Solution of the Problems – Exam (May 2012/13)

Question 1: Reheat Rankine cycle with two turbines:

(a) In order to fill the Table we need to calculate the thermodynamic properties for each stage of the cycle

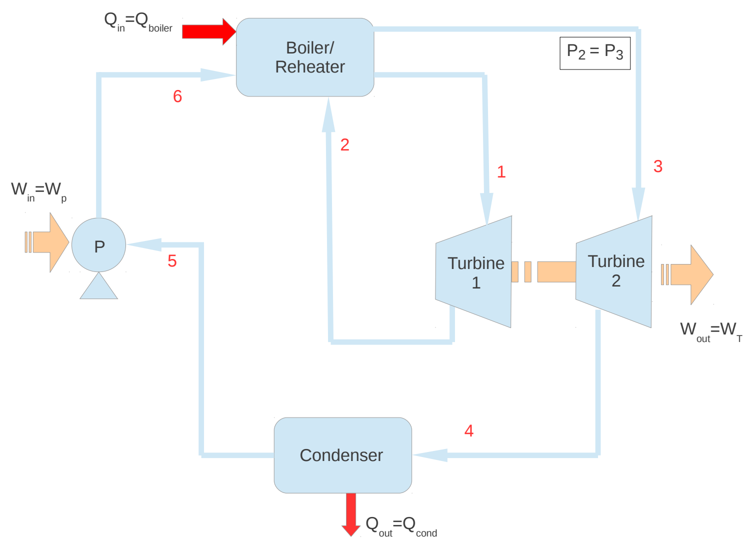


Figure 1: Reheat Rankine cycle with 2 turbines.

Stage 1: The fluid leaving the boiler towards the first turbine is at 40 bar and 370°C. This is well above the saturation temperature ($T_{\text{sat}} = 250.3$°C) and we can thus assume that the fluid is a superheated steam. At such pressure, the superheated steam tables (SHST) give,

T (°C)	**H (kJ/kg)**	**S (kJ/(kg.K))**
350	3092.5	6.582
400	3213.6	6.769

Thus, with linear interpolation at $T_1 = 370$°C: $H_1 = 3140.94\frac{kJ}{kg}$ and $S_1 = 6.6568\frac{kJ}{kg.K}$

Stage 3: The reheated steam is at 7 bar ($T_{\text{sat}} = 165$°C) and 370°C is also superheated and from SHST,

T (°C)	**H (kJ/kg)**	**S (kJ/(kg.K))**
350	3163.7	7.473
400	3268.7	7.635

At $T_1 = 370$°C: $H_3 = 3205.7\frac{kJ}{kg}$ and $S_3 = 7.5378\frac{kJ}{kg.K}$

Stage 2: After an isentropic expansion in the first turbine, therefore

$$P_{2s} = P_3 = 7 \text{ bar} \quad \text{and} \quad S_{2s} = S_1 = 6.6568 \frac{kJ}{kg.K}$$

The properties of the fluid can be calculated from the saturated steam tables:

	(kJ/kg)		(kJ/(kg.K))
H_f	697.1	S_f	1.9918
H_{fg}	2064.9	S_{fg}	4.7134
H_g	2762.0	S_g	6.7052

In order to calculate H_2, we need to calculate H_{2s} first,

$$H_{2s} = H_f + x_{2s} H_{fg}$$

Thus we should start calculating the quality of the steam flow, x_{2s}, given by

$$x_{2s} = \frac{S_{2s} - S_f}{S_{fg}} = \frac{6.6568 - 1.9918}{4.7134} = 0.9897$$

And now,

$$H_{2s} = 697.1 + 0.9897 \times 2064.9 = 2554.84 \frac{kJ}{kg}$$

We can finally calculate H_2 as,

$$H_2 = H_1 - (H_1 - H_{2s})\, \eta_{T1} = 3140.94 - (3140.94 - 2754.84) \times 0.84 = 2816.61 \frac{kJ}{kg}$$

Stage 4: The fluid that left the second turbine is a saturated steam at 0.10 bar with:

$$P_{4s} = 0.10 \text{ bar} \quad \text{and} \quad S_{4s} = S_3 = 7.5378 \frac{kJ}{kg.K}$$

From the saturated steam tables:

	(kJ/kg)		(kJ/(kg.K))		(m^3/kg)
H_f	191.8	S_f	0.649	V_f	0.0101
H_{fg}	2392.8	S_{fg}	7.501		
H_g	2584.7	S_g	8.150	V_g	14.67

The quality of the steam is

$$x_{4s} = \frac{S_{4s} - S_f}{S_{fg}} = \frac{7.5378 - 0.649}{7.501} = 0.9184$$

and the enthalpy,

$$H_{4s} = H_f + x_{4s} H_{fg} = 191.8 + 0.9184 \times 2392.8 = 2389.35 \frac{kJ}{kg}$$

Stage 5: The fluid after the condenser is a saturated water with the following characteristics:

$$P_5 = 0.10 \text{ bar}, \quad x_5 = 0.0, \quad V_5 = V_f = 0.0101\frac{m^3}{kg}, \quad H_5 = 191.8\frac{kJ}{kg} \text{ and } S_5 = 0.649\frac{kJ}{kg.K}$$

Stage 6: Finally, the fluid leaving the isentropic boiler feed pump has the following characteristics:

$$P_{6s} = P_6 = 40 \text{ bar}, \quad S_{6s} = S_5 = S_f = 0.649\frac{kJ}{kg.K} \text{ and } \; H_{6s} = H_{fg} = 1712.9\frac{kJ}{kg}$$

As the efficiency of the pump is 61%:

$$H_6 = H_5 + V_5\frac{(P_6 - P_5)}{\eta_P} = 191.8\frac{kJ}{kg} + \frac{0.0101\frac{m^3}{kg} \times (40 - 0.10)\,(\text{ bar})}{0.610} = 257.86\frac{kJ}{kg}$$

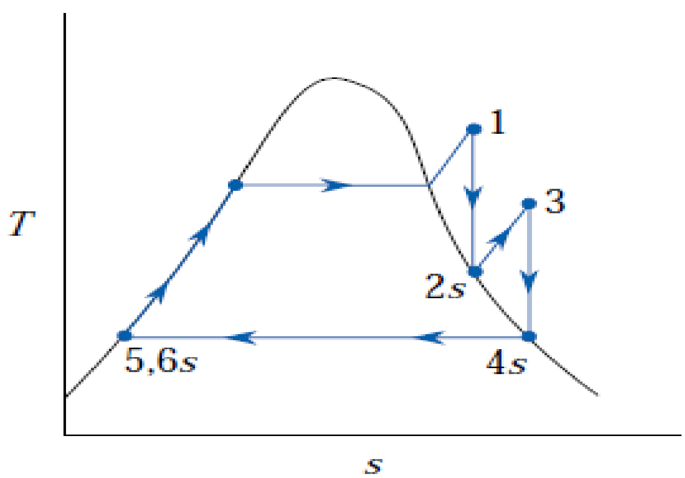
Thus the Table becomes:

Stage	**P (bar)**	**T (^{o}C)**	**State**	**H (kJ.kg^{-1})**	**S (kJ.(kg.K)$^{-1}$)**
1	40	370	superheated steam	(a) 3140.94	(b) 6.6568
2	–	–	(c) saturated steam	–	–
3	7	370	superheated steam	(d) 3205.7	(e) 7.5358
4	0.10	–	–	–	–
5	0.10	–	(f) saturated liquid(or water)	(g) 191.8	(h) 0.649
6	40	–	(i) saturated liquid(or water)	(j) 257.86	–

(b) The thermal efficiency of the cycle is:

$$\begin{aligned}
\eta_{\text{Thermal}} &= \frac{(H_1 - H_{2s})\,\eta_{\text{T1}} + (H_3 - H_{4s})\,\eta_{\text{T2}} - V_5\,(P_6 - P_5)\,\eta_{\text{P}}^{-1}}{(H_1 - H_6) + (H_3 - H_2)} \\
&= \frac{(3140.94 - 2754.84)\,0.84 + (3205.7 - 2389.35)\,0.80 - \dfrac{0.0101\,(40 - 0.10)}{0.610}\dfrac{100}{1.0}}{(3140.44 - 257.86) + (3205.7 - 2816.61)} \\
&= 0.2785
\end{aligned}$$

(c) Sketch of the Ts diagram


1
3
T
2s
5,6s
4s
s

Question 2:

(a)

$$421 \text{ billion kWh} = 421 \times 10^{12}\ \text{J.s}^{-1} \times 3600\ \text{s} = 1.5 \times 10^{18}\ \text{J of electricity.}$$

Efficiencies of generation depend on turbine performance not on whether nuclear and or chemical fuels were used, so the electricity above must have been obtained from about:

$$\left(1.5 \times 10^{18}/0.35\right)\ \text{J of heat} = 4.3 \times 10^{18}\ \text{J of heat}$$

If this had been raised from natural gas, the carbon dioxide release would have been:

$$\left[4.3 \times 10^{18}\ \text{J}/\left(889103\ \text{J.mol}^{-1}\right)\right] \times 0.044\ \text{kg.mol}^{-1} \times 10^{-3}\ \text{tonne.kg}^{-1} = 213\ \text{million tonnes}$$

(b) A supertanker, carrying 1 million barrels of oil, would if it blew up have a blast equivalent to that from the atomic bomb at Hiroshima. A blast of this magnitude was actually observed at a refinery fire in Venezuela in 2012 (either of these).

(c) Rate of supply of coke = $\left[300 \times 10^{6}\ \text{J. s}^{-1}/\left(25 \times 10^{6}\ \text{J.kg}^{-1}\right)\right] = 12\ \text{kg.s}^{-1}$
rate of production of carbon dioxide = $12\ \text{kg.s}^{-1} \times (44/12) \times 7 \times 24 \times 3600 \times 0.001$ tonne per week
= 26611 tonne per week.

For 10% mitigation of the carbon 10% of the heat must come from the citrus peel.
$10.8\ \text{kg.s}^{-1}$ of coke plus:
$(1.2 \times 25/7)\ \text{kg.s}^{-1}$ of citrus peel $= 4.3\ \text{kg.s}^{-1}$ of citrus peel
Ratio coke to citrus peel = $(10.8/4.3) = 2.5$

(d) Suppliers are monitored by the Forest Stewardship Council (FSC) for replacement of trees felled with new plantings.

Question 3: (a) The inlet has circumference $1\,\mathrm{m}$. Therefore the radius of the inlet $r_1 = 1/2\pi = 0.15915\,\mathrm{m}$ and the area of the inlet $A_1 = \pi r_1^2 = 0.07958\,\mathrm{m}^2$.
Similarly the outlet has circumference $0.6\,\mathrm{m}$. Therefore the radius of the outlet $r_2 = 0.6/2\pi = 0.09549\,\mathrm{m}$ and the area of the outlet $A_2 = \pi r_2^2 = 0.02865\,\mathrm{m}^2$. **[1 Mark of 4]**

Evaluating the mass flux at the inlet gives

$$\rho_1 = \frac{\dot{m}}{u_1 A_1} = \frac{4\,\mathrm{kg\,s^{-1}}}{30\,\mathrm{m\,s^{-1}} \times 0.07958\,\mathrm{m}^2} = 1.6755\,\mathrm{kg\,m^{-3}}.$$

[1 Mark of 4]

Rearranging the SFEE to give the gas velocity at the outlet implies

$$u_2^2 = u_1^2 + \frac{2\left(\dot{Q} - \dot{W}_s\right)}{\dot{m}} + 2\left(h_1 - h_2\right).$$

Therefore

$$\begin{aligned} u_2^2 =& 30^2 + \frac{2\left(-15000 - 30000\right)}{4} + 2\left(70000 - 40000\right) \\ =& 900 - 22500 + 60000 \\ =& 38400\,\mathrm{m^2\,s^{-2}}, \end{aligned}$$

giving a fluid velocity at the outlet of

$$u_2 = 195.959\,\mathrm{m\,s^{-1}}$$

[1 Mark of 4]

Now the gas density at the outlet can be calculated from the mass flux

$$\rho_2 = \frac{\dot{m}}{u_2 A_2} = \frac{4\,\mathrm{kg\,s^{-1}}}{195.959\,\mathrm{m\,s^{-1}} \times 0.02865\,\mathrm{m}^2} = 0.7125\,\mathrm{kg\,m^{-3}}.$$

Finally the difference in gas density the turbine is given by

$$\Delta\rho = \rho_1 - \rho_2 = 1.6755 - 0.7125 = 0.9630\,\mathrm{kg\,m^{-3}}.$$

[1 Mark of 4]

(b) The differential forms of for mass and energy conservation are

$$\begin{aligned} \frac{\mathrm{d}V}{V} - \frac{\mathrm{d}u}{u} - \frac{\mathrm{d}A}{A} =& 0, \\ \mathrm{d}h + u\,\mathrm{d}u =& 0. \end{aligned}$$

[1 Mark of 2]

Eliminating $\mathrm{d}u$ between these two expressions gives

$$\frac{\mathrm{d}V}{V} + \frac{\mathrm{d}h}{u^2} - \frac{\mathrm{d}A}{A} = 0$$

[1 Mark of 2]

The speed of sound is the distance travelled during a unit of time by a sound wave propagating through a compressible medium. The Mach number is the non-dimensional ratio of the speed of a body moving through a fluid to the local speed of sound. For an isentropic process, the speed of sound is given by

$$c = \left(\frac{\partial p}{\partial \rho}\right)^{1/2},$$

while the Mach number is defined to be

$$\mathrm{Ma} = \frac{u}{c},$$

[4 Marks]

For an isentropic process the specific volume $V = 1/\rho$, is a function of just pressure and therefore satisfies

$$\mathrm{d}V = \frac{\mathrm{d}V}{\mathrm{d}p}\mathrm{d}p.$$

In this expression the derivative can be written in terms of the speed of sound c since

$$\frac{\mathrm{d}V}{\mathrm{d}p} = \frac{\partial V}{\partial \rho}\frac{\partial \rho}{\partial p} = -\frac{V^2}{c^2},$$

and therefore

$$\mathrm{d}V = -\frac{V^2}{c^2}\mathrm{d}p.$$

[2 Marks of 3]

For a general thermodynamic process

$$\mathrm{d}h = T\,\mathrm{d}s + V\,\mathrm{d}p.$$

However for an isentropic process the entropy remains constant $\mathrm{d}s = 0$, and the enthalpy is a function of just pressure. Changes in enthalpy are not related to changes in entropy, and

$$\mathrm{d}h = V\,\mathrm{d}p.$$

[1 Mark of 3]

Hence if we eliminate $\mathrm{d}V$ and $\mathrm{d}h$,

$$-\frac{V}{c^2}\mathrm{d}p + \frac{V}{u^2}\mathrm{d}p - \frac{\mathrm{d}A}{A} = 0.$$

Collecting together terms involving $\mathrm{d}p$ gives

$$\frac{\mathrm{d}A}{A} = \left(\frac{V}{u^2} - \frac{V}{c^2}\right)\mathrm{d}p = \frac{V}{u^2}\left(1 - \frac{u^2}{c^2}\right)\mathrm{d}p.$$

Given the definition of Mach number

$$\frac{\mathrm{d}A}{A} = \frac{V}{u^2}\left(1 - \mathrm{Ma}^2\right)\mathrm{d}p.$$

For an isentropic process the speed of sound can be used to eliminate $\mathrm{d}p$, giving

$$\frac{\mathrm{d}A}{A} = \frac{V}{u^2}\left(1 - \mathrm{Ma}^2\right)c^2\mathrm{d}\rho.$$

Rearranging and using the definition of Mach number and specific density gives

$$\frac{1}{\left(1 - \mathrm{Ma}^2\right)A}\mathrm{d}A = \frac{1}{\rho\mathrm{Ma}^2}\mathrm{d}\rho.$$

If we're interested in changes along a pipe whose length is parameterized by x, then

$$\frac{1}{\left(1 - \mathrm{Ma}^2\right)A}\frac{\mathrm{d}A}{\mathrm{d}x} = \frac{1}{\rho\mathrm{Ma}^2}\frac{\mathrm{d}\rho}{\mathrm{d}x},$$

as required. **[5 Marks]**

For a supersonic diffuser $\left(1 - \mathrm{Ma}^2\right) < 0$, while $\frac{\mathrm{d}A}{\mathrm{d}x} > 0$, $A > 0$, $\rho > 0$ and $\mathrm{Ma}^2 > 0$. Therefore $\frac{\mathrm{d}\rho}{\mathrm{d}x} < 0$ and the gas density falls as gas flows along a supersonic diffuser. **[2 Marks]**

Question 4: Figure 2 shows the Ts diagram for the refrigeration cycle.

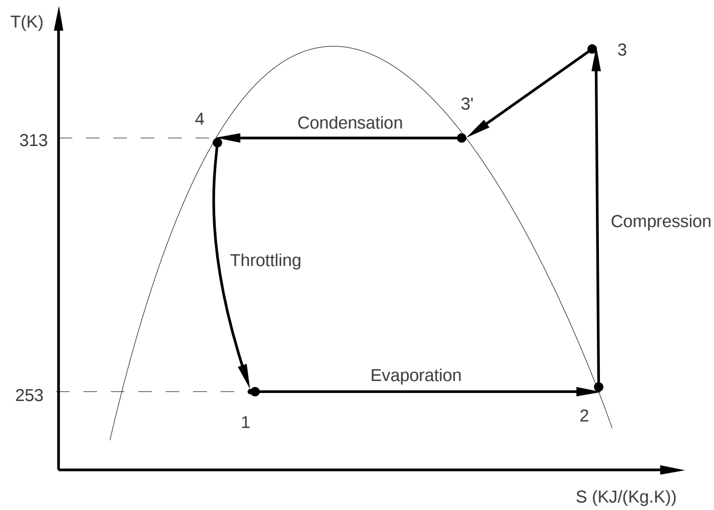


Figure 2: Refrigeration cycle – Question 4

From the given thermodynamic table for *Freon-12*,

T (°C)	P_s (bar)	V_g (m^3/kg)	H_f (kJ/kg)	H_g (kJ/kg)	S_f (kJ/(kg.K))	S_g (kJ/(kg.K))	*Specific Heat* (kJ/(kg.K))
-20	1.509	0.1088	17.8	178.61	0.073	0.7082	–
40	9.607	–	74.53	203.05	0.2716	0.682	0.747

$$H_2 = 178.61 \text{ kJ/kg}, \quad H_3' = 203.05 \text{ kJ/kg}, \quad H_{f4} = H_1 = 74.53 \text{ kJ/kg}$$

Calculating mass flow rate,

$$\text{Cooling Effect} = 20 = \dot{m}\,(H_2 - H_1) = \dot{m}\,(178.61 - 74.53) \Longrightarrow \dot{m} = 0.192 kg/s$$

As stage 2-3 is isentropic $\Rightarrow S_3 = S_2$ and

$$S_3' + C_p \ln\left(\frac{T_3}{T_3'}\right) = 0.7082$$

$$0.682 + 0.747 \ln\left(\frac{T_3}{313.15}\right) = 0.7082 \Longrightarrow T_3 = 330.95 \text{ K}$$

Now we can calculate the enthalpy of state 3:

$$H_3 = H_3' + C_p\,(330.95 - 313.15) = 203.05 + 0.747\,(330.95 - 313.15) = 216.34 kJ/kg$$

The power required by the machine is

$$\dot{m}\,(H_3 - H_2) = 7.25\ \text{kW} = 7244.91\ \text{W}$$

In order to calculate the piston displacement V, we first need to compute the volumetric efficiency,

$$\eta_{\text{vol}} = 1 + C - C\left(\frac{P_d}{P_s}\right)^{1/n} = 1 + 0.03 - 0.03\left(\frac{9.607}{1.509}\right)^{1.13} = 0.876$$

The volume of refrigerant at the intake condition is:

$$V_{\text{ref}}^{(\text{intake})} = \dot{m} \times v_g = 0.192 \times .1088 = 0.02089\frac{\text{m}^3}{\text{s}}$$

The swept volume can now be calculated as

$$V_{\text{swept}} = \frac{V_{\text{ref}}^{(\text{intake})}}{\eta_{\text{vol}}} = \frac{0.0289}{0.876} = 2.3847 \times 10^{-2}\frac{\text{m}^3}{\text{s}}$$

Finally, the piston displacement is:

$$V = \frac{V_{\text{swept}} \times 60}{300} = 4.769 \times 10^{-3}\ \text{m}^3$$

Question 5: The specific humidity ω is the ratio of the mass of water vapour m_v, to the mass of dry air m_a and satisfies the equation

$$\omega = \frac{m_v}{m_a}.$$

As both water vapour and dry air behave like ideal gases, in some arbitrary volume V,

$$\omega = \frac{m_v}{m_a} = \frac{\rho_v}{\rho_a} = \frac{p_v}{R_v T}\frac{R_a T}{p_a} = \frac{R_a p_v}{R_v p_a}.$$

The ratio of specific gas constants $R_a/R_v = 0.622$, while the partial pressures of dry air and water vapour satisfy $p_a = p - p_v$. Hence

$$\omega = \frac{0.622 p_v}{p - p_v}.$$

[4 Marks]

The saturation pressure of water

$$p_g = \varphi p_v.$$

Hence eliminating p_v from the previous expression gives

$$\omega = \frac{0.622\varphi p_g}{p - \varphi p_g}.$$

[2 Marks]

The heating and humidification are split into two steady process. Firstly a heater (with inlet properties labelled 1 and outlet properties labelled 2) and secondly a humidifier (with inlet properties labelled 2 and outlet properties labelled 3).

(a) The partial vapour pressure at the inlet 1, is

$$p_{v_1} = \varphi_1 p_{g_1} = \varphi p_{\text{sat @ 10°C}} = 0.25 \times 1.4028\,\text{kPa} = 0.3507\,\text{kPa}.$$

[1 Mark]

Hence the partial pressure of dry air is given by

$$p_{a_1} = p_1 - p_{v_1} = 100\,\text{kPa} - 0.3507\,\text{kPa} = 99.6493\,\text{kPa}.$$

[1 Mark]

The specific humidity is given by

$$\omega_1 = \frac{0.622 p_{v_1}}{p_1 - p_{v_1}} = \frac{0.622 \times 0.3507\,\text{kPa}}{100\,\text{kPa} - 0.3507\,\text{kPa}} = 0.00219\,\text{kg H}_2\text{O/ kg dry air}.$$

[1 Mark]

(b) Applying the mass and energy balances on the heating section gives

$$\begin{aligned}
\text{Dry air mass balance:}\quad & \dot{m}_{a_1} = \dot{m}_{a_2} = \dot{m}_a, \\
\text{Water mass balance:}\quad & \dot{m}_{a_1}\omega_1 = \dot{m}_{a_2}\omega_2, \quad \Rightarrow \quad \omega_1 = \omega_2, \\
\text{Energy balance:}\quad & \dot{Q} = \dot{m}_a h_2 - \dot{m}_a h_1.
\end{aligned}$$

[2 Marks]

The total specific enthalpy at 1, the inlet is

$$\begin{aligned}
h_1 = c_p T_1 + \omega_1 h_{g_1} =& \left(1.005\,\text{kJ/(kg K)} \times (12 + 173.15)\ \text{K}\right) \\
& + (0.00219 \times 2523\,\text{kJ/kg}) \\
=& 292.0988\,\text{kJ/kg}.
\end{aligned}$$

The total specific enthalpy at 2, the outlet of the heating is

$$\begin{aligned}
h_2 = c_p T_2 + \underbrace{\omega_2}_{=\omega_1} h_{g_2} =& \left(1.005\,\text{kJ/(kg K)} \times (20 + 173.15)\ \text{K}\right) \\
& + (0.00219 \times 2537\,\text{kJ/kg}) \\
=& 300.1693\,\text{kJ/kg}
\end{aligned}$$

[2 Marks]

The specific volume of dry air at 1, is given by

$$V_1 = \frac{R_a T_1}{p_{a_1}} = \frac{287.058\,\text{J/(kg K)}\,(12 + 273.15)\ \text{K}}{99649.3\,\text{Pa}} = 0.8215\,\text{m}^3\text{/kg}.$$

Therefore the mass flux of dry air through the inlet

$$\dot{m}_a = \frac{q_1}{V_1} = \frac{40\,\text{m}^3\text{/min}}{0.8185\,\text{m}^3\text{/kg}} = 48.6886\,\text{kg/min},$$

where $q_1 = 40\,\text{m}^3$/min is the total volume flux through the inlet. **[1 Mark]**

Hence the energy conservation equation gives the rate at which heat is transferred to the air

$$\dot{Q} = \dot{m}_a\left(h_2 - h_1\right) = 48.6958\,\text{kg/min}\,(300.1693\,\text{kJ/kg} - 292.0988\,\text{kJ/kg}) = 392.9488\,\text{kJ/min}.$$

[1 Mark]

(c) The mass balance for water in the humidifying section can be expressed as

$$\dot{m}_{a_2}\omega_2 + \dot{m}_w = \dot{m}_{a_3}\omega_3,$$

or

$$\dot{m}_w = \dot{m}_a\left(\omega_3 - \omega_2\right).$$

[2 Marks]

Here $\omega_2 = \omega_1$, while the specific humidity at 3, the outlet is given by

$$\omega_3 = \frac{0.622\varphi_3 p_{g_3}}{p_3 - \varphi_3 p_{g_3}} = \frac{0.622 \times 0.55 \times 2.9858}{100 - (0.55 \times 2.9858)} = 0.0104\,\text{kg H}_2\text{O/ kg dry air.}$$

[1 Mark]

Therefore the required mass flow rate of steam is

$$\dot{m}_w = \dot{m}_a\left(\omega_3 - \omega_2\right) = 48.6886\,\text{kg/min}\left(0.0104 - 0.00219\right) = 0.3990\,\text{kg/min}$$

[2 Marks]