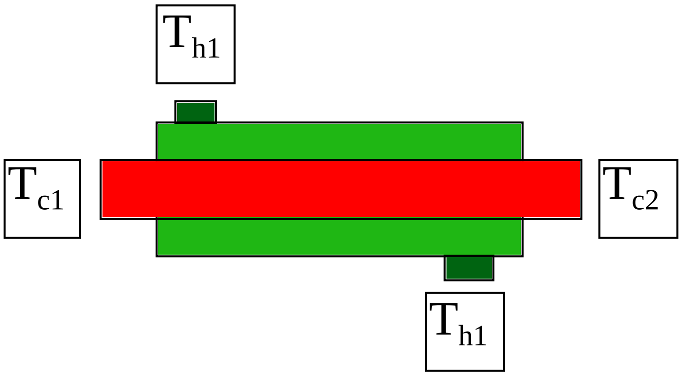
Heat Exchangers - Introduction

Concentric Pipe Heat Exchange



Energy Balance on Cold Stream (differential)

$$dQ_C = \left(wC_p\right)_C dT_C = C_C dT_C$$

Energy Balance on Hot Stream (differential)

$$dQ_H = \left(wC_p\right)_H dT_H = C_H dT_H$$

Overall Energy Balance (differential)

For an adiabatic heat exchanger, the energy lost to the surroundings is zero so what is lost by one stream is gathered by the other.

$$dQ_C + dQ_H = 0$$

Heat Exchange Equation

It follows that the heat exchange from the hot to the cold is expressed in terms of the temperature difference between the two streams.

$$dQ_H = U\left(T_H - T_C\right) dA$$

The proportionality constant is the "Overall" heat transfer coefficient (discussion later)

Solution of the Energy Balances

The Energy Balance on the two streams provides a delation for the differential temperature change.

$$dT_H = \frac{dQ_H}{C_H} \quad \text{and} \quad dT_C = \frac{dQ_C}{C_C}$$

However, we should recall that we have an adiabatic heat exchanger so that

$$d\left(\Delta T\right) = -\frac{dQ_H}{C_H}\left(1 + \frac{C_H}{C_C}\right)$$

Overall Energy balances on each stream

Hot Fluid

$$Q_H = C_H\left(T_{H1} - T_{H2}\right)$$

Cold fluid

$$Q_C = C_C\left(T_{C2} - T_{C1}\right)$$

Overall Energy balance on the Exchanger

$$Q_C + Q_H = 0$$

The equation for ΔT can be modified using the overall energy balances to yield

$$d(\Delta T) = \frac{dQ_H}{C_H} \frac{\Delta T_2 - \Delta T_1}{\left(T_{H1} - T_{H2}\right)}$$

The denominator is the energy lost by the hot stream, so

$$d(\Delta T) = \frac{dQ_H}{Q_H}\left(\Delta T_2 - \Delta T_1\right)$$

Application of the relation for energy transfer between the two streams yields

$$d(\Delta T) = -\frac{UdA\Delta T}{Q_H}\left(\Delta T_2 - \Delta T_1\right)$$

Integration of the relation is the basis of a design equation for a heat exchanger.

$$\ln\left(\frac{\Delta T_2}{\Delta T_1}\right) = \frac{UA}{Q_H}\left(\Delta T_2 - \Delta T_1\right)$$

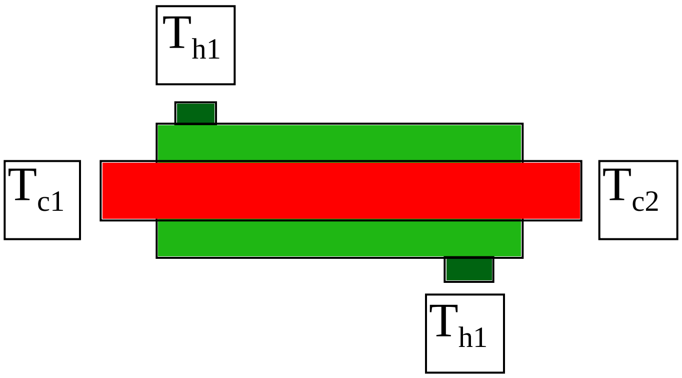
Rearrangement of the equation leads to

The Design Equation for a Heat Exchanger

$$Q_H = UA\frac{\left(\Delta T_2 - \Delta T_1\right)}{\ln\left(\frac{\Delta T_2}{\Delta T_1}\right)} = UA\Delta T_{lm}$$

Design of a Parallel Tube Heat Exchanger

The Exchanger



The Design Equation for a Heat Exchanger

$$Q_H = UA\frac{\left(\Delta T_2 - \Delta T_1\right)}{\ln\left(\dfrac{\Delta T_2}{\Delta T_1}\right)} = UA\Delta T_{lm}$$

Glycerin-water solution with a Pr = 50 (at 70 °C) flows through a set of parallel tubes that are plumbed between common headers. We must heat this liquid from 20 °C to 60°C with a uniform wall temperature of 100 °C. The flow rate, F, is 0.002 m^3/sec (31.6 gal/sec.).

- How many parallel tubes are required ?
- How do we select L and D for these tubes ?

<u>Data</u>
The heat capacity, C_p, is 4.2 kJ/kg-°K
The density, ρ, is 1100 kg/m^3
The liquid has a kinematic viscosity, $\nu = 10^{-3}$ cm^2/sec.

Step 1

Calculate the heat load

$$Q_c = \rho F C_p \left(T_{out} - T_{out}\right)$$

$$Q_c = \left(1100\ \frac{kg}{m^3}\right)\left(0.002\ \frac{m^3}{sec}\right)\left(4.2\ \frac{kJ}{kg{-}°K}\right)\frac{1°K}{1°C}\left(60-20\right)°C$$

$$Q_c = 369.6\ \frac{kJ}{sec} = 369.6\ kWatts$$

Step 2

Calculate the heat transfer coefficient

If the flow is laminar, likely since glycerin is quite viscous,
and the Re < 2000
the Nusselt number relation for laminar flow can be expressed as

$$Nu = \left[\left(3.66\right)^3 + \left(1.61\right)^3 Gz\right]^{1/3}$$

The Graetz number is

$$Gz = Re\ Pr\ \frac{D}{L}$$

If the flow is turbulent (Re > 2000), the Nusselt numberr is given by

$$Nu = 0.023\, Re^{0.8} Pr^{0.4}$$

We do not know the flow per tube and therefore we do not know the Re. However we don't need to know that. In Lecture 27 we observed for Heat Transfer in a Tube that

$$\frac{T - T_R}{T_1 - T_R} = \exp\left(-\frac{\pi D h z}{w C_p}\right) = \exp\left(-4 St\frac{z}{D}\right)$$

The definition of the Stanton Number is :

$$St = \frac{h}{\rho C_p U} = \frac{Nu}{RePr} = \frac{Nu}{Pe}$$

Given a Re and Pr, we can calculate the Nu and the Stanton Number, the latter prviding us with the temperature at length L from the previous equation. Let's examine several configurations at L/D = 50, 100, 200. The Excel table below can be used to specify a design chart.

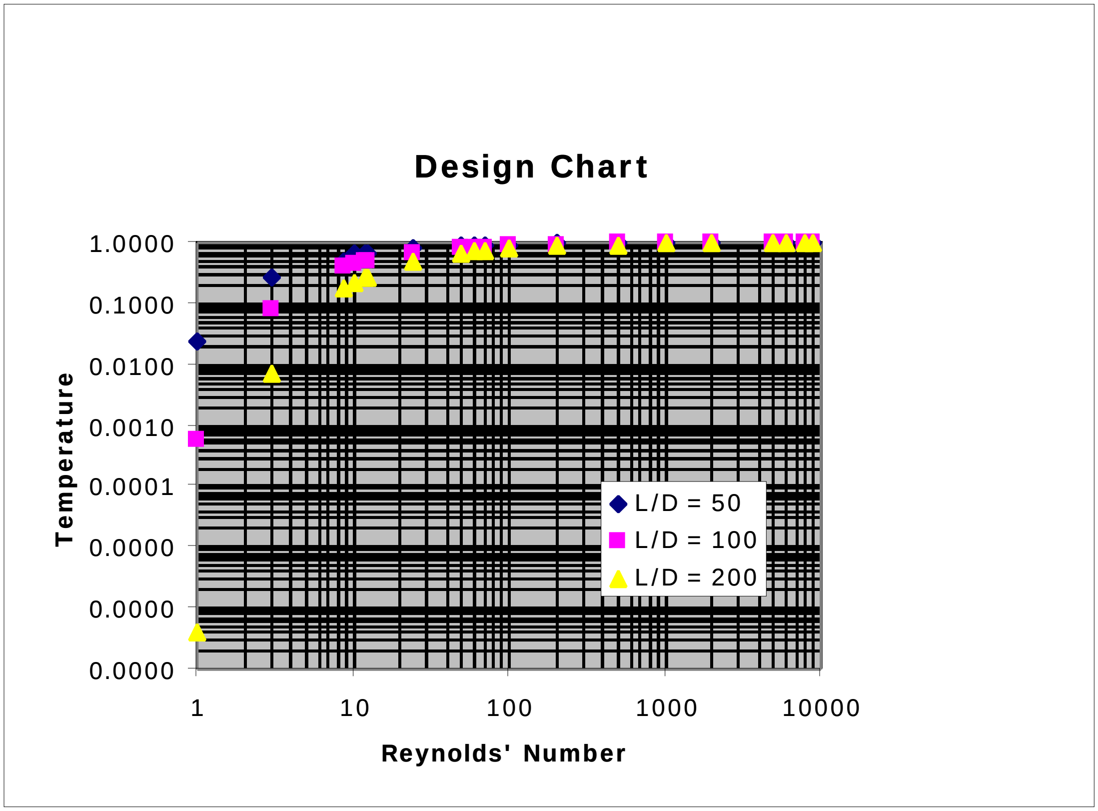
Design Chart			
Pr = 50		L/D = 50	
Re	Nu	St	θ_{cm}
1	3.7610	7.52E-02	0.0233
3	3.9482	2.63E-02	0.2682
6	4.1996	1.40E-02	0.4966
10	4.4940	8.99E-03	0.6380
20	5.0980	5.10E-03	0.7750
30	5.5852	3.72E-03	0.8301
100	7.7548	1.55E-03	0.9254
200	9.5962	9.60E-04	0.9532
500	12.8779	5.15E-04	0.9746
1000	16.1628	3.23E-04	0.9840
2000	20.3244	2.03E-04	0.9899
5000	100.1133	4.00E-04	0.9802
10000	174.3074	3.49E-04	0.9827
20000	303.4868	3.03E-04	0.9849
30000	419.7714	2.80E-04	0.9861

To obtain the numbers in the spreadsheet, we used the Nusselt number relation for laminar flow expressed as

$$Nu = \left[(3.66)^3 + (1.61)^3 Gz\right]^{1/3}$$

and for turbulent flow as

$$\mathrm{Nu} = 0.023\,\mathrm{Re}^{0.8}\mathrm{Pr}^{0.4}$$



Step 3

Calculate the Area required

Base case

D = 2 cm. and L = 100 D = 2 meters

For this case we observe that from the calculations for θ_{cm}

Reduced Temperature			
Re	L / D = 50	L / D = 100	L / D = 200
1	0.0233	0.0006	0.0000
3	0.2682	0.0789	0.0069
8.8	0.5000	0.3966	0.1718
10	0.6380	0.4387	0.2099
12	0.6800	0.4966	0.2682
12.3	0.6854	0.5042	0.2763
24.4	0.8040	0.6836	0.5017
50	0.8805	0.8073	0.6888
60	0.8945	0.8301	0.7254
70	0.9050	0.8473	0.7532
100	0.9254	0.8805	0.8073
200	0.9532	0.9254	0.8805
500	0.9746	0.9596	0.9358
1000	0.9840	0.9746	0.9596
2000	0.9899	0.9840	0.9746
5000	0.9802	0.9913	0.9862
6000	0.9809	0.9923	0.9878
8000	0.9819	0.9936	0.9899
9000	0.9824	0.9941	0.9907

We can observe that the flow rate per tube is given by

$$F_{nt} = \frac{F}{n_t}$$

so that the Reynolds' number is

$$Re = \frac{4F}{\pi D \upsilon n_t}$$

As a consequence we can observe that the total length of tubing is not dependent on D alone but on othere considerations that might set a condition for Re, e.g. a pressure drop limitation. Wv find that for this base case, we find

$$n_t L = A = \frac{4F}{\pi \upsilon \mathrm{Re}} \frac{L}{D}$$

We find that $\Theta_{cm} = 0.5$

L/D	***Re***	***$n_t L$***	***n_t***
50	8.8	14.47	14.46
100	12.3	20.70	10.35
200	24.4	20.87	5.21

Does it make sense?

Maximum Cooling Capacity of an Exchanger of Fixed Area

Water is available for use as a coolant for an oil stream in a double-pipe heat exchanger.

The flow rate of the water is 500 lb_m/hr.
The heat exchanger has an area of 15 ft^2.
The oil heat capacity, C_{po}, is 0.5 BTU/lb-°F
The overall heat transfer coefficient, U, is 50 BTU/hr-ft^2-°F

The initial temperature of the water, T_{w0}, is 100°F
The maximum temperature of the water is 210°F
The initial temperature of the oil, T_{w0}, is 250°F
The minimum temperature of the oil, T_{w0}, is 140°F

Estimate the maximum flow rate of oil that may be cooled assuming a fixed flow rate of water at 500 lb_m/hr

There are two possible modes of operation

Co-current flow
Counter-current flow

Let us look at both cases

Co-current flow

Constraints

$$T_w < 210 \; ; \; T_w < T_o \; ; \; T_o \geq 140$$

Energy balances

Oil

$$Q_o = F_o C_{po}\left(T_{o1} - T_{o2}\right) = F_o\left(0.5\right)\left(250 - T_{o2}\right)$$

Water

$$Q_w = F_w C_{pw}\left(T_{w1} - T_{w2}\right)$$
$$F_o C_{p0}\left(T_{o1} - T_{o2}\right) = 500(1.0)(210 - 100) = 55{,}000 \text{ BTU / hr}$$

Recall the Design equation

$$Q_H = UA\frac{\left(\Delta T_2 - \Delta T_1\right)}{\ln\left(\frac{\Delta T_2}{\Delta T_1}\right)} = UA\Delta T_{lm}$$

Now the ΔT_{lm} is given by

$$\Delta T_{lm} = \frac{\left(\Delta T_2 - \Delta T_1\right)}{\ln\left(\frac{\Delta T_2}{\Delta T_1}\right)} = \frac{Q_w}{UA} = \frac{55000}{(50)(15)} = 73.3$$

Using the temperatures, we obtain T_{0max} = 238.5 °F
and from the heat balance for oil, we obtain

$$F_o = \frac{C_{po}}{Q_o}\left(T_{o1} - T_{o2}\right) = \frac{\left(0.5\right)\left(250 - 238.5\right)}{55000} = 9560 \text{ lb / h}$$

<u>Counter-current Flow</u>

Constraints

$$T_w < 210 \; ; \; T_w < T_o \; ; \; T_o \geq 140$$

Energy balances

Oil

$$Q_o = F_o C_{po}\left(T_{o1} - T_{o2}\right) = F_o\left(0.5\right)\left(250 - T_{o2}\right)$$

Water

$$Q_w = F_w C_{pw}\left(T_{w1} - T_{w2}\right)$$

$$F_o C_{p0}\left(T_{o1} - T_{o2}\right) = 500(1.0)(210 - 100) = 55{,}000 \text{ BTU / hr}$$

Recall the Design equation

$$Q_H = UA\frac{\left(\Delta T_2 - \Delta T_1\right)}{\ln\left(\frac{\Delta T_2}{\Delta T_1}\right)} = UA\Delta T_{lm}$$

Now the ΔT_{lm} is given by

$$\Delta T_{lm} = \frac{\left(\Delta T_2 - \Delta T_1\right)}{\ln\left(\frac{\Delta T_2}{\Delta T_1}\right)} = \frac{Q_w}{UA} = \frac{55000}{(50)(15)} = 73.3$$

Using the temperatures, we obtain T_{0max} = 221 °F
and from the heat balance for oil, we obtain the oil flow rate as 3800 lbm/hr.

I thought that countercurrent flow was supposed to be more efficient. What is the problem ?