

Solution of the Problems – Exam (May 2012/13)

Question 1: Reheat Rankine cycle with two turbines:

- (a) In order to fill the Table we need to calculate the thermodynamic properties for each stage of the cycle

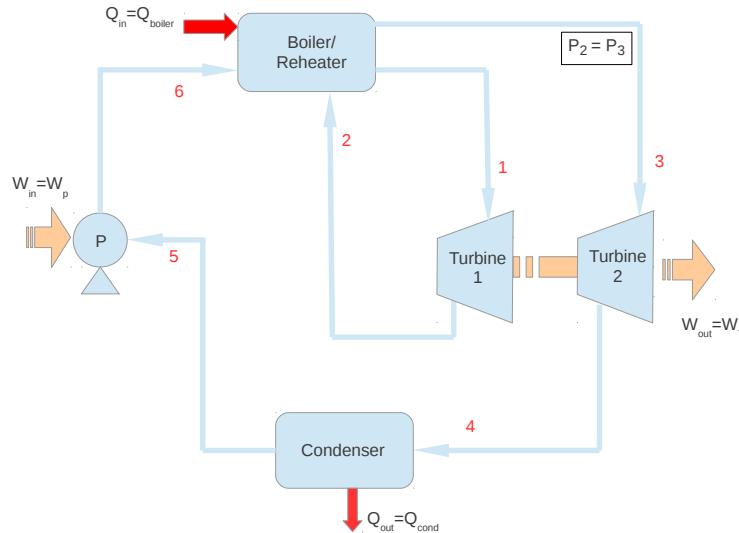


Figure 1: Reheat Rankine cycle with 2 turbines.

Stage 1: The fluid leaving the boiler towards the first turbine is at 40 bar and 370°C. This is well above the saturation temperature ($T_{\text{sat}} = 250.3^\circ\text{C}$) and we can thus confirm (as outlined in the given Table) that the fluid is superheated steam. At such pressure, the superheated steam tables (SHST) give,

T (°C)	H (kJ/kg)	S (kJ/(kg.K))
350	3092.5	6.582
400	3213.6	6.769

Thus, with linear interpolation at $T_1 = 370^\circ\text{C}$ for H_1 and S_1 ,

$$H_i = H_0 + \frac{T_i - T_0}{T_1 - T_0} (H_1 - H_0) = 3092.5 + \frac{370 - 350}{400 - 350} (3213.6 - 3092.5)$$

or

$$H_i = H_1 - \frac{T_1 - T_i}{T_1 - T_0} (H_1 - H_0) = 3213.6 - \frac{400 - 370}{400 - 350} (3213.6 - 3092.5)$$

This leads to $H_1 = 3140.94 \frac{\text{kJ}}{\text{kg}}$ (1 Mark) and $S_1 = 6.6568 \frac{\text{kJ}}{\text{kg.K}}$ (1 Mark).

Stage 2: Isentropic expansion in *Turbine 1* at $P_2 = P_3 = 7 \text{ bar} \Leftrightarrow S_{2s} = S_1 = 6.6568 \frac{\text{kJ}}{\text{kg.K}}$. The fluid can be either **saturated steam** or **2 phase (steam + liquid) fluid** (1 Mark). From the saturated water-steam table (SWST),

	(kJ/kg)		(kJ/(kg.K))
H_f	697.1	S_f	1.9918
H_g	2762.0	S_g	6.7052

We need to calculate the quality of the ideal fluid, X_{2s} ,

$$X_{2s} = \frac{S_{2s} - S_f}{S_g - S_f} = 0.9897$$

Now that we know X_{2s} , we can determine the ideal enthalpy,

$$X_{2s} = 0.9897 = \frac{H_{2s} - H_f}{H_g - H_f} \Leftrightarrow H_{2s} = 2740.73 \frac{kJ}{kg}$$

The efficiency associated with *Turbine 1* is 84% thus,

$$\eta_{T1} = \frac{H_2 - H_1}{H_{2s} - H_1} = 0.84 \Leftrightarrow H_2 = 2804.76 \frac{kJ}{kg}$$

Stage 3: The reheated steam is at 7 bar ($T_{sat} = 165^\circ\text{C}$) and 370°C is also superheated and from SHST,

T ($^\circ\text{C}$)	H (kJ/kg)	S (kJ/(kg.K))
350	3163.7	7.473
400	3268.7	7.635

From linear interpolation, at $T_1 = 370^\circ\text{C}$: $H_3 = 3205.7 \frac{kJ}{kg}$ (1 Mark) and $S_3 = 7.5378 \frac{kJ}{kg.K}$ (1 Mark)

Stage 4: The fluid that leaves the second turbine is a saturated steam at 0.10 bar with $S_{4s} = S_3 = 7.5378 \frac{kJ}{kg.K}$. At this pressure the SWST gives

$$\begin{aligned} V_f &= 0.00101 \frac{m^3}{kg} & H_f &= 191.8 \frac{kJ}{kg} & S_f &= 0.649 \frac{kJ}{kg.K} \\ V_g &= 14.67 \frac{m^3}{kg} & H_g &= 2584.7 \frac{kJ}{kg} & S_g &= 8.150 \frac{kJ}{kg.K} \end{aligned}$$

Calculating the quality of the fluid, X_{4s} ,

$$X_{4s} = \frac{S_{4s} - S_f}{S_g - S_f} = 0.9184$$

Now calculating the enthalpy of the ideal fluid,

$$X_{4s} = \frac{H_{4s} - H_f}{H_g - H_f} \Leftrightarrow H_{4s} = 2389.35 \frac{kJ}{kg}$$

For *Turbine 2*, the associated efficiency is of 80%,

$$\eta_{T2} = \frac{H_4 - H_3}{H_{4s} - H_3} = 0.80 \Leftrightarrow H_4 = 2552.69 \frac{kJ}{kg}$$

Stage 5: The fluid after the condenser is a **saturated water (liquid)** (1 Mark) at 0.10 bar with the following characteristics:

$$H_5 = 191.8 \frac{kJ}{kg} = H_f \quad (1 \text{ Mark})$$

$$S_5 = 0.649 \frac{kJ}{kg.K} = S_f \quad (1 \text{ Mark})$$

$$V_5 = 0.00101 \frac{m^3}{kg} = V_f$$

Stage 6: Finally, the fluid leaving the boiler feed pump is a **saturated water (liquid)** (1 Mark) at 40 bar. The pump has an efficiency of 61% and assuming that the water is nearly incompressible,

$$\begin{aligned} H_6 &\approx H_5 + V_5 \frac{P_6 - P_5}{\eta_P} \\ &\approx 191.8 \frac{kJ}{kg} + 0.00101 \frac{m^3}{kg} (40 - 0.10) \text{ bar} \times \frac{10^5 kg / (m.s^2)}{1 \text{ bar}} \times \frac{10^{-3} \frac{kJ}{kg}}{m^2/s^2} \times \frac{1}{0.61} \\ &\approx 198.41 \frac{kJ}{kg} \quad (1 \text{ Mark}) \end{aligned}$$

Thus the Table becomes:

Stage	P (bar)	T (°C)	State	H (kJ.kg ⁻¹)	S (kJ.(kg.K) ⁻¹)
1	40	370	superheated steam	(a) 3140.94	(b) 6.6568
2	—	—	(c) saturated steam	—	—
3	7	370	superheated steam	(d) 3205.7	(e) 7.5358
4	0.10	—	—	—	—
5	0.10	—	(f) saturated liquid (or water)	(g) 191.8	(h) 0.649
6	40	—	(i) saturated liquid (or water)	(j) 198.41	—

(b) The thermal efficiency of the cycle is expressed by

$$\eta_{\text{Thermal}} = \frac{(H_1 - H_{2s}) \eta_{T1} + (H_3 - H_{4s}) \eta_{T2} - V_5 (P_6 - P_5) \eta_P^{-1}}{(H_1 - H_6) + (H_3 - H_2)}$$

Before determine η_{thermal} , the following should have been calculated,

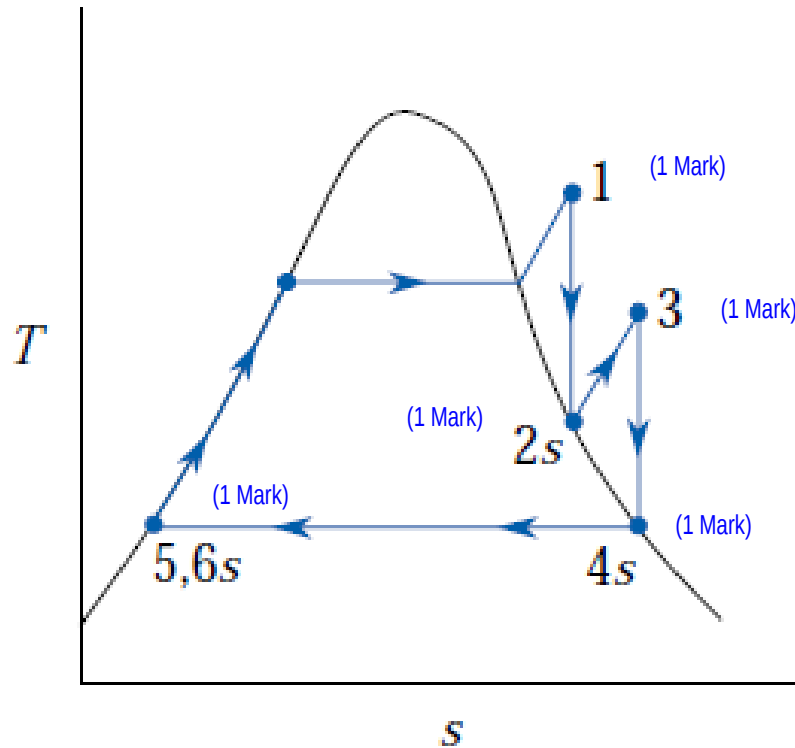
$$H_{2s} = 2740.73 \frac{kJ}{kg} \quad (1 \text{ Mark}) \quad H_2 = 2804.76 \frac{kJ}{kg} \quad (1 \text{ Mark})$$

$$H_{4s} = 2389.35 \frac{kJ}{kg} \quad (1 \text{ Mark}) \quad V_5 = 0.00101 \frac{m^3}{kg} \quad (1 \text{ Mark})$$

Now calculating η_{Thermal} ,

$$\begin{aligned} \eta_{\text{Thermal}} &= \frac{(3140.94 - 2740.73) 0.84 + (3205.7 - 2389.35) 0.80 - \frac{0.00101 (40 - 0.10) 10^5 10^{-3}}{0.61}}{(3140.94 - 198.41) + (3205.7 - 2804.76)} \\ &= 0.2939 \quad (1 \text{ Mark}) \end{aligned}$$

(c) Sketch of the Ts diagram



Question 2:

(a)

$$421 \text{ billion kWh} = 421 \times 10^{12} \text{ J.s}^{-1} \times 3600 \text{ s} = 1.5 \times 10^{18} \text{ J of electricity.}$$

Efficiencies of generation depend on turbine performance not on whether nuclear and or chemical fuels were used, so the electricity above must have been obtained from about:

$$(1.5 \times 10^{18} / 0.35) \text{ J of heat} = 4.3 \times 10^{18} \text{ J of heat}$$

If this had been raised from natural gas, the carbon dioxide release would have been:

$$[4.3 \times 10^{18} \text{ J} / (889103 \text{ J.mol}^{-1})] \times 0.044 \text{ kg.mol}^{-1} \times 10^{-3} \text{ tonne.kg}^{-1} = 213 \text{ million tonnes}$$

- (b) A supertanker, carrying 1 million barrels of oil, would if it blew up have a blast equivalent to that from the atomic bomb at Hiroshima. A blast of this magnitude was actually observed at a refinery fire in Venezuela in 2012 (either of these).
- (c) Rate of supply of coke = $[300 \times 10^6 \text{ J.s}^{-1} / (25 \times 10^6 \text{ J.kg}^{-1})] = 12 \text{ kg.s}^{-1}$
rate of production of carbon dioxide = $12 \text{ kg.s}^{-1} \times (44/12) \times 7 \times 24 \times 3600 \times 0.001 \text{ tonne per week}$
= 26611 tonne per week.

For 10% mitigation of the carbon 10% of the heat must come from the citrus peel.

10.8 kg.s⁻¹ of coke plus:

$$(1.2 \times 25/7) \text{ kg.s}^{-1} \text{ of citrus peel} = 4.3 \text{ kg.s}^{-1} \text{ of citrus peel}$$

$$\text{Ratio coke to citrus peel} = (10.8/4.3) = 2.5$$

- (d) Suppliers are monitored by the Forest Stewardship Council (FSC) for replacement of trees felled with new plantings.

Question 3: (a) The inlet has circumference 1 m. Therefore the radius of the inlet $r_1 = 1/2\pi = 0.15915$ m and the area of the inlet $A_1 = \pi r_1^2 = 0.07958$ m².

Similarly the outlet has circumference 0.6 m. Therefore the radius of the outlet $r_2 = 0.6/2\pi = 0.09549$ m and the area of the outlet $A_2 = \pi r_2^2 = 0.02865$ m². **[1 Mark of 4]**

Evaluating the mass flux at the inlet gives

$$\rho_1 = \frac{\dot{m}}{u_1 A_1} = \frac{4 \text{ kg s}^{-1}}{30 \text{ m s}^{-1} \times 0.07958 \text{ m}^2} = 1.6755 \text{ kg m}^{-3}.$$

[1 Mark of 4]

Rearranging the SFEE to give the gas velocity at the outlet implies

$$u_2^2 = u_1^2 + \frac{2(\dot{Q} - \dot{W}_s)}{\dot{m}} + 2(h_1 - h_2).$$

Therefore

$$\begin{aligned} u_2^2 &= 30^2 + \frac{2(-15000 - 30000)}{4} + 2(70000 - 40000) \\ &= 900 - 22500 + 60000 \\ &= 38400 \text{ m}^2 \text{ s}^{-2}, \end{aligned}$$

giving a fluid velocity at the outlet of

$$u_2 = 195.959 \text{ m s}^{-1}$$

[1 Mark of 4]

Now the gas density at the outlet can be calculated from the mass flux

$$\rho_2 = \frac{\dot{m}}{u_2 A_2} = \frac{4 \text{ kg s}^{-1}}{195.959 \text{ m s}^{-1} \times 0.02865 \text{ m}^2} = 0.7125 \text{ kg m}^{-3}.$$

Finally the difference in gas density the turbine is given by

$$\Delta\rho = \rho_1 - \rho_2 = 1.6755 - 0.7125 = 0.9630 \text{ kg m}^{-3}.$$

[1 Mark of 4]

(b) The differential forms of for mass and energy conservation are

$$\begin{aligned} \frac{dV}{V} - \frac{du}{u} - \frac{dA}{A} &= 0, \\ dh + u du &= 0. \end{aligned}$$

[1 Mark of 2]

Eliminating du between these two expressions gives

$$\frac{dV}{V} + \frac{dh}{u^2} - \frac{dA}{A} = 0$$

[1 Mark of 2]

The speed of sound is the distance travelled during a unit of time by a sound wave propagating through a compressible medium. The Mach number is the non-dimensional ratio of the speed of a body moving through a fluid to the local speed of sound. For an isentropic process, the speed of sound is given by

$$c = \left(\frac{\partial p}{\partial \rho} \right)^{1/2},$$

while the Mach number is defined to be

$$\text{Ma} = \frac{u}{c},$$

[4 Marks]

For an isentropic process the specific volume $V = 1/\rho$, is a function of just pressure and therefore satisfies

$$dV = \frac{dV}{dp} dp.$$

In this expression the derivative can be written in terms of the speed of sound c since

$$\frac{dV}{dp} = \frac{\partial V}{\partial \rho} \frac{\partial \rho}{\partial p} = -\frac{V^2}{c^2},$$

and therefore

$$dV = -\frac{V^2}{c^2} dp.$$

[2 Marks of 3]

For a general thermodynamic process

$$dh = T ds + V dp.$$

However for an isentropic process the entropy remains constant $ds = 0$, and the enthalpy is a function of just pressure. Changes in enthalpy are not related to changes in entropy, and

$$dh = V dp.$$

[1 Mark of 3]

Hence if we eliminate dV and dh ,

$$-\frac{V}{c^2} dp + \frac{V}{u^2} dp - \frac{dA}{A} = 0.$$

Collecting together terms involving dp gives

$$\frac{dA}{A} = \left(\frac{V}{u^2} - \frac{V}{c^2} \right) dp = \frac{V}{u^2} \left(1 - \frac{u^2}{c^2} \right) dp.$$

Given the definition of Mach number

$$\frac{dA}{A} = \frac{V}{u^2} (1 - \text{Ma}^2) dp.$$

For an isentropic process the speed of sound can be used to eliminate dp , giving

$$\frac{dA}{A} = \frac{V}{u^2} (1 - \text{Ma}^2) c^2 d\rho.$$

Rearranging and using the definition of Mach number and specific density gives

$$\frac{1}{(1 - \text{Ma}^2) A} dA = \frac{1}{\rho \text{Ma}^2} d\rho.$$

If we're interested in changes along a pipe whose length is parameterized by x , then

$$\frac{1}{(1 - \text{Ma}^2) A} \frac{dA}{dx} = \frac{1}{\rho \text{Ma}^2} \frac{d\rho}{dx},$$

as required.

[5 Marks]

For a supersonic diffuser $(1 - \text{Ma}^2) < 0$, while $\frac{dA}{dx} > 0$, $A > 0$, $\rho > 0$ and $\text{Ma}^2 > 0$. Therefore $\frac{d\rho}{dx} < 0$ and the gas density falls as gas flows along a supersonic diffuser. **[2 Marks]**

Question 4: Figure 2 shows the Ts diagram for the refrigeration cycle.

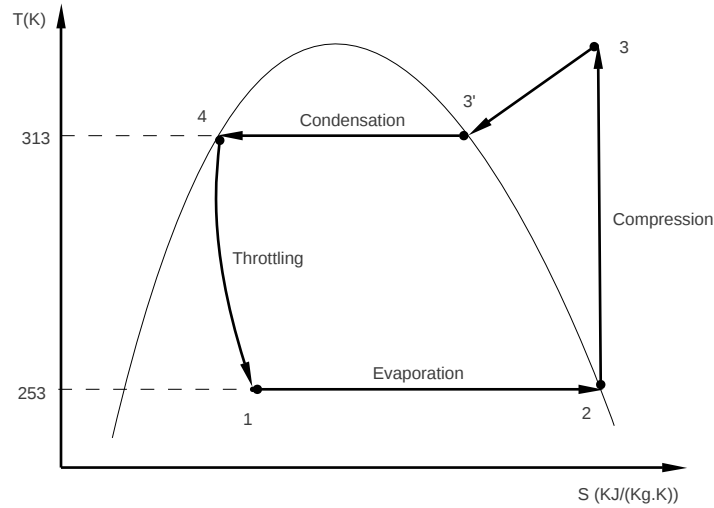


Figure 2: Refrigeration cycle – Question 4

Before solve the vapour-compressed refrigeration problem we should sketch the cycle (Fig. 3 – (1 Mark)), and calculate the enthalpies (and entropies) of the the different stages of the cycle based on the data obtained from the thermodynamic table (for Freon-12),

T (°C)	P_s (bar)	V_g (m ³ /kg)	H_f (kJ/kg)	H_g (kJ/kg)	S_f (kJ/(kg.K))	S_g (kJ/(kg.K))	Specific Heat (kJ/(kg.K))
-20	1.509	0.1088	17.8	178.61	0.073	0.7082	–
40	9.607	–	74.53	203.05	0.2716	0.682	0.747

Stage 2: At $P_2 = 1.509 \text{ bar}$,

$$H_2 = 178.61 \frac{\text{kJ}}{\text{kg}} = H_g \quad (1\text{Mark})$$

$$S_2 = S_g = 0.7082 \frac{\text{kJ}}{\text{kg.K}}$$

Stage 3: The refrigerant fluid undertakes an isentropic compression ($S_3 = S_2$) leading to a superheated fluid. Because we do not have the full saturation tables we need to use fundamental relationship to calculate the properties of the superheated fluid,

$$H_3 = H_3^{\text{sat}} + C_p (T_3 - T_3^{\text{sat}})$$

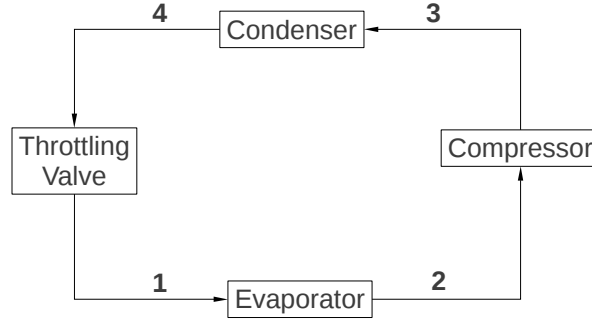


Figure 3: Sketch of the vapour-compressed refrigeration – Question 4.

where $T_3^{\text{sat}} = T^{\text{sat}}(P = 9.607 \text{ bar}) = 40^\circ\text{C} = 313.15 \text{ K}$. However we first need to calculate the temperature, T_3 , of the superheated fluid,

$$S_3 = 0.7082 = S_3^{\text{sat}} + C_p \ln \left(\frac{T_3}{T_3^{\text{sat}}} \right) = 0.682 + 0.747 \ln \left(\frac{T_3}{313.15} \right)$$

$$\Rightarrow T_3 = 324.33 \text{ K} \text{ (1 Mark)}$$

Now, replacing T_3 in the above equation for H_3 ,

$$H_3 = H_3^{\text{sat}} + C_p (T_3 - T_3^{\text{sat}}) = 203.05 + 0.747 (324.33 - 313.15) = 211.40 \frac{\text{kJ}}{\text{kg}} \text{ (1 Mark)}$$

Stage 4: Saturated liquid at the exit of the condenser ($P_4 = 9.607 \text{ bar}$) $\Rightarrow H_4 = H_f = 74.5 \frac{\text{kJ}}{\text{kg}}$ (1 Mark).

Stage 1: Isenthalpic expansion through the throttling valve, $H_1 = H_4 = 74.5 \frac{\text{kJ}}{\text{kg}}$ (1 Mark).

Now that we have all key-parameters,

(a) The power required by the compressor is

$$\mathcal{P} = \dot{m}_r (H_3 - H_2)$$

where \dot{m}_r is the mass flow rate of the refrigerant fluid that can be calculated with the help of the given refrigerant capacity, R_n ,

$$R_n = 20 = \dot{m}_r (H_2 - H_1) = \dot{m}_r (178.61 - 74.53) \Rightarrow \dot{m}_r = 0.1922 \frac{\text{kg}}{\text{s}} \text{ (1 Mark)}$$

and

$$\mathcal{P} = 0.1922 (211.40 - 178.61) \implies \mathcal{P} = 6.3022 \frac{kJ}{s} = 6302.2 \text{ W (3 Mark)}$$

- (b) In order to determine the piston displacement in the cylinder, we first need to calculate the volumetric efficiency given by

$$\eta_{vol} = 1 + C - C \left(\frac{P_d}{P_s} \right)^{1/n} = 1 + 0.03 - 0.03 \left(\frac{9.607}{1.509} \right)^{1/1.13} = 0.876 \text{ (2 Mark)}$$

The volume of the fluid at intake conditions is,

$$\dot{V}_r = \dot{m}_r V_g = 0.1922 \times 0.1088 = 0.0209 \frac{m^3}{s} \text{ (2 Mark)}$$

The swept volume rate is,

$$\dot{V}_{swept} = \frac{\dot{V}_r}{\eta_{vol}} = \frac{0.0209}{0.876} = 2.385 \times 10^{-2} \frac{m^3}{s} \text{ (2 Mark)}$$

The piston operates at 300 rpm, thus each stroke is completed in t_r ,

$$\begin{array}{rcl} 300 \text{ rotations} & \text{---} & 1 \text{ min} = 60 \text{ sec} \\ 1 \text{ rotation} & \text{---} & t_r \end{array}$$

(1)

$t_r = 0.2 \text{ s}$ (2 Mark). Thus, it takes 0.2 seconds to fill up the cylinder with Freon in the compressor operating with a reciprocating (or piston) engine. Now calculating the piston displacement – V_{swept} ,

$$\begin{array}{rcl} 2.385 \times 10^{-2} m^3 & \text{---} & 1 \text{ s} \\ V_{swept} & \text{---} & 0.2 \text{ s} \end{array}$$

$$V_{swept} = 4.771 \times 10^{-3} m^3 \text{ (2 Mark)}$$

Question 5: The specific humidity ω is the ratio of the mass of water vapour m_v , to the mass of dry air m_a and satisfies the equation

$$\omega = \frac{m_v}{m_a}.$$

As both water vapour and dry air behave like ideal gases, in some arbitrary volume V ,

$$\omega = \frac{m_v}{m_a} = \frac{\rho_v}{\rho_a} = \frac{p_v}{R_v T} \frac{R_a T}{p_a} = \frac{R_a p_v}{R_v p_a}.$$

The ratio of specific gas constants $R_a/R_v = 0.622$, while the partial pressures of dry air and water vapour satisfy $p_a = p - p_v$. Hence

$$\omega = \frac{0.622 p_v}{p - p_v}.$$

[4 Marks]

The saturation pressure of water

$$p_g = \varphi p_v.$$

Hence eliminating p_v from the previous expression gives

$$\omega = \frac{0.622 \varphi p_g}{p - \varphi p_g}.$$

[2 Marks]

The heating and humidification are split into two steady process. Firstly a heater (with inlet properties labelled 1 and outlet properties labelled 2) and secondly a humidifier (with inlet properties labelled 2 and outlet properties labelled 3).

(a) The partial vapour pressure at the inlet 1, is

$$p_{v_1} = \varphi_1 p_{g_1} = \varphi p_{\text{sat @ } 10^\circ\text{C}} = 0.25 \times 1.4028 \text{ kPa} = 0.3507 \text{ kPa}.$$

[1 Mark]

Hence the partial pressure of dry air is given by

$$p_{a_1} = p_1 - p_{v_1} = 100 \text{ kPa} - 0.3507 \text{ kPa} = 99.6493 \text{ kPa}.$$

[1 Mark]

The specific humidity is given by

$$\omega_1 = \frac{0.622 p_{v_1}}{p_1 - p_{v_1}} = \frac{0.622 \times 0.3507 \text{ kPa}}{100 \text{ kPa} - 0.3507 \text{ kPa}} = 0.00219 \text{ kg H}_2\text{O/ kg dry air}.$$

[1 Mark]

(b) Applying the mass and energy balances on the heating section gives

$$\text{Dry air mass balance: } \dot{m}_{a1} = \dot{m}_{a2} = \dot{m}_a,$$

$$\text{Water mass balance: } \dot{m}_{a1}\omega_1 = \dot{m}_{a2}\omega_2, \Rightarrow \omega_1 = \omega_2,$$

$$\text{Energy balance: } \dot{Q} = \dot{m}_a h_2 - \dot{m}_a h_1.$$

[2 Marks]

The total specific enthalpy at 1, the inlet is

$$\begin{aligned} h_1 &= c_p T_1 + \omega_1 h_{g1} = (1.005 \text{ kJ/(kg K)} \times (12 + 173.15) \text{ K}) \\ &\quad + (0.00219 \times 2523 \text{ kJ/kg}) \\ &= 292.0988 \text{ kJ/kg}. \end{aligned}$$

The total specific enthalpy at 2, the outlet of the heating is

$$\begin{aligned} h_2 &= c_p T_2 + \underbrace{\omega_2}_{=\omega_1} h_{g2} = (1.005 \text{ kJ/(kg K)} \times (20 + 173.15) \text{ K}) \\ &\quad + (0.00219 \times 2537 \text{ kJ/kg}) \\ &= 300.1693 \text{ kJ/kg} \end{aligned}$$

[2 Marks]

The specific volume of dry air at 1, is given by

$$V_1 = \frac{R_a T_1}{p_{a1}} = \frac{287.058 \text{ J/(kg K)} (12 + 273.15) \text{ K}}{99649.3 \text{ Pa}} = 0.8215 \text{ m}^3/\text{kg}.$$

Therefore the mass flux of dry air through the inlet

$$\dot{m}_a = \frac{q_1}{V_1} = \frac{40 \text{ m}^3/\text{min}}{0.8185 \text{ m}^3/\text{kg}} = 48.6886 \text{ kg/min},$$

where $q_1 = 40 \text{ m}^3/\text{min}$ is the total volume flux through the inlet.

[1 Mark]

Hence the energy conservation equation gives the rate at which heat is transferred to the air

$$\dot{Q} = \dot{m}_a (h_2 - h_1) = 48.6958 \text{ kg/min} (300.1693 \text{ kJ/kg} - 292.0988 \text{ kJ/kg}) = 392.9488 \text{ kJ/min}.$$

[1 Mark]

(c) The mass balance for water in the humidifying section can be expressed as

$$\dot{m}_{a2}\omega_2 + \dot{m}_w = \dot{m}_{a3}\omega_3,$$

or

$$\dot{m}_w = \dot{m}_a (\omega_3 - \omega_2).$$

[2 Marks]

Here $\omega_2 = \omega_1$, while the specific humidity at 3, the outlet is given by

$$\omega_3 = \frac{0.622\varphi_3 p_{g3}}{p_3 - \varphi_3 p_{g3}} = \frac{0.622 \times 0.55 \times 2.9858}{100 - (0.55 \times 2.9858)} = 0.0104 \text{ kg H}_2\text{O/ kg dry air.}$$

[1 Mark]

Therefore the required mass flow rate of steam is

$$\dot{m}_w = \dot{m}_a (\omega_3 - \omega_2) = 48.6886 \text{ kg/min} (0.0104 - 0.00219) = 0.3990 \text{ kg/min}$$

[2 Marks]