Q.1 **Question 1**

In an air-standard Otto cycle, the isentropic compression and expansion strokes are replaced by polytropic processes – $PV^{\gamma} = \text{constant}$, with $\gamma = 1.3$,

- **1–2:** Polytropic compression;
- **2–3:** Addition of heat at constant volume;
- **3–4:** Polytropic expansion and;
- **4–1:** Rejection of heat at constant volume.

The compression ratio $\left(\frac{V_1}{V_2} = \frac{V_4}{V_3}\right)$ is 9 for the modified cycle. At the beginning of compression, $P_1 = 1.0$ bar and $T_1 = 300K$. The maximum temperature during the cycle is $2000K$.

1. In Table 2, determine *(a)-(g)*. [7 marks]

	P **(bar)**	T **(K)**	$\mathbf{V}(m^3/kg)$	U (kJ/kg)
1	1	300	(a)	(b)
2	–	(c)	–	(d)
3	–	2000	–	(e)
4	–	(f)	–	(g)

Table 1: *Thermodynamic table of the modified air-standard Otto cycle.*

Solution:

The modified air-standard Otto cycle with the following initial conditions:

$$\gamma = 1.3, \quad \frac{V_1}{V_2} = \frac{V_4}{V_3} = 9, \quad P_1 = 1\ bar, \quad T_1 = 300\ K, \quad T_{max} = T_4 = 2000\ K$$

In order to fill Table 2, we need to calculate all relevant variables for each stage of the cycle (using the ideal-gas properties of air table):

[2/7] *(a)* **Stage 1:** *At* $T_1 = 300\ K$ *and* $P_1 = 1\ bar \Rightarrow U_1 = 214.07\frac{kJ}{kg}$. *From the ideal gas equation,*

$$V_1 = \frac{RT_1}{P_1} = 0.8610\frac{m^3}{kg}$$

[2/7] *(b)* **Stage 2:** $T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} = 300\,(9)^{0.3} = 580K \Rightarrow U_2 = 419.55\frac{kJ}{kg}$

[1/7] *(c)* **Stage 3:** $T_3 = 2000\ K \Rightarrow U_3 = 1678.7\frac{kJ}{kg}$;

[2/7] *(d)* **Stage 4:** $T_4 = T_3\left(\frac{V_3}{V_4}\right)^{\gamma-1} = 1034.57\ K \Rightarrow U_4 = 788.67\frac{kJ}{kg}$

	P (bar)	T (K)	$\mathbf{V}(m^3/kg)$	U (kJ/kg)
1	1	300	(a)0.8610	(b)214.07
2	–	(c)580	–	(d)419.55
3	–	2000	–	(e)1678.7
4	–	(f)1034.57	–	(g)788.67

Table 2: *Thermodynamic table of the modified air-standard Otto cycle.*

2. Calculate the mass-based heat (Q/m) and work (W/m) (both in kJ/kg) for each stroke in the cycle; [8 marks]

Solution:

For each stroke $i--j$:

[2/8] *(a)* **1–2:**

$$\frac{W_{12}}{m} = \int_1^2 PdV = \frac{R\left(T_2 - T_1\right)}{1-\gamma} = -267.85\frac{kJ}{kg}$$
$$\frac{Q_{12}}{m} = \left(U_2 - U_1\right) + \frac{W_{12}}{m} = -62.37\frac{kJ}{kg}$$

[2/8] *(b)* **2–3:**

$$\frac{W_{23}}{m} = 0$$
$$\frac{Q_{23}}{m} = \left(U_3 - U_2\right) + \frac{W_{23}}{m} = 1259.15\frac{kJ}{kg}$$

[2/8] *(c)* **3–4:**

$$\frac{W_{34}}{m} = \int_3^4 PdV = \frac{R\left(T_4 - T_3\right)}{1-\gamma} = 923.10\frac{kJ}{kg}$$
$$\frac{Q_{34}}{m} = \left(U_4 - U_3\right) + \frac{W_{34}}{m} = 33.07\frac{kJ}{kg}$$

[2/8] *(d)* **4–1:**

$$\frac{W_{41}}{m} = 0$$
$$\frac{Q_{41}}{m} = \left(U_4 - U_1\right) + \frac{W_{41}}{m} = -574.60\frac{kJ}{kg}$$

3. Calculate the thermal efficiency; [3 marks]

$$\eta = \frac{\left(W_{\text{cycle}}/m\right)}{\left(Q_{\text{in}}/m\right)}$$

Solution:

[3/3]

$$\frac{W_{cycle}}{m} = \frac{W_{12}}{m} + \frac{W_{34}}{m} = 655.25\frac{kJ}{kg}$$
$$\frac{Q_{in}}{m} = \frac{Q_{23}}{m} + \frac{Q_{34}}{m} = 1292.22\frac{kJ}{kg}$$
$$\eta = \frac{655.25}{1292.22} = 0.507$$

4. Calculate the mean effective pressure (in bar). [2 marks]

$$\text{MEP} = \frac{(W_{\text{cycle}}/m)}{V_1 - V_2}$$

Solution:

[2/2]

$$MEP = \frac{(W_{cycle}/m)}{V_1 - V_2} = \frac{(W_{cycle}/m)}{V_1\left(1 - \frac{V_2}{V_1}\right)} = 8.56\ bar$$

Also given:

- Isentropic relations for ideal gas:

$$TV^{\gamma-1} = \text{constant}$$
$$TP^{\frac{1-\gamma}{\gamma}} = \text{constant}$$
$$PV^{\gamma} = \text{constant}$$

- Molecular weight of air: $MW = 28.97\frac{kg}{kgmol}$;
- Polytropic relation for an ideal gas from state i to j:

$$\frac{W_{ij}}{m} = \int_i^j PdV = \frac{R\,(T_j - T_i)}{1-\gamma}$$

Q.2 Question 2

A vapour-compression refrigeration cycle is operated with Refrigerant R-134a as working fluid. Saturated vapour enters the compressor (with isentropic efficiency of 80%) at 2 bar, and saturated liquid exits the condenser at 8 bar. The mass flow rate of R-134a is 7 kg/min.

1. Sketch the schematic of the cycle indicating the numbering used in the calculation; [2 marks]

   Solution:

[2/2] *Figure. 1.*

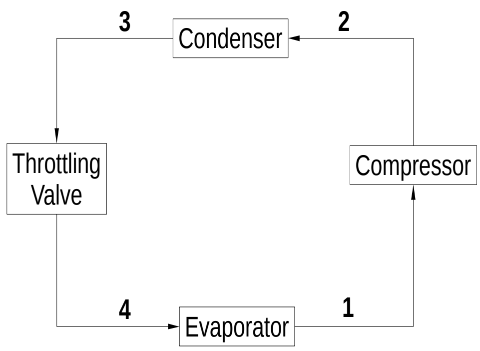

Figure 1: *Schematic of the vapour compression refrigeration cycle.*

2. Calculate all enthalpies in the cycle ($H_k\ \forall k \in \{1,2,3,4\}$); [8 marks]

   Solution:

- **Stage 1:** *Saturated vapour at* $P_1 = 2$ *bar* $\Rightarrow H_1 = 241.30\dfrac{kJ}{kg}$ *and* $S_1 = 0.9253\dfrac{kJ}{kg.K}$; [2/8]
- **Stage 2:** *Isentropic compression* ($S_{2s} = S_1$) *at* $P_2 = P_3 = 8$ *bar. At this pressure, the entropy of the saturated vapour is* $S_g = 0.9066\dfrac{kJ}{kg.K} << S_{2s}$, *therefore we can conclude that the fluid is at superheated state* $\Rightarrow$ *(via linear interpolation)* $T_{2s} = 36.59°C$ *and* $H_{2s} = 269.92\dfrac{kJ}{kg}$. *Now, in order to calculate* H_2, [2/8]

$$\eta_C = \frac{H_{2s} - H_1}{H_2 - H_1} \Rightarrow H_2 = 277.08\frac{kJ}{kg}$$

- [2/8] **State 3:** *Saturated liquid at* $P_3 = 8$ *bar* $\Rightarrow H_3 = 93.42\dfrac{kJ}{kg}$;
- [2/8] **State 4:** *Isenthalpic expansion:* $H_4 = H_3$.

3. Calculate the compressor power (W_{C}) in kW; [2 marks]
Solution:

[2/2]

$$W_C = \dot{m}_R \left(H_2 - H_1\right) = 4.17\ kW$$

4. Determine the refrigeration capacity (R_{n}) in *tons*; [2 marks]
Solution:

[2/2]

$$R_n = \dot{m}_R \left(H_1 - H_4\right) = 4.93\ tons$$

5. Calculate the coefficient of performance of the cycle (COP $= R_{\mathrm{n}}/W_{\mathrm{C}}$); [2 marks]
Solution:

[2/2]

$$COP = \frac{W_C}{R_n} = 4.13$$

6. Sketch the TS diagram, indicating all stages of the cycle. [4 marks]
Solution:

[4/4] *See Figure 2*

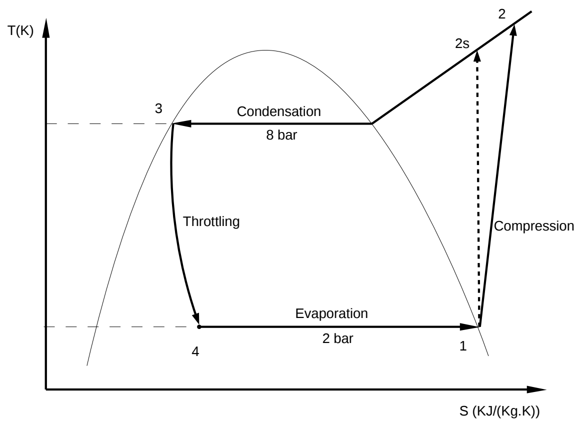

Figure 2: *TS diagram for the vapour compression refrigeration cycle.*

The efficiency of the compressor can be expressed as,

$$\eta_C = \frac{H_{ks} - H_{k-1}}{H_k - H_{k-1}}$$

where H_{ks} is the ideal enthalpy of the flow at stage k.

Q.3 Question 3

1. Saturated refrigerant R-134a vapour at $P_1 = 400\ kPa$ is compressed by a piston to $P_2 = 16$ bar in a reversible adiabatic process. Critical pressure and temperature of R-134a are 4.059 MPa and 101.06°C.

   (a) Calculate the work done by the piston; [6 marks]
   Solution:
   In order to calculate the work executed by the piston we need to calculate teh thermodynamic variables at states 1 *and* 2.

   i. **State 1:** *Saturated vapour at* $P_1 = 400\ kPa = 4\ bar \Rightarrow T_1 = T_{sat} = 8.93^oC$, $V_1 = V_g = 0.0509\frac{m^3}{kg}$, $H_1 = 252.32\frac{kJ}{kg}$, $S_1 = 0.9145\frac{kJ}{kg.K}$ *and*
[2/6] $U_1 = 231.97\frac{kJ}{kg}$.

   ii. **State 2:** *Adiabatic (i.e., isentropic) compression to* $P_2 = 16\ bar \Rightarrow S_2 = S_1 = 0.9145\frac{kJ}{kg.K}$. *At this pressure, the saturated vapour entropy is smaller than the prescribed entropy, i.e.,* $S_g = 0.8982\frac{kJ}{kg.K} << S_2$. *Therefore, the fluid in* 2 *is at superheated state, thus (via linear interpolation):* $T_2 = 61.96^oC << T_C$, $V_2 = 0.01254\frac{m^3}{kg}$, $H_2 = 280.77\frac{kJ}{kg}$ *and*
[2/6] $U_2 = 260.71\frac{kJ}{kg}$.
   Notice that $P_2 << P_C$ *and* $V_2 << V_1$.

[2/6] *Now, from the First Law:*

$$dU = dQ + dW \Rightarrow U_2 - U_1 = 0 + \Delta W \Rightarrow \Delta W = 28.74\frac{kJ}{kg}$$

   (b) Sketch the TS and PV diagrams including the constant pressure and temperature lines. [4 marks]
   Solution:
[4/4] *Figure 3.*

2. A reciprocating engines was designed to operate with the following stages:

   - 1–2: Isentropic compression;
   - 2–3: Addition of heat at constant volume;
   - 3–4: Addition of heat at constant pressure;
   - 4–5: Isentropic expansion and;
   - 5–1: Rejection of heat at constant volume.

   (a) Sketch the TS and PV diagrams for this cycle; [4 marks]
   Solution:
   This is a dual combustion cycle and the TS and PV diagrams are shown in
[4/4] *Fig. 4.*

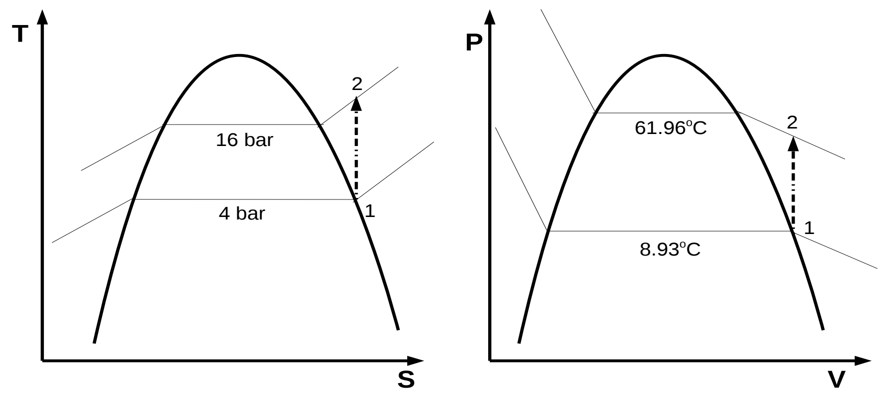

Figure 3: *TS and PV (rhs) diagrams.*

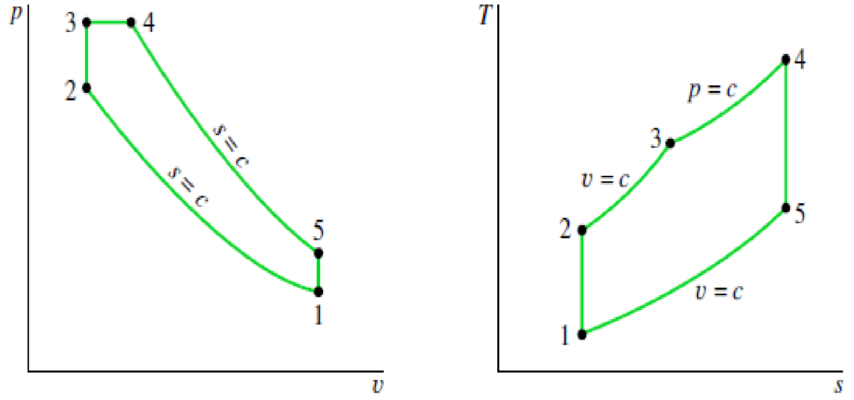

Figure 4: *Dual combustion cycle – PV and TS (rhs) diagrams.*

(b) For this cycle, define an expression for the net work (W_{net}) as a function of mass of air (m), heat capacities (C_p and C_v) and temperatures (T_j), [6 marks]

$$W_{\text{net}} = W_{\text{net}}\left(m, C_p, C_v, T_j\right) = \text{Heat Supplied} - \text{Heat Rejected}$$

Solution:

Thermal analysis, for m kg of air:

[2/6] i. *Total heat supplied (2–3 + 3–4):* $m\left[C_v\left(T_3 - T_2\right) + C_p\left(T_4 - T_3\right)\right]$

[2/6] ii. *Total heat rejected (5–1):* $mC_v\left(T_5 - T_1\right)$

[2/6] iii. *Net Work:* $W_{net} = m\left[C_v\left(T_3 - T_2\right) + C_p\left(T_4 - T_3\right) - C_v\left(T_5 - T_1\right)\right]$

Q.4 Question 4

(a) Energy conservation in a steady flow device implies

$$\dot{Q} - \dot{W}_s = \dot{m}\left(h_2 + \frac{u_2^2}{2} + gz_2\right) - \dot{m}\left(h_1 + \frac{u_1^2}{2} + gz_1\right),$$

where $\dot{Q}$ is the rate of heat addition, $\dot{W}_s$ is the rate of shaft work done by the fluid, h is the specific enthalpy and u is the fluid velocity. Briefly explain the physical significance of the fluxes making up the right-hand side of this equation. [3 marks]

Solution:

[3/3] *Enthalpy flux $\dot{m}h$, kinetic energy flux $\dot{m}u^2/2$ and potential energy flux $\dot{m}gz$.*

(b) An ideal gas flows through a steady flow device at a rate of 3 kg/s, and does work on a set of compressor blades at a rate of 28 kW. The gas has a temperature of 60°C when it enters the device through a circular inlet of diameter 50 cm (where the flow properties are denoted by a subscript 1) and exits the device at 50°C through a circular outlet of diameter 20 cm (where the flow properties are denoted 2). The inlet and outlet are at the same vertical height, while the pressure at the inlet is 100 kPa. Assuming no heat is added or removed from the device, then determine:

i) The inlet and outlet velocities; [9 marks]

Solution:

The density of the fluid at the inlet is

$$\rho_1 = \frac{p_1}{RT_1} = \frac{100000}{300 \times (60 + 273.15)} = 1.000550\, kg/m^3$$

[1/9]

The inlet and outlet areas are

$$A_1 = \frac{\pi d_1^2}{4} = \frac{\pi \times 0.5^2}{4} = 0.196350\, m^2,$$
$$A_2 = \frac{\pi d_2^2}{4} = \frac{\pi \times 0.2^2}{4} = 0.031416\, m^2.$$

[2/9]

The inlet velocity can now be obtained from the mass flux

$$u_1 = \frac{\dot{m}}{\rho_1 A_1} = \frac{3}{1.000550 \times 0.196350} = 15.270471\, m/s.$$

[1/9]

The outlet velocity can be obtained from the SFEE, which can be written in the form

$$u_2 = \sqrt{2\left[c_p\left(T_1 - T_2\right) + \frac{u_1^2}{2} - \frac{\dot{W}_s}{\dot{m}}\right]} = 39.57930\, m/s,$$

[5/9] *where we've used the fact the gas is an ideal gas and written $h = c_pT$.*

ii) The pressure at the outlet. [2 marks]

Solution:

The density at the outlet can be obtained from the mass flux

$$\rho_2 = \frac{\dot{m}}{u_2 A_2} = 2.412700\, kg/m^3.$$

[1/2]

The pressure at the outlet can subsequently be obtained from the ideal gas law

$$p_2 = \rho_2 R T_2 = 233899.2\, Pa.$$

[1/2]

You may assume the specific heat capacity of the gas $c_p = 1000\,\mathrm{J/(kg\ K)}$ and the specific gas constant $R = 300\,\mathrm{J/(kg\ K)}$.

(c) Downstream of the compressor, the gas flows isentropically into a difuser. Explain what is meant by an isentropic flow. [2 marks]

Solution:

[2/2] *A flow at constant entropy OR* $\mathrm{d}s = 0$.

(d) Isentropic flow along a diffuser satisfies the equation

$$\frac{1}{1-\mathrm{Ma}^2}\frac{1}{A}\frac{\mathrm{d}A}{\mathrm{d}x} = \frac{1}{\rho u^2}\frac{\mathrm{d}p}{\mathrm{d}x} = -\frac{1}{u}\frac{\mathrm{d}u}{\mathrm{d}x}.$$

With reference to this equation, explain how the pressure p, and flow velocity u vary if gas flows subsonically along the diffuser. [4 marks]

Solution:

[1/4] *In a diffuser* $\frac{\mathrm{d}A}{\mathrm{d}x} > 0$.

[1/4] *For subsonic flow* $\mathrm{Ma} < 1$ *and hence* $1 - \mathrm{Ma}^2 > 0$.

$A > 0$, $\rho > 0$ and $u > 0$, so $\frac{\mathrm{d}p}{\mathrm{d}x}$ has the same sign as $\frac{\mathrm{d}A}{\mathrm{d}x}$ and therefore the pressure
[1/4] *increases with flow along a subsonic diffuser.*

[1/4] *$\frac{\mathrm{d}u}{\mathrm{d}x}$ has the opposite sign to $\frac{A}{x}$ so flow velocities decrease with flow along the diffuser.*

Q.5 Question 5

(a) Air in an air-conditioning system is mixed adiabatically with air from outside in a steady process. If the inlets to the mixing chamber are labelled 1 and 2, and the outlet is labelled 3, then state equations corresponding to the mass conservation of dry air, the mass conservation of water vapour and the conservation of energy. Hence show that

$$\frac{\dot{m}_{a_2}}{\dot{m}_{a_1}} = \frac{h_3 - h_1}{h_2 - h_3} = \frac{\omega_3 - \omega_1}{\omega_2 - \omega_3}$$

where $\dot{m}$ is a mass flux, h is a specific enthalpy and ω is a specific humidity. [8 marks]
Solution:
In the mixing section

$$\begin{aligned} \textit{Conservation of dry air:} \qquad & \dot{m}_{a_1} + \dot{m}_{a_2} = \dot{m}_{a_3}, \\ \textit{Conservation of water vapour:} \qquad & \dot{m}_{a_1}\omega_1 + \dot{m}_{a_2}\omega_2 = \dot{m}_{a_3}\omega_3, \\ \textit{Conservation of energy:} \qquad & \dot{m}_{a_1}h_1 + \dot{m}_{a_2}h_2 = \dot{m}_{a_3}h_3. \end{aligned}$$

[3/8]

Using the dry air mass conservation equation to eliminate $\dot{m}_{a_3}$ from the other two expressions, gives

$$\begin{aligned} \dot{m}_{a_1}\omega_1 + \dot{m}_{a_2}\omega_2 &= \left(\dot{m}_{a_1} + \dot{m}_{a_2}\right)\omega_3, \\ \dot{m}_{a_1}h_1 + \dot{m}_{a_2}h_2 &= \left(\dot{m}_{a_1} + \dot{m}_{a_2}\right)h_3. \end{aligned}$$

[2/8]

Collecting all the terms involving $\dot{m}_{a_2}$ on the left-hand side and all the terms involving $\dot{m}_{a_3}$ on the right-hand side gives

$$\begin{aligned} \dot{m}_{a_2}\omega_2 - \dot{m}_{a_2}\omega_3 =& \dot{m}_{a_1}\omega_3 - \dot{m}_{a_1}\omega_1, \\ \dot{m}_{a_2}h_2 - \dot{m}_{a_1}h_3 =& \dot{m}_{a_2}h_3 - \dot{m}_{a_1}h_1. \end{aligned}$$

[1/8]

Rearranging

$$\begin{aligned} \dot{m}_{a_2}\left(\omega_2 - \omega_3\right) =& \dot{m}_{a_1}\left(\omega_3 - \omega_1\right), \\ \dot{m}_{a_2}\left(h_2 - h_3\right) =& \dot{m}_{a_2}\left(h_3 - h_1\right). \end{aligned}$$

[1/8]

Finally

$$\frac{\dot{m}_{a_2}}{\dot{m}_{a_1}} = \frac{\omega_3 - \omega_1}{\omega_2 - \omega_3}, \quad \textit{and} \quad \frac{\dot{m}_{a_2}}{\dot{m}_{a_2}} = \frac{h_3 - h_1}{h_2 - h_3},$$

[1/8] *which gives the necessary result.*

(b) Inlet 1 brings $25\,\mathrm{m}^3/\mathrm{min}$ of saturated air from the cooling section of the air conditioning system at a temperature of 7°C. Inlet 2 brings $40\,\mathrm{m}^3/\mathrm{min}$ of air from outside with a specific humidity of 0.01 kg water/kg dry air and a temperature of 39°C. Assuming the mixing is adiabatic and occurs at 1 atm, then determine:

i) The outlet mass flux of dry air; [4 marks]

Solution:

From the psychrometric chart the inlet gas streams (labelled 1 from the air-conditioning system and 2 from outside), have the properties

$$V_1 = 0.8\,m^3/kg \text{ dry air}, \quad \text{and} \quad V_2 = 0.9\,m^3/kg \text{ dry air}.$$

[2/4]

The inlet mass flow rates are

$$\dot{m}_{a_1} = \frac{\dot{V}_1}{V_1} = \frac{25}{0.8} = 31.25\,kg/min,$$
$$\dot{m}_{a_2} = \frac{\dot{V}_2}{V_2} = \frac{40}{0.9} = 44.44444\,kg/min.$$

[1/4]

Therefore mass conservation gives the outlet of mass flux of dry air to be

$$\dot{m}_{a_3} = \dot{m}_{a_1} + \dot{m}_{a_2} = 75.694444\,kg/min.$$

[1/4]

ii) The specifc humidity of the mixture; [5 marks]

Solution:

From the psychrometric chart

$$\omega_1 = 0.006\,kg \text{ water}/kg \text{ dry air}.$$

[1/5]

Now using the result from part (a),

$$\frac{\dot{m}_{a_2}}{\dot{m}_{a_1}} = \frac{\omega_3 - \omega_1}{\omega_2 - \omega_3}.$$

Rearranging

$$\left(1 + \frac{\dot{m}_{a_2}}{\dot{m}_{a_1}}\right)\omega_3 = \omega_1 + \omega_2\frac{\dot{m}_{a_2}}{\dot{m}_{a_1}}.$$

[2/5]

Now

$$\frac{\dot{m}_{a_2}}{\dot{m}_{a_1}} = \frac{44.44444}{31.25} = 1.42222222222,$$

which gives

$$\omega_3 = 0.008386\,kg \text{ water}/kg \text{ dry air}.$$

[2/5]

iii) The relative humidity and the dry-bulb temperature. [3 marks]

Solution:

The remain properties are determined by the intersection of straight line joining

[1/3] *1 and 2, and the* $\omega_3 = 0.008386\,kg\ water/kg\ dry\ air$*.*

This gives

$$\phi_3 = 35\%, \quad and \quad T_3 = 29^\circ C.$$

[2/3]

END OF PAPER