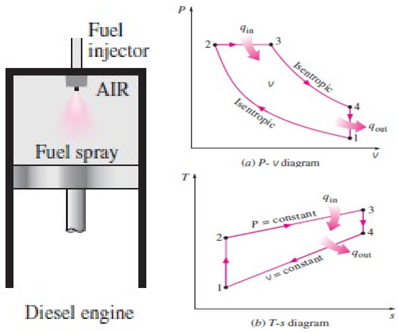
Q.1 Question 1

(a) The volume ratios of compression and expansion for a diesel engine are 15.3 and 7.5, respectively. The pressure and temperature at the beginning of the compression are 1 bar and 27 °C. Assume that the volume at the end of the isentropic compression is 1 m^3, determine:

(i) Mean effective pressure $\left(\mathrm{MEP} = \dfrac{\mathrm{W_{net}}}{\mathrm{V_{max}} - \mathrm{V_{min}}}\right)$; [7 marks]

Solution:

The problem gives:



- $\dfrac{V_1}{V_2} = 15.3 = r,\ \dfrac{V_4}{V_3} = 7.5;$
- $P_1 = 1\ bar;\ T_1 = 27^\circ C = 300.15\ K;\ V_2 = 1\ m^3$

The first step to solve the problem is to calculate T_2, P_2, T_3, T_4 and P_4:

[1/7] **1-2:** *adiabatic compression:*

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1} \rightarrow T_2 = 893.75\ K$$
$$P_1 V_1^{\gamma} = P_2 V_2^{\gamma} \rightarrow P_2 = 45.56\ bar$$

[0.5/7] **2-3:** *heat addition at constant pressure:*

$$\frac{V_2}{T_2} = \frac{V_3}{T_3} \rightarrow T_3 = T_2 \frac{V_3}{V_1}\frac{V_1}{V_2} = 1823.25\ K$$

[1/7] **3-4:** *adiabatic expansion:*

$$T_3 V_3^{\gamma-1} = T_4 V_4^{\gamma-1} \rightarrow T_4 = 814.37\ K$$
$$P_3 V_3^{\gamma} = P_4 V_4^{\gamma} \rightarrow P_4 = 2.71\ bar$$

[1.5/7] *Now calculating MEP:*

$$MEP = \frac{W_{net}}{V_{max} - V_{min}} = \frac{m\left[C_p\left(T_3 - T_2\right) - C_v\left(T_4 - T_1\right)\right]}{V_1 - V_2}$$

now we need to calculate the mass (m) via the equation of state of ideal gas at
[1/7] *state 2:*

$$m = \frac{P_2 V_2}{R T_2} MW = 17780.98 g \approx 17.78\ kg$$

[2/7] *with the mass of air, the MEP is 7.02 bar*

(ii) Cycle efficiency $\left(\eta_{\text{Diesel}} = \dfrac{\text{W}_{\text{net}}}{\text{Heat Supplied}}\right)$. [3 marks]

Solution:

[3/3] *The efficiency of the Diesel engine is given by:*

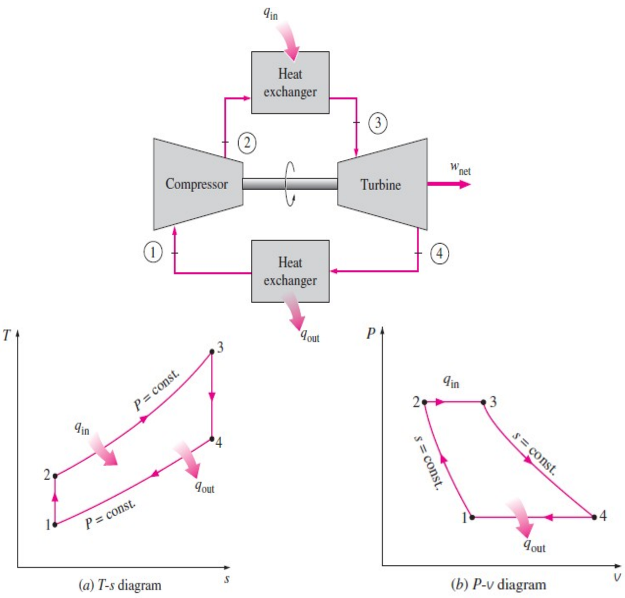
$$\eta_{Diesel} = \frac{W_{net}}{Heat\ Supplied} = \frac{m\left[C_p\left(T_3 - T_2\right) - C_v\left(T_4 - T_1\right)\right]}{mC_p\left(T_3 - T_2\right)} = 0.6048$$

(b) In an ideal air-standard Brayton cycle, air enters the compressor at 0.1 MPa and 15°C and leaves at 1 MPa. Assuming that the maximum temperature of the cycle is 1100°C:

(i) Sketch Pv (pressure × specific volume) and Ts (temperature × specific entropy) diagrams for the cycle, numbering each stage; [2 marks]

Solution:

[2/2] *Schematics, Pv and Ts diagrams:*



(ii) Calculate the temperature of the fluid leaving the compressor and the turbine; [2 marks]

Solution:

Calculating T_2 and T_4:

[1/2] **1-2:** *isentropic compression:*

$$T_1 P_1^{\frac{1-\gamma}{\gamma}} = T_2 P_2^{\frac{1-\gamma}{\gamma}} \rightarrow T_2 = 556.33\ K$$

[1/2] **3-4:** *isentropic expansion:*

$$T_3 P_3^{\frac{1-\gamma}{\gamma}} = T_4 P_4^{\frac{1-\gamma}{\gamma}} \rightarrow T_4 = 711.22\ K$$

(iii) Determine the efficiency of the cycle $\left(\eta_{\text{Brayton}} = \dfrac{\text{W}_{\text{net}}}{\text{Heat Supplied}}\right)$; [2 marks]

Solution:

[0.5/2] *Work required by the compressor*

$$W_C = h_2 - h_1 = C_p\left(T_2 - T_1\right) = 269.52\ kJ.kg^{-1}$$

[**0.5/2**] *And the work produced by the turbine:*

$$W_T = h_4 - h_3 = C_p\left(T_4 - T_3\right) = -665.24\ kJ.kg^{-1}$$

[**1/2**] *The efficiency is given by:*

$$\eta_{Brayton} = \frac{W_{net}}{heat\ supplied} = \frac{|W_T + W_C|}{C_p\left(T_3 - T_2\right)} = 0.4821$$

(iv) For the efficiency of the compressor and the turbine of 80% and 85%, respectively, and the pressure drop between the compressor and the turbine of 15 kPa, calculate the work in the compressor and turbine, and the efficiency of the cycle. [4 marks]

Solution:
As the cycle is no longer ideal, temperature after the turbine calculated in (b) is T_{2s} (i.e., calculated from isentropic compression) and actual T_2 can be recal-
[**1/4**] *culated as*

$$\eta_c^{(actual)} = \frac{h_{2s} - h_1}{h_2 - h_1} = \frac{T_{2s} - T_1}{T_2 - T_1} = 0.80\ \rightarrow T_2 = 623.38\ K$$

And the flow across the turbine,

$$\eta_T^{(actual)} = \frac{h_3 - h_4}{h_3 - h_{4s}} = \frac{T_3 - T_4}{T_3 - T_{4s}}$$

[**0.5/4**] *However: $\Delta P = P_2 - P_3 = 15\ kPa \rightarrow P_3 = 9.85\ bar$. Thus*

$$T_3 P_3^{\frac{1-\gamma}{\gamma}} = T_{4s} P_4^{\frac{1-\gamma}{\gamma}} \rightarrow T_{4s} = 714.30\ K$$

[**0.5/4**] *Now calculating T_4,*

$$\eta_T^{(actual)} = \frac{T_3 - T_4}{T_3 - T_{4s}} \quad \rightarrow \quad T_4 = 813.13\ K$$

[**1/4**] *And the power in the turbine and compressor:*

$$W_C^{(actual)} = Cp\left(T_2 - T_1\right) = 336.91\ kJ.kg^{-1}$$
$$W_T^{(actual)} = Cp\left(T_4 - T_3\right) = -562.82\ kJ.kg^{-1}$$

[**1/4**] *And the efficiency:*

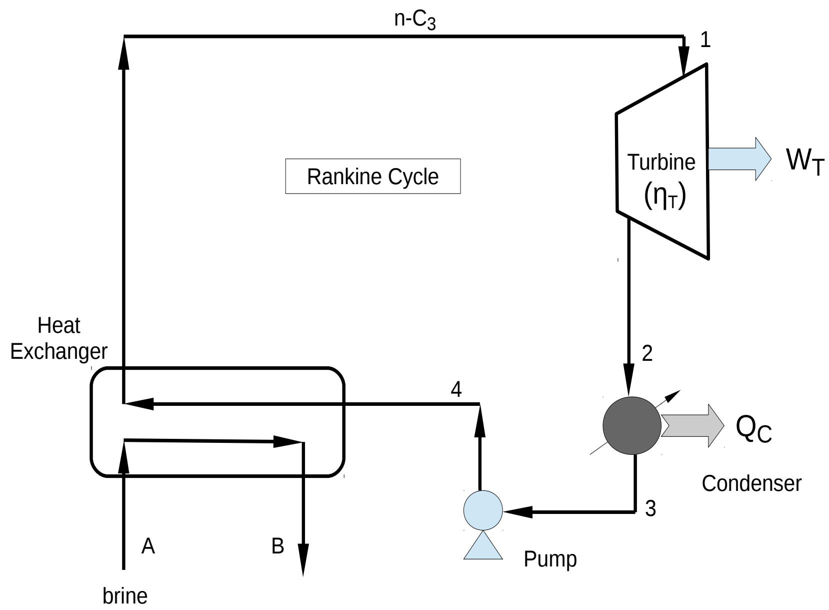
$$\eta_{Brayton}^{(actual)} = \frac{W_{net}}{Q_{in}} = 0.30$$

Assume that air behaves as an ideal gas with the following properties: MW = 29 g.mol^{-1}, C_p = 1.005 kJ.(kg.K)$^{-1}$ and C_v = 0.718 kJ.(kg.K)$^{-1}$, where MW is the molar mass and C_p and C_v are the heat capacities at constant pressure and volume, respectively. Also, given the following relations for isentropic operations:

$$TV^{\gamma-1} = \text{constant},\ \ TP^{\frac{1-\gamma}{\gamma}} = \text{constant and}\ \ PV^{\gamma} = \text{constant},\ \ \text{with}\ \gamma = \frac{C_p}{C_v}$$

Q.2 Question 2

(a) A geothermal power station operating with Rankine cycle uses propane (n-C_3) as working fluid to produce power (W_T) in a turbine with efficiency (η_T) of 90%. Propane is vaporised by geothermal water (i.e., brine, $A - B$ in the diagram) at 90°C. After condensed, propane is driven to a heat exchanger and the cycle continues. The mass flow rate of propane ($\dot{m}_{C3}$) is 250 kg.s^{-1} and the heat capacity at constant pressure (C_p) of brine is 3565.5 J.(kg.K)$^{-1}$. Conditions for propane and brine flows are described in the Table below.



Stage	P (bar)	T (^{o}C)	State	Quality	h (kJ.kg^{-1})	s (kJ.(kg.K)$^{-1}$)
1	40	100	**(a)**	–	**(b)**	**(c)**
2	10	–	–	**(d)**	**(e)**	**(f)**
3	–	–	**(g)**	–	**(h)**	**(i)**
4	**(j)**	–	**(k)**	–	**(l)**	–
A	–	90	–	–	–	–
B	–	30	–	–	–	–

Table 1: *Conditions of propane and brine in the cycle.*

(i) In this Table, determine *(a)-(l)*. [6 marks]

Solution:

In order to fill the Table we need to calculate the thermodynamic properties for each stage of the cycle:

Stage 1: *At* $P_1 = 16$ *bar,* $T_1 = 100^{o}C > T_{sat}(P_1) = 93.38^{o}C$. *Therefore the*
[0.5/6] *fluid is at* **superheated state (SHS)** . *From the superheated table for n-*C_3 *at* P_1 *and* T_1*, we can obtain:*
[0.5/6] $\mathbf{h_1 = 549.7\ kJ.kg^{-1}}$ *and*
[0.5/6] $\mathbf{s_1 = 1.70\ kJ.(kg.K)^{-1}}$.

Stage 2: *At* $P_2 = 10$ *bar, the fluid is wet vapour after the isentropic expansion. We should first calculate the quality of the vapour in an ideal expansion (using values of entropy/enthalpy obtained from the saturated n-*C_3 *table at* P_2.

$$x_{2s} = \frac{s_{2s} - s_f}{s_g - s_f} = \frac{1.7 - 0.618}{1.723 - 0.618} = 0.9792$$

now to calculate the ideal enthalpy,

$$x_{2s} = 0.9792 = \frac{h_{2s} - h_f}{h_g - h_f} = \frac{h_{2s} - 166.1}{497.9 - 166.1} \iff h_{2s} = 491\frac{kJ}{kg}$$

[0.5/6] *As the efficiency of the turbine is of 90%,*

$$\eta_{Turbine} = 0.90 = \frac{h_2 - h_1}{h_{2s} - h_1} \iff \mathbf{h_2} = \mathbf{496.87\frac{kJ}{kg}}$$

[0.5/6] *Calculating the actual quality using the expressions above,* $\mathbf{x_2 = 0.9969}$ *and*
[0.5/6] $\mathbf{s_2 = 1.7196}$ $kJ.(kg.K)^{\mathbf{-1}}$.

Stage 3: *At* $P_3 = P_2 = 10$ *bar, the fluid leaving the condenser towards the*
[0.5/6] *pump is* **saturated liquid** *, and the enthalpy and specific volume are the same of the liquid phase obtained from the saturated table:*
[0.5/6] $\mathbf{h_3} = h_f\,(P = 10\ bar) = \mathbf{166.1\ kJ.kg^{-1}}$
[0.5/6] $\mathbf{s_3} = s_f\,(P = 10\ bar) = \mathbf{0.618\ kJ.(kg.K)^{-1}}$
$v_3 = v_f\,(P = 10\ bar) = 2.043 \times 10^{-3}\ m^3.kg^{-1}$

Stage 4: *The fluid leaving the pump suffered a isentropic compression and we*
[0.5/6] *assumed it is incompressible with* $\mathbf{P_4 = P_1 = 40}$ *bar. As there is no heat loss in the pump, we can assume* $dH \approx VdP$*, therefore*

$$\mathbf{h_4} = h_3 + v_3\,(P_4 - P_3) == \mathbf{172.23\frac{kJ}{kg}}$$

[0.5/6] *As* $h_4 < h_{f,4}\,(P = 40\ bar) = 401\frac{kJ}{kg}$*, the fluid is a* **subcooled liquid state**
[0.5/6] **(SLS)**.

Thus the Table becomes:

Stage	P (bar)	T (^oC)	State	Quality	h ($kJ.kg^{-1}$)	s ($kJ.(kg.K)^{-1}$)
1	*40*	*100*	**SHS**	–	**549.7**	**1.70**
2	*10*	–	–	**0.9969**	**496.87**	**1.7196**
3	–	–	**Sat Liq.**	–	**166.1**	**0.618**
4	**40**	–	**SLS**	–	**172.23**	–
A	–	*90*	–	–	–	–
B	–	*30*	–	–	–	–

(ii) Calculate the power produced by the turbine (W_T) and the heat extracted in the condenser (Q_C) in MW. [2 marks]

Solution:

$$\mathbf{W_T} = \dot{m}_{C3}\left(h_2 - h_1\right) = -13207.5\frac{kJ}{s} = \mathbf{13.21MW}$$

[1/2]

$$\mathbf{Q_C} = \dot{m}_{C3}\left(h_3 - h_2\right) = -82692.5\frac{kJ}{s} = \mathbf{82.69MW}$$

[1/2]

(iii) Calculate the mass flow rate of brine (A-B) in $kg.s^{-1}$. [2 marks]

Solution:

[1/2] *The heat extracted by the n-C_3 $\left(\dot{Q}_{HE,nC3}\right)$ fluid in the heat exchanger can be easily calculated by*

$$\mathbf{\dot{Q}_{HE,nC3}} = \dot{m}_{C3}\left(h_1 - h_4\right) = \mathbf{94367.5\frac{kJ}{s}}$$

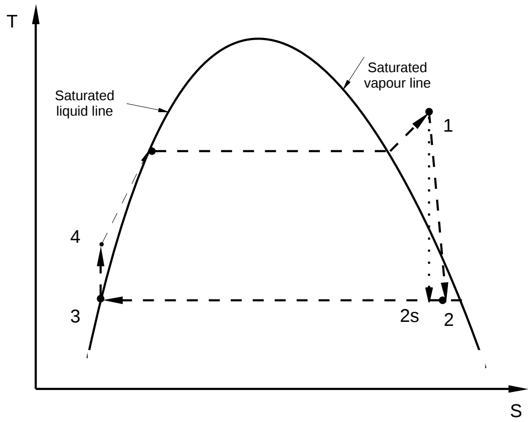
[1/2] *Assuming all heat is transferred from the brine to the propane (i.e., no heat losses to the environment),*

$$\dot{Q}_{HE,nC3} + \dot{Q}_{HE,Brine} = 0$$
$$\dot{Q}_{HE,Brine} = -94367.5 = \dot{m}_{Brine}C_{p,Brine}\left(T_B - T_A\right) \quad \rightarrow \quad \dot{\mathbf{m}}_{Brine} = \mathbf{441.12}\ kJ.kg^{-\mathbf{1}}$$

(iv) Sketch the Ts (temperature $\times$ specific entropy) diagram for the process indicating the liquid and vapour saturated lines and each stage of the n-C_3 Rankine cycle. [3 marks]

Solution:

[3/3]



To solve this problem, you should assume that the saturated liquid streams are incompressible, and therefore $dh = vdP$ (where h, v and P are specific enthalpy, volume and pressure, respectively). Quality of the vapour is expressed as

$$x_j = \frac{\Psi_j - \Psi_f}{\Psi_g - \Psi_f} \quad \text{with } \Psi = \left\{h, s\right\},$$

where s is the entropy. Efficiency of the turbine (η_{Turbine}) is given by,

$$\eta_{\text{Turbine}} = \frac{h_2 - h_1}{h_{2s} - h_1}$$

where h_{2s} is the enthalpy of stream 2 assuming ideal turbine performance (i.e., reversible expansion).

(b) Air contained in a piston-cylinder system undergoes three consecutive processes,

- Process 1–2: Isobaric cooling with P_1=69 kPa and V_1=0.11 m^3;
- Process 2–3: Isochoric heating with P_3=345 kPa;
- Process 3–1: Polytropic expansion, with $PV = constant$.

(i) Calculate V_2 (in m^3). [2 marks]
Solution:
For Process 2–3: $V_2 = V_3$. *However the expansion 3–1 follows* $PV = constant$,

$$P_1V_1 = P_3V_3 \Longrightarrow V_3 = \frac{P_1V_1}{P_3} = \mathbf{0.022}\ m^{\mathbf{3}} = \mathbf{V_2}$$

[2/2]

(ii) Calculate the work (in kJ) for each process. [3 marks]
Solution:
Process 1–2:

$$\mathbf{W_{1-2}} = \int_{V_1}^{V_2} PdV = P\left(V_2 - V_1\right) = -6072J \Rightarrow -\mathbf{6.072kJ}$$

[1/3] *Process 2–3:* $V_2 = V_3 \Longrightarrow \mathbf{W}_{2-3} = \mathbf{0}$

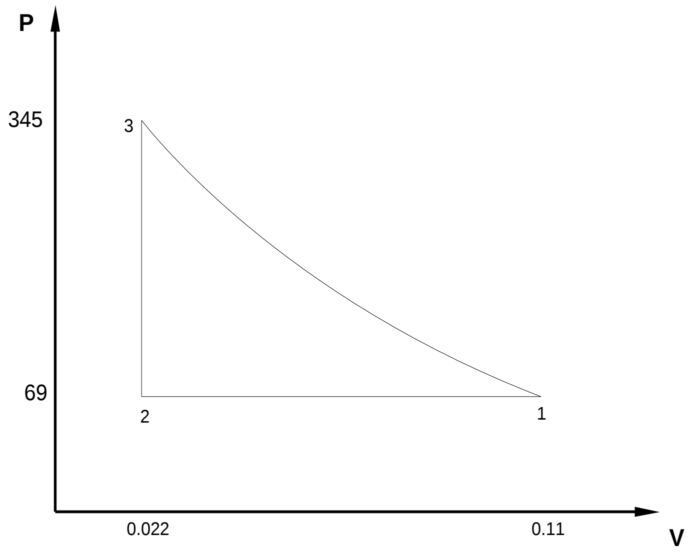
[1/3] *Process 3–1:* $PV = C$

$$\mathbf{W_{31}} = \int_{V_3}^{V_1} PdV = \int_{V_3}^{V_1} \frac{C}{V} dV = P_1V_1 \ln\frac{V_1}{V_3} = 12220J \Rightarrow \mathbf{12.22kJ}$$

[1/3]

(iii) Sketch the PV (pressure × volume) diagram for these processes. [2 marks]
Solution:

[2/2]


P
345
69
3
2
1
0.022
0.11
V

Q.3 Question 3

(a) The energy conservation equation for a steady flow device with one inlet and one outlet can be written in the form:

$$\dot{Q} - \dot{W}_s = \dot{m}\left(c_p T_{\text{outlet}} + \frac{u_{\text{outlet}}^2}{2}\right) - \dot{m}\left(c_p T_{\text{inlet}} + \frac{u_{\text{inlet}}^2}{2}\right).$$

(i) Explain what the fluxes on the right-hand side of this equation represent. [2 marks]

Solution:

[1/2] $\dot{m}c_pT = \dot{m}h$ *is a flux of enthalpy;*

[1/2] $\frac{1}{2}\dot{m}u^2$ *is a flux of kinetic energy.*

(ii) What assumptions are required to derive this equation? [4 marks]

Solution:

- *The flow is steady and continuous;*
- *The rate of heat addition and the rate of shaft work is are both constant;*
- *The velocity and temperature profiles are uniform across the inlet and outlet cross section;*
- *The change in gravitational energy is negligible;*
- *No energy transfer due to viscous effects, electrodynamics, magnetism, surface tension, nuclear or chemical reactions.*

[4/4] *One mark for each (up to four).*

(b) The equation given above is valid for steady flow devices with one inlet and one outlet. Under the same modelling assumptions, state a modified version of this formula that is valid for steady flow devices with one inlet (whose properties are labelled 1) and two outlets (labelled 2 and 3). Derive an equation that represents steady mass conservation in this case. [4 marks]

Solution:

Energy conservation:

$$\dot{Q} - \dot{W}_s = \dot{m}_2\left(c_p T_2 + \frac{u_2^2}{2}\right) + \dot{m}_3\left(c_p T_3 + \frac{u_3^2}{2}\right) - \dot{m}_1\left(c_p T_1 + \frac{u_1^2}{2}\right).$$

[3/4]

Mass conservation:

$$\dot{m}_1 = \dot{m}_2 + \dot{m}_3.$$

[1/4]

(c) Gas in a steady flow device with a circular inlet and two circular outlets does shaft work at a rate of 400 kW. The remaining known inlet and outlet properties are:

Property	Inlet 1	Outlet 2	Outlet 3	Units
Diameter, d	0.05	0.2	0.5	m^2
Volume flux, q	1.8	0.5	20	m^3/s
Temperature, T	80	70	30	°C
Pressure, p	200		110	kPa

If the gas behaves like an ideal gas with a gas specific gas constant $R = 250$ J/(kg.K) and specific heat capacity $c_p = 1000$ J/(kg.K), then:

(i) Calculate the fluid velocity through each inlet and outlet; [3 marks]
Solution:
[1/3] *The area of the inlet* $A = \pi d^2/4$ *and therefore* $A_1 = 7.853 \times 10^{-3}\,m^2$, $A_2 = 3.141 \times 10^{-2}\,m^2$ *and* $A_3 = 1.963 \times 10^{-1}\,m^2$.
[1/3] *The velocity through each inlet and outlet* $u = q/A$.
[1/3] *Therefore* $u_1 = 178.25\,m/s$, $u_2 = 50.930\,m/s$ *and* $u_3 = 10.186\,m/s$.

(ii) Calculate the gas pressure at outlet 2; [5 marks]
Solution:
The density at inlet 1 and outlet 3 are

$$\rho_1 = \frac{p_1}{RT_1} = \frac{400000}{250 \times (80 + 273.15)} = 4.5307\,kg/m^3,$$
$$\rho_3 = \frac{p_3}{RT_3} = \frac{120000}{250 \times (30 + 273.15)} = 1.5834\,kg/m^3.$$

[1/5]

[1/5] *The mass flux at inlet 1 is* $\dot{m}_1 = \rho_1 q_1 = 4.5307 \times 1.4 = 6.3429\,kg/s$, *while the mass flux at outlet 2 is* $\dot{m}_3 = \rho_3 q_3 = 1.5834 \times 2 = 3.1667\,kg/s$.
[1/5] *Mass conservation gives* $\dot{m}_2 = \dot{m}_1 - \dot{m}_3 = 6.3429 - 3.1667 = 3.1762\,kg/s$.
[1/5] *The density at outlet 2 is* $\rho_2 = \dot{m}_2/q_2 = 3.1762/1.6 = 1.9851\,kg/m^3$.
[1/5] *The pressure at outlet 2 is* $p_2 = \rho_2 R T_2 = 1.9851 \times 250 \times (20 + 273.15) = 145500\,Pa$.

(iii) Determine whether the steady flow device heats the gas or whether the gas heats the steady flow device. [2 marks]
Solution:
Rearranging the energy equation

$$\begin{aligned}\dot{Q} =& \dot{m}_2\left(c_pT_2 + \frac{u_2^2}{2}\right) + \dot{m}_3\left(c_pT_3 + \frac{u_3^2}{2}\right) - \dot{m}_1\left(c_pT_1 + \frac{u_1^2}{2}\right) + \dot{W}_s \\ =&(3.1762 \times 294400) + (3.1667 \times 303200) + (6.3429 \times 369000) + 400000 \\ =&960164.279690529 + 935211.868120291 - 2340770.89614347 + 400000 \\ =& -45394\,W.\end{aligned}$$

[1/2]

[1/2] *The rate of heat addition to the gas is* $-45\,kW$ *and therefore the gas gives off heat to the surroundings at a rate of* $45\,kW$.

Q.4 Question 4

(a) A change in enthalpy $\mathrm{d}h$, entropy $\mathrm{d}s$ and pressure $\mathrm{d}p$ in an ideal gas are related through

$$\mathrm{d}h = T\mathrm{d}s + V\mathrm{d}p,$$

where T is the absolute temperature and V is the specific volume.

Show that the change in entropy between the inlet and outlet of a compressor is given by

$$s_{\text{outlet}} - s_{\text{inlet}} = c_p \ln\left(\frac{T_{\text{outlet}}}{T_{\text{inlet}}}\right) - R\ln\left(\frac{p_{\text{outlet}}}{p_{\text{inlet}}}\right).$$

Here c_p is the specific heat capacity at constant pressure and R is the specific gas constant, which can both be assumed to be constant. [6 marks]

Solution:

From the ideal gas equation

$$V = \frac{RT}{p},$$

while the gas specific enthalpy satisfies

$$\mathrm{d}h = c_p\,\mathrm{d}T.$$

[2/6]

If we eliminate the change in specific enthalpy $\mathrm{d}h$*, the specific volume* V *and rearrange, then*

$$\mathrm{d}s = c_p\frac{\mathrm{d}T}{T} - R\frac{\mathrm{d}p}{p}.$$

[1/6]

Integrating from the inlet condition to the outlet condition

$$\int_{s_{inlet}}^{s_{outlet}} \mathrm{d}s = \int_{T_{inlet}}^{T_{outlet}} c_p\frac{\mathrm{d}T}{T} - \int_{p_{inlet}}^{p_{outlet}} R\frac{\mathrm{d}p}{p}.$$

Assuming c_p *and* R *are constant, then*

$$[s]_{s_{inlet}}^{s_{outlet}} = c_p\left[\ln T\right]_{T_{inlet}}^{T_{outlet}} - R\left[\ln p\right]_{p_{inlet}}^{p_{outlet}},$$

i.e.

$$s_{outlet} - s_{inlet} = c_p \ln\left(\frac{T_{outlet}}{T_{inlet}}\right) - R\ln\left(\frac{p_{outlet}}{p_{inlet}}\right).$$

[3/6]

(b) If the flow through the compressor is isentropic, then show that

$$\frac{T_{\text{outlet}}}{T_{\text{inlet}}} = \left(\frac{p_{\text{outlet}}}{p_{\text{inlet}}}\right)^{1-1/\gamma},$$

where γ is the ratio of the specific heat capacities. [4 marks]

Solution:

If the flow is isentropic, then $s_{outlet} - s_{inlet} = 0$ and

$$\ln\left(\frac{T_{outlet}}{T_{inlet}}\right) = \frac{R}{c_p}\ln\left(\frac{p_{outlet}}{p_{inlet}}\right).$$

[1/4]

By definition

$$\frac{R}{c_p} = \frac{c_p - c_v}{c_p} = 1 - \frac{c_v}{c_p} = 1 - \frac{1}{\gamma}.$$

[1/4]

Using properties of logarithms

$$\ln\left(\frac{T_{outlet}}{T_{inlet}}\right) = \ln\left[\left(\frac{p_{outlet}}{p_{inlet}}\right)^{1-1/\gamma}\right].$$

[1/4]

Finally exponentiating both sides

$$\frac{T_{outlet}}{T_{inlet}} = \left(\frac{p_{outlet}}{p_{inlet}}\right)^{1-1/\gamma}.$$

[1/4]

(c) An ideal gas with $R = 260\,\mathrm{J/(kg\,K)}$ and $c_p = 900\,\mathrm{J/(kg\,K)}$ flows through a well designed compressor in which the steady flow energy equation is given by

$$\dot{W}_s = \dot{m}\left(h_{\text{inlet}} - h_{\text{outlet}}\right).$$

The compressor is doing work on the gas at a rate of 500 kW, while the mass flux through the compressor is 4 kg/s. If the compressor has an inlet with pressure $p_{\text{inlet}} = 100\,\mathrm{kPa}$ and temperature $T_{\text{inlet}} = 25°\mathrm{C}$, and an outlet with pressure $p_{\text{outlet}} = 350\,\mathrm{kPa}$, then:

(i) Calculate the actual gas temperature at the outlet. [3 marks]

Solution:

As $\mathrm{d}h = c_p\mathrm{d}T$, for the compressor

$$\dot{W}_s = \dot{m}c_p\left(T_{inlet} - T_{outlet}\right),$$

and hence

$$T_{outlet} = T_{inlet} - \frac{\dot{W}_s}{\dot{m}c_p} = (25 + 273.15) - \frac{-500000}{4 \times 900} = 437.04\,\mathrm{K} = 138.9°\mathrm{C}.$$

[3/3]

(ii) Determine the isentropic efficiency of the compressor. [7 marks]

Solution:

The predicted gas temperature assuming isentropic compression is

$$T_{outlet,isentropic} = T_{inlet}\left(\frac{p_{outlet}}{p_{intlet}}\right)^{1-1/\gamma}.$$

[1/7]

However, to use this formula we first must calculate the ratio of specific heats γ. *The specific heat capacity at constant volume*

$$c_v = c_p - R = 900\,\mathrm{J/(kg\,K)} - 260\,\mathrm{J/(kg\,K)} = 640\,\mathrm{J/(kg\,K)}.$$

Therefore

$$\gamma = \frac{c_p}{c_v} = \frac{900\,\mathrm{J/(kg\,K)}}{640\,\mathrm{J/(kg\,K)}} = \frac{45}{32} = 1.4063.$$

[1/7]

Now we can calculate the predicted isentropic compression temperature

$$T_{outlet,isentropic} = 298.15\,\mathrm{K} \times \left(\frac{350000\,\mathrm{Pa}}{100000\,\mathrm{Pa}}\right)^{13/45} = 428.164\,\mathrm{K}.$$

[2/7]

The isentropic efficiency of the compressor

$$\begin{aligned}\eta_s &= 100\% \times \left(\frac{T_{outlet,isentropic} - T_{inlet}}{T_{outlet,actual} - T_{inlet}}\right)\\ &= 100\% \times \left(\frac{428.164 - 298.15}{437.039 - 298.15}\right)\\ &= 100\% \times 0.936.\end{aligned}$$

[3/7] *Therefore the compressor has an isentropic efficiency of* 94%.

Q.5 Question 5

An air stream with relative humidity of 0.01 kg water/kg dry air passes through a duct at the rate of $1.2\,\mathrm{kg\,s^{-1}}$. In the duct the air is first heated from 5°C to 20°C by heating coils and then humidified via the injection of hot steam. Air leaves the device at 25°C with a relative humidity of 60%. The air pressure is 101.325 kPa throughout.

(a) Determine the rate of heat addition in the heating section. [5 marks]

Solution:

In the simple heating section no water is added so $\omega_2 = \omega_1 = 0.01$ kg water/kg dry air.

The enthalpies have contributions from dry air and water vapour with

$$\begin{aligned} h_1 =& c_p T_1 + \omega_1 h_{g_1} = 1.005 \times (5 + 273.15) + (0.01 \times 2510.1) = 304.64\,kJ/kg \\ h_2 =& c_p T_2 + \omega_2 h_{g_2} = 1.005 \times (20 + 273.15) + (0.01 \times 2537.4) = 320.00\,kJ/kg. \end{aligned}$$

[3/5]

Conservation of energy implies the rate of heat addition

$$\dot{Q} = \dot{m}_a\left(h_2 - h_1\right) = 1.2 \times (320.00 - 304.64) = 18.42\,kJ/s.$$

[2/5]

(b) Determine the specific humidity of the air after the humidifying section. [2 marks]

Solution:

[2/2] *From the psychrometric chart $\omega_3 = 0.012$ kg water/kg dry air.*

(c) Determine the mass flow rate of steam required in the humidifying section. [5 marks]

Solution:

Conservation of water vapour implies

$$\dot{m}_a\omega_2 + \dot{m}_w = \dot{m}_a\omega_3.$$

[2/5]

Rearranging this expression the mass rate of steam required

$$\dot{m}_w = \dot{m}_a\left(\omega_3 - \omega_2\right) = 1.2 \times (0.012 - 0.01) = 0.0024\,kg/s.$$

[3/5]

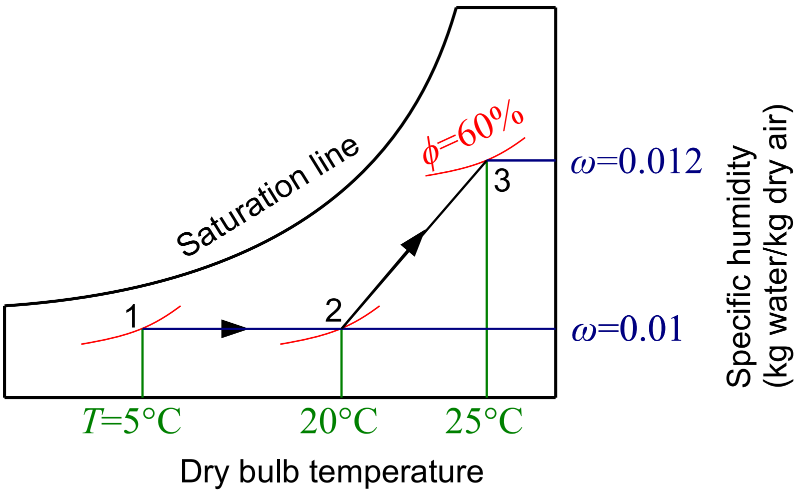
(d) Explain why adding steam may be beneficial in this process. [2 marks]

Solution:

As temperature increases the lower limit of the humidity range acceptable for human comfort also increases. Therefore if the temperature is increased and steam is not added (to increase the humidity), then the resulting air stream may have a humidity level that is uncomfortable for humans. **[3/3]**

(e) Sketch and identify the evolution of the different stages of this process on rough axes of a psychrometric chart. [6 marks]

Solution:



[**6**/**6**] *Simple heating occurs between 1 and 2. Humidification occurs between 2 and 3.*

You may assume that $c_p = 1005\,\mathrm{J/(kg\ K)}$, $R_a = 287\,\mathrm{J/(kg\ K)}$, while the enthalpy of saturated water vapour is 2510.1 kJ/kg at 5°C and 2537.4 kJ/kg at 20°C.

END OF PAPER