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NEW PASSIVE TRACKING ALGORITHMS USING POWER INFORMATION WITH APPLICATION TO PASSIVE SONAR

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ABSTRACT

Tracking systems make both detection and position estimation of targets through time association of the received data , performed from the targets kinematic models . In Passive situations , the bearing information do not permit to track low Signal to Noise Ratio (SNR) targets . The received signals power can however be used for the data association as it is in the Tracks Decision Rule .
This will be checked with the association test power , especially when the signal power estimate is biased . We then present a tracks initiation algorithm using a global Maximum Likelihood Estimate (MLE) , and a filter derived from the Probabilistic Data Association Filter (PDAF) , including the signal Power and Detection probability (Pd) estimates . Results will be presented in the frame of a Monte Carlo simulation with a Track Detection criterion .

1. Introduction

This paper will cope with the single source Tracking Problem , generally presented as a filtering of tracks updated with measurements present in a gate around the predicted value , at each scan one coming from the target (if detected) and the others being false alarms , originating from self or medium noise .
This is known as the uncertain correlation problem solved by associating the correct updates .
Both data association and filtering are generally made only with the target estimated kinematics and measured positions , as bearing (θ) in passive broadband sonar . Then one uses the track power in order to make a decision on tracks confidence .
Let z be the measurement vector , X the state vector .
Assuming Linear Constant Speed (LCS) motion , we but cannot choose the natural cartesian equation $\ddot{x} = \ddot{y} = 0$ (1) in unobservable passive systems .
We use the approximated model : $\theta^{(r+1)} = 0$ (2)
with the state vector $X = (\theta,\dot{\theta},\dots,\theta^{r})^t$, $r = 1,2,3$
The obtained linear filter is preferred to the one issued from the exact equation $\ddot{\theta} = 3\dot{\theta}/\dot{\theta}^2 - 2\dot{\theta}^3$ (3)
The motion model at scan k is so

$$X(k+1) = A(k)\,X(k) + V(k) \qquad (4)$$

If r = 2 , $C^t(k) = (1,0,0)$, $A(k) = \begin{bmatrix} 1, T, T^2/_2 \\ 0, 1, T \\ 0, 0, 1 \end{bmatrix}$ (6)

where C^t is C matrix transpose ,
T is the time analysis period , V and W are centered gaussian processes with known covariance
The uncertain correlation modifies (5), as each z in $Z_k = (z_1,\dots,z_m)$, the population in the gate , is a candidate for z(k) .
A Nearest Neighbour Association (NNA) method chooses one particular z_i in Z_k , then (4),(5) are solved by the classical Kalman Filter , while a Bayesian or Probabilistic method ([1]) computes an average of all the association hypothesis :
$X(k) = E(X(k)/ Zk) = \sum_i X(k,i)\,\beta(k,i)$ (7) , where

$\beta(k,i) = P(\ z(k) = z_i)$ are computed using Baye's rule and a priori knowledge of Pd .
It seems natural in order to select z_i and compute $\beta(k,i)$ to use all the available information , say both the position and the SNR as is done in the tracks decision rule .
This will be checked in the next section .
We then present a track initiation technique using sliding bearing gates and MLE , and also a track statistics calculation .
In a fourth part , we develop the PDASI algorithm derived from the Probabilistic Data Association (PDA , [1]) and using the Signal Information , including estimates of the signal power and of Pd which is not a priori known in automatic surveillance systems .
Detection curves are then presented for NNA , PDA, and PDASI , using our initiation algorithm .

2. Data association tests

Given a measurement z , we test that it is a false alarm or a true one . We will compare two tests . The first one uses only the bearing and the second one uses both the bearing and the SNR .
The first test considers $z = \theta$, with hypothesis
H_1 : θ is from the target, normally distributed with known mean and unknown covariance $\sigma_\theta{}^2$.
H_0 : θ is a false alarm , with uniform bearing in the prediction gate of width $\Delta = 2\rho\sigma_\theta$.
In the second test , $z' = (\theta,s)^t$.
Then $H'_0 = H_0$ and s follows a reduced normal law
$H'_1 = H_1$ and s is normal with mean s_0 and covariance σ^2 ($N(s,\sigma)$) , $s \neq 0$, $\sigma > 1$
At low SNR and with a large number of frequency channels , $\sigma \simeq 1$, but the mean s is unknown .
Consider first we get a biased estimate for s .
We compare the two ratios :

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$$\Lambda = \frac{p(z/H_1)}{p(z/H_0)} \quad , \quad \Lambda' = \frac{p(z'/H'_1)}{p(z'/H'_0)} \qquad (8)$$

with levels $N = P(\Lambda > \eta \,/\, H_0)$ (9)

$N' = P(\Lambda' > \eta \,/\, H'_0)$ (10)

and powers $P = P(\Lambda > \eta \,/\, H_1)$ (11)

$P' = P(\Lambda' > \eta \,/\, H'_1)$ (12)

σ_θ is assumed known (it is in fact line estimated) b and b_n are the biases under H_1 and H_0.

one obtains $P = 2\ F(\ N\Delta/2\sigma_\theta\ ) - 1$ (13)

$$N' = P(\ \omega^2 - 2ub_n < S_0\ ) \qquad (14)$$

$$P' = P(\ v^2 - 2u(s+b) < S_1\ ) \qquad (15)$$

with u,v following the reduced normal law (N(0,1) , with repartition function F) ,

ω uniform in $[-\rho,\rho]$,

$$S_0 = -\ b_n^{\,2} - 2\ \ln(\ \Delta^{-1}\ \sigma_\theta\ \eta\ \sqrt{2\pi}\ ) \qquad (16)$$

$$S_1 = -\ b^2 - 2\ \ln(\ \Delta^{-1}\ \sigma_\theta\ \eta\ \sqrt{2\pi}\ ) \qquad (17)$$

and , using characteristic functions :

$$N' = 1/\rho_1\ \int_{0<t<\rho 1} F(S_0/\sqrt{2b_n} - t^2)\ dt \qquad (18)$$

$$\rho_1 = 1/\sqrt{2b_n} \qquad (19)$$

N' is thus easily computed , but the estimation of P' requires Monte Carlo runs .

FIG. 1 shows the results for test 1 compared to test 2 with 3 db SNR , and a bias b on s of 0 , 20% and 40% (we fixed the gate with $\rho = 3.$) .

We did not consider here the false alarms density , and so characterize the entire behaviour of the association function .

To do so , it would be necessary to compute the likelihood assuming known the distribution $\mu(m)$ of the number of false alarms in the gate (Poisson) and in each case compute both P'(m) and N'(m) .

Note when there are a lot of false alarms the correct association is not the crucial feature .

It is however possible to keep the approach of [2] in order to analyze and compare estimation errors with PDAF convergence criterion between PDA and PDASI presented in section 4 .

σ_θ generally depends on s ($\sigma_\theta = \sigma_\theta(s)$) ,

(13) becomes $\quad P = 2\ F(\ \Delta N/2\sigma_\theta(s)\ ) - 1 \qquad (20)$

where $\Delta/\sigma_\theta(s) \neq \Delta/\hat{\sigma}_\theta$ ($\hat{\sigma}_\theta = f(s^*) = f(s+b)$) (21)

the same changes affect Λ' , and results are not very different from those shown in FIG. 1

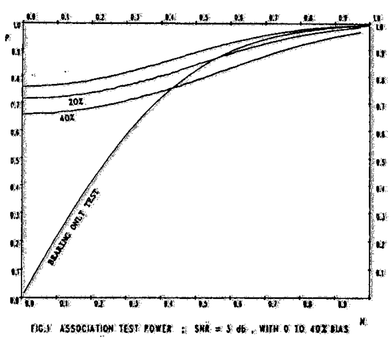

FIG.1 ASSOCIATION TEST POWER : SNR = 3 db , WITH 0 TO 40% BIAS

If we consider the signal estimate is random we can so modify our second test .

We present its results in FIG. 2 with signal estimated within 10 scans , for 0 to 3 db SNR .

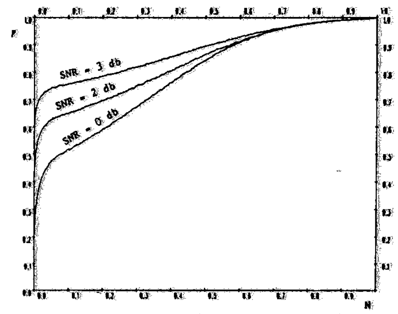

FIG. 2 ASSOCIATION TEST POWER (BEARING AND ESTIMATED ENERGY)

3. Tracks initiation

We first describe the tracks construction as a reduction of the hypothesis number , then tracks selection with MLE .

3.1 Tracks creation

The target motion is assumed LCS (or more generally smooth) so as we can use model (2) with r = 1 . It is possible to bound the distance between the true and approximated models .

Let us consider a n scan frame and a sliding bearing gate : $\theta \in [\theta_c + kT\ \dot{\theta}_c - \rho, \theta_c + kT\ \dot{\theta}_c + \rho]$ where θ_c moves along the bearing space (from 0 to 360° for a cylindrical array) , and $\dot{\theta}_c$ takes arbitrary values (if $\dot{r} = 0$, then $\theta_c = 0$) .

For each couple $(\theta_c, \dot{\theta}_c)$ we consider all possible associations . we say we found a track each time we can choose n_1 ($n_1 \leq n$) points from different scans in the gate . When using sorted bearings , θ_c is naturally sampled , but it is necessary to quantify $\dot{\theta}_c$ with bounds depending on the targets kinematics . The gate is sized with a detection criterion , given Pd for some minimal SNR .

It is possible to increase the number of parameters to be estimated , taking r > 1 , with polynomial approximation , but it implies to increase the frame length .

The number of false tracks created with this method is easily calculated (see Appendix 1) .

3.2 Tracks selection

We obtain a list of tracks : $\gamma = (t_1,\dots,t_m)$ which are not in general consistent (by construction) . Each hypothesis H consists in a subset γ_H of γ . The points not in γ_H are assumed to be false alarms . We thus get a partitioning of the data .

Each track is characterized by its parameters : original bearing θ , $\dot{\theta}$, and SNR s .

Thus the MLE is the Maximum of $L(Z\,/\,H,\theta,\dot{\theta},s)$.

$$\max_{H,\theta,\dot{\theta},s} L(Z/\,H,\theta,\dot{\theta},s) = \max_{H} L(Z/\,H) \qquad (22)$$

$$L(Z/\,H) = \max_{\theta,\dot{\theta},s} L(Z/\,H,\theta,\dot{\theta},s) \qquad (23)$$

Here , $Z = (z_1,..z_p)$ is the set of measurements in the frame , with $z_i = (\theta_i, s_i)$.
Without using signals , the MLE is a linear regression estimate on θ and $\dot{\theta}$. In general , we can write

$$L(\theta_i, s_i / H,\theta,\dot{\theta},s) = L(\theta_i / H,\theta,\dot{\theta},s,s_i)\ L(s_i / H,s) \quad (24)$$

Moreover , the variance of θ_i can be written
$\sigma_{\theta i}{}^2 = g(\theta,s,s_i)$ (25) , as the bearing error depends on the target strength , the corresponding peak when used for conditionning , and the bearing when the array is not cylindrical .
This function can be calculated in broadband , as the (integrated) output signals are gaussian, from their two first moments and the bearing estimation rule .
For each t in γ , we compute

$$l(t) = \max_{\theta,\dot{\theta},s} L(t / \theta,\dot{\theta},s) \quad (26)$$

Then , let $l_1,..,l_m$ be the likelihoods of tracks $t_1,..,t_m$ defined as (26)

for $H = (t_{i1},..t_{i m_H})$, $m_H \le m$,

$$L(Z/ H) = \prod_{j=1}^{m_H} l_{i_j} \cdot \prod_{z \in H} L(z/ H_0) \quad (27)$$

If we consider as tracks the single measurements with $t_{m+k} = \{z_k\}$, $k = 1,..,p$

then for each H , $L(Z/ H) = \prod_{t \in H} L(t/ H)$ (28)

we set $q = m + p$, and $l = (l_1,..,l_q)^t$,
we will conclude as in [3] . The hypothesis H is associated to a vector :
$\tau = (\tau_1,..,\tau_q)^t$, where τ_i is 1 if t_i is in H , and 0 otherwise . Then , $L(Z/ H) = \tau^t.l$ (29)

The problem is to maximize this product , with the constraint (assuming resolved targets) that the tracks are disjoint , which may be written

$$A.\tau \le 1 \quad (30)$$

Where A is the matrix (a_{ij}) , $1 \le i \le p$, $1 \le j \le q$
$a_{ij} = 1$ if z_i is in t_j , 0 otherwise
and 1 stands for the q vector made of 1 .
It is solved with a classical 0-1 Programming algorithm .

Resolution of (25) leads to a real functional equation (with a polynomial approximation for function g in (25)) . A more simple approach (also used for beginning the preceding search) consists in separately estimating s (the mean of s_i in the Gaussian case) , and $(\theta,\dot{\theta})$ as a Linear Regression , with weights as (25) .

Consider the choice between the hypothesis
H1 : { t is a true track } and
H0 : { t is made of false alarms , uniform in $[-\rho\sigma,\rho\sigma]$ }
(so not using the s parameter value) .
Asymptotically (for large n) , if $\theta = 0$, one would choose H1 if

$$\rho \le \sqrt{6 \ln(\Delta/\sigma\sqrt{2\pi})} \quad (31)$$

(31) will generally be realized if Δ is the entire volume , and thus creates too much false tracks .

4. Filtering

Derived from (7) , the PDA Filter equations are Kalman filter's , except the estimated covariance which is increased in order to hold the uncertain correlation contained in Pd and $\beta(k,i)$.
The main approximation of the PDA is that the state estimate is Gaussian , which permits the recursion , since the processes are not Markovian like in Linear Filtering and an optimal solution would thus need all the given data computation .
When using the signal power estimate, we likewise must know its law , exactly or approximately .
The assumption we make , and will question later is that it is Gaussian too .
We can then calculate the parameters :
Let m_k be the number of points in the gate , s_{k-1} the estimated SNR , $X_{k/k-1}$ the predicted target position , P_k its covariance , and let $H_{k,i}$ be the assumption that $z(k) = z_i$. Then ,

$$\beta(k,i) = P(H_{k,i} / m_k, Z_k, X_{k/k-1}, P_k, s_{k-1}) \quad (32)$$

$$= \alpha\, P(Z_k / S_k, H_{k,i}, m_k, X_{k/k-1}, P_k, s_{k-1}) \cdot$$

$$p(S_k / H_{k,i}, m_k, s_{k-1}) \cdot P(H_{k,i}/m_k, s_{k-1}) \quad (33)$$

We assume $Pd = \phi(s)$ (34)
If we use a threshold detection on the (Gaussian) signal ,

$$Pd = \int_{y \ge \xi} N(s,y)\, dy \quad (35)$$

where N(s,y) is the s mean normal law value in y .
s_{k-1} is assumed Gaussian , with covariance $\sigma(k)^2$

$$P(H_{k,i}/m_k, a_{k-1}) = \int_s p(s/s_{k-1})\ P(H_{k,i}/s,m_k) \quad (36)$$

$$= \int_s p(s_{k-1}/s)\ \phi(s)/m_k\, ds \quad , \quad i \ne 0$$

$$\int_s p(s_{k-1}/s)\ [1 - \phi(s)]\, ds \quad , \quad i = 0 \quad (37)$$

The calculation of the second factor of (34) gives

$$P(S_k/H_{k,i}, m_k, s_{k-1}) = e^{-1/2\ [s_{k-1}{}^2/\sigma(k)^2 + s_i{}^2/\sigma_i{}^2]}$$

$$\cdot\ e^{1/2\ \sigma^2\ [s_{k-1}/\sigma(k)^2 + s_i/\sigma_i{}^2]^2} \cdot \sigma/\sqrt{2\pi} \quad (38)$$

with $1/\sigma^2 = 1/\sigma(k)^2 + 1/\sigma_i{}^2$

The first factor is derived as usually , with σ_θ depending on s .

We use as the estimate for s at scan k :

$$k\, s_k + (k-1)s_{k-1} + \sum_i \beta(k,i)\, s_{k,i} \quad (39)$$

We did not assume Pd known but the knowledge of the detection function ϕ in (34) is necessary .
The estimated SNR s_k is asymptotically normal (it can be seen from (39) with the Lindeberg condition for the Central Limit Theorem [4] , for large k).
Moreover, although the estimate s_0 issued from the track initiation algorithm is itself Gaussian , the assumption is not yet true when beginning the β parameters computation (33) . The use of a global estimate for s_k is then required , with for example the study of the (bimodal) signal-noise mixture law , obtained when taking all the points in the successive bearing gates .

5. Simulation Results

We show the detection results for different tracking methods , as the tracks Pd , function of the SNR at a given false tracks rate (Pft) .
Pd is the average percent time of presence for tracks in a gate around the true target position . Pd and Pft were estimated with 100 scans 50 Monte Carlo trials . The tracking input data are the output of a 48 sensors , 100 beams cylindrical array (with a 7.6° 3 db directivity) . The filters are of order 2 , with no model noise .
We compare results for NNA , PDA , and PDASI with our initiation method (frame lengths of 2, 4, 6) and the signal integration curve with a false alarm probability taken equal to Pft .

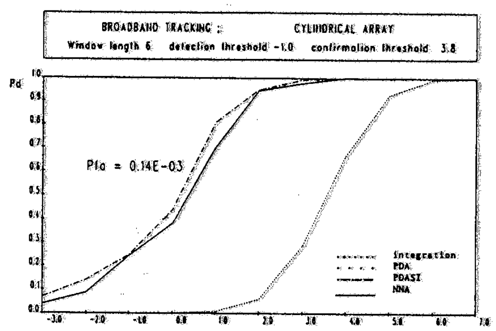

FIG 3 Tracks Detection Curves (NNA ,PDA ,PDASI)

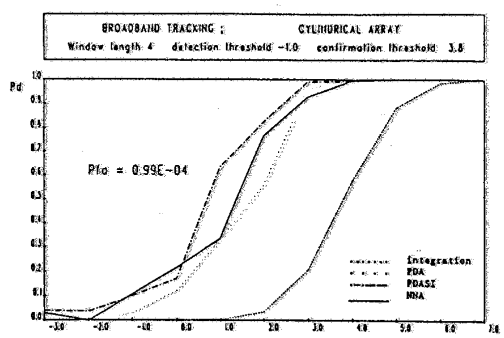

FIG 4 Tracks Detection Curves (NNA ,PDA ,PDASI)

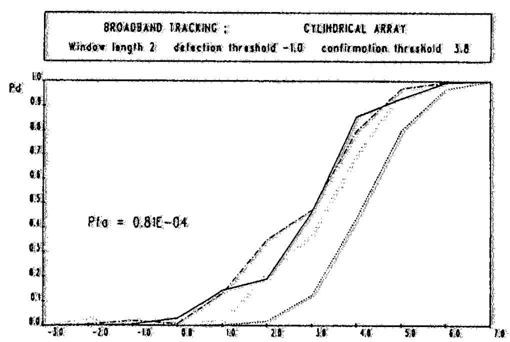

FIG 5 Tracks Detection Curves (NNA ,PDA ,PDASI)

6. Conclusion

We derived both initiation (MLE) and Filtering algorithms for tracking low level targets , and showed the interest for using the SNR parameter in the data association . We saw there is a visible improvement for 1 to 3 db signals, which are hardly detectable with classical methods , with a 3 db gain towards the integration function.

7. Acknowledgement

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8. References

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9. Appendix Tracks statistics calculation

Using the tracks construction presented in 3.1 , we can calculate the number N_t of false tracks .
Assume $n_1 = n-1$, for simplification , and let δ be the gate width , an Δ the entire region .
m_k is the number of points at scan k , with $E(m_k) = \mu_k$, uniformly distributed in Δ .
P_k is the probability that k points from different scans are in some identical gate .

we have $N_t = N_{t1} + N_{t2} - N_{t3}$ (A1)

where N_{t1} is the number of n-sets in some gate
N_{t2} is the number of (n-1)-sets
N_{t3} is the number of (n-1)-sets in N1

then $n_t = E(N_t) = n_{t1} + n_{t2} - n_{t3}$ (A2)

$$n_{t1} = \sum_{m_1} \ldots \sum_{m_n} P(m_1)\ldots P(m_n)\, E(N_{t1} / m_1, \ldots, m_n) \quad \text{(A3)}$$

$$= \mu_1 \ldots \mu_n P_n \quad (= \mu_n P_n \text{ if } \mu_k \text{ is constant })$$

similarly , $n_{t2} = n\, \mu^{n-1} P_{n-1}$ (A4)

We can show that $P_k = k\, \delta^{k-1}/\Delta^{k-1} - (k-1)\delta^k/\Delta^k$ (A5)

$n_{t3} = n_{t2} \cdot Q$, and $Q = 6 \sum_m P(m)$.

$$\int\ldots\int_{0 \le x1 \le \ldots \le xn-1 \le \delta} \Delta^{-3}\ [1 - (1 - (2\delta + x_1 - x_{n-1})/\Delta)^m] \quad \text{(A6)}$$

and assuming $\delta < \Delta$, we get

$$Q \approx 6\, \mu\, \Delta^{-4} \int\ldots\int_{0 \le x1 \le \ldots \le xn-1 \le \delta} (2\delta + x_1 - x_{n-1})\, dx \quad \text{(A7)}$$

The final result is :

$$nt \approx n(n-1)\, \mu^{n-1}\, \delta^{n-2}/\Delta^{n-2}\ (1 - \mu\delta/\Delta) \quad \text{(A8)}$$