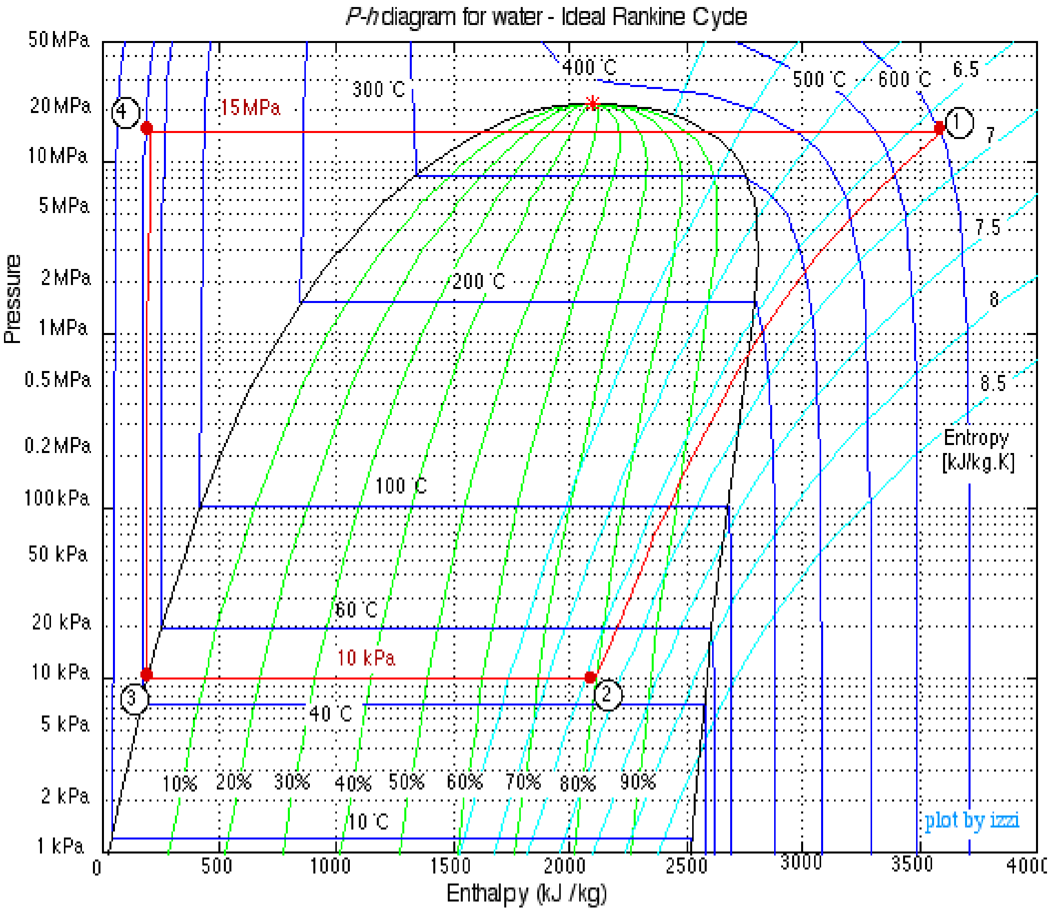
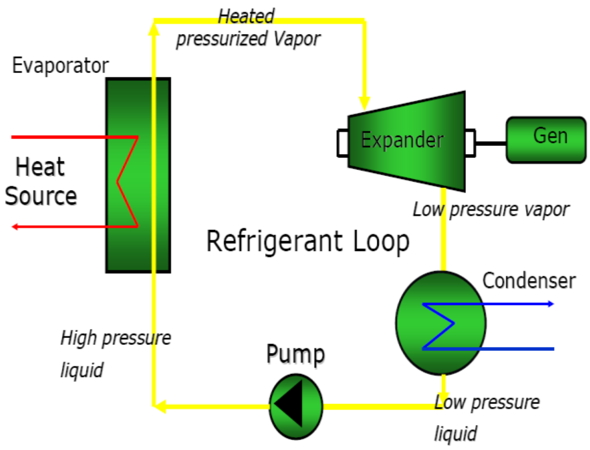
Question 1

The Organic Rankine Cycle (ORC) uses a heat source to heat an organic motive fluid – point 4 to point 1. The motive fluid is evaporated and is then expanded through a turbine, which generates power – point 1 to point 2. The motive fluid is condensed in a heat exchanger using a cooling medium (cooling water, seawater, air) – point 2 to point 3, from where it is pumped back to the evaporator – point 3 to point 4.





THE STUDENT'S CALCULATION MAY BE SLIGHTLY DIFFERENT DUE TO CHART READING. PROVIDED IT IS CLEAR THE STUDENT UNDERSTANDS THE METHODOLOGY FULL MARKS SHOULD BE GIVEN.

Water flow	0.184	m3/s	Given
Water Density	1000.000	kg/m3	Given
Water Mass flow	184.000	kg/s	Volume flow x density
Water Specific Heat	4.200	kJ/kgDegC	Given
Water Inlet	95.000	DegC	Given
Water Outlet	75.000	DegC	Given
Heat removed from water	15456.000	kJ/s	Mass flow x specific heat x Water Temperature drop

The iPentane enters the cycle pump as a saturated liquid at 100kPa - point 3 on chart.
The pump operates isentropicaly hence point 4 follows the isentropic line to 350 kPa.
Point 4 is the pump outlet/evaporator inlet condition.

Enthalpy at 4	495.000	kJ/kg

The i-Pentane is heated until it is a saturated vapour - point 1. Follow constant pressure line.

Enthalpy at 1	895.000	kJ/kg

The mass flow of i-Pentane can now be calculated -
The heat removed from the water is the amount of heat absorbed by the i-Pentane.
The mass flow of i-Pentane is the heat absorbed divided the enthalpy difference from Point 4 to 15456/(895-495)

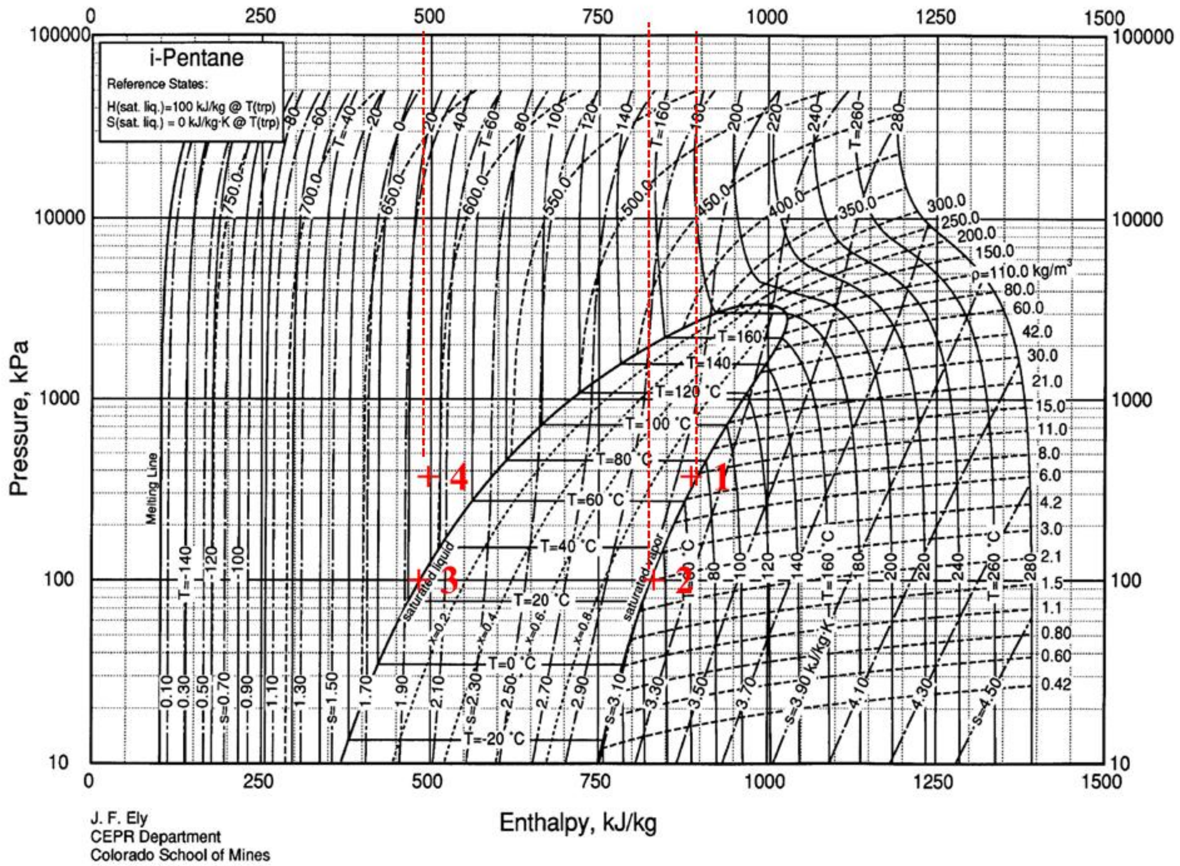
Mass flow of i-P	38.640	kg/s

Point 1 to point 2 is an isentropic expansion of the i-Pentane.

Enthalpy at 2	830.000	kJ/kg

Isentropic enthalpy change over expander - Point 1 -Point 2 (895-830 = 65 kJ/kg)
Multiply by the mass rate of i-P gives the theoretical power expander power output (65x38.64)

Isentropic Turbine Power	2511.600	kJ/s	kW
Turbine efficiency	0.750		
Actual power recovered	1883.700	kJ/s	kW
Generator efficiency	0.900		
Electrical power produced	1695.330	kJ/s	kW



Question 2

(a)

Student requires to construct the following table and demonstrate the Titan as the preferred selection.

		Machine Efficiency		Rel CO2 Produced	
Year	**Load MW**	Mars	Titan	Mars	Titan
1	22	29.4	30.6	0.748299	0.718954
2	22	29.4	30.6	0.748299	0.718954
3	22	29.4	30.6	0.748299	0.718954
4	24	30.2	31.1	0.794702	0.771704
5	24	30.2	31.1	0.794702	0.771704
6	24	30.2	31.1	0.794702	0.771704
7	24	30.2	31.1	0.794702	0.771704
8	26	30.8	28.2	0.844156	0.921986
9	28	31.2	29.1	0.897436	0.962199
10	28	31.2	29.1	0.897436	0.962199
11	24	30.2	31.1	0.794702	0.771704
12	24	30.2	31.1	0.794702	0.771704
13	24	30.2	31.1	0.794702	0.771704
14	24	30.2	31.1	0.794702	0.771704
15	24	30.2	31.1	0.794702	0.771704
16	24	30.2	31.1	0.794702	0.771704
17	24	30.2	31.1	0.794702	0.771704
18	24	30.2	31.1	0.794702	0.771704
19	24	30.2	31.1	0.794702	0.771704
20	24	30.2	31.1	0.794702	0.771704
	Ave	30.21	30.68	0.667073	0.523414

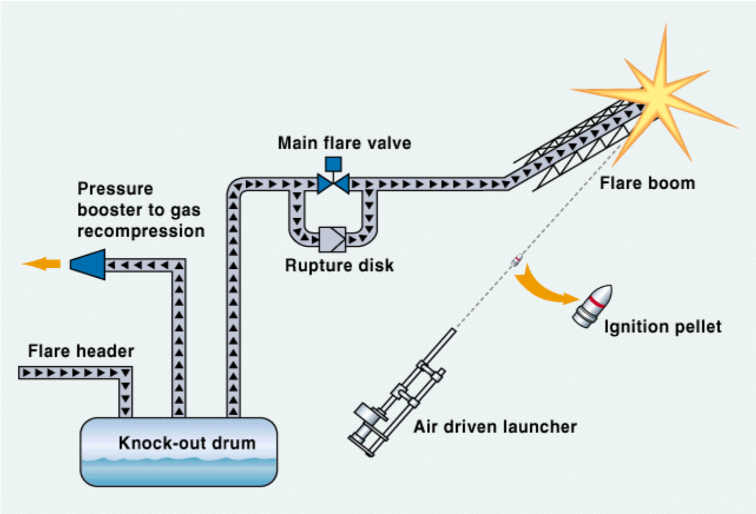
(b)

Install a device to isolate the flare stack from the flare gas collection system. Normally a valve with a bypass bursting disc.

Providing a means of returning the gas normally flared to the process.

Ensuring that under upset conditions, when large flows are sent to the flare, the gas is safely discharged via the flare tip and burnt. This requires a system to reliably open the flare stack valve and an automatic ignition system.

Purging the flare stack with nitrogen to prevent air ingress and flame back.


Main flare valve
Pressure
booster to gas
recompression
Flare boom
Rupture disk
Ignition pellet
Flare header
Air driven launcher
Knock-out drum

Question 3

(a) The poles of the characteristic equation of the system need to be investigated with the Routh-Hurwitz stability criterion. Specifically, the characteristic equation is:

$$1+G(s)=0 \Rightarrow 1+G_c(s)\cdot G_f(s)G_p(s)G_m(s)=0 \Rightarrow 1+\frac{K_c\cdot\left(1+\frac{1}{\tau_I s}\right)\cdot 1\cdot K_p\cdot 1}{(\tau_a s+1)(\tau_b s-1)}=0$$

This can further be manipulated into:

$$1+\frac{K_c\cdot\left(1+\frac{1}{\tau_I s}\right)\cdot 1\cdot K_p\cdot 1}{(\tau_a s+1)(\tau_b s-1)}=0 \Rightarrow 1+\frac{K_cK_p\left(s+\frac{1}{\tau_I}\right)}{s(\tau_a s+1)(\tau_b s-1)}=0 \Rightarrow$$

$$s(\tau_a s+1)(\tau_b s-1)+K_cK_p\left(s+\frac{1}{\tau_I}\right)=0 \Rightarrow \tau_\alpha\tau_b s^3+(\tau_b-\tau_\alpha)s^2+(K_cK_p-1)s+\frac{K_cK_p}{\tau_I}=0$$

The characteristic equation has now been expanded in a polynomial form:

$a_0s^3+a_1s^2+a_2s+a_3=0$, with: $a_0=\tau_\alpha\tau_b$, $a_1=\tau_b-\tau_\alpha$, $a_2=K_cK_p-1$ and $a_3=\dfrac{K_cK_p}{\tau_I}$

So the Routh array can be constructed:

1	a_0	a_2
2	a_1	a_3
3	$(a_1a_2-a_0a_3)/a_1$	0
4	a_3	

This becomes:

1	$\tau_\alpha\tau_b$	K_cK_p-1
2	$\tau_b-\tau_\alpha$	$\dfrac{K_cK_p}{\tau_I}$
3	$\dfrac{(\tau_b-\tau_\alpha)(K_cK_p-1)-\tau_\alpha\tau_b\dfrac{K_cK_p}{\tau_I}}{\tau_b-\tau_\alpha}$	0
4	$\dfrac{K_cK_p}{\tau_I}$	

According to the Routh-Hurwitz stability criterion all coefficients of the characteristic equation need to be positive and, additionally, all elements of the first column of the Ruth array need also to be positive for the system to be stable:

- $a_0>0 \Rightarrow \tau_\alpha\tau_b>0\ (valid\ always)$
- $a_1>0 \Rightarrow \tau_b-\tau_\alpha>0 \Rightarrow \tau_b>\tau_\alpha$

- $$a_2 > 0 \Rightarrow K_c K_p - 1 > 0 \Rightarrow K_c > \frac{1}{K_p}$$
- $$a_3 > 0 \Rightarrow \frac{K_c K_p}{\tau_I} > 0 \Rightarrow K_c > 0$$
- $$\frac{(\tau_b - \tau_\alpha)(K_c K_p - 1) - \tau_\alpha \tau_b \frac{K_c K_p}{\tau_I}}{\tau_b - \tau_\alpha} > 0 \Rightarrow (\tau_b - \tau_\alpha)(K_c K_p - 1) > \tau_\alpha \tau_b \frac{K_c K_p}{\tau_I} \Rightarrow$$

  $$\tau_I > \frac{K_c K_p \tau_\alpha \tau_b}{(\tau_b - \tau_\alpha)(K_c K_p - 1)}$$

Considering the above the system is stable when:

$\tau_b > \tau_\alpha$, $K_c > \frac{1}{K_p}$ and $\tau_I > \frac{K_c K_p \tau_\alpha \tau_b}{(\tau_b - \tau_\alpha)(K_c K_p - 1)}$

(b) The open-loop transfer function needs to be brought in the proper form first:

$$G(s) = G_c(s) \cdot G_f(s) G_p(s) G_m(s) = K_c \cdot \left(1 + \frac{1}{0.25s}\right) \cdot 1 \frac{1}{(s+1)(s+2)} \cdot 1 = \frac{K_c(s+4)}{s(s+1)(s+2)}$$

We see that there are three poles, so n=3. These poles are: p_1=0, p_2=-1 and p_3=-2. There is one zero, specifically z_1=-4, so m=1.

We then apply the 7 rules to construct the Root-Locus:

1. n=3, so the Root-Locus has three branches.
2. m=1, hence n-m=2, so there are 2 asymptotes in the Root-Locus.
3. The Root-Locus is symmetrical to the Re-axis.
4. Part of the Re-axis belongs to the Root-Locus, specifically intervals [-4, -2] and [-1, 0].
5. The asymptotes form the following angles with the positive direction of the *Re*-axis:

   $\varphi_i = \frac{2k+1}{n-m} \cdot \pi, \quad k = 0, \ldots, n-m-1$, hence: φ_1=π/2 and φ_2=3π/2.

   The centre of gravity of these asymptotes is:

   $$\gamma = \left[\sum_{i=1}^{n} p_i - \sum_{j=1}^{m} z_i\right] \Big/ (n-m) = \frac{0 - 1 - 2 + 4}{3 - 1} = 0.5$$

6. The departure/arrival points of branches can be found through the solution of:

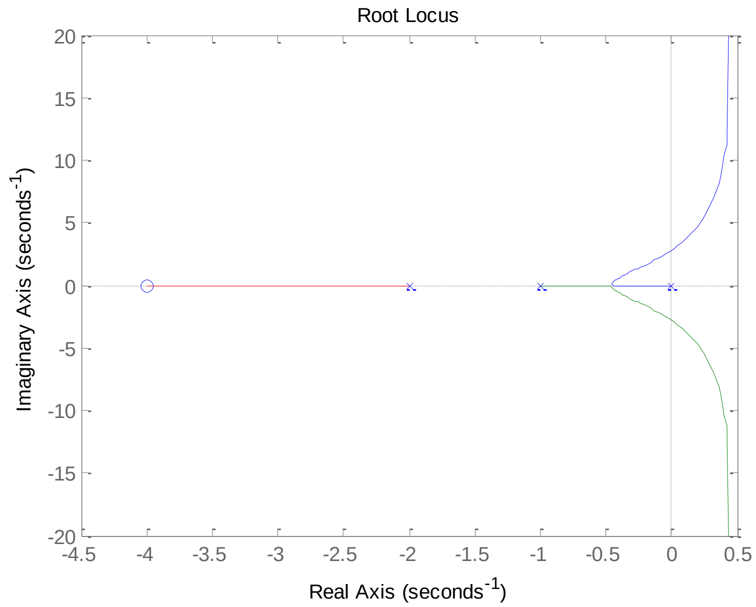
   $$\sum_{i=1}^{n} \frac{1}{(s_0 - p_i)} = \sum_{j=1}^{m} \frac{1}{(s_0 - z_j)} \Rightarrow \frac{1}{s_0} + \frac{1}{s_0 + 1} + \frac{1}{s_0 + 2} = \frac{1}{s_0 + 4} \Rightarrow$$

   $$2s_0^3 + 11s_0^2 + 20s_0 + 8 = 0 \Rightarrow \begin{cases} s_{0,1} = -0.55 \\ s_{0,2} = -2.475 + 1.074i \\ s_{0,3} = -2.475 - 1.074i \end{cases}$$

Point $s_{0,1}$ belongs to the *Re*-axis and the Root-Locus, so it is a valid departure/arrival point. The other solutions $s_{0,2}$ and $s_{0,3}$ do not belong to the *Re*-axis, so they are rejected. Branches depart from the *Re*-axis at point $s_{0,1}$ forming an angle of $(\pm\pi/2)$ with it.

7. The Root-Locus branches emanate from three simple poles and arrive at a simple zero on the Re-axis so they form 0 or π angles with its positive direction.

The final complete Root-Locus is shown below.



To find the exact points that the Root-Locus intersects the *Im*-axis, the Routh-Hurwitz stability criterion needs to be applied. Specifically, the characteristic equation of the system is considered:

$$1 + G(s) = 0 \Rightarrow 1 + \frac{K_c(s+4)}{s(s+1)(s+2)} = 0 \Rightarrow s(s+1)(s+2) + K_c(s+4) = 0 \Rightarrow$$

$$s^3 + 3s^2 + (K_c + 2)s + 4K_c = 0$$

The characteristic equation has now been expanded in a polynomial form:

$a_0 s^3 + a_1 s^2 + a_2 s + a_3 = 0$, with: $a_0 = 1$, $a_1 = 3$, $a_2 = K_c + 2$ and $a_3 = 4K_c$

So the Routh array can be constructed:

1	a_0	a_2
2	a_1	a_3
3	$(a_1a_2-a_0a_3)/a_1$	0
4	a_3	

This becomes:

1	1	$K_c + 2$
2	3	$4K_c$
3	$\frac{3(K_c+2) - 4K_c}{4K_c}$	0
4	$4K_c$	

According to the Routh-Hurwitz stability criterion all coefficients of the characteristic equation need to be positive and, additionally, all elements of the first column of the Ruth array need also to be positive for the system to be stable:

- $a_0 > 0 \Rightarrow 1 > 0 \ (valid\ always)$
- $a_1 > 0 \Rightarrow 3 > 0 \ (valid\ always)$
- $a_2 > 0 \Rightarrow K_c + 2 > 0 \Rightarrow K_c > -2$
- $a_3 > 0 \Rightarrow 4K_c > 0 \Rightarrow K_c > 0$
- $\dfrac{3(K_c + 2) - 4K_c}{4K_c} > 0 \Rightarrow 3(K_c + 2) > 4K_c \Rightarrow K_c < 6$

It can be seen that for the system to be stable it must be: $0<K<6$. The critical stability point is found for line n in the Routh array when all coefficients are zero (n=3), so where $\dfrac{3(K_c + 2) - 4K_c}{4K_c} = 0 \Rightarrow K_c = 6$, by the solution of:

$$C \cdot s^2 + D = 0 \Rightarrow 3 \cdot s^2 + 4 \cdot 6 = 0 \Rightarrow s = \pm 2.828i$$

Question 4:

For $\alpha = 20°\text{C.m}^2$, $T_{\text{wall}} = 200°$ C, $L = 0.3$ m and $T_{\text{amb}} = 20°$ C. Using central difference scheme for the second-order derivative:

$$\frac{T_{i+1} - 2T_i + T_{i-1}}{(\Delta x)^2} - \alpha T_i = -\alpha T_{\text{amb}} \quad i = 1, 2, \cdots$$

Defining $\beta = \alpha T_{\text{amb}}$, the equation can be rearranged to

$$\frac{1}{(\Delta x)^2} T_{i-1} - \left[\frac{2}{(\Delta x)^2} + \alpha\right] T_i + \frac{1}{(\Delta x)^2} T_{i+1} = -\beta$$

- For node $i = 2$:
$$\frac{1}{(\Delta x)^2} T_1 - \left[\frac{2}{(\Delta x)^2} + \alpha\right] T_2 + \frac{1}{(\Delta x)^2} T_3 = -\beta$$

- For node $i = 3$:
$$\frac{1}{(\Delta x)^2} T_2 - \left[\frac{2}{(\Delta x)^2} + \alpha\right] T_3 + \frac{1}{(\Delta x)^2} T_4 = -\beta$$

- $T(x = 0) = T_1 = T_{\text{wall}}$;

- And for the second boundary condition:
$$\frac{\partial T}{\partial x}(x = L) = 0$$
that can be discretised with backward difference method
$$\frac{\partial T}{\partial x} = \frac{T_i - T_{i-1}}{\Delta x}$$
where $i = 4$:
$$\frac{T_4 - T_3}{\Delta x} = 0 \to -\frac{1}{\Delta x} T_3 + \frac{1}{\Delta x} T_4 = 0$$

- The system of equations can be written in matricial form as
$$\begin{pmatrix} -\gamma & \frac{1}{(\Delta x)^2} & 0 \\ \frac{1}{(\Delta x)^2} & -\gamma & \frac{1}{(\Delta x)^2} \\ 0 & -\frac{1}{(\Delta x)^2} & \frac{1}{(\Delta x)^2} \end{pmatrix} \begin{pmatrix} T_2 \\ T_3 \\ T_4 \end{pmatrix} = \begin{pmatrix} -\beta - \frac{1}{(\Delta x)^2} T_1 \\ -\beta \\ 0 \end{pmatrix}$$
where $\gamma = \dfrac{2}{(\Delta x)^2} + \alpha$, leading to: $T_2 = 151.71°\text{C}$, $T_3 = 129.76°\text{C}$ and $T_4 = 129.76°\text{C}$.

Question 5:

1. Expanding a function $\underline{u}$ at x_{i+1} about the point x_i (assuming regular grid):

$$u\left(x_i + \Delta x_i\right) = u\left(x_i\right) + \Delta x_i \left.\frac{\partial u}{\partial x}\right|_{x_i} + \frac{\left(\Delta x_i\right)^2}{2!} \left.\frac{\partial^2 u}{\partial x^2}\right|_{x_i} + \frac{\left(\Delta x_i\right)^3}{3!} \left.\frac{\partial^3 u}{\partial x^3}\right|_{x_i} + \cdots$$

The Taylor's expansion can be rearranged as,

$$\frac{u\left(x_i + \Delta x_i\right) - u\left(x_i\right)}{\Delta x_i} - \left.\frac{\partial u}{\partial x}\right|_{x_i} = \frac{\Delta x_i}{2!} \left.\frac{\partial^2 u}{\partial x^2}\right|_{x_i} + \frac{\left(\Delta x_i\right)^2}{3!} \left.\frac{\partial^3 u}{\partial x^3}\right|_{x_i} + \cdots$$

The rhs of the equation is the truncation error of the series and the equation can be rewritten as

$$\left.\frac{\partial u}{\partial x}\right|_{x_i} = \frac{u_{i+1} - u_i}{\Delta x} + \mathcal{O}\left(\Delta x\right)$$

2. The system of algebraic equations can be represented in matricial form as

$$\begin{pmatrix} 2 & 0 & 1 & 0 \\ 0 & 4 & 1 & 1 \\ 1 & 0 & 2 & 0 \\ 0 & 0 & 3 & 4 \end{pmatrix} \begin{pmatrix} \mathcal{C}_1 \\ \mathcal{C}_2 \\ \mathcal{C}_3 \\ \mathcal{C}_4 \end{pmatrix} = \begin{pmatrix} 6.35 \\ 9.00 \\ 3.95 \\ 4.11 \end{pmatrix}$$

i.e., $\underline{\mathcal{A}}\mathcal{C} = b$.

(a) A matrix $\mathcal{A}$ fulfill the conditions for convergence in any iterative method if any of the conditions below is true:

i. strictly diagonal dominant, i.e.,

$$|a_{ii}| > \sum_{j=1, j\neq i}^{n} |a_{ij}| \quad i \in \{1, 2, \cdots, n\};$$

ii. symmetric, i.e., $\mathcal{A}^T = \mathcal{A}$ or;

iii. positive definite, i.e., $z^T \mathcal{A} z > 0$ for any matrix-column z.

Matrix $\mathcal{A}$ satisfies condition (i) above, therefore it will converge regardless the iterative method used.

(b) In order to calculate the solution of the linear system using $\mathcal{C} = \underline{\mathcal{A}}^{-1} b$, we need to invert $\mathcal{A}$ using Gauss-Jordan method:

$$\left|\begin{array}{cccc|cccc} 2 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 4 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 2 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 3 & 4 & 0 & 0 & 0 & 1 \end{array}\right| \Longrightarrow \left|\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 2/3 & 0 & -1/3 & 0 \\ 0 & 1 & 0 & 0 & 1/48 & 1/4 & -1/24 & -1/16 \\ 0 & 0 & 1 & 0 & -1/3 & 0 & 2/3 & 0 \\ 0 & 0 & 0 & 1 & 1/4 & 0 & -1/2 & 1/4 \end{array}\right|$$

thus,

$$\mathcal{A}^{-1} = \begin{pmatrix} 2/3 & 0 & -1/3 & 0 \\ 1/48 & 1/4 & -1/24 & -1/16 \\ -1/3 & 0 & 2/3 & 0 \\ 1/4 & 0 & -1/2 & 1/4 \end{pmatrix}$$

Now calculating $\mathcal{C} = \underline{\mathcal{A}}^{-1} b$ $\left(\text{in mg.l}^{-1}\right)$

$$\begin{pmatrix} \mathcal{C}_1 \\ \mathcal{C}_2 \\ \mathcal{C}_3 \\ \mathcal{C}_4 \end{pmatrix} = \begin{pmatrix} 2/3 & 0 & -1/3 & 0 \\ 1/48 & 1/4 & -1/24 & -1/16 \\ -1/3 & 0 & 2/3 & 0 \\ 1/4 & 0 & -1/2 & 1/4 \end{pmatrix} \begin{pmatrix} 6.35 \\ 9.00 \\ 3.95 \\ 4.11 \end{pmatrix} = \begin{pmatrix} 2.9167 \\ 1.9608 \\ 0.5167 \\ 0.70000 \end{pmatrix}$$