TOOLS FOR MHD SIMULATION OF HOT DENSE PLASMA

S.Semushin, B.Etlicher

Laboratoire de Physique des Milieux Ionisés, Laboratoire CNRS, Ecole Polytechnique
91128 Palaiseau, France
e-mail: sem@lpmi.polytechnique.fr

The complexity of physical problems in plasma demands special tools for its analysis. The primary source of data is, of coarse, the physical experiment. But, sufficiently complete comprehension of physical phenomenon is actually impossible without the numerical simulation, which is more and more often named as computational experiment. Moreover, computational experiment usually provides more complete set of data, which becomes credible only after verification by experiment

The processes under consideration (such as different Z-pinches, plasma opening switches - POS, plasma focus and others) have some common features. Fist of all, they are non stationary and subjected to divers instabilities. Another common feature is a big difference in time and spatial scales, and small local details are very often significant for global process development. This means, that some kind of adaptation is inevitable for numerical simulations. Traditional lagrangian approach can be also considered as adaptation, based on the mass, but it is not applicable to the problems with high level of convection or mixing.

Two numerical methods for MHD plasma simulation and corresponding codes are presented here. The first code ASTRE uses original adaptive mesh refinement algorithm [1, 2], the other one, Z+ is based on classical arbitrary lagrangian-eulerian algorithm [3, 4]. These two approaches are complimentary. Each of them has its own advantages and specific application domains, but each of them covers the majority of the problems under consideration

Physical models

The choice of physical model is always a compromise between the effectiveness of computation and resulting accuracy. Here the standard description of two temperature plasma is used. The influence of more sophisticated terms was also estimated, but these terms were not included into all computations. Thus, the governing equations are:

$$
\begin{aligned}
&\frac{\partial \rho}{\partial t} + div(\rho u) = 0 \qquad\qquad \frac{\partial \rho u}{\partial t} + Div(\rho u u) + grad(p) + div\hat{\pi} = \frac{j \times B}{c} \\
&\frac{\partial \rho \varepsilon_e}{\partial t} + div(\rho u \varepsilon_e) + p_e \; divu + divW_e = jE^* - S_{rad} + Q_{ei} \\
&\frac{\partial \rho \varepsilon_i}{\partial t} + div(\rho u \varepsilon_i) + p_i \; divu + \hat{\pi}\, divu + divW_i = -Q_{ei} \\
&\frac{\partial B}{\partial t} = -crotE\,, \quad E = E^* - \frac{u \times B}{c}\;; \qquad j = \frac{c}{4\pi} rotB\,, \quad divB = 0 \\
&W_{e,i} = -k_{e,i}\, gradT_{e,i}\,, \qquad\qquad n_i = \rho / m_i ; n_e = Zn_i
\end{aligned}
\tag{1}
$$

where ρ, P- plasma density and pressure, n_e, n_i- electron and ion densities, Z – ionisation, u – velocity, E, B – electric and magnetic fields, $\varepsilon_e, \varepsilon_i$, T_e, T_i- electron and ion specific internal energies and temperatures, $W_{e,i}$- heat fluxes, Q_{ei}, S_{rad}- exchange term and radiation sources. Omh's law is used either in form of classical conductivity or with the Hall terms (3).

$$E^* = \frac{j}{\sigma} \tag{2}$$

$$E^* = \frac{j}{\sigma} + \frac{j \times B}{cen_e} - \frac{grad\, p_e}{en_e} \tag{3}$$

This system of differential equation is completed by equations of state and transport coefficients definition. Transport coefficients are usually estimated by Braginskii theory [8].

For equation of state a set of physical model was used. The simplest approach is an analytical model, for example [3]. More accurate ones are, of coarse, tables like SESAME [9], or THERMOS [10]. The physical content includes filters to import different tabular data,

The ionisation level can be taken either from Thomas-Fermi approximation or from tables, for example [9], or more widely applicable [10].

There is a set of approximation for radiation transport. The simplest model is the radiation losses. Two methods of radiation transport simulation are more complicated and more time-consuming. They include the ray tracing algorithm and quasi-diffusion approach [12]. The latter is more efficient, but it is valid only for optically dense plasma. Now it is adapted for general case by special form of flux correction.

In magnetic implosion problems we have the domains with quite different relation between magnetic and thermal energies. It is easy to show, that for small values of parameter $\beta = P/(B^2/8\pi)$ the numerical approximations based on internal energy equations are more favourable. But in this case the special attention must be paid on the conservation of total energy.

Code ASTRE

This code uses a rectangular eulerian mesh. Its main advantages are efficiency, robustness and very simple changing of considered problem. A resolution of local small detail of the flow is based on original adaptive mesh refinement algorithm [1, 2]. The local adaptive procedure was specially constructed for essentially non-linear problems of plasma dynamics. In such algorithm the cells can be subdivided or merged, depending on the gradient of the variables. The algorithm is explicit, but the computations are very effective due to the different time steps for cells of different size and due to the possible temporal refinement. The code name (Adaptive Spatial-Temporal REfinement) reflects this feature.

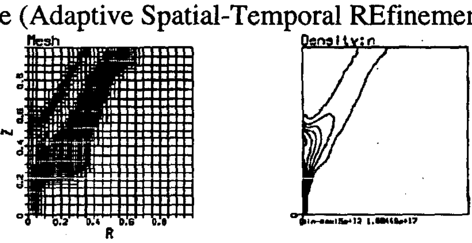

Fig.1 Adaptive mesh for high zippering Z-pinch simulation

This method allows solving the problems in region of complex form. Such opportunity is provided by special boundary conditions treatment. Conservative spatial approximation minimises error in total energy.

New approximation of Hall term was proposed in [2]. Such treatment of this complicated term gives monotone solution even for the ideally conducting plasma, without any conductive diffusion. This approximation lightens the constraints of time step and it extends the application domain to the rare plasmas. The latter future facilitates the POS simulations.

Now code exists in one- and two-dimensional versions, and in Cartesian and cylindrical coordinates for r-z and r-φ planes. Wide choice of boundary condition allows easy changing of considered problem. In eulerian simulations the vacuum treatment is always complicated. The vacuum is distinguished from plasma by density threshold, and it is considered as frozen plasma with very small electrical conductivity.

Code Z+

The other code, Z+, uses classical method with lagrangian and eulerian stages. Lagrangian stage is based on [3], eulerian – on slightly modernised version of [4]. Fully conservative algorithm [3, 4] keeps precise balance of total energy for any strategy of mesh motion between lagrangian and eulerian. The remeshing strategy depends of the problem under consideration, but mesh motion is defined automatically, without any manual control.

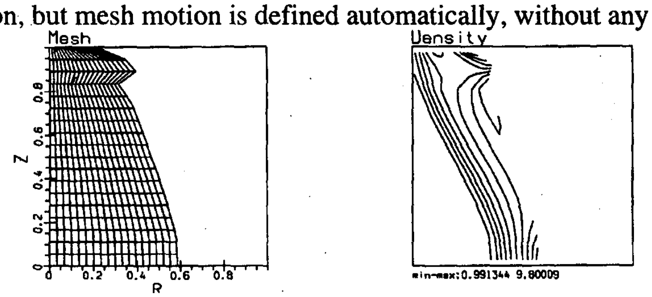

Fig.2 ALE mesh for Z-pinch simulation

The code Z+ was initially constructed for Z-pinches simulation, but later it was ported to other problems, due to the wide choice of boundary conditions. This code allows simple multi-material simulations and it has a full set of radiation transport models.

Realisation of codes

The complexity of the processes often requires the modifications of the physical models and of the numerical approximations. Thus, the code structure should be adapted for such evolution. The codes are written in modern style in C language. The single source of physical models is shared between the codes. Different compilations of the same source files produce 1D or 2D versions. The latter property simplifies the development and support of the codes.

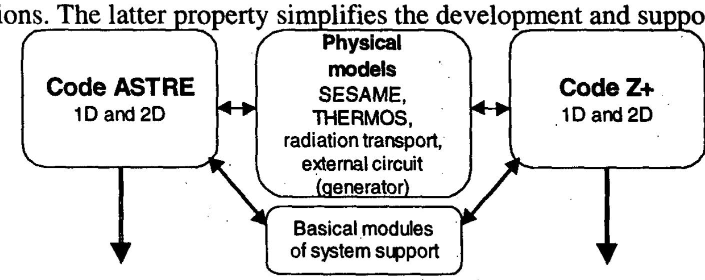

Fig.3 Interaction of codes and physical models

Examples of simulations

A lot of simulations were done by these codes in one and two dimensions. Z-pinch implosions were modelled in different configurations (Fig.1, 2, for example), including instability analysis with randomly perturbed initial data [6]. Typical parameters are current from 50-100 kA till 50 MA, implosion time – from 100 ns to a few microseconds. Optimizations of geometry and density distribution were done. An example of multi-wire liner compression [6] is presented either in r-z plane or in polar coordinates r-φ plane. The simulations of plasma focus present another type of magnetic compression problems. These codes were also used for optimization of gas-puff production for further compression in Z-pinch [5]. In this case the simulations were started by gas dynamics version, but than the data were transmitted to MHD code for quality estimation of achieved density distribution.

Plasma opening switches were another important objects for simulation [7]. Their geometrical parameters and position of plasma guns was analyzed and optimized.

Conclusion

These two codes share wide choice of different physical models for equations of state, transport coefficients, optical properties. Such construction provides interesting opportunity for verification of simulation data, namely, either the same physical model can be used by different numerical approaches, or a variety of physical models can be checked with the same numerical conditions.

The authors appreciate the fruitful physical discussions with Alexandre. Chuvatin.

References.

[1] S.A.Semushin, Finite Volume Adaptive Mesh Refinement algorithm for gas dynamics and MHD problems. *Finite Volumes for Complex Applications*, Paris, HERMES, 1996, p. 659

[2] S.Semushin, Adaptive Mesh Refinement approach for simulation of gas dynamics and magneto hydrodynamics problems. *Proceedings of 15th IMACS Word Congress. vol.3,* 1997, 109-114.

[3] V.A.Gasilov, A.Yu.Krukovskii, A.A.Otochin, The program package for calculations of 2D axially symmetrical MHD flows of radiative plasma, *Preprint N162, Keldysh Inst. Appl. Math*ematics, Moscow, 1990, (in Russian).

[4] V.A.Gasilov, A.Yu.Krukovskii, A.A.Otochin, Al.A.Otochin, The Conservative Lagrangian-Eulerian Scheme for 2D Axially Symmetrical MHD Flows, *Preprint N5, Keldysh Inst. Appl. Mathematics,* Moscow, 1991, (in Russian).

[5] S.Semushin, B.Etlicher, C.Rouillé. Optimisation of gas target production for Z-pinch implosions. *Proceeding of BEAMS'96 Int. Conf.*, p.758-761.

[6] S.Semushin, B.Etlicher, Simulation of multi-wire liner dynamics during Z-pinch implosion. *These Proceedings.*

[7] S.Semushin, A.Chuvatin Hall MHD simulation of plasma behaviour in an opening switch. *These Proceedings.*

[8] S.I.Braginskii, Transport Processes in a plasma. *In: Reviews of Plasma Physics,* Vol.1, Consultants Bureau, New York, (1965).

[9] SESAME: The Los Alamos National Laboratory Equation Of State Database, Report LA-UR-92-3407, Los-Alamos, 1992.

[10] A.F.Nikifirov, V.G.Novikov, A.D.Solomyannaya Self consistent hydrogen-like average atom model for the matter with given temperature and density. High Temperature, **34**, (1996), 1. (in Russian).

[11] D.E.Post, R.V.Jensen, C.B.Tarter et al. Steady-state radiative cooling rates for low-density, high-temperature plasmas, *Atomic Data and Nuclear Data Tables,* **20**, (1977), 397-439.

[12] B.N.Chetverushkin. Mathematical Modelling of radiation gas dynamics problems. Nauka, Moscow, 1985, (in Russian).