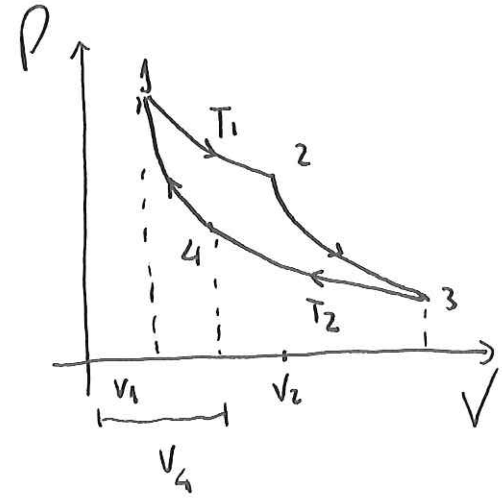
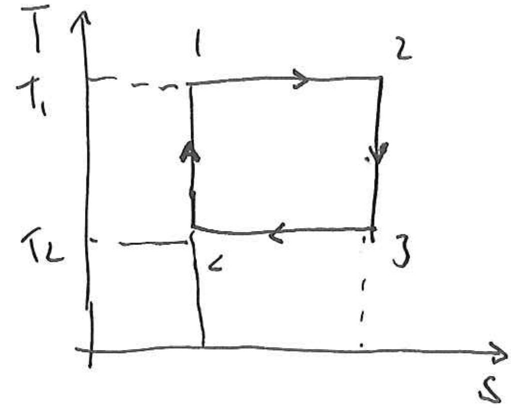
Gas Power Cycles

Problem 01



$$\frac{V_3}{V_1} = r \quad \text{and}$$

compression ratio $r = V_4/V_1$



In the isotherms, since

compression ratio = expansion ratio

$$\frac{V_3}{V_4} = \frac{V_2}{V_1}$$

But we can redefine V_3/V_4 with the known variables:

$$\frac{V_3}{V_4} = \frac{V_3}{V_1} \times \frac{V_1}{V_4} = r \cdot \frac{1}{r} = r/r$$

The work done per unit mass of gas is

$W =$ heat supplied - heat rejected

$$= RT_1 \ln(\kappa/r) - RT_2 \ln(\kappa/r)$$

$$= R(T_1 - T_2)\ln(\kappa/r)$$

However we know that the isentropic change

$$\frac{T_1}{T_2} = \left(\frac{V_4}{V_1}\right)^{\gamma-1} = r^{\gamma-1}$$

Thus

$$W = RT_2\,(r^{\gamma-1} - 1)\ln(\kappa/r)$$

The maximum work is obtained by differentiating

$\underline{W}$ wrt $\underline{r}$:

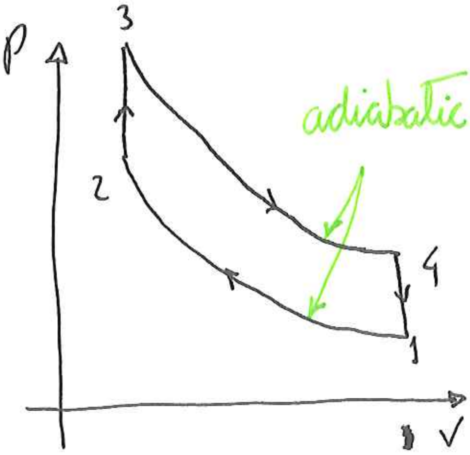
$$\frac{dW}{dr} = RT_2\left[(r^{\gamma-1} - 1)\,\frac{r}{\kappa}\,(-\kappa r^{-2}) + \ln\frac{\kappa}{r}\,(\gamma-1)r^{\gamma-2}\right]$$

$$= 0$$

This equation becomes

$$\boxed{(\gamma-1)\ln(\kappa/r) + \frac{1}{r^{\gamma-1}} - 1 = 0}$$

Problem 02:



Given: $\frac{V_1}{V_2} = \frac{V_4}{V_3} = r = 6$

$P_1 = 0.1\ MPa$

$T_1 = 300.15\ K$; $T_3 = 1842.15\ K$

(a) T_4, P_4 : ?

For the compression 1-2

$$P_1 V_1^{\gamma} = P_2 V_2^{\gamma} \quad \therefore \quad P_2 = P_1 \left(\frac{V_1}{V_2}\right)^{\gamma} = 12.29\ \text{bar}$$

and

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = 6^{\gamma-1} = 2.0477$$

$$T_2 = 614.61\ K$$

For the constant-volume process 2-3:

$$\frac{P_2}{T_2} = \frac{P_3}{T_3} \quad \therefore \quad P_3 = \frac{P_2 T_3}{T_2} = 36.82\ \text{bar}$$

And for the expansion 3-4:

$$\frac{T_3}{T_4} = \left(\frac{V_4}{V_3}\right)^{\gamma-1} = 2.0477$$

$$T_4 = T_3 \left(\frac{V_3}{V_4}\right)^{\gamma-1} = \boxed{899.63\,K = T_4}$$

and

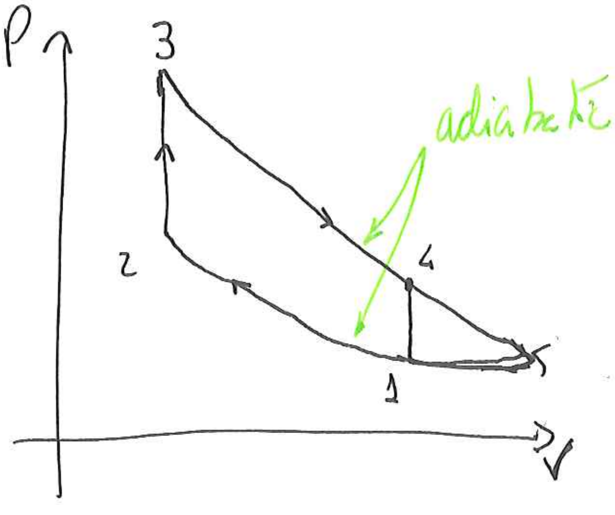
$$P_3 V_3^{\gamma} = P_4 V_4^{\gamma} \therefore P_4 = P_3 \left(\frac{V_3}{V_4}\right)^{\gamma}$$

$$\boxed{P_4 = 2.997 \text{ bar}}$$

(b) We can use a heat rejection at constant pressure after further expansion —

We saw this as the

Atkinson cycle



$\frac{01/03}{05}$

(c)

$$\eta_{otto} = 1 - \frac{1}{r^{\gamma-1}} = 0.5116 \therefore \boxed{51.16\%}$$

$$\eta_{Atkinson} = \frac{\text{Heat Supplied} - \text{Heat Rejected}}{\text{Heat Supplied}}$$

$$\eta_{Atkinson} = \frac{C_v(T_3 - T_2) - C_p(T_5 - T_1)}{C_v(T_3 - T_2)}$$

$$= 1 - \frac{C_p}{C_v}\frac{(T_5 - T_1)}{(T_3 - T_2)} = 1 - \gamma\frac{T_5 - T_1}{T_3 - T_2}$$

The isentropic process 3-5

$$\frac{T_5}{T_3} = \left(\frac{P_5}{P_3}\right)^{\frac{\gamma-1}{\gamma}} \therefore T_5 = 657.47\ K$$

$$\eta_{Atkinson} = 1 - 1.4\ \frac{657.47 - 300.15}{1842.15 - 614.61}$$

$$\eta_{Atkinson} = 0.5925 \therefore \boxed{59.25\%}$$

↳ Improvement of $\boxed{8.08\%}$

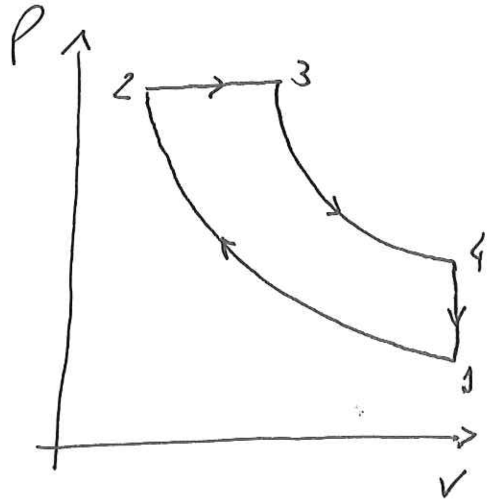
Problem 03

Given: $V_1/V_2 = 15.3$

$V_4/V_3 = 7.5$

$P_1 = 1$ bar

$T_1 = 300.15\ K$



(a) MEP : ?

$$MEP = \frac{\text{Work done by the cycle}}{\text{Swept volume}} = \frac{W}{V_1 - V_2}$$

$$= \frac{m C_p (T_3 - T_2) - m C_v (T_4 - T_1)}{V_1 - V_2}$$

Before we are able to calculate MEP, we need to determine all properties of the cycle, i.e., T_i, V_i, and T_i $(i = 1, 4)$. Thus

(i) adiabatic compression 1-2 :

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma - 1} \therefore T_2 = 893.75\ K$$

$$P_1 V_1^{\gamma} = P_2 V_2^{\gamma} \therefore P_2 = 45.56 \text{ bar}$$

(ii) Constant pressure (2-3):

$$\frac{V_2}{T_2} = \frac{V_3}{T_3} \therefore T_3 = \frac{V_3}{V_2} T_2 = \frac{V_1}{V_2}\frac{V_3}{V_4} T_2$$

$$T_3 = \frac{15.3}{7.5} T_2 \therefore T_3 = 1823.25 \text{ K}$$

Let's assume that $V_2 = 1 m^3$, thus the mass of air is (assuming ideal gas behaviour)

$$m = \frac{P_2 V_2}{R T_2} = \frac{45.56 \text{ bar} \times 1 m^3}{8.314 \times 10^{-5} \frac{m^3 . bar}{K . gmol} \times 893.75 K \times \frac{1 gmol}{29 g}}$$

$$m = 17792.512 \text{ g} = 17.79 \text{ Kg}$$

(iii). For adiabatic expansion (3-4):

$$\frac{T_4}{T_3} = \left(\frac{V_3}{V_4}\right)^{\gamma - 1} = \left(\frac{1}{7.5}\right)^{\gamma-1} = 0.4467$$

$$T_4 = 814.37 K$$

Thus

$$MEP = \frac{m\left[\overset{1.005\,\frac{KJ}{Kg.K}}{C_p}(T_3 - T_2) - \overset{0.718\,\frac{KJ}{Kg.K}}{C_v}(T_4 - T_1)\right]}{V_1 - V_2}$$

$$MEP = \frac{10051.64\ KJ}{\underbrace{14.3\ V_2}_{1m^3}} = 702.91\ \frac{KJ}{m^3} = \boxed{7.029\ bar}$$

(b) $\frac{P_2}{MEP}$: ?

$$\boxed{\frac{P_2}{MEP} = 6.48}$$

(c)

$$\eta = \frac{\text{Work done}}{\text{heat supplied}} = \frac{10051.64}{mC_p(T_3 - T_2)}$$

$$\boxed{\eta = 0.6049 \therefore 60.49\%}$$

(d) Fuel consumption : ? $\eta_T = 0.5\,\eta_{ideal}$

$$\eta_T = 0.5 \times 0.6049 = 0.3024 \therefore 30.24\%$$

However if the mechanical efficiency is 0.8, the "real" thermal efficiency is

$$\eta_T' = \eta_T \times \eta_M = 0.3024 \times 0.8 = 0.2419$$

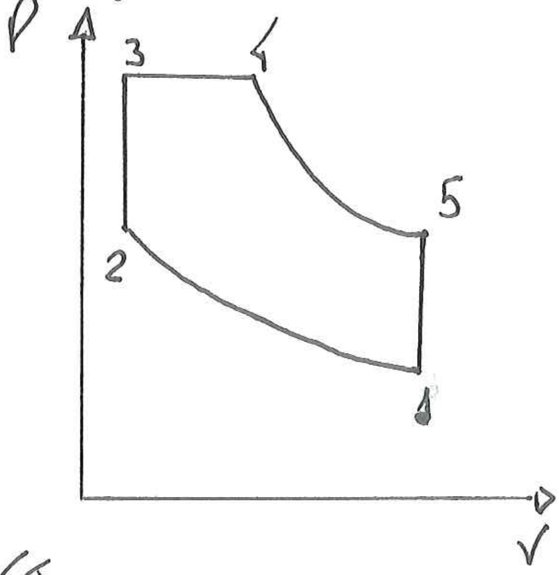
$$\eta_T' = \frac{\text{Brake Power}}{\dot{m}_f \times C}$$

→ effective power output of the engine.

↳ calorific heat of the oil

$$0.2419 = \frac{1}{\frac{\dot{m}_f}{3600} \, 42000} \quad \therefore \quad \boxed{\dot{m}_f = 0.354 \ \text{Kg}/\text{KWh}}$$

Problem 04: Air-Standard Dual-Cycle



Given: • compression ratio : $V_1/V_2 = 9$

• $P_1 = 100\ KPa$

$T_1 = 300K$

$V_1 = 14\ l = 0.014 m^3$

• $Q_{34} = Q_{23} = \frac{22.7}{2} KJ = 11.35\ KJ$

Compute: T_i ? MEP_{dual} ?

W_{cycle}/m ?

η_{dual}

(a) Thermal analysis of all stages:

• Stage 01: from "ideal-gas properties of air" (table in attachment)

$T_1 = 300K$ $\quad$ $u_1 = 214.07\ KJ/Kg$

$P_1 = 100 KPa$ $\quad$ $v_{r_1} = 621.2$ (dimensionless)

↳ relative specific volume (see note)

· <u>Stage 02</u> : isentropic compression

$$\frac{V_{r_2}}{V_{r_1}} = \frac{V_2}{V_1} \therefore V_{r_2} = V_{r_1}\left(\frac{V_2}{V_1}\right) = 621.2 \times \frac{1}{9}$$

$$V_{r_2} = 69.022$$

From the table:

T (K)	U (KJ/Kg)	V_r
700	512.33	69.76
710	520.23	67.07

With linear interpolation:

$$T_2 = 702.74\ \text{K}$$

$$U_2 = 514.50\ \text{KJ/Kg}$$

· <u>Stage 03</u> : For <u>2-3</u>, heat addition at constant volume : $U_3 = \frac{Q_{23}}{m} + U_2$

$$Q_{23} = m\,(U_3 - U_2) \therefore$$

As we know all relevant variables at <u>Stage 01</u>, we can calculate the <u>mass</u>, m, of the fluid:

$$PV = nRT = \frac{m}{\overline{MW}} RT$$

$\overline{MW} \longrightarrow$ molecular weight

$\hookrightarrow 28.97\ Kg/Kgmole$

$$m = \frac{P_1 V_1}{R T_1} \overline{MW}$$

$$m = (100\ KPa)(0.014\,m^3)\left(\frac{1}{300K}\right)\left(\frac{Kgmole \cdot K}{8.314\ KJ}\right) \times \left(28.97\ \frac{Kg}{Kgmole}\right) \times \left(\frac{10^3\ N/m^2}{1\ KPa}\right)\left(\frac{1\ KJ}{10^3\ Nm}\right)$$

$$\boxed{m = 0.01626\ Kg}$$

Thus in

$$U_3 = \frac{Q_{23}}{m} + U_2 = \frac{11.35\ KJ}{0.01626\ Kg} + 514.50\ KJ/Kg = 1212.53\ KJ/Kg$$

From the Table:

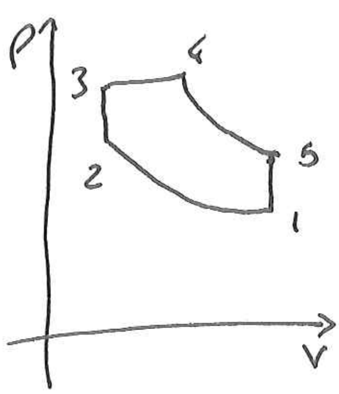
T(K)	H(KJ/Kg)	U(KJ/Kg)
1500	1635.97	1205.41
1520	1660.23	1223.87

Through linear interpolation:

$T_3 = 1507.71\ K$

$H_3 = 1645.33\ KJ/Kg$

- State 04: For constant pressure heat addition



$$Q_{34} = m\,(u_4 - u_3)$$

$$u_4 = \frac{Q_{34}}{m} + u_3$$

$$u_4 = \frac{11.35\ kJ}{0.01626\ kg} + 1645.33\ \frac{kJ}{kg} = \underline{2343.36\ kJ/kg}$$

From the table:

T(K)	u (kJ/kg)	v_r
2050	2314.6	2.555
2100	2377.7	2.356

Through linear interpolation:

$T_4 = 2072.79\ K$

$v_{r_4} = 2.4643$

- State 05: isentropic expansion

$$\frac{v_{r5}}{v_{r4}} = \frac{V_5}{V_4} \therefore v_{r5} = v_{r4}\frac{V_5}{V_4} = v_{r4}\frac{V_1}{V_4} = v_{r4}\left(\frac{V_1}{V_2}\,\frac{V_3}{V_4}\right)$$

$$\text{As } P_3 = P_4 \Rightarrow \frac{V_3}{V_4} = \frac{T_3}{T_4}$$

$$V_{r5} = V_{r4}\left(\frac{V_1}{V_2}\frac{T_3}{T_4}\right) = 2.4648\left(9 \times \frac{1507.71}{2072.79}\right)$$

$$\boxed{V_{r5} = 16.13}$$

From the table:

T (K)	U (KJ/Kg)	V_r
1140	880.35	16.946
1160	897.91	16.064

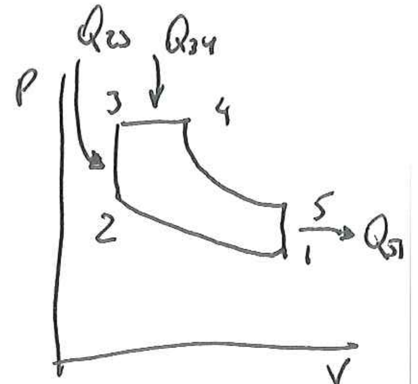
Through linear interpolation

$T_5 = 1158.50\,K$

$U_5 = 896.60\,KJ/Kg$

Now all $\underline{T_i}$ are calculated, and W_{cycle}/m:

$$\frac{W_{cycle}}{m} = \frac{Q_{cycle}}{m} = \frac{Q_{23} + Q_{34} - Q_{51}}{m}$$



$$\frac{W_{cycle}}{m} = \frac{22.70 - m(U_5 - U_1)}{m}$$

$$= \frac{22.70 - 0.01626\,(896.60 - 214.07)}{0.01626} = 713.53\,KJ/Kg$$

And the efficiency:

$$\eta = \frac{W_{cycle}}{Q_{in}} = \frac{11.60}{22.70} = 0.511 \hookrightarrow 51.1\%$$

$Q_{in} \downarrow Q_{34} + Q_{23}$

And the Mean Effective Pressure (MEP):

$$MEP = \frac{W_{cycle}}{V_1(1 - V_2/V_1)} = \frac{11.60\,KJ}{0.014\,m^3(1-1/9)} \times \frac{10^3\,Nm}{1\,KJ} \frac{1\,KPa}{10^3\,N/m^2}$$

$$MEP = 932.14\ KPa$$

Back from the Past - Or defining V_2

From the second law we know that for ideal gas:

$$\begin{cases} dU = TdS - PdV & (1) \\ dH = TdS + VdP & (2) \end{cases}$$

Replacing $dU = C_v dT$, $dH = C_p dT$ and $PV = RT$ in (1) and (2):

$$\begin{cases} dS = C_v \dfrac{dT}{T} + R\dfrac{dV}{V} & (3) \\ dS = C_p \dfrac{dT}{T} - R\dfrac{dP}{P} & (4) \end{cases}$$

with: $C_p + C_v = R$. Integrating (3-4) from (T_1, P_1) to (T_2, P_2) conditions

$$\begin{cases} S(T_2, V_2) - S(T_1, V_1) = \displaystyle\int_{T_1}^{T_2} C_v \frac{dT}{T} + R \ln\left(\frac{V_2}{V_1}\right) & (5) \\ S(T_2, P_2) - S(T_1, P_2) = \displaystyle\int_{T_1}^{T_2} C_p \frac{dT}{T} - R \ln\left(\frac{P_2}{P_1}\right) & (6) \end{cases}$$

Now, if we define

$$S^{\circ}(T) = \int_0^T C_p \frac{dT}{T}$$

as the entropy at 1 atm and temperature T.

Also assuming that temperature is continuous and differentiable through all domain:

$$\int_{T_1}^{T_2} C_p \frac{dT}{T} = \int_0^{T_2} C_p \frac{dT}{T} - \int_0^{T_1} C_p \frac{dT}{T} = S^{\circ}(T_2) - S^{\circ}(T_1)$$

Thus, replacing in ~~(5) and~~ (6):

$$S(T_2, P_2) - S(T_1, P_1) = S^{\circ}(T_2) - S^{\circ}(T_1) - R \ln\left(\frac{P_2}{P_1}\right) \quad (7)$$

~~And using the same as above in (5)~~:

$\cancel{S(T_2, P_2)} -$

Eqn (7) is valid if we assume that C_p is a function of T, i.e. $C_p = C_p(T)$ and we only need to integrate the $C_p(T)\frac{dT}{T}$ term. However if C_p is constant, the (5) and (6) can be easily integrated resulting in:

$$\begin{cases} S(T_2,V_2) - S(T_1,V_1) = C_v \ln\left(\frac{T_2}{T_1}\right) + R\ln\left(\frac{V_2}{V_1}\right) & (8) \\ S(T_2,P_2) - S(T_1,P_1) = C_p \ln\left(\frac{T_2}{T_1}\right) + R\ln\left(\frac{P_2}{P_1}\right) & (9) \end{cases}$$

Now for isentropic processes — $\underline{S_2 = S_1}$, Eqn. (8) becomes

$$C_v \ln\left(\frac{T_2}{T_1}\right) + R\ln\left(\frac{V_2}{V_1}\right) = 0 \qquad (\cdot 1/C_v)$$

$$\ln\left(\frac{T_2}{T_1}\right) = -\frac{R}{C_v}\ln\left(\frac{V_2}{V_1}\right) = \ln\left(\frac{V_1}{V_2}\right)^{R/C_v}$$

Since: $R = C_p - C_v$, $\gamma = C_p / C_v$ $\Big\}$ $\dfrac{R}{C_v} = \gamma - 1$

$$\left(\frac{T_2}{T_1}\right)_{s=const} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} \qquad (10)$$

Applying the same procedure to Eqn. (9):

$$\left(\frac{T_2}{T_1}\right)_{s=Const} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \qquad (11)$$

Now (10)=(11):

$$\left(\frac{P_2}{P_1}\right)_{s} = \left(\frac{V_1}{V_2}\right)^{\gamma} \qquad (12)$$

Eqns (10-12) - isentropic relations for ideal gas may also be rewritten as:

$$\boxed{\begin{aligned} T V^{\gamma-1} &= \text{constant} \\ T P^{\frac{1-\gamma}{\gamma}} &= \text{constant} \\ P V^{\gamma} &= \text{constant} \end{aligned}} \qquad (13)$$

In most cases specific heats are not constant, thus we need to use ~~...~~ Eqn (7),

$$S(T_2,P_2)-S(T_1,P_1)=S^\circ(T_2)-S^\circ(T_1)-R\ln\left(\frac{P_2}{P_1}\right) \quad (7)$$

and for an isentropic process 1-2:

$$0=S^\circ(T_2)-S^\circ(T_1)-R\ln(P_2/P_1)$$

$$S^\circ(T_2)=S^\circ(T_1)+R\ln(P_2/P_1) \quad (14)$$

Now ~~taking~~ ~~...~~ making P_2/P_1 explicit:

$$\exp\left[\frac{S^\circ(T_2)-S^\circ(T_1)}{R}\right]=\frac{P_2}{P_1}$$

$$P_2/P_1=\frac{\exp[S^\circ(T_2)/R]}{\exp[S^\circ(T_1)/R]} \quad (15)$$

The term $\exp[S^\circ/R]$ is the relative pressure $\underline{\underline{P_r}}$; therefore

$$\boxed{\left(\frac{P_2}{P_1}\right)_s=\frac{P_{r_2}}{P_{r_1}}} \quad (16)$$

So S° is a function of the temperature only, P_r is also a function of temperature only. P_r can be obtained from tabulated tables.

Now using the ideal-gas equation:

$$\frac{PV}{T} = \text{constant}$$

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \quad \therefore \quad \frac{V_2}{V_1} = \frac{T_2}{T_1}\,\frac{P_1}{P_2} = \frac{T_2}{T_1}\,\frac{P_{r1}}{P_{r2}}$$

$$V_2/V_1 = \frac{T_2 / P_{r2}}{T_1 / P_{r1}}$$

The term T/P_r is called relative specific volume v_r, and just as the relative pressure is a function of the temperature only:

$$\boxed{\left(\frac{V_2}{V_1}\right)_s = \frac{V_{R2}}{V_{R1}}} \qquad (17)$$