A fuzzy-optima definition based Multiobjective optimization of a racing car tyre-suspension system

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***Abstract*— When dealing with multiobjective optimization of the tyre-suspension system of a racing car, a large number of design variables and a large number of objectives have to be taken into account. Two different models have been used, both validated on data coming from an instrumented car, a differential equation based physical model and a neural network purely numerical model. Up to 23 objective functions have been defined, at least 14 of which showing to be in strict clash each other. The equivalent scalar function based formulation is intentionally avoided due to its well known limitations. A fuzzy definition of optima, being a generalization of Pareto-optimality, is applied to the problem. The result of such an approach is that subsets of Pareto-optimal solutions (being on such a problem a big portion of the entire search space) can be properly selected as a consequence of inputs from the designer. The obtained optimal solutions are compared with the reference vehicle and with the optima previously obtained with design of experiments techniques and different MO optimization strategies.**

I. Introduction

The notion of Pareto Optimality and the related notion of Pareto dominance, are the basic concepts for the development of multiobjective optimization theory and algorithms, both in the community of "classical" multiobjective optimization [1] and in the community of evolutionary multiobjective optimization (EMO) [2]. When looking at both classical multiobjective optimization and at the twenty years of EMO history, it can be easily concluded that almost all algorithms have aimed at converging towards either a particular Pareto-optimal solution (classical methods) or towards the entire Pareto-optimal front, which is the set of optimal solutions in the Pareto sense (EMO methods) [3]. This is essentially due to the fact that both classical and evolutionary multiobjective optimization algorithms have been mostly developed and addressed to engineering design problems. In such area the number of objectives is usually not huge but the computation of such objectives is time-consuming and highly complex [4].

Nevertheless, when problems with a number of objectives relatively larger than two or three were considered in the above framework, the limitations of Pareto optimality came out—both for classical and evolutionary methods—to give an unmanageable and large set of Pareto Optimal solutions; researchers subsequently developed a wide variety of techniques for the visualization, additional classification and choice of Pareto optimal solutions [5].

The reason for the limitations that one encounters when using the PO definition in multi-criteria optimization problems, is that Pareto's definition [6] captures the notion of "optimality" in a narrowly prescribed sense, several different concepts of "optimality" can thus be introduced and considered [7]. More into details, in Pareto's definition, when comparing two solutions, the following three aspects are not taken into consideration: the number of improved (or decreased) objectives, the relevance of such improvements (or decreases) and the decision maker's preferences between objectives (if any). All three issues are crucial in the human decision making process and may lead to several degrees of dominance, when two solutions are compared and, consequently, to several degrees of optimality among Pareto optimal solutions.

The first issue, regarding the number of improved objectives, is tackled in [8] where the authors also consider an additional weighting procedure for the incorporation of preferences (the third issue) as crisp weighting coefficients. The joint consideration of both the number of improved objectives and preferences gives a very compact definition of optimality, but a more simple and effective way for a proper numerical formalization of decision maker's preferences, is the use of the fuzzy set theory. This leads to a very general definition of fuzzy dominance and optimality where the decision maker's linguistical knowledge of preferences is numerically formalized; a widely developed theory on this is already available in literature [9], and [10]. The natural application of such work is in the field of many-criteria decision making (MCDM) problems where the decision maker's preferences have a leading role in the decision making process; an example of such a preference-based linguistic decision making procedure can be found in [11].

Differently, the aim of the present work is to exclude preferences from the optimization process and to consider fuzzy memberships as a tool for the numerical formalization and treatment of the size of improvements in the dominance definition [12]. The natural field of application of such an approach is mostly multiobjective optimal design with several criteria (more than three), where objective functions are numerical crisp functions, being eventually very complex and computationally very expensive. In the case of problems where preferences are to be included in the optimization,

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a good overview of current research on this topic in the EMO community is given in [13] and [14] and a technique for transforming non-crisp (qualitative) relationships between objectives into quantitative attributes is presented in [15]. Moreover, when tackling conflicting design problems with multiobjective optimization, it is often important to remove any bias deriving from the decision maker's preferences and to compute "unbiased" solutions. An example of such a problem, can be found in [16] where the optimal design of a controller for a gas turbine is formulated as a 6-objective multiobjective optimization problem.

Another example, related to industrial design, can be found in [5], where a 4-objective problem is tackled and a complex strategy, based on self-organizing maps, is developed for the visualization of PO solutions and for the *a posteriori* selection. Unlike what happens in multi-criteria decision making, when dealing with MO, even four objectives only are sufficient for an unaffordable complexity in the space of Pareto-optimal solutions. Several examples of design problems with a huge number of objectives can be found in the automotive design field [17]. In [18] a 17 objective problem is tackled, for the design of a full car chassis where no preference-biased solutions are required. PO solutions are computed and the difficulty encountered in the choice among them is clearly outlined. The same happens in [17] where a 14 objective problem is formalized for the optimal design of the tyre-suspension system of a racing car. When dealing with such problems, the automotive expert would be strongly interested in computing a sampling of the entire Pareto front, without introducing any preference-based bias, but this may be unpractical due to the high number of objectives. For this reason, the computation of a proper sub-set of the PO front, corresponding to a higher degree of optimality and based on the definitions given in the present paper, gives a wide variety of choice possibilities with no preference based biases. The extended definitions of k-optimality and fuzzy-optimality can be applied to multiobjective optimization problems at two levels. Level 1: a-posteriori selection of non-dominated solutions obtained via available MOEAs. Level 2: development of new algorithms directly converging towards the k-front (or fuzzy front) Results will be shown related to the first approach. Fuzzy-optimality and k-optimality are thus linked to a complex set of techniques in order to make practical the procedure of automated optimal design on such problems. The entire procedure requires the exploitation of differential modelling, NN modelling, design of experiments, statistical correlation analysis, Evolutionary multiobjective optimization and validation (through comparison with real data) of each step. The overall schema of the computational procedure is shown in figure 1. Although the main subject of the paper is fuzzy and k optimality, some details about the entire procedure and its blocks will be given.

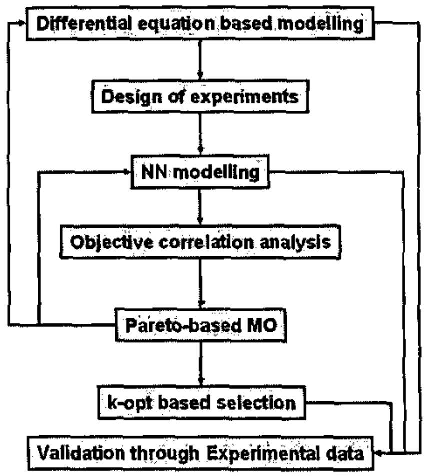

Fig. 1. Block diagram of the modelling + design of experiments + optimization + validation strategy.

II. Fuzzy Definition of Optima for Multiobjective Optimization

The following multi-objective optimization problem is considered [1], [19], [20]:

Def. 2.1 (Multi-criteria optimization problem): Let $\mathbf{V} \subseteq K_1 \times K_2 \times \ldots \times K_N$ and $\mathbf{W} \subseteq O_1 \times O_2 \times \ldots \times O_M$ be vector spaces, where K_i, O_j (with $i = 1, \ldots, N$ and $j = 1, \ldots, M$) are (continuous or finite) fields. Let $N, M \in \mathbb{N}$, and $\mathbf{f} : \mathbf{V} \mapsto \mathbf{W}$ be a mapping. A *Non-linear constrained multi-criteria (minimum) optimization problem* with M objectives is defined as:

$$\min_{\mathbf{v}\in\boldsymbol{\Omega}} \mathbf{f}(\mathbf{v}) \triangleq \{f_1(\mathbf{v}), ..., f_M(\mathbf{v})\}, \tag{1}$$

where $\boldsymbol{\Omega}$ is a nonempty subset of $\mathbf{V}$.
The set $\boldsymbol{\Omega}$ is referred to as *design (domain) search space*, or *feasible region*. Its image $\boldsymbol{\Omega}_{\mathrm{O}} = \mathbf{f}(\boldsymbol{\Omega})$ is referred to as *objective (domain) search space*, or *feasible objective region*. The set $\boldsymbol{\Omega}$ is defined and bounded by two *constraint functions* $\mathbf{g} : \mathbf{V} \mapsto \mathbb{R}^p, \mathbf{h} : \mathbf{V} \mapsto \mathbb{R}^q$, where $p, q \in \mathbb{N}$ as follows:

$$\boldsymbol{\Omega} = \{\mathbf{v} \in \mathbf{V} \mid \mathbf{g}(\mathbf{v}) \leq 0 \wedge \mathbf{h}(\mathbf{v}) = 0\}. \tag{2}$$

When design optimization is concerned, $\mathbf{V} = \mathbb{R}^N$ and $\mathbf{W} = \mathbb{R}^M$; consequently each f_i is a mapping from $\mathbb{R}^N$ to $\mathbb{R}$. Moreover, for better clarity and simplicity and with no loss of generality, we assume that all objective functions are to be minimized. If an objective function f_i is to be maximized, all formulas still holds for the minimization of $-f_i$. For the reader's convenience, we recall here the well known definition of Pareto dominance (definition 2.2), Pareto optimum (definition 2.3) and Pareto set and front (definition 2.4 in a multi-criteria (M criteria) optimization problem. $\mathbf{v}$ is the decision (or design) variables vector, it belongs to the search space Ω.

Def. 2.2 (Pareto dominance): For any two points (candidate solutions) $\mathbf{v}_1, \mathbf{v}_2 \in \boldsymbol{\Omega}$, $\mathbf{v}_1$ is said to *dominate* $\mathbf{v}_2$ in the Pareto sense (P-dominate) if and only if the following conditions hold

$$\begin{cases} f_i(\mathbf{v}_1) \leq f_i(\mathbf{v}_2) & \text{for all } i \in \{1, 2, ..., M\} \\ f_j(\mathbf{v}_1) < f_j(\mathbf{v}_2) & \text{for at least one } j \\ & \in \{1, 2, ..., M\} \end{cases} \tag{3}$$

Def. 2.3 (Pareto optimality): $\mathbf{v}^* \in \Omega$ is *Pareto-optimal* (P-optimal) if there is no $\mathbf{v} \in \Omega$ such that $\mathbf{v}$ P-dominates $\mathbf{v}^*$.

Def. 2.4 (Pareto set and front): We call *Pareto Optimal Set ($\mathcal{S}_\mathrm{P}$)* and *Pareto Optimal Front ($\mathcal{F}_\mathrm{P}$)* the set of Pareto-optimal solutions in design variables domain and objective functions domain, respectively.

When considering a big number of objectives, the Pareto definition of optimality in a multi-criteria optimization problem can be unsatisfactory due to essentially three reasons:

P1 the number of improved objective functions values is not taken into account,

P2 the (normalized) relevance of the improvements is not taken into account.

P3 no preference among objectives is considered.

Such issues are essential decision elements when looking for the best solution, and they are implicitly included in the common-sense notion of optimality.

When no preferences are prescribed by the designer (P3), in order to account for P1 and P2, in the following sections we will give two more general definitions of optimum for a multi-criteria optimization problem. One Pi issue at a time will be taken into account, and the definitions will be introduced in such a way that the classical Pareto optimum definition will be a special case for both the definitions.

A. *P1: Taking into account the number of improved objectives: k-optimality*

When the Pareto optimality definition is considered, two solutions $\mathbf{v}_1$ and $\mathbf{v}_2$ (to be compared) are equivalent, if at least in one objective the first solution is better than the second one, and at least in one objective the second one is better than the first one (or if they are equal in all the objectives). Indeed a more general definition, being able to cope with a wider variety of problems, should take into account the number of objectives where the first candidate solution is better than the second one and viceversa. Let us first introduce the following functions, which relate a natural number to each couple of points in Ω.

$$n_b(\mathbf{v}_1, \mathbf{v}_2) \triangleq |\{i \in \mathbb{N} | i \leq M \wedge f_i(\mathbf{v}_1) < f_i(\mathbf{v}_2)\}| \tag{4}$$

$$n_e(\mathbf{v}_1, \mathbf{v}_2) \triangleq |\{i \in \mathbb{N} | i \leq M \wedge f_i(\mathbf{v}_1) = f_i(\mathbf{v}_2)\}| \tag{5}$$

$$n_w(\mathbf{v}_1, \mathbf{v}_2) \triangleq |\{i \in \mathbb{N} | i \leq M \wedge f_i(\mathbf{v}_1) > f_i(\mathbf{v}_2)\}| \tag{6}$$

For each couple of points $\mathbf{v}_1, \mathbf{v}_2 \in \Omega$, the function n_b counts those objectives where $\mathbf{v}_1$ is better than $\mathbf{v}_2$, n_e counts those objectives where the two solutions are equal, and n_w counts those objectives where $\mathbf{v}_1$ is worse than $\mathbf{v}_2$. To lighten the mathematical notation, from now on we will consider a generic couple of points and we will write simply n_b, n_e and n_w instead of $n_b(\mathbf{v}_1, \mathbf{v}_2), n_e(\mathbf{v}_1, \mathbf{v}_2)$ and $n_w(\mathbf{v}_1, \mathbf{v}_2)$. Obviously, the following inequalities hold

$$n_b + n_w + n_e \quad = M, \tag{7}$$

$$0 < n_b, n_w, n_e \quad < M \tag{8}$$

We are now able to give a first new definition of dominance and optimality, namely (1-k)-dominance and k-optimality:

Def. 2.5 ((1-k)-dominance): $\mathbf{v}_1$ is said to *(1-k)-dominate* $\mathbf{v}_2$ if and only if:

$$\begin{cases} n_e < M \\ n_b \geq \dfrac{M - n_e}{k+1}, \end{cases} \tag{9}$$

where $0 \leq k \leq 1$.

As can be easily seen, 1-dominance (i.e., $k = 0$) is nothing else than Pareto-dominance (definition 2.2). Ideally k may assume any value in $[0, 1]$, but because n_b has to be a natural number, only a limited number of optimality degrees need to be considered. In fact in equation (9) the second inequality is equivalent to

$$n_b \geq \left\lceil \frac{M - n_e}{k+1} \right\rceil,$$

where $\lceil \cdot \rceil$ is the ceiling operation ($\lceil x \rceil$ is the smallest integer greater than the real x). After defining (1-k)-dominance, the following definition of optimality can be given:

Def. 2.6 (k-optimality): $\mathbf{v}^*$ is *k-optimum* if and only if there is no $\mathbf{v} \in \Omega$ such that $\mathbf{v}$ k-dominates $\mathbf{v}^*$

The meaning of the terms "(1-k)-dominance" and "k-optimality"is that the former is a loose version of Pareto dominance (1-dominance), while the latter is a strong version of Pareto optimality (0-optimality). Concepts of $\mathcal{S}_P$ and $\mathcal{F}_P$ can be now easily extended in the following way:

Def. 2.7 (k-optimal set and front): We call the *k-optimal set ($\mathcal{S}_k$)* and the *k-optimal front ($\mathcal{F}_k$)* the set of k-optimal solutions in the design variables domain and in the objective functions domain, respectively.

Several $\mathcal{S}_k$ sets and $\mathcal{F}_k$ fronts are thus introduced, one for each value of k; moreover, as evident, $\mathcal{S}_0$ is the Pareto Optimal set ($\mathcal{S}_\mathrm{P}$), and $\mathcal{F}_0$ is the Pareto Optimal Front ($\mathcal{F}_\mathrm{P}$).

B. *P2: Taking into account the size of improvements: Fuzzy optimality*

A simple and effective extension of of (1-k)-dominance and k-optimality is to substitute crisp relations in Eqs. 6 with fuzzy ones. As a first step, for each objective function, the degree of improvement (or equality) between two points $\mathbf{v}_1$ and $\mathbf{v}_2$ is taken into account. In order to do this fuzzy numbers and fuzzy arithmetic will be considered. As a second step, we extend the dominance relation itself to a fuzzy relation. We remark that, in this work, fuzzy set theory is not related to treatment of preferences among objectives, but it is related to the size of the improvements, with the underlying hypothesis that all objectives have equal importance.

The standard way to introduce fuzzy arithmetic (here being the objective domain search space Ω_O), is to relate to each point in Ω_O three fuzzy sets (fuzzy number), one for equality, one for "greater than" and one for "less than". For coherence with the terminology used so far, we refer to the membership function as μ_e, μ_w (where w means "worst" and where we are talking about minimization problems) and μ_b respectively.

The fuzzy definition of n_b, n_w and n_e (now with superscript F) are the following

$$n_b^F(\mathbf{v_1}, \mathbf{v_2}) \triangleq \sum_{i=1}^{M} \mu_b^{(i)}(f_i(\mathbf{v_1}) - f_i(\mathbf{v_2})) \tag{10}$$

$$n_w^F(\mathbf{v_1}, \mathbf{v_2}) \triangleq \sum_{i=1}^{M} \mu_w^{(i)}(f_i(\mathbf{v_1}) - f_i(\mathbf{v_2})) \tag{11}$$

$$n_e^F(\mathbf{v_1}, \mathbf{v_2}) \triangleq \sum_{i=1}^{M} \mu_e^{(i)}(f_i(\mathbf{v_1}) - f_i(\mathbf{v_2})) \tag{12}$$

In order n_b^F, n_w^F and n_e^F to be a meaningful extension of n_b, n_w and n_e, the membership functions $\mu_b^{(i)}, \mu_w^{(i)}$ and $\mu_e^{(i)}$ have to satisfy the Ruspini condition [21] and thus, in all the domain, they must sum up to 1 (For a detailed description of the Ruspini condition role in fuzzy membership function definition, see [22]). In fact, under this hypothesis, the following holds

$$n_b^F + n_e^F + n_w^F = \sum_{i=1}^{M}(\mu_i^b + \mu_i^w + \mu_i^e) = M \tag{13}$$

Two possible different membership shapes, linear and gaussian, can be considered. For both of them The shape definition requires some parameters being ε_i and γ_i for the linear one and σ_i for the gaussian one. Although the definition of such parameters is to be carefully considered, their intended meaning is clear and closely related to the physical properties of the system. They thus can be provided by the human decision-maker knowledge on the system.

- ε_i defines in a fuzzy way the practical meaning of equality and it can thus be considered the tolerance on the i-th objective, that is the interval within which an improvement on objective i is meaningless (in other words the maximum imperceptible improvement on objective i).
- γ_i can be defined as a relevant, but not big, size improvement for objective i.
- σ_i evaluation requires a combination of the two aforementioned parameters (ε_i) and γ_i; the following formula for the membership μ_e can be used:

$$\mu_e^{(i)} = \exp\left(-\frac{\ln \chi}{\varepsilon_i^2}(f_1^i - f_2^i)^2\right) \tag{14}$$

where χ is an arbitrary parameter ($0.8 < \chi < 0.99$). μ_b and μ_w can be univocally derived from Ruspini's condition.

With such a fuzzy definition of n_b^F, n_e^F and n_w^F, both (1-k)-dominance and k-optimality definition can be easily extended.

C. *k-optimality with fuzzy numbers of improved objectives: fuzzy optimality*

At first the following extended definitions of k-dominance and k-optimality can be given. n_b, n_e and n_w are replaced by n_b^F, n_e^F and n_w^F in definitions 2.5 and 2.6.

Def. 2.8 ((1-k_F)-dominance): $\mathbf{v}_1$ is said to *(1-k_F)-dominate* $\mathbf{v}_2$ if and only if:

$$\begin{cases} n_e^F < M \\ n_b^F \geq \dfrac{M - n_e^F}{k_F + 1}, \end{cases} \tag{15}$$

where $0 \leq k_F \leq 1$.

With the above definition, once k_F is provided, it is easy to check whether a candidate solution (1-k_F)-dominates or not another solution. A simple extension can be given for definition 2.6 as follows:

Def. 2.9 (k_F-optimality): $\mathbf{v}^*$ is *k_F-optimum* if and only if there is no $\mathbf{v} \in \Omega$ such that $\mathbf{v}$ k_F-dominates $\mathbf{v}^*$

The parameter k (now called k_F) has the same meaning as in the previous case ($0 \leq k_F \leq 1$), but now continuous degrees of optimality and dominance are introduced (k_F-dominance and k_F-optimality).

Moreover, an extension of $\mathcal{S}_k$ and $\mathcal{F}_k$ can be defined as follows:

Def. 2.10 (k_F-optimal set and front): We call $\mathcal{S}_{k_F}$ and $\mathcal{F}_{k_F}$ the set of k_F-optimal solutions in the design variables domain and in the objective functions domain, respectively.

III. The System Models and Optimization Problem

A. *Differential equation based model*

A vehicle model, divided into modules, has been constructed. Each module is an object which interacts with the other ones by means of input/output variables. The result is a 18 d.o.f. model that simulates satisfactory the actual vehicle dynamic behavior. The car chassis is considered as a rigid body (6 d.o.f.). The tyre forces transmitted by the suspension system are applied to the vehicle body by considering two resultant forces and two torques. The vehicle body is subject to aerodynamic lift and drag forces. Front and rear independent suspension systems have a double wishbone layout. Shock absorbers are modelled as non-linear components; the forces transmitted by the shock absorber are function of the deformation speed. The non-linearity due to bump stop characteristics has been included. The torques-speed characteristic of the differential has been accurately modelled. A model of braking system is included by considering different distribution of braking force at the front and rear axle. The tyre characteristics have been implemented by using the *Pacejka's Magic Formulæ*[23]. The transient tyre behavior (relaxation length effect) is given by

$$\sigma \frac{dF_y}{dt} + vF_y = vF_{y,stat}(\alpha, F_z, \gamma) \tag{16}$$

where $sigma$ is the relaxation length, F_y is the lateral force, α is the slip angle, F_z is the vertical load acting on the tyre and γ is the camber angle. Combined effects of longitudinal and lateral slip have been considered as described in [24]. The

	1	2	6	8	11	13
μ_r	0.0882	0.0389	0.0543	0.0602	0.0821	0.0351
σ_r	0.2022	0.0710	0.0679	0.0928	0.1531	0.0469
μ_a	0.0325	0.0146	0.0188	0.0198	0.0189	0.0138
σ_a	0.0373	0.0117	0.0157	0.0163	0.0166	0.0113
$\mu_{r,v}$	0.1032	0.0489	0.0595	0.0635	0.0894	0.0415
$\sigma_{r,v}$	0.3374	0.2034	0.0869	0.1060	0.0907	0.0480
$\mu_{a,v}$	0.0366	0.0165	0.0179	0.0231	0.0212	0.0172
$\sigma_{a,v}$	0.0437	0.0139	0.0146	0.0191	0.0194	0.0146
	15	17	18	20	22	23
μ_r	0.0643	0.0322	0.0368	0.0470	0.0671	0.0589
σ_r	0.1155	0.0560	0.0434	0.0725	0.1183	0.0630
μ_a	0.0262	0.0151	0.0147	0.0213	0.0268	0.0232
σ_a	0.0215	0.0123	0.0123	0.0200	0.0219	0.0187
$\mu_{r,v}$	0.0714	0.0363	0.0425	0.0473	0.0765	0.0639
$\sigma_{r,v}$	0.0984	0.0601	0.0544	0.0582	0.1569	0.0658
$\mu_{a,v}$	0.0334	0.0166	0.0170	0.0320	0.0305	0.0277
$\sigma_{a,v}$	0.0300	0.0140	0.0147	0.0273	0.0246	0.0232

TABLE I

ERROR AVERAGE $\mu_\bullet$ AND STANDARD $\sigma_\bullet$ DEVIATIONS FOR NN APPROXIMATION OF TWELVE OBJECTIVES, $\bullet_r$: RELATIVE, $\bullet_a$: ABSOLUTE, $\bullet_v$: ON VALIDATION SET.

variable name	minval	maxval	unit
kant	100000.0	300000.0	[N/m]
kpost	150000.0	350000.0	[N/m]
krolant	10000.0	70000.0	[N/m]
krolpost	10000.0	70000.0	[N/m]
smbumpant	5000.0	25000.0	[Ns/m]
smrebant	20000.0	100000.0	[Ns/m]
smbumppost	5000.0	25000.0	[Ns/m]
smrebpost	20000.0	100000.0	[Ns/m]
toeant	-0.0015	-0.002618	[rad]
toeant	0.0015	0.002618	[rad]
toepost	-0.005	-0.010472	[rad]
toepost	0.005	0	[rad]
camberant	0.0	0.05236	[rad]
camberant	0.06	0.0872665	[rad]
camberpost	0.0	0.043633	[rad]
camberpost	0.05	0.0785398	[rad]
biant	0.9	1.1	[-]
bipost	0.9	1.1	[-]
ciant	0.85	1.15	[-]
cipost	0.85	1.15	[-]
maxfzant	0.85	1.15	[-]
maxfzpost	0.85	1.15	[-]

TABLE II

DESIGN VARIABLES, LOWER/UPPER VARIATION RANGES.

vehicle model has been extensively validated, see [25], [18] for more details.

B. Design of Experiments and NN based model

Once a satisfactorily uniform and large set of training points has been selected through a proper design of experiments technique (see [26], [27] for more details), three MLPs (Multi-Layer Perceptrons) have been trained for the approximation of objectives being grouped into three sets (being defined by common features of objectives belonging to the sets). Mean values and standard deviations on both training and generalization errors are shown in Table I. When using the MLPs model for the optimization the one-shot approach (compute the model once and then optimize) can be very time consuming due to the high accuracy required in the model that cannot be improved once the optimization is started. This is why results obtained with an iterative interpolation-optimization strategy [28] will be shown in the conference presentation.

C. Optimization problem formulation

The design variables that have be considered in the optimisation process are 22, as reported in Table II. The objective functions are obtained from the simulation of a total number of six running situations. The maneuvers considered are the steady state cornering with a path radius of $50\,m$ and $120\,m$ respectively (ISO 4138 standard), j-turn maneuver (ISO 7401 standard), power on-off while steering, braking on a bend and passing over a non symmetric kerb. A total of 23 objective functions have been considered, as they are reputed fully representative of the actual vehicle behavior. The objective functions are reported in Table III. From a general point of view, when such a big number of objective is considered, some of them may be treated as constrains. The approach has been discarded due to the arbitrariness in the choice and for

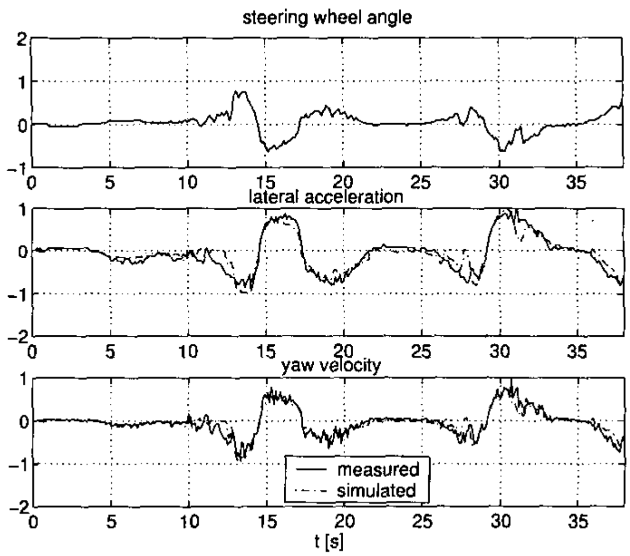

Fig. 2. Example of the vehicle model validation.

the same reason the scalar function approach has been also discarded.

D. Objective correlation analysis

In order to reduce, if possible, the number of performance indexes that have to be taken into account, the correlation between performance indices has been investigated. The statistic we used to measure correlation between each objective couple is the Spearman's non-parametric correlation statistic [29]. Spearman's correlation is used in preference to the other commonly applied non-parametric correlation statistics (e. g. Kendall's tau [29]) because the latter proved to be too

	Symbol	Performance index	Goal
		Steady-state turning 50 m	
Y^1	$a_{c,50\,m}$	Maximum centripetal acceleration	Maximise
Y^2	$\beta_{1.3\,g,50\,m}$	Steady state sideslip angle at c.g	Minimise
Y^3	$P_{1.3\,g,50\,m}$	Power required (steady state)	Minimise
		Steady-state turning120 m	
Y^4	$a_{c,120\,m}$	Maximum centripetal acceleration	Maximise
Y^5	$\beta_{1.3\,g,120\,m}$	Steady state sideslip angle at c.g	Minimise
Y^6	$P_{1.3\,g,120\,m}$	Power required (steady state)	Minimise
		J-turn	
Y^7	$t_{\dot\psi}$	Yaw velocity peak response time	Minimise
Y^8	$O_{\dot\psi}$	Yaw velocity overshoot	Minimise
Y^9	$SS_{\dot\psi}$	Steady state yaw velocity	Maximise
Y^{10}	t_β	Sideslip angle at centre of gravity peak response time	Minimise
Y^{11}	O_β	Sideslip angle at centre of gravity overshoot	Minimise
Y^{12}	MAX_β	Maximum sideslip angle at centre of gravity	Minimise
Y^{13}	SS_β	Steady state sideslip angle at centre of gravity	Minimise
Y^{14}	t_{a_c}	Centripetal acceleration peak response time	Minimise
Y^{15}	O_{a_c}	Centripetal acceleration overshoot	Minimise
Y^{16}	MAX_{a_c}	Maximum centripetal acceleration	Maximise
Y^{17}	SS_{a_c}	Steady state centripetal acceleration	Maximise
Y^{18}	$RMS(\dot\beta)$	Root mean square of d/dt sideslip angle at c.g.	Minimise
		Power on-off	
Y^{19}	$RMS(\dot\psi)$	Root mean square of d/dt sideslip angle at c.g.	Minimise
		Braking on a bend	
Y^{20}	δF_z	Vertical load variation on the rear DX wheel	Maximise
Y^{21}	δF_z	Vertical load variation on the rear SX wheel	Maximise
		Passing on a non symmetric kerb	
Y^{22}	$RMS(F_{z,f})$	Root mean square of the vertical load of the excited front wheel	Minimise
Y^{23}	$RMS(F_{z,r})$	Root mean square of the vertical load of the excited rear wheel	Minimise

TABLE III

PERFORMANCE INDICES.

computationally demanding for the analysis of large distance matrices. Calculation of Spearman's correlation statistic is based on the ranking of the distances within each of the two distance matrices. Two alternative algorithms can be used to calculate Spearman's statistic, the choice depends on the presence of ties in the ranking of the two matrices. In case of relevant incidence of ties, the following general formula can be used

$$r = \frac{1 - \left(6D + (T_x + T_y)/2\right)/(n^3 - n)}{(1 - \frac{T_x}{n^3 - n})(1 - \frac{T_y}{n^3 - n})} \tag{17}$$

where n is the number of elements, $T_{x,y} = \sum t_{x,y}^3 - t_{x,y}$, $t_{x,y}$ being the number of ties of a particular value of variable x or y and where the summation is over all tied values of variable x or y respectively, where $D = \sum_{i=1}^{n}(Rx_i - Ry_i)^2$, Rx_i being the vector of ranks of the observation x. Ties are treated by midranking. If on the other hand there are no ties, r is calculated as follows: $r = 1 - 6\frac{D}{n^3 - n}$. Some correlation analysis results are shown in table IV. A correlation parameter close to 1 means maximum correlations. Once an element is found (say line i column j, see • in Table IV) to be close to one, one of the two objectives is purged from the final set of truly conflicting criteria. This procedure allows to select 13 objective from the initial set of 24.

	F_{18}	F_{19}	F_{20}	F_{21}	F_{22}	F_{23}
F_1	0.4742	-0.0324	0.0528	0.0583	-0.1789	-0.0984
F_2	• 0.9177	0.4558	0.1424	0.1969	-0.0338	0.0506
F_3	-0.2890	0.0455	-0.0807	-0.1158	0.0505	0.0155
F_4	0.4027	0.0179	0.0884	0.0910	-0.1208	-0.0889
F_5	• 0.9170	0.4626	0.1387	0.1914	-0.0309	0.0526
F_6	-0.2888	0.0234	-0.0828	-0.1248	0.0289	-0.0037
F_7	0.5064	0.0922	0.0815	0.0613	-0.2565	-0.1365
F_8	-0.4047	-0.048	-0.0653	-0.0737	0.1954	0.1575
F_9	0.8402	0.2647	0.1037	0.1275	-0.1309	-0.0325
F_{10}	0.7186	0.2109	0.0609	0.0759	-0.1861	-0.0914
F_{11}	0.7124	0.2199	0.3404	0.3531	-0.2989	0.3569
F_{12}	• 0.9622	0.3913	0.1732	0.2140	-0.1254	0.0715
F_{13}	• 0.9544	0.3942	0.1315	0.1753	-0.0834	0.0151
F_{14}	0.3006	-0.0243	-0.0768	-0.1105	-0.2696	-0.2377
F_{15}	0.4941	0.2116	0.5390	0.5703	-0.0391	0.7307
F_{16}	0.8778	0.2834	0.2203	0.2481	-0.1249	0.1396
F_{17}	0.8246	0.2503	0.1004	0.1228	-0.1312	-0.0364
F_{18}	1.0000	0.4467	0.2033	0.2522	-0.0796	0.1424
F_{19}		1.0000	0.1707	0.2090	0.0610	0.0363
F_{20}			1.0000	0.9842	-0.4026	0.5201
F_{22}				1.0000	-0.3712	0.5235
F_{23}					1.0000	-0.0721
F_{24}						1.0000

TABLE IV

SPEARMAN CORRELATION COEFFICIENTS MATRIX (PARTIAL). 0: NO CORRELATION, 1: TOTAL CORRELATION

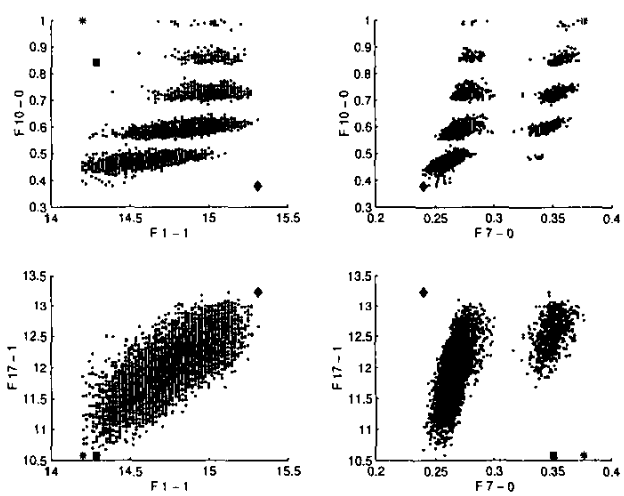

Fig. 3. Preliminary objective search space sampling (some 2D projections). ■: nadir projection, ∗: distopia projection, ◆: utopia projection

IV. RESULTS

At first NSGAII [30] has been run on the entire 13 objective functions space and a big number of non-dominated

	k	n_s	n_b		f	n_s	n_b
	0(Pareto)	631701	1		0(Pareto)	631701	1
	0.25	6635	3		0.25	5851	3
	0.35	1116	4		0.5	115	6
	0.42	119	5		0.55	53	6.3871
	0.5	10	6		0.6	16	6.75
	0.65	3	7		0.65	3	7.091
					0.8	1	8
	k	n_s	n_b		f	n_s	n_b
	0(Pareto)	16152	1		0(Pareto)	16152	1
	0.5	1052	6		0.4	105	5.143
$\Rightarrow$	0.6	256	8		0.5	20	6
	0.7	64	9		0.6	7	6.75
					0.7	2	7.412
				$\Rightarrow$	0.8	1	8

TABLE V

EFFECTS OF K-OPTIMALITY AND FUZZY-OPTIMALITY BASED A-POSTERIORI SELECTION ON PARETO-BASED NON-DOMINATED SOLUTIONS FROM SEARCH SPACE SAMPLING (ABOVE) AND FROM OPTIMIZATION (BELOW). K:LEVEL OF K-OPTIMALITY, F: LEVEL OF FUZZY OPTIMALITY, n_s: NUMBER OF K-OPTIMAL (OF FUZZY-OPTIMAL) SOLUTIONS, n_b: MINIMAL NUMBER OF IMPROVED OBJECTIVES WITHIN THE CORRESPONDING K-OPTIMAL (OF FUZZY-OPTIMAL) SET.

solutions (16152) have been obtained. Moreover utopia, distopia, nadir and payoff matrix have been computed through single-objective evolutionary computation (ES). After that k-dominance and fuzzy-dominance has been considered as an a-posteriori selection rule among Pareto-based non-dominated solutions. The results of different selections at different values of k and k_F are shown in table V. For the sake of comparison a selection has also been done on a set of 631701 non-dominated solutions corresponding to a search space sampling of 10^6 points. As can be seen the number of selected non-dominated solutions becomes smaller for high values of k, making possible the final solution selection. Moreover column three of table V shows the meaning of k-optimality that is the minimum number of improved objective functions that one counts when considering all possible couples of k-optimal solutions at different values of k. When fuzzy optimality is used for the selection n_b becomes a real number (see section II). Focusing the attention on results corresponding to k=0.6 (Table V, $\Rightarrow$), a comparison in objective space domain is shown in figure 4 between k-optimal solutions, reference vehicle and two different optima corresponding to previously computed minima by using scalar function approach. One possible combination of three among the 13 conflicting objectives is shown. As it can be seen, almost all k-optimal solutions dominates the reference solution and are significantly close to the utopia point. Moreover a set of 256 solutions is in some way practical for the final choice by the designer. k_F-optimality can be used for the choice of one final solution. This can be seen from the last line in Table V ($f = 0.8$). The selected solution is shown in figure 5 and 6 in the phenotypic space of the sideslip angle at c.g. (versus time) and centripetal acceleration (versus time) respectively. The comparison is performed with the reference vehicle and with previous optima (see above). The selected

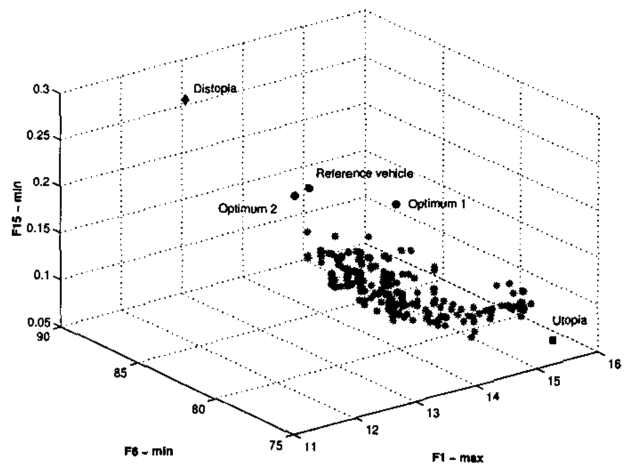

Fig. 4. Comparison in objective functions space of the selected k-optimal solution with the reference vehicle and with two previously computed minima of equivalent scalar function value.

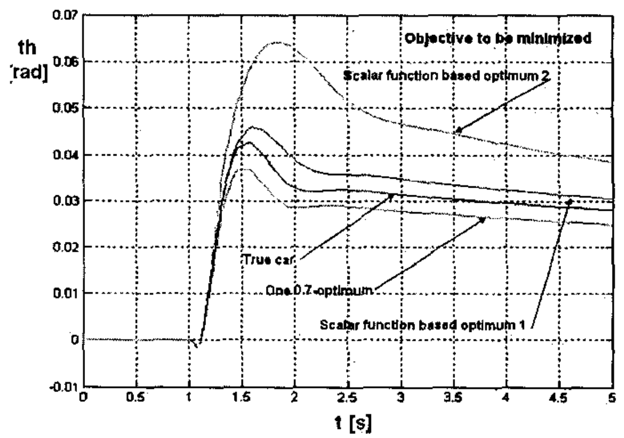

Fig. 5. One k-optimal solution in the phenotipic space of sideslip angle at c.g.

k-optimal solution shows a relevant improvement on both the objectives with respect to the reference vehicle and to both previously computed optima.

V. CONCLUSION

The multiobjecive optimization of the tyre-suspension system of a racing car has been tackled. The problem has shown to be highly challenging mainly due to the high number of objectives to be considered (totally 24 with 13 conflicting each other). Several approaches have been followed involving state of the art evolutionary optimization algorithms (NSGAII) both on the differential model and on the NN model. Nevertheless the number of objectives is so large, that a huge number of non-dominated solutions have been obtained among which it is difficult to make any a-posteriori choice. In order to solve the problem a different definition of optimality (previously

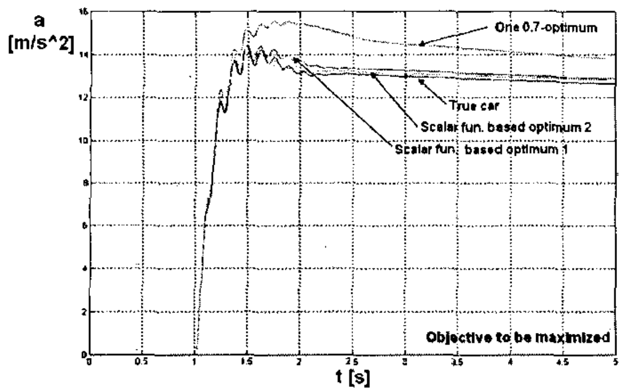

Fig. 6. One k-optimal solution in the phenotipic space of centripetal acceleration.

developed) has been applied to the problem. It has shown to be significantly flexible both as an a-posteriori selection rule among non-dominated solutions and as an extension to Pareto dominance definition to be included in the available multiobjective evolutionary algorithms. Some results are shown and compared to the actual vehicle behaviour and to some additional optima obtained through a classical scalar function approach.

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