Answers 1

Q1.
For S_1:

$$mass\ \ flux = \rho u A \quad = 1000 \times 2 \times \frac{\pi \times 0.1^2}{4} \quad = 15.7 \text{ kg s}^{-1}$$

This must be the same as the mass flux through S_2. (Otherwise, mass would accumulate between these surfaces.)

To calculate the mass flux in general, use the component of velocity *normal* to the area (or, equivalently, the projected area normal to the velocity):

$$mass\ \ flux = \rho u_n A \quad = \rho \mathbf{u} \bullet \mathbf{A}$$

Answer: 15.7 kg s^{-1} (both surfaces).

Q2.

Steady-state momentum principle:

force (on *fluid*) = (*momentum flux*)$_{out}$ – (*momentum flux*)$_{in}$

where, for a uniform velocity:

momentum flux = *mass flux* × *velocity*

For the x-component of momentum and the fluid in a control volume encompassing the region shown in the question,

$$-F = 0 - (\rho U A)U$$

Hence,

$$F = \rho U^2 A \quad = 1000 \times 8^2 \times \frac{\pi \times 0.1^2}{4} \quad = 503 \text{ N}$$

Answer: 503 N.

Q3.

(a) Volume of room:

$$V = 30 \times 8 \times 5 \quad = 1200 \text{ m}^3$$

If ϕ is the concentration (expressed as mass of toxin per mass of fluid) then the initial mass of gas in the room is

mass of fluid × concentration = mass of toxin

$$\Rightarrow \qquad (\rho V)\phi_0 = 2 \text{ kg}$$

$$\Rightarrow \qquad \phi_0 = \frac{2}{1.2 \times 1200} \quad = 1.389 \times 10^{-3}$$

Answer: ϕ_0 = 1390 ppm.

(b)

change in amount of toxin = amount in – amount out

or, as a rate equation:

rate of change of amount of toxin = rate of entering – rate of leaving

$$\frac{\mathrm{d}}{\mathrm{d}t}(\rho V\phi) = 0 - (\rho uA)\phi$$

$$\Rightarrow \qquad \frac{\mathrm{d}\phi}{\mathrm{d}t} = -(\frac{uA}{V})\phi, \qquad \phi = \phi_0 \text{ at } t = 0$$

$$\Rightarrow \qquad \frac{\mathrm{d}\phi}{\mathrm{d}t} = -\lambda\phi \qquad \text{where} \qquad \lambda = \frac{uA}{V} \quad = \frac{0.5 \times 6}{1200} \quad = 0.0025 \text{ s}^{-1}$$

This is exponential decay, with solution

$$\phi = \phi_0 e^{-\lambda t}$$

$$\Rightarrow \qquad e^{\lambda t} = \frac{\phi_0}{\phi}$$

$$\Rightarrow \qquad t = \frac{1}{\lambda}\ln\frac{\phi_0}{\phi}$$

For the required concentration (ϕ = 1 ppm):

$$t = \frac{1}{0.0025}\ln(1389) \quad = 2895 \text{ s} \quad \approx 48 \text{ min}$$

Answer: 48 minutes.

Q4.

The rate at which chemical enters the river (as mass of chemical per unit time) is

$$S = \frac{2.5}{3600} \quad = 6.944 \times 10^{-4} \text{ kg s}^{-1}$$

At steady state, the flux of chemical through a downstream section equals the rate at which it enters the river. If ϕ is pollutant concentration (here, mass of chemical per *volume* of water):

$$(uA)\phi = S$$

$$\Rightarrow \qquad \phi = \frac{S}{uA} \quad = \frac{6.944 \times 10^{-4}}{0.3 \times 5 \times 2} \quad = 2.31 \times 10^{-4} \text{ kg m}^{-3}$$

Answer: 2.31×10^{-4} kg m^{-3}.