A Study on Association among Dispatching Rules in Manufacturing Scheduling Problems

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Abstract - There are many scheduling strategies, called dispatching rules, for manufacturing systems. However, those rules can be applied mainly to simple manufacturing systems. In this paper we propose a way to evaluate the effectiveness of such dispatching rules in the environment of complex manufacturing systems in the sense that there exist two performance measures. Defining a new concept of closeness of dispatching rules with respect to a performance measure, we can find a set of close dispatching rules for the performance measure. This will give us some guidance about which dispatching rules may apply at each dispatching point in a manufacturing system under two or more performance measures in practice. For the purpose a technique of data mining plays an important role.

1. Introduction

Many dispatching rules are derived for manufacturing scheduling tasks[2][3][6]. In many cases, however, it is difficult to find which dispatching rule fits to a given manufacturing system. The reason is that a manufacturing system has two or more performance measures in practice. For example, due date is one of the concerns and machine utilization is also one of them. Hence, the selection of dispatching rules are performed basically by a trial and error approach that is based on the experience of manufacturing staffs. There is no sound theoretical way to select an appropriate dispatching rule for complex manufacturing systems in practice.

In this paper, we propose a way to obtain some knowledge to select dispatching rules for a given manufacturing system. The basic idea is explained in the following sections. The next section describes a motivated example. The motivated example leads to the definition of the closeness of dispatching rules with respect to a performance measure. A technique of data mining, called Apriori algorithm[1], is utilized to obtain some associations among dispatching rules and performance measures. This will show that we may select an appropriate dispatching rule in a quantitative manner. Finally, some examples are given to illustrate the idea given in this paper.

2. Motivated Example

Figure 1 shows an example of applying two dispatching rules: *EDD* (Earliest Due Date First) and *SPT* (Shortest Processing Time First). Suppose that two jobs *A* and *B* are processed by a machine. The processing time of job *A* is 5 time units. The due date of job *A*, d_A, is 7 time units. The processing time of job *B* is 3 time units. The due date of job *B*, d_B, is 8 time units. The tardiness of job *A* is defined as $T_A = max(0, c_A - d_A)$, where c_A is the completion time of job *A*. Similarly, the tardiness of job *B* is defined as $T_B = max(0, c_B - d_B)$, where c_B is the completion time of job *B*.

When we apply *EDD* to this example, job *A* is selected at first for processing because the due date is earlier than the one of job *B*. Then, following job *A*, job *B* will be processed. In this example, *EDD* produces an optimal schedule with respect to the maximum tardiness $T_{max} = max(T_A, T_B)$.

On the other hand, if we apply *SPT*, job *B* is selected at first for processing because the processing time is shorter than the one of job *A*. In this case, *SPT* produces an optimal schedule with respect to the mean flow time, $F_{avg} = (c_A + c_B)/2$. The readers are referred to Conway, *et al* [3] for the details about the dispatching rules, attributes and performance measures.

In this example, if our objective is only to minimize the maximum tardiness, then we apply *EDD* to obtain

$$T_{max} = 0 \quad (\text{in this case } F_{avg} = 6.5). \tag{1}$$

If our objective is only to minimize the mean flow time, then we apply *SPT* to obtain

$$F_{avg} = 5.5 \quad (\text{in this case, } T_{max} = 1.0) \tag{2}$$

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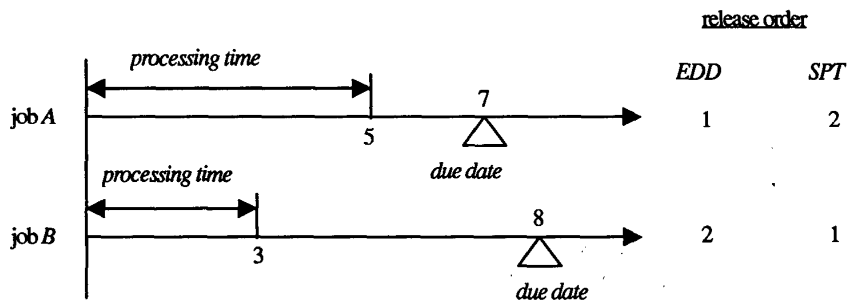

Figure 1. An example of dispatching rules

Here, a question is which dispatching rule should be applied to the system if our objectives are both the minimization of the maximum tardiness and the minimization of the mean flow time.

Let us consider the case that we are allowed to have the allowance of one time unit of the tardiness (T_{max}=1.0 in equation (2)) in practice. Then *SPT* will create a close result to *EDD* with respect to the maximum tardiness under the allowance. Hence, we may also apply *SPT* to minimize the maximum tardiness in the practical sense, while minimizing the mean flow time. That is, for the purpose of the minimization of the maximum tardiness, we may apply both *EDD* and *SPT*. We will write this pattern as

$$\{T_{max}\} \Rightarrow \{\, EDD, SPT \,\}. \tag{3}$$

On the other hand, if the mean flow time should be less than or equal to 5.5, then we may only apply *SPT*. This pattern can be written as

$$\{F_{avg}\} \Rightarrow \{\, SPT \,\} \tag{4}$$

As a result of patterns (3) and (4), we may conclude that we may select *SPT* for both the performance measures because *SPT* appears in both patterns. This will be written as

$$\{\, T_{max}, F_{avg} \,\} \Rightarrow \{\, SPT \,\}. \tag{5}$$

Or, for example, if we are allowed to have the allowance of one time unit of the mean flow time (F_{avg}=6.5 in equation (1)) in practice, then we may also apply *EDD* to minimize the mean flow time under the allowance. That is, for the purpose of the minimization of the mean flow time, we may apply both *EDD* and *SPT*. We will write this pattern as

$$\{F_{avg}\} \Rightarrow \{\, EDD, SPT \,\}. \tag{6}$$

On the other hand, if the maximum tardiness should be less than or equal to 0.0, then we only apply *EDD*. This pattern can be written as

$$T_{max} \Rightarrow \{\, EDD \,\} \tag{7}$$

From patterns (6) and (7), we conclude that we may select *EDD* for both the performance measures because *EDD* appears in both patterns. This will be written as

$$\{\, T_{max}, F_{avg} \,\} \Rightarrow \{\, EDD \,\}. \tag{8}$$

Following the above example, we will discuss in this paper a policy for selecting an appropriate set of dispatching rules such as in the pattern (5) or (8). For the purpose we will introduce a concept of closeness among dispatching rules in the next section.

3. Closeness of Dispatching Rules

Let P^Q and P^R denote real numbers when the performance measure P is evaluated under dispatching rules Q and R, respectively.

Definition:

Suppose that Q and R are given dispatching rules. If the dispatching rule Q satisfies

$$P^Q - \alpha \leqq P^R$$

for some real number $\alpha > 0$, then the dispatching rule Q is said to be close to the dispatching rule R with respect to the performance measure P under a given allowance α. We will write this as follows:

$$Q = R/(\alpha, P). \qquad \square$$

For example, if the dispatching rule *EDD* is close to *SPT* with respect to the mean flow time under some $\alpha > 0$, then the followings hold.

$$F_{avg}^{EDD} - \alpha \leqq F_{avg}^{SPT}$$
$$EDD = SPT/(\alpha, F_{avg})$$

4. Mining of Rules

Consider two-machine flow shop scheduling problem. Let Z be a given set of dispatching rules for each machine:

$$Z = \{ EDD, SLACK, SPT \}. \tag{9}$$

Then the total of nine combinations of dispatching rules can be applied to the flow shop. For example, *EDD* is applied to the first machine, and *SLACK* (Shortest *SLACK* Time First) is applied to the second machine. We write this kind of combinations of dispatching rules *EDD&SLACK*, and so on.

Suppose that one of the objectives is to minimize the maximum tardiness and the other is to minimize the mean flow time. The values of performance measures by using each combination of dispatching rules are evaluated by using a discrete event simulation[5]. At first, a set of n jobs is created. For the set each of nine combinations of dispatching rules is applied to obtain the maximum tardiness of the jobs. By using these nine simulation runs, a set will be created as follows:

$$D_e = \{ T_{max}, z \in Z \times Z \mid z = e/(\alpha, T_{max}) \} \tag{10}$$

where α is a given positive real number and $e \in Z \times Z$ is the best combination of dispatching rules that gave the minimum value of the maximum tardiness among a set of nine simulation runs. Note that D_e at least includes the combination e. A set (10) is called a transaction. These simulation runs are repeated until we obtain N_e such transactions.

The same procedure is performed to obtain N_s transactions of the form:

$$D_s = \{ F_{avg}, z \in Z \times Z \mid z = s/(\beta, F_{avg}) \} \tag{11}$$

where β is a given positive real number and $s \in Z \times Z$ is the best combination of dispatching rules that gave the minimum value of the mean flow time among a set of nine simulation runs.

4.1 Data Mining

We adapt the data mining technique[1]. The item set for the above two-machine flow shop is defined by

$$I = \{ T_{max}, F_{avg}, EDD\&EDD, \cdots, SPT\&SPT \}.$$

Then transactions D_s and D_e are included in the item set I as subsets. A transaction D_e is , for example, of the form:

$$\{ T_{max}, EDD\&EDD, EDD\&SPT \} \subset I.$$

An association rule is defined as

$$X \Rightarrow Y$$

where $X \subset I$, $Y \subset I$ and $X \cap Y = \phi$. In this paper, an association rule is given by, for example,

$$\{ T_{max} \} \Rightarrow \{ EDD\&EDD, SPT\&SPT \}.$$

The '*support*' of an association rule $X \Rightarrow Y$ is defined by the number of occurrences of $X \cup Y$ divided by the number of all transactions.

The '*confidence*'' of an association rule is defined by the number of occurrences of $X \cup Y$ divided by the number of occurrences of X.

Apriori algorithm[1] finds association rules that satisfy at least given values of support and confidence, which are called *minimal support* and *minimal confidence*, respectively. In general, Apriori algorithm generates two types of association rules:

{Performance measures} ⇒ {Dispatching rules}
{Dispatching rules} ⇒ {Performance measures}

Note that we are only interested in the first association above. Hence, we always set the minimal confidence equal to 1.0.

4.2 An Example: Flow-Shop of 2 Machines

Consider a two-machine flow shop scheduling problem. The set of dispatching rules, Z, is given by equation (9). Then the set of all the combinations of dispatching rules to be applied to this flow shop are given by $Z \times Z$. Each simulation run deals with 20 jobs. These jobs are divided into four groups in equal probability. The routes of the job groups are given by

job group 1: machine 1
job group 2: machine 2
job group 3: machine 1 → machine 2
job group 4: machine 2 → machine 1.

The processing times at each machine are given by uniform distributions:

machine 1: uniform(70, 400)
machine 2: uniform(300, 800).

The due dates of jobs are defined by

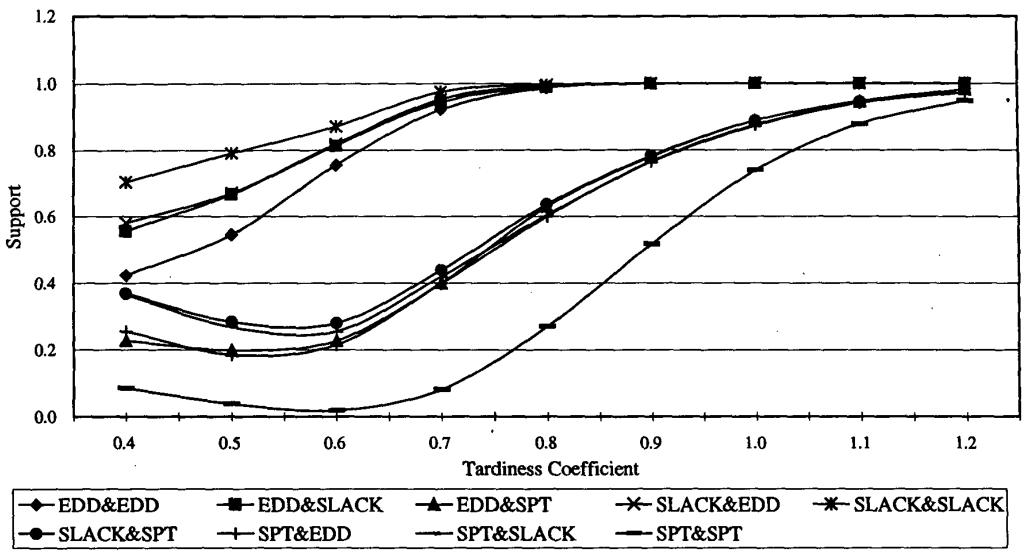

Figure 2. Support for the maximum tardiness (α = 20.0)

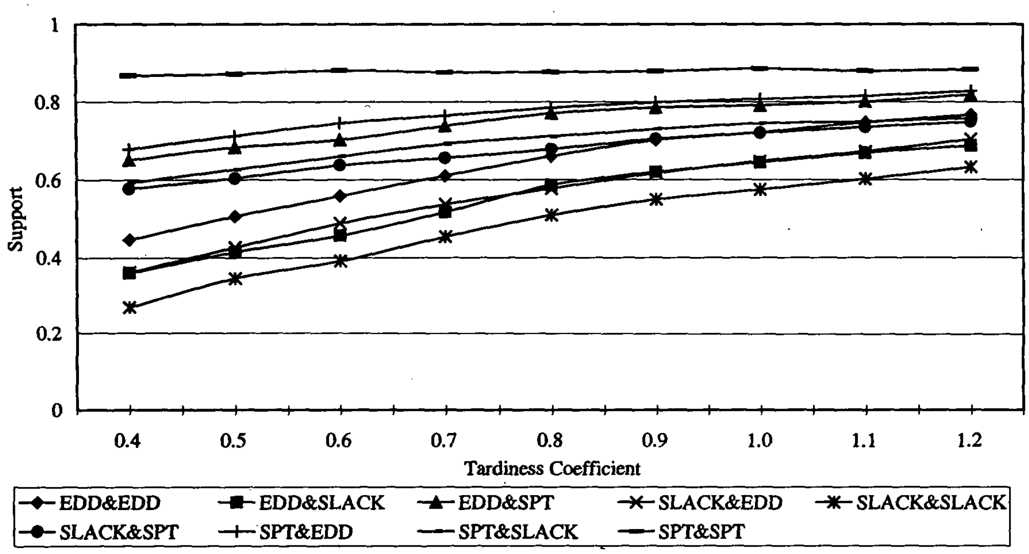

Figure 3. Support for the mean flow time (β = 50.0)

$$(\text{total processing time}) \times \text{uniform}(7, 13) \times \gamma \qquad (12)$$

where γ is called a tardiness coefficient and varied 0.1 each from 0.4 to 1.2. A set of simulation consists of nine runs; for a set of 20 jobs nine combinations of dispatching rules are applied, where two performance measures, the maximum tardiness and mean flow time, are evaluated for each run. In this experiment the maximum tardiness are, for example, given by

(EDD&EDD, EDD&SLACK, EDD&SPT, SLACK&EDD, SLACK&SLACK, SLACK&SPT, SPT&EDD, SPT&SLACK, SPT&SPT)

= (1727, 1727, 1858 1743, 1743, 1912, 2443, 2443, 2443)

where the minimum value is 1727 by *EDD&EDD*. Hence, if an allowance, α, is set to 20.0, then *EDD&EDD* is close to itself, *EDD&SLACK*, *SLACK&EDD*, and *SLACK&SLACK* by definition. This data creates a following transaction:

$$D_e = \{ T_{max}, z \in Z \times Z \mid z = EDD\&EDD/(20.0, T_{max}) \}$$
$$= \{ T_{max}, EDD\&EDD, EDD\&SLACK, SLACK\&EDD, SLACK\&SLACK \}.$$

In this experiment the simulations are repeated 1000 sets. The allowances are set to $\alpha=20.0$ and $\beta=50.0$ for the maximum tardiness and mean flow time, respectively. At the tardiness coefficient $\gamma=0.4$ and minimal support = 0.4, the following association rules are found by applying Apriori algorithm:

$$\{T_{max}\} \Rightarrow \{ EDD\&EDD, EDD\&SLACK, SLACK\&EDD, SLACK\&SLACK\}$$
$$\{ F_{avg} \} \Rightarrow \{ EDD\&EDD, EDD\&SPT, SLACK\&SPT, SPT\&EDD, SPT\&SLACK, SPT\&SPT \}$$

The common combination of dispatching rules in the above two association rules is only *EDD&EDD*. This is a kind of what we would like to have. That is, we have

$$\{ T_{max}, F_{avg} \} \Rightarrow \{ EDD\&EDD \}$$

with minimal support = 0.4. We may say that the combination of dispatching rules, *EDD&EDD*, will perform good to minimize both the maximum tardiness and the mean flow time in practical sense; that is, under the allowances $\alpha=20.0$ and $\beta=50.0$.

Figures 2 shows the values of supports for the combinations of dispatching rules with respect to the various tardiness coefficients, γ, in equation (12), where the performance measure is the maximum tardiness. Figure 3 shows the same except for the performance measure. In Figure 3 the performance measure is the mean flow time.

5. Simulation Experiment

In this section, we consider a random-routed job shop of 5 machines[3]. At first, we apply *SPT* and *D/OPN* (Due date per operations remaining) and obtain some associations of the dispatching rules and performance measures. Then we apply a linear combinations of the dispatching rules to the job shop. Following the simulation experiments, we examine some associations among dispatching rules and performance measures.

5.1 Random-routed job shop of 5 machines

Consider a job shop that consists of 5 machines. The routes of the jobs are determined randomly, while each job must pass all the machines. The transfer times between machines are assumed to be zero. There is no setup time for operations and no machine breakdown. The jobs arrive at the shop dynamically, where their arrival rate is exponentially distributed. The processing times are obtained from the uniform distribution between 1 and 9 time units. The due dates of the jobs are 9 times the sum of the processing times (TWK method[3]). The utilization factor, ρ, is defined by

$$\rho = (\text{average arrival rate of jobs}) \times (\text{average service time}).$$

5.2 First Experiment

A set of 10,000 jobs is created with the utilization factor 0.9. Applying at first *SPT* to all machines of the model described in the section 5.1, we measure the flow time and lateness of each job. Then we repeat the simulation experiment by using the same job set with *D/OPN* instead of *SPT*. The dispatching rule *D/OPN* gives the highest priority to the job with the smallest value of the following quantity:

$$\frac{d_i - t}{g_i - J + 1}$$

where t is the time at which a selection for machine assignment is to be made, i is the index over the jobs to be processed by the shop, J is the specific value of operation, g_i is the total number of operations on job i, and d_i is the due date of job i [4].

Apriori algorithm is then applied to the produced transactions. The results of Apriori algorithm are the following association rules:

{ flow time } $\Rightarrow$ { *SPT* } with *support = 0.75*
{ flow time } $\Rightarrow$ { *D/OPN* } with *support = 0.25*
{ lateness } $\Rightarrow$ { *SPT* } with *support = 0.76*
{ lateness } $\Rightarrow$ { *D/OPN* } with *support = 0.24*.

This indicates that *SPT* produced better flow time for *75%* of jobs and *EDD* produced better flow time for *25%* of jobs. Also, *SPT* produced better lateness for *76%* of jobs and *EDD* produced better lateness for *24%* of jobs.

5.3 Second Experiment

In this experiment the set of dispatching rules is defined by

$$Z = \{ 0.6SPT+0.4D/OPN, 0.7SPT+0.3D/OPN, 0.8SPT+0.2D/OPN, 0.9SPT+0.1D/OPN\}$$

where the elements are linear combinations of *SPT* and

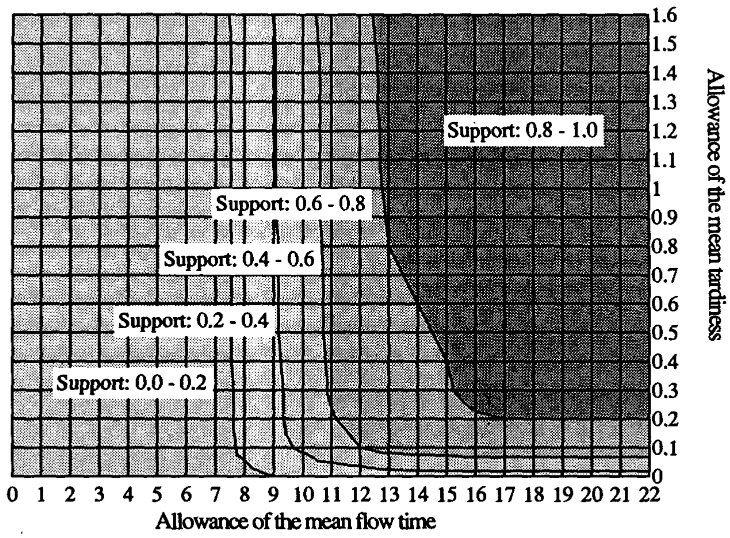

Figure 4. Landscape of support of the association rule (utilization factor ρ=0.9)

D/OPN[4]. This rule gives the highest priority to the job with the smallest weighted sum of processing time and due-date-per-operation:

$$u \cdot p_{i,J} + (1-u) \times \frac{d_i - t}{g_i - J + 1}$$

where $p_{i,J}$ is the processing time for the J^{th} operation of job i and u $(0.0 \leqq u \leqq 1.0)$ is the weight. As in the section 4, the transactions are defined as follows:

$$D_e = \{ z \in Z \times Z \times Z \times Z \times Z \times Z \mid z = e/(\alpha, T_{avg}) \}$$
$$D_s = \{ z \in Z \times Z \times Z \times Z \times Z \times Z \mid z = s/(\beta, F_{avg}) \}$$

where T_{avg} and F_{avg} are the mean tardiness and mean flow time, respectively.

In this experiment, we created 100 sets of 1,000 jobs. For each job set, we apply total of 1024 combinations of the given dispatching rules to evaluate the mean flow time and mean tardiness. When we set α=0.3 and β=16.0, we obtain the following association rule with support≧0.8.

$\{ T_{avg}, F_{avg} \}$
$\Rightarrow$ { Apply 0.7*SPT*+0.3*D/OPN* at all machines }

Figure 4 shows the detailed landscape of the support of this association rule with respect to various allowance values.

5. Conclusion

A concept of closeness among dispatching rules is defined in this paper. Using this concept, we have shown the possibility of selecting a dispatching rule that minimizes some performance measures in practical sense. Here, the support of the association rule plays an important role in quantitative manner.

It is not obvious, however, to determine appropriate values for allowance values to find an effective combination of dispatching rules. Furthermore, it is necessary to clarify the rationale of the given value of minimal support.

References

[1] Agrawal, R. and Srikant, R.: Fast Algorithm for Mining Association Rules, Proc. Of the 20th VLDB Conf., Santiago, Chile, 487-499, 1994.

[2] Blackstone, J.H., Phillips,D.T.,and Hogg,G.L.: A state-of-the-art survey of dispatching rules for manufacturing job shop operations. Int. J. Prod. Res., 20, pp.27-45, 1982.

[3] Conway, R., Maxwell, W. and Miller, L. : *Theory of Scheduling*, Addison-Wesley, 1967.

[4] Fukui, H. and Hashimoto, F.: A Study on the Dispatching Rule for Minimizing Tardiness, J.of Japan Industrial Management Association, Vol.46, No.4, pp.248-253, 1995 (in Japanese).

[5] Kelton, D., Sadowski, R. and Sadowski, D. : *Simulation with Arena*, WCB/McGraw-Hill., 1998.

[6] Panwalker,S.S., and Iskander, W.: A survey on scheduling rules. Operations Research, 25, pp.45-61, 1977.