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MULTITARGET TRACKING USING DOMINANT PROBABILITY DATA ASSOCIATION

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Abstract

A new suboptimal approach to the probability data association of multitarget tracking, the Dominant Probability Data Association (DPDA), is presented in this paper. In view of the fact that the case where many targets cross together or move in a very "small" neighborhood, rarely occurs for most practical multitarget tracking enviroments we may define a dominant joint event and corresponding dominant joint probability. Using Bayesian rule, we can deduce a formula of the dominant joint probabilities without calculating the all joint probabilities of all joint events such as in joint probability data association (JPDA). So, the DPDA can avoid the problem of combinatarial "explosion" in JPDA. In addition, we prove that the top limit of performance of DPDA is equal to that of JPDA and the low limit is not lower than that of probability data association (PDA) and that the event with low limit is the one with very small probability. Monte Carlo simulation results give out inspiring performance.

1. INTRODUCTION

In most practical multiple target tracking enviroments, there is no probability that many targets cross together or move in a very "small" neighborhood. If so, the group target tracking method would be considered. Assuming that there are T targets in a surveillance region and Tc(Tc$=1,2,\cdots,$T) targets in a "small" neighborhood of the region, let p(Tc/T) represent the probability of the event that Tc of T targets appear in the "small" neighborhood. Obviously for Tc/T$=1/$T, we will have p(Tc/T)$\approx$ 1; and for Tc/T$=1$ and large number T, p(Tc/T)$\gtrsim 0$. Further more, p(Tc/T) will decrease sharply along with Tc/T increasing. This fact gives us an inspiration that those joint events with very small joint probabilities may be left out. Hereon we define the joint events with Tc$\leqslant 2$ as dominant joint events and corresponding joint probabilities as dominant joint probabilities. The rest of joint events with Tc>2 will be disregarded. Using Bayesian rule we can deduce a fomula calculting marginal probability. where there is no need to calculate all joint probabilities of all joint events thus avoiding the combinatorial "explosion" problem existing in JPDA. For the case of Tc$\leqslant 2$, the results of DPDA will be the same as JPDA; and for Tc$=$T,T>2 its performances will be not lower than that of PDA[1].

In ordinary PDA only the case of p(1/T)$=1$ is considered. That is, there is only one target in validation region. The ordinary PDA uses a normalized version of the simple likelihood function as the association probability and is, in general, a suboptimal estimation procedure. When there are multiple targets in a validation region and the gates of targets are over lapped each other, the JPDA[2] will be used. In JPDA, it is assumed that p(Tc/T)$=1$, for Tc$=1,2,\cdots,$T. That is to say all the joint events are of equal "importance" and we should consider all joint events without regarding their real occurring probabilities. The definition of joint event in JPDA is as follows:

$$\varepsilon=\prod_{r=1}^{m(k)}\varepsilon_{rt_r} \tag{1}$$

where

$\varepsilon_{rt}\triangleq\{$measurement r originated from torget t$\}$
$r=0,1,2,\cdots,m(k);\ t=1,\cdots,T$

and t_r is the index of the target with which the measurement r is associated in the event under consideration. In order to reduce the number of joint events following feasibility rules are used.

(1) One measurement can have only one source;
(2) No more than one measurement can originate from one target.

However the number of feasible joint events can be as large as the product of the numbers of returns in the validation regions. The increae in complexity for the computation of association probabilities may prove significant for a number of targets even in moderately dense clutter.

There have been some efforts to approach the performance of the JPDA by imitating its properties via ad hoc association rules[3] or its modified rules[4] and via combinatorial methods[5,6]. However, the effectiveness of these approximations in tracking multiple targets in the presence of clutter will not be guaranteed. The neural solution[7] to the multitarget tracking data association problem is of good performance. However for real-time multitarget tracking by a microprocessor associated with an airborne track-while-scan radar system it has some problems remain to be solved.

In fact, There is no need to consider the joint events with very small p(Tc/T). In JPDA the definition of feasible joint event encompasses almost everything and leads to heavy and complicated computations. In this paper, section 2 presents a formulation of estimation. Section 3 deduces the formula of marginal probability in DPDA and section 4 shows the simulation results of typical example . The last is conclusion.

2. STATE ESTIMATION

The target dynamics are modelled as follows:

$$x(k+1)=F(k)x(k)+v(k) \tag{2}$$

where $x(k)$ is n-dimension state vector, $F(k)$ is transition matrix and $v(k)$ is assumed to be a normally distributed process noise, with zero mean and known variance

$$E(v(k)v^{\tau}(1))=Q(k)\delta_{k1} \tag{3}$$

The initial state vector is also assumed to be normally distributed with mean $\hat{x}(0/0)$ and covariance $p(0/0)$, and independent of $v(k)$.

The measurement equations are:

$$Z(k)=H(k)x(k)+w(k) \tag{4}$$

where $z(k)$ is r-dimension measurement vactor, $H(k)$ is a known $(r\times n)$ matrix and $w(k)$ is assumed to be a normally distributed measurement noise, independent of $v(k)$ and $\hat{x}(0/0)$, with zero mean and known variance

$$E(w(k)w^{\tau}(1))=R(k)\delta_{k1} \tag{5}$$

The set of validated measurements at time k is denoted by

$$Z(k)\triangleq\{Z_i(k)\}_{i=0}^{m(k)} \tag{6}$$

The cumulative set of measurements up to time k is denoted by

$$Z^k\triangleq\{Z(j)\}_{j=1}^{k}$$

The minimum variance estimate or conditional mean is therefore

$$\hat{x}(k|k)=\int x(k)P[x(k)/Z^k]dx(k) \tag{8}$$

The above equation is the mathematical basis of the optimal estimate in the m. m. s. e. sense. Further more we assume that

$$P[x(k)/Z^{k-1}]=N[x(k);\hat{x}(k|k-1),P(k|k-1)] \tag{9}$$

Define dominant joint event, corresponding to target;

$$\varepsilon_i(j)\triangleq\varepsilon_{ij}\cap(\bigcap_{\substack{r=1\\ i\neq 1}}^{m(k)}\varepsilon_{rt_r})$$

$$i=0,1,\cdots,m(k),t=1,2,\cdots,T \tag{10}$$

A dominant joint event will satisfy following rules

rule 1. the number of clutter q satisfies

$$m(k)-T\leqslant q\leqslant m(k)$$

rule 2. For target $j(j\neq 0)$, if return $i\neq 1$, then $i_j\neq 1_j$;

rule 3. For return i, if target $t\neq j$, then $t_i\neq j_i$;

Define the marginal association probabilities:

$$\beta_r^t(k)\triangleq P\{\varepsilon_r(t)/Z^k\} \qquad \begin{matrix} r=0,1,\cdots,m(k)\\ t=1,\cdots,T\end{matrix} \tag{11}$$

According to the assumptions of dominant joint event mentioned above, for $Tc>2$, $P(Tc/T)\doteq 0$, these events are mutually exclusive and exhanstive, and hence:

$$\sum_{r=0}^{m(k)}\beta_r^t(k)=1,t=1,2,\cdots,T \tag{12}$$

Using the results of PDAF we can get the following estimation algorithm directly:

$$\hat{x}^t(k|k)=\hat{x}^t(k|k-1)+K^t(k)\sum_{r=0}^{m(k)}\beta_r^t(k)v_r^t(k) \tag{13}$$

$$P^t(k|k)=\beta_0^t(k)P^t(k|k-1)+[1-\beta_0^t(k)]P_c^t(k|k)+\tilde{P}^t(k) \tag{14}$$

$$\tilde{P}^t(k)=K^t(k)[\sum_{r=1}^{m(k)}\beta_r^t(k)v_r^t(k)-v_r^t(k)v_r^{t'}(k)]K^{t'}(k) \tag{15}$$

$$P_c^t(k|k)=[I-K^t(k)H^t(k)]P^t(k|k-1) \tag{16}$$

$$P^t(k+1|k)=F^t(k)P^t(k|k)F^{t'}(k)+Q^t(k) \tag{17}$$

where t is the index of the *t*th filter of the target being tracked.

3. DOMINANT PROBABILITY DATA ASSOCIATION

Using Bayesian rule, we can get the formula of dominant probability data association:

$$P\{\varepsilon_i(j)/Z^k\}=\frac{1}{C_1}P\{Z(k)/\varepsilon_i(j),Z^{k-1}\}P\{\varepsilon_i(j)/Z^{k-1}\}$$

$$=\frac{1}{C_1}P\{Z_i(k)/\varepsilon_{ij},Z^{k-1}\}P\{\tilde{Z}_i(k)/\xi,Z^{k-1}\}P\{\varepsilon_i(j)/Z^{k-1}\} \tag{18}$$

where c_1 is normalization constant, $Z_i(k)\triangleq Z(k)-\{Z_i(k)\}$, $\xi\in F_{ij}$, $F_{ij}\triangleq\{\bigcap_{\substack{r=0\\ r\neq i\\ t_r\neq j}}^{m(k)}\varepsilon_{rt_r}\}$, t_r is the index of the target with which measurement r is associated in the event under consideration and $r=0,1,\cdots m(k)$, $r\neq i$.

$$P\{\varepsilon_i(j)/Z^{k-1}\}=P\{\varepsilon_i(j)\}$$

$$=P\{\varepsilon_i(j)/\delta[\varepsilon_i(j)],\Phi[\varepsilon_i(j)]\}P\{\delta[\varepsilon_i(j)],\Phi[\varepsilon_i(j)]\} \tag{19}$$

where $\delta[\varepsilon_i(j)]$ is the number of targets associated with a real return (i. e being dectected), and $\Phi[\varepsilon_i(j)]$ is the number of clutter, both are the function of $\varepsilon_i(j)$.

$$P\{\varepsilon_i(j)/\delta[\varepsilon_i(j)],\Phi[\varepsilon_i(j)]\}=[P_{m(k)}^{m(k)}{}_{-\Phi[\varepsilon_i(j)]}]^{-1}=\frac{\Phi[\varepsilon_i(j)]}{m(k)!} \tag{20}$$

$$P\{\delta[\varepsilon_i(j)],\Phi[\varepsilon_i(j)]\}=\prod_{t=1}^{T}(P_D^t)^{\delta_t}(1-P_D^t)^{1-\delta_t}\mu_F(\Phi) \tag{21}$$

where P_D^t is the detection probability, δ_t is the indicator of association of target t, and

$$\delta_t=\begin{cases}1 & \text{if target t is associated with a real return}\\ 0 & \text{if target t is not associated with any real return}\end{cases}$$

We assume that clutter is of Poission distribution.

$$\mu_F(\Phi)=e^{-\lambda v}\frac{(\lambda V)^{\Phi}}{\Phi!}$$

Where v is the volume of the entire surveillance region and λ is the density of clutter.

$$P\{\varepsilon_i(j)/Z^{k-1}\}=\frac{e^{-\lambda v}}{m(k)!}(\lambda V)^{\Phi}\prod_{t=1}^{T}(P_D^t)^{\delta_t}(1-P_D^t)^{1-\delta_t} \tag{22}$$

From (18) and (19), we can obtain

$$P\{\varepsilon_{ij}/Z^k\}=\sum_{\varepsilon_i(j)\in E_{ij}}P\{\varepsilon_i(j)/Z^k\}$$

$$=\frac{1}{C_1}P\{Z_i(k)/\varepsilon_{ij},Z^{k-1}\}\sum_{\xi\in F_{ij}}[P\{\tilde{Z}_i(k)/\xi,Z^{k-1}\}P\{\varepsilon_{ij}\cap\xi/Z^{k-1}\}\}]$$

$$=\frac{1}{C_1}P\{Z_i(k)/\varepsilon_{ij},Z^{k-1}\}\sum_{\xi\in F_{ij}}\{(\prod_{\substack{l=0\\ l\neq j}}^{m(k)}P\{\tilde{Z}_{il}(k)/\xi_{lt_l},Z^{k-1}\})\frac{e^{-\lambda v}}{m(k)!}(\lambda V)^{\Phi}\prod_{t=1}^{T}[(P_D^t)^{\delta_t}+(1-P_D^t)^{1-\delta_t}]\} \tag{23}$$

where $E_{ij}\triangleq\{\varepsilon_i(j)|i,j\}$.

$$P\{\tilde{Z}_{ij}(k)/\xi_{lt_l},Z^{k-1}\}=\begin{cases}N_{t_l}[Z_l(k)], & \text{if l is a return from target}\\ V^{-1}\quad, & \text{if l is a clutter}\end{cases}$$

$$\prod_{\substack{l=0\\ l\neq j}}^{m(k)}P\{\tilde{Z}_{il}(k)/\xi_{t_l},Z^{k-1}\}\frac{e^{-\lambda v}}{m(k)!}(\lambda V)^{\Phi}\prod_{t=1}^{T}[(P_D^t)^{\delta_t}(1-P_D^t)^{1-\delta_t}]$$

$$=\prod_{\substack{l=0\\ l\neq i}}^{m(k)} P'\{\tilde{Z}_{il}(k)/\xi_{lt_l}, Z^{k-1}\}\frac{e^{-\lambda v}}{m(k)!}\prod_{t=1}^{T}(1-P_D)$$

$$=C_2\prod_{\substack{l=0\\ l\neq i}}^{m(k)} P'\{\tilde{Z}_{il}(k)/\xi_{lt_l}, Z^{k-1}\} \tag{24}$$

where $C_2=\frac{e^{-\lambda v}}{m(k)!}\prod_{t=1}^{T}(1-P_D^t)$.

$$P'\{\tilde{Z}_{ij}(k)/\xi_{lt_l}, Z^{k-1}\} = \begin{cases} N_{t_l}[t_l(k)]\frac{P_D^t}{1-P_D^t}, & \text{if } l \text{ is a return from target} \\ \lambda, & \text{if } l \text{ is a clutter} \end{cases} \tag{25}$$

from (25), we have

$$\prod_{\substack{l=0\\ l\neq i}}^{m(k)} P'\{\tilde{Z}_{il}(k)/\xi_{lt_l}, Z^{k-1}\}=\prod_{\substack{t=1\\ t\neq j}}^{T} q_t(\xi, t) \tag{26}$$

where

$$q_t(\xi, t)=\lambda^{n_t}\{N_{t_l}[Z_{l_t}(k)]\frac{P_D^t}{1-P_D^t}\}^{\Phi_t[\varepsilon_t(j)]} \tag{27}$$

where $\Phi_t[\varepsilon_i(j)]=\begin{cases}0 & \text{when } n_t=m_t(k)\\ 1 & \text{when } n_t<m_t(k)\end{cases}$

$n_t(\xi)\triangleq$ the number of clutters among the returns associated with target. Form the definition (10) and the reule 3 we know that ξ_{lt_l} $(t_l=1,\cdots,T; l\neq i)$ are not correlated with each other. So from (23)、(24)、(25) we will have

$$\sum_{\xi\in F_{ij}}[P\{\tilde{Z}_i(k)/\xi, Z^{k-1}\}\cdot P\{\varepsilon_{ij}\cap\xi/Z^{k-1}\}]$$

$$=C_2\sum_{\xi\in F_{ij}}[\prod_{\substack{t=1\\ t\neq j}}^{T} q(\xi)]$$

$$=C_2\prod_{\substack{t=1\\ t\neq j}}^{T}[\sum_{\xi_{lt}\in F_{ijt}} q_t(\xi_{lt})]$$

$$=C_2\prod_{\substack{t=1\\ t\neq j}}^{T}[\sum_{\xi_{lt}\in F_{ijt}} q_t(\xi_{lt})+q_t(\xi_{it})-q_t(\xi_{it})] \tag{28}$$

where

$F_{ijt}\triangleq\{\xi_{lt}/l\neq i$, return l is associated with target t, $t\neq j\}$ and

$$q_{ti}=\lambda^{n_{t_l}}\left[N_{t_l}[Z_i(k)]\frac{P_D^t}{1-P_D^t}\right]^{\Phi_t[\varepsilon_t(j)]} \tag{29}$$

$n_t(\xi)$ can be approximately calculated as follows

$$n_t(\xi)=m_t(k)-f_l(T, a_t) \tag{30}$$

where $m_t(k)$ is the number of returns associated with target t and a_t is the number of returns associated with target t, which are in the correlation association area.

$$f_l(T,a_t)=\begin{cases} T-1, & \text{if } a_t\geqslant T \text{ and return } l \text{ is only associated with target } t\\ T, & \text{if } a_t\geqslant T \text{ and retrun } l \text{ is associated with target } t \text{ and some other targets}\\ a_t, & \text{if } a_t<T \text{ and retrun } l \text{ is associated with target } t \text{ and some other targets}\\ a_t-1, & \text{if } a_t<T \text{ and return } l \text{ is only associated with target } t.\\ 0, & \text{if } a_t=0 \end{cases} \tag{31}$$

from (28) and (29) we have

$$\sum_{\xi_{lt}\in F_{ijt}} q_t(\xi_{lt})+q_{ti}=\sum_{l=0}^{m_t(k)} q_{tl}\triangleq C_3(t) \tag{32}$$

Finally from (23)、(28)、(31)、(32), we have

$$P\{\varepsilon_{ij}|Z^k\}=\frac{C_2}{C_1}P\{Z_i(k)/\varepsilon_{ij}, Z^{k-1}\}\prod_{\substack{t=1\\ t\neq j}}^{T}[1-\frac{q_{ti}}{C_3(t)}]C_3(t)$$

$$=\frac{C_2}{C_1}N_j[Z_i(k)]\prod_{\substack{t=1\\ t\neq j}}^{T}[1-\frac{q_{ti}}{C_3(t)}]C_3(t)$$

$$=C_4(j)N_j[Z_i(k)]\prod_{\substack{t=1\\ t\neq j}}^{T}[1-\frac{q_{ti}}{C_3(t)}] \tag{33}$$

where $C_4(j)\triangleq\prod\frac{C_2\prod_{\substack{t=1\\ t\neq j}}^{T}C_3(t)}{C_1}$.

From (33) we know, when $T_c\leqslant 2$, $P\{\varepsilon_{ij}|Z^k\}$ will be the same as JPDA; when $T_c>2$ and $P(Tc/T)=0$, $P\{e_{ij}/Z^k\}$ will be approximately equal to PDA. That is to say, in DPDA, the top limit of performance will be equal to that of JPDA and the low limit will be not lower than that of PDA and also, the events with low limit will be the events with very small occuring probabilities.

The difference in form between the (33) and the ad hoc formula in SPDA[3] is that, in DPDA, we modify both of the numerator and denominator under the definition of dorminant joint events. However, the ad hoc formula[3] is experientially modified in the denominator only.

4. MONTE CARLO SIMULATION

100 runs of Monte Carlo simulations also have been done for 10 crossing targets shown in Fig. 1 and Fig. 2.

The dynamics of the target and the measurement equations are:

$$x_i^t(k+1)=F_i^t(k)x_i^t(k)+U_i^t(k)\bar{a}_i^t(k)+v_i^t(k) \qquad i=1,2$$

$$Z^t(k)=H^t(k)x_i^t(k)+w_i^t(k)$$

Here we use the model which presented in the Ref [8].

The variance of the process noise is Q_i^t.

The variance of measurement noise is

$$R(k)=\mathrm{diag}(r_x \quad r_y)$$

The true measurement is detected at any given time with probability $P_D=0.9$ and falls into gate with probability $P_G=0.9$. The number of clutter points is Possion-distributed and their locations in the measurement space are uniformly distributed over a very broad region about the actual track. The clutter density is $\lambda=0.5\text{km}^{-2}$ with varying gate sizes, this will range from 0.2 to 2.0 false detections per gate.

In Tab. 1 and Tab. 2, the root-mean-square (rms) values of DPDA are equal to that of JPDA and the percentage of lost tracks 6.7% is also equal to JPDA.

5. CONCLUSIONS

A new suboptimal approach to the probability data association is presented in this paper based upon the definition of dominant joint events and corresponding dominant joint probabilities. The approach of DPDA is developed for the application to real-time multitarget tracking by a microprocess or associated with an airborne track-while-scan radar system. The performance of DP-DA is superior to other simplified JPDA algorithms and the computation berdurn is only a little great than that of PDA.

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Tab. 1

Target \ Err	X		Y	
	ME	RMSE	ME	RMSE
1	−0. 002	0. 050	0. 019	0. 048
2	−0. 041	0. 109	0. 006	0. 049
3	−0. 009	0. 076	−0. 009	0. 079
4	0. 043	0. 076	0. 028	0. 069
5	−0. 025	0. 125	−0. 038	0. 099
6	−0. 023	0. 054	−0. 022	0. 058
7	0. 001	0. 082	0. 016	0. 076
8	0. 016	0. 075	−0. 035	0. 074
9	−0. 009	−0. 045	0. 046	0. 092
10	−0. 003	0. 054	−0. 022	0. 061

Tab. 2

Target \ Err	X		Y	
	ME	RMSE	ME	RMSE
1	0. 062	0. 182	−0. 022	0. 172
2	−0. 0095	0. 127	−0. 027	0. 173
3	0. 0014	0. 120	0. 018	0. 137
4	0. 0027	0. 120	−0. 020	0. 118
5	0. 007	0. 134	0. 000	0. 122
6	0. 026	0. 121	−0. 0038	0. 113
7	0. 020	0. 180	0. 035	0. 222
8	−0. 014	0. 118	0. 043	0. 135
9	0. 0177	0. 123	−0. 001	0. 107
10	−0. 001	0. 106	0. 018	0. 119

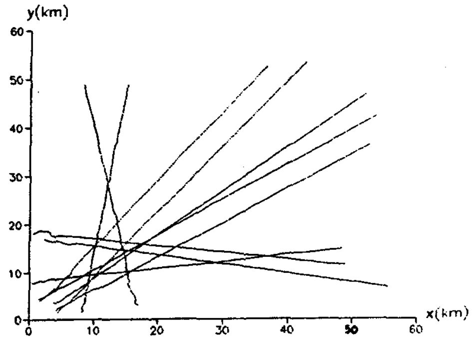

Fig. 1 10 Nonmerneuver Crossing Targets

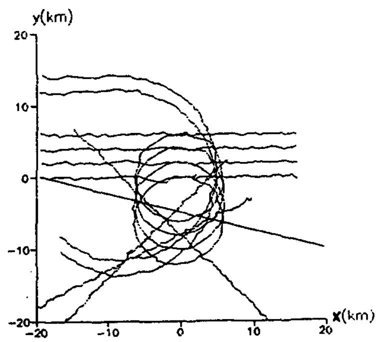

Fig. 2 10 Merneuver Crossing Targets