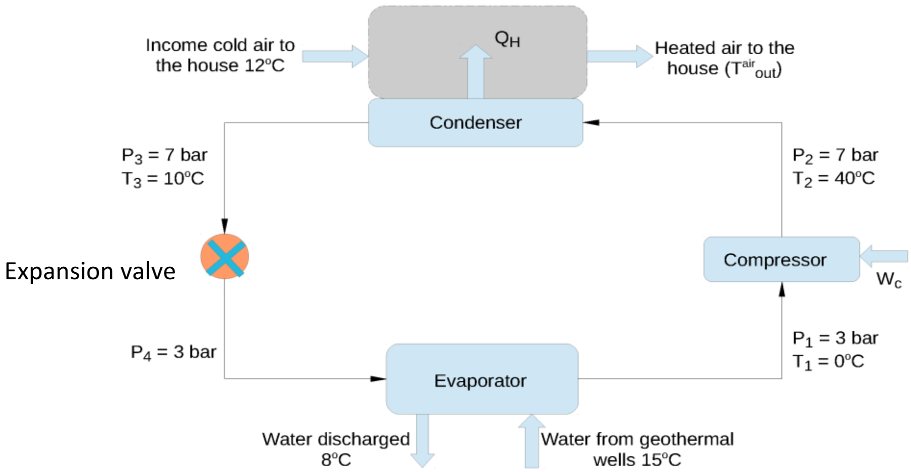
RESIT PAPER

PROBLEM 1

An engineer decided to use a geothermal source from the yard to keep her Scottish house warm during the winter. The designed heat pump extracts water from the well at 15°C and discharges at 8°C. As working fluid, she decided to use propane (C_3) - see Figure below, that will transfer heat into a constant stream of cold air (m_{air} = 2 kg/s, $C_{p,air}$ = 1.004 kJ/(kg.K)) at 12°C.



a) Calculate enthalpies and entropies of streams 1-4. **[8 Marks]**

b) For a mass flow rate of C_3 (m_{C3}) of 0.1 kg/s, calculate the required water flow rate (m_W). The heat capacity (C_P) of water is 4.1813 kJ/(kg.K). **[2 Marks]**

c) Assuming that all heat extracted in the condenser is transferred to the air stream (m_{air} = 2 kg/s), calculate the temperature of this heated stream $\left(T_{\text{out}}^{\text{air}}\right)$. **[5 Marks]**

d) Nearby the engineer's house, a geothermal reservoir was mapped and the following data was gathered,

Temperature (°C)	25	40	63	100	155	245
Depth (m)	0	200	400	600	800	1000

i. Calculate the temperature gradient of this reservoir. **[2 Marks]**

ii. Binary cycle geothermal power plants operate at a temperature above 100°C. Assuming there is no heat losses in the production well, what is the ideal depth for a source geothermal fluid of 143°C?

[3 Marks]

iii. Describe how binary cycle geothermal power plants operate. **[5 Marks]**

To solve this problem, you should assume that the saturated liquid streams are incompressible, and therefore *dh = vdP* (where h, v and P are specific enthalpy, specific volume and pressure, respectively). Quality of the vapour stream is expressed as,

$$x_j = \frac{\psi_i - \psi_f}{\psi_g - \psi_f} \quad \text{with } \psi = h, s$$

where s is the specific entropy.

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Solution: (a)

Stage 1: is at $P_1 = 3\ bar\ and\ T_1 = 0℃ \Rightarrow T_{sat} = -14.16℃ < T_1, thus\ C_3\ is\ at$
[1/8] *superheated state (SHF). Thus, from SHF table,* $\boldsymbol{h_1 = 477.1\ kJ.kg^{-1}}$ *and*
[1/8] $\mathbf{s_1 = 1.851 kJ.(kg.K)^{-1}}$

Stage 2: $At\ P_2 = 7\ bar\ and\ T_2 = 40℃ \Rightarrow T_{sat} = 13.41℃ < T_2, thus\ C_3$ is at
[1/8] superheated state (SHF). Thus, from SHF table $\boldsymbol{h_2 = 534.8\ kJ.kg^{-1}}$ and
[1/8] $\boldsymbol{s_2 = 1.901 kJ.(kg.K)^{-1}}$.

Stage 3: $At\ P_3 = 7\ bar\ and\ T_3 = 10℃ \Rightarrow T_{sat} = 13.41℃ > T_2, thus\ C_3$ is
[1/8] a sub-cooled fluid. Thus, from saturated table, $\boldsymbol{h_3 = 129.6\ kJ.kg^{-1}}$ and
[1/8] $\boldsymbol{s_2}$**=0.495** **kJ**$(\boldsymbol{kg.K})^{-1}$.

[1/8] **Stage 4:** At
$P_4 = 3\ bar \Rightarrow Isenthalpic\ expansion, thus\ \boldsymbol{h_4 = h_3 = 129.6\ kJ.kg^{-1}}$.
In order to calculate the entropy, we first need to calculate the quality of the fluid,

$$x_4 = \frac{h_4 - h_f}{h_g - h_f} = \frac{129.6 - 60.3}{453.6 - 60.3} = 0.1762$$

$$x_4 = 0.1762 = \frac{s_4 - s_f}{s_g - s_f} = \frac{S_4 - 0.244}{1.762 - 0.244} \Rightarrow \boldsymbol{s_4 = 0.5115 \frac{kJ}{kg.K}}$$

[1/8]

Solution: (b)

The heat exchange in the evaporator can be expressed as:

$$-\dot{m}_{C3}(h_1 - h_4) = (h_w^{out} - h_w^{in}) = \dot{m}_w C_{p,w}\left(T_w^{out} - T_w^{in}\right)$$

$$-10^{-1}(477.1 - 129.6) = \dot{m}_w \times 4.1813 \times (8 - 15) \Rightarrow \boldsymbol{\dot{m}_w = 1.1873\ \frac{kg}{s}}$$

[2/2]

Solution: (c)

The heat exchange in the condenser is expressed as

$$\boldsymbol{Q_H} = \dot{m}_{C3}(h_2 - h_3) = \boldsymbol{40.52 \frac{kJ}{kg}}$$

[2/5] *Assuming that there is no heat loss,*

$$Q_4 = 40.52 = \dot{m}_{air} C_{p,air}\left(T_{out}^{air} - T_{in}^{air}\right) = 2 \times 1.004 \times \left(T_{out}^{air} - 12\right) \Rightarrow \boldsymbol{T_{out}^{air} = 32.18}℃$$

[3/5]

Solution: (d - i)

$$\boldsymbol{\nabla T} = \frac{\partial T}{\partial z} = \frac{T_n - T_1}{z_n - z_1} = \frac{245 - 25}{1000 - 0} = \boldsymbol{0.22 \frac{℃}{m}}$$

[2/2]

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Solution: (d – ii)

[3/3] The depth can be obtained via linear interpolation at $100 \leq T \leq 155℃. At$ 143℃ the depth is **756.36m**

Solution: (d – iii)

- Binary cycle geothermal power plants are often operated between $107 \leq T \leq 182℃$;

[1/5]

- The produced hot brine vaporises the working fluid and is isentropically expanded in the turbine;

[2/5]

- Working fluids often used in geothermal plants are organic chemical species with low boiling temperature.

[2/5]