Problem 1: A gas is confined in a vertical 0.47 m diameter cylinder by a piston. On the piston rests a weight and the combined mass of the piston and weight is 150 kg. The local acceleration of gravity is 9.81 m.s^{-2} and the ambient pressure is 101.57 kPa.

(a) What is the total force exerted on the gas by the atmosphere, the piston and the weight assuming no friction between the piston and the cylinder?

(b) What is the pressure of the gas?

(c) The gas in the cylinder is heated and expands pushing the piston and weight upward. Calculate the work done by the gas if the piston and weight are raised 0.83 m. What is the change in potential energy of the piston and weight?

Problem 2: In a closed system (kinetic and potential energy are constant) three consecutive processes are done by an ideal gas (10 moles, MW = 24.945 g.mol^{-1}):

Initial conditions:	P_1 = 1 bar, T_1 = 300K
Process 1→2:	Reversible isothermal compression, v_2 = 0.1m^3.kg^{-1}
Process 2→3:	Isochoric colling, P_3 = 2 bar
Process 3→4:	Isobaric heating, T_4 = 600 K

(a) Calculate the initial volume V_1^t and specific volume v_1 of the gas.

(b) Calculate the pressure P_2 after the first process.

(c) Calculate the temperature T_3 after the second process.

(d) Calculate the final specific volume v_4.

(e) What forms of energy are present in transit across the system's boundary during the first process? Calculate the values.

(f) Which kind of process can we use to reach the initial state?

(g) Draw a Pv diagram with all processes 1→2→3→4→1.

Problem 3: One mole of an ideal gas with C_P = (7/2)R and C_V = (5/2)R expands form P_1 = 8 bar and T_1 = 600 K to P_2 = 1 bar by each of the following paths:

(a) Constant volume.

(b) Constant temperature.

(c) Adiabatically.

Assuming mechanical reversibility, calculate W, Q, ΔU, ΔH $\left(\text{in J.mol}^{-1}\right)$ for each process. Sketch each path on a single PV diagram.

Problem 4: A Carnot engine receives 250 kJ.s^{-1} of heat from a heat-source reservoir at 525°C and rejects heat to a heat-sink reservoir at 50°C. What are the power developed and the heat rejected?

Problem 5: Assuming $S = S\left(P, V\right)$ and taking into consideration that,

$$\left(\frac{\partial S}{\partial T}\right)_V = \frac{C_V}{T} \quad \text{and} \quad \left(\frac{\partial S}{\partial T}\right)_P = \frac{C_P}{T}$$

Prove that

$$\mathrm{d}S = \frac{C_V}{T}\left(\frac{\partial T}{\partial P}\right)_V \mathrm{d}P + \frac{C_P}{T}\left(\frac{\partial T}{\partial V}\right)_P \mathrm{d}V$$

Problem 6: Large quantities of liquefied natural gas (LNG) are shipped by ocean tankers. At the unloading port, provisions are made for vaporisation of the LNG so that it may be delivered to pipelines as gas. The LNG arrives in the tanker at atmospheric pressure and 113.7 K, and represents a possible heat sink for use as cold reservoir of a heat engine. For unloading vapour LNG at 9000 $m^3.s^{-1}$ (measured at 298.15 K and 1.0133 bar) and assuming the availability of an adequate heat source at 303.15 K:

(a) What is the maximum possible power ?

(b) What is the rate of heat transfer from the heat source?

Assume that LNG at 298.15 K and 1.0133 bar is an ideal gas with the molar mass of 17 g.mol^{-1}. Also assume that the LNG vaporises only, absorbing only its latent heat of 512 kJ/kg at 113.7 K.

Problem 7: Given saturated ammonia $(\mathrm{NH_3})$ vapour at $P_1 = 200kPa$ compressed by a piston to $P_2 = 1.6MPa$ in a reversible adiabatic process,

(a) Find the work done per unit mass;

(b) Sketch the $T - s$ and $P - v$ diagrams.

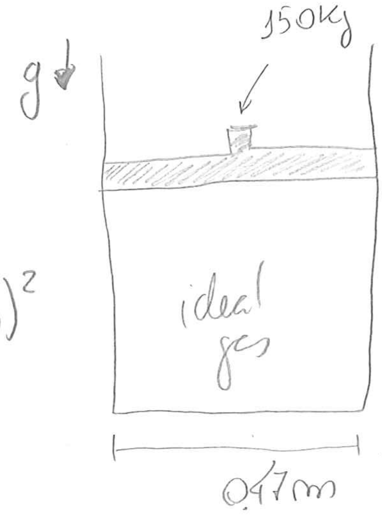
Given the following critical conditions for ammonia: T$_c$ = 132.4°C and P$_c$ = 112.8 bar.

Tutorial 02

Problem 01

(a) $F_{piston} = mg = 150\,Kg \times 9.81\,\frac{m}{s^2} =$

$= 1471.5\ N$



$F_{atm} = P.A = 101.57 \times 10^3\ Pa \times \frac{\pi}{4}(0.47m)^2$

$= 17621.31\ Pa.m^2 = 17621.31\ N$

$F_{TOTAL} = F_{piston} + F_{atm} = \underline{19092.81\ N}$

(b) $P_{gas} = \frac{F}{A} = \frac{19092.81\ N}{\pi/4\ (0.47\,m)^2} = 110051.78\ N/m^2 = 110051.78\ Pa$

$= \underline{110.05\ KPa}$

piston displacement

(c) $W = F \times \Delta d = 19092.81\ N \times 0.83m$

$W = 15847.03\ N.m = \underline{15847.03\ J}$

The potential energy is:

$\Delta E_p = mg\Delta d = 150\,Kg \times 9.81\,\frac{m}{s^2} \times 0.83m$

$\Delta E_p = 1221.35\ Kg\frac{m^2}{s^2} = \underline{1221.35\ J}$

Problem 02

(a) $P_1 V_1^* = m R T_1 = \frac{m}{\overline{MW}} R T_1$

$$\frac{V_1^*}{m} = v_1 = \frac{R T_1}{\overline{MW} \times P_1} = 8.314 \frac{J}{\cancel{mol.K}} \times 300\cancel{K} \times \frac{1}{24.945 \, g/\cancel{mol}} \times 1/1bar$$

$$v_1 = 99.97 \frac{\cancel{J}}{bar.g} \times \frac{1N.m}{1\cancel{J}} \times \frac{1Kg\,m/s^2}{1N} \times \frac{1bar}{10^5 N/m^2} \times \frac{1N}{1Kg\,m/s^2} \times \frac{1000 g}{1Kg}$$

$\downarrow$

$[m^3/Kg]$

$$v_1 = 0.9997 \; m^3/Kg$$

And $V_1^t/m = v_1 \Rightarrow V_1^t = v_1 \times m = v_1 \times m \times \overline{MW}$

$$V_1^t = 0.9997 \frac{m^3}{\cancel{Kg}} \times 10 \, \cancel{mol} \times 24.945 \frac{\cancel{g}}{\cancel{mol}} \times \frac{1\cancel{Kg}}{1000\cancel{g}}$$

$$\underline{V_1^t = 0.24945 \; m^3}$$

(b) Reversible isothermal compression: $(1 \rightarrow 2)$

PV: constant $(T_1 = T_2)$

$$P_1 V_1 = P_2 V_2 \;\therefore\; P_2 = \frac{P_1 V_1}{V_2} = 1 bar \times \frac{0.9997 \; m^3/Kg}{0.1 \; m^3/Kg}$$

$$\underline{P_2 = 9.997 \; bar}$$

(c) Isochoric cooling: (2 → 3)

P/T: constant ($V_2 = V_3$)

$$\frac{P_2}{T_2} = \frac{P_3}{T_3} \therefore T_3 = \frac{P_3 T_2}{P_2} = \frac{2\,\text{bar}}{9.997\,\cancel{\text{bar}}} \times 300\,K$$

$$\underline{T_3 = 60.02\,K}$$

(d) Isobaric heating: (3 → 4)

$\frac{V}{T}$: constant ($P_3 = P_4$)

$$\frac{V_3}{T_3} = \frac{V_4}{T_4} \therefore V_4 = \frac{V_3 T_4}{T_3} = 0.1\,m^3/Kg \times \frac{600\,K}{60.02\,K}$$

$$V_4 = \underline{0.9997\,m^3/Kg}$$

(e) Reversible isothermal process (1 → 2):

$$W = -P\,dV = -\frac{RT}{V}\,dV = -RT(n)\ln\frac{V_2}{V_1}$$

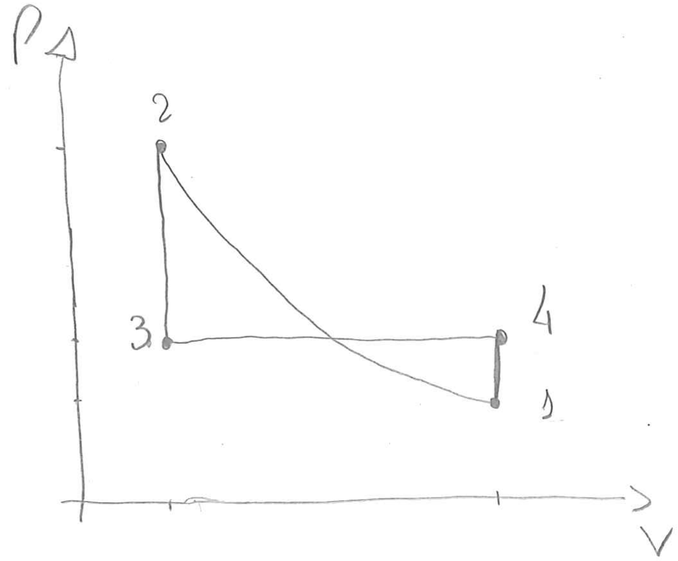
this will ensure that the whole term is in $\underline{J}$

$$W = -8.314\,\frac{J}{\cancel{mol}.\cancel{K}} \times 300\,\cancel{K} \times 10\,\cancel{mol} \times \ln\left(\frac{0.1\,m^3/Kg}{0.9997\,m^3/Kg}\right)$$

$$\underline{\underline{W = 57423.59\,J = 57.42\,KJ = Q}}$$

(f) Isochoric cooling

(g)



Problem 03 Ideal gas $\begin{cases} P_1 = 8 \text{ bar} \\ T_1 = 600 K \end{cases} \Rightarrow P_2 = 1 \text{ bar}$

$C_p = 7/2\, R$ and $C_v = 5/2\, R$

(a) Constant volume $(dV = 0)$

$\boxed{W = 0}$

$T_2 = \frac{P_2}{P_1} T_1 = \frac{1}{8} \times 600 = 75 K$

$\Delta U = Q = C_v \Delta T = \frac{5}{2} \times 8.314 \frac{J}{mol.\cancel{K}} \times (75 - 600) \cancel{K}$

$\boxed{\Delta U = -10912.13 \text{ J/mol} = Q}$

$\Delta H = C_p \Delta T = \frac{7}{2} \times 8.314 \frac{J}{mol.\cancel{K}} \times (75 - 600) \cancel{K}$

$\boxed{\Delta H = -15276.98 \text{ J/mol}}$

(b) Constant temperature $(dT = 0)$

$\boxed{\Delta U = 0 = \Delta H}$

$Q = -W = -RT_1 \ln (P_2/P_1)$

$\boxed{Q = -W = -10373.09 \text{ J/mol}}$

(c) Adiabatically

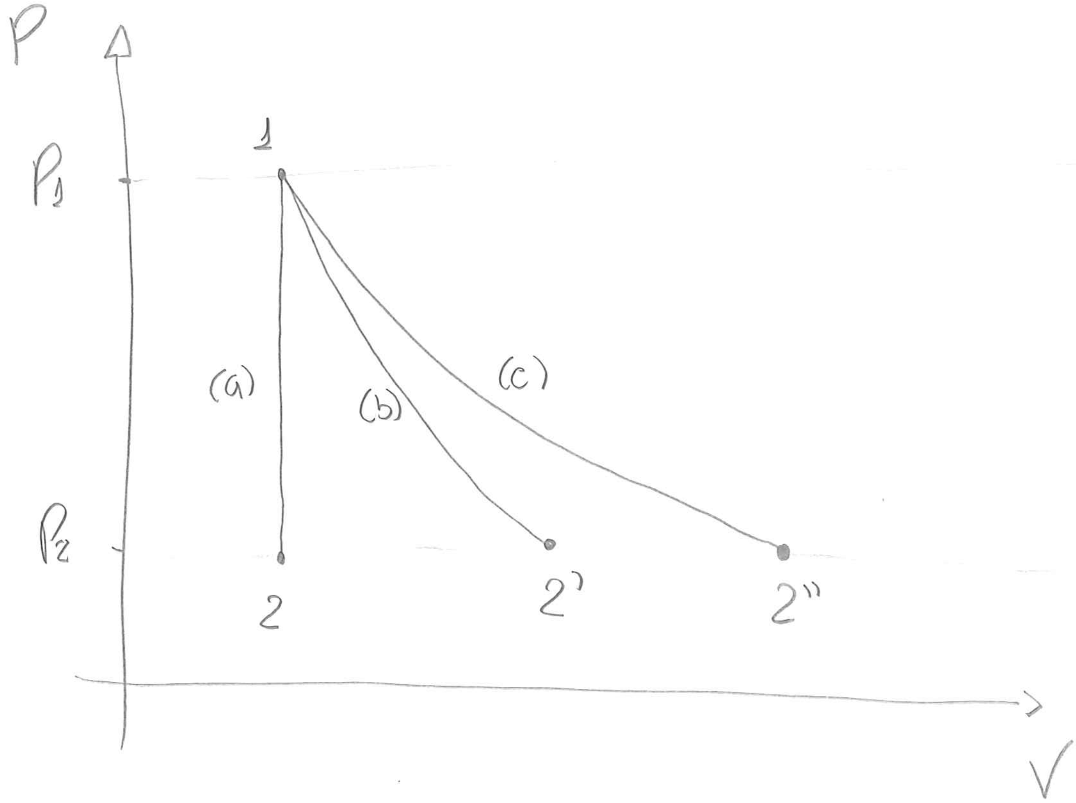
$$\boxed{Q = 0}$$

$$\gamma = C_p / C_v \; ; \quad T_2 = T_1 (P_2/P_1)^{\frac{\gamma-1}{\gamma}} = 600 (1/8)^{\frac{1.4-1}{1.4}} = 331.23 K$$

$$\gamma = \frac{7/2 R}{5/2 R} = 1.4 \qquad \Delta T = T_2 - T_1 = -268.77 K$$

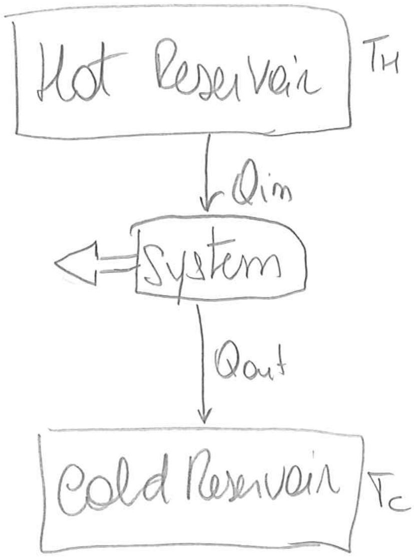
$$\boxed{W = \Delta U =} C_v \Delta T = \frac{5}{2} R \Delta T = \boxed{-5586.38 \ J/mol}$$

$$\boxed{\Delta H =} C_p \Delta T = 7/2 R \Delta T = \boxed{-7820.94 \ J/mol}$$



Problem 04 Carnot engine

$$\eta_{Carnot} = 1 - \frac{T_C}{T_H}$$



$W_{cycle} = Q_{in} - Q_{out}$

$$\eta_{Carnot} = 1 - \frac{(50 + 273.15)K}{(525 + 273.15)K} = 0.5951$$

$$\underline{\underline{59.5\ \%}}$$

$$\eta_{Carnot} = 1 - \frac{T_C}{T_H} = \frac{W_{cycle}}{Q_{in}}$$

$$W_{cycle} = Q_{in}\,\eta_{Carnot} = 250\ \frac{KJ}{s} \times 0.5951 = 148.775\ \frac{KJ}{s}$$

$$\boxed{W_{cycle} = 148.775\ KW}$$

heat rejected

$$W_{cycle} = Q_{in} - Q_{out}$$

$$Q_{out} = Q_{in} - W_{cycle} = 250 - 148.775 = 101.225\ KJ/s$$

$$\boxed{Q_{out} = 101.225\ KW}$$

Problem 05:

$$S = S(P, V)$$

$$dS = \left(\frac{\partial S}{\partial P}\right)_V dP + \left(\frac{\partial S}{\partial V}\right)_P dV$$

using chain rule

$$dS = \underbrace{\left(\frac{\partial S}{\partial T}\right)_V}_{C_V/T} \left(\frac{\partial T}{\partial P}\right)_V dP + \underbrace{\left(\frac{\partial S}{\partial T}\right)_P}_{C_P/T} \left(\frac{\partial T}{\partial V}\right)_P dV$$

$$\boxed{dS = \frac{C_V}{T}\left(\frac{\partial T}{\partial P}\right)_V dP + \frac{C_P}{T}\left(\frac{\partial T}{\partial V}\right)_P dV}$$

Problem 06: LNG: $\begin{cases} MW = 17\ g/mol & P = 1.0133\ bar \\ \dot{V} = 9000\ m^3/s & T = 298.15\ K \end{cases}$

Calculating the mass flow rate of LNG, assuming it behaves like an ideal gas:

$$P\dot{V} = \dot{m}_{N26} RT = \frac{\dot{m}_{LNG}}{MW} RT \therefore \dot{m}_{LNG} = \frac{P\dot{V}}{RT} MW$$

$$\dot{m}_{LNG} = 1.0133\ bar \times 9000\ \frac{m^3}{s} \times 17\ \frac{g}{\cancel{mol}} \times \frac{1}{8.314\ J/\cancel{mol}.\cancel{K}} \times \frac{1}{298.15\ \cancel{K}}$$

[Kg/s]

$$\dot{m}_{LNG} = 62.54\ \frac{g.\cancel{bar}.\cancel{m^3}}{\cancel{J}.s} \times \frac{10^5\ \cancel{N/m^2}}{1\ \cancel{bar}} \times \frac{1\ \cancel{J}}{1\ \cancel{N}\cancel{m}} \times \frac{1\ Kg}{1000\ \cancel{g}}$$

$$\boxed{\dot{m}_{LNG} = 6254.39\ Kg/s}$$

$\begin{cases} T_H = 303.15\ K \\ T_C = 113.7\ K \end{cases}$ $\Rightarrow$ $\dot{Q}_C = 512\ \frac{KJ}{Kg} \times \dot{m}_{LNG}$

$$\dot{Q}_C = 3202247.68\ \frac{KJ}{s} = 3202.25\ MW$$

KW

Maximum power of a thermal cycle is given by the Carnot engine:

$$\frac{\dot{W}}{\dot{Q}_C} = \frac{\dot{Q}_H - \dot{Q}_C}{\dot{Q}_C} = \frac{\dot{Q}_H}{\dot{Q}_C} - 1 = \frac{T_H}{T_C} - 1$$

Thus, the power (or work) can be calculated from

$$\dot{W} = \dot{Q}_C \left(\frac{T_H}{T_C} - 1 \right) = 3202247.68 \, \frac{KJ}{s} \left(\frac{303.15 \, K}{113.7 \, K} - 1 \right)$$

$$\dot{W} = 5335671.27 \, KJ/s = 5335.67 \, MW$$ (a)

↳ Max Power

And the heat:

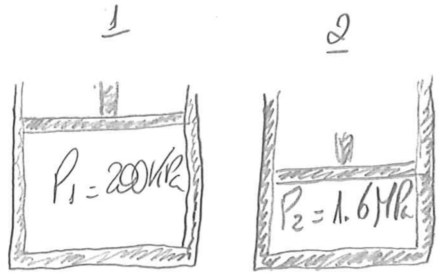
$$\dot{Q}_H = \dot{Q}_C + \dot{W}$$

$$\dot{Q}_H = (3202247.68 + 5335671.27) \, KJ/s$$

$$\dot{Q}_H = 8537918.95 \, KJ/s = 8537.92 \, MW$$ (b)

Problem 07: NH_3 (ammonia)

Reversible and Adiabatic Process $\Rightarrow$ Isentropic Process



At 200KPa (2bar) the specific volume and entropy of saturated vapour ammonia are: ($T_1 = T_{sat} = -18.86°C$)

$$v_g = 0.5946\ m^3/Kg = v_1$$
$$s_g = 5.5969\ KJ/Kg.K = s_1$$
$$u_g = 1300.39\ KJ/Kg = u_1$$

As compression 1-2 is isentropic:

$$\boxed{s_2 = s_1}$$

At $P_2 = 1.6MPa = 16$ bar, the entropy of saturated vapour ammonia is

$$s_g(P = 16\ bar) = 4.8542\ KJ/Kg.K < s_2$$

This indicates that ammonia is at superheated state. Knowing P_2 and s_2, from superheated table we can infer that $120 \leq T \leq 140°C$. Then through linear interpolation for $s_2 = 5.5969$ kJ/kg.K:

T (°C)	u (kJ/kg)	v (m³/kg)	s (kJ/kg.K)
120	1516.34	0.11268	5.5008
140	1556.14	0.11974	5.6276

$T_2 = 135.16\,°C$ $\qquad v_2 = 0.11439\ m^3/kg$

$u_2 = 1546.50\ kJ/kg$

$T_2 > T_c$ but $P_2 < P_c$ and also $v_2 < v_1$

From the 1st law:

$$\Delta u = \cancel{q}^{\,0\ (\text{isentropic})} - w = u_2 - u_1 \therefore \quad \text{(a)}\ \boxed{w = u_1 - u_2 = -246.11\ \frac{kJ}{kg}}$$