Problem 1: A closed system contains 1 mol of nitrogen $\left(\text{MW} = 28 \text{ g. mol}^{-1}\right)$. Using the ideal gas law, calculate the missing *PVT* parameter for the following data given. Give your results in **SI units**. The universal gas constant is R = 8.314 $\text{J.gmol}^{-1}.\text{K}^{-1}$.

(a) P = 1 atm, T = 0°C

(b) V^t = 12.85 ft^3; T = 59°F

(c) P = 2.5×10^9 $\text{g.m}^{-1}.\text{s}^{-2}$; T = 650.50°R

(d) V^t = 1.3×10^{-12}Gl; P= 500 psi

Problem 2: In a coal-fired power station water-steam system is used to produce electricity. Determine the enthalpy $\left(\text{kJ.kg}^{-1}\right)$ and entropy $\left(\text{kJ.kg}^{-1}.K^{-1}\right)$ of the system under the following conditions (a-h):

Pressure (bar)	**Temperature**	**Enthalpy kJ.kg^{-1}**	**Entropy $\text{kJ.kg}^{-1}.\text{K}^{-1}$**	**State**
150.0	733.15 K	(a)	(b)	(c)
37.5	–	(d)	(e)	liquid water
142.6	375.00°C	(f)	(g)	(h)

Problem 3: The *Angle Falls* in Venezuela are the worlds highest waterfalls (∼1000 m). Take the amount of 1 kg of water as the system flowing over the waterfall. Assume that is does not exchange energy with its surroundings.

(a) Calculate the potential energy of the water at the top of the falls with respect to the base of the falls. Assume gravity acceleration as 9.81 m.s^{-2}.

(b) What is the energy balance that applies during the water falling down? What is the kinetic energy of the water just before it strikes down?

(c) When striking down the energy is converted to internal energy. Calculate the temperature change with the heat capacity 4184 $\text{J.kg}^{-1}.\text{K}^{-1}$.

Problem 4: A hydroturbine operates with a head of 160 ft of water. Inlet and outlet conduits are 78.74 inches in diameter. Calculate the maximum mechanical power (in kW) that can be developed by the turbine for an inlet velocity of 18 km.h^{-1}.

Problem 5: Given *Ar* at P_1 = 140 kPa, T_1 = 10°C, V_1 = 200 liters which undergoes a polytropic compression to P_2 = 700 kPa, T_2 = 180 °C, find Q_{1-2}. Given MW = 39.948 kg.kgmole^{-1}, R = 0.2081 $\text{kJ.kg}^{-1}.\text{K}^{-1}$ and C_V = 0.312 $\text{kJ.kg}^{-1}.\text{K}^{-1}$.

Problem 6: Given air (assuming ideal gas behaviour) expanding reversibly and adiabatically from $T_1 = 450K$ and $V_1 = 3.0 \times 10^{-3} m^3$ to the final volume, $V_2 = 5.0 \times 10^{-3} m^3$. T and V relationship for constant heat capacities is represented by $\dfrac{T_2}{T_1} = \left(\dfrac{V_1}{V_2}\right)^{\gamma-1}$.

(a) Derive a relationship between T and P; Assuming that $C_p = 5.0cal.\left(mol.K\right)^{-1}$ and $C_v = 3.0cal.\left(mol.K\right)^{-1}$;

(b) Calculate T_2;

(c) Calculate the work done during the process and;

(d) Determine the enthalpy change.

Problem 7: Gaseous CO_2 ($m_{CO_2} = 4g$) is contained in a vertical piston-cylinder assembly by a piston of mass 50 kg and having a face area of 1.0×10^{-2} m^2. The CO_2 initially occupies a volume of 5×10^{-3} m^3 and has a specific internal energy of 657 kJ.kg^{-1}. The atmosphere exerts a pressure of 100 kPa on the top of the piston. Heat transfer in the amount of 1.95 kJ occurs slowly from the CO_2 to the surroundings, and the volume of the CO_2 decreases to 2.5×10^{-3} m^3. Friction between the piston and the cylinder wall can be neglected. The local acceleration of gravity is 9.81 m.s^{-2}. For the CO_2, determine (a) the pressure in kPa and (b) the final specific internal energy in kJ.kg^{-1}.

Problem 8: CO gas contained within a piston-cylinder assembly undergoes three processes in series:

- Process 1-2: expansion from p_1 = 5 bar, V_1 = 0.2 m^3 to V_2 = 1.0 m^3, during which the pressure-volume relationship is pV = constant.
- Process 2-3: constant volume heating from state 2 to state 3, where p_3 = 5 bar.
- Process 3-1: constant pressure compression to the initial state.

Sketch the processes in series on p-V coordinates and evaluate the work for each process, in kJ.

Tutorial 01

P1 ($PV^t = nRT$) $n = 1$ gmol $R = 8.314$ J/gmol.K

(a) $P = 1$ atm ; $T = 0°C$ ∴ $V^t = ?$

$$V^t = \frac{nRT}{P} = 1\,\cancel{gmol} \times 8.314\frac{J}{\cancel{gmol.K}} \times (0+273.15)\cancel{K} \times \frac{1}{1\,atm}$$

$$V^t = 2270.9691\,\frac{\cancel{J}}{\cancel{atm}} \times \frac{1\,N.m}{1\,\cancel{J}} \times \frac{1\,\cancel{atm}}{1.01325\times10^5\,\cancel{N}/m^2}$$

$[m^3]$ $V^t = 2.2412\times10^{-2}\,m^3$

(b) $V^t = 12.85\,ft^3$; $T = 59°F$ ∴ $P = ?$

$\hookrightarrow 288.15\,K$

$$P = \frac{nRT}{V^t} = 1\,\cancel{gmol} \times 8.314\frac{J}{\cancel{gmol.K}} \times 288.15\cancel{K} \times \frac{1}{12.85\,ft^3}$$

$$P = 186.43\,\frac{\cancel{J}}{\cancel{ft^3}} \times \frac{1\,Nm}{1\,\cancel{J}} \times \frac{1\,\cancel{ft^3}}{2.832\times10^{-2}\,m^3}$$

$[Pa = N/m^2]$

$P = 6582.98\,N/m^2 = 6.58$ KPa

(c) $P = 2.5\times10^9\,g/ms^2$; $T = 650.50°R$ ∴ $V^t = ?$

$\hookrightarrow 361.39\,K$

$$V^t = \frac{nRT}{P} = 1\,\cancel{gmol} \times 8.314\frac{J}{\cancel{gmol.K}} \times 361.39\cancel{K} \times \frac{1}{2.5\times10^9\,\frac{g}{ms^2}}$$

$$V^t = 1.2018\times10^{-6}\ \frac{\cancel{J}}{\cancel{g}/m\cancel{s^2}} \times \frac{1\ \cancel{N}.m}{1\ \cancel{J}} \times \frac{1\ \cancel{kg}\ m/\cancel{s^2}}{1\ \cancel{N}} \times \frac{1000\ \cancel{g}}{1\ \cancel{kg}}$$

$[m^3]$ $\quad V^t = 1.2018\times10^{-3}\ m^3$

(d) $V^t = 1.3\times10^{-12}\ Gl$; $P = 500\ psi$ ∴ $T = ?$

$$T = \frac{PV}{mR} = 500\ psi \times 1.3\times10^{-12}\ Gl \times \frac{1}{g\cancel{mol}} \times \frac{g\cancel{mol}.K}{8.314\ J}$$

$$T = 7.81814\times10^{-11}\ \cancel{psi}\times\cancel{Gl}\times\frac{K}{\cancel{J}} \times \frac{6.895\times10^3\ \cancel{N}/\cancel{m^2}}{1\ \cancel{psi}} \times \frac{10^9\ \cancel{l}}{1\ \cancel{Gl}} \times$$

$[K]$ $$\times \frac{10^{-3}\ \cancel{m^3}}{1\ \cancel{l}} \times \frac{1\ \cancel{J}}{1\ \cancel{N}\cancel{m}}$$

$T = 0.5391\ K$

P2

(1) At 150 bar, the saturation temperature (T_{sat}) is 342.1°C <<< T = 733.15 K (= 460 °C). Thus the water is at superheated steam (c) state.

The superheated steam table (SST) at 150 bar looks like:

	450	500
H (KJ/Kg)	3156.2	3308.6
S (KJ/Kg.K)	6.140	6.344

But we want H and S at 460 °C.
A linear interpolation between 450 and 500 °C

$$\underbrace{500-450}_{T_2 - T_1} \;\_\_\_\; \underbrace{3308.6-3156.2}_{H_2 - H_1}$$

$$\underbrace{500-460}_{T_2 - T^*} \;\_\_\_\; 3308.6 - H^*$$

$$(3308.6 - H^*)(500-450) = (500-460)(3308.6-3156.2)$$

$$H^* = 3186.68 \text{ KJ/Kg} \quad (a)$$

Using the same procedure for entropy (S):

$$S^* = 6.1808 \text{ KJ/Kg.K} \quad (b)$$

(2) At 37.5 bar and considering that the fluid is liquid, by simple inspection in the saturated steam table:

$$\begin{cases} H^* = 1069.0 \ \text{KJ/Kg} & (\underline{d}) \\ S^* = 2.7618 \ \text{KJ/Kg.K} & (\underline{e}) \end{cases}$$

(3) At 142.6 bar, the saturation temperature (T_{sat}) is $337.7 \leq T_{sat} \leq 338.8$. This is lower than the established temperature, therefore the fluid is at superheated state ($\underline{h}$). From the SST (at 375 °C):

P (bar) T_{sat} (°C)		T = 375 °C
140 (336.6)	H (KJ/Kg)	2894.5
	S (KJ/Kg.K)	5.782
150 (342.1)	H (KJ/Kg)	2858.4
	S (KJ/Kg.K)	5.703

Now we need to proceed with a linear interpolation in the pressure range:

$$\underbrace{150-140}_{P_2 - P_1} \quad — \quad \underbrace{2858.4-2894.5}_{H_2 - H_1}$$

$$\underbrace{150-142.6}_{P_2 - P^*} \quad — \quad 2858.4 - H^*$$

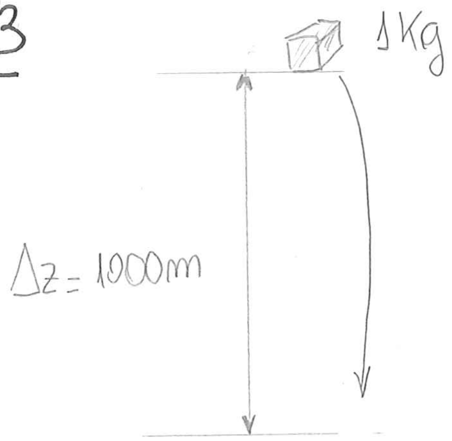
$$(2858.4 - H^*)(150-140) = (150-142.6)(2858.4 - 2894.5)$$

$$H^* = 2885.11 \ KJ/Kg \ (g)$$

With the same procedure for entropy:

$$S^* = 5.7615 \ KJ/Kg.K \ (g)$$

P3



potential energy

(a) $E_p = m \cdot g \cdot \Delta z$

$E_p = 1\,Kg \times 9.81\,\frac{m}{s^2} \times 1000\,m$

$E_p = 9810\,\underbrace{\frac{Kg\,m^2}{s^2}}_{N.m\,=\,J} = 9.81\,KJ$

internal energy   Kinetic energy

(b) $\Delta U + \Delta E_p + \Delta E_K = 0$ (energy conservation)

$\hookrightarrow$ during fall down: $\Delta U = 0$

$E_p + E_K = \text{const.} \Rightarrow E_p(z) + E_K(z) = E_p(\Delta z)$

Just before hitting the bottom: $z = 0$

$E_K = E_p(\Delta z) = 9.81\,KJ$

(c) $E_K = \Delta U$

$\Delta U = m\,C_p\,\Delta T = Q$

$\Delta T = \frac{\Delta U}{m\,C_p} = \frac{9810\,\cancel{J}}{1\,\cancel{Kg} \times 4184\,\frac{\cancel{J}}{\cancel{Kg}\,K}}$ $\therefore \Delta T = 2.34\,K$

P4

$$\begin{cases} \Delta z = 160\ ft \\ \rho_{water} = 1000\ Kg/m^3 \\ D = 78.74\ in \\ v = 18\ Km/h \end{cases}$$

The mechanical power can be defined here as,

$$\dot{W} = \dot{m}\, g\, \Delta z \quad [KW]$$

Thus, we first need to calculate the mass flow rate:

$$\dot{m} = \rho\, u\, \overbrace{A}^{\pi D^2/4} = 1000\ \frac{Kg}{m^3} \times 18\ \frac{Km}{h} \times \frac{\overbrace{\pi}^{3.1415}}{4} \times (78.74\ in)^2$$

converting units to SI:

$$\dot{m} = 1000\frac{Kg}{m^3} \times 18\frac{\cancel{Km}}{\cancel{h}} \times \frac{\pi}{4}\ 78.74^2\ \cancel{in^2} \times \frac{1000\, \cancel{m}}{1\, \cancel{Km}} \times \frac{6.452\times10^{-4}\, m^2}{1\ \cancel{in^2}} \times \frac{1\cancel{h}}{3600 s}$$

$$\dot{m} = 15708.41\ Kg/s$$

The power is thus,

$$\dot{W} = 15708.41\ \frac{Kg}{s} \times 9.81\ \frac{m}{s^2} \times 160\ \cancel{ft} \times \frac{0.3048\, m}{1\, \cancel{ft}}$$

$$\dot{W} = 7515124.52\ \frac{\cancel{Kg}.\cancel{m^2}}{s^3} \times \frac{1\, J}{1\, \cancel{Nm}} \times \frac{1\, \cancel{N}}{1\ \cancel{Kg\frac{m}{s^2}}}$$

$$\dot{W} = 7515124.52\ \overbrace{J/s}^{W} = 7515.12\ KW$$

P5

$$Ar \begin{cases} P_1 = 140 \text{ KPa} \\ T_1 = 10°C \\ V_1 = 200 \, l \end{cases} \xrightarrow[\text{expansion}]{\text{polytropic}} \begin{cases} P_2 = 700 \text{ KPa} \\ T_2 = 180 °C \end{cases}$$

$$Q_{12} = ?$$

From the 1st law $\Delta U = U_2 - U_1 = Q_{12} - W_{12}$

For ideal gases, we need to calculate $\Delta U = \Delta U(\Delta T)$ and we can easily compute W_{12} from its definition as $\int_{V_1}^{V_2} P dV$. Using the following data for noble gas Argon:

$MW = 39.948$ Kg/Kgmol ; $C_v = 0.312$ KJ/Kg.K

The mass of Argon can be calculated from state 1:

$$n = \frac{m}{MW} = \frac{P_1 V_1}{R T_1} \therefore m = \frac{P_1 V_1}{R T_1} \times MW$$

$$m = \frac{140 \text{ KPa} \times 200 \, l}{8.314 \frac{KJ}{Kmol\,K} \times 283.15 K} \times 39.948 \frac{Kg}{Kmol} \times \frac{1000 \text{ Pa}}{1 \text{ KPa}} \times \frac{1 \text{ N/m}^2}{1 \text{ Pa}} \times \frac{1 \text{ KJ}}{1000 \text{ J}}$$

$\downarrow$ [Kg]

$$\times \frac{1 J}{1 N m} \times \frac{10^{-3} m^3}{1 \, l} = 0.4751 \text{ Kg}$$

Now, the volume at state 2:

$$P_2 V_2 = \frac{m}{MW} R T_2$$

$$V_2 = \frac{m}{MW}\frac{RT_2}{P_2} = \frac{0.4751\,\cancel{Kg}}{39.948\,\frac{\cancel{Kg}}{Kgmol}} \times 8.314\frac{\cancel{KJ}}{\cancel{Kmol}.\cancel{K}} \times 453.15\cancel{K} \times \frac{1}{700\,\cancel{KPa}} \times \frac{1\,\cancel{KPa}}{1000\,\cancel{Pa}} \times$$

$$\times \frac{1\,\cancel{Pa}}{1\,\cancel{N}/m^2} \times \frac{1000\,\cancel{J}}{1\,\cancel{KJ}} \times \frac{1\,\cancel{N}m}{1\,\cancel{J}} = 6.40\times10^{-2}\,m^3$$

Now, for polytropic processes,

$$P_i V_i^n = \text{constant} = C$$

$$\frac{P_1}{P_2} = \left(\frac{V_2}{V_1}\right)^n$$

$$\ln(P_1/P_2) = \ln(V_2/V_1)^n$$

$$n = \frac{\ln(P_1/P_2)}{\ln(V_2/V_1)} = 1.412$$

Thus, the work in polytropic processes can be described as

$$W_{12} = \int_{V_1}^{V_2} P\,dV = \int_{V_1}^{V_2} \frac{C}{V^n}\,dV = \frac{C}{1-n} V^{(1-n)}\Big|_{V_1}^{V_2}$$

$$W_{12} = \frac{C}{1-n}\left[V_2^{(1-n)} - V_1^{(1-n)}\right]$$

As $C = P_1 V_1^n = P_2 V_2^n$

$$W_{12} = \frac{P_2 V_2^n V_2^{1-n} - P_1 V_1^n V_1^{1-n}}{1-n} = \frac{P_2 V_2 - P_1 V_1}{1-n}$$

$$W_{12} = \frac{1}{1-1.412} \left[(700 \times 6.40 \times 10^{-2}) - (140 \times 2 \times 10^{-1}) \right] KPa \, m^3 \times$$

$$\times \frac{1000\,Pa}{1\,KPa} \times \frac{1\,N/m^2}{1\,Pa} \times \frac{1\,J}{1\,Nm} = -40776.70\,J$$

$$W_{12} = -40.78\,KJ$$

The work is negative ⇒ Ar was worked upon in compression.
From the 1st law,

$$U_2 - U_1 = Q_{12} - W_{12}$$

$$Q_{12} = U_2 - U_1 + W_{12}$$

Assuming that Ar behaves like an ideal gas, i.e., $u = u(T)$ (with $u = U/m$)

$$\frac{du}{dT} = C_v \therefore du = C_v dT \therefore \Delta u = \frac{\Delta U}{m} = C_v \Delta T$$

Thus, $Q_{12} = m C_v (T_2 - T_1) + W_{12}$

$$Q_{12} = 0.4751\,Kg \times 0.312 \frac{KJ}{Kg\,K} \times 170\,K + (-40.78\,KJ) = -15.58\,KJ$$

Heat was lost from the system although temperature increased. This is because the raise in internal energy was mainly due to work.

P6

$$\text{Air} \left\{ \begin{array}{l} T_1 = 450K \\ V_1 = 3\times10^{-3} m^3 \end{array} \right. \Rightarrow V_2 = 5\times10^{-3} m^3$$

The total change in energy, ΔE, can be split into

$$\Delta E = \underset{\substack{\text{kinetic}\\ \text{energy}}}{\Delta E_K} + \underset{\substack{\text{potential}\\ \text{energy}}}{\Delta E_p} + \underset{\substack{\text{internal}\\ \text{energy}}}{\Delta U} = Q - W$$

Assuming that in the expansion process, kinetics and potential energies do not change

$$\Delta E = \Delta U = Q - W$$

or in differential form

$$dU = dQ - dW$$

As the process is adiabatic, ie, $dQ = 0$

$$dU = -dW = -PdV$$

Since the gas is ideal, $dU = C_v dT = -PdV$ and $V = RT/P$, and applying the chain rule to $V = V(T,P)$

$$dV = \frac{\partial V}{\partial T} dT + \frac{\partial V}{\partial P} dP = \frac{R}{P} dT - \frac{RT}{P^2} dP$$

Thus replacing dV,

$$C_v dT = -PdV = -P\left[\frac{R}{P}dT - \frac{RT}{P^2}dP\right]$$

$$C_v dT = -RdT + \frac{RT}{P}dP$$

Since $C_p - C_v = R$, replacing C_v in the equation above

$$C_p dT = \frac{RT}{P}dP$$

$$\frac{dT}{T} = \frac{R}{C_p}\frac{dP}{P} = \frac{C_p - C_v}{C_p}\frac{dP}{P} = \frac{\gamma - 1}{\gamma}\frac{dP}{P} \qquad (\gamma = C_p/C_v)$$

Now integrating from state 1 to 2:

$$\ln\frac{T_2}{T_1} = \left(\frac{\gamma-1}{\gamma}\right)\ln\frac{P_2}{P_1}$$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \qquad (a)$$

(b) T_2 is obtained from (see lecture notes)

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} \quad \therefore \quad \frac{T_2}{450K} = \left(\frac{3\times10^{-3}\,m^3}{5\times10^{-3}\,m^3}\right)^{\left(\frac{5\,cal/mol.K}{3\,cal/mol.K}-1\right)}$$

$$T_2 = 320.12\,K$$

The work done during the process is

$$W = -\Delta U = -C_v \Delta T = -3 \frac{cal}{mol.K} (320.12 - 450) K$$

$$W = 389.64 \ cal/mol \times \frac{4.184 J}{1 cal}$$

$$W = 1630.25 \ J/mol \qquad (c)$$

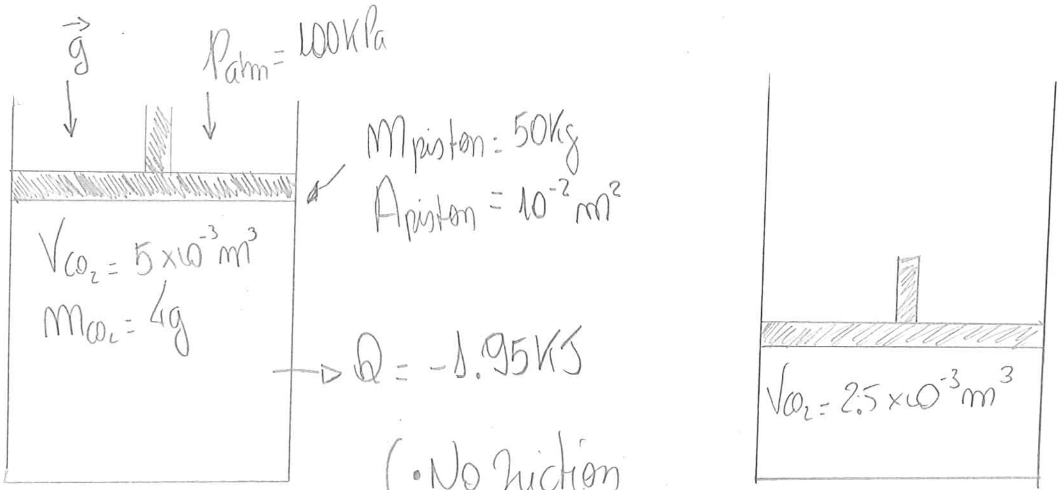
↳ $W>0$ indicates that in the expansion process, work is done by the system.

(d) The enthalpy change of the gas can be calculated as,

$$\Delta H = C_p \Delta T = 5 \frac{cal}{mol.K} \times (320.12 - 450) K \times \frac{4.184 J}{1 cal}$$

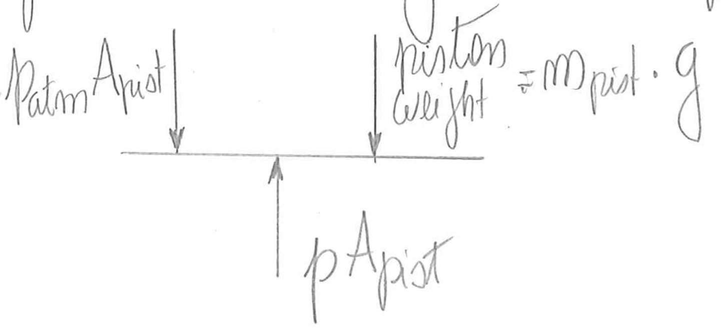
$$\Delta H = -2717.09 \ J/mol$$

P7



- No friction (piston x cylinder)
- $g = 9.81 m s^{-2}$

Let's assume that potential and kinetic energy are negligible. Also, as there is no friction and the piston <u>is NOT</u> accelerated, the force exerted by the CO_2 in the cylinder on the bottom of the piston is equal to the <u>weight</u> of the piston (+) the force exerted by the atmosphere on the top of the piston:



The force balance at the surface of the piston is:

$$p A_{pist} = p_{atm} A_{pist} + m_{pist} g \quad \times (1/A_{pist})$$

$$p = p_{atm} + \frac{m_{pist}}{A_{pist}} g = 100 KPa + \frac{50 Kg}{10^{-2} m^2} \times 9.81 \frac{m}{s^2} \times \frac{1 Pa}{1 N/m^2} \times \frac{1N}{1 Kg \frac{m}{s^2}} \times \frac{1 KPa}{1000 Pa}$$

$$p = 149.05 KPa \quad \underline{(a)}$$

(b) $V_1 = 5\times10^{-3}\,m^3$ $\quad\rightarrow\quad$ $V_2 = 2.5\times10^{-3}\,m^3$

$u_1 = 657\ KJ/Kg$ $\qquad$ $u_2 = ?$

The work can be calculated through (assuming constant pressure):

$$W_{12} = \int_{V_1}^{V_2} p\,dV = p(V_2 - V_1) = 149.05\ \cancel{KPa} \times (2.5-5)\times10^{-3}\,\cancel{m^3} \times$$

$$\frac{1000\ \cancel{Pa}}{1\ \cancel{KPa}} \times \frac{1\ \cancel{N/m^2}}{1\ \cancel{Pa}} \times \frac{1\,J}{1\ \cancel{N\,m}} = -372.63\ J$$

The energy balance for CO_2:

$$\Delta U = Q - W = -1.95\times10^3\,J - (-372.63)\,J$$

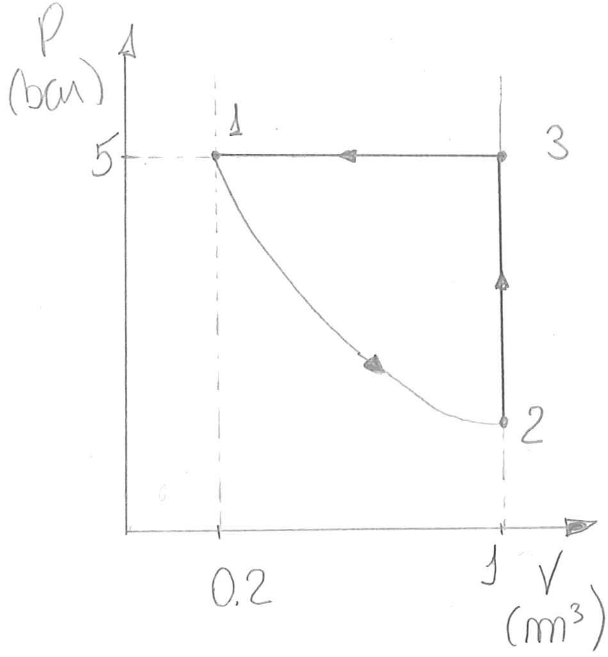
$$\Delta U = -1577.25\ J$$

Then with $\Delta U = m(u_2 - u_1)$ $\therefore$ $u_2 = \dfrac{\Delta u}{m} + u_1$

$$u_2 = \frac{-1577.25\ J}{4\times10^{-3}\,Kg} + 657\times10^3\,\frac{J}{Kg}$$

$$u_2 = 262687.5\ \frac{J}{Kg} = 262.69\ \frac{KJ}{Kg}$$

P8



(a) 1-2 : $PV = \text{constant} = K$

$p = K/V$

$$W_{12} = \int_{V_1}^{V_2} P\,dV = \int_{V_1}^{V_2} \frac{K}{V}\,dV =$$

$$= K \ln \frac{V_2}{V_1} = p_1 V_1 \ln \frac{V_2}{V_1}$$

$$= 5\ \text{bar} \times 0.2\ m^3 \times \ln \frac{1}{0.2}$$

$$W_{12} = 1.609\ \text{bar}.m^3 \times \frac{10^5\ N/m^2}{1\ \text{bar}} \times \frac{1\ J}{1\ Nm}$$

$$W_{12} = 1.61 \times 10^5\ J$$

(b) 2-3: The piston does not move (V constant). Therefore

$W_{23} = 0$

(c) 3-1 : $W_{31} = \int_{V_3}^{V_1} P\,dV$

As pressure is constant between 3-1 (i.e., $p_3 = p_1 = 5$ bar)

$$W_{31} = p_1 (V_1 - V_3) = 5\ \text{bar}\ (0.2 - 1)\ m^3 \times \frac{10^5\ N/m^2}{1\ \text{bar}} \times \frac{1\ J}{1\ Nm}$$

$$W_{31} = -4 \times 10^5\ J$$