Structural Properties of Network Attractor Associated with Neuronal Dynamics Transition

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Abstract

We have found that single neuronal activities in various regions in the brain commonly exhibit the distinct dynamics transition from the white to the 1/f spectral profiles during sleep cycle in cats. Computer simulations using the neural network model which has the asynchronous state transition rule and the symmetry connection showed that the globally applied inhibitory input could induce that transition. The structure of network attractor was suggested to vary associated with the change in inhibitory level. In this paper, in order to examine the robustness of the dynamics transition, the symmetry network structure is newly extended to the asymmetrically connected network model. Under the weak inhibition, the network state intermittently jumps in and out of a few restricted metastable equilibrium states. Under the strong inhibition, the network sticks around the state where a low random activity prevails ("0" state). These dynamical features are similar to the symmetry case. However, the dynamics of the network is modified dependent on an additonal parameter:ρ which means the proportion of excitatory neurons. As the global inhibitory input changes, some asymmetry networks show the complicated dynamics transition. The difference between attractors of the symmetry and the asymmetry networks is also discussed.

1 Introduction

Concerning the biological function of paradoxical sleep(PS), Crick and Mitchson(1983) proposed the reverse learning theory that the spurious memories or learning are filtered out during PS. This theory suggests the close relation between PS and the memory and learning. However, the physiological mechanism mediates between them has been no yet known. Our physiological findings and the modeling study described below might clarify the possible functions of dreaming and PS.

In cat's central nervous system, we have found the following phenomena concerning dynamics of single neuronal spike trains during sleep cycle. i) During slow-wave sleep(SWS), neurons showed almost flat spectral profiles. ii) During paradoxical sleep(PS), dynamics of neuronal activities showed 1/f-like spectral profiles. This phenomenon has been found in various regions of the brain such as the mesencephalic reticular formation(MRF)[1] and the hippocampal cells[2]. Pharmacological study suggested that the globally working serotonergic system is associated with the dynamics transition[2].

We successfully simulated the dynamics transition by using a symmetrically connected neural network model including a globally applied inhibitory input which is supposed to be mediated by serotonergic system[3]. We have also suggested that a variation of network attractor related to the inhibitory level could contribute to this transition. In this paper, for the purpose of studying the above dynamics transition under more general situation the network structure is extended to an asymmetrically connected one. Dynamics of an asymmetry neural network is newly investigated. This asymmetricity follows the rule which approximately reflects the characteristics of synaptic contacts between neurons. Computer simulations again shows the inhibitory input could change the neuronal dynamics from the white to the 1/f under more realistic situation. The geometry of the network attractor realizing the dynamics transition will be discussed in detail.

2 Asymmetrically Connected Neural Network Model

Serotonin is one of biochemical substances which works as neuro-transmitter and/or neuro-modulator. It's likely that this serotonin neuron contributes the dynamics transition of the neuronal activities in the

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brain during sleep cycle. The system of serotonergic neurons has several unusual features. Serotonin fibers are known to make few normal synaptic contact with target neurons, and to give modulatory effects via varicosities on them like a hormone. Almost all CNS regions receive serotonergic afferents. In almost cases, neurophysiological effect of serotonin is supposed to be inhibition. A serotonin-containing neuron shows periodic discharge pattern(about 3 spikes/s) during waking, reduces the firing rate during SWS and then shows little discharge during PS. Above facts prompt us to have the working hypothesis as follows. During SWS neurons are inhibited strongly. During PS neurons are released from inhibition. In brief, we speculate that the change of this global inhibitory input induces the neuronal dynamics transition during sleep cycle.

The neural network model we use consists of fully interconnected neuron-like elements (abbreviated as "neuron")[4]. For the i-th neuron, the evolution rule of its state is defined as follows.

$$u_i(t+1) = \sum_{j=1}^{N} w_{ij} x_j(t) - h + \varepsilon_i(t+1) \tag{1}$$

$$x_i(t+1) = g(u_i(t+1)) \tag{2}$$

$$g(x) = \begin{cases} 1 & , x \geq 0 \\ 0 & , x < 0 \end{cases} \tag{3}$$

$$i = 1, 2, \cdots, N$$

where N(= 100) denotes the number of neurons contained in the network, and t is a discrete time. $\{\varepsilon_i(t)\}$ denotes random perturbations which are assumed as mutually independent, zero-mean white Gaussian noises with an identical variance σ^2, $h(> 0)$ represents the inhibitory input which is fixed here independent of neurons for the sake of simplicity. Besides, synaptic weights $\{w_{ij}\}$ are defined by

$$w_{ij} = \begin{cases} \frac{1}{N} \sum_{m=1}^{M} (2x_i^{(m)} - 1)(2x_j^{(m)} - 1) & , i \neq j \\ 0 & , i = j \end{cases} \tag{4}$$

where, as is the case of an associative memory[5], $x_i^{(m)}$ indicates the i-th neuron's state of the m-th memorized pattern, and M is the number of memorized patterns. In symmetry neural network, the symmetry condition

$$w_{ij} = w_{ji} \tag{5}$$

is satisfied.

As easily known in symmetry network, a neuron has both of inhibitory and excitatory synapses. From the physiological point of view, however, the symmetricity is hardly supposed to be satisfied in actual neural networks. Here, the network structure is extended to an asymmetrically connected one. In this network, an excitatory neuron makes only positive synaptic contacts with target neurons, and an inhibitory neuron does only negative[6]. Although this rule might not always be retained in the actual neural network, the further biological reality could be added to the model rather than the symmetry case. We call this network asymmetry neural network here. According to the following equation, the synaptic weights of the asymmetry neural network $\{w'_{ij}\}$ are determined[6],

$$w'_{ij} = 2w_{ij}\theta(\zeta_j w_{ij}) \tag{6}$$

where w_{ij} denotes the corresponding synaptic weight of the symmetry network, and $\theta(x)$ is a Heaviside step function, i.e.,

$$\theta(x) = \begin{cases} 1 & x \geq 0 \\ 0 & x < 0 \end{cases} \tag{7}$$

The parameter ζ_j indicates the property of the j-th neuron:+1 for excitatory, and -1 for inhibitory. Prior to the above operation, the neuron type(inhibitory or excitatory) is assigned at random on each neuron with certain proportions ρ(excitatory) and $1 - \rho$(inhibitory).

3 Simulation Results

A PSD of a time series: $x_i(t)$ is calculated to characterize the dynamics of a neuron in our simulation. The evolving operation is performed in the asynchronous (cyclic) manner[4]. Sequentially evolved states of 11000 are subject to spectrum analyses except for the initial 1000 states. The memorized patterns and the initial states are given as equiprobable binary random sequences.

In Figs.1A, 1B and 2A, 2B typical PSD profiles and raster plots of single neurons with weak and strong inhibitory inputs are presented both for the asymmetry networks with $\rho = 0.4$ and 0.6. All the simulations presented here are performed with M = 20. The raster plot of $x_i(t)$ is shown together with the corresponding spectrum, where a dot indicates $x_i(t) = 1$. The rate of the number of excitatory states to the total data length("mean rate") is reduced as the inhibitory input increases. Through simulations presented here, an inhibitory input is manipulated so that the mean rate in the strong inhibition case is reduced to about 10% of the weak inhibition case. Power spectral density profiles become flat at least in the lower frequency range under strong inhibition. As seen in both figures, clustered active periods still survive under strong inhibition, which result in not totally flat PSD. Nevertheless, roughly speaking, the transition of PSD profile from the 1/f to the white are observed similar to the symmetry case.

The behavior of the asymmetry network possibly depends on the geometrical structure of the network attractor, because the network state is supposed to wander among equilibrium states, which construct the attractor, driven by a random perturbation. This speculation is examined in detail here. Under the weak inhibition, activities of all neurons in the networks are presented in Figs.1C and 2C together with the corresponding behavior of direction cosines in Figs.1D and 2D. The direction cosine here represents the "similarity" between the current network state $\mathbf{x}(t)$ and one of certain equilibrium states $\mathbf{x}^*$ which are found in the network under no perturbation, i.e.,

$$direction \quad cosine = \frac{\mathbf{x}^{*\prime}\mathbf{x}(t)}{|\mathbf{x}^*||\mathbf{x}(t)|} \tag{8}$$

$$\begin{aligned} \mathbf{x}'(t) &= [x_1(t), x_2(t), \ldots, x_N(t)] \\ \mathbf{x}^{*\prime} &= [x_1^*, x_2^*, \ldots, x_N^*] \end{aligned}$$

Therefore, a value close to 1 means good agreement between both vectors.

The plots of all neuronal activities in the network explicitly indicate that some ordered and disordered patterns alternatively appear with varied durations. The direction cosine shows that these ordered patterns almost correspond to one of the equilibrium states which are a priori known under the condition of no perturbation. In other words, the network state is supposed to stay in some meta-stable states and jump to the other states accidentally. In particular, the network in the case of $\rho = 0.4$ switches between ordered and disordered states more frequently than the case of $\rho = 0.6$. The following description is possible concerning the generation mechanism of the 1/f fluctuation of neuronal activity and the dynamics transition. Under the weak inhibition, the network state altenatively visits globally attractive states and stay in them. Since each neuronal state is almost fixed during this stay, this maintained state with varied periods possibly contributes to the low frequency components resulting in the 1/f-like PSD. On the other hand, under the strong inhibition, the "0" state only becomes attractive and the random activities directly driven by the noise are supposed to dominate, which correspond to the disordered states.

Fig.3 gives an example of simulation results with $\rho = 0.5$ and $M = 30$. Also in this case, the transition of PSD profile from the 1/f to the white are observed responsible for the global inhibitory input level. As seen in Figs.3E and 3F, the network state visits only around several meta-stable states which are near located in the state space. This observation seems to be quite different from the previous two examples where certain meta-stable states and "0" states are alternatively visited. Figure 3F suggests that the neighboring states around a certain meta-stable state are highly attractive. Interestingly, when the inhibitory input becomes large($h = 0.54$), meta-stable states and "0" states seem to be attractive as is the previous cases($\rho = 0.6$ and 0.4). Although the behavior of the network state is different each other, neurons can exhibit the 1/f PSD profiles as shown in Figs.3A and 3C. Under the inhibitory input $h = 0.58$, the network tends to visit around "0" state which is attractive under the strong inhibition and the neuron shows the white activity as seen in Figs.3D, 3I and 3J. The above results especially in Fig.3 suggest that the globally applied inhibitory input modifies the structure of the network attractor which

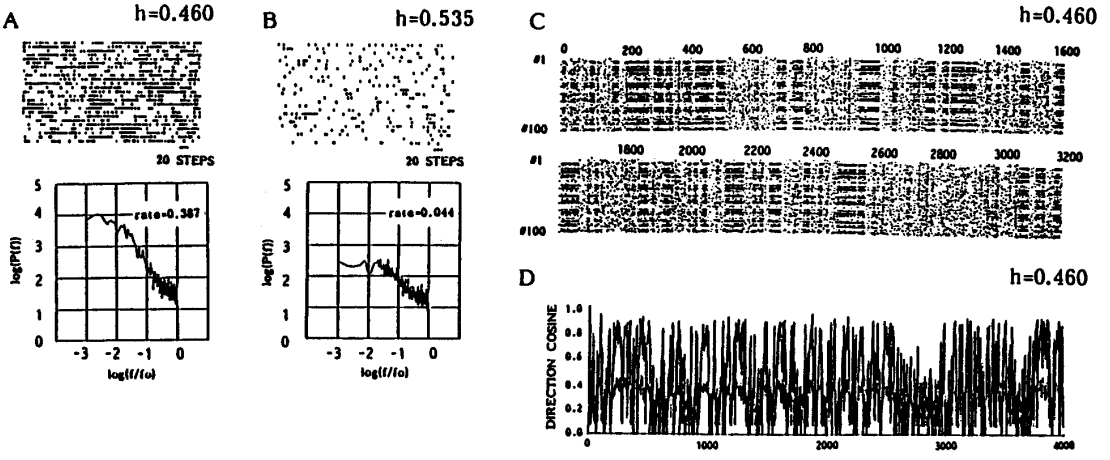

Fig.1 Simulation results on asymmetry neural network($\sigma = 0.25$, $\rho = 0.4$, $M = 20$)
A and **B** : single neuronal activities and PSDs under the inhibitory inputs **A** $h = 0.460$ and **B** $h = 0.535$, respectively.
C : plot of all neuronal activities in the network under the inhibitory input $h = 0.460$.
D : direction cosine under the inhibitory input $h = 0.460$.

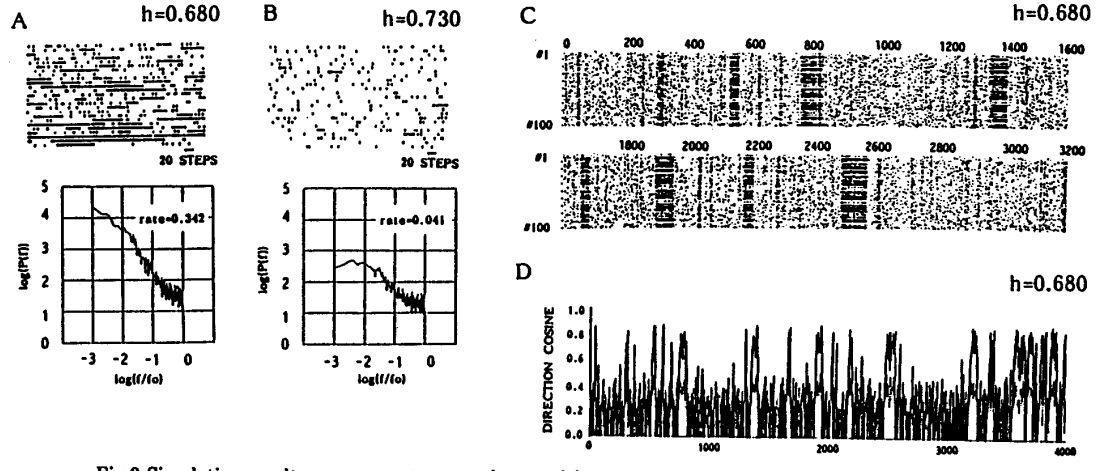

Fig.2 Simulation results on asymmetry neural network($\sigma = 0.33$, $\rho = 0.6$, $M = 20$)
A and **B** : single neuronal activities and PSDs under the inhibitory inputs **A** $h = 0.680$ and **B** $h = 0.730$, respectively.
C : plot of all neuronal activities in the network under the inhibitory input $h = 0.680$.
D : direction cosine under the inhibitory input $h = 0.680$.

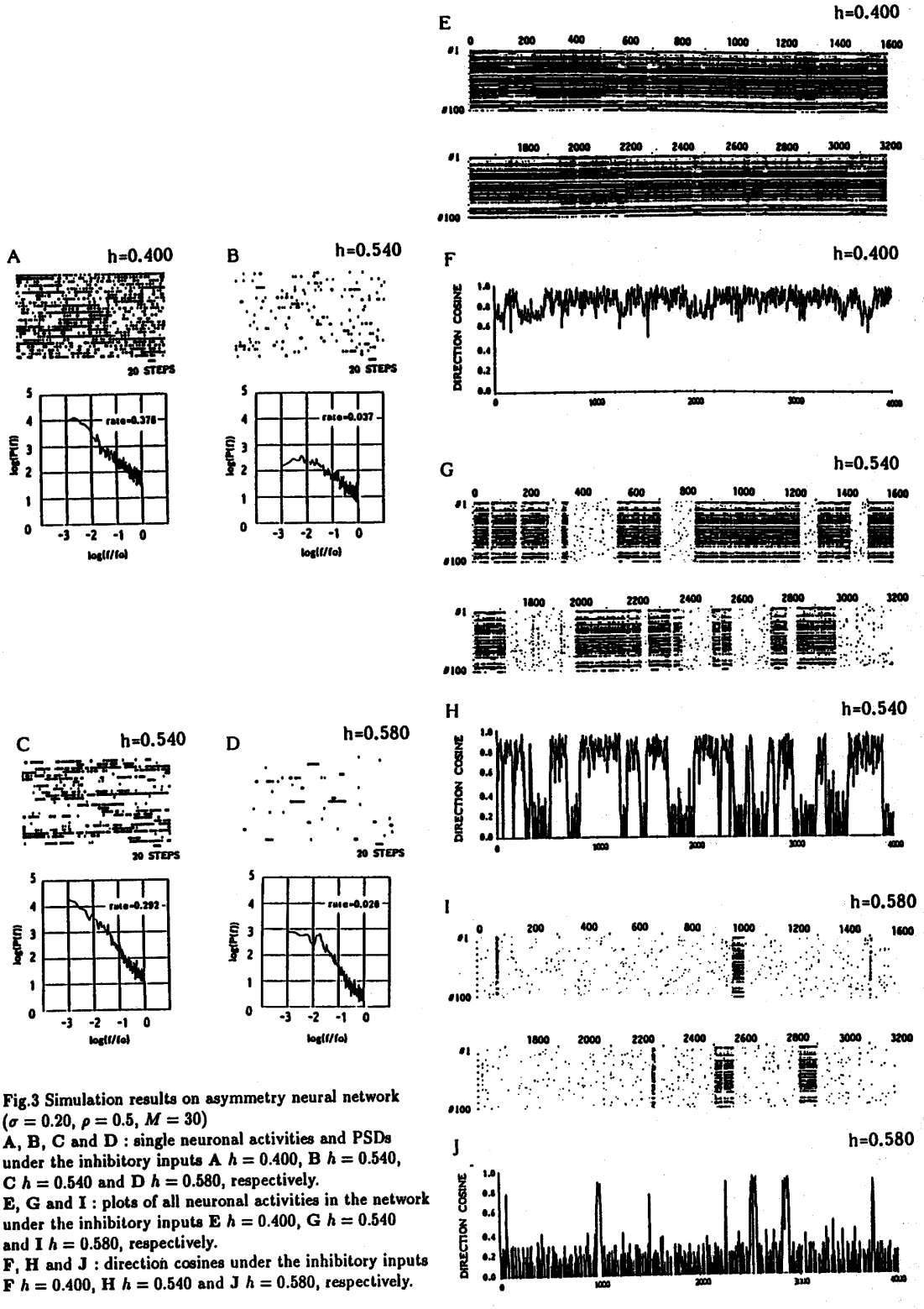

Fig.3 Simulation results on asymmetry neural network ($\sigma = 0.20$, $\rho = 0.5$, $M = 30$)
A, B, C and D : single neuronal activities and PSDs under the inhibitory inputs A $h = 0.400$, B $h = 0.540$, C $h = 0.540$ and D $h = 0.580$, respectively.
E, G and I : plots of all neuronal activities in the network under the inhibitory inputs E $h = 0.400$, G $h = 0.540$ and I $h = 0.580$, respectively.
F, H and J : direction cosines under the inhibitory inputs F $h = 0.400$, H $h = 0.540$ and J $h = 0.580$, respectively.

determines the dynamical features of neuronal activities.

4 Discussion

This study is highly inspired by the research of Sawada concerning Boltzmann machine[7], and also by the spin glass studies(e.g.,[8]). They have commonly found that the "temperature" of the system could induce the dynamics transition of each element contained in the system from the white to the 1/f. We have reproduced this phenomenon on the basis of the physiological reality by applying the global inhibition and the random perturbation in the network.

In this paper, the symmetry network including the global inhibitory input was extended to the asymmetrically connected one which the further biological reality could be added. The dynamics of a single neuronal activity generated by this asymmetry network has been investigated. Within the presented simulations, it is confirmed in more general way that the global inhibitory input could induce the neuronal dynamics transition we concern. Under the weak inhibition, simulation results appear to suggest that a few states are involved in the generation of the 1/f dynamics. The network repeatedly visits a few limited equilibrium states including memorized states and stays for varied durations. That is, when each neuron exhibits 1/f-like activity, the network state wanders among the meta-stable equilibrium states, which coincides with the assumption validating the learning rule for the long term memory maintenance[10]. In contrast, under the strong inhibition, the "0" state are only attractive;the network stick around the state where a low random activity prevails.

References

[1] M. Yamamoto, H. Nakahama, K. Shima, T. Kodama, H. Mushiake, "Markov-dependency and spectral analyses on spike counts in mesencephalic reticular neurons during sleep and attentive states" *Brain Res.*, vol.366, pp. 279–289, 1986.

[2] H. Mushiake, T. Kodama, K. Shima, M. Yamamoto, H. Nakahama, "Fluctuations in spontaneous discharge of hippocampal theta cells during sleep-waking states and PCPA-induced insomnia" *J. Neurophysiol.*, vol.60, pp. 925–939, 1988.

[3] M. Nakao, T. Takahashi, Y. Mizutani, M. Yamamoto, "Simulation study on dynamics transition in neuronal activity during sleep cycle by using asynchronous and symmetry neural network model" *Biol. Cybernetics*, vol.63, pp. 243–250, 1990.

[4] J. J. Hopfield, "Neural networks and physical systems with emergent collective computational abilities" *Proc. Natl. Acad. Sci. USA*, vol.79, pp. 2254–2258, 1982.

[5] T. Kohonen, *Associative Memory.* Springer, Berlin, Heidelberg, New York, 1977.

[6] S. Shinomoto, "A cognitive and associative memory" *Biol. Cybernetics*, vol.56, pp. 1–10, 1987.

[7] Y. Sawada, "Memory and fluctuation in a neuronal network model" *Proc. JSMEBE Prof. Group Meeting on Acquisit. Anal. Biol. Sig.*, 1986.

[8] E. Marinari, G. Paladin, G. Parisi, A. Vulpiani, "1/f noise, disorder and dimensionality" *J. Physique*, vol.45, pp. 657–663, 1984.

[9] F. Crick, G. Mitchison, "The function of dream sleep" *Nature*, vol.304, pp. 111–114, 1983.

[10] S. Shinomoto, "Memory maintenance in neural networks" *J. Phys. A: Math. Gen.*, vol.20, pp. L1305–L1309, 1987.