Problem 1: In a consultancy company, a junior engineer was hired to prepare the initial design of a new geothermal facility in France. After the initial field survey, tests indicated that the geothermal fluid temperature at the top of the production well will be at 120ºC.

a) A binary cycle power plant (Fig. 1) operating with propane was chosen to extract heat from the geothermal fluid. Assume that propane at the output of the heat exchanger (coordinate 1) is at 40 bar and 100ºC and that the fluid leaves the turbine (coordinate 2) at 0.50 bar. Calculate the power (in MW) produced by the turbine assuming that the mass flow rate of the propane is 10 $kg.s^{-1}$. **[10 Marks]**

b) After a few calculations the engineer obtained the specific enthalpy of the propane entering the heat exchanger (coordinate 3) as -35 $kJ.kg^{-1}$. Assuming that the re-injection temperature of the geothermal fluid is 65ºC, calculate the mass flow rate (in 10 $kg.s^{-1}$) of the geothermal fluid. For your calculations, assume that the heat capacity at constant pressure (C_p) of the geothermal fluid is 4.18 $kJ.(kg.^{o}C)^{-1}$, and that there are no heat losses and pressure drop in the heat exchanger. **[6 Marks]**

c) After re-injection in the subsurface, geothermal fluid exchanges heat with saturated rock in the geological formation. What are the main heat transfer mechanisms in the subsurface? **[9 Marks]**

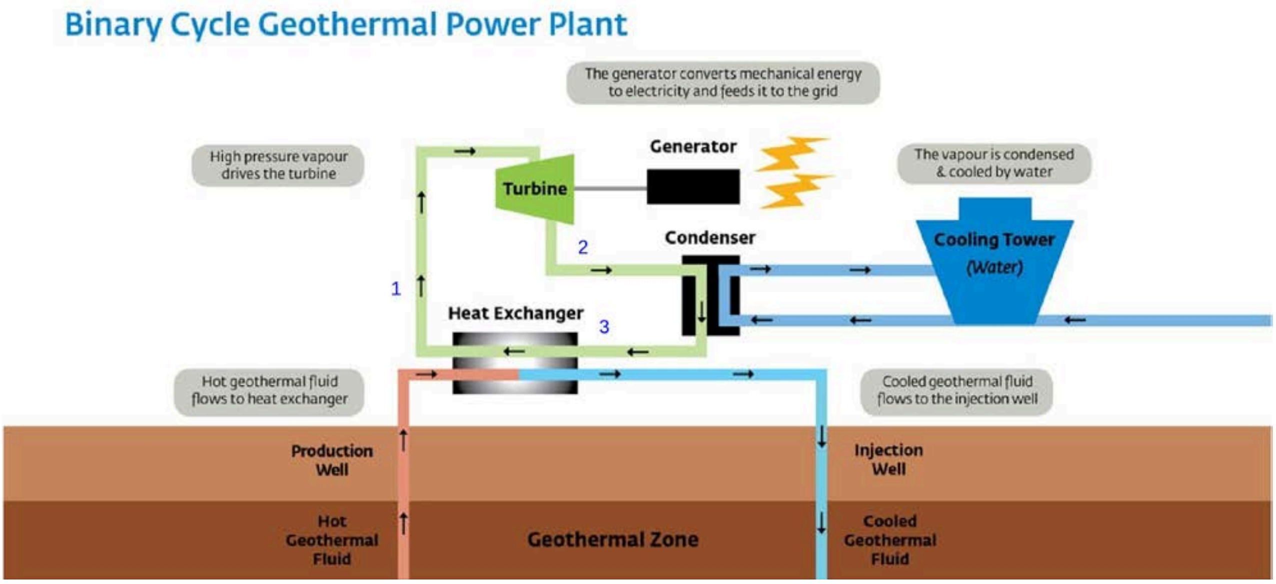

Figure 1: Sketch of the binary power cycle.

Relevant equations for this problem:

i. Quality of the vapour:

$$x_i = \frac{\Psi_i - \Psi_f}{\Psi_g - \Psi_f}, \qquad with\ \Psi = h, s,$$

where h and s are the specific enthalpy and entropy, respectively.

ii. Heat transfer rate:

$$Q = \dot{m}.C_p \Delta T$$

where $\dot{m}$ is the mass flow rate, C_p is the heat capacity at constant pressure and T is the temperature.

Solution:

(a) From the superheated table, at P_1=40 bar and T_1 = 100°C ➔ s_1 = 1.70 kJ.(kg.K)$^{-1}$, and h1=549.7 kJ.kg^{-1}. **[2 Marks]**

Pressure at the output of the turbine is P_2=0.50 bar. The fluid suffers an isentropic expansion in the turbine, therefore s_2=s_1.Specific enthalpy and entropy for liquid and vapour propane at this temperature are s_f = -0.085 kJ.(kg.K)$^{-1}$, s_g = 1.841 kJ.(kg.K)$^{-1}$, h_f = -19.5 549.7 kJ.kg^{-1} and h_g = 412.8549.7 kJ.kg^{-1}. As $s_2 < s_g$, we can conclude that the fluid is within the dome (i.e., two-phase fluid). **[2 Marks]**

As such, quality of the fluid needs to be calculated,

$$x_2 = \frac{\Psi_2 - \Psi_f}{\Psi_g - \Psi_f} = \frac{s_2 - s_f}{s_g - s_f} = 0.9268$$ **[2 Marks]**

With the quality of the vapour we can calculate the specific enthalpy h_2,

$$x_2 = \frac{h_2 - h_f}{h_g - h_f} = 0.9268 \implies h_2 = 381.1556\ kJ.kg^{-1}$$ **[2 Marks]**

Power in the turbine can now be calculated as

$$\dot{W}_T = \dot{m}_{c3}(h_2 - h_1) = -1685.44\ kJ.s^{-1}$$

Power produced by the turbine is 1.69 MW. **[2 Marks]**

(b) Assuming that there are no heat losses in the heat exchanger,

$$\begin{cases} \dot{Q}_{C3} + \dot{Q}_w = 0 \implies \dot{Q}_{C3} = -\dot{Q}_w \\ \dot{m}_{C3}(h_1 - h_3) = -\dot{m}_w C_p \Delta T = -\dot{m}_w C_p\left(T_w^{out} - T_w^{in}\right) \end{cases}$$ **[3 Marks]**

Leading to $\dot{m}_w$=25.43kg.s^{-1}. **[3 Marks]**

(c) There are three main heat exchange mechanisms in geothermal reservoirs

i. Thermal conduction (that occurs between materials at the same phase); **[3 Marks]**

ii. Thermal convection (or inter-phase heat transfer that occurs between materials at different phases); **[3 Marks]**

iii. Thermal radiation from Earth's core to rock formations at relatively large depths. **[3 Marks]**

Problem 2: A study was conducted to assess the feasibility of using local geothermal resources to continuously power a small village.

a) In order to assess the natural regeneration of the subsurface fluid (due to rainwater) a survey was conducted in an area near the water-table. After a week of rain, water accumulated in an area located in the top of a water-table. The accumulated water slowly percolates the sandstone formation (15 m thick) of porosity 0.45 and mean particle diameter (d_s) of 0.25 mm. Calculate the volumetric flow rate (in $m^3.s^{-1}$) of an area of (10 X 10) m^2 using the Carman-Koneny equation,

$$\frac{dp}{dz} = -180\, u\, \frac{\mu(1-\phi)^2}{d_s^2 \phi^3},$$

where u is the fluid velocity. Dynamic viscosity (μ) and density (ρ) are $0.89X10^{-3}$ Pa.s and 998 $kg.m^{-3}$, respectively. z is the vertical depth, also the hydrostatic pressure in a fluid can be determined by

$$p = p_0 + \rho g z,$$

where g = 9.81 $m.s^{-2}$ is the acceleration due to gravity. **[10 Marks]**

b) During the geophysics and geochemistry surveys, data collected from wells revealed that the geothermal reservoir would produce dry saturated water at 95ºC. What are the pressure (in bar) and specific enthalpy (in $kJ.kg^{-1}$) of the fluid at this condition? **[7 Marks]**

c) A binary cycle geothermal plant was chosen design to produce power. Describe how this power plant operates. **[8 Marks]**

Solution:

a) In this problem we need to calculate the volumetric flow rate of the rainwater through rock layers of sandstone formation

$$\dot{Q} = |u| \text{ x Area}$$

The Carman-Koneny equation gives a relationship between pressure drop through the porous media and velocity, however, the pressure drop is not known but can be obtained by manipulating the hydrostatic pressure equation in differential form,

$$\frac{dp}{dz} = \rho g, \textbf{ [2 Marks]}$$

this equation indicates that the reference origin is in the surface, therefore

$$\frac{dp}{dz} = \rho g = -180\, u\, \frac{\mu(1-\phi)^2}{d_s^2 \phi^3} \textbf{ [2 Marks]}$$

Resultando em u = -1.1506 X 10^{-3} m.s^{-1}. **[3 Marks]**

The negative velocity indicates that the fluid flows downwards. Finally the volumetric flow rate is (for an area of 100 m^2)

$$\dot{Q} = |u| \text{ x Area} = 1.1506\text{X } 10^{-1} \text{ m}^3.\text{s}^{-1} \textbf{ [3 Marks]}$$

b) At 95ºC, the coordinates for dry (i.e., quality of vapour is 100%) and saturated steam is exact at the saturated vapour line. **[1 Mark]**

Thus from the saturated water-steam table, the pressure is 0.8455 bar **[3 Marks]**
and the specific enthalpy is 2668.1 kJ.kg^{-1}. **[3 Marks]**

c) Binary cycle operation:

- Binary cycles are operated between 107 and 182ºC; **[2 Marks]**
- Produced hot brine vaporises the working fluid in a heat exchanger; **[1 Mark]**
- The working fluid is isentropically expanded in the turbine; **[1 Mark]**
- After the expansion, the working fluid is condensed in a secondary heat exchanger; **[1 Mark]**
- And isentropically compressed in a pump prior to its return the main heat exchanger; **[1 Mark]**
- Working fluid often used in geothermal plants are organic chemical species with low boiling temperature. **[2 Marks]**