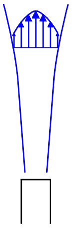
7. Turbulence

Turbulent Jet

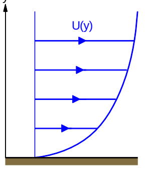
Instantaneous



Mean

Turbulent Boundary Layer

Instantaneous



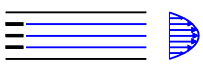
Mean

What is Turbulence?

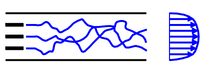
- A “random”, 3-d, time-dependent eddying motion with many scales, superposed on an often drastically simpler **mean** flow
- A solution of the Navier-Stokes equations
- The natural state at **high Reynolds numbers**
- An efficient transporter and **mixer**
- A major source of **energy loss**
- A significant influence on **drag**:
  - **increases frictional drag** in non-separated flow
  - delays, or sometimes prevents, boundary-layer separation on curved surfaces, so **reducing pressure drag**.
- “*The last great unsolved problem of classical physics*”

Flow Regimes

- **Laminar:**
  - smooth
  - **no mixing** of fluid
  - momentum transfer by **viscous forces**



- **Turbulent:**
  - chaotic
  - **mixing** of fluid
  - momentum transfer mainly by **net effect of intermingling**

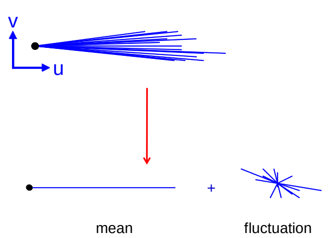


Regime determined by **Reynolds number**: $\mathrm{Re} = \frac{\rho U L}{\mu} = \frac{UL}{\nu}$

‘Low’ Re $\Rightarrow$ laminar; ‘high’ Re $\Rightarrow$ turbulent

‘High’ or ‘low’ depends on the flow and the choice of U and L

Reynolds’ Decomposition



$$u = \bar{u} + u'$$
$$v = \bar{v} + v'$$
$$p = \bar{p} + p'$$

Alternative Notations

- **Overbar** for mean, **prime** for fluctuation:
  $\bar{u} + u'$ $\qquad -\rho\overline{u'v'}$
- **Upper case** for mean, **lower case** for fluctuation:
  $U + u$ $\qquad -\rho\overline{uv}$

Reynolds Averaging

$$u = \bar{u} + u' \qquad \overline{u'} = 0$$

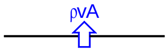
Variance: $\overline{u^2} = \bar{u}^2 + \overline{u'^2}$

Covariance: $\overline{uv} = \bar{u}\,\bar{v} + \overline{u'v'}$

Effect of Turbulence on the Mean Flow

(i) Mass

Mass flux: $\rho v A$

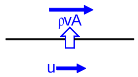


Average mass flux: $\rho \bar{v} A$

$\boxed{\text{The mean velocity satisfies the same continuity equation as the instantaneous velocity}}$

Effect of Turbulence on the Mean Flow

(ii) Momentum



(x-)momentum flux: $(\rho v A)u \quad = \rho(uv)A$

Average momentum flux : $\overline{(\rho v A)u} \quad = \rho(\overline{u}\,\overline{v} + \overline{u'v'})A$

extra term

Net rate of transport of momentum $\rho\overline{u'v'}A$ from LOWER to UPPER ...

... is equivalent to $-\rho\overline{u'v'}A$ from UPPER to LOWER ...

... has the same **dynamic** effect as a **stress** $-\rho\overline{u'v'}$

The **mean** velocity satisfies the same momentum equation as the **instantaneous** velocity, except for additional **apparent** stresses; e.g.

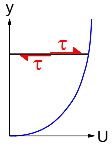
$-\rho\overline{u'v'}$ $\qquad$ $-\rho\overline{u'u'}$

These are called the **Reynolds stresses**

Total Stress

In simple shear:

$$\tau = \underbrace{\mu\frac{\partial \overline{u}}{\partial y}}_{\text{viscous stress}} \underbrace{- \rho\overline{u'v'}}_{\text{turbulent stress}}$$



τ may be regarded as EITHER:

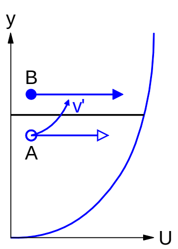
- the **force** per unit area exerted by upper on lower fluid

OR

- the **rate of transport of momentum** per unit area from upper to lower fluid

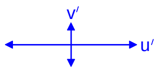
The **dynamic** effect (average rate of transfer of momentum) is the same

Turbulent Transport



Velocity Fluctuations

Normal stresses: $\overline{u'^2}, \overline{v'^2}, \overline{w'^2}$



Shear stresses: $\overline{v'w'}, \overline{w'u'}, \overline{u'v'}$

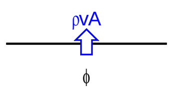
Turbulent kinetic energy: $k = \frac{1}{2}(\overline{u'^2} + \overline{v'^2} + \overline{w'^2})$

Turbulence intensity: $\dfrac{\textit{root-mean-square fluctuation}}{\textit{mean velocity}} = \dfrac{u'_{rms}}{U} = \dfrac{\sqrt{\frac{2}{3}k}}{U}$

Effect of Turbulence on the Mean Flow

(iii) General Scalar

Scalar flux: $(\rho v A)\phi = \rho(v\phi)A$



Average scalar flux: $\overline{(\rho v A)\phi} = \rho(\bar{v}\,\bar{\phi} + \overline{v'\phi'})A$

extra term

The **mean** concentration satisfies the same transport equation as the **instantaneous** concentration, except for the addition of **turbulent fluxes**

$\rho\overline{v'\phi'}$

The Closure Problem

- To close the **mean** flow equations one must specify the **Reynolds stresses**

  $-\rho\overline{u'^2}, -\rho\overline{u'v'}, \ldots$

  and other turbulent fluxes

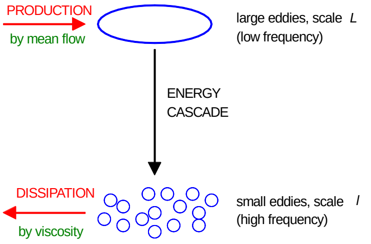
- A means of doing so is called a **turbulence model** or **turbulence closure**

- The simplest model assumes that the turbulent stress can be modelled similarly to the viscous stress. In simple shear:

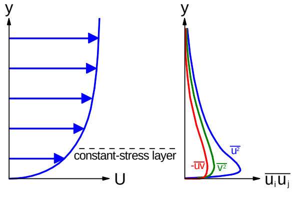
  $$-\rho\overline{u'v'} = \mu_t \frac{\partial U}{\partial y}$$

  This is called an **eddy-viscosity model**

Turbulence Energy



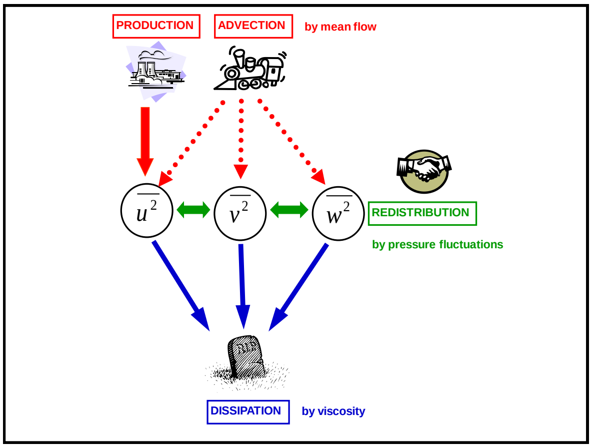
Anisotropy



$$\overline{u'^2} \neq \overline{v'^2} \neq \overline{w'^2}$$

Turbulence Transport

- Turbulence is transported by the flow
- Each individual Reynolds stress satisfies its own scalar-transport equation
- The "source" term is a balance between:
  - **production** by mean-velocity gradients
  - **dissipation** by viscosity
  - **redistribution** by pressure fluctuations
- Turbulence **anisotropy** is a result of:
  - anisotropic production by particular velocity gradients
  - selective damping of wall-normal fluctuations



Production of Turbulence Energy

$$P_{11} = -2\left(\overline{uu}\frac{\partial U}{\partial x} + \overline{uv}\frac{\partial U}{\partial y} + \overline{uw}\frac{\partial U}{\partial z}\right)$$

$$P_{12} = -\left(\overline{uu}\frac{\partial V}{\partial x} + \overline{uv}\frac{\partial V}{\partial y} + \overline{uw}\frac{\partial V}{\partial z}\right) - \left(\overline{vu}\frac{\partial U}{\partial x} + \overline{vv}\frac{\partial U}{\partial y} + \overline{vw}\frac{\partial U}{\partial z}\right)$$

$$P_{ij} = -\left(\overline{u_i u_k}\frac{\partial U_j}{\partial x_k} + \overline{u_j u_k}\frac{\partial U_i}{\partial x_k}\right)$$

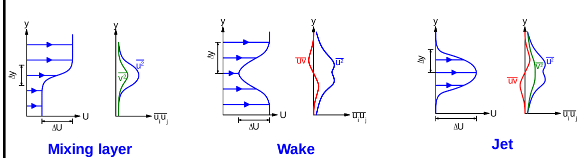
$$P^{(k)} = \tfrac{1}{2}(P_{11} + P_{22} + P_{33})$$

In **simple shear**: $P_{11} = -2\overline{uv}\frac{\partial U}{\partial y}, \quad P_{22} = P_{33} = 0$

$P^{(k)} = -\overline{uv}\frac{\partial U}{\partial y}$

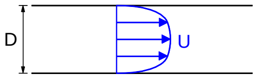
Energy is preferentially supplied to the streamwise normal stress $\overline{uu}$

Free Shear Flows

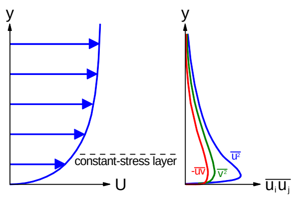


- Maximum turbulence where $|\partial U/\partial y|$ largest
- $\overline{uv}$ has the opposite sign to $\partial U/\partial y$ and vanishes where this is zero
- Turbulence is anisotropic: $\overline{u^2} > \overline{v^2}$

Wall-Bounded Flows



Pipe flow

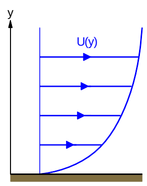


Flat-plate boundary layer

Parameters in Wall-Bounded Flows

Wall shear stress: τ_w

Friction velocity: $u_\tau = \sqrt{\tau_w/\rho}$



Non-dimensional height: $y^+ = \dfrac{y u_\tau}{\nu}$

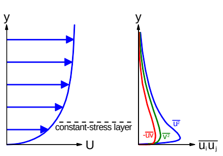
Non-dimensional velocity: $U^+ = \dfrac{U}{u_\tau}$

$\Big\}$ **wall units**

Key Regions of a Turbulent Boundary Layer

Viscous sublayer ($y^+ < 5$):
- viscous stress dominant

$$\tau_w = \mu \frac{\partial U}{\partial y} \quad \Rightarrow \quad U = \frac{\tau_w}{\mu} y$$



Log layer ($30 < y^+ < 500$):
- direct effect of viscosity negligible

$$\frac{\partial U}{\partial y} \propto \frac{u_\tau}{y} \quad \Rightarrow \quad U = u_\tau\left(\frac{1}{\kappa}\ln\frac{y u_\tau}{\nu} + B\right)$$

In **wall units**:

$$U^+ = \frac{U}{u_\tau}, \qquad y^+ = \frac{y u_\tau}{\nu}$$

$$\boxed{\begin{array}{ll} \text{(viscous sublayer)} & U^+ = y^+ \\ \text{(log layer)} & U^+ = \dfrac{1}{\kappa}\ln y^+ + B \end{array}}$$

Summary (1)

- Turbulence is a 3-d, time-dependent eddying motion with many scales, causing continuous mixing
- Any instantaneous flow variable can be decomposed as **mean** + **fluctuation**
- The process of averaging turbulent variables is called **Reynolds averaging** and leads to the **Reynolds-averaged Navier-Stokes (RANS) equations**
- The **product** of turbulent fluctuations makes a net contribution to the transport of momentum and other quantities via **Reynolds stresses** and **turbulent fluxes**

Summary (2)

- A **turbulence model** is a means of specifying turbulent fluxes in order to close the mean-flow equations
- Turbulent energy is:
  - **generated** (anisotropically, at large scale) by mean-velocity gradients
  - **redistributed** amongst components by pressure fluctuations
  - **dissipated** (at small scale) by viscosity
- Turbulence modelling is guided by theory and experiments for fundamental **free** and **wall-bounded** flows: in particular, by the **log law**