Region-of-Interest Imaging in Cone Beam Computerized Tomography

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Abstract

Imaging a sectional region within an object with a detector just big enough to cover the sectional region-of-interest is analyzed. We show that with some suitable choice of scanning configuration and with an innovative method of data combination, all the Radon data can be obtained accurately. The algorithm is mathematically exact, and requires no iterations and no additional measurements. The method can be applied to inspect portions of large industrial objects in industrial imaging, as well as to image portions of human bodies in medical diagnosis.

I. Introduction

Cone beam x-ray computerized Tomography takes much shorter time than scanning the 3-dimensional object slice-by-slice with conventional fan beam x-ray, and promises to generate reconstructed images with sharper contrast, and better spatial resolution between slices. In order to have complete information to reconstruct a 3-dimensional object in cone beam scanning, each plane intersecting the object must intersect an x-ray source. With the complete cone beam data the object can be reconstructed using one of a number of exact algorithms [1,2,3] in the case of small objects that completely fit inside the FOV of the detector at all view angles.

Some objects which are of interest in medical as well as industrial inspections, however, are in the form of a relatively small sectional region in a long object. It is therefore more practical to employ a detector just big enough to cover the sectional region rather than to cover the entire object. However, such arrangement presents serious difficulties for the image reconstruction problem. From the perspective of reconstructing the entire object, some of the cone beam data penetrating portions of the object other than the region-of-interest are missing because of the insufficient size of the detector. From the perspective of reconstructing the region-of-interest, some of the x-ray paths penetrate other portions of the object as well as the region-of-interest, and thus the cone beam data collected no longer represent the region-of-interest exclusively but are corrupted by the overlaying materials. In this paper region-of-interest imaging with a detector just big enough to cover the sectional region is analyzed, and a method to image the region without corrupted or missing data is presented.

II. Data Corruption

Because of the one-to-one correspondence between the points in Radon space and the planes of integration in the object space, the data corruption situation in the former can be analyzed by studying the corresponding situation in the latter. All the planes that intersect the region-of-interest can be classified into the following 3 types:

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1. those that intersect the region-of-interest only;
2. those that intersect the region-of-interest and also either the region above or the region below, but not both;
3. those that intersect the region-of-interest and also both the region above and the region below.

Consider a region-of-interest with a cylindrical support with parallel top and bottom surfaces. The cylinder has height $2b$ and radius a. Take any vertical plane in Radon space containing the z axis. Without loss of generality label the horizontal axis as the y axis. The projection of the cylinder on the vertical plane is a rectangle of dimension $2a$ x $2b$, as illustrated in Figure 2. It can be easily shown from geometry that the boundary of the support of the Radon transform of the region-of-interest on the vertical plane is the curve A shown in Figure 2. In polar coordinate a point (r,θ) in the first quadrant on curve A is given by the equation:

$$r = \sqrt{a^2 + b^2}\cos(\theta - \tan^{-1}\frac{b}{a}) \qquad \theta \in \left[0, \frac{\pi}{2}\right]$$

The curve A in other quadrants are obtained by folding the curve in the first quadrant about the y axis and z axis respectively.

The three regions in Radon space which correspond to these three types of planes are illustrated in Figure 3 for the cylindrically shaped region-of-interest. With otherwise complete (for small objects) cone beam scanning configurations, only the Radon data (more precisely, the radial derivative of the Radon data) corresponding to the first type of planes of integration can be computed without corruption. The region in Radon space corresponding to this type of plane is indicated in the figure and is characterized by the equation:

$$r_{\max} = \sqrt{a^2 + b^2}\cos\left[\tan^{-1}\frac{a}{b} + \frac{\pi}{2} - \theta\right] \tag{1}$$

III. Elimination of Data Corruption

Usually, the cone beam data for the second and third types of planes are corrupted. However, by suitably manipulating x-ray beam coverage from various source positions, it would be possible to avoid data corruption by overlying material and at the same time achieve complete Radon space coverage. Consider the second type of planes. Take any scan path which includes a scan on the top level plane of the region-of-interest, viz. the $z = b$ level, or a scan on the bottom level plane of the region-of-interest, viz. the $z = -b$ level. The scan path on the top or bottom planes can take the form of any closed curve, not necessarily circular nor convex. For the ease of illustration, however, we assume these scan paths to be circular in the discussion below.

Consider the case of the top level, the $z = b$ level. In Figure 4 is illustrated a plane of integration intersecting the region-of-interest and its top surface, but not the bottom surface. When the x-ray source is located at this level, each x-ray path originating from the source intersects either only the region of the object above the $z = b$ level, or only the region below it, or only the $z = b$ level itself; but the path will not intersect more than one of these three regions. Which one of these three regions the path intersects can be identified from the position on the detector the x-ray path intersects. Thus the unwanted contribution of the portion of the object above the $z = b$ level to the computation of Radon derivatives can be eliminated by discarding all the x-ray data whose paths traverse the portion above the $z = b$ level.

With another innovative procedure, even data corruption due to the third type of planes can be eliminated. This is achieved by properly combining the cone beam data from the top scan and the bottom scan for each plane of integration intersecting both the top level and the bottom level of the region-of-interest. The method is illustrated in Figure 5, which features a plane of integration through the object intersecting portions of it above the $z = b$ level and below the $z = -b$ level, together with the two x-ray source positions T and B on the plane which are located on the top scan and the bottom scan respectively. We shall refer to the portion of the plane of integration covered by the cone beam rays emitted at source position T between the top level and the line TB as the partial plane A, and the portion of the plane covered by the cone beam rays emitted at source position B between the bottom level and the line TB shall be referred to as the partial plane B. Note that both partial planes do not intersect the portion of the object above the top level and that below the bottom level, and in combination they constitute the cross section of the region-of-interest intersected by the plane of integration.

If for the source position T only the portion of the cone beam data on the plane between the top level and the line TB is used in computing the Radon derivative datum, one can obtain the radial derivative of the planar integral of the object density on partial plane A. Similarly, for the source position B the computation of the Radon derivative datum using only the cone beam data on the plane generated between the bottom level and the line TB yields the radial derivative of the planar integral of the object density on partial plane B. Their sum gives the radial derivative of the planar integral of the density over the cross section of the region-of-interest. It should be noted that the method of obtaining the Radon transform of the object by combining cone beam data from the top and bottom level scans is possible if the operation which computes the function of the Radon transform from cone beam data is linear and local, such as Radon derivative computation [2,3].

The algorithm is mathematically exact, and requires no iterations and no additional measurements. The size of objects to be inspected with cone beam CT is no longer limited by the detector dimension.

References:

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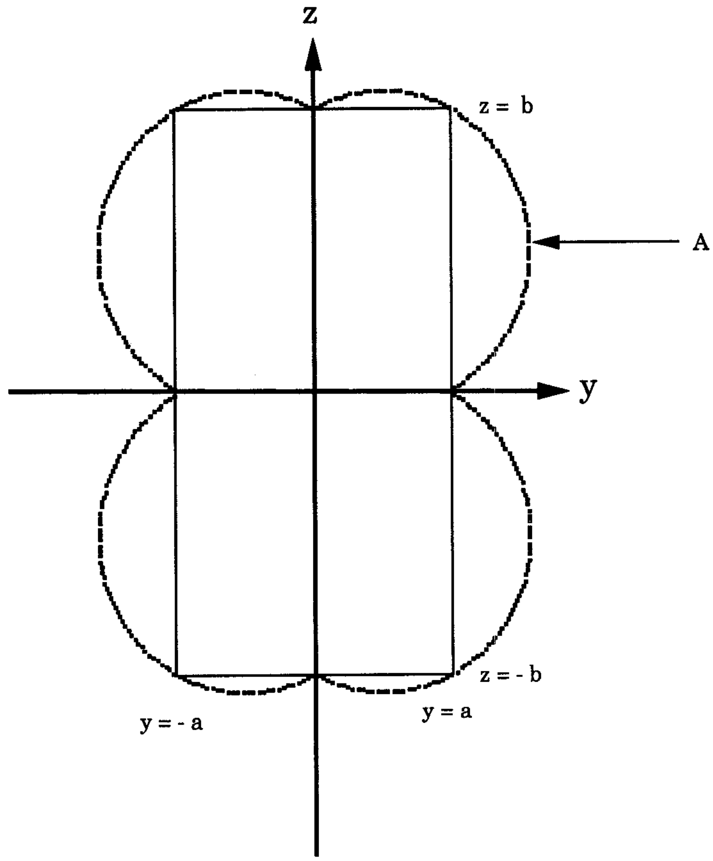

Figure 1. The vertical middle cross section of the Radon transform of the cylindrical region-of-interest.

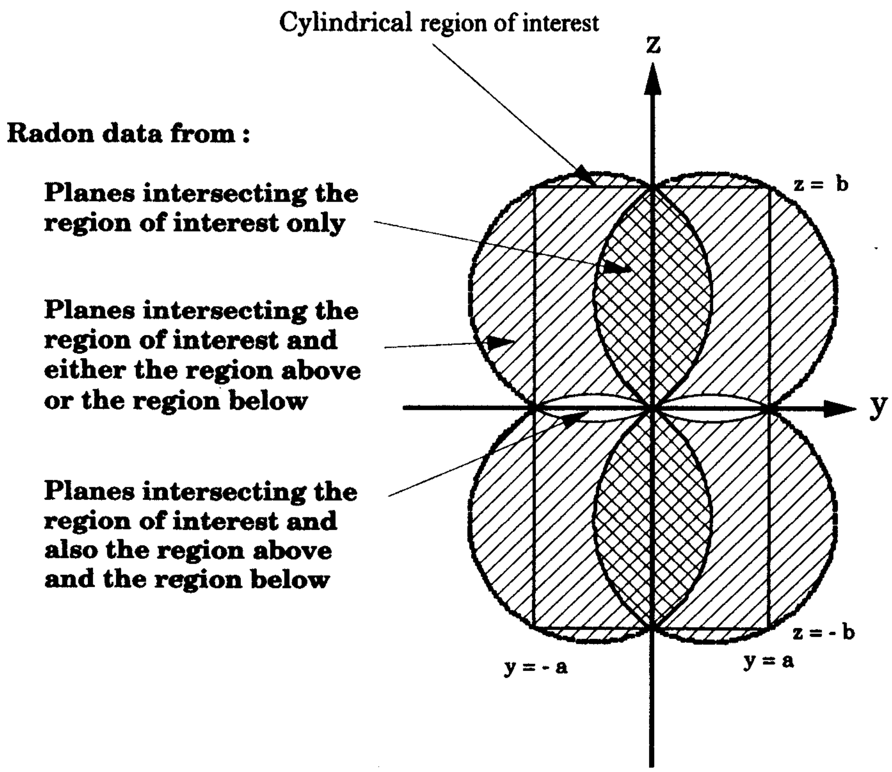

Figure 2. Radon data corresponding to: *a*. the planes that intersect the region-of-interest only; *b*. the planes that intersect the region-of-interest and also either the region above or the region below, but not both; and *c*. the planes that intersect the region-of-interest and also both the region above and the region below.

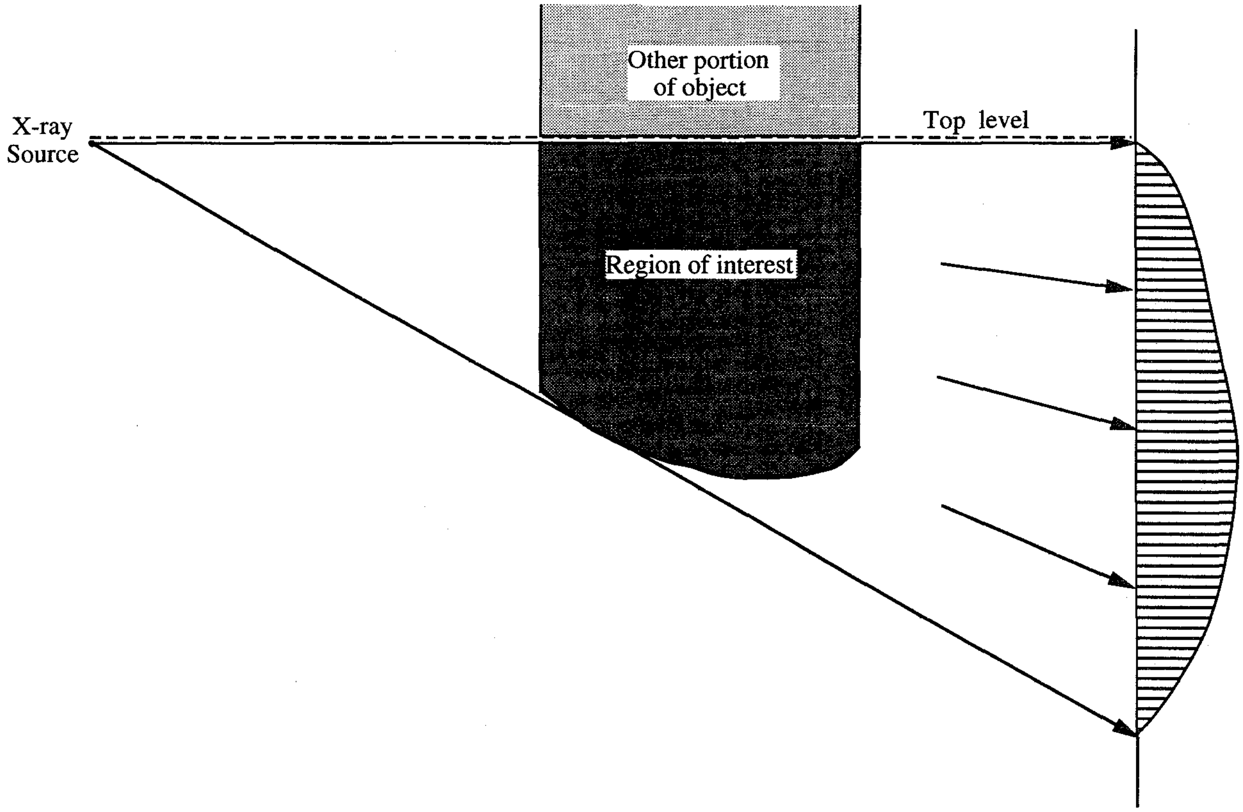

Figure 3. Eliminating cone beam data contamination for the planes that intersect the region-of-interest and also the region above.

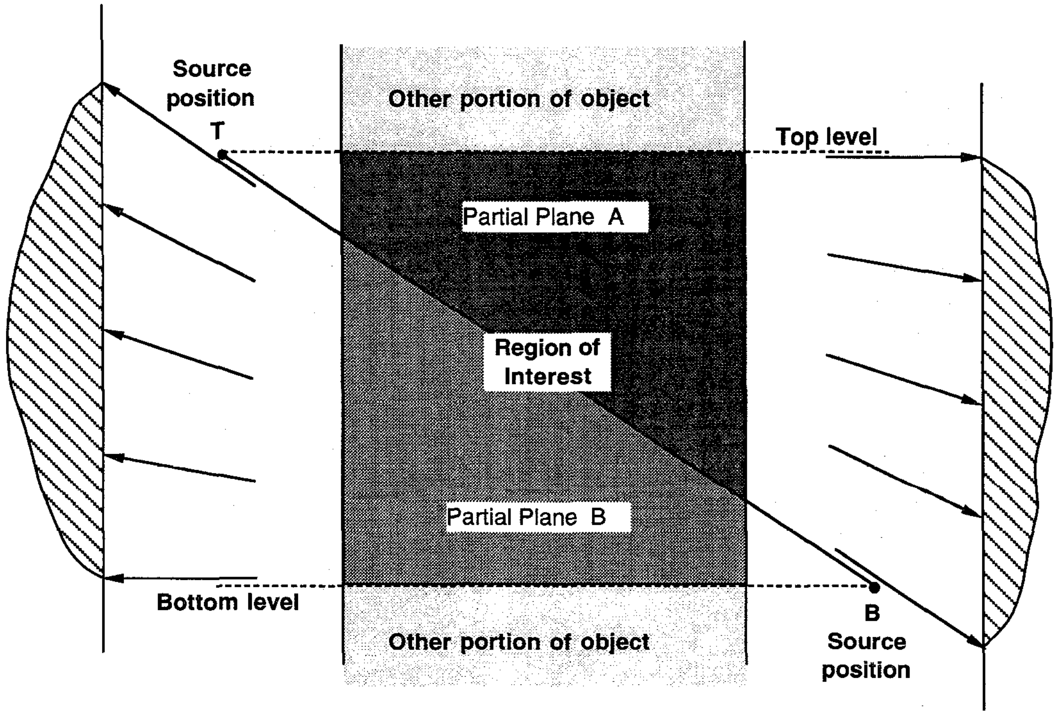

Figure 4. Eliminating cone beam data contamination for the planes that intersect the region-of-interest and also both the region above and the region below. This is achieved by suitably combining cone beam data from the top and the bottom scans.