Effectiveness Concept for Heat Exchangers

The Design Equation for a Heat Exchanger

$$Q_H = UA\frac{\left(\Delta T_2 - \Delta T_1\right)}{\ln\left(\frac{\Delta T_2}{\Delta T_1}\right)} = UA\Delta T_{lm}$$

A typical problem in the analysis of a heat exchanger is the Performance calculation. That is, we are asked , given inlet conditions to evaluate how the exchanger performs, that is what are the outlet temperatures. With the equation given above, the solution may be reached only by trial-and-error.

Effectiveness

An alternate approach lies in the notion of exchanger effectiveness, E.

$$E = \frac{\text{actual heat transfer}}{\text{maximum possible heat transfer}}$$

Overall Energy Balance

The actual heat transfer is given by the energy balance

$$(wC_p\,\Delta T)_{hot} = (wC_p\Delta T)_{cold}$$

The maximum possible temperature rise is the difference between the temperatures of the two entering streams (T_{hin} - T_{cin}). Which fluid undergoes the maximum temperature rise ? Of course, it is the one with the least heat capacitance

$$(wC_p)_{min}\,(\Delta T)_{max} = (wC_p)_{max}\,(\Delta T)_{min}$$

It follows then that

$$Q_{max} = (wC_p)_{min}\,(T_{hin} - T_{cin})_{max}$$

Definitions of Effectiveness

For the Double-Pipe Heat Exchanger there are four possible cases:

	Co-Current	Counter-Current
Hot Fluid minimum	Case 1	Case 3
Cold Fluid minimum	Case 2	Case 4

Case 1- Co-Current flow, hot fluid minimal

$$E = \frac{(wC_p)_h (T_{h1} - T_{h2})}{(wC_p)_h (T_{h1} - T_{c1})} = \frac{(T_{h1} - T_{h2})}{(T_{h1} - T_{c1})}$$

Case 2- Co-Current flow, cold fluid minimal

$$E = \frac{(wC_p)_c (T_{c2} - T_{c1})}{(wC_p)_c (T_{h1} - T_{c1})} = \frac{(T_{c2} - T_{c1})}{(T_{h1} - T_{c1})}$$

Case 3- Counter-Current flow, hot fluid minimal

$$E = \frac{(wC_p)_h (T_{h1} - T_{h2})}{(wC_p)_h (T_{h1} - T_{c2})} = \frac{(T_{h1} - T_{h2})}{(T_{h1} - T_{c2})}$$

Case 4- Counter-Current flow, cold fluid minimal

$$E = \frac{(wC_p)_c (T_{c1} - T_{c2})}{(wC_p)_c (T_{h1} - T_{c2})} = \frac{(T_{c1} - T_{c2})}{(T_{h1} - T_{c2})}$$

Number of Transfer Units

Recall the definition of the ratio of thermal capacitances

$$R = \frac{(WC_p)_c}{(WC_p)_h} = \frac{C_c}{C_h} = \frac{T_{h1} - T_{h2}}{T_{c2} - T_{c1}}$$

Also we can reexamine the Design Equation and rewrite it in the following form:

$$\ln\left(\frac{(T_{h2} - T_{c2})}{(T_{h1} - T_{c1})}\right) = -\frac{UA}{C_c}(1 + R)$$

or

$$\frac{(T_{h2} - T_{c2})}{(T_{h1} - T_{c1})} = e^{-\frac{UA}{C_c}(1+R)}$$

We need to express this temperature ratio in terms of the effectiveness, E.

A good deal of algebra leads for Case 1 to

$$E = \frac{1 + e^{-\frac{UA}{C_c}(1+R)}}{1 + R}$$

For case 2 the equation is the relation is very similar and indeed would be the same if R were replaced by $R_{min} = C_{min}/C_{max}$.

$$E = \frac{1 - e^{-\frac{UA}{C_{min}}(1+R_{min})}}{1 + R_{min}}$$

For case 3 and case 4, the equation can be expressed as a single relation.

$$E = \frac{1 - e^{-\frac{UA}{C_{min}}\left(1 - R_{min}\right)}}{1 + R_{min}e^{-\frac{UA}{C_{min}}\left(1 - R_{min}\right)}}$$

We can define a dimensionless group as the Number of Transfer Units (NTU)

$$NTU = \frac{UA}{C_{min}}$$

This whole concept can be extended to all kinds of exchanger configurations, e.g.,.shell and tube with n tube passes and one shell pass; a cross-flow exchanger.

Cross-flow Heat Exchangers

Types -	Mixed Un-Mixed
Example	Automobile radiator

Cross-flow
Mixed/Unmixed Exchanger
unmixed stream- minimal stream

$$E = R\left\{1 - \exp\left(-R\left[1 - e^{-NTU}\right]\right)\right\}$$

mixed stream- minimal stream

$$E = 1 - \exp\left(-R\left[1 - e^{-RNTU}\right]\right)$$

Unmixed/Unmixed Exchanger
Cross-flow

$$E = 1 - \exp\left[R(NTU)^{0.22} \left(\exp\left[-R(NTU)^{0.78} \right] - 1 \right) \right]$$