ROYAL INSTITUTE OF TECHNOLOGY

SCIENCE AND ART

A Short Course on Heat Transfer

Intended as a repetition from previous courses

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Course Contents,

based on Holman's book Heat Transfer

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Chapter 2: Steady-State Conduction - One Dimension
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(Chapter 11: Mass Transfer)

Part 1

Introduction

Introduction

What is heat?

Heat is energy transfer caused by temperature difference!

The four laws of thermodynamics:

- ***Zeroth law:***

If two bodies both are in thermal equilibrium with a third body, they are also in thermal equilibrium with each other, and they then are said to have the same temperature.

• ***First law: (Energy principle)***
Energy cannot be generated or destroyed, only converted to different forms.

(”Energy consumption” is the transfer of ”prime” energy to thermal energy in the surrounding)

•***Second law:***
Heat cannot by itself pass from one body to another body with higher temperature.

(Entropy [disorder] strives to a maximum in a closed system. Shows the *direction* of time.)


POOF

Third law:

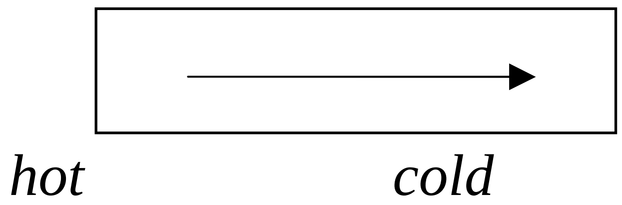
The entropy of a pure, crystalline material takes its lowest value at absolute zero temperature, where it is 0.

(There is a lowest limit to the temperature, 0K = -273.15°C. World record: 0.00000017 K = 170 nanokelvin)

Three modes of heat transfer:

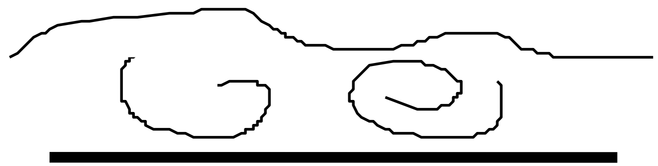
- Conduction

Through solid bodies and ”still” fluids



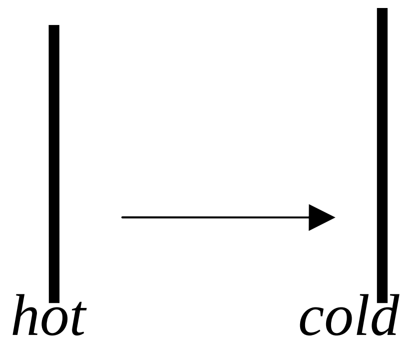
- Convection

Through moving fluids
(also boiling and condensation)



- Radiation

  Between surfaces, through gas or vacuum



What do we know about conduction?

Conduction, thermal conductivity, Fourier´s law:

Fourier’s law:

$$q = -k \cdot A \cdot \delta T / \delta x$$

where, q = heat flow (W)

A = area perpendicular to heat flow (m^2)

$\delta T/\delta x$ = temperature gradient in the direction of heat flow(°C/m)

k = thermal conductivity (W/(m °C))

Fourier´s law is the *defining equation* for the thermal conductivity.

Conduction, thermal conductivity, Fourier´s law:

For *one-dimensional heat transfer* (a plane wall,) with *constant thermal conductivity*, Fourier´s law is simplified to

$$q = k \cdot A \cdot \Delta T / \delta$$

where ΔT = temperature difference (°C)

δ = distance or thickness (m).

fig. 1

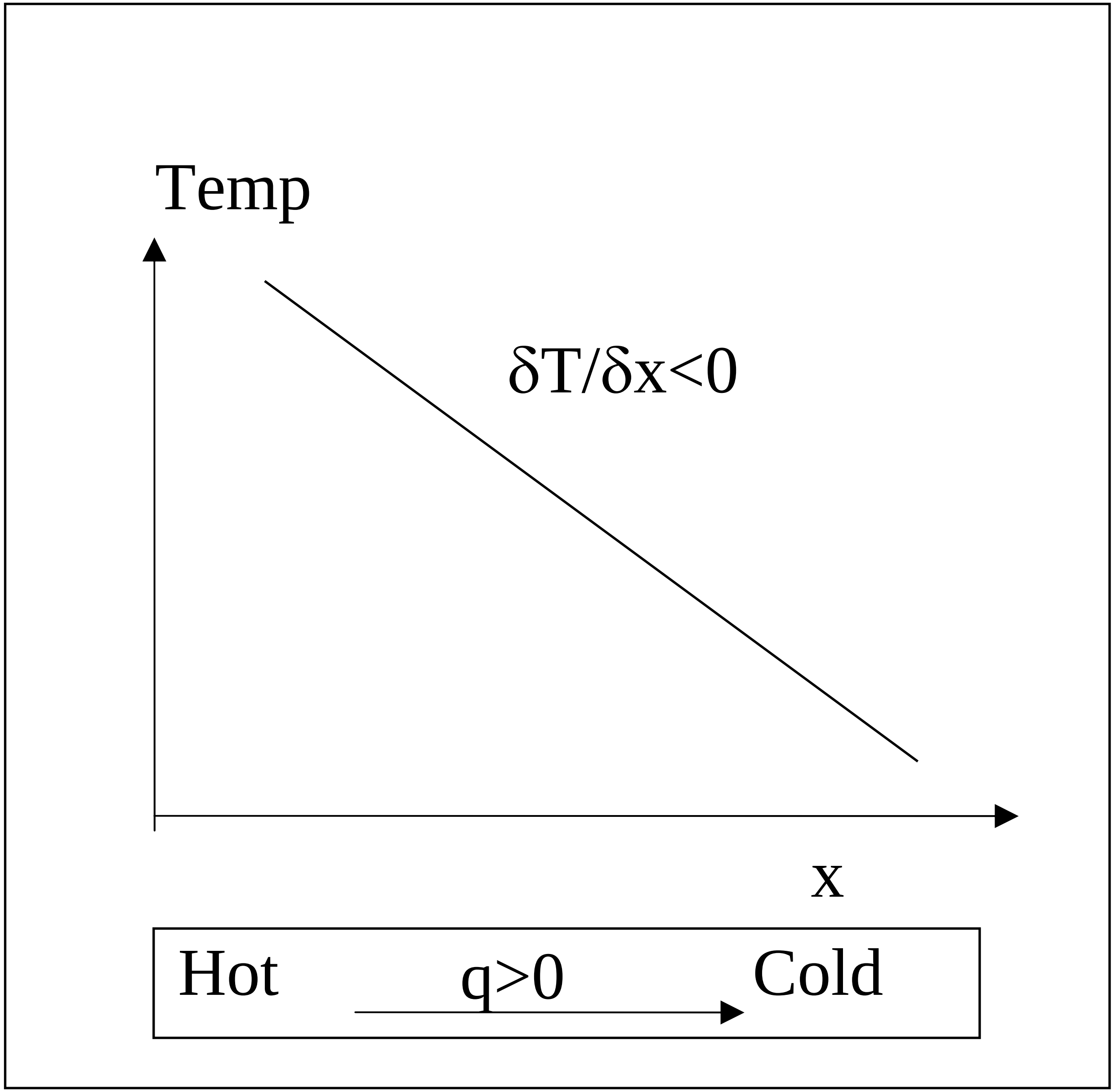


Fig. 2 Heat transfer through a plane wall

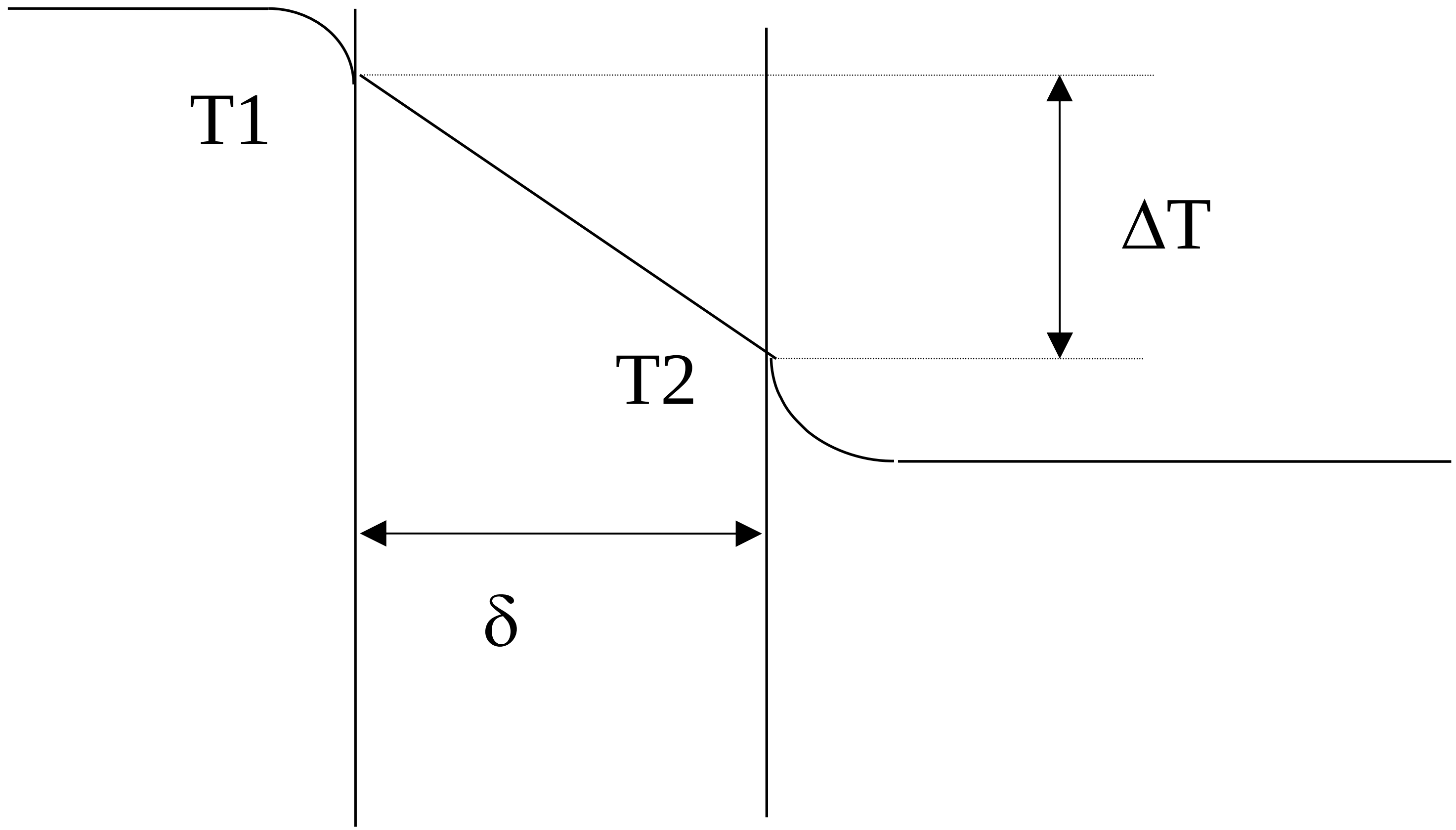


Table 1

Material	**Thermal conductivity (20°), (W/m·°C)**
Diamond, type IIa	2600
Copper	386
Iron, wrought, 0.5% C	60
Stainless steel, 18/8	16.3
Brick	0.69
Water	0.6
Pine wood,	0.15/0.33 (cross/along fibres)
Cork	0.045
Glass wool	0.038
Mineral wool	0.04
Polyurethane	0.02
Air	0.026
Argon gas	0.018

Example to solve:

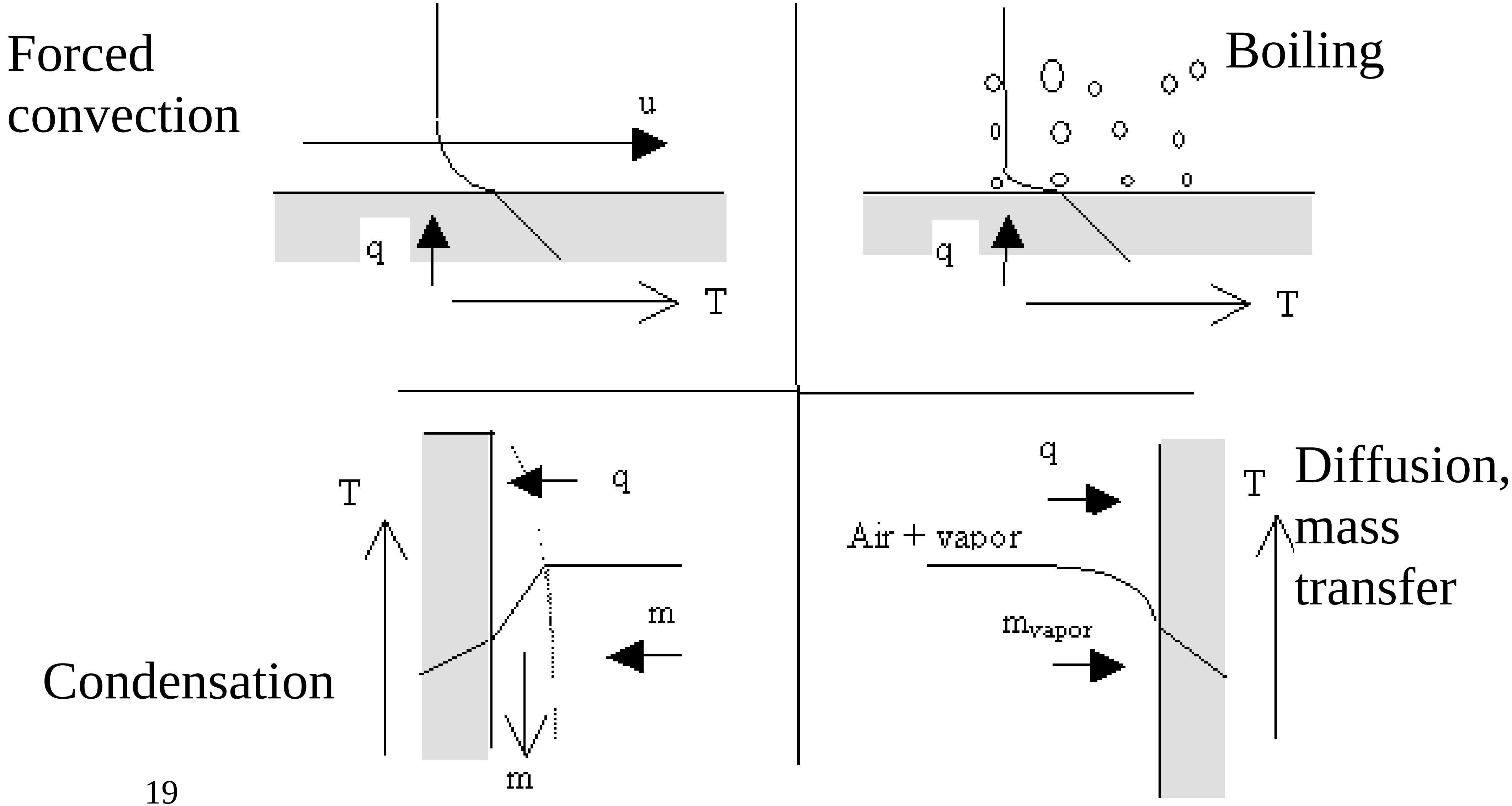
Conduction:

Calculate the heat flow per square meter (heat flux) through a mineral wool insulation, 5 cm thick, if the temperatures on the two surfaces are 30 and 200°C, respectively.

Heat transfer by convection, Newton’s law of cooling

Convection is a general term for heat transfer through a moving fluid.

Fig 3, Different types of convection heat transfer



What do we know about convection?

All types of convection are governed by *Newton's law of cooling:*

$$q = h \cdot A \cdot \Delta T$$

where A = surface area where convection takes place (m^2)

ΔT = temperature difference (°C)

h = heat transfer coefficient ($W/(m^2 \cdot °C)$)

Newton's law of cooling is the defining equation for the heat transfer coefficient h.

Table 2

Type of flow	Approximate heat transfer coefficient ($W/(m^2 \cdot °C)$)
Turbulent flow in tubes, (diameter ≈ 50 - 25 mm)	
Water (0.5 - 5 m/s)	1500 - 20000
Air (1 - 10 m/s)	10 - 50
Laminar flow in tubes, (diameter ≈ 50 - 10 mm)	
Water	50 - 250
Air	2 - 15
Air flow past plates (1 - 10 m/s)	10 - 50

Type of flow	Approximate heat transfer coefficient ($W/(m^2 \cdot °C)$)
Natural convection	
Water	200 - 1000
Air	2 - 10
Condensation	
Water	5000 - 15000
Refrigerants	1000 - 5000
Boiling	
Water	1000 - 40000
Refrigerants	200 - 5000

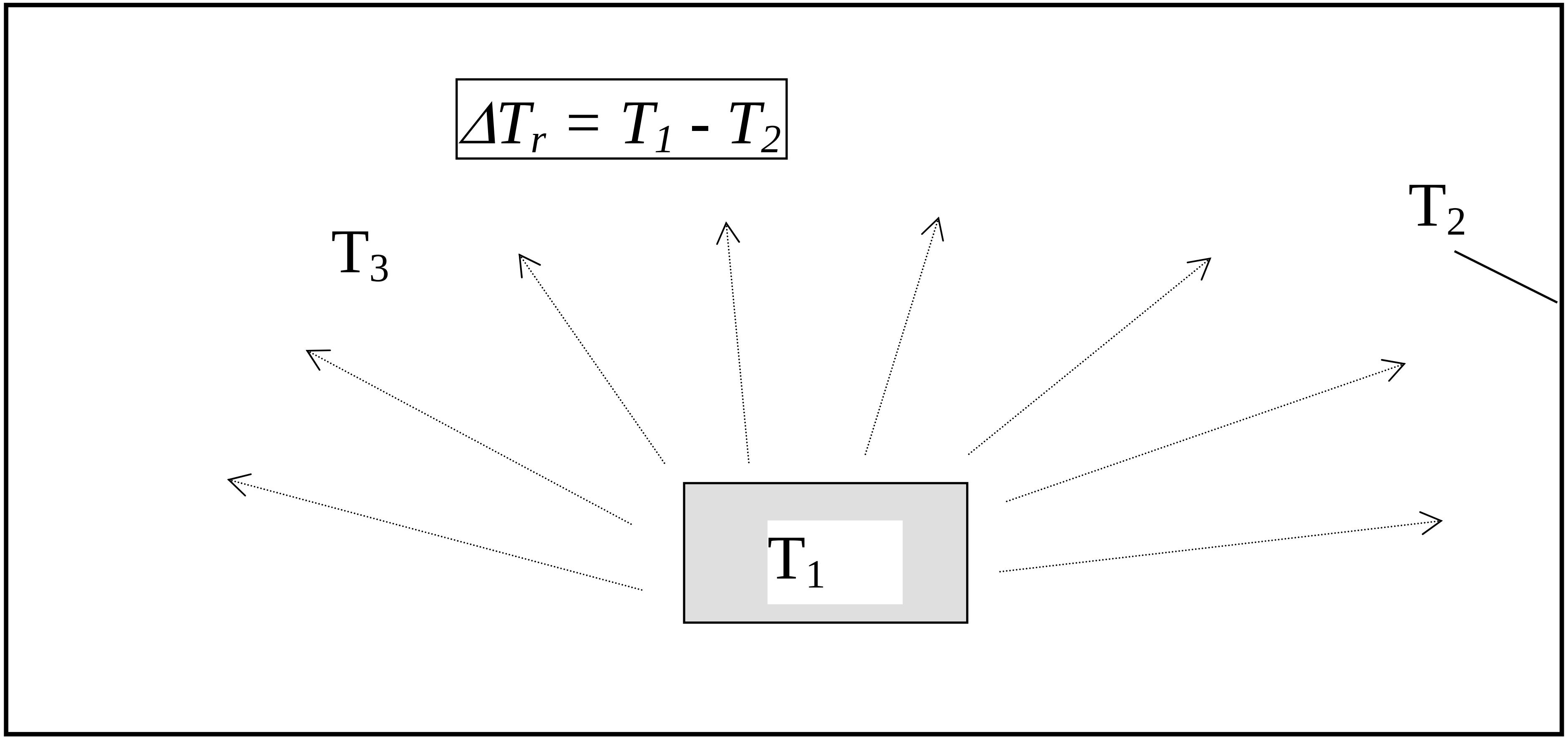
Example to solve:

Convection:
What is the approximate temperature difference between a hot plate and the surrounding air if the heat flux from the plate is 800 W/m^2 ? Assume that the air is flowing past the surface with a velocity of 5 m/s giving a heat transfer coefficient of 20 $W/(m^2K)$.

Radiation

- All bodies send out energy in the form of electromagnetic radiation.
- The wavelength and intensity is dependent on the temperature of the surface.
- Radiation may be transferred through vacuum, but also through air.

- fig. 4, Heat transfer by radiation
-



Heat transfer by radiation between a small body and an isothermal environment may be calculated by *Newton's law of cooling*, if we define a radiation heat transfer coefficient:

$$q_r = h_r \cdot A \cdot \Delta T_r$$

where q_r = heat flow due to radiation (W)

h_r = radiation heat transfer coefficient (W/(m^2·°C))

A = surface area of the small body (m^2)

ΔT_r = temp. difference between surfaces (°C).

- h_r is a function of the *geometry*, the *emissivity* of the surfaces and on the *temperatures of the surfaces*.

- For radiation heat transfer between non-polished surfaces with temperatures between 0 and 100°C, the radiation heat transfer coefficient is usually between 4.5 and 12 W/(m^2 °C) (≈5 at room temperature, ≈25 at 200°C)

Summing convection and radiation modes of heat transfer

- If the temperature differences for convection and radiation are equal then the h-values may be added to a *total* heat transfer coefficient.

$$h_c + h_r = h_{tot}$$

- If the temperature differences are not the same, the two modes have to be treated separately.

Example to solve:

Convection + radiation:

A cold bottle of beer (+5°C) is placed in a room where the temperature of the air and of the walls is 25°C. Calculate the approximate heat flux caused by radiation and by natural convection

Overall heat transfer coefficient

We define the *overall heat transfer coefficient* by the equation

$$q = U \cdot A \cdot \Delta T_{tot}$$

where U = the overall heat transfer coefficient (W/(m^2·°C))

A = surface area on either side of the wall (m^2)

ΔT_{tot} = difference between the fluid temperatures sufficiently far from the wall.

Relation between U, h and k

- U can be related to the h_1, h_2 in the fluids and to k and δ of the wall through the temperature differences:

- ΔT on either side of a wall can be written, according to Newton's law of cooling as, respectively:

$$\Delta T_1 = q/(h_1 \cdot A_1)$$

$$\Delta T_2 = q/(h_2 \cdot A_2)$$

- ΔT in the wall is, according to Fourier´s law:

$$\Delta T_W = q \cdot \delta/(k \cdot A_W)$$

- The total temperature difference is

  $$\Delta T_{Tot} = q/(U \cdot A)$$

- But

  $$\Delta T_{tot} = \Delta T_1 + \Delta T_2 + \Delta T_W$$

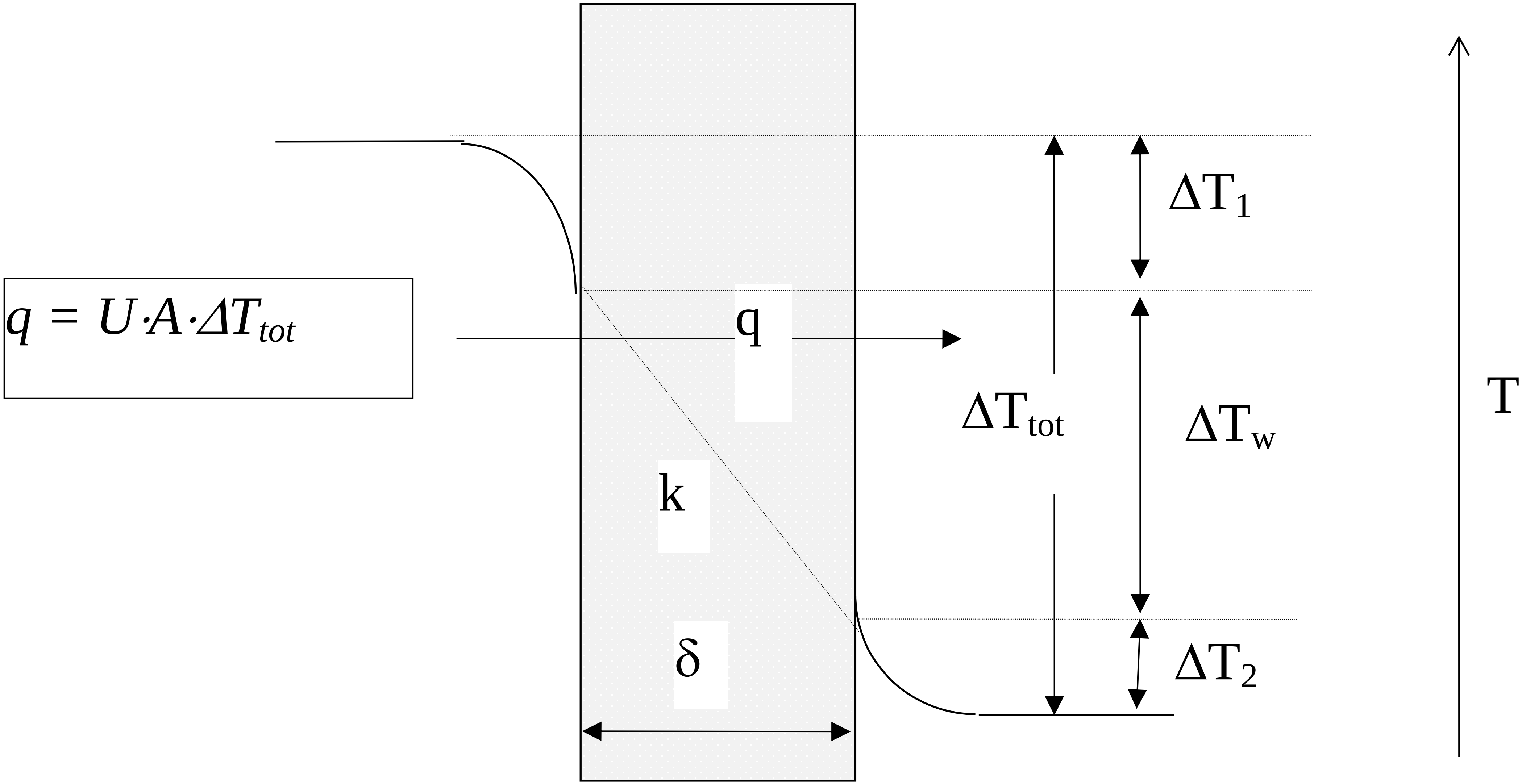
- q (at steady state) must be equal at the inside, the outside and in the wall, thus
- $1/(U \cdot A) = 1/(h_1 \cdot A_1) + \delta/(k \cdot A_W) + 1/(h_2 \cdot A_2)$

- For the case of a plane wall, the areas are also equal, and the relation is even simpler.

- If the wall has two or more layers of different materials, additional terms of $[\delta/(k \cdot A_W)]$ have to be added.

- For a curved surface, the surfaces are not the same. The overall heat transfer coefficient may be referred to any of the surfaces, but when specifying the U-value, it *must always be stated* to which area it is connected.

Fig. 5


$q = U \cdot A \cdot \Delta T_{tot}$
q
k
δ
ΔT_1
ΔT_{tot}
ΔT_w
ΔT_2
T

Heat transfer resistance

- *Thermal resistances*, are defined analogous to electric resistances.

- The thermal correspondence to Ohms law shows how the thermal resistance has to be defined:

Ohms law: Voltage = Current · Resistance

Thermal analogy gives: $\Delta T = q \cdot R_{th}$

- Thermal resistance in the fluid

  $R = 1/(h \cdot A_1)$

- Thermal resistance in the wall

  $R = \delta/(k \cdot A)$

- Total thermal resistance

  $R_{tot} = 1/(U \cdot A)$

- The total thermal resistance is thus the sum of the resistances

  $R_{tot} = R_1 + R_W + R_2$

Example to solve:

Overall heat transfer coefficient:
The heat flux through a plane wall is 5000 W/m^2 and the overall heat transfer coefficient is 1000 $W/(m^2 \cdot °C)$. Calculate the temperature difference!

Part 2

More about convection

Definition of dimensionless parameters

- A large number of parameters are needed to describe heat transfer.

- Parameters may be grouped together to form a small number of *dimensionless similarity parameters*.

These give simpler and more general equations by which heat transfer coefficients may be calculated.

Reynolds number

The Reynolds number is defined as

$$Re = u \cdot x / \nu$$

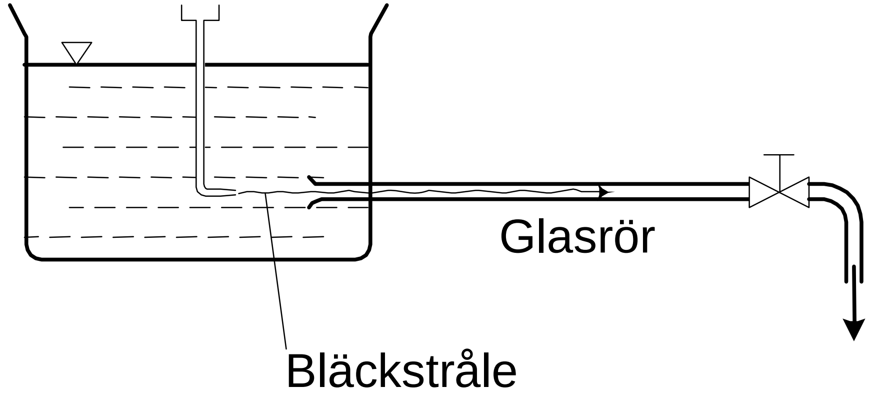
where u = velocity of fluid (m/s)

x = characteristic length (m). (For a tube, $x = d$).

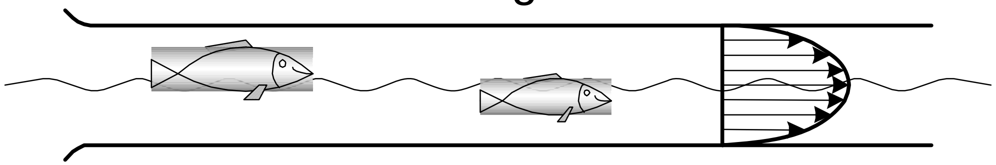
ν = kinematic viscosity of fluid (m^2/s)

- Reynolds number determines whether the fluid flow is laminar or turbulent.
- Reynolds number determines the ratio of the inertia and viscous forces in the flow.

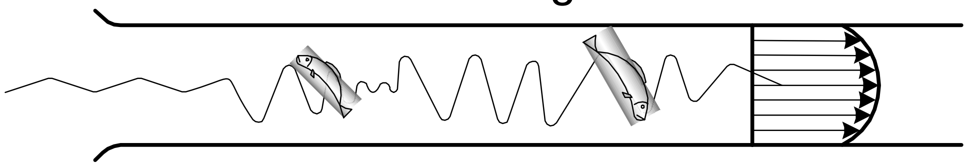
Reynolds number



Laminärt strömningssätt



Turbulent strömningssätt



Nusselt number

The Nusselt number is defined as

$$Nu = h \cdot x / k$$

where h = surface heat transfer coefficient ($W/(m^2 \cdot °C)$)

x = characteristic length (m).

k = thermal conductivity ($W/(m \cdot °C)$)

- The Nusselt number: a *dimensionless temperature gradient* at the surface.
- From Nu, the heat transfer coefficient can easily be calculated.

Example to solve:

Water ($k = 0.608$ W/(m·K)) is flowing through a tube with inner diameter 15 mm. The Nusselt number is found to be 70. What is the heat transfer coefficient?

Prandtl number

The definition of the Prandtl number is

$$Pr = \nu/\alpha = c_p \cdot \mu/k$$

where ν = kinematic viscosity of fluid (m^2/s)

α = thermal diffusivity (m^2/s)

c_p = specific heat (J/kg·°C)

μ = dynamic viscosity (N·s/m^2)

k = thermal conductivity (W/(m·°C))

- The Prandtl number is a thermodynamic property of the fluid.

Grashof number

The Grashof number is defined as

$$Gr = g \cdot \beta \cdot \Delta T \cdot x^3 / \nu^2$$

where g = acceleration of gravity (m/s^2)

β = volumetric thermal expansion coeff. (1/°C)

For (ideal) gases, ($\beta = 1/T_{abs}$).

ΔT = temperature difference between surface and fluid (°C)

x = characteristic length (m).

ν = kinematic viscosity of fluid (m^2/s)

- The Grashof number indicates, in *free convection*, whether the flow is laminar or turbulent.
- It is the ratio between buoyancy and viscous forces.

Graetz number

The Graetz number is defined as

$$Gz = Re \cdot Pr \cdot d/x$$

where d = (hydraulic) diameter of channel (m)
x = distance from entrance of channel (m)

- Gz is used when calculating heat transfer in laminar tube flow.

Can be interpreted as a dimensionless, inverted length (distance from entrance).

Rayleigh number

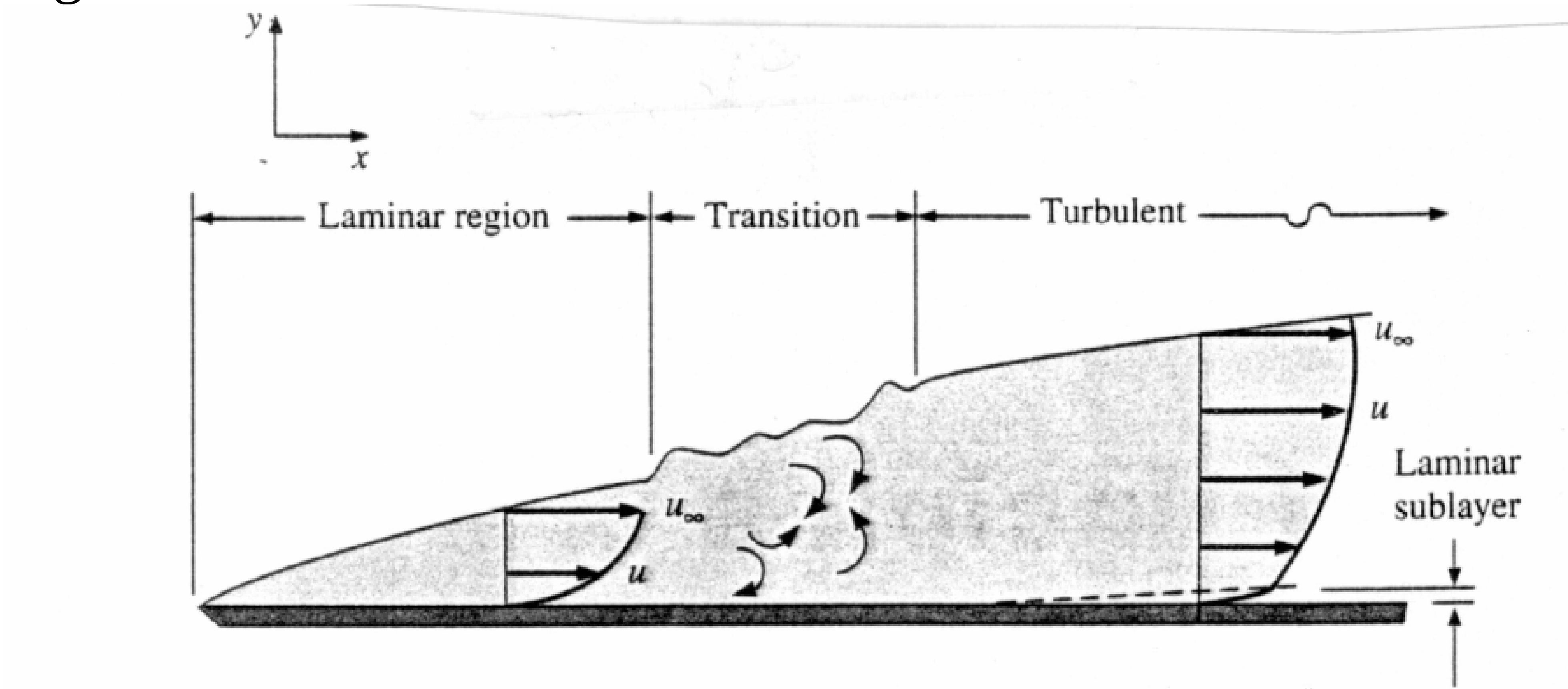
The Rayleigh number is defined as

$$Ra = Gr \cdot Pr$$

The Rayleigh number often appears in equations for free convection.

Laminar and turbulent flow, velocity boundary layer

Fig. 6



- The part of the flow where the velocity is influenced by the surface is called the boundary layer.

- Near the front edge of the plate the thickness of the boundary layer is thin and it then grows successively thicker.

- As long as the boundary layer is thin, there is no mixing between layers at different distances from the plate. The flow is then said to be *laminar*.

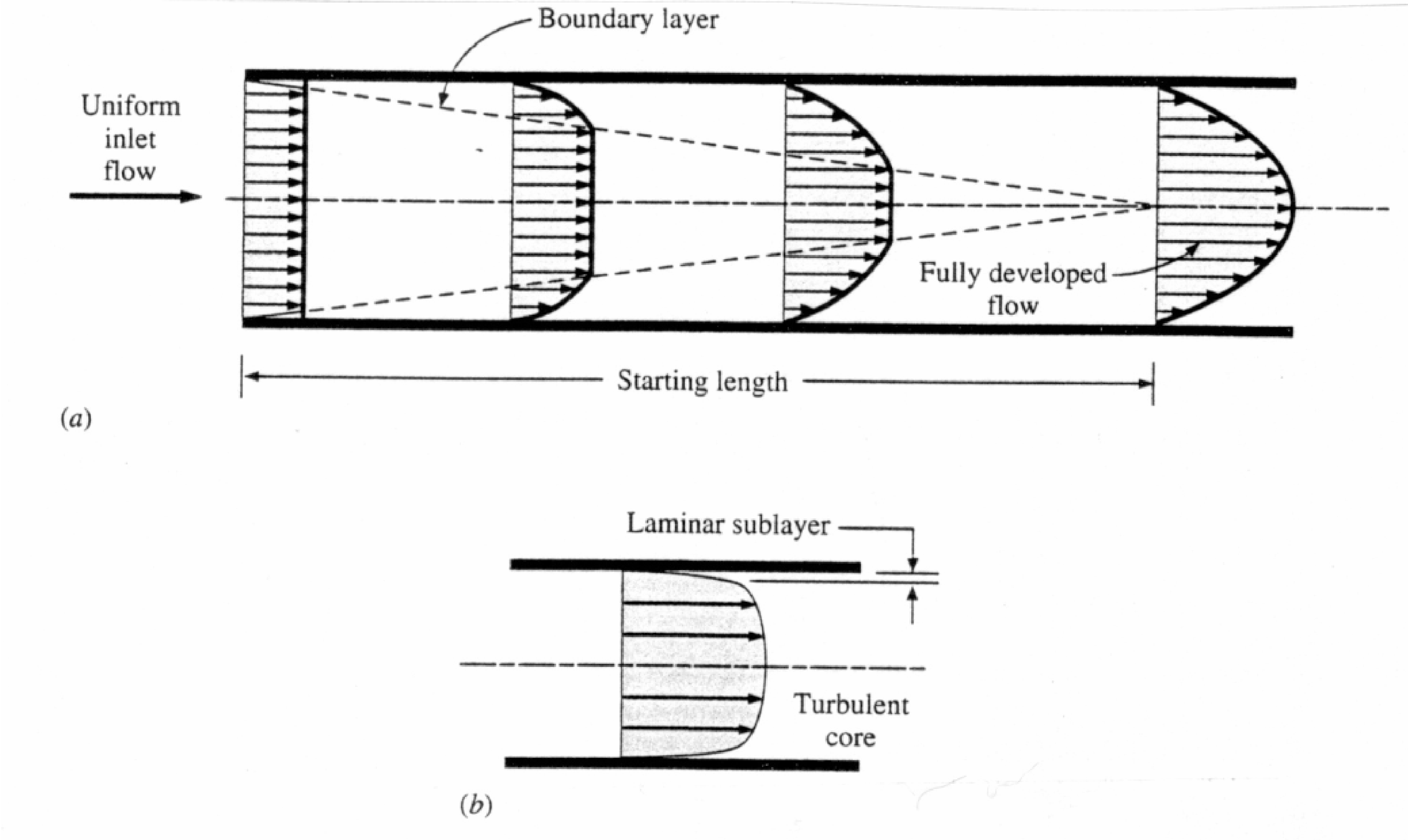
- At some distance from the leading edge, the laminar layer will become unstable, and eddies will develop, mixing the different layers. The flow is then becoming *turbulent*.

- Because of the mixing, the difference in velocity between layers is much smaller in turbulent flow than in laminar, and the velocity profile thus much flatter.

- Even in the turbulent region there is a *laminar sub-layer* closest to the surface. In this sub-layer, the temperature profile is nearly linear.

- The type of flow, laminar or turbulent, can be determined from the Reynolds number calculated with the distance from the leading edge as the characteristic length. Transition will occur at $Re \approx 5 \cdot 10^5$.

Fig. 7


Boundary layer
Uniform inlet flow
Fully developed flow
Starting length
(a)
Laminar sublayer
Turbulent core
(b)

- *In tube flow*, at some distance from the entrance, the boundary layers from opposite sides will meet. At this point the *flow is fully developed*.

- The fully developed flow may be turbulent or laminar.

- The Reynolds number is calculated using the tube diameter as the characteristic length. Transition takes place at approximately Re ≈ 2300.

- The heat transfer coefficients are generally higher in turbulent than in laminar flow.

Example to solve:

Water ($\nu = 0.86 \cdot 10^{-6}$ m^2/s) flows through a tube with the diameter 12 mm at a velocity of 2 m/s. Determine if the flow is laminar or turbulent!

Thermal boundary layer

- At heated (or cooled) surfaces, a *thermal boundary layer* will form in which the temperature change from the wall temperature to the temperature of the undisturbed fluid.
- *In laminar flow* the velocity- and the thermal boundary layers will be similar, but the thicknesses will not necessarily be the same.
- The relative thickness of the thermal and velocity boundary layers is *in laminar flow* related to the Prandtl number by $\delta_v / \delta_{th} \approx Pr^n$ where n is a positive exponent (≈1/3).

- For *gases*, the Prandtl number is usually between 0.7 and 1, and in laminar flow the thermal and velocity boundary layer thicknesses are thus approximately equal.

- For *liquid metals*, the Pr<<1 and the thermal boundary layer (in laminar flow) is considerably thicker than the velocity boundary layer, while for *oils*, Pr>>1 and the velocity boundary layer is the thickest. (For water, Pr range from 13.4 at 0°C to 1.75 at 100°C).

- For *turbulent boundary layers*, the mixing within the layer will result in more or less equal thicknesses of the velocity and turbulent layers.

Forced and free convection

- Fluid flow may be caused by

  >*force* (by a fan or a pump or any other means external to the fluid itself).

  >*free convection* (the movement is caused by temperature induced density differences in the fluid).

- In forced convection, the type of flow (turbulent or laminar) is determined from the *Reynolds number*

- In free convection, the type of flow is determined by the *Grashof number*

- The dimensionless equations are different for the two cases:

  >In forced convection: $Nu = f(Re, Pr)$

  >In free convection: $Nu = f(Gr, Pr)$

Methods for calculating heat transfer coefficients (in one phase flow)

Forced convection

Turbulent flow in tubes and channels

The heat transfer coefficient is generally referred to the logarithmic mean temperature difference (LMTD) defined as:

$$\vartheta_{ln} = [(T_{h2} - T_{c2}) - (T_{h1} - T_{c1})] / ln[(T_{h2} - T_{c2}) / (T_{h1} - T_{c1})]$$

$$= \frac{\Delta T_2 - \Delta T_1}{\ln \frac{\Delta T_2}{\Delta T_1}}$$

where ΔT_1 *and* ΔT_2 *are the temperature differences at each end of the heated section (inlet and outlet)*

Turbulent flow in tubes and channels

Fully developed turbulent flow in tubes,
the Dittus - Boelter equation:

$$Nu = 0.023 \cdot Re^{0.8} \cdot Pr^{n}$$

where $n = 0.4$ *for heating of the fluid*
$n = 0.3$ *for cooling of the fluid*

Valid in smooth tubes for $Re > 10000$
$0.6 < Pr < 100$.

Fluid properties at bulk temperature (≈average of inlet and outlet temperatures).

May, for fluids with low viscosity ($\mu < 2 \cdot \mu_{H2O}$), be used when Re> 2300, that is for the whole turbulent region.

Entrance region, turbulent flow

For the *entrance region* and for short tubes ($10<L/d<400$):

$$Nu = 0.036 \cdot Re^{0.8} \cdot Pr^{1/3} \cdot (d/L)^{0.055}$$

where d = diameter of tube (m)

L = length of tube (m)

Thermal properties at bulk temperature.

Non-circular cross-sections, hydraulic diameter

Calculate the diameters as *hydraulic diameters*, D_H , defined as

$$\boxed{D_H = 4 \cdot A/P}$$

where A = cross sectional area of the flow (m^2)

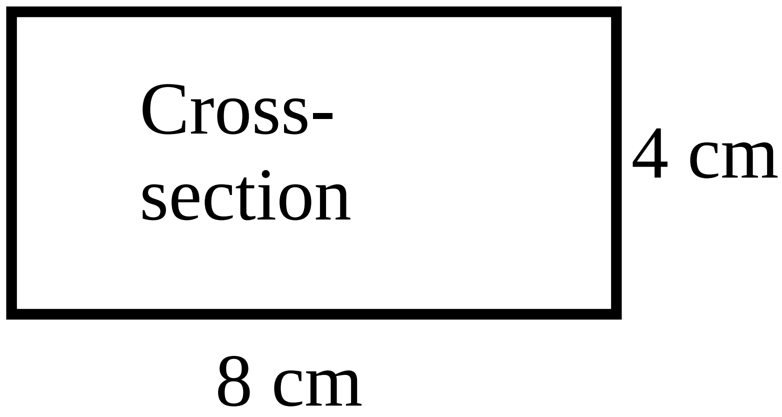
P = wetted perimeter (m)

Example:

$A = 4\text{ cm} \cdot 8\text{ cm} = 32\text{ cm}^2$

$P = 4 + 4 + 8 + 8 = 24\text{ cm}$

$\Rightarrow D_H = 4 \cdot 32 / 24 = 5.33\text{ cm}$



Laminar flow in tubes and channels

When the temperature field is fully developed, the *Nusselt number is constant!*

The value of the constant depends on the boundary conditions and on the shape of the cross section

The temperature field is considered to be fully developed for Gz <10.

Nu number in fully developed laminar flow in tubes and channels:

Geometry of cross section	Nu (Constant wall temp)	Nu (Constant heat flux)
Triangular (equilat.)	2.47	1.89
Square	2.98	3.09
Circular	3.66	4.36
Two infinite plates	7.54	8.24

Entrance region in laminar flow

For the entrance region in laminar flow (Gz>10), (average Nusselt number)

$$Nu = 1.86 \cdot Gz^{1/3} \cdot (\mu / \mu_w)^{0.14}$$

where μ = the dynamic viscosity of the fluid at the mean bulk temperature

μ_w = the dynamic viscosity of the fluid at the wall temperature

Fig. 9 (Fig. 6-5 in Holman)

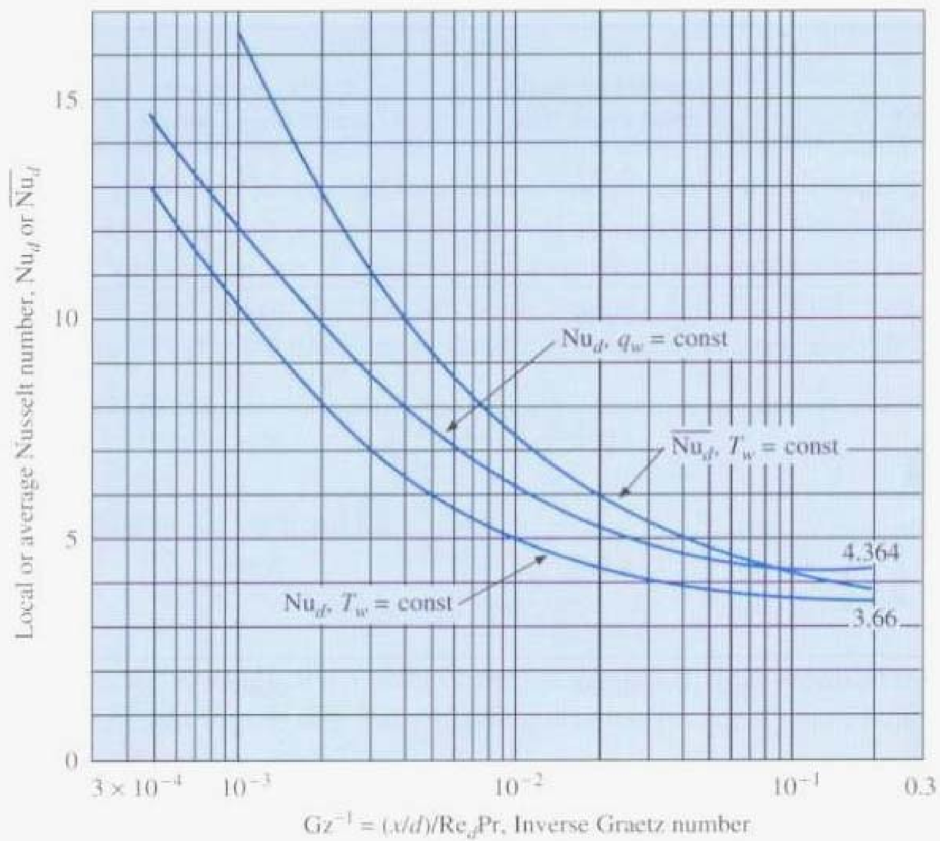


Figure 6-5 | Local and average Nusselt numbers for circular tube thermal entrance regions in fully developed laminar flow.

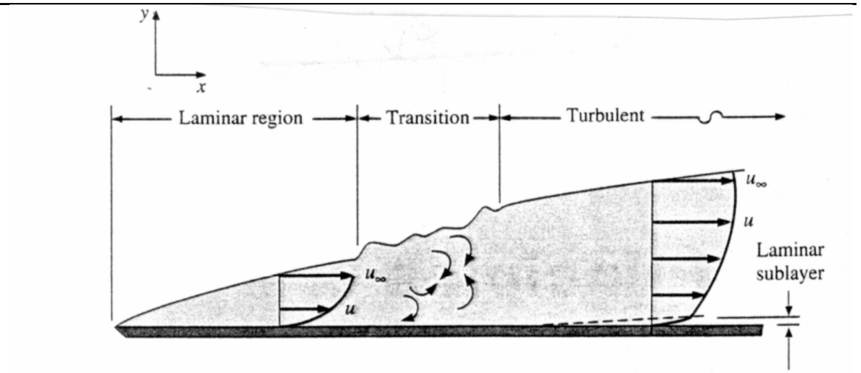
Flow across a plate, laminar and turbulent flow

Plate heated (or cooled) right from the leading edge, *laminar* flow ($Re_x < 5 \cdot 10^5$, 0.6<Pr<60, isothermal surface):

$Nu_x = 0.332 \cdot Re_x^{1/2} \cdot Pr^{1/3}$	Local Nu number at *x*.
$Nu_L = 0.664 \cdot Re_L^{1/2} \cdot Pr^{1/3}$	Average Nu number for distance 0 to L from edge.

Thermodynamic properties at *film temperature* (average of wall and free stream temperatures).

Turbulent flow across plates



For $(5 \cdot 10^5 < Re < 10^7)$

$Nu_x = 0.0296 \cdot Re_x^{0.8} \cdot Pr^{1/3}$	Local Nusselt number at *x*

Average Nu number for *both* the laminar part of the flow closest to the edge and the turbulent part $(5 \cdot 10^5 < Re < 10^7)$

$$Nu_L = (0.037 \cdot Re_L^{0.8} - 871) \cdot Pr^{1/3}$$

Forced convection across single cylinders

Average Nusselt number for the circumference:

$$Nu = C \cdot Re^{n} \cdot Pr^{1/3}$$

Reynolds number (*x*=*d*)	*C*	*n*
0.4 - 4	0.989	0.330
4 - 40	0.911	0.385
40 - 4000	0.683	0.466
4000 - 40’000	0.193	0.618
40’000 - 400’000	0.0266	0.805

Fluid properties at *film temperature*.

Forced convection across tube banks

Average Nusselt number:

$$\boxed{Nu = C \cdot Re^{n} \cdot Pr^{1/3}}$$

Constant *C* and the exponent *n* are dependent on the ratios of the tube spacings (normal and perpendicular to flow) and the tube diameter

(See table 6-4 in Holman).

The Reynolds number should be calculated using the maximum velocity occurring in the tube bank.

Example to solve:

Find the average Nusselt no as air flows past a 0.3 m heated plate at 0.5 m/s. For the air, $\nu = 15.7 \cdot 10^{-6}$ m^2/s, Pr = 0.72.

Free convection

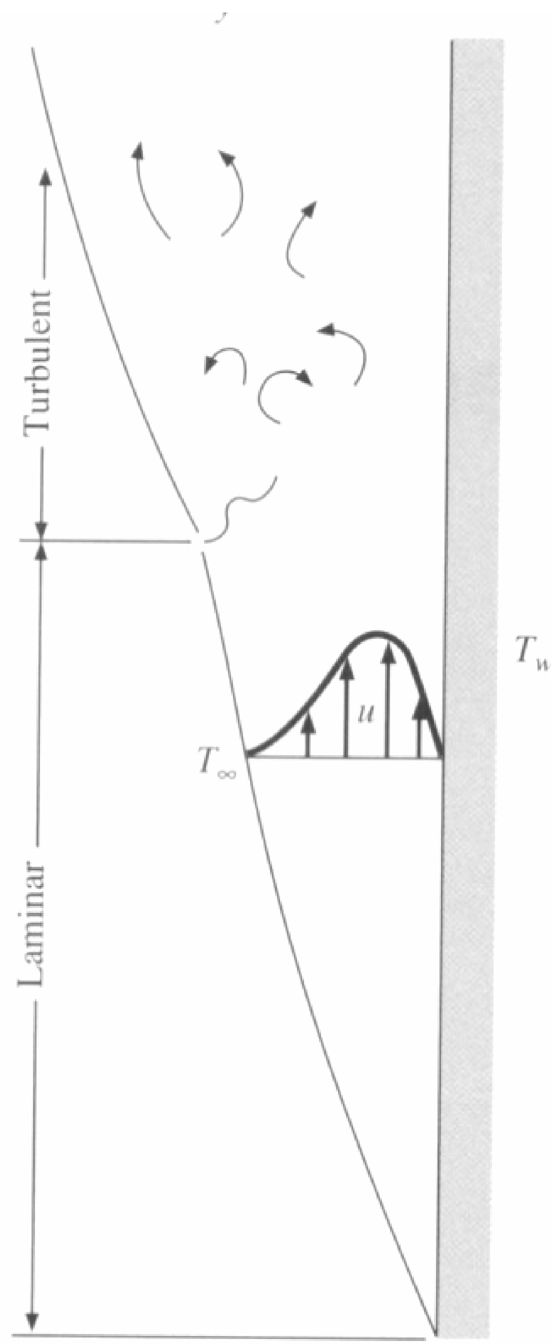
- In free convection the fluid flow is induced by density differences caused by temperature differences.

- The flow is caused by buoyancy forces, is dependent on gravitation to appear.

- The thermal and velocity boundary layers in free convection are quite different from those in forced convection.

- The *critical Grashof number*, where the transition occurs from laminar to turbulent flow depends on the geometry.

Free convection boundary layer on a vertical flat plate



General correlation for free convection

In many different geometries, the *average Nusselt number* may be calculated by as

$$\boxed{Nu = C \cdot (Gr \cdot Pr)^m}$$

The constant *C* and the exponent *m* depend on the geometry and on the size of (Gr·Pr).

Thermodynamic properties at *film temperature*.

Constant C and exponent m in general corr. for free conv.

Geometry	$Gr \cdot Pr$	C	m
Vertical plates	10^4 - 10^9	0.59	1/4
and cylinders	10^9 - 10^{13}	0.10	1/3
Horizontal cylinders	10^4 - 10^9	0.53	1/4
	10^9 - 10^{12}	0.13	1/3
Upper surface of heated plates, or	$2 \cdot 10^4$ - $8 \cdot 10^6$	0.54	1/4
lower surface of cooled plate	$8 \cdot 10^6$ - 10^{11}	0.15	1/3
Lower surface of heated plates, or upper surface of cooled plates	10^5 - 10^{11}	0.27	1/4

Free convection - $Gr \cdot Pr/(\Delta T \cdot L^3)$ tabulated

- The group

$$\underline{Gr \cdot Pr/(\Delta T \cdot L^3) = g \cdot \beta / \nu^2}$$

is only dependent on material properties and may be found in the *Collection of formulas and tables* for different fluids.

Free convection - The exponent m

- The exponent *m* is often 1/3 (turbulent flow) or ¼ (laminar flow).

- With $m = 1/3$, *h* is independent of *L* as it cancels out of the equation:

$$Nu = C \cdot (Gr \cdot Pr)^{1/3}$$

$$h \cdot L/k = C \cdot [(g \cdot \beta \cdot \Delta T \cdot L^3/\nu^2) \cdot Pr]^{1/3}$$

$\Rightarrow$ *h* independent of *L*

Free convection - Vertical plates, vertical cylinders

- Isothermal vertical plates: L = height of plate.

- Vertical cylinders: treat as vertical plates, *if*
  $D/L \geq 35/Gr^{1/4}$

- For Gr < 10^4, use graphical solution (fig. 12)

Free convection, simplified correlations for air

For a given fluid, the dimensionless equation 34 may be rewritten to give the heat transfer coefficient directly. Two cases have to be discerned, laminar and turbulent:

For laminar flow:

$$h = K_l \cdot (\Delta T/H)^{1/4} \qquad \text{for } 10^4 < Gr \cdot Pr < 10^8$$

For turbulent flow

$$h = K_t \cdot \Delta T^{1/3} \qquad \text{for } 10^8 < Gr \cdot Pr < 10^{12}$$

Where K_l and K_t from Table 6 *for air* (assuming SI-units).

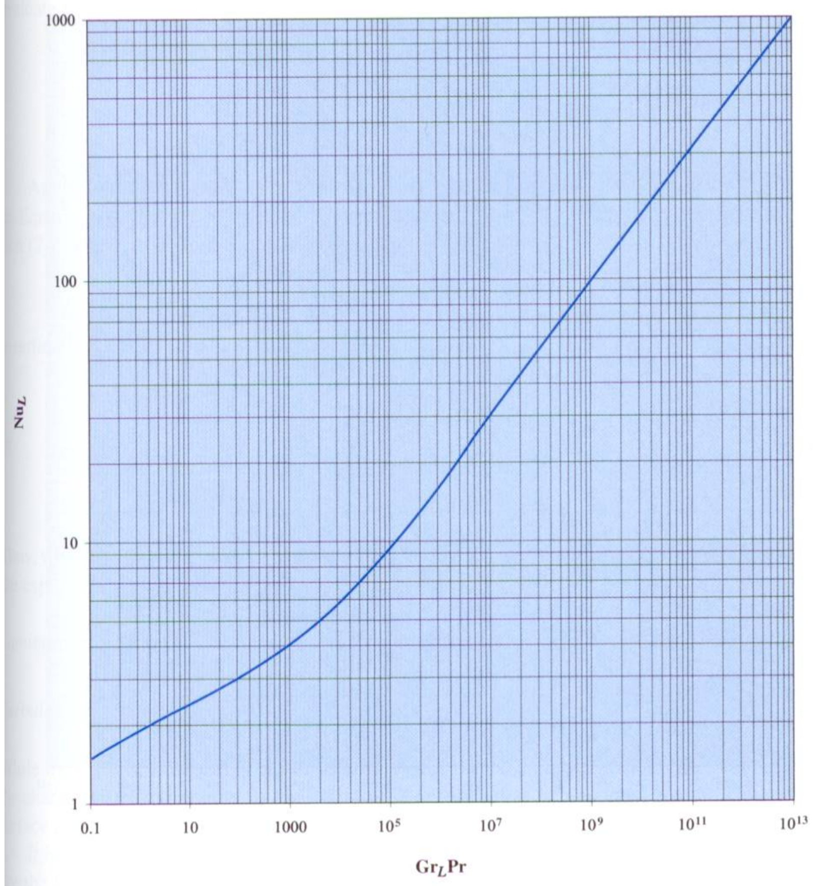
Free convection, simplified correlations for air

Table 3: Constants K_l and K_t for air

T_{film}	$(Gr \cdot Pr)/(\Delta T \cdot H^3)$	K_l	K_t
-50	$34.8 \cdot 10^7$	1.57	1.88
0	$14.5 \cdot 10^7$	1.49	1.66
50	$6.75 \cdot 10^7$	1.41	1.48
100	$3.47 \cdot 10^7$	1.35	1.33
200	$1.18 \cdot 10^7$	1.27	1.14
300	$5.1 \cdot 10^6$	1.21	1.01
400	$2.54 \cdot 10^6$	1.15	0.91
600	$0.85 \cdot 10^6$	1.06	0.76

Vertical
Isothermal
Plates,
Nu vs Gr*Pr

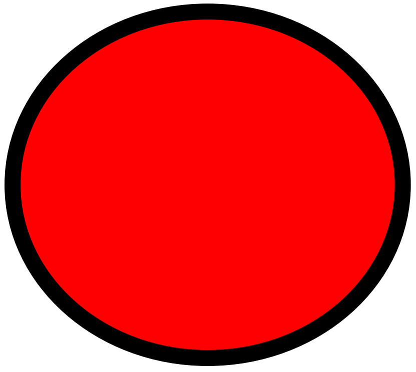
Figure 7-5 | Free-convection heat transfer from vertical isothermal plates.



Free convection from horizontal cylinders

- Constants *C* and *m* from table.

- Tube diameter is characteristic length.



Free convection from horizontal surfaces

Two cases:

1. Top side of a heated plate *and* bottom side of a cooled plate. (Gravitation will force fluid away from surface)

2. Bottom side of a heated surface *and* top side of a cooled plate. (A stable layer of fluid will form, decreasing the heat transfer).

- *C* and *m* from table. *C* is exactly twice as high in case 1.

- Characteristic length: $L = A/P$
  where *A* is the area and *P* the perimeter of the surface.

Free convection in between vertical plates

Correlation for the average Nusselt number:

$$Nu_{as} = \left[\frac{C_1}{(Ra_s \cdot s / L)^2} + \frac{C_2}{(Ra_s \cdot s / L)^{1/2}} \right]^{-1/2}$$

where s = distance between plates (m)

L = plate height (m)

Ra_s = (Gr · Pr), with s as characteristic length.

$Nu_{a,s}$ = average Nusselt no, with s as the characteristic length.

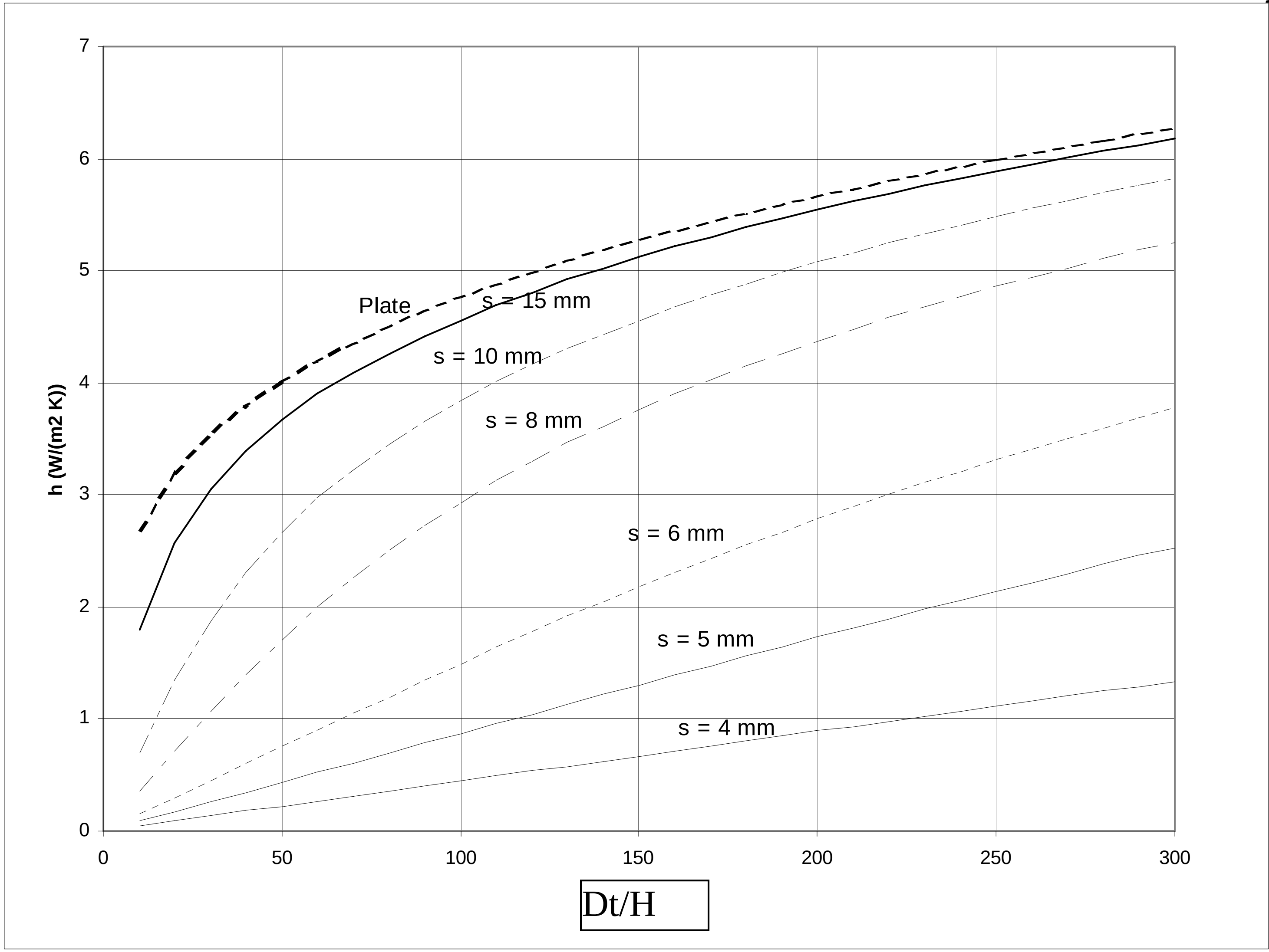
- ΔT in Gr is the temperature difference between the fin surface and the undisturbed air.

Free convection in between vertical plates

Constants C_1 and C_2

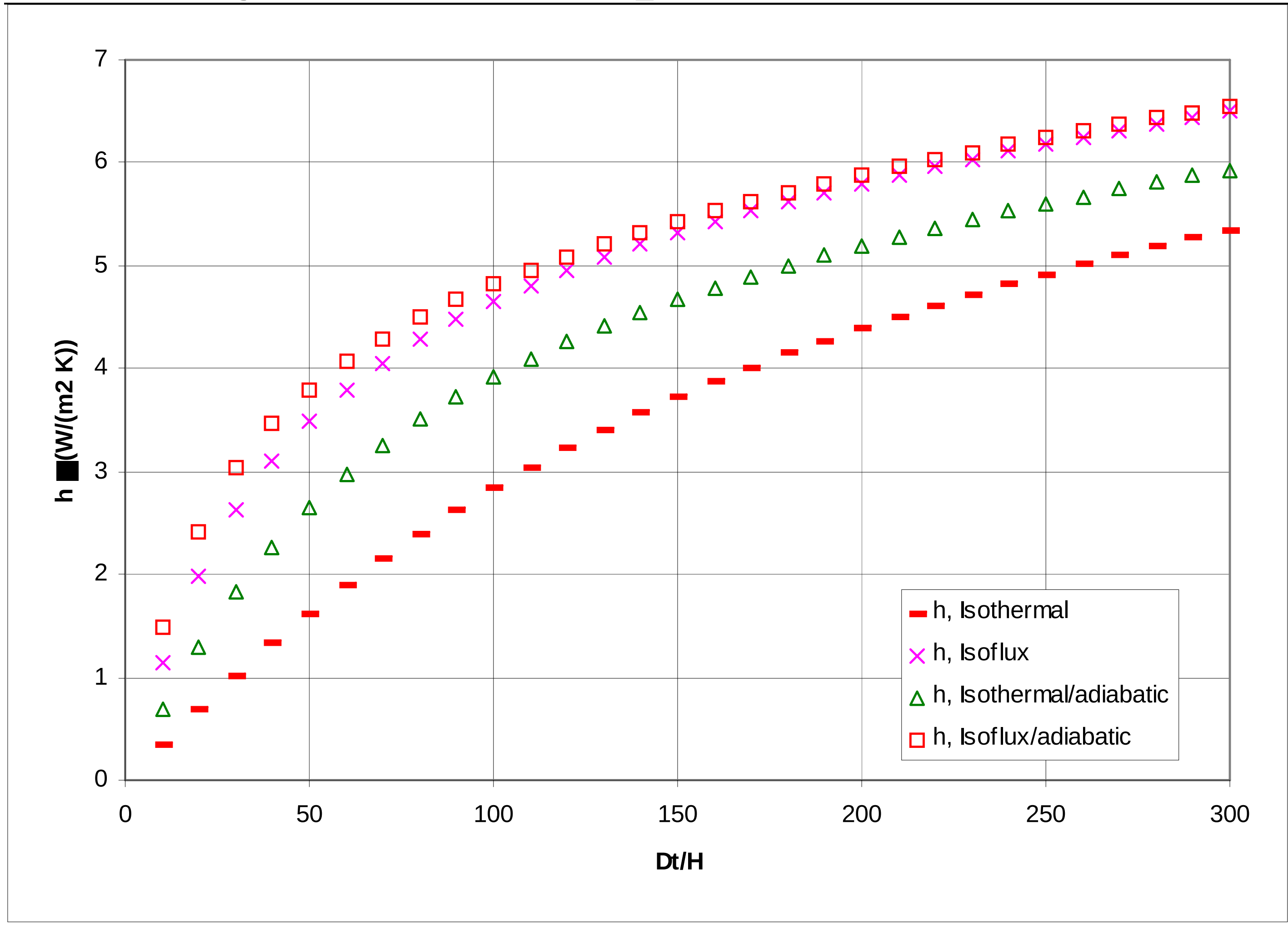
Boundary condition	C_1	C_2
Symmetric isothermal plates	576	2.87
Symmetric isoflux plates	48	2.51
Isothermal/adiabatic plates	144	2.87
Isoflux/adiabatic plates	24	2.51

Free convection in between vertical isothermal plates

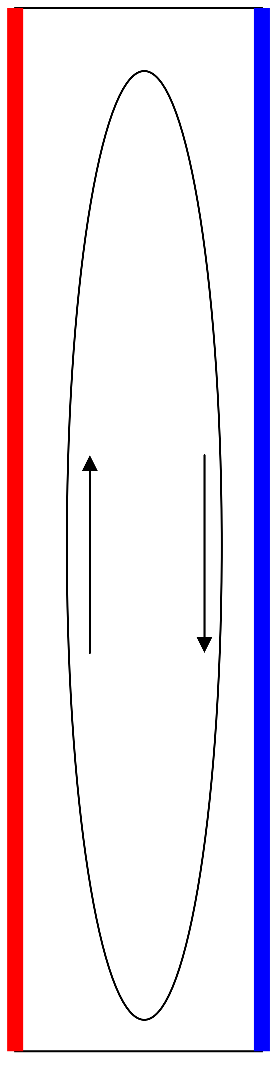


(air @ 60°C)

Free convection in between vertical plates, different boundary conditions, (plate distance 8 mm, air @ 60°C)



Flow in an enclosed space



Free convection in enclosed spaces (fig 15)

Define a heat transfer coefficient using the surface temperatures between which heat is transferred:

$$h = q/(A \cdot (T_1 - T_2))$$

Use the distance δ between the surfaces as the characteristic length

The Nusselt number is expressed as the ratio between an *apparent, or effective thermal conductivity* k_e and the normal conductivity k.

$$Nu = k_e / k$$

We get:

$$q/A = h \cdot (T_1 - T_2) = (Nu \cdot k/\delta) \cdot (T_1 - T_2) = k_e / \delta \cdot (T_1 - T_2)$$

with $Nu \cdot k = k_e$

For vertical enclosures and constant heat flux

$$\boxed{Nu = 0.42 \cdot (Gr \cdot Pr)^{1/4} \cdot Pr^{0.012} \cdot (L/\delta)^{-0.30}}$$

Conditions: $10^4 < Gr \cdot Pr < 10^7$

$1 < Pr < 20000$

$10 < L/\delta < 40$

where δ = the distance between the plates (m)

L = the height of the enclosure (m)

Vertical enclosures - Larger values of Gr·Pr

$$\boxed{Nu = 0.046 \cdot (Gr \cdot Pr)^{1/3}}$$

Conditions $10^6 < Gr \cdot Pr < 10^9$

$1 < Pr < 20$

$1 < L/\delta < 40$

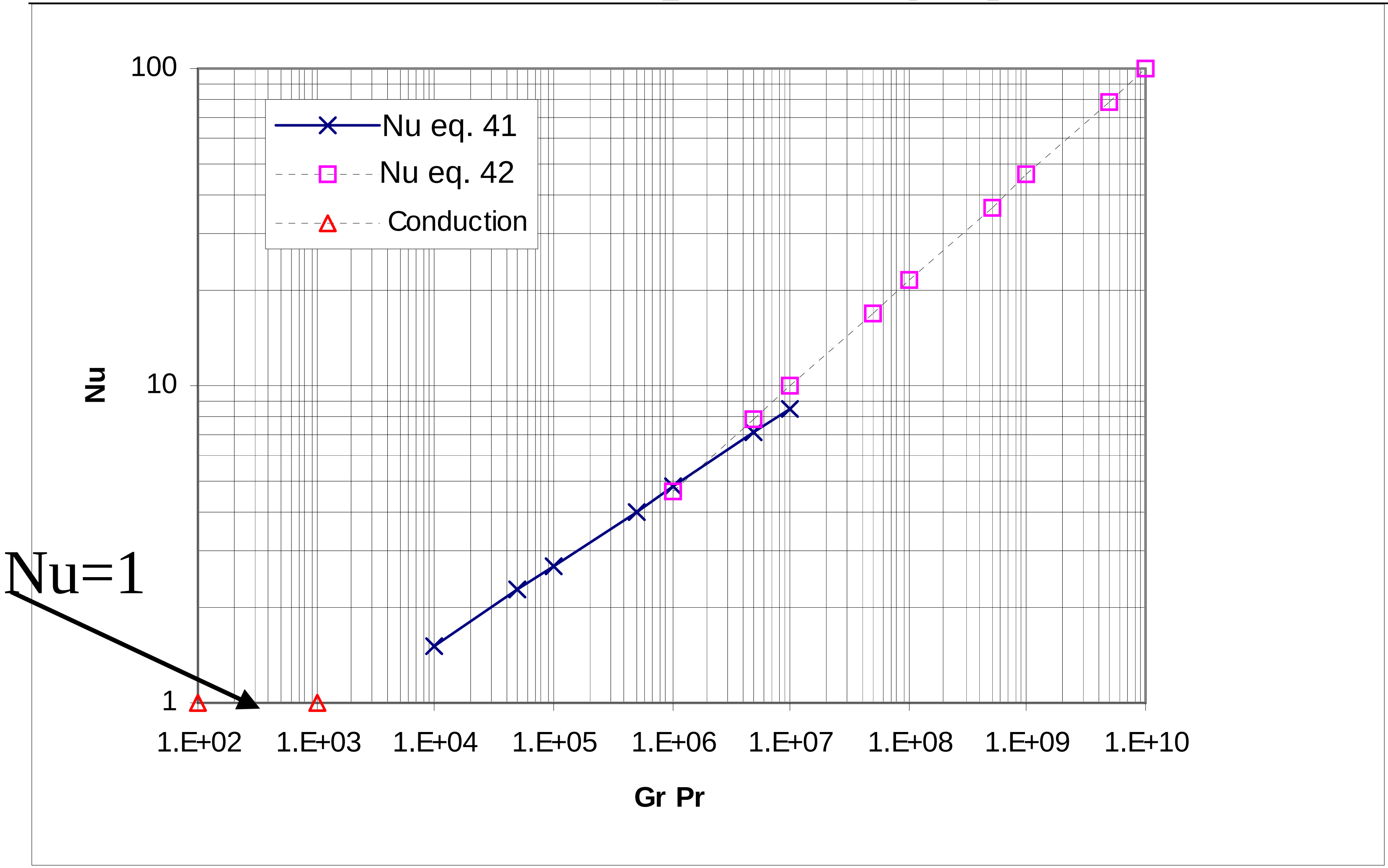
Use δ, as characteristic length in Nu and Gr,

ΔT = difference between the surfaces.

Vertical enclosures - Low Gr · Pr

At $Gr \cdot Pr < 10^3$ there is no free convection and heat is transferred by conduction only ($Nu = 1$).

Vertical enclosures, comparison of eqs. 41 and 42.

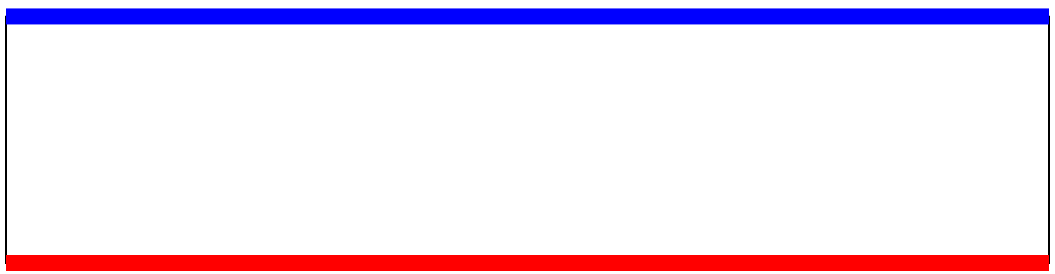


Horizontal enclosures

Two cases:

1. If the top plate is the hotter, a stable situation will occur and heat is transferred by pure conduction, and thus Nu = 1 (or $k_e /k =1$).

2. If the bottom plate is the hotter, pure conduction will occur at Gr < 1700, while at higher values, *convection cells* will occur, increasing heat transfer. See table 7-3 in Holman.



Combined free and forced convection

To determine which type of convection that is dominant, the following inequality may be used.

$$\boxed{Gr / Re^2 > 10}$$

If this criterion is fulfilled, free convection is dominant.

Example to solve:

Find the Nusselt no at a heated vertical plate, if the Grashof no is known to be $1 \cdot 10^8$ and Pr is 1.

Part 3

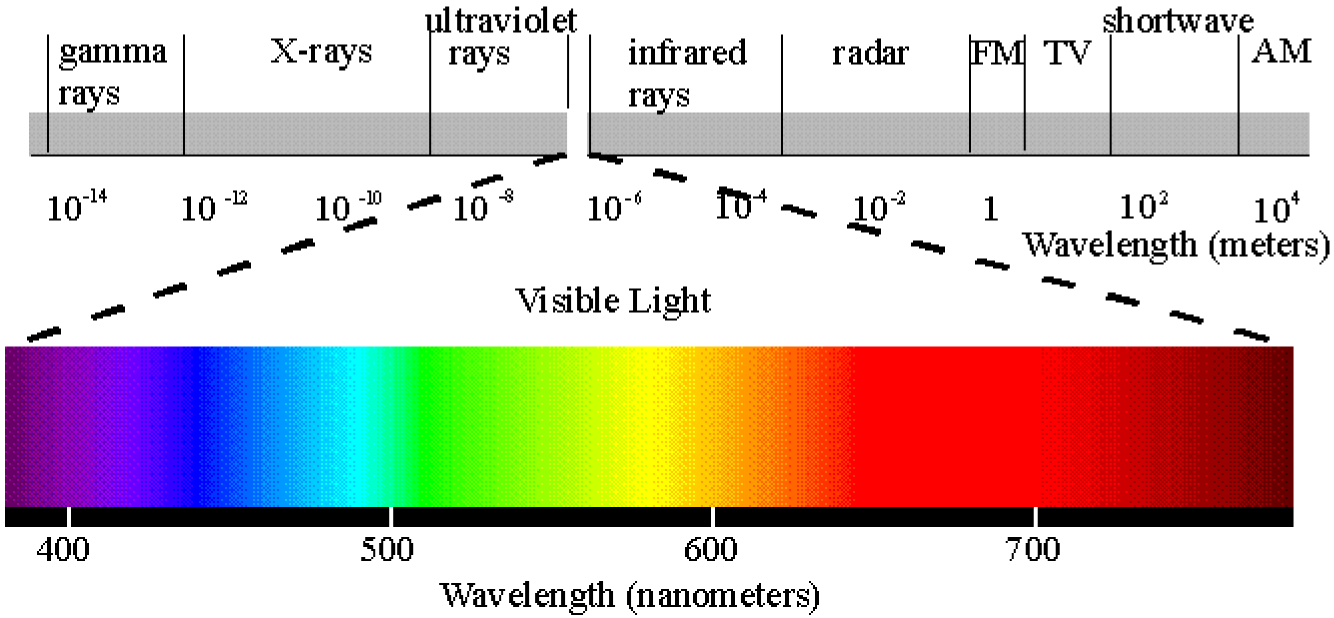
Radiation

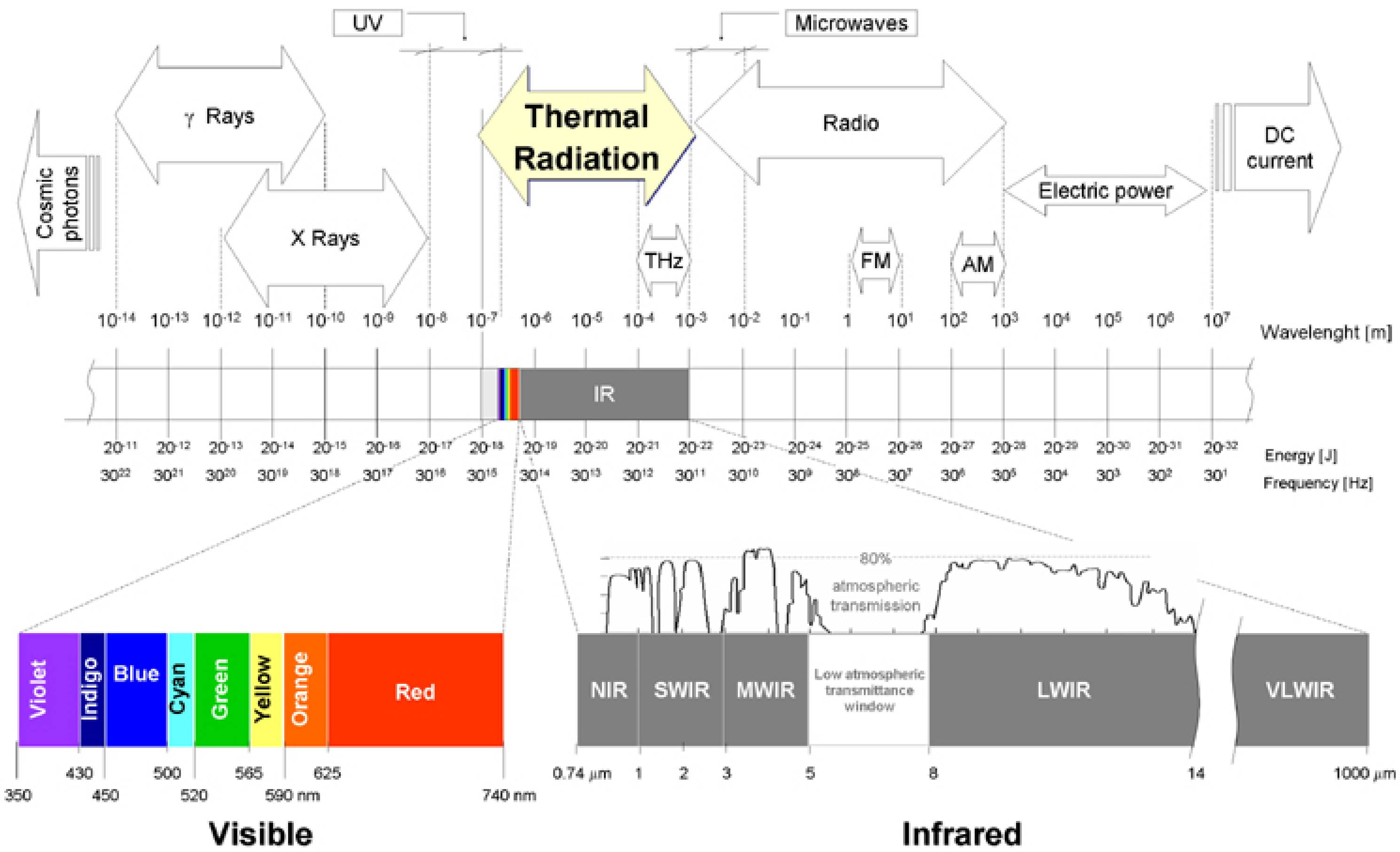
Radiation

Thermal radiation.:
”That electromagnetic radiation emitted by a body as a result of its temperature”.

Thermal radiation is restricted to a limited range of the electromagnetic spectrum.

Radiation


gamma rays
X-rays
ultraviolet rays
infrared rays
radar
FM
TV
shortwave
AM
10^{-14}
10^{-12}
10^{-10}
10^{-8}
10^{-6}
10^{-4}
10^{-2}
1
10^{2}
10^{4}
Wavelength (meters)
Visible Light
400
500
600
700
Wavelength (nanometers)



UV
Microwaves
Cosmic photons
γ Rays
X Rays
Thermal Radiation
Radio
DC current
Electric power
THz
FM
AM
10^{-14} 10^{-13} 10^{-12} 10^{-11} 10^{-10} 10^{-9} 10^{-8} 10^{-7} 10^{-6} 10^{-5} 10^{-4} 10^{-3} 10^{-2} 10^{-1} 1 10^{1} 10^{2} 10^{3} 10^{4} 10^{5} 10^{6} 10^{7}
Wavelenght [m]
IR
20^{-11} 20^{-12} 20^{-13} 20^{-14} 20^{-15} 20^{-16} 20^{-17} 20^{-18} 20^{-19} 20^{-20} 20^{-21} 20^{-22} 20^{-23} 20^{-24} 20^{-25} 20^{-26} 20^{-27} 20^{-28} 20^{-29} 20^{-30} 20^{-31} 20^{-32}
Energy [J]
30^{22} 30^{21} 30^{20} 30^{19} 30^{18} 30^{17} 30^{16} 30^{15} 30^{14} 30^{13} 30^{12} 30^{11} 30^{10} 30^{9} 30^{8} 30^{7} 30^{6} 30^{5} 30^{4} 30^{3} 30^{2} 30^{1}
Frequency [Hz]
Violet
Indigo
Blue
Cyan
Green
Yellow
Orange
Red
350 430 450 500 520 565 590 nm 625 740 nm
Visible
80%
atmospheric transmission
NIR
SWIR
MWIR
Low atmospheric transmittance window
LWIR
VLWIR
0.74 μm 1 2 3 5 8 14 1000 μm
Infrared

Blackbody radiation

A *blackbody* is a perfect radiator.

Three characteristics:

- It absorbs all incident radiation
- It radiates more energy than any real surface at the same temperature
- The emitted radiation is independent of direction

Also:
Blackbody radiation obey certain simple laws

Stefan-Boltzmann’s law

The total power radiated from a *blackbody* is calculated from *Stefan-Boltzmann´s law*:

$$\boxed{E_b = \sigma \cdot T^4}$$

where E_b = total power radiated per unit area from a *blackbody* (W/m^2)

$\sigma = 5.669 \cdot 10^{-8}$ W/(m$^2 \cdot$K^4). (Stefan-Boltzmann constant)

T = absolute temperature (K)

Planck distribution law

The wavelength distribution of emitted blackbody radiation is determined from the *Planck distribution law* (fig. 19):

$$E_{\lambda,b}(\lambda,T)=\frac{C_1}{\lambda^5\cdot\left[\exp(C_2/(\lambda\cdot T))-1\right]}$$

where $C_1 = 2\pi\cdot h\cdot c_o^{\,2} = 3.742\cdot 10^8\ W\cdot\mu m^4 / m^2$

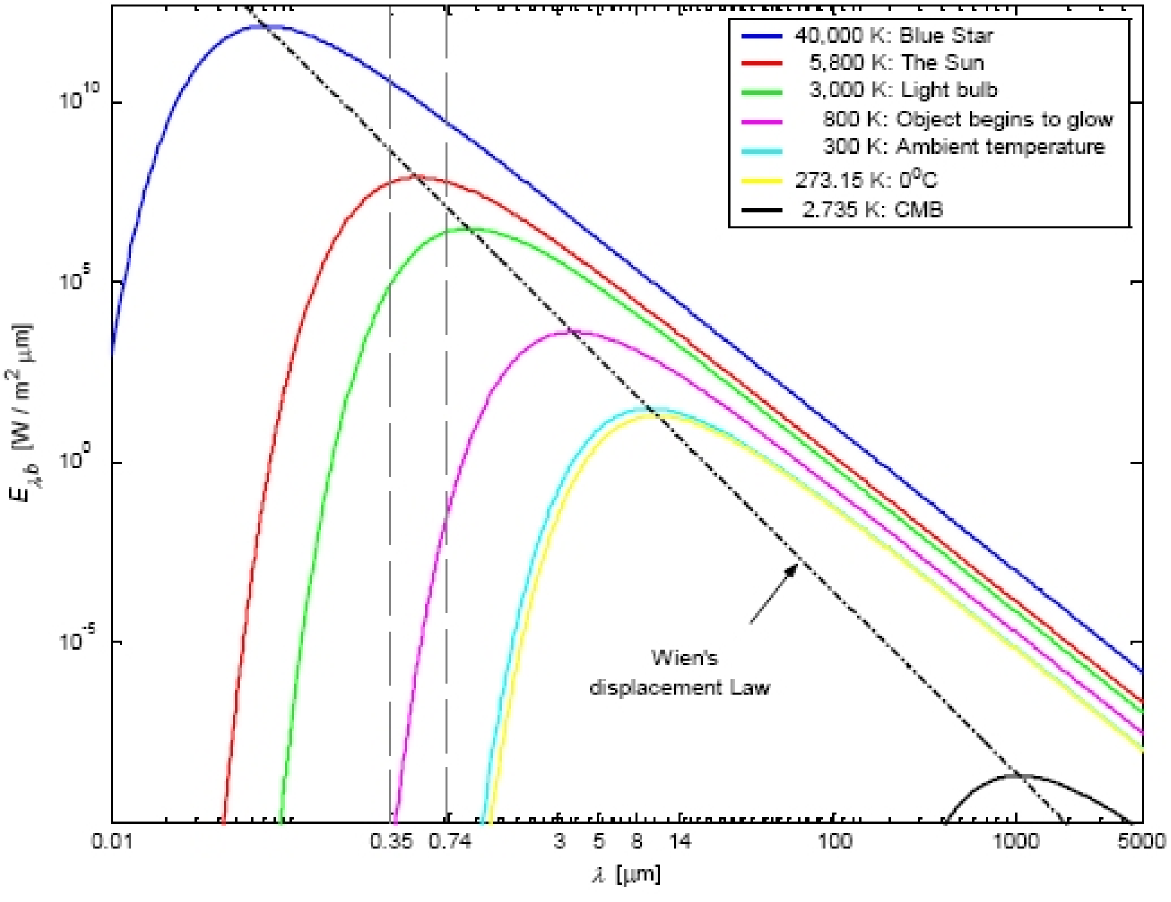
$C_2 = (h\cdot c_o / k) = 1.439\cdot 10^4\ \mu m\ K$

λ = *wavelength* (μm)

T = *absolute temperature* (T)

h = Planck constant

k = Boltzmann constant
c_0 = speed of light in vacuum



Result of increasing temperature on radiation

- Higher intensity
- Shorter wavelength ⇔ higher frequency

Wien’s displacement law

The wavelength of maximum emissive power is determined by *Wien’s displacement law:*

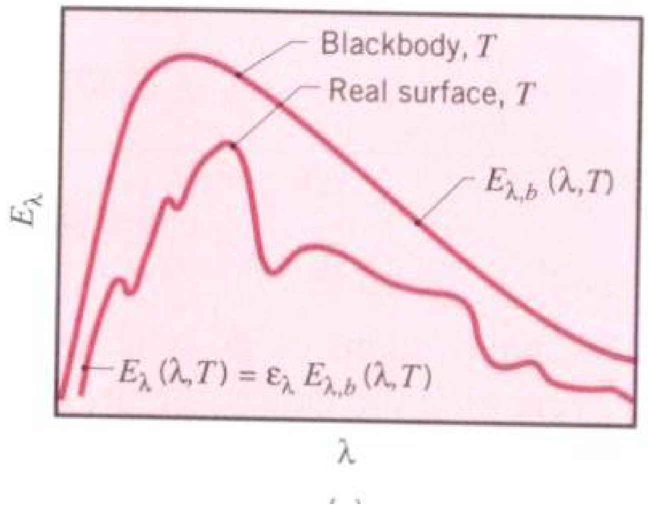
$$\lambda_{max} \cdot T = C_3 = 2897.8\ \mu m\ K$$

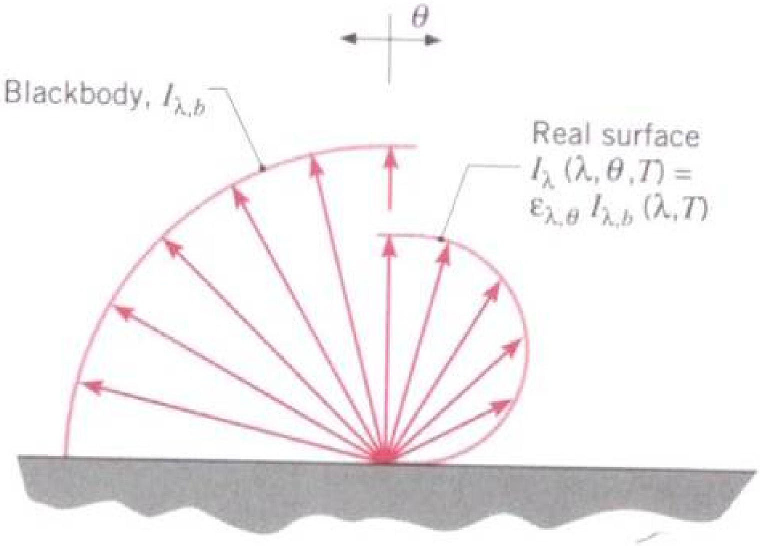
Radiation from real surfaces

Real surfaces:

- emit and absorb less than blackbodies
- reflect radiation
- emit and absorb differently depending on angle and wavelength
- do not obey the simple laws

Blackbody and real surface emissions





Spectral emissivity and total emissivity

To account for ”real surface”- behavior we introduce the *spectral emissivity*, defined by

$$E_\lambda(\lambda,T) = \varepsilon_\lambda(\lambda,T) \cdot E_{\lambda,black}(\lambda,T)$$

and the *total emissivity* defined by

$$E = \varepsilon \cdot E_b = \varepsilon \cdot \sigma \cdot T^4$$

(ε = integrated average)

Gray diffuse body

To simplify matters it is common to assume the emissivity to be independent on wavelength and direction. Such a surface is called a *gray diffuse body*, for which

$$E_\lambda(\lambda,T) = \varepsilon_\lambda \cdot E_{\lambda,black}(\lambda,T)$$

$$\varepsilon_\lambda = constant = \varepsilon \qquad\qquad (0 < \varepsilon < 1)$$

Absorptivity, reflectivity, transmittivity

Incident radiation may be *absorbed*, *reflected* or *transmitted*

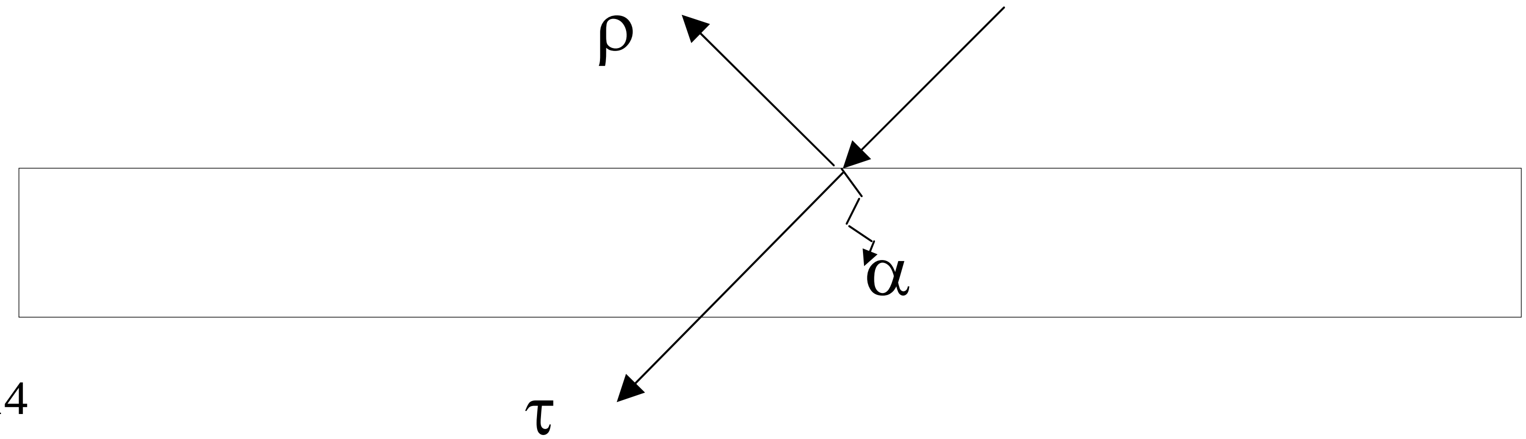
We define

Absorptivity α: Fraction of incident radiation absorbed

Reflectivity ρ: Fraction of incident radiation reflected

Transmittivity τ: Fraction of incident radiation transmitted

thus $\alpha + \rho + \tau = 1$



Kirchhoff's identity

The (total) emissivity and the (total) absorptivity of a surface are equal at equal temperatures (wavelengths):

$$\varepsilon = \alpha$$

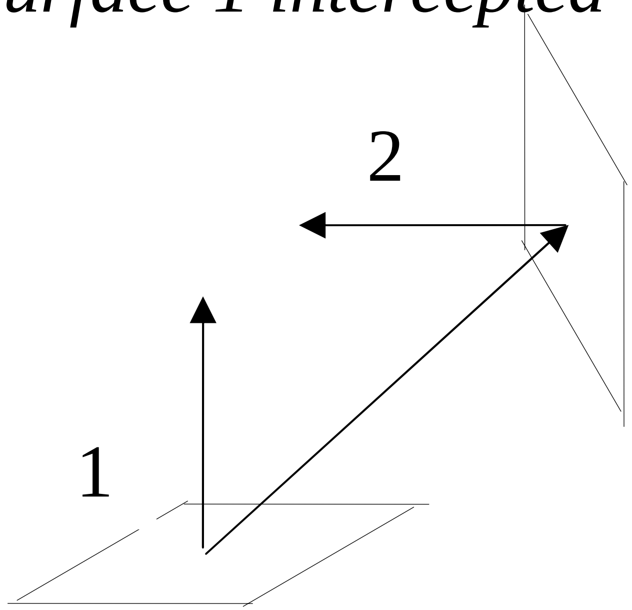
Radiation exchange between blackbodies

To calculate radiation exchange we must take into account

- surface areas
- surface geometries
- position in relation to each other

This is done by the *shape factor*, F_{12}

> F_{12} = *fraction of radiation leaving surface 1 intercepted by surface 2.*



Net exchange of radiation, law of reciprocity

Net exchange of radiation between blackbodies:

$$q_{1\text{-}2} = F_{12} \cdot A_1 \cdot (E_{b1} - E_{b2}) = F_{12} \cdot A_1 \cdot \sigma \cdot (T_1^4 - T_2^4)$$

Law of reciprocity

$$F_{12} \cdot A_1 \cdot = F_{21} \cdot A_2$$

Shape factors

Shape factors are often difficult to calculate.
See diagrams and formulas in

Holman, Figs 8.12-8.16 and
CFT pp. 45-47

Important special case:
Small (convex) surface (1) surrounded by other surface (2):
⇨ $F_{12} = 1$.

Radiation exchange between real surfaces, simple case

For the special case of F_{12} = 1 the exchange is calculated as

$$q_{1\text{-}2} = \varepsilon_1 \cdot A_1 \cdot (E_{b1} - E_{b2}) = \varepsilon_1 \cdot A_1 \cdot \sigma \cdot (T_1^4 - T_2^4)$$

Comparing to Newton's law of cooling:

$$q = h_r \cdot A_1 \cdot (T_1 - T_2)$$

⇨ $h_r = \varepsilon_1 \cdot \sigma \cdot (T_1^4 - T_2^4) / (T_1 - T_2) = \varepsilon_1 \cdot h_{r,\, black}$

⇨ $h_{r,\, black} = f(T_1, T_2)$, from table!

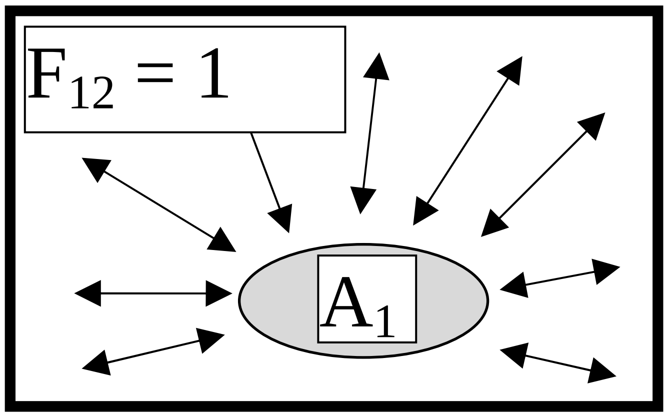


Table 8 (p.30 in CFT)

Black body radiation heat transfer coefficient, h_s, with $F_{12}=1$:

t_1 C°	t_2, °C: -50	-10	0	10	20	50	100	200	400	800
-50	2.5	3.2	3.5	3.7	4.0	4.8	6.4	10.8	25.2	87.9
-10		4.1	4.3	4.6	4.9	5.8	7.5	12.1	27.7	92.4
0			4.6	4.9	5.2	6.0	7.8	12.6	28.3	93.6
10				5.2	5.4	6.4	8.2	12.9	28.8	94.7
20					5.7	6.6	8.5	13.4	29.5	95.8
50						7.6	9.6	15.3	31.5	99.2
100							11.8	17.4	35.1	106
200								24.0	43.9	121
400									69.2	159
800										281

Total emissivities of selected materials

Material	Temp (°C)	Emissivity ε
Aluminum, commercial sheet	100	0.09
Copper, polished	100	0.052
Iron, dark-gray surface	100	0.31
Glass, smooth	22	0.94
Snow-white enamel varnish	23	0.906
Black shiny lacquer	24	0.875
Roofing paper	21	0.91
Porcelain, glazed	22	0.92
Red brick	23	0.93
Al-paints	100	0.27-0.67

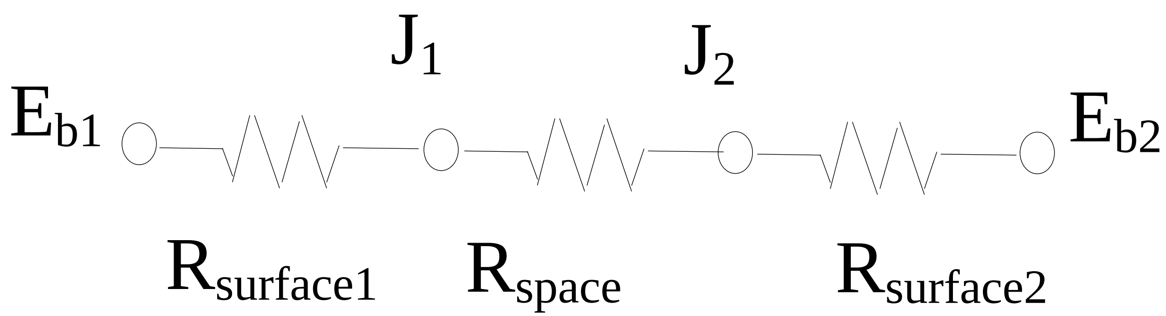
Radiation exchange calculated by resistance networks

Consider the blackbody emissive power as the driving potential

$$q_{1\text{-}2} = (E_{b1} - E_{b2}) / R_{rad} = \sigma \cdot (T_1^4 - T_2^4) / R_{rad}$$

Consider the radiation resistance as the sum of surface and space resistances

$$R_{rad} = R_{surface1} + R_{space} + R_{surface2}$$



Radiation exchange by resistance networks, assumptions

Assume:

- All surfaces are gray,
- All surfaces are uniform in temperature.
- Reflective and emissive properties are constant over the surfaces.

Define:

- *Irradiation*, G = total radiation / (unit time, unit area)
- *Radiosity*, J = total radiation leaving /(unit time, unit area) (including reflected radiation)

Assume these properties are uniform over each surface.

Surface resistance:

Radiosity = emitted radiation + radiation reflected:

$$J = \varepsilon \cdot E_b + \rho \cdot G$$

where ε = emissivity

ρ = reflectivity of surface

Assume surfaces opaque (τ=0)

=> $\rho = 1 - \alpha = 1 - \varepsilon$

(as $\alpha = \varepsilon$).

=> $J = \varepsilon \cdot E_b + (1 - \varepsilon) \cdot G$

or $G = (J - \varepsilon \cdot E_b) / (1 - \varepsilon)$

The net energy leaving the surface per unit area:

$$q / A = J - G = \varepsilon \cdot E_b + (1 - \varepsilon) \cdot G - G =$$
$$= \varepsilon \cdot E_b - \varepsilon \cdot G = \varepsilon \cdot (E_b - G)$$

=> $$q = \frac{\varepsilon \cdot A}{1-\varepsilon} \cdot (E_b - J) = \frac{(E_b - J)}{(1-\varepsilon) / (\varepsilon \cdot A)}$$

Consider $(E_b - J)$ as the driving potential.

=> $R_{surface} = (1-\varepsilon)/(\varepsilon \cdot A)$

Space resistance:

Radiation from 1 to 2 (per unit time): $J_1 \cdot A_1 \cdot F_{12}$.

Radiation from 2 to 1 (per unit time): $J_2 \cdot A_2 \cdot F_{21}$.

Net exchange of radiation:

$$q_{12} = J_1 \cdot A_1 \cdot F_{12} - J_2 \cdot A_2 \cdot F_{21} = (J_1 - J_2) \cdot A_1 \cdot F_{12} = (J_1 - J_2) \cdot A_2 \cdot F_{21}$$

(as $A_1 \cdot F_{12} = A_2 \cdot F_{21}$).

Consider $(J_1 - J_2)$ as the driving potential,

$=>$ $\quad R_{space} = 1/(A_1 \cdot F_{12})$

Heat exchange due to radiation, two bodies

$$q = \frac{E_{b1} - E_{b2}}{R_{rad}} = \frac{E_{b1} - E_{b2}}{R_{surface1} + R_{space} + R_{surface2}} = \frac{\sigma \cdot (T_1^{\;4} - T_2^{\;4})}{\frac{1-\varepsilon_1}{\varepsilon_1 \cdot A_1} + \frac{1}{A_1 \cdot F_{12}} + \frac{1-\varepsilon_2}{\varepsilon_2 \cdot A_2}}$$

Special case, two parallel infinite plates

$F_{12} = 1$ and $A_1 = A_2$

$$\frac{q}{A} = \frac{\sigma \cdot (T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}$$

Special case, two concentric cylinders or spheres

$F_{12} = 1$

$$\frac{q}{A_1} = \frac{\sigma \cdot (T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{A_1}{A_2} \cdot (\frac{1}{\varepsilon_2} - 1)}$$

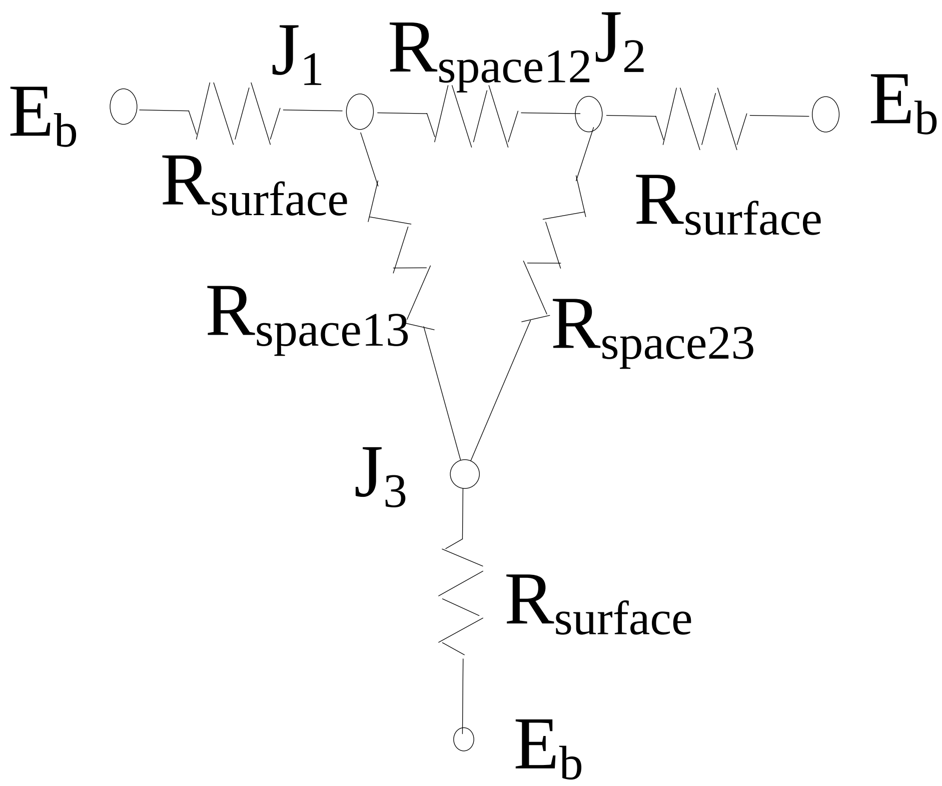
If $A_1 << A_2$ $\qquad$ $q / A_1 = \varepsilon_1 \cdot \sigma \cdot (T_1^4 - T_2^4)$

Three body problem

- Calculate all resistances.
- For the nodes J_1, J_2 and J_3, the sum of the energy flows into each of the nodes must be zero.

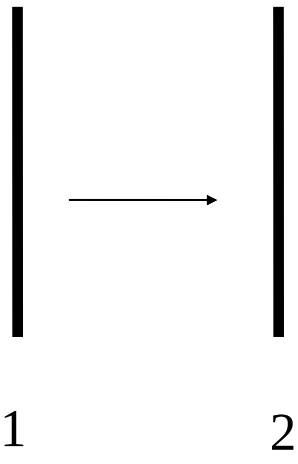
Example, node J_1

$$\frac{E_{b1}-J_1}{\left(\frac{1-\varepsilon_1}{\varepsilon_1 \cdot A_1}\right)}+\frac{J_2-J_1}{A_1 \cdot F_{12}}+\frac{J_3-J_1}{A_1 \cdot F_{13}}=0$$

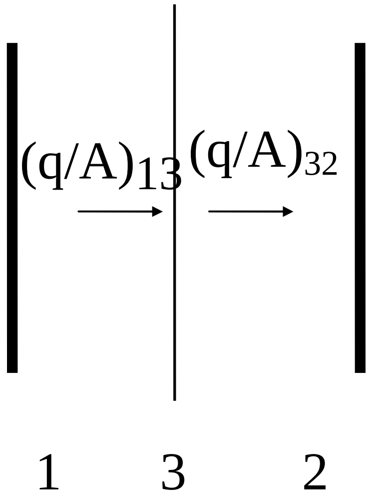


- Solve J_1 -J_3 !
- Calculate the heat exchanges!

Radiation shields



No shield



With shield

$$\frac{q}{A} = \frac{\sigma \cdot (T_1^4 - T_2^4)}{1/\varepsilon_1 + 1/\varepsilon_2 - 1}$$

$$(q/A)_{1\text{-}3} = (q/A)_{3\text{-}2} = (q/A)$$

$$\frac{q}{A} = \frac{\sigma \cdot (T_1^4 - T_3^4)}{1/\varepsilon_1 + 1/\varepsilon_3 - 1} = \frac{\sigma \cdot (T_3^4 - T_2^4)}{1/\varepsilon_3 + 1/\varepsilon_2 - 1}$$

Everything is known except the temperature T_3.

Simplest case, all emissivities equal:

$$T_1^4 - T_3^4 = T_3^4 - T_2^4 \Rightarrow T_3^4 = \tfrac{1}{2} \cdot (T_1^4 + T_2^4)$$

$$\Rightarrow \quad \frac{q}{A} = \frac{\tfrac{1}{2} \cdot \sigma \cdot (T_1^4 - T_2^4)}{1/\varepsilon_3 + 1/\varepsilon_2 - 1}$$

Emissivities assumed equal, => heat flux *reduced to half*

Multiple shields of equal emissivities

It may be shown by similar reasoning that the heat flux will be reduced to

$$(q / A)_{\text{with shields}} = \frac{1}{n+1} \cdot (q / A)_{\text{without shields}}$$

where n is the number of shields.

Different emissivities

Two plates with equal emissivities ε_1

One shield of different emissivity ε_2:

$$(q/A)_{\text{with shields}} = \frac{1}{2} \cdot \frac{\frac{2}{\varepsilon_1} - 1}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1} (q/A)_{\text{without shields}}$$

$\varepsilon_2 << \varepsilon_1$ => largest decrease in heat flux

Assume $\varepsilon_1 = 1$. =>

$$(q/A)_{\text{with shields}} = \frac{1}{2} \cdot \frac{2-1}{1 + \frac{1}{\varepsilon_2} - 1} (q/A)_{\text{without shields}} = \frac{1}{2} \cdot \varepsilon_2 \cdot (q/A)_{\text{without shields}}$$

thus factor equal to *one half times the emissivity of the shield.*

Part 4

Conduction, Boiling, Condensation and Diffusion

Conduction

Introduction

Fourier's law:

$$q = -k \cdot A \cdot \delta T / \delta x$$

For a plane wall, the temperature gradient is constant

$$q = k \cdot A \cdot \Delta T / \delta$$

Conduction in cylindrical shells

When heat is conducted through a wall which is not plane, the temperature gradient will no longer be constant through the wall.

$$q = -k \cdot A \cdot \delta T/\delta x$$

At any section, q is constant

Assume k constant

$$\Rightarrow (A \cdot \delta T/\delta x) = \text{constant.}$$

Thus, if the area A changes with x, as it will for a curved surface, the temperature gradient must also change.

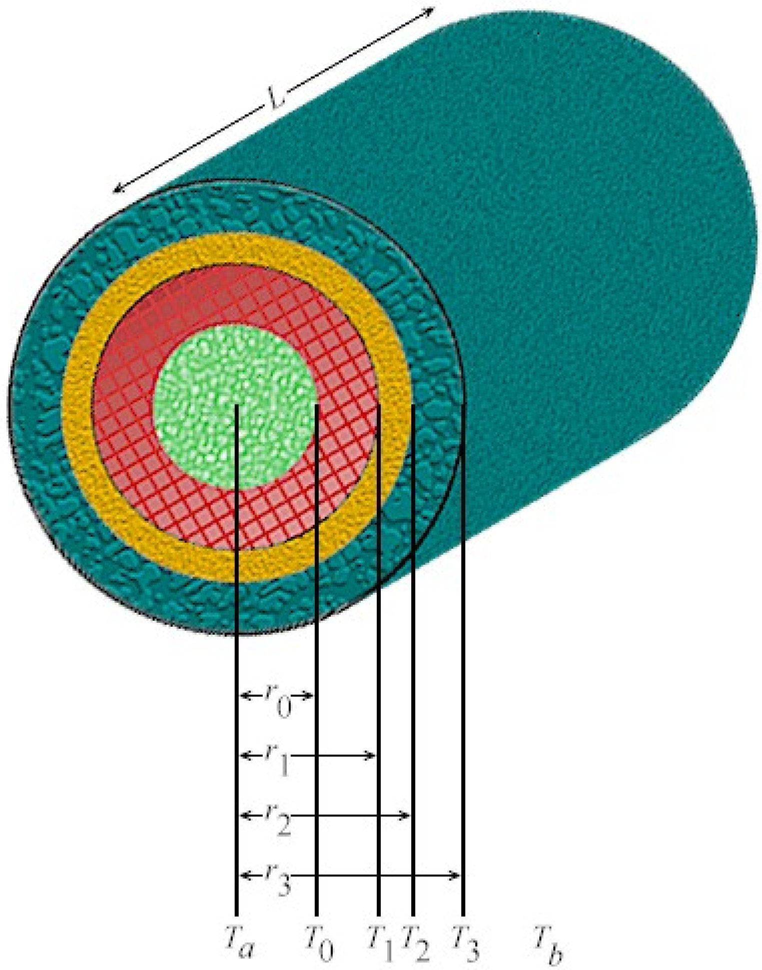
Conduction in cylindrical shells

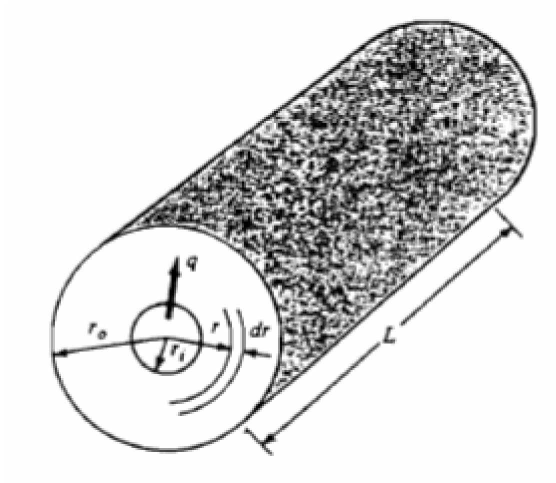
The simplified version of Fourier´s law may still be used if we use an *average area* (or average diameter)

For a cylindrical wall the area is calculated as the *logarithmic mean* of the inside and outside surface.

$$\boxed{\begin{aligned} A_{ln} &= (A_2 - A_1)/ln(A_2/A_1) = \\ &\pi \cdot L \cdot d_{ln} = \pi \cdot L \cdot (d_2 - d_1) / ln(d_2 / d_1) \end{aligned}}$$

Conduction in cylindrical shells





Conduction in fins

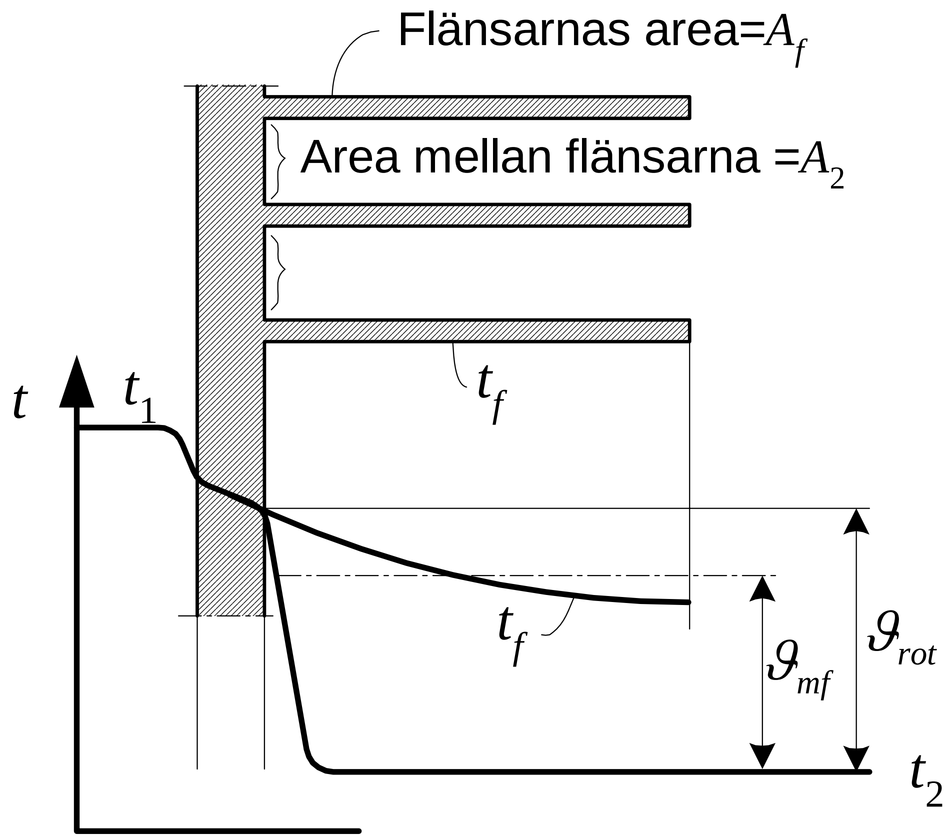


Fig 11.5 i termo

Definition of fin efficiency

Because of the lower ΔT, lower heat flux from fin

⇒ The fin surface is less *efficient* than the base surface,

We define a *fin efficiency* η_f as the ratio between the *actual heat transferred and the heat which would be transferred if the whole fin had the base temperature.*

$$\boxed{\eta_f \equiv \Delta T_{m,\,fin} / \Delta T_{base}}$$

(Assuming equal *h* on fin and base)

Conduction in fins

For straight fins with constant cross section:

$$\eta_f \equiv \tanh(m \cdot L) / (m \cdot L)$$

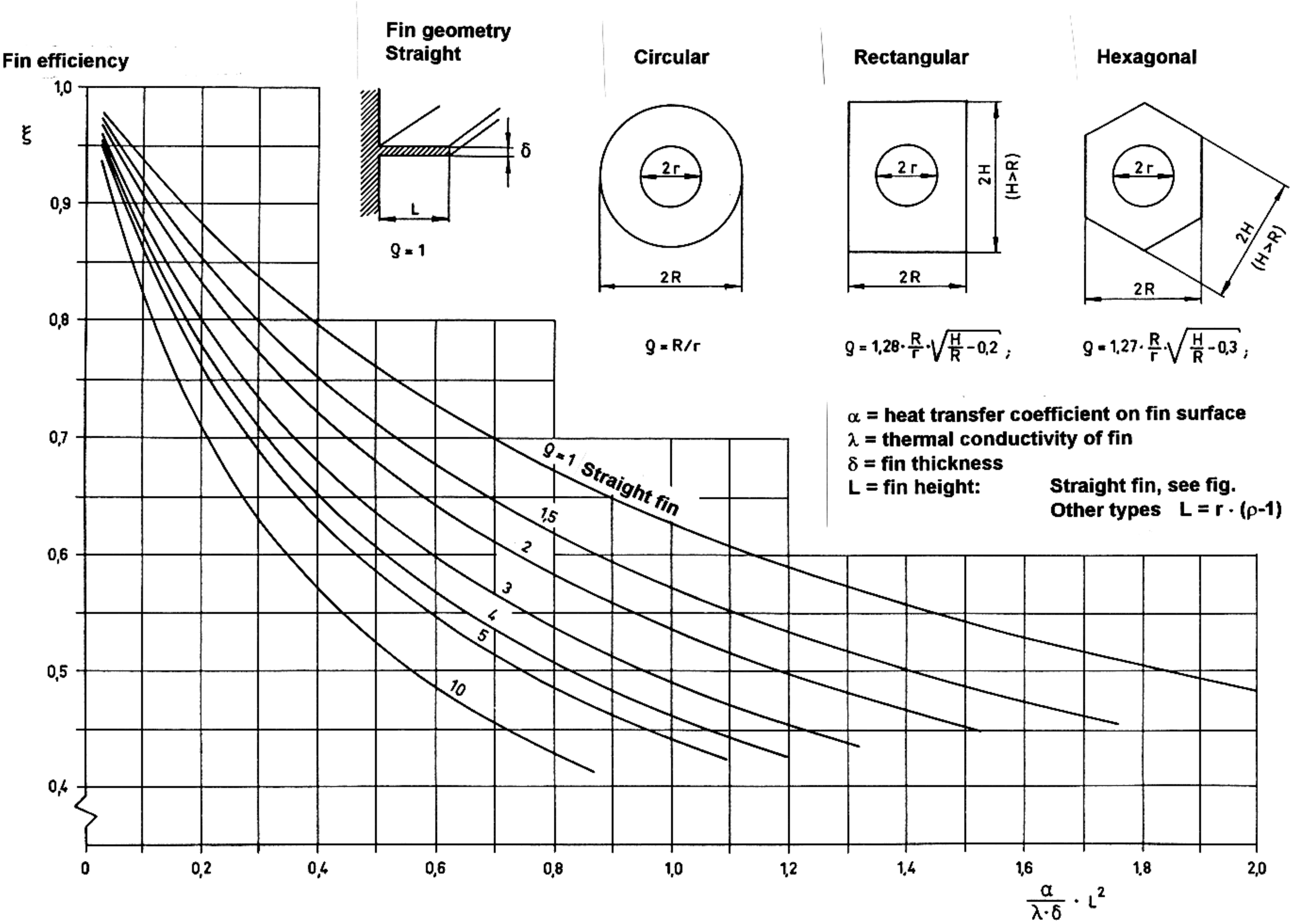
where L = the length of the fin (m)

$m = [(h \cdot P)/(k \cdot A)]^{1/2}$

where P = perimeter of the fin (m)

A = cross section area perpendicular to heat flow (m^2)

For many types of fins, the fin efficiency may be estimated from the following diagram.



Fin efficiency
ξ
Fin geometry
Straight
δ
L
ρ = 1
Circular
2r
2R
ρ = R/r
Rectangular
2r
2H
(H>R)
2R
$\rho = 1{,}28 \cdot \frac{R}{r} \cdot \sqrt{\frac{H}{R} - 0{,}2}$;
Hexagonal
2r
2H
(H>R)
2R
$\rho = 1{,}27 \cdot \frac{R}{r} \cdot \sqrt{\frac{H}{R} - 0{,}3}$;
α = heat transfer coefficient on fin surface
λ = thermal conductivity of fin
δ = fin thickness
L = fin height:
Straight fin, see fig.
Other types L = r · (ρ-1)
ρ = 1 Straight fin
1,5
2
3
4
5
10
1,0
0,9
0,8
0,7
0,6
0,5
0,4
0
0,2
0,4
0,6
0,8
1,0
1,2
1,4
1,6
1,8
2,0
$\frac{\alpha}{\lambda \cdot \delta} \cdot L^2$

Overall heat transfer coefficient

For a finned surface, the area is calculated as

$$A_{tot} = A_{base} + \eta_f \cdot A_{fin}$$

The U-value of a finned tube is calculated as:

$$1/(U \cdot A) = 1/(h_1 \cdot A_1) + \delta/(k \cdot A_{ln}) + 1/[h_2 \cdot (A_{base} + \eta_f \cdot A_{fin})]$$

where A_1 = inside surface area (m^2)

A_{base} = outside surface area *in between fins* (m^2)

A_{ln} = logarithmic mean area of tube (m^2)

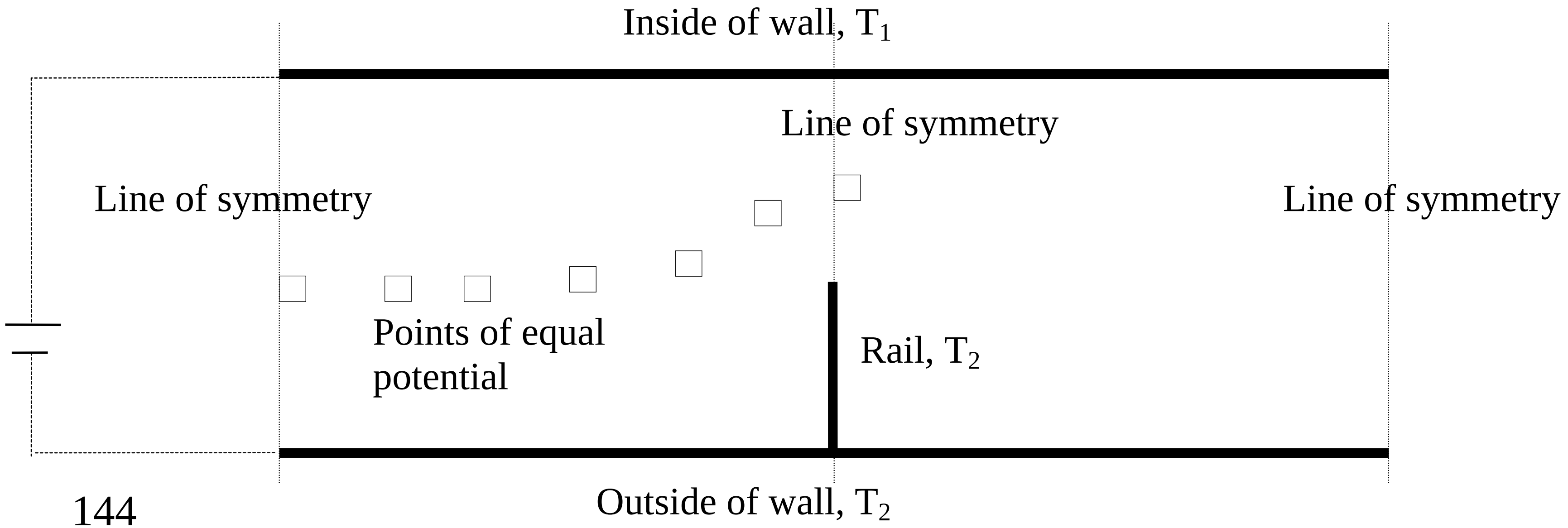
A_{fin} = fin area (m^2)

Electric analogy for solving 2-d conduction problems

Make electric model of 2-d conduction problem in:

- Electrolyte tray
- Thin conducting foil
- Resistance mesh

Boundary conditions: Isothermal or adiabatic

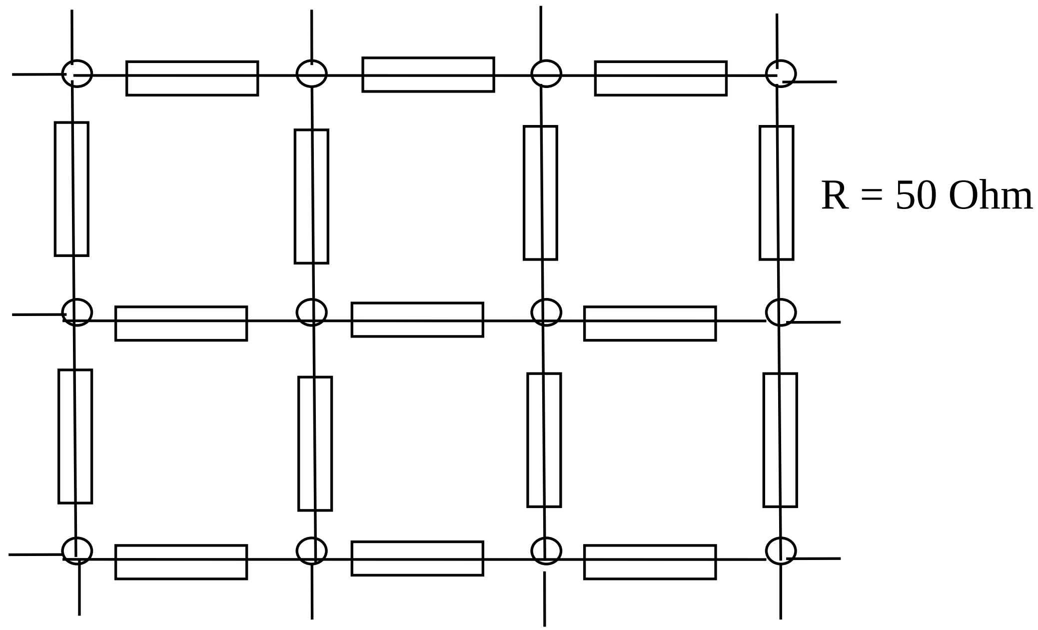


Electric analogy models…

Electrolyte tray or foil - *continuous models*
Resistance mesh: *discreet model*

Measurements show : Potential in node = mean of the four surrounding nodes

⇨Measurements not necessary!
Calculate!



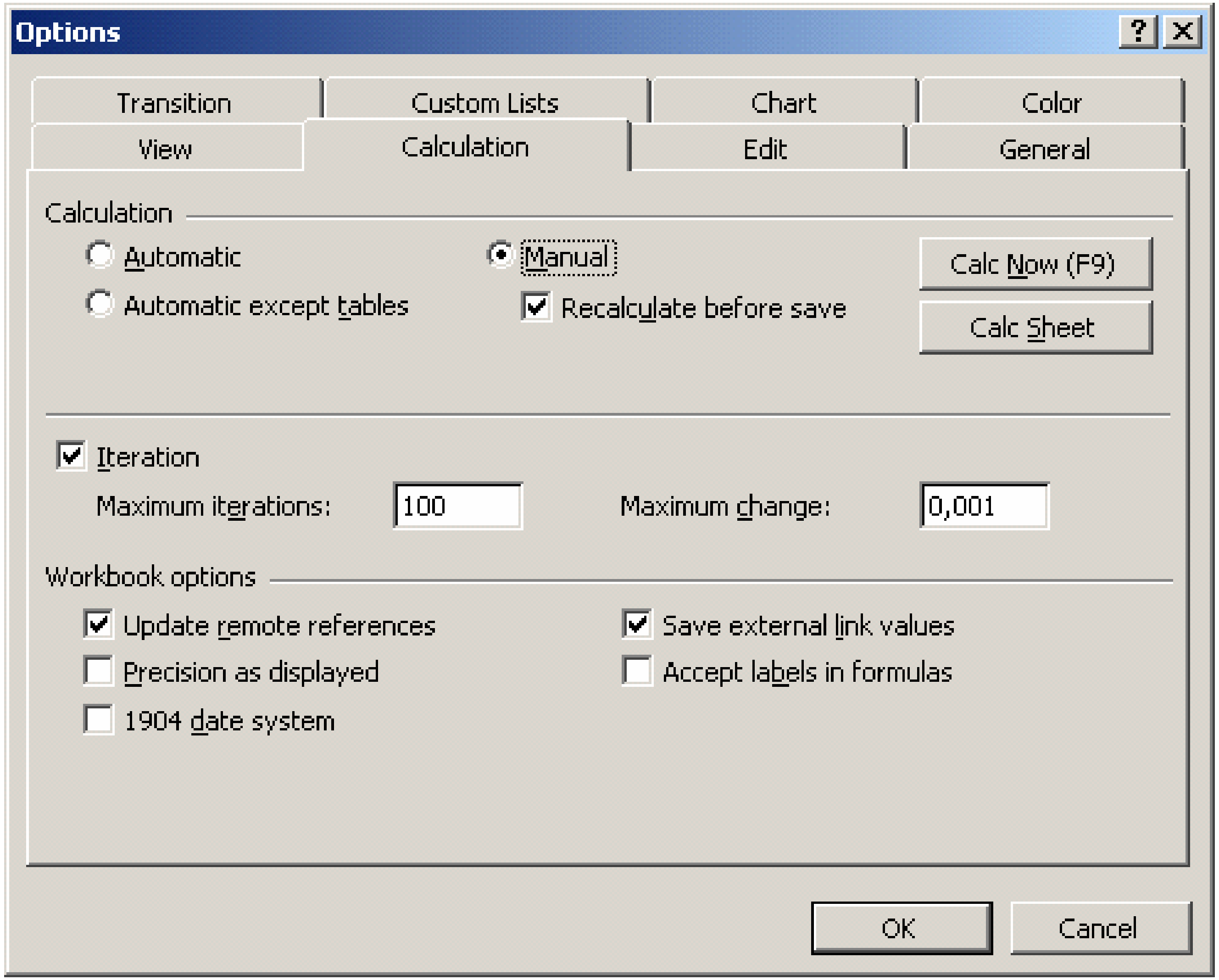
Excel model of wall with rail, horizontal cut

	A	B	C	D	E	F	G	H	I	J	K	L
1	200	200	200	200	200	200	200	200	200	200	200	200
2	188.7026	188.6737	188.5876	188.447	188.2564	188.0241	187.7626	187.49	187.2305	187.013	186.8668	186.8151
3	177.463	177.4044	177.2299	176.9438	176.5547	176.0775	175.5362	174.9668	174.419	173.9545	173.6391	173.5268
4	166.3407	166.2511	165.9838	165.5437	164.941	164.1949	163.3379	162.4221	161.5243	160.747	160.2085	160.0137
5	155.3975	155.2755	154.9105	154.3061	153.4708	152.4232	151.1985	149.8595	148.5091	147.3007	146.434	146.1111
6	144.6982	144.5429	144.0767	143.2993	142.2131	[illegible]	[illegible]	[illegible]	135.3518	133.5128	132.1159	131.5626
7	134.3096	134.1213	133.5542	132.6013	131.2538	[illegible]	[illegible]	[illegible]	122.077	119.283	116.9541	115.9075
8	124.2975	124.0787	123.4173	122.298	120.6961	118.578	115.906	112.6502	108.8248	104.5881	100.51	98.15913
9	114.723	114.4787	113.7382	112.4775	110.6545	108.2053	105.0376	101.0218	95.98379	89.73458	82.33865	75.70911
10	105.637	105.375	104.5793	103.2192	101.2393	98.55121	95.01738	90.41565	84.35401	76.02783	63.40091	40
11	97.07489	96.8051	95.98494	94.58083	92.53218	89.74285	86.06506	81.26942	74.98878	66.6218	55.23719	40
12	89.05243	88.78556	87.97451	86.58699	84.56579	81.82295	78.2306	73.60818	67.7099	60.23341	50.92604	40
13	81.56375	81.31023	[illegible]	[illegible]	[illegible]	[illegible]	71.42625	67.22281	62.00923	55.67591	48.23357	40
14	74.58213	74.35108	[illegible]	[illegible]	[illegible]	[illegible]	65.49901	61.84761	57.42829	52.22745	46.33231	40
15	68.06261	67.86134	67.25212	66.21895	64.736	62.76959	60.28217	57.24033	53.62889	49.47328	44.86824	40
16	61.94566	61.77957	61.27757	60.42882	59.21651	57.62016	55.61977	53.20265	50.37367	47.16856	43.66736	40
17	56.1609	56.03372	55.64977	55.00226	54.08106	52.87477	51.3741	49.57686	47.49456	45.15991	42.63266	40
18	50.63054	50.54463	50.28553	49.84939	49.23071	48.42378	47.42502	46.23611	44.86782	43.34387	41.70337	40
19	45.27201	45.22874	45.09834	44.87908	44.5686	44.16462	43.66609	43.07473	42.39673	41.64439	40.83694	40
20	40	40	40	40	40	40	40	40	40	40	40	40

=(D5+E6+C6+D7)/4

=(A13+B14+B14+A15)/4

Tools/Alternatives/Calculate/*Iteration*



Transient heat transfer - Lumped capacitance method

Use when temperature of the body is approx. uniform.

I.E. if $R_{th,\ surface} >> R_{th,\ internal} \Leftrightarrow h << k / L$

We define the Biot number (Bi) as

$$Bi = h \cdot L / k$$

where h = heat transfer coefficient

k = thermal conductivity *of solid*

L = characteristic length *of solid body*

= V/A =volume of body /exterior surface area

Transient heat transfer - Lumped capacitance method…

Method can be used when Bi < 0.1.

After a sudden change in ambient temperature:

$$\Delta T = \Delta T_0 \cdot exp(-Bi \cdot Fo)$$

where ΔT = temperature difference at time τ.

ΔT_0= temperature difference at τ =0.

$Fo = k \cdot \tau / (\rho \cdot c_p \cdot L^2)$ = Fourier number (dim. less time)

τ = time after step-change in ambient temperature

k = thermal conductivity of solid body

ρ = density of solid body

c_p = specific heat of solid body

Transient heat transfer - Lumped capacitance method…

Product of the Biot and Fourier:

$$Bi \cdot Fo = \frac{h \cdot (V/A)}{k} \cdot \frac{k \cdot \tau}{\rho \cdot c_p \cdot (V/A)^2} = \frac{h \cdot A}{\rho \cdot c_p \cdot V} \cdot \tau$$

Numerator: = $1/R_{th,\,\mathrm{convection}}$
Denominator: C_{th} Heat capacity of the solid

$R_{th} = 1/(h \cdot A)$ and $C_{th} = m \cdot c_p = \rho \cdot V \cdot c_p$

=> $\mathrm{Bi} \cdot \mathrm{Fo} = \tau / (\mathrm{R} \cdot \mathrm{C})_{\mathrm{th}}$

Transient heat transfer - Lumped capacitance method…

$$=> \qquad \frac{\Delta T}{\Delta T_0} = e^{-\frac{\tau}{R_{th} \cdot C_{th}}}$$

Compare equation for discharge of a condenser through a resistor!

Lumped capacitance method handy for quick estimates!

Transient heat transfer - Heisler charts

For simple geometries, use diagrams (Heisler charts) where

$$\Delta T/\Delta T_0 = f\ (Fo, Bi)$$

Diagrams are found in:

- Collection of formulas and tables, pp. 42-43,
- book by Holman, fig. 4-7 - 4-13.

Note that the definition of the characteristic length used in Bi and Fo in the charts are not the same as used above!

Charts valid for Fo > 0.2.

Two- or three dimensional problems: multiply the solutions for the corresponding one-dimensional cases.

Examples:
The temperature in the center of a short cylinder:

$$(\Delta T/\Delta T_0)_{\text{center, short cylinder}} = (T/\Delta T_0)_{\text{center, infinite cylinder}} \cdot$$
$$\cdot(\Delta T/\Delta T_0)_{\text{center, infinite wall}}$$

Temperature in the center of a cube:

$$(\Delta T/\Delta T_0)_{\text{center, cube}} = [(\Delta T/\Delta T_0)_{\text{center, infinite wall}}]^3$$

Boiling

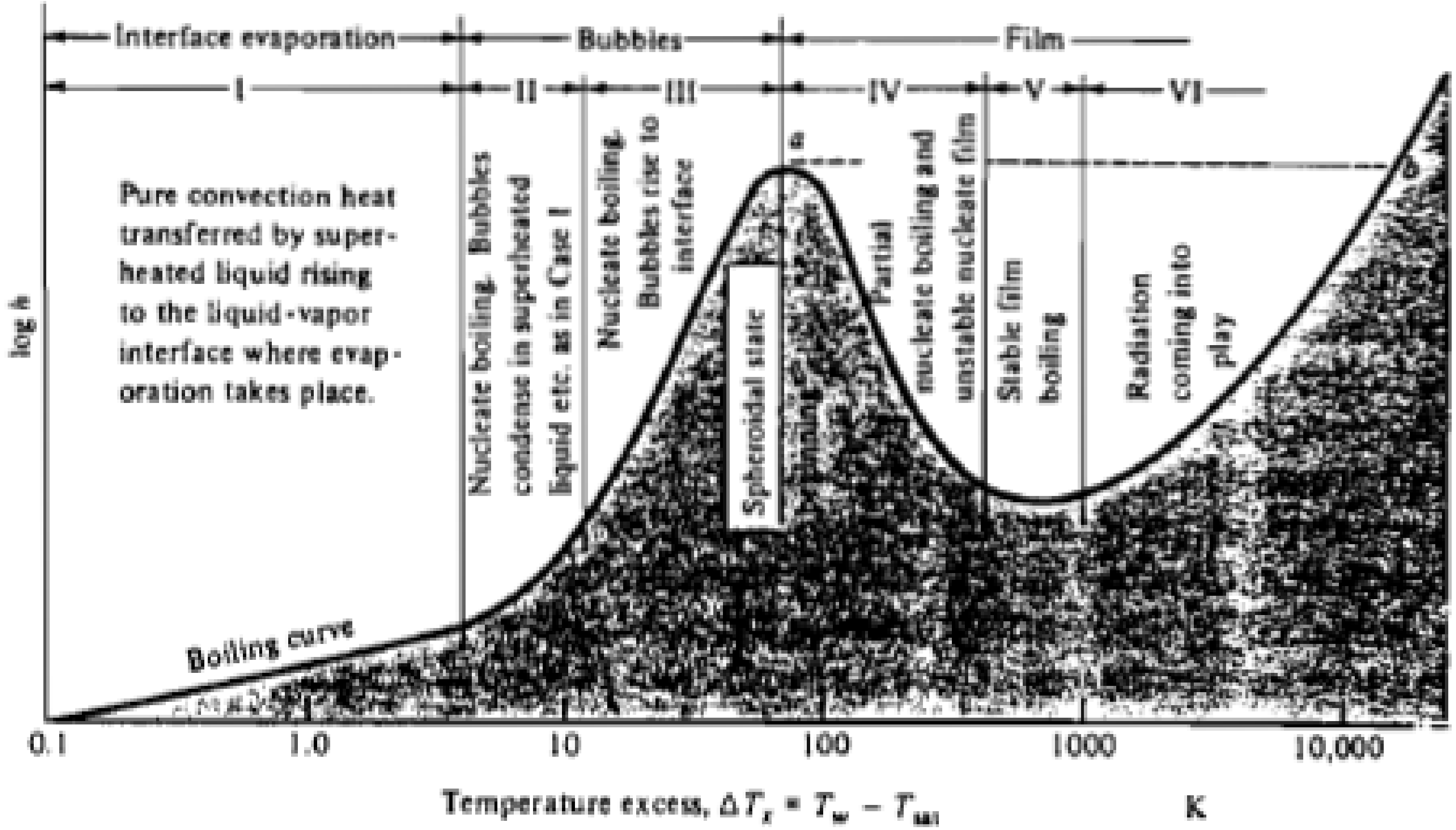
Two types of boiling:

pool boiling, (as on the outside of tubes)

and *flow boiling* (boiling inside tubes)

Pool Boiling

The boiling curve



Pool Boiling

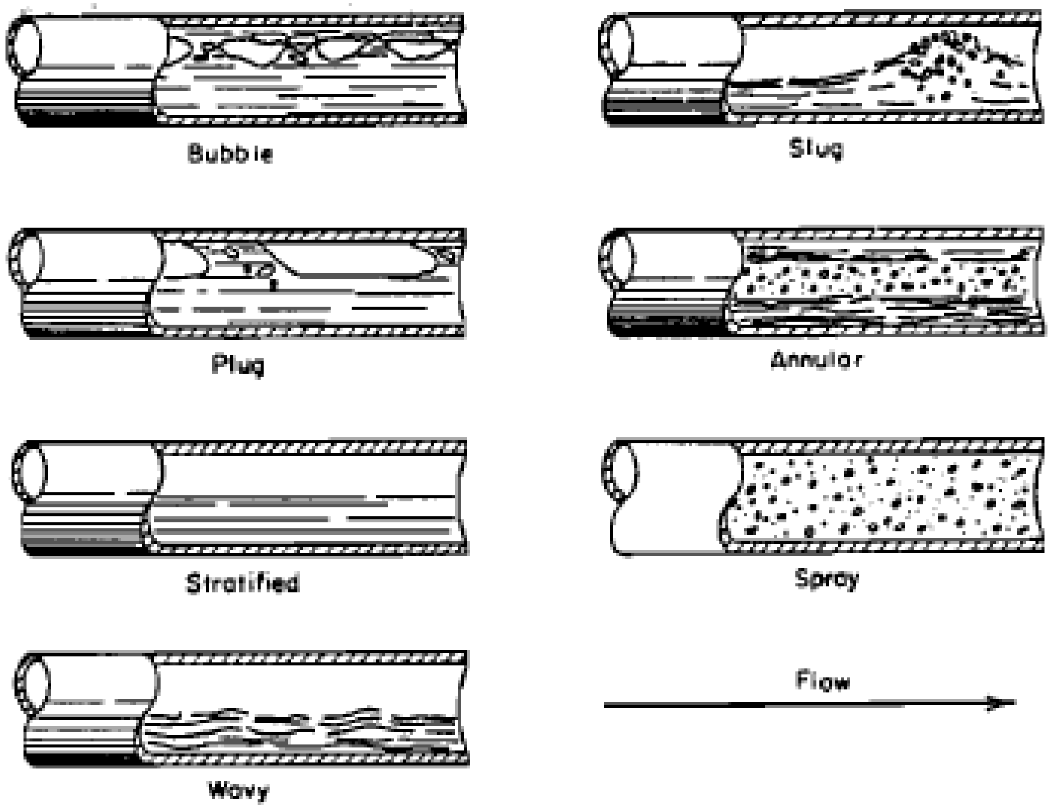
A *simple* correlation for nucleate pool boiling:

$$\boxed{h_{nb} = C \cdot 55 \cdot p_r^{(0.12 - 0.2 \cdot \log10\, Rp)} \cdot (-\log_{10} p_r)^{-0.55} \cdot M^{-0.5} \cdot (q/A)^{0.67}}$$

where C = 1 for horizontal, plane surface
C = 1.7 for horizontal copper cylinders
$p_r = p/p_{critical}$ = reduced pressure
R_p = Surface roughness (μm) (about 1 for many technically smooth surfaces)
M = molecular weight of the fluid (kg/kmol)
q/A = heat flux (W/m^2)

Flow boiling

Flow regimes in flow boiling



Flow boiling

Flow boiling of refrigerants in horizontal tubes:

Complete evaporation: $Nu_m = 1.0 \cdot 10^{-2} \cdot (Re^2 \cdot K_f)^{0.4}$

Incomplete evaporation: $Nu_m = 1.1 \cdot 10^{-3} \cdot Re \cdot K_f^{\,0.5}$

where $Re = 4 \cdot m' /(\pi \cdot d \cdot \mu_l)$

where m' = mass flow (kg/s)

d = tube diameter (m)

μ_l = dynamic viscosity of liquid (Ns/m^2)

$$K_f = \Delta i/(L \cdot g)$$

where Δi = specific enthalpy difference across tube (J/kg)

L = tube length (m)

g = acceleration of gravity (m/s^2)

The equations give *average* Nusselt number for the tube.

Condensation

Two types of condensation:

Film condensation (when the liquid wets the surface)

Drop condensation (opposite)

Drop condensation best but difficult to achieve. Calculate for film condensation!

Heat transfer resistance in condensation is entirely due to conduction of heat through the liquid.

$\Rightarrow$ Heat transfer coefficient may be calculated from the film thickness as

$$h = k/\delta$$

where k = thermal resistance of liquid (W/(m·°C))

δ = film thickness (m)

Film thickness in *laminar* flow at a distance x from the top

$$\boxed{\delta = \{4 \cdot \mu \cdot k \cdot x \cdot \Delta T / [g \cdot h_{fg} \cdot \rho \cdot (\rho - \rho_v)]\}^{1/4}}$$

where μ = dynamic viscosity of liquid (Ns/m^2)

x = distance from top of surface (m)

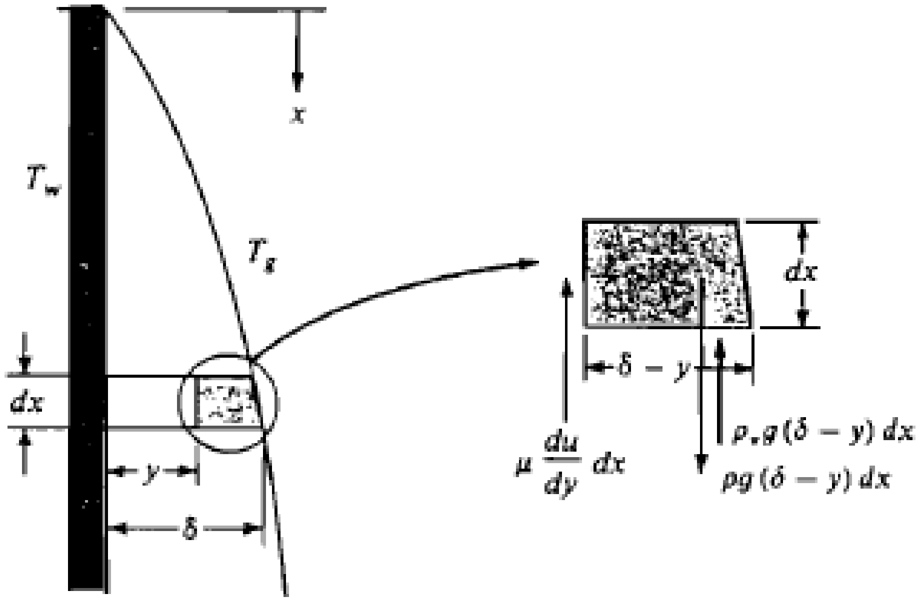
ΔT = temperature difference between surface and vapour (°C)

g = acceleration of gravity (m/s^2)

h_{fg} = heat of vaporisation (J/kg)

ρ = density of liquid (kg/m^3)

ρ_v = density of vapour (kg/m^3)



Condensation

This gives the *local* heat transfer coefficient

$$h = \{g \cdot h_{fg} \cdot \rho \cdot (\rho - \rho_v) \cdot k^3 / [4 \cdot \mu \cdot x \cdot \Delta T]\}^{1/4}$$

and by integrating, the *average* heat transfer coefficient

$$\begin{aligned} h_{av} &= 4/3 \cdot h_L = \\ &= 4/3 \cdot \{g \cdot h_{fg} \cdot \rho \cdot (\rho - \rho_v) \cdot k^3 / [4 \cdot \mu \cdot L \cdot \Delta T]\}^{1/4} = \\ &= 0.943 \cdot \{g \cdot h_{fg} \cdot \rho \cdot (\rho - \rho_v) \cdot k^3 / [\mu \cdot L \cdot \Delta T]\}^{1/4} \end{aligned}$$

Fluid properties at film temperature.

Condensation

Alternative eq if q/A rather than ΔT is known:

$$h_{av} = 0.924 \cdot k \cdot \{g \cdot h_{fg} \cdot \rho \cdot (\rho - \rho_v) / [\mu \cdot L \cdot q/A]\}^{1/3}$$

For single horizontal tubes:

exchange *L* for *d*,

and the constant 0.943 is changed to 0.725.

(For *n* horizontal tubes placed on top of each other, the diameter *d* should be multiplied by *n*).

For condensation inside horizontal tubes,

The liquid will more or less fill the tube.

It has been suggested that could be used in this case too, but with the constant changed from 0.943 to 0.555.

As a rule of thumb, condensation heat transfer coefficients of refrigerants (not NH_3) is about 2000 $W/(m^2 \cdot K)$.

Condensation

In turbulent flow (Re>1800), plane vertical surfaces

$$\boxed{Nu = 0.0030 \cdot G \cdot C_v^{-1/2} \qquad \textit{for } Re > 1800}$$

where $Nu = h \cdot L / k$

$G = g \cdot L^3 / \nu^2$

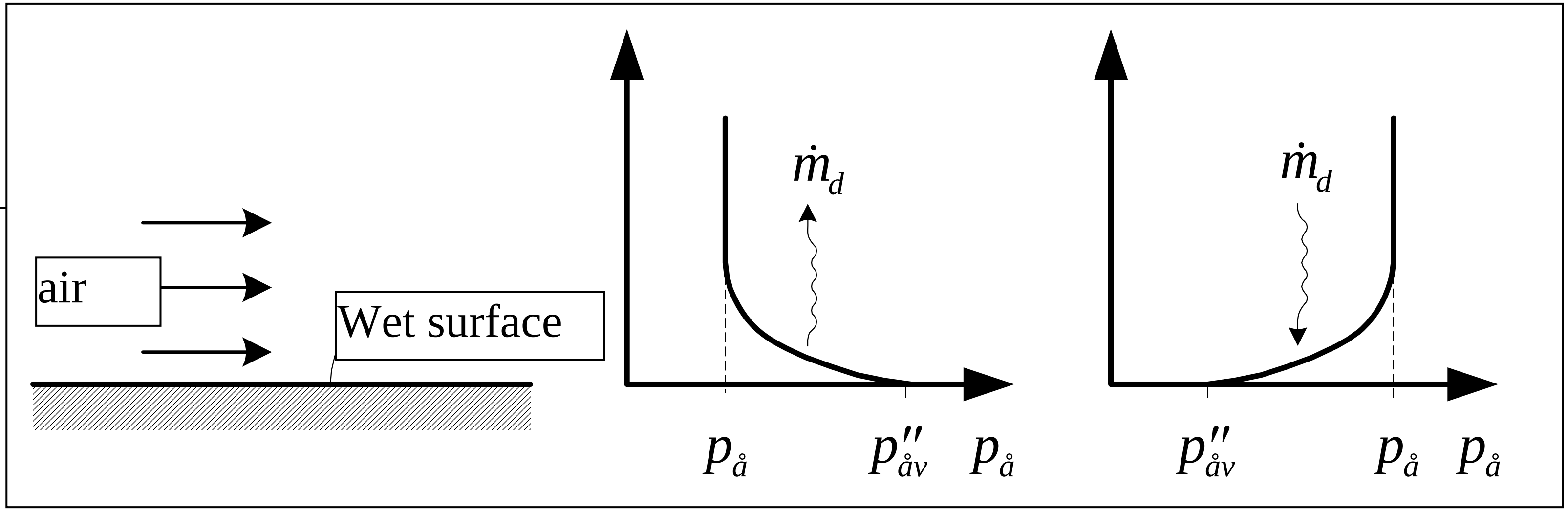
$C_v = g \cdot \rho \cdot L^3 / (k \cdot \nu \cdot \Delta T)$

$Re = 4 \cdot m' / (P \cdot \mu)$

where m' = mass flow (kg/s)

P = wetted perimeter (= width for plane wall, = $\pi \cdot d$ for vertical tube)

Diffusion



Water or frost collects on cold surfaces,
and
wet surfaces dry up:

In these processes water vapor is transferred by diffusion and convection to or from the surface.

Heat of vaporization is transferred to or from the surface.

Diffusion

The heat flow is calculated as:

$$\boxed{q_d = m'_d \cdot h_{fg}}$$

where m' = mass of water changing phase per unit time

We define a *diffusion heat transfer coefficient* h_d as

$$\boxed{q_d = h_d \cdot A \cdot \Delta T}$$

where A = wet area (m^2)

ΔT = temp diff between wet surface and air

Diffusion

The ratio between the diffusion heat transfer coefficient h_d and the convection heat transfer coefficient h_c may be calculated by the equation

$$\boxed{h_d / h_c \approx C \cdot \Delta p / \Delta T}$$

where C = 1520 above a wet surface and
C = 1750 above a frozen surface

$\Delta T = T''_{vw} - T_v$, temperature difference between wet (frozen) surface and air (°C)

$\Delta p = p''_{vw} - p_v$ = difference in partial pressure of water vapour between the surface and the free air (in bar)

p''_{vw} = saturation pressure of water at the temperature of the surface (from steam table).

$p_v = \varphi \cdot p''_v$

φ = relative humidity (%)

p''_v = saturation pressure of water at the temperature of the free air (from steam table).

Diffusion

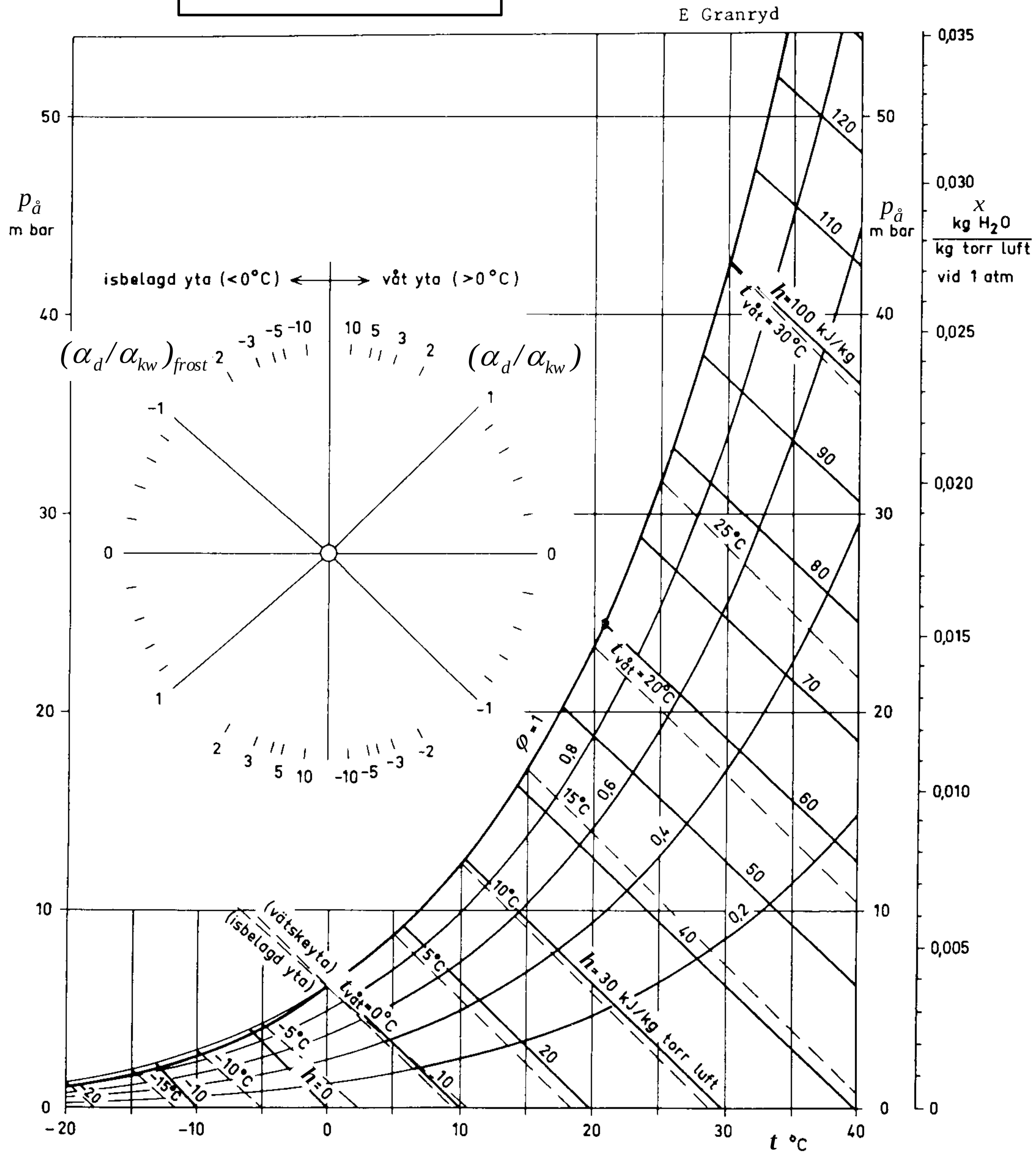
Note that h_d / h_c may be positive *or negative*, indicating heat flow in opposite directions.

The ratio may also be attained directly from the T, p_v diagram (fig 28)

To find the *heat transfer* by diffusion, we first calculate the convection heat transfer coefficient according to Lesson 2, then calculate the ratio h_d / h_c.
Then we can calculate q_d and q_{tot} as $h_{tot} = h_c + h_d = h_c (1 + h_d/h_c)$

Diffusion

$t, p_å$ - diagram för fuktig luft



Part 5

Heat exchangers

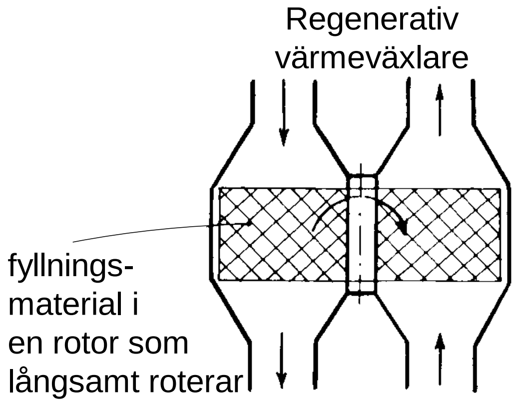
Heat exchangers

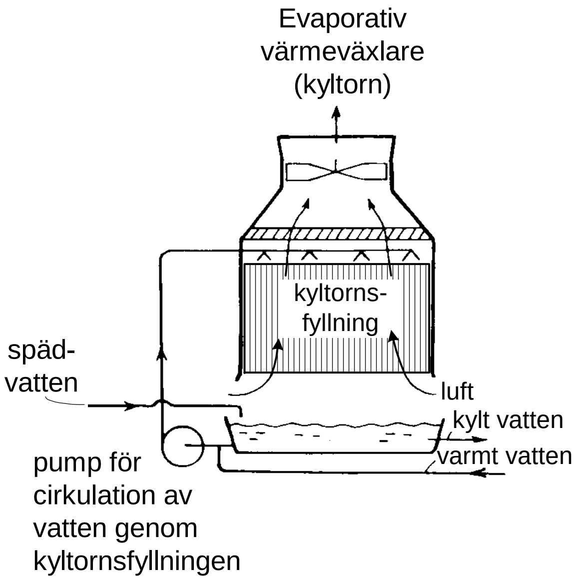
Three main types of heat exchangers:
In *recuperative heat exchangers*, the heat is transferred from one fluid to the other through a dividing wall.

In *regenerative heat exchangers*, heat is transferred via a moving solid part, (a rotating wheel with a large number of narrow channels).

In *evaporative heat exchangers*, hot water is cooled by partly evaporating the water.

Heat exchangers





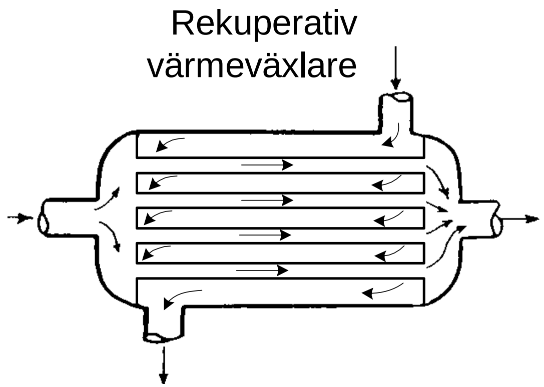
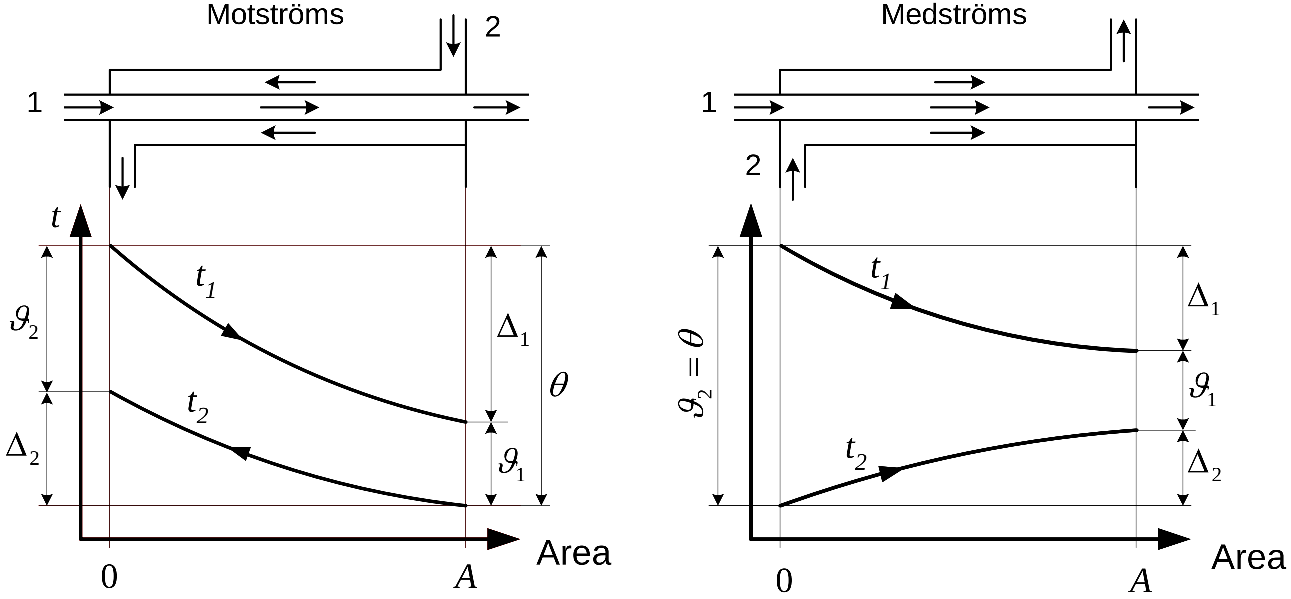


Fig 29 (Fig. 11.18 i termo)

Heat exchangers

Two basic types of recuperative, Counterflow and parallel flow (fig 30).



a) Counterflow

b) Parallel flow

Heat exchangers

The heat exchange rate from flow:

$$\boxed{q = m_h \cdot c_h \cdot (T_{h1} - T_{h2}) = m_c \cdot c_c \, (T_{c1} - T_{c2})}$$

where m = mass flow (kg/s)
c = heat capacity (J/(kg·°C))
index h and c refers to the hot and cold fluid
index 1 and 2 refers to the two ends.

The product ($m \cdot c$) is called the *heat capacity rate* and is often written as C.

Heat exchangers

Heat exchange rate from size:

$$q = U \cdot A \cdot \vartheta_{ln}$$

where ϑ_{ln} = *log mean temp difference* (or LMTD)
(For parallel flow and counter flow heat exchangers)

$$\vartheta_{ln} = [(T_{h2} - T_{c2}) - (T_{h1} - T_{c1})] / \ln[(T_{h2} - T_{c2}) / (T_{h1} - T_{c1})]$$

$$= (\Delta T_2 - \Delta T_1) / \ln(\Delta T_2 / \Delta T_1)$$

Heat exchangers

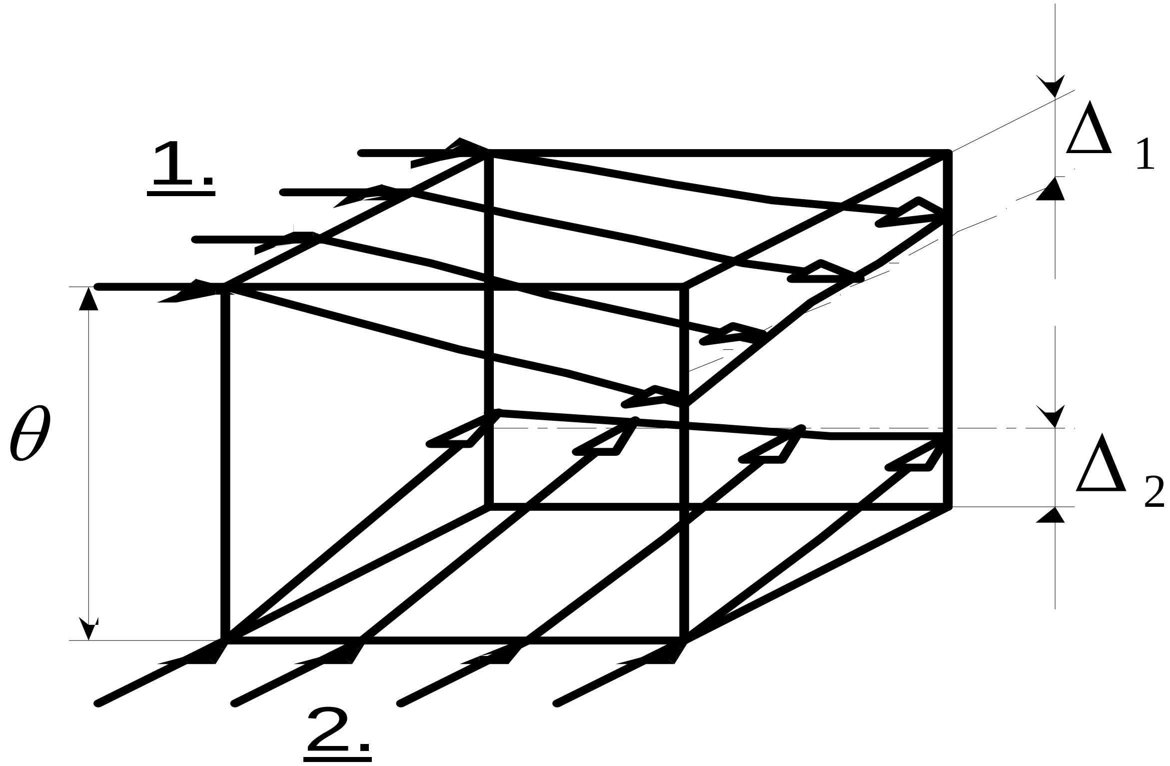
A third basic type of flow is crossflow.
The temperatures of both fluids at the outlets are different from one side to the other of the flow channel.

In most heat exchangers, the flow is neither purely parallel, counterflow or crossflow, but rather a mixture of these types.

The logarithmic mean temperature difference may still be used, if corrected by factor F.:

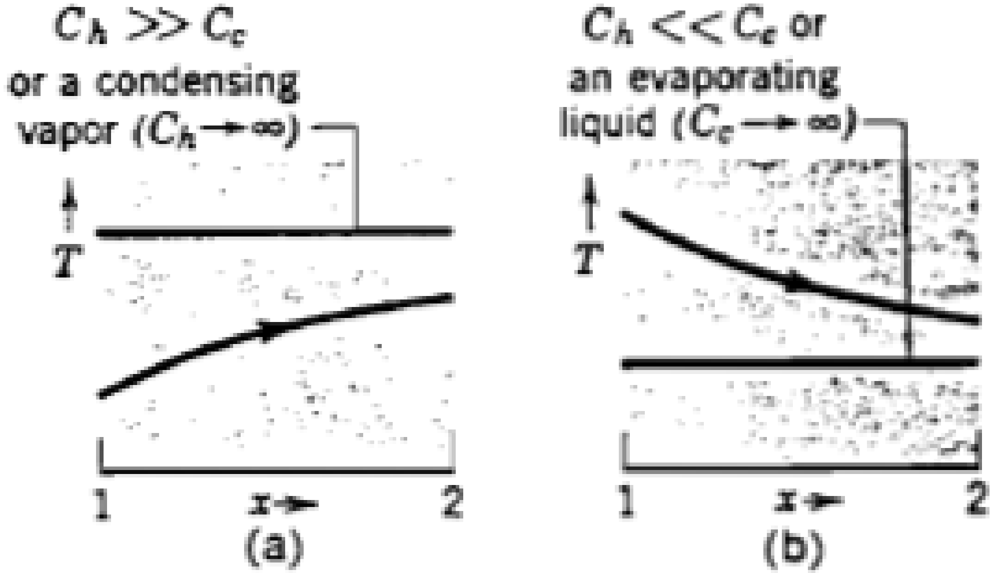
$$\boxed{q = U \cdot A \cdot \underline{F} \cdot \vartheta_{ln}}$$

Cross flow heat exchanger, temperature profile



A fourth basic type: Condensers and evaporators:
the *temperature of one fluid is constant*

The logarithmic mean temperature may be used without any correction (F=1)



Temperature effectiveness, NTU-method:

The *temperature effectiveness* is defined as

$$\boxed{\varepsilon = \text{actual heat transfer / maximum possible heat transfer}}$$

The maximum possible heat transfer would result if the temperature of *the fluid with the lowest heat capacity rate (C =m·c)* at the outlet of the heat exchanger reach the inlet temperature of the other fluid.

(This fluid is referred to in Holman as the *minimum fluid*. =
= the fluid with largest temperature change).

The definition gives

$$\boxed{\varepsilon = C_{min} \cdot \Delta T_{min} \ / \ C_{min} \cdot \Delta T_{max} = \Delta T_{min} \ / \ \Delta T_{max}}$$

where ΔT_{min} = temperature change of *minimum fluid*.
ΔT_{max} = the difference between the *inlet temperatures*.

Heat exchangers

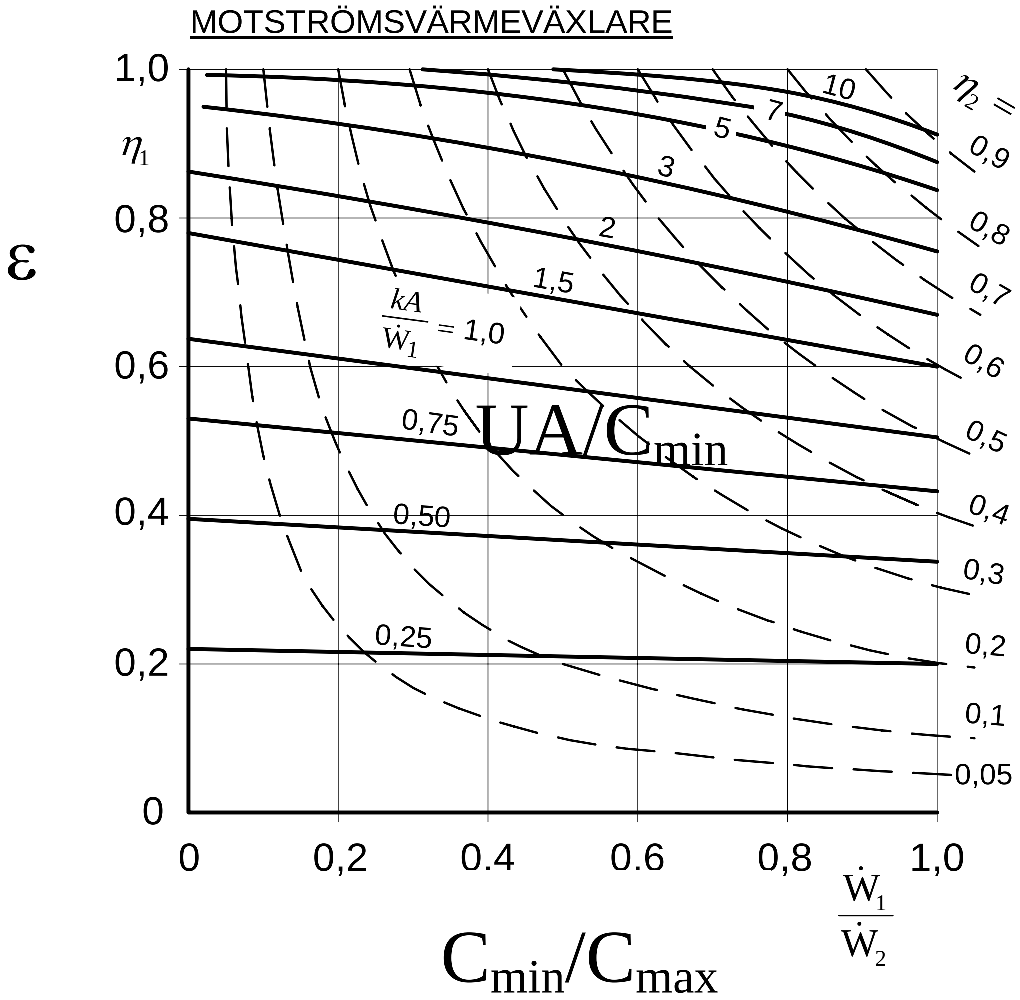
The *temperature effectiveness* is a function of *only two variables*: UA/C_{min} and C_{min} / C_{max}

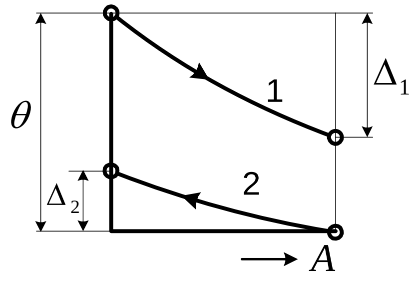
$$\boxed{\varepsilon = f(C_{min}/C_{max}\, ,\, UA/C_{min}\,)}$$

UA/C_{min} is called the *number of transfer units* (NTU).

See fig. 33.
Equations for *f* are found in table 10-3 i Holman.

Heat exchangers





$$\eta_1 = \frac{\Delta_1}{\theta}; \ \eta_2 = \frac{\Delta_2}{\theta}$$

$$\dot{W}_1 = \dot{m}_1 \cdot c_{p_1}$$

$$\dot{W}_2 = \dot{m}_2 \cdot c_{p_2}$$

$$(\eta_2 = \eta_1 \cdot \dot{W}_1 / \dot{W}_2)$$

fig. 11.25 i Termo

Lesson 6

Methods of enhancing heat transfer

Enhanced heat transfer

What is enhanced heat transfer?

”An enhanced heat transfer surface has a special surface geometry that provides a higher $h{\cdot}A$ value per unit base surface area than does a plain surface.”

When should it be applied?

When should enhanced heat transfer be applied?

To the side of a heat exchanger where the dominant heat transfer resistance lies.

It is the overall heat transfer resistance which determines the temperature difference.

Example:
Heat exchanger with surface areas equal on both sides ($1m^2$).
Total heat transfer resistance (wall is neglected):

$$1/(UA) = 1/(h_1 \cdot A) + 1/(h_2 \cdot A)$$

Assume $h_1 = 50\ W/(m^{2} \cdot °C)$ (forced flow air)
and $h_2 = 1000\ W/(m^{2} \cdot °C)$ (forced flow water)

The overall heat transfer resistance is

$$1/(UA)=1/(50 \cdot 1) + 1/(1000 \cdot 1) = 0.02 + 0.001 = 0.021°C/W$$

If we double the heat transfer coefficient on the water side from 1000 to 2000 we would get

$$1/(UA)=1/(50 \cdot 1)+1/(2000 \cdot 1)=0.02 + 0.0005=0.0205°C/W$$

No improvement!

If we double the heat transfer coefficient on the air side, we would get

$$1/(UA)=1/(100\cdot 1)+1/(1000\cdot 1) = 0.01+0.001 = 0.011°C/W$$

A reduction to half of the original total heat transfer resistance.

Other aspects of enhanced heat transfer:

The costs of enhanced surfaces are usually higher than for the plain surfaces.

The enhanced surfaces are also usually *more sensitive to fouling*, which may be important in some applications.

What is enhanced heat transfer good for?

Enhanced heat transfer may be used for any of four reasons:

1. To reduce the temperature difference between the fluids at a given heat exchanger size and capacity.

2. To reduce the size of the heat exchanger at a given capacity and temperature difference.

3. To increase the capacity of a given surface area.

4. To reduce the pumping power for a given capacity and temperature difference.

Enhanced heat transfer in gas flow on finned surfaces

Louvered fins
Offset strip fins

Enhancement is achieved by making the fins short in the flow direction.

In this way the boundary layer is not allowed to grow

Single-phase flow inside tubes

Surface with small protrusions, to disturb the boundary layer. (”dimples”, grooves or minute fins).

Turbulator in the tube, inducing large undulations in the flow.

Enhancement of boiling heat transfer

Pool boiling

Enhanced surfaces act by facilitating the nucleation of vapour bubbles.

Nucleation is facilitated by porous surface structure.

Small amounts of vapour is trapped during boiling.

Evaporation takes place in thin liquid films in the structure.

Enhancement larger for single tubes than for tube bundles.

Flow boiling

Nucleation is often suppressed by the convection.

Evaporation takes place at the vapour liquid interface.

A porous coating will induce nucleate boiling and enhance heat transfer.

Most of the porous surfaces can not be made inside tubes.

Enhancements generally increase the convection in the fluid.

”Micro-fins” (0.2mm), give h-values 2-4 times smooth tube.

Star shaped aluminium insert: divides the tube into several channels. Area is increased,

Hydraulic diameter is decreased,

Enhanced heat transfer in condensation

Heat transfer resistance in condensation is due to the resistance in the liquid film.

Enhanced heat transfer is achieved by reducing the thickness of this film.

Surface structure where surface tension act to gather the liquid at certain locations while keeping other areas free of liquid.

Surfaces often have fins with narrow spacing, surface tension drags liquid from tips to ”valleys” in between fins.

Enhanced heat transfer in condensation inside tubes

Micro-fins have been shown to increase the heat transfer coefficients by a factor of 2 to 3.

Home assignments:

Exercise 1: e
Exercise 2: e
Exercise 3: c
Exercise 4 : d
Exercise 5 : c
Exercise 6 : c
Exercise 7 : a, b
Exercise 8 : c, d
Exercise 9 : a, b
Exercise 10 : d
Exercise 11 : c, d
Exercise 12 : a, b
Exercise 13 : a, b
Exercise 14 : b, d
Exercise 15 : a, c