**ECE 3522: Stochastic Processes in Signals and Systems**

# Computer Assignment (CA) No. 13: Maximum Likelihood Decoding

In this assignment, we will build a simulation of a pattern recognition system that classifies data using a simple maximum likelihood classifier. The process we will follow is this:

* Generate two classes of data using multivariate Gaussian distributions.
* Transform the data using principal components analysis.
* Classify the data using a maximum likelihood decoder.
* Compute the corresponding error rate.

We will work in two dimensions so that you can check your results with the pattern recognition applet. We will assume each class is equally likely to keep things simple.

(1) Write a function that generates *N* vectors of dimension two. The inputs to this function are a mean vector, a covariance matrix, and the number of points, N. The output, ***X***, is an *Nx2* matrix:

**X** = get\_grvs(*μ*, ***C***, *N*)

Generate two classes of data, *ω1* and *ω2*, with the following parameters:



Plot a scatter plot of the data for *N* = 10,000 and *σ2*= 1.0. Explain why your plot makes sense.

(2) Compute the covariance matrix for each class. Verify that it matches what you specified.

(3) Transform the data using Principal Components Analysis. Compute a transformation matrix, ***V1***, for class 1, transform ***X1*** into ***Y1***, and compute the covariance of ***Y1***. Verify that it is an identity matrix. Repeat for class 2.

(4) Transform ***μ1*** using ***V1*** and compare this to the mean of ***Y1***. Repeat for ***μ1*** *and* ***Y2***. Justify your observations.

(5) For each vector in ***Y1***, compute the distance from ***μ1*** and ***μ2*** and choose the class corresponding to the smaller distance. Count this as an error every time class 2 is chosen. Repeat this for class 2, and count as an error any time a vector from class 2 is closer to the mean of class 1. The total error rate is the sum of these two counts divided by the total number of points.

(6) Repeat steps (1)-(5) for  and *N = 10,000*. Plot the error rate as a function of . Explain whether your plot makes sense. Draw a diagram similar to what you observed in the Java applet and explain what you have accomplished in this assignment. (Note: we have used a slightly different approach in this assignment where each class is modeled by its own covariance matrix. This is known as class-dependent Principal Components Analysis.)

Describe how this assignment relates to the problem of sending digital data over a communications channel. What would happen if you used four classes instead of two?