. Let $\mu_{k-1|k-1}^i$ be the conditional probability of $m_{k-1} = m^i$ given the cumulative measurements z^{k-1}, and $\hat{x}_{k-1|k-1}^i$ and $P_{k-1|k-1}^i$ be the state estimate and error covariance of x_{k-1} for filter with model m^i. Each cycle of the IMM algorithm consists of the following steps:

1. Interaction or model-conditioned re-initialization

Predicted model probability:

$$\mu^i_{k|k-1} \equiv P\{m^i_k \mid z^{k-1}\} = \sum_{j=1}^{r} p_{ji}\mu^j_{k-1|k-1} \tag{16}$$

Mixing weight:

$$\mu^{j|i}_{k-1|k-1} \equiv P\{m^j_{k-1} \mid m^i_k, z^{k-1}\} = p_{ji}\mu^j_{k-1|k-1} \big/ \mu^i_{k|k-1} \tag{17}$$

Mixing estimate:

$$\bar{x}^i_{k-1|k-1} \equiv E[x_{k-1} \mid m^i_k, z^{k-1}] = \sum_{j=1}^{r} \hat{x}^i_{k-1|k-1}\mu^{j|i}_{k-1|k-1} \tag{18}$$

Mixing covariance:

$$\bar{P}^i_{k-1|k-1} = \sum_{j=1}^{r}[P^j_{k-1|k-1} + (\bar{x}^i_{k-1|k-1} - \hat{x}^j_{k-1|k-1})(\bar{x}^i_{k-1|k-1} - \hat{x}^j_{k-1|k-1})']\mu^{j|i}_{k-1|k-1} \tag{19}$$

2. Model-conditioned filtering. For each $i = 1,2,\ldots,r$, the estimate $\bar{x}^i_{k-1|k-1}$ and covariance $\bar{P}^i_{k-1|k-1}$ are used to initialize a standard Kalman filter to process the measurement z_k to obtain the updated state $\hat{x}^i_{k|k}$ and covariance $P^i_{k|k}$ according to equations (7) and (8).
3. Model probability update. The model likelihood $L^i_k \equiv p(z_k \mid m^i_k, z^{k-1}) = N(v_k;0,S^i_k)$, where $N(v;0,S)$ denotes a normal distribution in v with zero mean and covariance S. The model probability is then updated by

$$\mu^i_{k|k} = \mu^i_{k|k-1}L^i_k \Big/ \sum_j \mu^j_{k|k-1}L^j_k \tag{20}$$

4. Estimate fusion. The overall estimate and covariance are given by

$$\hat{x}_{k|k} = E[x_k \mid z^k] = \sum_{i=1}^{r} \hat{x}^i_{k|k}\mu^i_{k|k} \tag{21}$$

$$P_{k|k} = \sum_{i=1}^{r}[P^i_{k|k} + (\hat{x}_{k|k} - \hat{x}^i_{k|k})(\hat{x}_{k|k} - \hat{x}^i_{k|k})']\mu^i_{k|k} \tag{22}$$

The IMM algorithm is one of the most popular algorithms for tracking maneuvering targets because of its relatively simple implementation and its ability to handle complicated dynamics.

Variable Structure Interacting Multiple Models – While IMM has been successfully used in several applications, having a fixed model set has its disadvantages. The applicable target motion models will depend on the situation. Keeping all the needed models throughout the tracking period can incur extra computational load and degrade accuracy. In tracking multiple targets, the same estimator with the same set of models is used for all targets even though each target may need different models.

The variable structure multiple model (VSMM) estimator has been proposed [54], [55] for situations where the model space is large and the set of likely models varies with time or state. Since the optimal VSMM estimator is infeasible, a variable structure interacting multiple model (VS-IMM) estimator has been developed. It has a two level structure:

- Model set adaptation. This determines which model set to use at each time using both information contained in the measurements as well as prior knowledge. Different ways of terminating existing models and initializing new models have been proposed.
- Model set conditioned estimation. This consists of the same steps as fixed model set IMM and an additional step to assign initial probabilities to the new models and initialize the filters.

In [48], [49], the VSIMM approach is used to track ground targets moving over roads and open field. The target motion models reflect the mobility of a target for different conditions. In open field, the process noise has equal covariances in both X and Y directions. For a target moving on a road, the noise is higher along the road and lower orthogonal to the road due to the road constraint.

Models are added or removed by examining the location of the target track. At each time, the track is tested for proximity to road segments, intersections, etc. For example, when the track moves from one road segment A into a road segment B, the model corresponding to segment B is added. Similarly, when a target moves near an intersection, the models for the roads at the intersection are added to the model set.

3.4 Nonlinear Estimation

The Kalman filter is the basis of the single target state estimation algorithm described above. For the Kalman filter to be applicable, both the target motion model and the sensor measurement model have to be linear. Strictly speaking, for the Kalman filter to be a conditional mean estimate, the noises also have to be Gaussian. Since many dynamic models or observation models do not satisfy the linear assumptions, approaches for estimating the state of nonlinear systems have been developed. The following sub-subsections review some of these approaches.

Extended Kalman Filter – When the non-linearity of the dynamic and observation models is not too severe, these models can be linearized about a nominal trajectory, and then a Kalman filter can be developed with the linearized model. Specifically, let the target motion model and measurement model be

$$x_{k+1} = f_k(x_k) + g_k(x_k)w_k \quad (23)$$

and

$$z_k = h_k(x_k) + v_k \quad (24)$$

In the extended Kalman filter, the nominal target trajectory is usually chosen to be the best estimate of x_k given the cumulative measurements z^k. Thus time update is given by

$$\hat{x}_{k+1|k} = f_k(\hat{x}_{k|k}) \quad (25)$$

$$P_{k+1|k} = F_k(\hat{x}_{k|k})P_{k|k}F_k'(\hat{x}_{k|k})' + G_k(\hat{x}_{k|k})Q_kG_k'(\hat{x}_{k|k}) \quad (26)$$

where $F_k(x)$ and $G_k(x)$ are the derivatives of $f_k(.)$ and $g_k(.)$ evaluated at x. The measurement equation is approximated by

$$z_k = h_k(\hat{x}_{k|k-1}) + H_k(\hat{x}_{k|k-1})(x_k - \hat{x}_{k|k-1}) + v_k \quad (27)$$

where $H_k(x)$ is the partial derivative of $h_k(.)$ evaluated at x. The standard Kalman filter measurement update equation can then be used to updated estimate.

The extended Kalman filter is the most popular nonlinear filter commonly used. Its operation is similar to the usual Kalman filter except that the Kalman filter gain depends on the most recent estimate and thus cannot be computed ahead of time. For target tracking, if the appropriate coordinate system is used, the target motion equation is linear. For radar measurements, the measurement equation is nonlinear because the range and azimuth are nonlinear functions of the target coordinates in x and y.

Gaussian Sum Approximation [2], [80] – The extended Kalman filter assumes that the conditional probability distribution can be approximated reasonably accurately by a Gaussian distribution. When this approximation is not valid, e.g., when the conditional distribution is multi-modal, the conditional probability distribution of x given the cumulative measurements z^k can be approximated by a sum of Gaussian distributions, i.e.,

$$p(x_k \mid z^k) \approx \sum_{i=1}^{m} \alpha_{k|k}^i N(x_k - \overline{m}_{k|k}^i; 0, B_{k|k}^i) \quad (28)$$

where the weights $\alpha_{k|k}^i$'s of the individual Gaussian distributions with means $\overline{m}_{k|k}^i$ and covariance $B_{k|k}^i$ sum to one. The Gaussian sum filter operates as a collection of extended Kalman filters. In time update, the weights stay the same and the mean and covariance of each Gaussian distribution is propagated by the time update equation. In measurement update, the means and covariances are updated by the usual Kalman filter, but the weights are updated using the innovations of the measurements for the different filters. Note that the Gaussian sum filter has almost the same structure as the static multiple model filter, except that there is only one model evaluated at different nominal trajectories.

Nonlinear Filtering – In a Bayesian framework, nonlinear estimation is conceptually very simple [38]. The target dynamics is represented by the Markov transition probability $p(x_{k+1} \mid x_k)$ and the measurement model is given by the conditional probability $p(z_k \mid x_k)$. Then the optimal Bayesian estimate is given by the following equations:

Time update:

$$p(x_{k+1} \mid z^k) = \int p(x_{k+1} \mid x_k) p(x_k \mid z^k) dx_k \quad (29)$$

Measurement update:

$$p(x_k \mid z^k) = C^{-1} p(z_k \mid x_k) p(x_k \mid z^{k-1}) \quad (30)$$

where C is a normalization constant.

This optimal nonlinear filtering algorithm has nice features such as the ability to update the probability distribution of the states due to non-detections (so called negative evidence). However, implementation is computationally intensive since it requires discretization of the state space and performing the integration by a summation. Thus even though the algorithm has been known for many years, it has seldom been used. Reference [74] describes how this algorithm can be used to track targets moving over terrain. The time update equations propagate the probability distribution by considering the effects of terrain on target speed and direction of movement. The measurement update equations update the distributions when measurements are received.

This general algorithm was also discussed in [64] and used to formulate the multiple hypothesis tracking problem. Reference [81] presents more details on this approach and its use in multiple target tracking motivated by work in tracking submarines.

4. Data Association

When the origins of the measurements are uncertain, e.g., when clutter or multiple targets are present, the measurements have to be associated with other measurements or tracks before the target state estimates can be generated. Association is what distinguishes target tracking from traditional state estimation and establishes tracking as a separate discipline.

Association can be between two sets of measurements to establish target tracks (track initiation or formation), between measurements and existing tracks (track continuation or maintenance), or between tracks from different sensors (track fusion in a distributed architecture). Historically, track formation (initiation) and maintenance (continuation) were at first treated as separate problems. However, the more recent algorithms usually integrate these functions into a single framework.

Algorithms can also be classified according to whether they focus on single targets (uncoordinated association) or consider explicitly the presence of multiple targets (coordinated association) and whether association decisions are made using single or multiple scans of data. The early algorithms tend to focus on single scan and uncoordinated associations, while the recent algorithms deal with multiple scans of data and coordinated association. In general, algorithms that consider multiple targets and use multiple scans of data perform better but require more computations.

4.1 Single Target (Uncoordinated) Track Formation

These track formation algorithms initiate tracks from sequences of measurements without considering competition from other tracks. Thus the resulting tracks may share the same measurements.

M out of N Test – The earliest track formation algorithms [25] are based on the following simple logic. A track is tentatively initiated from a single measurement. A validation gate is then established around this measurement and a measurement falling inside this gate becomes part of the track. When there are m detections out of n scans of measurements, then the track is formed or confirmed. This method is very simple but does not provide a score on the confidence of the track.

Likelihood (Ratio) Test – Let $\theta^{k,l}$ be a track l consisting of the measurement sequence $z^{k,l} = \{z_{0,j_l}, z_{1,j_l}, \ldots, z_{k,j_l}\}$. A tentative track can be scored by the likelihood of the track, [78] defined as

$$\Lambda(\theta^{k,l}) = p(z^{k,l} \mid \theta^{k,l}) = \prod_{i=1}^{k} p(z_{i,j_l} \mid z^{k-1,l}, \theta^{k,l}) \tag{31}$$

where $p(z_{i,j_l} \mid z^{k-1,l}, \theta^{k,l}) = N(v_{i,j_l}; 0, S_{i,j_l})$ is the probability distribution of the innovations process. The log likelihood can be computed recursively from the previous log likelihood and the measurement residual. Alternatively, one can compute the likelihood ratio given by

$$LR = \frac{p(z^{k,l} \mid \theta^{k,l})}{p(z^{k,l} \mid FA)} \tag{32}$$

where $p(z^{k,l} \mid FA)$ is the likelihood that all the measurements in the sequence are false alarms. Again, the likelihood ratio is converted to a log likelihood ratio. In the likelihood tests, tracks are declared as confirmed (or deleted) when the likelihood or ratio exceeds (or falls below) a certain threshold.

4.2 Single Target (Uncoordinated) Track Maintenance

As in Sections 4.1, these algorithms associate measurements with the existing tracks without considering the presence of other tracks. Thus a measurement may be associated with multiple tracks.

Nearest Neighbor – When there are multiple measurements within the validation gate of a track, the measurement that is closest (according to some distance measure) to the track is associated with the track. This approach makes a hard decision based on a single scan and is very easy to implement. However, it does not perform well in high density situations.

Track Splitting [79] – This is basically applying the likelihood function (or ratio) approach to track maintenance. For every measurement that falls in the validation gate, the track is split. Each track is scored using a likelihood as discussed before. The track is pruned when the likelihood falls below a threshold. This approach makes soft decisions based upon multiple scans of data. Because of its computational requirements and limited performance, this approach is no longer popular.

Probabilistic Data Association (PDA) [12], [13] – Instead of associating a single measurement with a track, this approach probabilistically associates all measurements in the validation gate. If the number of validated measurements is m_k, the updated state estimate for the track is

$$\hat{x}_{k|k} = \sum_{i=0}^{m_k} \beta_k^i \hat{x}_{k|k}^i \tag{33}$$

where $\beta_k^i \equiv P\{\theta_k^i \mid Z^k\}$ is the conditional probability of the event θ_k^i that measurement $z_{k,i}$ is the correct measurement (originating from the target) and $\hat{x}_{k|k}^i = E[x_k \mid \theta_k^i, Z^k]$ is the updated state estimate given the association. The state estimate update has the same form as the usual Kalman filter, i.e.,

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + W_k v_k \tag{34}$$

except that the measurement residual is the weighted sum of the measurement residuals

$$v_k = \sum_{i=0}^{m_k} \beta_k^i v_k^i \tag{35}$$

The covariance update equation is also similar to the Kalman filter update equation except for another term which depends on the measurement residuals. Thus the Kalman filter gain cannot be computed a priori. The need to associate measurements has turned the problem into a nonlinear estimation problem.

The PDAF is an all-neighbors association algorithm. Its logic is similar to GPB1 where a single "lumped" state estimate is used as sufficient statistics. It is fairly easy to implement and has been shown to perform better than the nearest neighbor approach in high clutter. Augmented PDAF's that use features in addition to kinematics measurements have also been developed.

The PDA approach was originally developed for track maintenance for a fixed number of targets. By combining with interacting multiple models in the so called interacting multiple models PDAF (IMMPDAF) [7], it has been modified to handle track formation as well.

Optimal Bayesian Approach [77] – The PDAF is a suboptimal approach since the association event only considers the current measurements. The optimal approach will consider all possible association hypotheses up to the current time. Let the total number of measurement sequences of possible tracks up to time k be

$$L_k = \prod_{j=1}^{k} (1+m_j) \tag{36}$$

The additional 1 in the product corresponds to the absence of measurement from that scan in the track. Then the state estimate is

$$\hat{x}_{k|k} = \sum_{l=0}^{L_k} \beta^{k,l} \hat{x}^l_{k|k} \tag{37}$$

where $\beta^{k,l} \equiv P\{\theta^{k,l} \mid Z^k\}$ is the conditional probability that the measurement sequence is the correct sequence (originating from the target), and

$$\hat{x}^i_{k|k} = E[x_k \mid \theta^{k,l}, Z^k] = E[x_k \mid z^{k,l}] \tag{38}$$

is the updated state estimate given the association sequence. The optimal algorithm is not feasible since the number of filters grows exponentially, as in the optimal multiple model algorithm. One approximate solution approach is the n-scan algorithm that combines all tracks with identical measurement sequences during the previous n scans, similar to GPBn in multiple models estimation. The PDAF corresponds to the situation when n=0.

4.3 Multiple Target Track Maintenance

Association performance can be improved when the algorithms consider explicitly the presence of multiple targets and recognize that a single measurement cannot belong to multiple tracks.

Optimal Assignment – The optimal assignment approach, also sometimes called global nearest neighbor, is the coordinated version of nearest neighbor. Instead of selecting the measurement that is closest to a track, this approach selects the set of measurements that is closest to the set of tracks according to some global distance measure subject to the constraint that two tracks do not share a single measurement, and two measurements do not appear in the same track. The likelihood of a measurement $z_{k,j}$ originating from the track i with previous measurement sequence $z^{k,i}$ is

$$p(z_{k,j} \mid z^{k,i}) = N(v_{k,j|k,i}; 0, S_{k,j|k,i}) \tag{39}$$

where $v_{k,j|k,i}$ and $S_{k,j|k,i}$ are the residual or innovations and covariance of measurement $z_{k,j}$ in track i. The joint likelihood of each set of associations is given by

$$\prod_{i,j} p(z_{k,j} \mid z^{k,i}) \tag{40}$$

By taking the logarithm and substituting in the Gaussian distribution, the log likelihood function of the joint association is

$$\sum_{i,j} v_{k,j|k,i}{}' S^{-1}_{k,j|k,i} v_{k,j|k,i} + \ln |S_{k,j|k,i}| \tag{41}$$

With the constraint that each track can be associated with at most one measurement, and each measurement with at most one track, the optimization problem is the so called assignment problem which has been solved by methods such as the Munkres [66], [23] or Hungarian [50] algorithms. Recent algorithms such as the JVC algorithm first proposed in [39] and enhanced in [24] and the auction algorithm [15] have been shown to be highly effective in solving the assignment algorithm arising from data association problems.

Optimal assignment is a single (coordinated) hypothesis, single scan approach to track maintenance. It is generally used in conjunction with a track formation approach discussed before. Measurements that are not assigned to any existing tracks are used to initiate new tracks. This approach is still very popular because of its simplicity and the availability of good assignment algorithms.

Joint Probabilistic Data Association (JPDA) [32], [33] – This is the extension of PDA to multiple targets. The tracks for a known number of targets are assumed to have been initiated and the problem is to associate the measurements to the tracks. As in PDA, the estimate for each target is given by a single mean and covariance.

The state estimation equation looks exactly the same as PDA, except that the measurement and target association probabilities are now computed jointly across all targets. Specifically, for target t, the estimate is

$$\hat{x}^t_{k|k} = \sum_{j=0}^{m_{k,t}} \beta_k^{j,t} \hat{x}^{j,t}_{k|k} \tag{42}$$

where $m_{k,t}$ is the number of measurements in the validation gate for target t, $\beta_k^{j,t}$ is the (marginal) probability for the event that measurement j originates from target t, and $\hat{x}^{j,t}_{k|k}$ is the estimate obtained by updating the target state with measurement j. The key to JPDA is computing the probability for the joint association event

$$\theta_k = \bigcap_{j=1}^{m_k} \theta_k^{j,t_j} \tag{43}$$

such that all the associations are feasible, i.e., each measurement can originate from at most one target and each target can generate at most one measurement.

JPDA can be viewed as a special case of the multiple hypothesis approach when hypotheses are combined as soon as they are formed. As in PDA, it is an all neighbors approach, as compared to optimal assignment, which is global nearest neighbor. The JPDAF is a very popular association algorithm with many applications. One of its disadvantages is that since models interference is assumed to be random, it does not work well when there is persistent interference, e.g., from another nearby target.

4.4 Multiple Scan Coordinated Association

The track maintenance algorithms in Section 4.3 base the association decisions on only one scan of measurements. Since both the measurement and the target motion models have uncertainty, the single scan decisions may not be the correct associations. Thus association performance can be improved by using multiple scans of data.

The core of all multiple scan algorithms is the evaluation of track likelihoods, which can be used for both track formation and maintenance. Thus multiple scan algorithms generally can be used for both track formation and association.

Integer Programming – The earliest multiple scan algorithm was found in [62]. Let N be the total number of measurements and L be the total number of tracks formed (the tracks are assumed to satisfy the constraint that each track cannot contain more than one measurement from the same scan). For each track i, let φ_i be an N-dimensional indicator vector whose k-th element is 1 if the k-th measurement is in the track, and 0 otherwise. Let ρ be a L-dimensional vector with elements 1 or 0 that indicate whether a particular track is in the scene hypothesis. Then the feasibility condition that a single measurement cannot belong to two tracks becomes

$$A\rho \equiv [\varphi_1 \ldots \varphi_L]\rho \le 1 \qquad (44)$$

Let the score of a track i be λ_i. This score can be computed as before (31), e.g., as a likelihood. The total score for a particular assignment is given by $\lambda'\rho$ and the problem of finding the best scene hypothesis is the integer programming problem

$$\min_{\rho} \quad \lambda'\rho \qquad (45)$$

$$\text{subject to } A\rho \le 1 \qquad (46)$$

This integer programming problem can be solved by branch and bound or other methods. Although this approach was the first multiple scan algorithm, interest soon shifted to other multiple scan algorithms such as multiple hypothesis tracking described below.

Multiple Hypothesis Tracking (MHT) – Multiple hypothesis tracking delays making hard decisions when there is not sufficient information to make a good decision. Alternative hypotheses are formed to represent the ambiguities and each hypothesis is evaluated. MHT originated with the work in [75] and a general formulation was given in [63], [64]. In MHT, there are two levels of hypotheses: track hypotheses containing measurements originating from the same targets and scene hypotheses of consistent tracks (with no measurements) that explain the origins of all cumulative measurements.

There are two alternative implementations of MHT: the original hypothesis-oriented approach of [75] and expanded in [64] and the track-oriented approach in [51]. Both approaches require the formation and evaluation of hypotheses at the track and scene level. Received measurements are first associated with the current tracks to form new track hypotheses. A measurement may be associated with a current track (i.e., from a previously detected target), used to form a new track (i.e., from a target that has not been detected before), or not used in a track at all (i.e., false alarm). At the same time, a track may not be associated with any measurements (i.e., not detected). The result is a set of updated track hypotheses along with the likelihoods of these associations.

These track hypotheses are then used to form scene hypotheses by incorporating the feasibility constraints and the probabilities of the hypotheses evaluated. One formula for recursively computing the probability of the scene hypothesis is given by [64]:

$$P(\Lambda \mid Z) = C^{-1} P(\overline{\Lambda} \mid \overline{Z}) L_k^{FA}(Z_k \mid \overline{\Lambda}) \prod_{l \in \Lambda} L_k(z_{k,l} \mid \theta^{k,l}, \overline{Z}) \qquad (47)$$

where C is a normalization constant, Λ is a scene level hypothesis given the cumulative measurements Z, $\overline{\Lambda}$ is the parent hypothesis given the previous measurements $\overline{Z}$, $L_k^{FA}(Z_k \mid \overline{\Lambda})$ is the likelihood of the false alarms in the current measurements Z_k as hypothesized by Λ, and $L_k(z_{k,l} \mid \theta^{k-1,l}, \overline{Z})$ is the likelihood of associating the measurement $z_{k,l}$ with the track $\theta^{k-1,l}$. The hypothesis evaluation equation can also be expressed in batch form in terms of the track likelihoods.

The MHT is conceptually simple but computationally intensive since the number of hypotheses grows exponentially. Thus sophisticated hypothesis management techniques are needed to reduce the combinatorics. Common techniques include pruning to remove low probability hypotheses, clustering to decompose the problem, and combining or merging to replace similar hypotheses by a single hypothesis.

In the hypothesis oriented approach [75], [64], the most current scene hypotheses and the measurements are used to generate new scene level hypotheses. Most of the hypothesis management operations are performed on scene hypotheses. The track oriented approach [51], [20] does not maintain scene hypotheses from scan to scan. Only high likelihood track hypotheses are maintained and used to form scene hypotheses. The probabilities of the scene hypotheses are then used to further prune the track hypotheses.

The track-oriented approach is computationally more efficient since scene hypotheses are only formed for high likelihood tracks. In the hypothesis-oriented approach, the expansion of the hypothesis may result in the generation of many low probability hypotheses which will eventually be pruned any way. However, the use of Murty's method [67] for finding the n-best solutions of an assignment problem can be used to find high ranking scene hypotheses directly without having to expand all hypotheses [28], [29].

In recent years, MHT has become quite popular because of advances in computing hardware and algorithms. Simulation studies have shown that it can produce much better performance than more traditional algorithms in difficult situations [26]. MHT has also been combined with more sophisticated models to handle maneuvering targets, e.g., IMM/MHT as discussed in Chapter 16 of [20].

Multi-Dimensional Assignment – Traditional MHT requires the explicit expansion and evaluation of many hypotheses. Successful implementation requires the use of sophisticated hypothesis management techniques to handle the combinatorics. During the last decade, alternative optimization based methods that do not require the explicit expansion and evaluation of hypotheses have been developed [30], [68]-[72]. Such algorithms are easier to implement and computationally more efficient.

Suppose that there are N scans of data with m_k measurements in each scan. To account for missing detections, we define an additional dummy measurement indexed by 0 in each set k. A track hypothesis can be represented by an indicator function $z_{i_1 i_2 \ldots i_N}$ where:

$$z_{i_1 i_2 \ldots i_N} = \begin{cases} 1 & \text{if measurements } i_1, i_2, \ldots, i_N \text{ belong to track} \\ 0 & \text{otherwise} \end{cases}$$

Let $c_{i_1 i_2 \ldots i_N}$ be the cost of forming the track $z_{i_1 i_2 \ldots i_N}$, e.g., the negative of the logarithm of the track likelihood define in (31). Then the problem of selecting the scene hypothesis to maximize the probability or likelihood of the hypothesis can be formulated as the following optimization problem:

$$\min \sum_{i_1=0}^{m_1} \cdots \sum_{i_N=0}^{m_N} c_{i_1 i_2 \ldots i_N} z_{i_1 i_2 \ldots i_N} \tag{48}$$

$$\text{subject to} \sum_{i_1=0}^{m_1} \cdots \sum_{i_{k-1}=0}^{m_{k-1}} \sum_{i_{k+1}=0}^{m_{k+1}} \sum_{i_N=0}^{m_N} z_{i_1 i_2 \ldots i_N} \tag{49}$$

for all $i_k = 1,2,\ldots\ldots, m_k$, and $k = 1,2,\ldots\ldots, N$. The constraints in (49) specify that each measurement can belong to only one track. There are no constraints on the dummy measurements since several tracks may be undetected in a single scan. The cost $c_{00\ldots0}$ is defined to be zero.

Equations (48) and (49) define an N dimensional assignment which is a natural extension of the two dimensional assignment problem described in Section 4.3. While the 2-D assignment problem can be solved in polynomial time, the N-D assignment is *N-P* hard, i.e., the solution time cannot be bounded by a polynomial of fixed order. Thus sub-optimal approaches have been developed to solve the assignment problem.

Although there are differences in the specific steps, the existing approaches to N-D assignment are all based upon Lagrangian relaxation. The main idea is to use Lagrangian multipliers to move constraints into the cost to be minimized until the problem becomes a 2-D assignment problem which can be solved efficiently by algorithms such as the auction algorithm. A main challenge in Lagrangian relaxation is updating the Lagrangian multipliers so that the constraints are satisfied. Different approaches of handling this problem are discussed in [68]-[72].

Multi-dimensional assignment can be used for both track initiation and track continuation by using a sliding window over the different scans of data. The solution from one time can be used as a "warm start" for the solution of the next time [71]. Multi-dimensional assignment has become quite popular during the last few years because it provides a computationally attractive alternative to MHT when there is a need to use multiple scans of data for association. Algorithms for finding the m-best assignments have also been developed for situations when knowing the best assignment is not enough [73].

4.5 Tracking Without Data Association

Most multi-target tracking algorithms follow the structure of Figure 1 and perform association before target state estimation. In recent years, several approaches have been proposed to perform tracking without an explicit association function. Instead of dealing with individual target states and individual measurements, these approaches treat all targets and measurements as components of one system, and estimate the system state directly without explicitly forming association hypotheses.

Symmetric Measurement Equations (SME) [42]-[43]– In this approach the original measurements on the targets are converted into a new set of measurements that are symmetric functions of the original measurements. For example, for two targets with measurements y_1 and y_2, the symmetric measurement vector $Z = (z_1, z_2)$ is given by

$$z_1 = y_1 + y_2 \tag{50}$$

$$z_2 = y_1 y_2 \tag{51}$$

Since the new measurements do not depend on specific origins of the measurements, data association is not necessary for estimating the target state. Because the new measurement model $Z = f(X) + V$ is always nonlinear in the joint state X of all the targets, the extended Kalman filter is used to estimate the joint target state.

The SME approach assumes that the number of targets is known a priori. Thus it is really an algorithm for track continuation or maintenance. The cost of avoiding data association is a nonlinear estimation problem with higher dimension. Thus, even though the SME approach has potential, it may be applicable only to tracking problems with a small number of targets.

Multi-target Nonlinear Filtering [45], [46], [57]-[59], [81] – Suppose the individual target states are replaced by a multi-target state X_k, and the individual measurements are replaced by a joint measurement Z_k. Suppose the individual target motion and measurement models can be aggregated into a multi-target motion model given by the conditional probability $p(X_{k+1} \mid X_k)$ and a measurement model given by the likelihood $p(Z_k \mid X_k)$. Then, the same nonlinear filtering equations (29), (30) developed for a single target can be used (conceptually at least) for tracking multiple targets, i.e.,

$$p(X_{k+1} \mid Z^k) = \int p(X_{k+1} \mid X_k) p(X_k \mid Z^k) dX_k \tag{52}$$

$$p(X_k \mid Z^k) = C^{-1} p(Z_k \mid X_k) p(X_k \mid Z^{k-1}) \tag{53}$$

The simplicity of these equations is deceptive. Discretization of the target state space is needed to represent the probability densities. Since the joint target state space is the direct product of the individual target spaces, the number of cells needed can be very high. For example, if each target state is discretized into N cells, the joint state space for two targets will have N^2 cells. Also, if the number of targets is unknown, the joint target state needs to represent that information.

The prior conditional probabilities also need to be defined. In particular, the likelihood function $p(Z_k \mid X_k)$ now contains all the information on association. The Unified Tracking Algorithm approach [81] computes the likelihood directly using fairly basic probability techniques and applies it to situations when ordinary association is not meaningful, e.g., measurement strength that depends on two targets. On the other hand, the Finite-Set Statistics (FISST) approach [57], [58] is based on random set theory.

Although multi-target nonlinear filtering has potential, its mathematical foundation is quite complicated and not widely understood. Reference [59] presents some issues in problem formulation and compares some algorithms. The computational requirements are quite substantial. Reference [81] states that the algorithm "is computationally infeasible for problems involving even moderate numbers of targets". It also suggests that the algorithm should be part of a system with a standard tracking method and used only when targets cross or merge.

5. Conclusions

Ground target tracking has become an important problem recently because airborne platforms such as Joint STARS can provide measurements on vehicles moving over a large area. Since such measurements have become available only recently, very few tracking algorithms have been developed specifically for ground targets, and most tracking algorithms have their origins in air targets.

Ground target tracking is considerably more difficult than air target tracking because of higher target motion variability, higher target density, and possible miss-detection due to terrain masking and minimum detection velocity for MTI measurements. Thus, many tracking algorithms developed for air targets will not be suitable for ground targets.

Our review of tracking algorithms over the last four decades suggests that the state-of-the-art in tracking is such that high performance tracking of ground targets is possible. Target state estimation algorithms have progressed from Kalman filters for targets with fairly stationary dynamics to multiple models estimation for highly maneuverable targets. At the same time, multiple scan association algorithms are now available to handle the high target density typically found for ground targets. Thus we believe that ground tracking algorithms can be developed using the more advanced techniques currently available.

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Chee-Yee Chong *received his S.B., S.M., and Ph.D. degrees in electrical engineering from the Massachusetts Institute of Technology. Currently he leads fusion technology development at Booz Allen & Hamilton. His research interests include centralized and distributed tracking and fusion, resource planning and scheduling, reasoning with uncertainty, distributed decision making, integration of system techniques with artificial intelligence, and the application of the above to large scale systems. Previously he was with Advanced Decision Systems in Mountain View, CA, and before that on the faculty of the Georgia Institute of Technology. In addition to being an IEEE member for many years, he also helped to start the International Society of Information Fusion in 1998.*

David A. Garren *earned the B.S. degree in mathematics and physics from Roanoke College in 1986 and the M.S. and Ph.D. degrees in physics from the College of William and Mary in 1988 and 1991. He has thirteen years experience in analysis and algorithms of multi-sensor multi-target tracking, image/signal processing, and physics. He is currently the U.S. Government core team lead for the Precision Fire-Control Tracking segment of DARPA's Affordable Moving Surface Target Engagement (AMSTE) program.*

Timothy P. Grayson *received his B.S. degree from the University of Dayton, and M.A. and Ph.D. degrees from the University of Rochester. Currently he is DARPA program manager, Special Projects Office supporting various programs in the general area of moving target engagement, including the Affordable Moving Surface Target Engagement (AMSTE) program. His technical interests include statistical GMTI and SAR radar processing, tracking, complex signal processing, combat ID systems, electro-optic sensors, lasers and fundamental statistical optics. Previously he worked for SAIC as a senior system engineer and before that for Wright Laboratory, Wright-Patterson AFB.*