Forecasts which we can easily check follow

and the corresponding h-step-ahead forecast error is

$$e_{t+h,t} = y_{t+h} - y_{t+h,t} = \varepsilon_{t+h} + b_1 \varepsilon_{t+h-1} + \dots + b_{h-1} \varepsilon_{t+1},$$

with variance

$$\sigma_h^2 = \sigma^2 \left(1 + \sum_{i=1}^{h-1} b_i^2 \right).$$

The key property of optimal forecast errors, from which all others follow (including those cataloged below), is that they should be unforecastable on the basis of information available at the time the forecast was made. This **unforecastability principle** is valid in great generality; it holds, for example, regardless of whether linear-projection optimality or conditional-mean optimality is of interest, regardless of whether the relevant loss function is quadratic, and regardless of whether the series being forecast is stationary.

Many tests of aspects of optimality are based on the unforecastability principle. 1-step-ahead errors, for example, had better be white noise, because otherwise we could forecast the errors using information readily available when the forecast is made. Indeed at least four key properties of optimal forecasts, which we can easily check, follow immediately from the unforecastability principle:

a. Optimal forecasts are unbiased

b. Optimal forecasts have 1-step-ahead errors that are white noise

c. Optimal forecasts have h-step-ahead errors that are at most $MA(h-1)$

d. Optimal forecasts have h-step-ahead errors with variances that are non-decreasing in h and that converge to the unconditional variance of the process.

11.1.1 Are errors zero-mean?

If the forecast is unbiased, then the forecast error has a zero mean. A variety of tests of the zero-mean hypothesis can be performed, depending on the assumptions we're willing to maintain. For example, if $e_{t+h,t}$ is Gaussian white noise (as might be reasonably the case for 1-step-ahead errors), then the standard t-test is the obvious choice. We would simply regress the forecast error series on a constant and use the reported t-statistic to test the hypothesis that the population mean is zero. If the errors are non-Gaussian but remain iid, then the t-test is still applicable in large samples.

If the forecast errors are dependent, then more sophisticated procedures are required. We maintain the framework of regressing on a constant, but we must "correct" for any serial correlation in the disturbances. Serial correlation in forecast errors can arise for many reasons. Multi-step-ahead forecast errors will be serially correlated, even if the forecasts are optimal, because of the forecast-period overlap associated with multi-step-ahead forecasts. More generally, serial correlation in forecast errors may indicate that the forecasts are suboptimal. The upshot is simply that when regressing forecast errors on an intercept, we need to be sure that any serial correlation in the disturbance is appropriately modeled. A reasonable starting point for a regression involving h-step-ahead forecast errors is $MA(h-1)$ disturbances, which we'd expect if the forecast were optimal. The forecast may, of course, *not* be optimal, so we don't adopt $(h-1)$ disturbances uncritically; instead, we try a variety of models using the AIC and SIC to guide selection in the usual way.

11.1.2 Are 1-step-ahead errors white noise?

Under various sets of maintained assumptions, we can use standard tests of the white noise hypothesis. For example, the sample autocorrelation and partial autocorrelation functions, together with Bartlett asymptotic standard errors, are often useful in that regard. Tests based on the first autocorrelation (e.g., the Durbin-Watson test), as well as more general tests, such as the Box-Pierce and Ljung-Box tests, are useful as well.

11.1.3 Are h-step-ahead errors are at most $MA(h-1)$?

The $(h-1)$ structure implies a cutoff in the forecast error's autocorrelation function beyond displacement $h-1$. This immediately suggests examining the statistical significance of the sample autocorrelations beyond displacement $h-1$ using the Bartlett standard errors. In addition, we can regress the errors on a constant, allowing for $MA(q)$ disturbances with $q > (h-1)$, and test whether the moving-average parameters beyond lag $h-1$ are zero.

11.1.4 Are h-step-ahead error variances non-decreasing in h?

It's often useful to examine the sample h-step-ahead forecast error variances as a function of h, both to be sure they're non-decreasing in h and to see their *pattern*, which may convey useful information.

11.1.5 Are errors orthogonal to available information?

The tests above make incomplete use of the unforecastability principle, insofar as they assess only the *univariate* properties of the errors. We can make a more complete assessment by broadening the information set and assessing optimality with respect to various sets of information, by estimating regressions of the form

$$e_{t+h,t} = \alpha_0 + \sum_{i=1}^{k} \alpha_i x_{it} + u_t.$$

The hypothesis of interest is that all the α's are zero, which is a necessary condition for forecast optimality (orthogonality) with respect to available

information.

The particular case of testing optimality with respect to $y_{t+h,t}$ is very important in practice. (Note that $y_{t+h,t}$ is obviously in the time-t information set.) The relevant regression is

$$e_{t+h,t} = \alpha_0 + \alpha_1 y_{t+h,t} + u_t,$$

and optimality corresponds to $(\alpha_0, \alpha_1) = (0, 0)$.

If the above regression seems a little strange to you, consider what may seem like a more natural approach to testing optimality, regression of the realization on the forecast:

$$y_{t+h} = \beta_0 + \beta_1 y_{t+h,t} + u_t.$$

This is called a "**Mincer-Zarnowitz regression**." If the forecast is optimal with respect to the information used to construct it, then we'd expect $(\beta_0, \beta_1) = (0, 1)$, in which case

$$y_{t+h} = y_{t+h,t} + u_t.$$

Note, however, that if we start with the regression

$$y_{t+h} = \beta_0 + \beta_1 y_{t+h,t} + u_t,$$

and then subtract $y_{t+h,t}$ from each side, we obtain

$$e_{t+h,t} = \alpha_0 + \alpha_1 y_{t+h,t} + u_t,$$

where $(\alpha_0, \alpha_1) = (0, 0)$ when $(\beta_0, \beta_1) = (0, 1)$. Thus, the two approaches are identical. We can regress the error on an intercept and the forecast and test (0, 0), or we can regress the realization on an intercept and the forecast and test (0, 1).

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