Statistical Approach to Track-to-Track Association Problem

A. Pinsky
Avionic Systems Engineering
Lahav Division
Israel Aircraft Industries Ltd
apinsky@hotmail.com

ABSTRACT

We have applied techniques, developed in [1], to samples of track measurements and have obtained decision rule that are asymptotically close to decision rule characteristics (probability of false objects generation is not greater than any given significance level, and possibility to omit existing objects decreases with increase of observation time). The recursive computation algorithm that implements the above decision rule is developed, and objects location estimates are also obtained at every time step.

1. INTRODUCTION

In [1] Sensor Fusion (SF) problem of instantaneous measurements, received from multiple independent sensors is developed. This decision rule assures that the probability of false object generation is not higher than any given significance level, while possibility to omit existing objects decreases, when more precise measurements are available. More precise instantaneous measurements can be obtained only when sensor-to-object distance decreases. However, when tracks are available, similar effect can be achieved by statistical average of measurements of their non-variable parameters. In this paper invariability of track "sources" is regarded as such parameter.

The purpose of this research is to propose an algorithm that extends decision rule for instantaneous measurements to sequences of track measurements, and to obtain their consistency conditions. This algorithm has been developed in recursive form, namely, every current decision is based on the previous one.

In the second section the general track-to-track association problem is formulated as the classic statistical analysis problem.

In the third section an example of two track association is presented.

In the forth section the main statement of the desired decision rule and object location estimates are given.

The algorithm implementing this decision rule is presented in section 5.

2. SF PROBLEM FORMULATION

Tracks received from n independent sensors can be represented as independent sequences of measurements of some moving object locations. These independent sequences of measurements are denoted by $Z_i(k)$ $i=1...n$. Every measurement, received at the k time ($k=1,2,...$), can be represented in the following form:

$$\vec{Z}_i(k) = \vec{\Theta}_i(k) + \vec{\varepsilon}_i(k),$$

$$\vec{\Theta}_i(k) \in \{\vec{\Theta}_1(k), \dots, \vec{\Theta}_m(k)\}, \tag{1}$$

$$\varepsilon_i(k) \cong N(\vec{0}, \Sigma_i(k)) \tag{2}$$

where $\vec{\Theta}_i(k)$ is an actual location of the object at the time k, m is the unknown number of the objects, $\{\vec{\varepsilon}_i(k)\}$ is the sequence of independent normal distributed random vectors with vector zero mean and covariance matrix $\Sigma_i(k)$.

The problem is to estimate m, to cluster the tracks corresponding to the same sources, and to estimate the locations of the objects at each time step.

3. CONSISTENCY CONDITIONS FOR DECISION RULES FOR THE n = 2 CASE

In this case two tracks received from two independent sensors are available. The fact that two tracks $\vec{Z}_1(k), \vec{Z}_2(k)$ $k=1,2,...$ correspond to the same source, is equivalent to set of equations in $3x2$-dimensional parameter space Θ of mean vectors $\vec{\Theta}^2(k) = \{\vec{\Theta}'_1(k), \vec{\Theta}'_2(k)\}'$:

$$\vec{\Theta}_1(k) = \vec{\Theta}_2(k) \tag{3}$$

or

$$\vec{\Theta}_1(k) - \vec{\Theta}_2(k) = \vec{0} \tag{4}$$

Thus, in this case the track-to-track association problem is equivalent to testing problem hypothesis about mean

0-7803-4449-9/98/$10.00 1998 IEEE

vectors $\vec{\Theta}^2(k)=\{\vec{\Theta}'_1(k),\vec{\Theta}'_2(k)\}'$ that are situated in the same linear subspace for every $k=1,2,\ldots$.

Denote $\vec{Y}(k)$ the following sequence of vector sample variables

$$\vec{Y}(k)=\vec{Z}_1(k)-\vec{Z}_2(k) \tag{5}$$

Then $\vec{Y}(k)$ is normal distributed random vector with vector mean $\vec{\Delta}(k)$ and covariance matrix $\Sigma(k)$, where

$$\vec{\Delta}(k)=\vec{\Theta}_1(k)-\vec{\Theta}_2(k) \tag{6}$$

$$\Sigma(k)=\Sigma_1(k)+\Sigma_2(k) \tag{7}$$

Under hypothesis

$$\vec{\Delta}(k)=\vec{0} \tag{8}$$

both tracks are of the same origin.

Let us consider the following recursive estimation of $\vec{\Delta}(k)$:

$$\hat{\vec{\Delta}}(1)=\vec{Y}(1)$$

$$\hat{\vec{\Delta}}(k)=\frac{k-1}{k}\hat{\vec{\Delta}}(k-1)+\frac{1}{k}\vec{Y}(k) \tag{9}$$

Covariance matrix $\Sigma_\Delta(k)$ of this estimation is calculated in the following manner:

$$\Sigma_{\hat{\Delta}}(1)=\Sigma(k)$$

$$\Sigma_{\hat{\Delta}}(k)=\frac{(k-1)^2}{k^2}\Sigma_{\hat{\Delta}}(k-1)+\frac{1}{k^2}\Sigma(k) \tag{10}$$

We will introduce the following sequence of statistics:

$$U((\vec{Z}\,_1^k)^2)=\hat{\vec{\Delta}}'(k)\Sigma_{\hat{\Delta}}^{-1}(k)\hat{\vec{\Delta}}(k),\quad k=1,2,\ldots, \tag{11}$$

where:

$$(Z_1^k)^2=(Z_1(1),Z_1(2),\ldots,Z_1(k);$$
$$Z_2(1),Z_2(2),\ldots,Z_2(k))$$

Since the estimation (9) is unbiased, the sequence of statistics (11) represents the sequence of deviation estimation (9) from the parameter subspace $\{\vec{\Delta}(k)=\vec{0}\}$.

We will introduce the following sequence of the decision rules:

$$\Phi((\vec{Z}_1{}^k)^2)=\begin{cases}1, & U(\vec{Z}^k{}_1)^2>K_\alpha\\ 0, & otherwise\end{cases}, \tag{12}$$

where 1 represents rejected hypothesis, 0 - accepted one, and K_α is the α - quantile of the central χ^2 distribution with 3 degrees of freedom.

Statement 1. Sequence of the decision rules (12) for the problem (3) is **similar.** It is also **consistent** if the condition

$$\lim_{k\to\infty}\bar{\vec{\Delta}}'(k)\Sigma^{-1}{}_{\hat{\Delta}}(k)\bar{\vec{\Delta}}(k)=0 \tag{13}$$

where

$$\bar{\vec{\Delta}}(k)=\frac{\sum_1^k\vec{\Delta}(k)}{k} \tag{14}$$

is fulfilled (this statement is a special case of the theorem below).

4. CONSISTENCY CONDITIONS FOR DECISSION RULES IN GENERAL CASE.

The fact that p tracks $\vec{Z}_{i_1}(k),\ldots\vec{Z}_{i_p}(k),k=1,2,\ldots$ correspond to the same source is equivalent to the set of *p-1* equations in the *3n*-dimensional parameter space Θ of mean vectors $\vec{\Theta}^n(k)=\{\vec{\Theta}'_1(k),\ldots,\vec{\Theta}'_n(k)\}'$:

$$\vec{\Theta}_{i_1}(k)-\vec{\Theta}_{i_2}(k)=\vec{0},\ldots,\vec{\Theta}_{i_1}(k)-\vec{\Theta}_{i_p}(k)=\vec{0} \tag{15}$$

or in the vector-matrix notation

$$A_p\vec{\Theta}^n(k)=\vec{0} \tag{16}$$

where A_p is corresponding to *3(p-1)x3n* matrix (see [1]). Let us formulate definitions, that extend the corresponding definitions of [1] to our case.

Definition 1. Sequence ***a*** of matrix $\{A_k\}$ $k=1,2,\ldots$, is called asymptotically **similar** to matrix *A1* (***a*** $\prec A1$) if $A_k, A1\in\mathbf{A}$ and $kernel(A1)\subseteq\bigcap_k kernel(A_k)$.

Let ***a*** = $\{A_k\}$ be some sequence of matrix and $\Phi(k):(\vec{Z}_1^k)\to A_k$ is corresponding sequence of decision rules, and let *A1* be matrix related to an actual solution.

Definition 2. Sequence of decision rules $\Phi(k):(\vec{Z}_1^k)^n\to A_k$ will be called asymptotically **similar,** if for any ***a*** $=\{A_k\}\prec A1$

$$\lim_{k\to\infty}P(A_k/A1)\le\alpha, \tag{17}$$

α - significance level, and will be called **consistent,** if for any ***a*** = $\{A_k\}$, which is not asymptotically similar to A1,

$$\lim_{k\to\infty} P(A_k / A1) \to 0 \qquad (18)$$

is fulfilled.

*This similarity definition means that probability of false objects generation is always less than α, while consistency means that possibility to omit objects **decreases with increasing of observation time**.*

We will introduce the following recursive sequence of vector statistics

$$\bar{\bar{Z}}^n(k) = \frac{k-1}{k}\bar{\bar{Z}}^n(k-1) + \frac{1}{k}Z^n(k), \quad k = 1,2,..., \qquad (19)$$

where:

$$\vec{Z}^n(k) = \{\vec{Z}_1(k),...,\vec{Z}_n(k)\}$$

$$(\vec{Z}_1^k)^n = \{\vec{Z}^n(1),...,\vec{Z}^n(k)\}$$

This sequence member distribution is normal

$$\bar{\bar{Z}}^n(k) \cong N(\vec{\vartheta}^n(k), \bar{\Sigma}(k)) \qquad (20)$$

with

$$\vec{\vartheta}^n(1) = \vec{\Theta}^n(1)$$

$$\vec{\vartheta}^n(k) = \frac{k-1}{k}\vec{\vartheta}^n(k-1) + \frac{1}{k}\vec{\Theta}^n(k) \qquad (21)$$

and

$$\bar{\Sigma}(1) = \Sigma_1^n(1)$$

$$\bar{\Sigma}(k) = \frac{(k-1)^2}{k^2}\bar{\Sigma}(k-1) + \frac{1}{k}\Sigma_1^n(k) \qquad (22)$$

where

$$\Sigma_1^n(k) = \begin{Vmatrix} \Sigma_1(k) & & 0 \\ & \Sigma_p(k) & \\ 0 & & \Sigma_n(k) \end{Vmatrix}, \qquad (23)$$

$\Sigma_i(k)$ is covariance matrix of vector measurement $\vec{Z}_i(k)$. The tested hypothesis with respect to statistics (19) can be rewritten in the form:

$$A_p\vec{\vartheta}^n(k) = \vec{0}, \; k = 1,2,..., \qquad (24)$$

We will include the sequence of statistics

$$U_{A_k}((\bar{\bar{Z}}_1^k)^n) = [A_k(\bar{\bar{Z}}_1^k)^n(k)]^t[A_k\Sigma_1^n(k)A_k^t]^{-1}$$
$$A_k[(\bar{\bar{Z}}_1^k)^n(k)] \qquad (25)$$

Denote by $B(\alpha, (\vec{Z}_1^k)^n)$ (compare with [1]) a subset of sequences of matrices, such as $U_{A_k}((\vec{Z}_1^k)^n) < K_\alpha^r$, where K_α^r is the α - quantile of the central χ^2 distribution with r degrees of freedom and $r = rank\, A_k$. We are looking for sequence of decision rules, that satisfies (17) and (18). Such sequence is represented in the following theorem.

Theorem: Sequence of decision rules, such, as every member of it is represented by:

$$\Phi(k) : (\vec{Z}_1^k)^n \to \hat{A}_k$$

$$rank\hat{A}_k = \max_{A\in B(\alpha,(\vec{Z}_1^k)^n)}(rankA) \qquad (26)$$

is asymptotically similar with significance level α. If the condition

$$\lim_{k\to\infty}\{(A_k\vec{\vartheta}^n(k))^t(A_k\bar{\Sigma}(k)A_k^t)^{-1}(A_k\vec{\vartheta}^n(k))\}^{-1} = 0 \qquad (27)$$

is fulfilled, the sequence is also **consistent.**

Proof: Proof of the first part of the theorem follows from the corresponding statement [1] and the limit theorems.

To prove the second part of the theorem it is sufficient to show that the condition (27) replaces the condition $\max_{1\le i\le n}(trace\Sigma_i) \to 0$ in [1].

If some matrix $\hat{A}_k$ has already been found, and tracks $\{\vec{Z}_1^k{}_{i_1},...,\vec{Z}_1^k{}_{i_p}\}$ correspond to the same cluster, then at every time k the object location estimate $\hat{\bar{\bar{Z}}}$ is calculated as follows:

$$\hat{\bar{\bar{Z}}}(k) = W_{i_1}(k)\bar{\bar{Z}}_{i_1}(k) + W_{i_2}(k)\bar{\bar{Z}}_{i_2}(k) + ... + W_{i_p}(k)\bar{\bar{Z}}_{i_p}(k) \qquad (28)$$

$$W_{i_m}(k) = \Sigma^{-1}{}_{i_m}(k)(\Sigma^{-1}{}_{i_1}(k) + ... + \Sigma^{-1}{}_{i_p}(k))^{-1}, \quad 1 \le m \le p \qquad (29)$$

5. THE DECISION RULE ALGORITHM

Statistic (25) can be represented at every time step k in the following form (compare with (12) in [1]):

$$U_{A_k}((\bar{\bar{Z}}_1^k)^n) = \sum_{p=1}^{m} U_{A_{k_p}}((\bar{\bar{Z}}_1^k)^n) \qquad (30)$$

where every item can be interpreted as p-cluster "weight" and the sum represents the "weight" of an arrangement. Then according to the decision rule (26), for all arrangements, whose weights do not exceed their corresponding threshold, it has already been found an arrangement with minimum number of clusters. Let matrix $\hat{A}_k$

$$\hat{A}_k = \left\| \begin{matrix} \hat{A}_{k_1} \\ \hat{A}_{k_1} \\ \\ \\ \hat{A}_{k_m} \end{matrix} \right\| \qquad (31)$$

be matrix corresponding to this arrangement. The algorithm works as follows:

- At the time step $k+1$ the new "average track measurement" (19) is calculated
- Then "average track measurement" covariance matrices (23) are calculated.
- Now build new arrangements (see 5.[1]) with the above averages, according to (31).
- Update the clusters from the previous step with new arrangement (some previous clusters may be separated).
- The separated clusters, which are considered as an ordinary elements, each with its own "weight" and location in cluster "center", merge in according with the decision rule (26).

Merger of the clusters is performed using the following equations:

$$W = W_i + W_j + (\vec{r}_i - \vec{r}_j)^t (\Sigma_{\vec{r}_i} + \Sigma_{\vec{r}_j})^{-1} (\vec{r}_i - \vec{r}_j)$$
$$r = (\Sigma^{-1}{}_{\vec{r}_i} + \Sigma^{-1}{}_{\vec{r}_j})^{-1} (\Sigma^{-1}{}_{\vec{r}_i} \vec{r}_i + \Sigma^{-1}{}_{\vec{r}_j} \vec{r}_j) \qquad (32)$$
$$\Sigma = (\Sigma^{-1}{}_{\vec{r}_i} + \Sigma^{-1}{}_{\vec{r}_j})^{-1}$$

where W, r, Σ W_k, r_k, Σ_{r_k} - "new cluster" ,"old clusters"($k = I,j$) weights, centers, and matrix covariance of centers correspondingly.

ACKNOWLEDGMENT

The author would like to thank Dr. Michael Eskin and Mr. Arieh Tsentsiper for their useful comments and suggestions and for constant support in carrying out the work reported in this paper.

REFERENCES

[1] A. Pinsky, M. Eskin, Y. Soroka, Statistical Cluster Analysis Approach to Sensor Fusion Problem, NAECON 97', pp. 809-814

AUTHOR BIOGRAPHY

Alexander Pinsky was born on December 25, 1946. He received the B.S. and M.S. degrees in mathematics from The Moscow State University in 1968 and Ph.D degree in cybernetics from The Moscow Institute of Radio and Telecommunications USSR, in 1972.

From 1972 to 1990 he was Assistant Professor at the Department of Mathematics of The Moscow Radio and Telecommunications Institute, USSR.

From 1992 to the present time he works as researcher at the Avionics Systems Engineering dept., IAI. His research interests include estimation theory, hypothesis testing and sensor fusion problems.

Address: Avionic Systems Engineering, Dept. 2641, Lahav Division, Israel Aircraft Industries Ltd., Ben-Gurion Int'l Airport, 70100 ISRAEL

Fax: 972-3-935-3196