# Attitude Determination of Small Spinning Spacecraft Using Three Axis Magnetometer and Solar Panels Data

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Abstract-An algorithm for attitude determination of a spinning spacecraft in Low Earth Orbit using three axis magnetometer and solar panels data is proposed. With this algorithm, the solar panels necessary for power generation are also used as a sensing system for attitude determination. The algorithm employs voltage, current and temperature measurements from all of the solar panels to reconstruct the sun direction, which is based on an accurate solar panels mathematical model and requires some data filtering, according to the motion of spinning spacecraft. The sun direction determination accuracy is improved by exploiting a known relationship between sun and magnetic field motion in the spacecraft reference frame. It is shown, by numerical simulation, that the spacecraft attitude can be determined within a few degrees of accuracy in a reasonable time (on the order of one min), which is usually adequate for small, low cost, missions.

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# 1. Introduction

In small and micro-scale spacecraft on-board resources to perform the mission tasks are typically very limited. Power and weight limitations are often most stringent, while continuous improvements in the field of computing systems might give the possibility of inexpensively using very powerful computing systems. For future spacecraft this trend is very likely to continue, leaving power and weight as the main limiting factors to micro spacecraft design. In the field of attitude determination this might allow the on board implementation of very complex attitude determination

algorithms, reducing the sensing hardware weight and power.

In this paper an attitude determination system for small spinning spacecraft is proposed, using three axis magnetometer and solar panels data. By using such data the solar panels necessary for power generation are also used as a sensing system for attitude determination, eliminating the need for sun sensor usually employed in spinning spacecraft attitude determination. The price paid for the achieved reduction in weight and power is the increased computation and memory storage necessary for the attitude determination algorithm.

Most of the algorithm effort is devoted to the evaluation of the direction of the sun, based on the measured by solar panels voltages, currents and temperatures. A mathematical model of the solar panels is employed that includes the temperature and space radiation degradation effects on the electrical characteristics. Solar panel geometry is also included in the estimation process, assuming the spacecraft to be spinning without nutation. The three axis magnetometer data reduction requires less computing effort, since fewer data are present and the magnetometer is inherently a much more accurate sensor than the solar panels.

The estimated sun direction and the earth magnetic field direction given by magnetometer are then used to evaluate the body axis orientation, using the classic cone intersection algorithm.

The proposed algorithm will be used as the attitude determination system for the UNIversity SATellite (UNISAT) microsatellite, under construction at Scuola di Ingegneria Aerospaziale, Università di Roma "la Sapienza". This microsatellite is an octagonal prism weighting less than 20kg. The solar panels, made of silicon solar cells, are body mounted on the side faces and on the bases. A miniature three axis magnetometer is used for the magnetic field determination. The impact on the satellite weight is negligible, since the magnetometer weight is only few grams. Moreover the solar panels data used by the algorithm are available on board, since they are used for the satellite health monitoring, and there is no need for additional telemetry channels.

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The algorithm performance has been evaluated by numerical simulation. Measurements are simulated adding Gaussian noise to the theoretical values. Results are discussed, showing that spin axis can be determined within a few degrees of accuracy in a short time (on the order of 1 min), which fits the mission requirements very well.

# 2. Solar Panel Mathematical Model

To evaluate the angle between the sun and a solar panel, an accurate mathematical model of the solar panel itself is needed, including temperature, space radiation degradation and sun angle effects on the solar panel i-v curve. The solar panel mathematical model used in the attitude determination algorithm is briefly described here, with some details on the UNISAT silicon solar panels, which are useful in understanding the sun direction evaluation procedure.

A single diode solar panel equivalent circuit can be assumed as in Fig. 1[1].

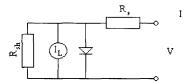


Figure 1 – Solar panel equivalent circuit

From this model, connecting many cells in series, the i-v curve shown in Fig. 2 is obtained for the UNISAT solar panels.

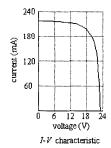


Figure 2 – UNISAT solar panel i-v curve at room temperature (25°C)

### Temperature Effect

The temperature effect on the solar panel's i-v curve is shown in Fig. 3, where voltages and currents are those of the UNISAT solar panels.

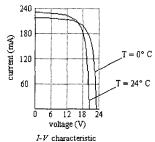


Figure 3 - Temperature effect on solar panel i-v curve

The open circuit voltage linearly decreases temperature, while the short circuit current increases with the temperature logarithm. The rate of change with temperature can be accurately predicted by solar cell theory [1] and it has been confirmed by tests run in a solar simulator for the UNISAT solar panels, including solar cell cover material effect.

# Sun Angle Effect

The effect of sun angle on the spacecraft short circuit current and open circuit voltage is [1]:

$$I_{sc} = I_{sc} = \cos \sigma \tag{1}$$

$$I_{SC} = I_{SC} \cos \sigma \tag{1}$$

$$V_{OC} = V_{OC} - V_{T} \log(\cos \sigma) \tag{2}$$

where  $\sigma$  is the angle between the sun and the direction normal to the solar panel (Fig. 4).

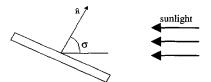


Figure 4 - Sun angle definition

Fig. 5 and Fig. 6 show the theoretical values of  $I_{sc}$ ,  $V_{oc}$  given respectively by Eq. 1 and Eq. 2 (dotted line), and those (cross black symbol) experimentally obtained using a solar simulator calibrated at power flux of  $1 \frac{KW}{m^2}$ .

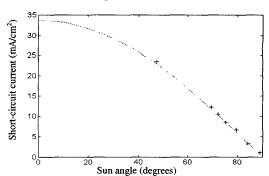


Figure 5 – Mathematical prediction and experimental test of the effect of sun angle on short circuit current

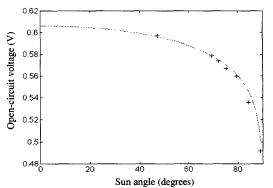


Figure 6 - Mathematical prediction and experimental test of the effect of sun angle on open circuit voltage

The voltage variation with the sun angle is relatively low, being roughly ten percent for a sun angle between  $0^{\circ}$  and  $80^{\circ}$ . The current is properly what is sensing the angle, and, as discussed in the following section, the sun angle is very sensitive to current noise.

# Space Radiation Degradation Effect

Exposure for a long time to the Space Radiation environment reduces the solar panel's performances [1]. Depending on the charged particle flux, the solar panel's maximum power reduction can reach a few percent per year. This process occurs on time scales orders of magnitude longer than attitude determination times. However, for long lived spacecraft, the on board solar panel model should include this effect. It can be theoretically evaluated [1], or calculated during the mission, by processing data gathered on board. In this paper solar panel degradation is neglected, since the expected lifetime for UNISAT is roughly one year.

# Connection of solar panels to the battery

The UNISAT solar panels are connected in parallel, all feeding the battery directly through a diode, as shown in Fig. 7.

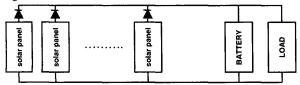


Figure 7 - UNISAT solar panel parallel connection

Because of this parallel connection, all the solar panels exposed to sunlight are nominally at the same voltage, imposed by the battery and load conditions. It means that only a single voltage has to be measured for sun angle determination. The sun direction measurement accuracy depends, of course, on the accuracy of the solar panel i-v curve stored on board, including all the effects previously discussed. The UNISAT solar panel mathematical model parameters have been obtained through extensive testing, using a calibrated solar simulator.

# 3. SUN ANGLE MEASUREMENT FOR A SINGLE SOLAR PANEL

The sun angle measurement for a single panel is based on temperature, voltage and current data. The i-v curve at the measured temperature is selected, according to the solar panel mathematical model, as shown in Fig. 8.

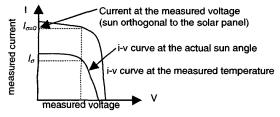


Figure 8 – Sun angle measurement for a single solar panel

This is used to evaluate the current  $I_{c=0}$  (sun orthogonal to the solar panel) at the measured voltage. For the sun angle cosine law given by Eq. 1, the sun angle can be approximately evaluated as:

$$\sigma = \cos^{-1}(I_{\sigma}/I_{\sigma=0}) \tag{3}$$

# 4. Evaluation of the Sun Direction

UNISAT has the shape of an octagonal prism. The solar panels are mounted on the external surface of the spacecraft as shown in Fig. 9.

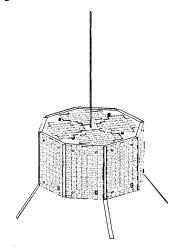


Figure 9 – Solar panels on board UNISAT

The spacecraft reference frame ijk with k along the spin axis and ij rotating at the spin angular rate is introduced. The sun direction is expressed in the spacecraft reference frame by the co-elevation angle  $\theta$ , between the sun and the spin axis, and the in plane angle  $\alpha$ . The angles  $\theta_B$ ,  $\alpha_B$ , define the magnetic field orientation in the same reference frame, as shown in Fig. 10.

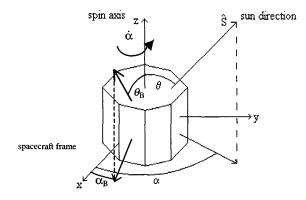


Figure 10 - Sun direction angular parameters

From the geometry in Fig. 11, the unit vector normal to the i-th panel forms the angle  $\alpha_i$  with the projection of the sun in the plane orthogonal to the spin axis.

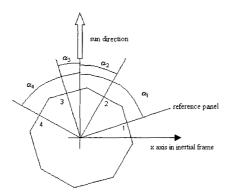


Figure 11 - Lateral solar panels in the sunlight.

The sun angle of each lateral panel can be then expressed as:

$$\cos\sigma_i = \sin\theta\cos\alpha_i \tag{4}$$

The sun angle of the upper base of the prism is  $\theta$ . For the cosine law applied to the upper panel, we can write Eq. 4 as

$$\cos \sigma_i = \cos \alpha_i \sqrt{1 - \left(\frac{I_{up}}{I_{\sigma = oup}}\right)^2} = \frac{I_i}{I_{\sigma = o_i}}$$
 (5)

where  $I_i$  is lateral panel measured current,  $I_{\sigma=0}$  i the lateral panel zero sun angle current, and  $I_{up}$ ,  $I_{\sigma=0}$  up, the same parameters for the upper solar panel.

The maximum number of sun illuminated lateral panels is four, therefore we have four equations like Eq. 5.

The angle  $\alpha_i$  is calculated with the least-squares method, minimizing the function:

$$J = \frac{1}{2} \left[ (A\cos\alpha_{1} - B_{1})^{2} + (A\cos\alpha_{2} - B_{2})^{2} + (A\cos\alpha_{3} - B_{3})^{2} + (A\cos\alpha_{4} - B_{4})^{2} \right]$$

where

$$A = \sqrt{1 - \left(\frac{I_{\text{sup}}}{I_{\sigma = o_{\text{sup}}}}\right)^2} \tag{6}$$

$$B_{i} = \frac{I_{i}}{I_{\sigma = o_{i}}} \qquad i = 1, 2, 3, 4. \tag{7}$$

Taking into account that the following geometrical constraint holds among angles  $\alpha_i$ :

$$\alpha_i = \alpha_1 + i \frac{\pi}{4}$$

The solution for  $\alpha_l$  is

$$tg\alpha_{1} = -\frac{\frac{I_{3}}{I_{\sigma=o_{3}}} + \frac{\sqrt{2}}{2} \left( \frac{I_{2}}{I_{\sigma=o_{2}}} + \frac{I_{4}}{I_{\sigma=o_{4}}} \right)}{\frac{I_{1}}{I_{\sigma=o_{1}}} + \frac{\sqrt{2}}{2} \left( \frac{I_{2}}{I_{\sigma=o_{2}}} - \frac{I_{4}}{I_{\sigma=o_{4}}} \right)}$$
(8)

The angle measurement given by (8) is affected by errors due to noise in the measure of the panels data. As discussed before, angle measurements are mostly affected by current noise. This is confirmed by numerical results obtained simulating the UNISAT solar panel behavior. The influence of different errors is analyzed assuming noise only on one of the data being measured. The angle measurement error distributions due to noise only in voltage and current are shown in Fig.12 and Fig.13.

While voltage noise effects are very low, leading to negligible angle errors even in the presence of quite high noise, a 50 mA standard deviation noise in the current leads to huge errors in the angle evaluation, as high as 34° standard deviation.

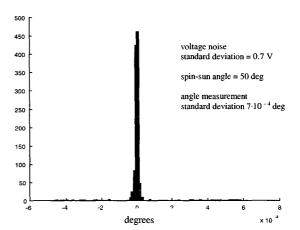


Figure 12 - In plane sun angle measurements error distribution for noise voltage measurements

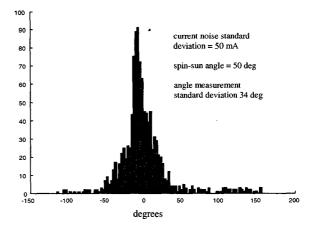


Figure 13 - In plane sun angle measurements error distribution for noise current measurements

# 5. Data Filtering for a Spinning Spacecraft

As shown in the previous section, a single sun direction measurement, obtained by gathering information from several solar panels, can be affected by a significant error due to measurement noise or poor solar panel behavior modeling. To remove the noise effect, different filtering schemes can be used, exploiting the fact that the spacecraft

can be assumed to be spinning at a constant angular rate for short periods of time, on the order of a few minutes, which are enough for attitude determination, as shown in the following. In such a period of time also the magnetic field is assumed to be constant in the inertial reference frame.

Sun-spin axis and magnetic field-spin axis angle estimation

Assuming fixed spin axis, sun, and magnetic field, the sunspin axis and the magnetic field-spin axis angles are constant, and they can be efficiently estimated using a recursive least square estimator [2], simply averaging the measured data as soon as they are available.

Fig. 14 and Fig. 15 show the sun-spin axis angle and the magnetic field-spin axis angle estimation.

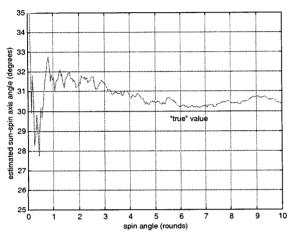


Figure 14 - Sun-spin axis angle estimation

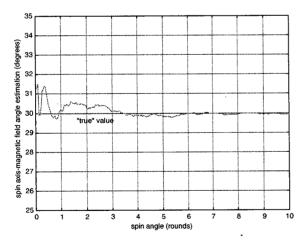


Figure 15 - Magnetic field-spin axis angle estimation

Simulated measurement noise standard deviation in the solar panel voltage is 0.5 V, in solar panel current is 50 mA, in solar panel temperature is 20°C. For the magnetic field direction a conservative value of 2° is assumed, which takes into account for magnetometer inaccuracy and for the error in the Earth magnetic field model. The spacecraft spin rate is 5rpm. Measurements are taken every half a second.

In plane sun angle estimation by sun measurement Kalman filtering

Assuming a fixed spin axis and a fixed sun direction in an inertial reference frame, attitude motion can be described by the following equation

$$\begin{bmatrix} \alpha \\ \Delta \alpha \end{bmatrix}_{k+1} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \alpha \\ \Delta \alpha \end{bmatrix}_{k}$$
 (9)

in which  $\alpha$  is the spin angle, increasing by the constant amount  $\Delta\alpha$  at each measurement, assuming the measurements are taken at constant time intervals. In a state space description of the system motion, the in plane sun angle  $\alpha$  measurement at time k is

$$z_k = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \alpha \\ \Delta \alpha \end{bmatrix}_k \tag{10}$$

A Kalman filter [2] can then be set up for optimal estimation of  $\alpha$  and  $\Delta\alpha$  from equations (9) and (10), using only data from solar panels. This is the simplest method of in plane sun angle estimation, but, since measurements are very noisy, long times could be needed for filter convergence.

In plane sun angle estimation by sun and magnetic field measurement Kalman filtering

Magnetometer measurements are typically much more accurate than solar panel measurements, and, assuming a constant magnetic field, the same relations (9) and (10) hold for the magnetic field motion with respect to the spinning spacecraft:

$$\begin{bmatrix} \alpha_B \\ \Delta \alpha_B \end{bmatrix}_{k+1} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \alpha_B \\ \Delta \alpha_B \end{bmatrix}_k \tag{11}$$

and

$$z_{Bk} = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \alpha_B \\ \Delta \alpha_B \end{bmatrix}_k \tag{12}$$

where  $\alpha_B$  is the in plane angle between the reference panel and the magnetic field. Since the sun and the magnetic field are fixed in an inertial reference frame, the angle increment between two angle measures must be the same for the sun and the magnetic field. This can be exploited in the sun estimation, using the estimated  $\Delta\alpha_B$  obtained from the magnetic field (much more accurate than the one from the sun) as a  $\Delta\alpha$  measure to be added in the sun estimation process in equation (10). Then the sun and magnetic field estimations are coupled, using the following equation for the sun measurements instead of (10):

$$z_k = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \alpha \\ \Delta \alpha_B \end{bmatrix}_k \tag{13}$$

In plane sun angle estimation using the constant phase angle with the magnetic field

In the method just described, it is implicitly assumed that the in plane angle difference between the sun and the magnetic field  $\beta$  is constant, as shown in Fig. 16.

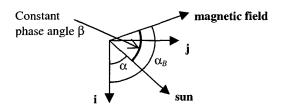


Figure 16 – In plane phase angle between sun and magnetic field

This constant can be efficiently estimated by recursive least square processing of the difference between the measured magnetic field and sun angles. The sun angle is then evaluated as:

$$\alpha = \alpha_R - \beta \tag{14}$$

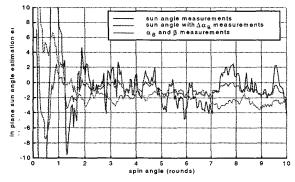
The last two algorithms are based on the same principle, but the last one is more efficient, since it requires less computation.

Comparison among the estimation procedures

The three proposed procedures for the in plane sun angle estimation have been compared through numerical simulation. The results are shown in Fig. 17.

The sun angle estimation using only solar panels data converges in few spin periods, starting with errors of tens of degrees and reaching errors of the order of 5°.

The estimation improves when using also the magnetic field measured  $\Delta\alpha$  in the filtering process (Eq. (9) and (13)). Convergence is obtained in about two spin periods and the estimation error does not exceed 2°. This method, here shown in the assumption of constant magnetic field, can easily be generalized to the real in orbit situation of slowly varying magnetic field, provided that a magnetic field model



**Figure 17** – In plane sun angle estimation errors. Comparison among different estimation procedures

is available on-board. The performances are not expected to degrade significantly. In fact, the conservative value of 2° assumed here for the magnetic field direction measurements noise standard deviation, takes into account for errors in the Earth magnetic field mathematical model and eventually for the filter delay in following the magnetic field motion.

The method using the phase angle  $\beta$  has an estimation error of about 3°. This method cannot be extended to the situation of moving magnetic field, because it is based on the assumption of a constant phase angle  $\beta$ .

Thus the best performing sun estimation method seems to be the one using equations (9) and (13), capable to estimate the sun direction within 2° in the assumptions made before.

#### 6. ATTITUDE DETERMINATION

Attitude determination of a spinning satellite consists of the estimation of the spin axis and the spin angle, or, as in many small missions, spin axis and spin angular rate.

Knowing the directions of two external reference vectors, in our case the sun and the magnetic field, the spin axis can be determined using the cone intersection algorithm proposed in [3] (Fig. 18). The angle  $\varphi_I$  between the estimated sun direction and the unknown spin axis determines a cone whose axis is the sun direction, while the angle  $\varphi_2$  between the magnetic field and the same unknown vector determines another analogous cone. The two possible solutions for the unknown axis are determined by the intersection of two cones, as shown in Fig. 18, where  $\hat{S}$  and  $\hat{B}$  are respectively the unit vectors of the sun and the magnetic field directions and  $\hat{N}$  is the unit vector

$$\hat{N} = (\hat{B} \times \hat{S}) / |\hat{B} \times \hat{S}|$$

The two solutions ambiguity is usually resolved by the fact that the true spin axis does not move in the inertial frame while  $\hat{B}$  varies along the orbit. The false solution is the one following the motion of  $\hat{B}$ . This procedure is very efficient, but requires time scales on the order of the motion of  $\hat{B}$  in orbit to discriminate the correct solution. Moreover it cannot be used in our hypothesis of fixed  $\hat{B}$  (short times for attitude determination). The complete attitude determination can be obtained following [4]. The cones formed from all three spacecraft axis  $(\hat{i}, \hat{j}, \hat{k})$  with  $\hat{S}$  and  $\hat{B}$  are considered, obtaining eight possible solutions for the triad  $\hat{i}, \hat{j}, \hat{k}$ . The unique correct solution satisfying the property that  $\hat{i}, \hat{j}, \hat{k}$  is

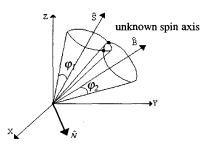


Figure 18 - Cone intersection algorithm geometry

an orthogonal right reference frame can be obtained as follows:

$$\begin{bmatrix} \hat{i} \\ \hat{j} \\ \hat{k} \end{bmatrix} = M^{-1} \begin{bmatrix} \hat{B} \\ \hat{S} \\ \hat{N} \end{bmatrix}$$
 (15)

where M is

$$M = \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{23} & m_{33} \end{bmatrix}$$

and

 $m_{11}=\sin\theta_B\cos\alpha_B$  ,  $m_{12}=\sin\theta_B\sin\alpha_B$  ,  $m_{13}=\cos\theta_B$  ,

 $m_{21} = \sin\theta \cos\alpha$ ,  $m_{22} = \sin\theta \sin\alpha$ ,  $m_{23} = \cos\theta$ ,

 $m_{31} = \sin \theta_B \sin \alpha_B \cos \theta - \sin \theta \sin \alpha \cos \theta_B$ ,

 $m_{32} = \sin\theta\cos\alpha\cos\theta_B - \sin\theta_B\cos\alpha_B\cos\theta$ ,

 $m_{33} = \sin \theta_B \cos \alpha_B \sin \theta \sin \alpha - \sin \theta \cos \alpha \sin \theta_B \sin \alpha_B$ 

The results obtained following Eq.(15) are shown in Fig.19.

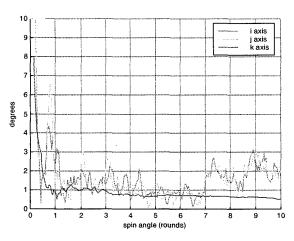


Figure 19 - Spacecraft reference axis estimation errors.

The spin axis  $\hat{k}$  is estimated with an accuracy better than 1°, while the spin angle error (axis  $\hat{i}$  and  $\hat{j}$ ) is on the order of 3°.

As a final remark, it is pointed out that the best part of the orbit to perform the attitude determination in Low Earth Orbit is a few minutes before the satellite enters the eclipse or a few minutes after coming out of the Earth shadow. In fact the sunlight reflected by the Earth on the solar panels is the main source of noise for the sun angle measurement and in those points of the orbit there is no Earth albedo. The Moon albedo is instead always negligible.

# 7. CONCLUSIONS

An attitude determination algorithm for spinning spacecraft with body mounted solar panels is proposed. The algorithm uses data from a three axis magnetometer and detects the direction of the sun by solar panel voltage, current and temperature measurements. In this way the hardware for sun

determination usually present on board can be removed, with some saving in power and weight. According to numerical simulation, the proposed attitude determination algorithm can determine attitude within a few degrees of accuracy, which is often enough for many small, low cost, missions.

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Fabio Bolotti is a aerospace engineer, who graduated in July 1999 from Università di Roma "la Sapienza". While at the University, he took courses on electronics, avionics, structures, materials, propulsion, spacecraft dynamics and aerospace systems. He worked on the attitude determination subsystem for the microsatellite UNISAT, and



he is part of the team GAUSS (Gruppo di Astrodinamica dell'Università degli Studi "la Sapienza"), now building the microsatellite UNISAT, to be launched shortly. He is currently involved in research activities in attitude determination.