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A Study of Association Model Based on Synergetics

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Abstract

In this study we shall show an association process of synergetic neural networks. The present association model is based on a top down formulation of the dynamic rule of analogue neural network. It is found that a complete association can be accomplished up to the same number of the embedded patterns as the number of neurons. In practice an association process is simulated for the practical images with 256 gray scale levels and 256×256 size. In addition searching process of the embedded patterns is also realized by means of controlling the attraction parameters. Finally a stochastic model for the dynamic process is also proposed for the association and the searching of the embedded patterns.

§1. INTRODUCTION

Recently the associative model based on the autocorrelation characteristics have been extensively investigated. Since the pioneering works by Anderson[1], Kohonen[2], and Nakano[3], many researchers have theoretically analyzed such an association model of connected neurons with a certain autocorrelation matrix[4-6]. Up to date, however, it has been reported that the capacity of the autocorrelation associative model is not so large and at most ~0.15N where N is the number of neurons[1-6]. This is considered to be as the result of the many spurious memory states increasing exponentially with N. In addition, as is well known nowadays, the performance is usually depressed especially for the strongly correlated (or non-orthogonal) embedded patterns (M).

In this work, let us propose a new association model, which is available for $M \leq N$, on the basis of the synergetics[7-9]. In the present model a complete association of an embedded patter can be accomplished in the order parameter space[9]. Here a stochastic process is involved for the control of the attraction parameters. Controlling a threshold of the switching of the attraction parameters in a stochastic manner, it is found that the present model can be continuously changed between an associative model and a searching model in the order parameter space.

§2. DYNAMICS OF SYNERGETIC ASSOCIATIVE MODEL

In the present model, the internal states and the output of the *i* th neuron parameter are denoted as q_i and p_i ($1 \le i \le N$), respectively. Denoting the embedded vectors as $v^{(r)}_i$ ($1 \le r \le M$, $1 \le i \le N$), the discrete time and the analogue neuron model is assumed.

$$q_i(n+1)=kq_i(n)+(1-k)p_i(n) \quad , \tag{1}$$

$$p_i(n)=q_i(n)\left\{1+\sum_{r=1}^{M} \lambda^{(r)}(n)\xi^{(r)}(n)v^{(r)}_i-\sum_{r\neq r'}^{M}\xi^{(r')}(n)^2\xi^{(r)}(n)v^{(r)}_i -\langle q^{\dagger}, q\rangle^2 q_i\right\} , \tag{2}$$

where k is a constant between 0 and 1, $\lambda^{(r)}$ is called the attraction parameter, and the symbol $\langle\, , \rangle$ stands for the innerproduct as

$$\langle a,b\rangle=\sum_{i=1}^{N} a_i b_i \quad . \tag{3}$$

The state of the *i* th neuron q_i can be expressed in terms of the linear combination of the embedded pattern vectors $v^{(r)}_i$ ($1 \le r \le M$, $1 \le i \le N$) as follows

$$q_i(n)=\sum_{r=1}^{M} \xi^{(r)}(n)v^{(r)}_i+\eta_i(n) \quad , \tag{4a}$$

and

$$\xi^{(r)}(n)=\sum_{i=1}^{N} v^{(r)\dagger}_i q_i(n)=\langle v^{(r)\dagger},q(n)\rangle \quad , \tag{4b}$$

where $\eta_i(n)$ is the residual error or noise due to the incompleteness of the vector set $v^{(r)}_i$, and $v^{(r)\dagger}_i$ is the adjoint vector of $v^{(r)}_i$ and defined as

$$v^{\dagger\,(r)}_i=\sum_{r'=1}^{M} a_{rr'} v^{(r')}_i \quad ; \tag{5}$$

here $a_{rr'}$ is given by

$$a_{rr'}=\left(\sum_{i=1}^{N} v^{(r)}_i v^{(r')}_i\right)^{-1} \quad . \tag{6}$$

To assure the existence of $a_{rr'}$, $M \le N$ has to be assumed. Then $\eta_i(n)$ can be expanded as

$$\eta_i(n)=\sum_{r=1}^{N-M} \zeta^{(r)}(n)u^{(r)}_i \tag{7}$$

Here the vector set $u^{(r)}_i$ and $u^{(r)\dagger}_i$ have to satisfy the followings.

$$\sum_{r=1}^{M} u^{(r)\dagger}{}_k v^{(r)}{}_i = \sum_{r=1}^{M} v^{(r)\dagger}{}_k u^{(r)}{}_i = 0 \quad . \tag{8}$$

$$\sum_{r=1}^{M} u^{(r)\dagger}{}_k u^{(r)}{}_i = \delta_{ki} \quad . \tag{9}$$

Substituting eq.(2) into eq.(1), we readily have

$$q_i(n+1) = q_i(n) + (1-k)\left\{ \sum_{r=1}^{M} \lambda^{(r)}(n)\xi^{(r)}(n) v^{(r)}{}_i - \sum_{s\neq r'} \xi^{(r)}(n)^2 \xi^{(r)}(n) v^{(r)}{}_i - \langle q^{\dagger}, q\rangle^2 q_i \right\} . \tag{10}$$

Then making the innerproduct with $v^{(r)\dagger}{}_i$ and $u^{(r)\dagger}{}_i$, one has

$$\xi^{(r)}(n+1) = \xi^{(r)}(n) + (1-k)\left\{ \lambda^{(r)}(n)\xi^{(r)}(n) - \sum_{s\neq r'}^{M} \xi^{(r)}(n)^2 \xi^{(r)}(n) - \left(\sum_{s=1}^{M} \xi^{(s)}(n)^2 + \sum_{s=1}^{N-M} \zeta^{(s)}(n)^2 \right) \xi^{(r)} \right\} ,$$

and (11)

$$\zeta^{(r)}(n+1) = \zeta^{(r)}(n) - (1-k)\left(\sum_{s=1}^{M} \xi^{(s)}(n)^2 + \sum_{s=1}^{N-M} \zeta^{(s)}(n)^2 \right) \zeta^{(r)} \quad , \tag{12}$$

respectively . Therefore, in the time evolution process defined by eqs.(11) and (12), the residual noise $\zeta^{(r)}(n)$ or $\eta_i(n)$ is assured to vanish as $n \to \infty$ as fas as $0<k<1$. Hence, in such a condition, eq.(11) can be reduced to

$$\xi^{(r)}(n+1) = \xi^{(r)}(n) + (1-k)\left\{ \lambda^{(r)}(n) - 2\sum_{s=1}^{M} \xi^{(s)}(n)^2 + \xi^{(r)}(n)^2 \right\} \xi^{(r)} \tag{13}$$

From this, if $\lambda^{(r)}(n)=1$, one can confirm that $\xi^{(r)}(n)=0$ or 1 with

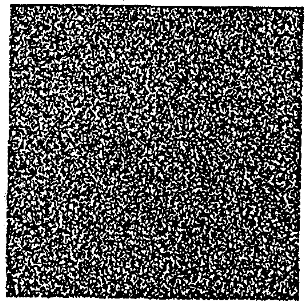
$$\sum_{r=1}^{M} \xi^{(r)}(n)^2 = 1 \quad . \tag{14}$$

Therefore only one pattern can be selected through the association process. Finally the resultant pattern $q_i^*(n)$ can be derived as

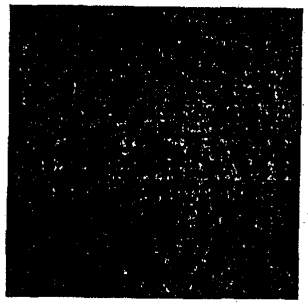
$$q_i^*(n) = \sum_{r=1}^{M} \xi^{(r)}(n) v^{(r)}{}_i \quad . \tag{15}$$

§3. SIMULATION RESULTS

In this section let us show an example of the simulation results of our neural model. 12 (=M) images with 256×256 (=N) size and 256 gray scale levels were embedded for the association simulation. Then a random pattern was set as the initial pattern for the association process. The resultant association process is presented in Fig.1 (a)-(c). The time dependence of the order parameters is also given in Fig.2. Finally the completely associated image is shown in Fig.3. From this one can confirm a complete association without any trapping at a spurious pattern.



(a) Initial Image



(b) Intermidiate Image 1

(b) Intermidiate Image 2

Fig.1 Association Process

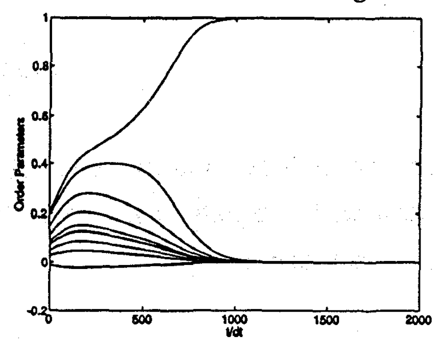


Fig.2 Time Dependence of Order Parameters

Fig.3 Associated Image
(256 gray scal levels, 256×256 size)

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