

# EG3029 Chemical Thermodynamics

## Vapour/Liquid Equilibrium of Mixtures

# General Remarks

## Introduction

- In most technical processes there are no pure substances but mixtures of various species
  - Chemical reactors
  - Separation units (distillation, absorption, extraction)
- Phase equilibrium necessary for quantitative treatment
- Here: only non-reacting systems will be considered

# General Remarks

## Introduction

- Equilibrium is a static condition: balance of all potentials
- Measures of composition

– Mass or mole fraction

$$x_i = \frac{m_i}{m} = \frac{\dot{m}_i}{\dot{m}}$$

$$x_i = \frac{n_i}{n} = \frac{\dot{n}_i}{\dot{n}}$$

– Molar concentration

$$C_i = \frac{x_i}{V}$$

– Molar concentration in flow processes

$$C_i = \frac{\dot{n}_i}{q}$$

– Molar mass of mixtures

$$M = \sum_i x_i M_i$$

# General Remarks

## Phase Rule and Duhem's Theorem

- Phase Rule for multi-phase systems at equilibrium

$$F = 2 - \pi + N$$

$F$  degrees of freedom  
 $\pi$  number of phases  
 $N$  number of chem. species

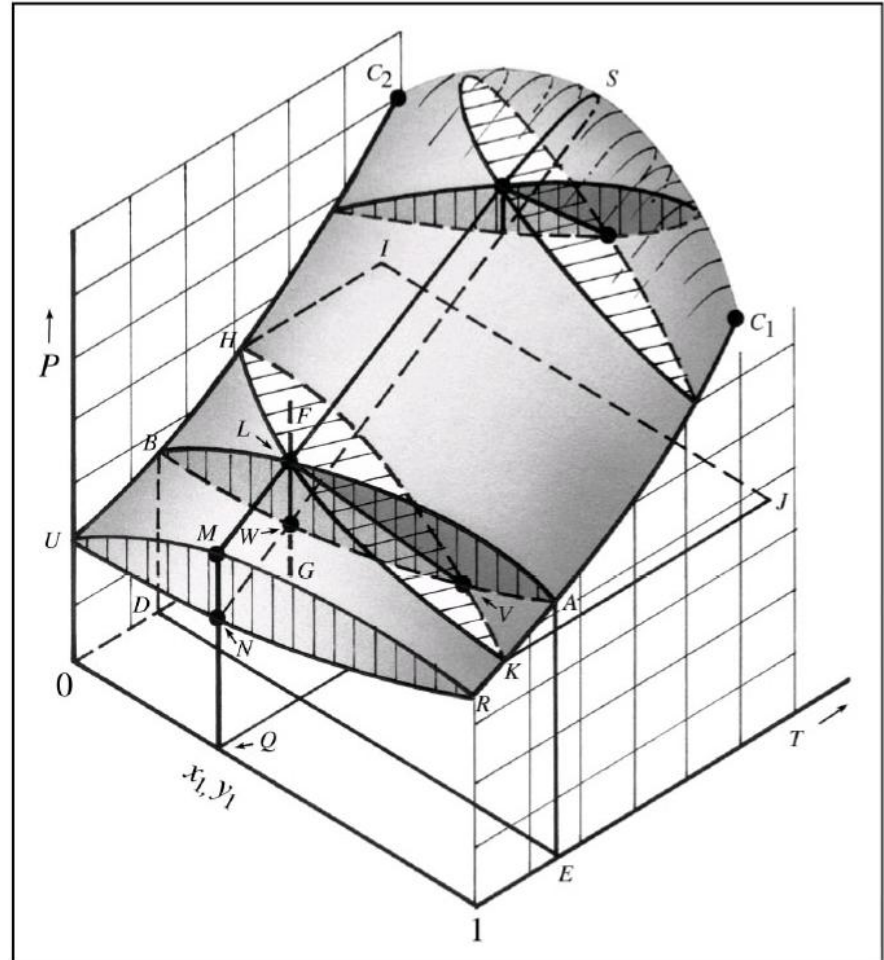
- Duhem's Theorem:

***For any closed system formed initially from given masses of prescribed chemical species, the equilibrium state is completely determined when any two independent variables are fixed.***

# VLE

## Qualitative Behaviour

- Systems comprised of 2 chemical species

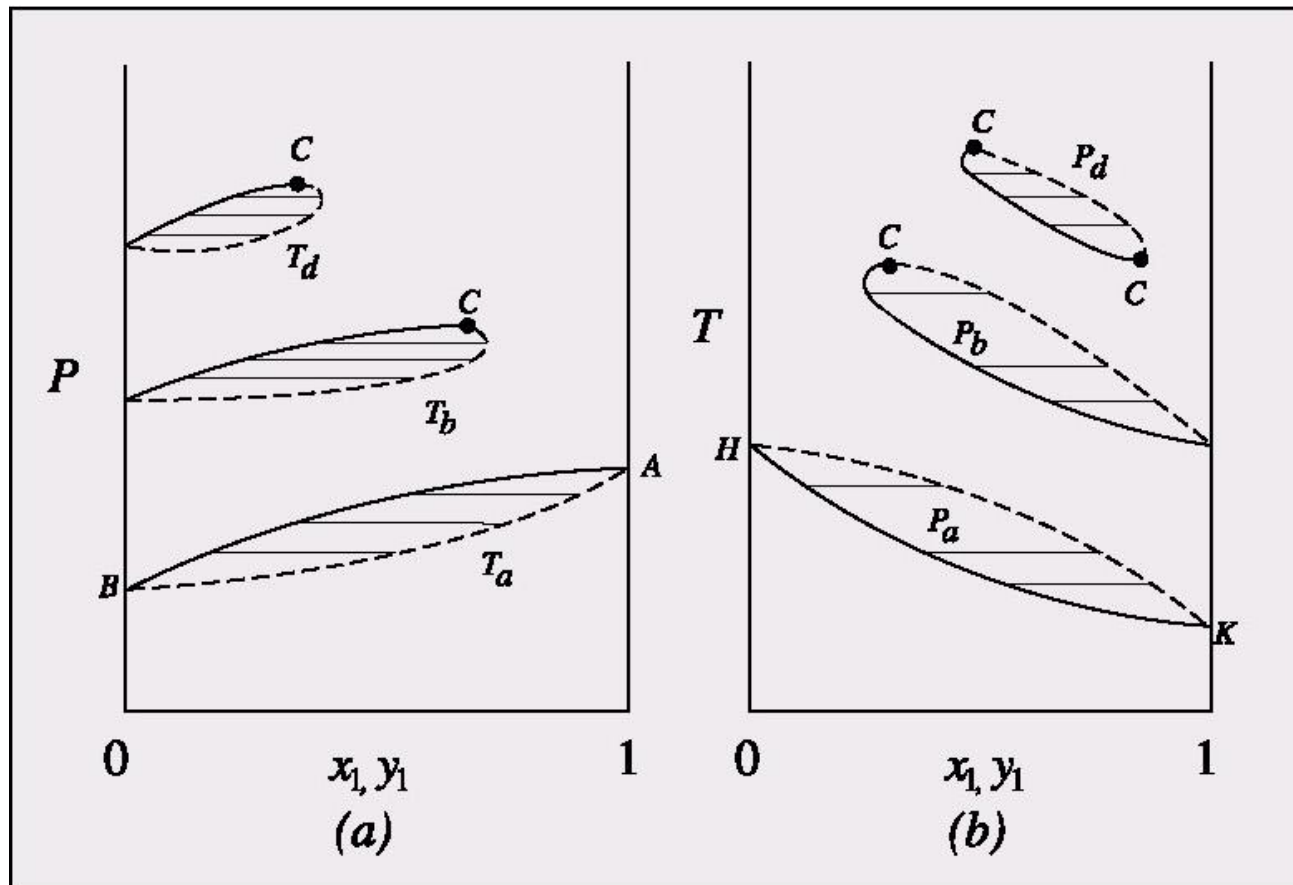


$P$   $T$   $xy$  diagram for VLE

# VLE

## Qualitative Behaviour

- $P_{xy}$  and  $T_{xy}$  diagrams



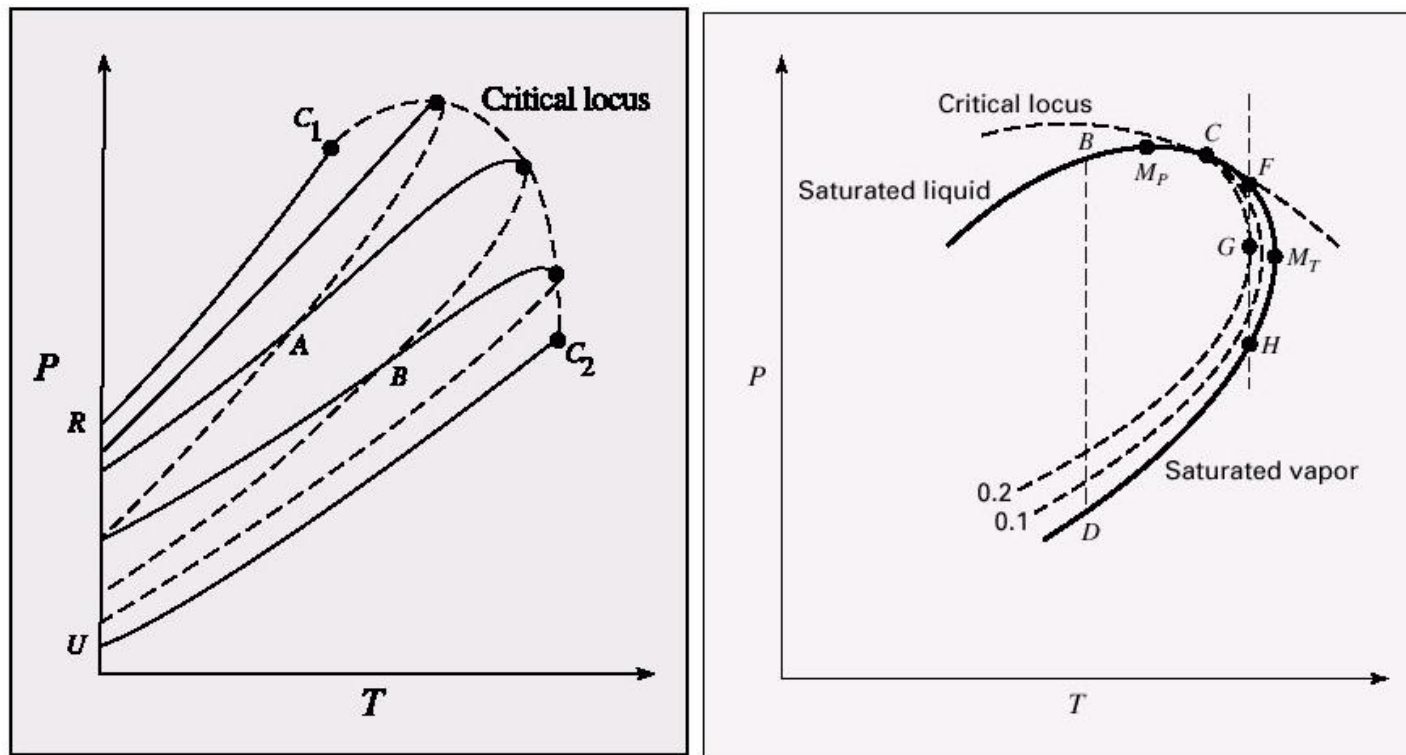
(a)  $P_{xy}$  diagram for three temperatures.  
 (b)  $T_{xy}$  diagram for three pressures.

Solid line: saturated liquid (bubble line);  
 dashed line: saturated vapour (dew line)

# VLE

## Qualitative Behaviour

- $P$   $T$  diagrams at fixed composition

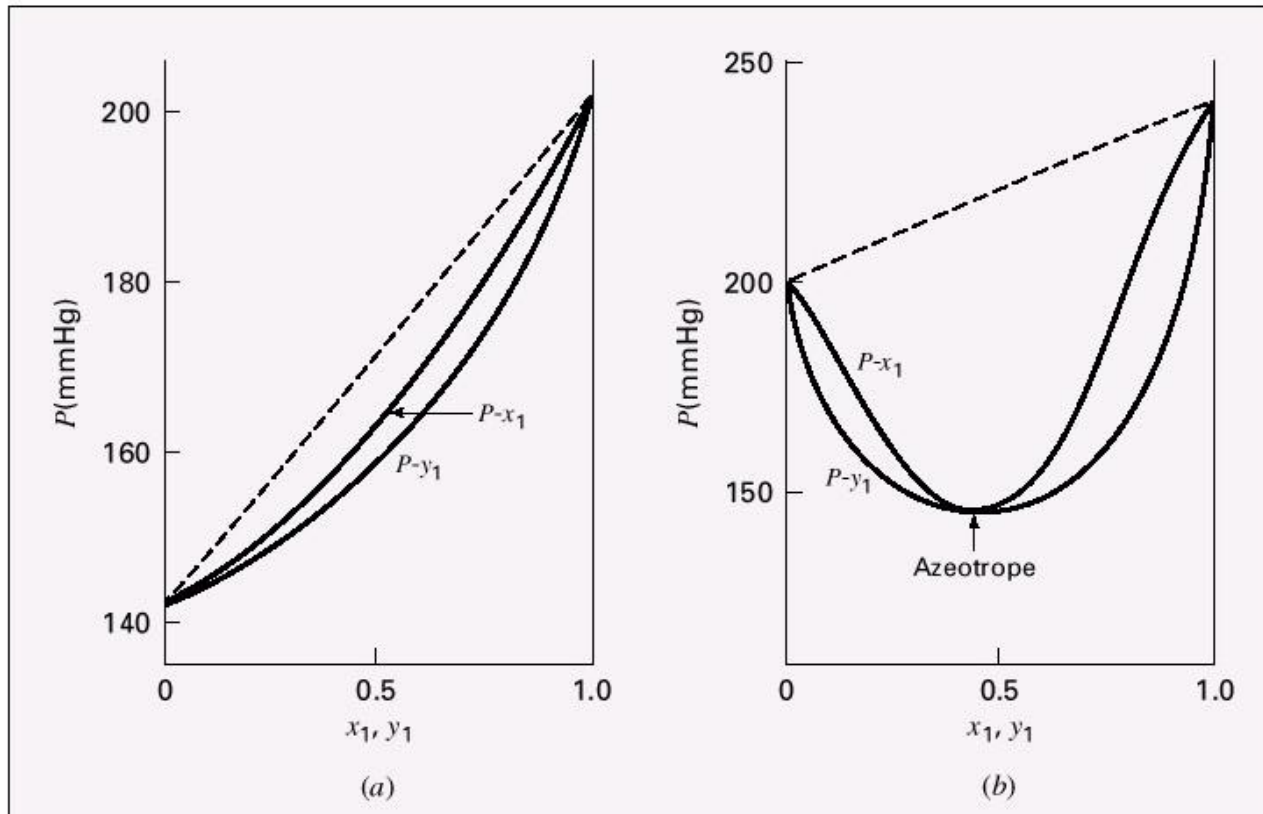


Solid line: saturated liquid (bubble line); dashed line: saturated vapour (dew line)

# VLE

## Qualitative Behaviour

- $P$   $xy$  diagrams at constant  $T$



(a) Tetrahydrofuran (1) and carbon tetrachloride (2) at  $30^\circ\text{C}$

(b) Chloroform (1) and tetrahydrofuran (2) at  $30^\circ\text{C}$

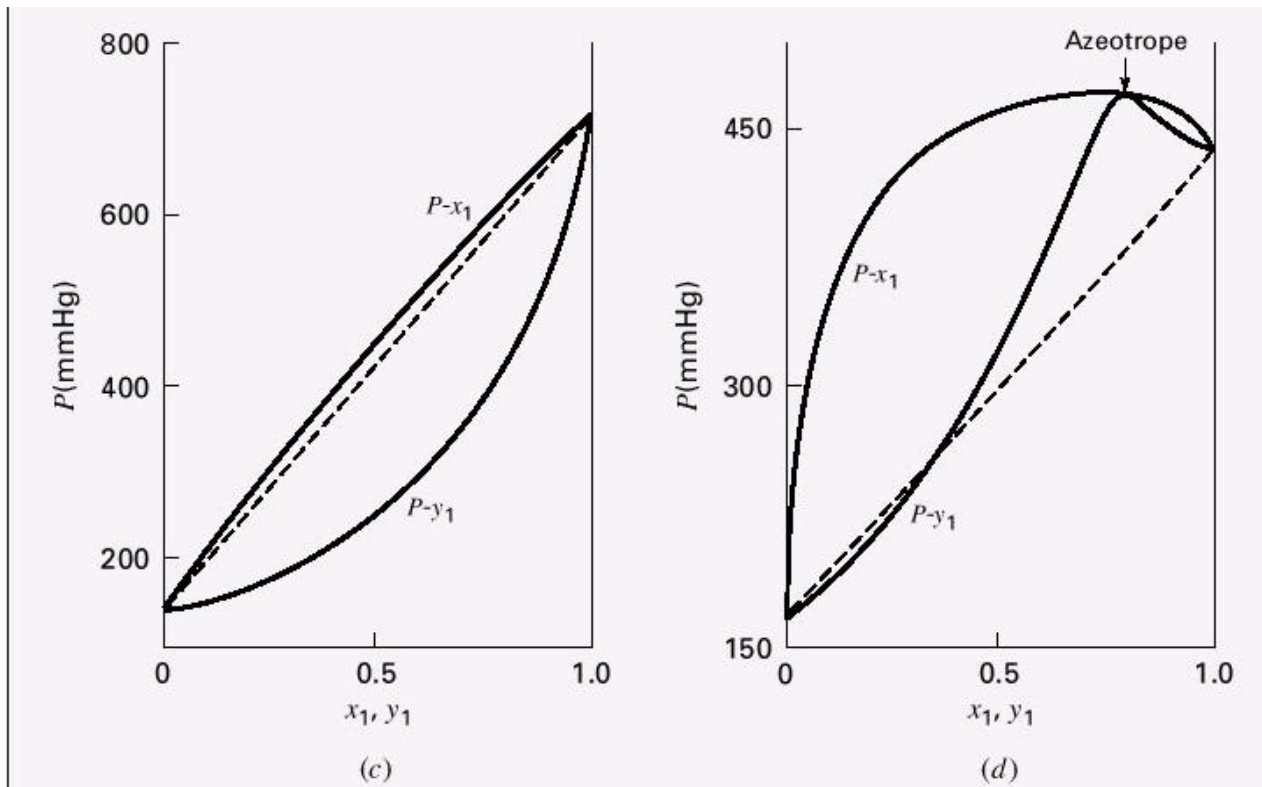
Dashed line:  $P$   $x$  relation for Raoult's law



# VLE

## Qualitative Behaviour

- $P$   $xy$  diagrams at constant  $T$



(c) Furan (1) and carbon tetrachloride (2) at 30°C

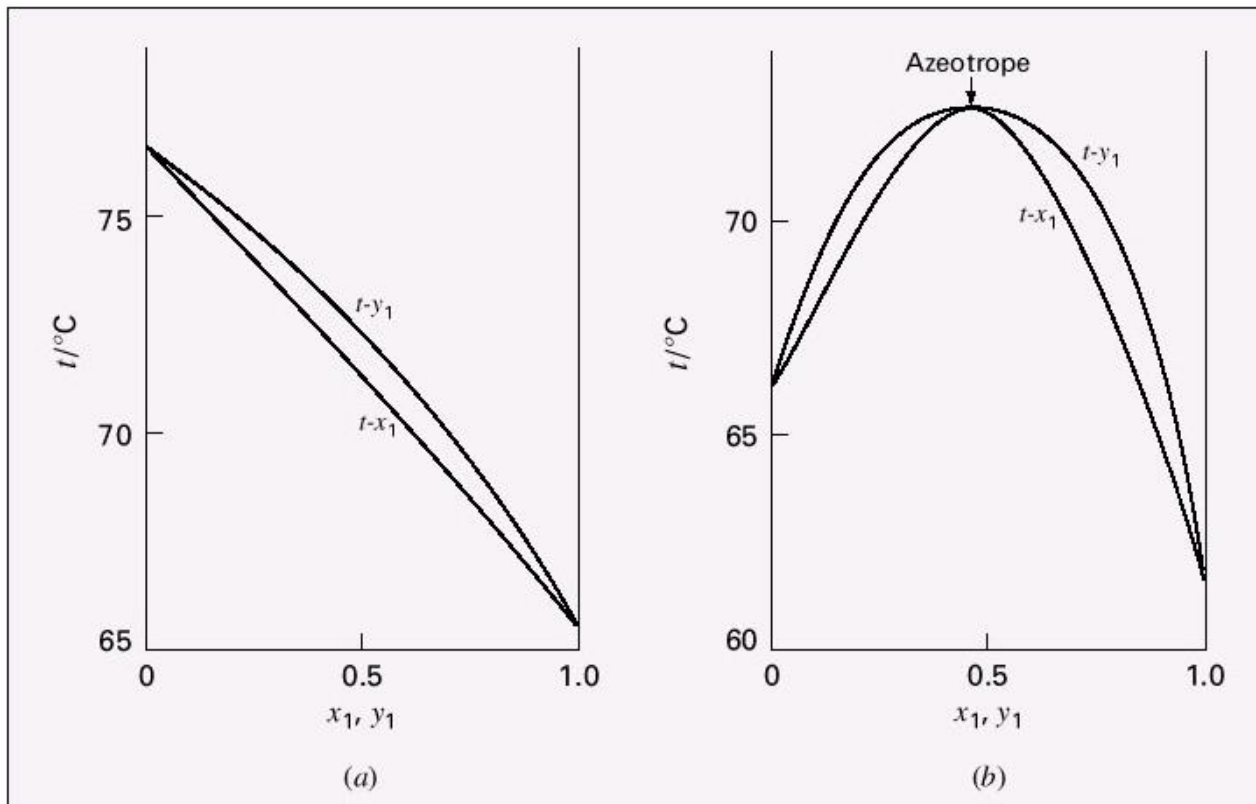
(d) Ethanol (1) and toluene (2) at 65°C

Dashed line:  $P$   $x$  relation for Raoult's law

# VLE

## Qualitative Behaviour

- $T_{xy}$  diagrams at constant  $P$

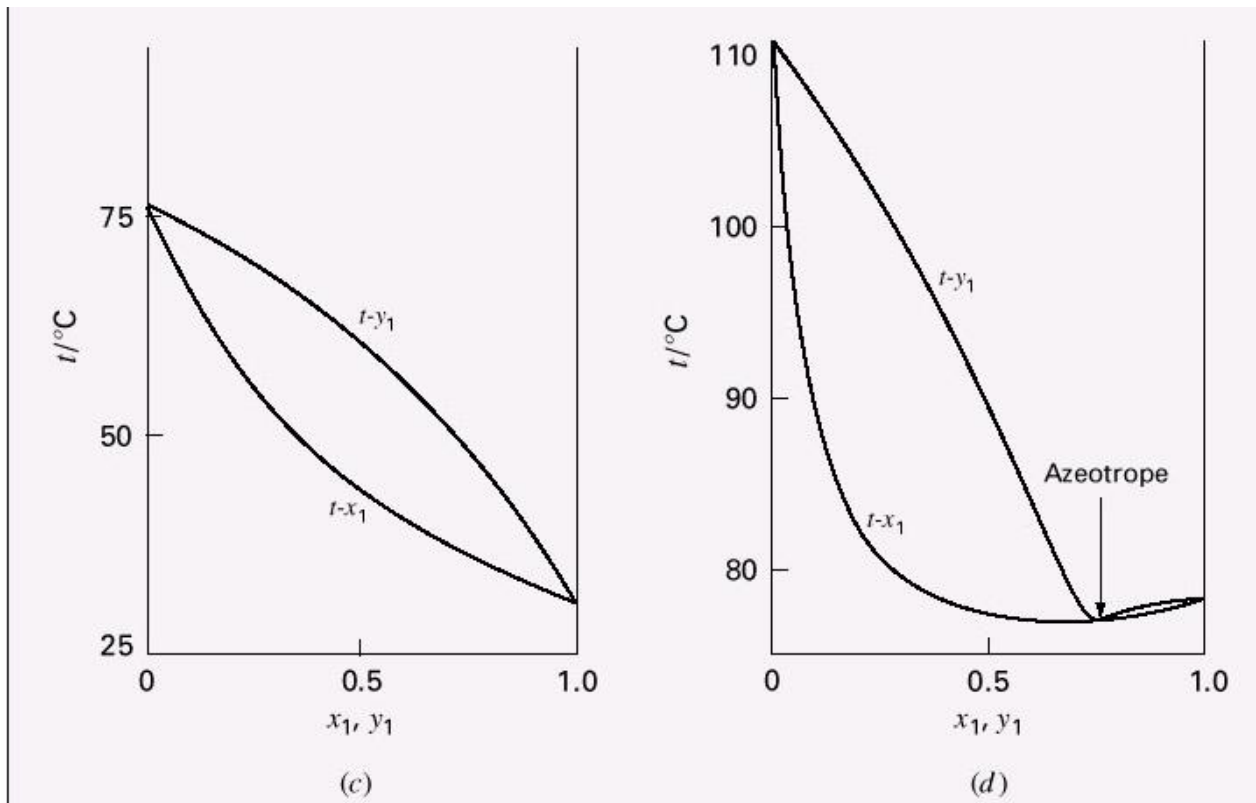


- (a) Tetrahydrofuran (1) and carbon tetrachloride (2) at 1 atm
- (b) Chloroform (1) and tetrahydrofuran (2) at 1 atm

# VLE

## Qualitative Behaviour

- $T_{xy}$  diagrams at constant  $P$

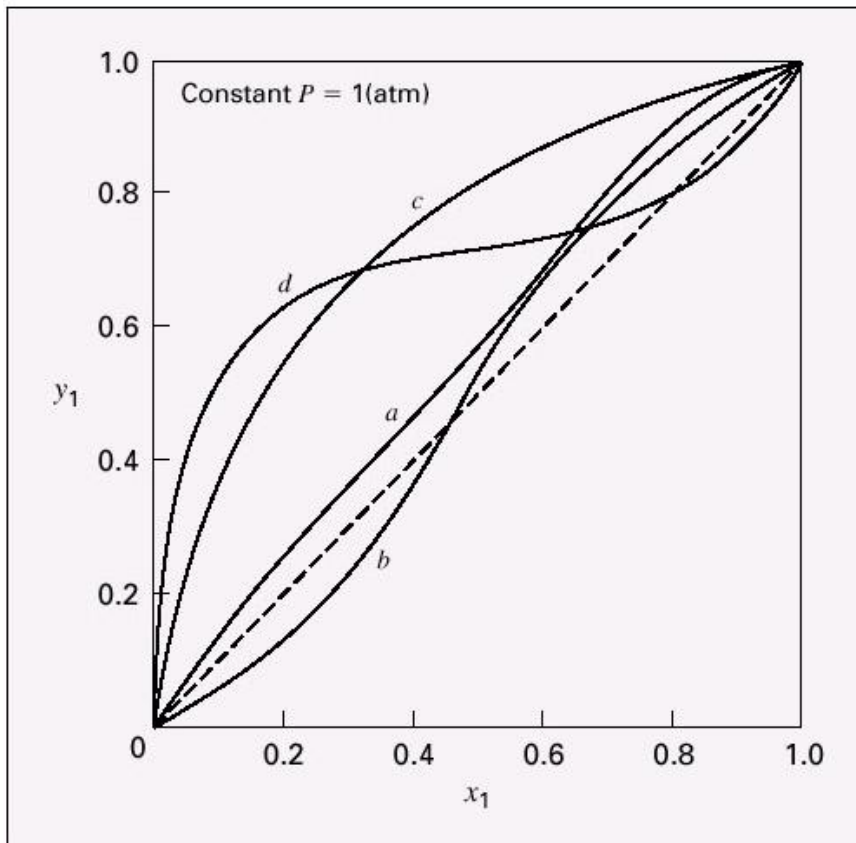


- (c) Furan (1) and carbon tetrachloride (2) at 1 atm
- (d) Ethanol (1) and toluene (2) at 1 atm

# VLE

## Qualitative Behaviour

- $y$  $x$  diagrams at constant  $P$



$y$  $x$  diagrams at 1 atm

(a) Tetrahydrofuran and carbon tetrachloride

(b) Chloroform and tetrahydrofuran

(c) Furan and carbon tetrachloride

(d) Ethanol and toluene

# VLE

## Simple Models: General

- Important goal: mathematical description of VLE
  - Prediction of vapour and liquid compositions at given temperature and pressure
  - Basis for process modelling
- Two simple models for VLE:
  - Raoult's law
  - Henry's law

# VLE

## Simple Models: Raoult's Law

- Two major assumptions:
  - Vapour phase is an ideal gas
  - Liquid phase is an ideal solution
- Mathematical expression:

$$y_i P = x_i P_i^{sat} \quad (i = 1, 2, \dots, N)$$

$x_i$  liquid phase mole fraction  
 $y_i$  gas phase mole fraction  
 $P_i^{sat}$  vapour pressure  
 $N$  number of chem. species

- Only valid for limited number of systems
- Valid for any species present at  $x \rightarrow 1$

# VLE

## Simple Models: Raoult's Law

- Ideal gas mixture properties:

- Dalton's law:

$$P = \sum_i P_i = \sum_i y_i P$$

- Amagat's law:

$$V^t = \sum_i V_i^t = \sum_i y_i V^t$$

- Kay's rule:

$$T'_c = \sum_i y_i T_{c,i}$$

$$P'_c = \sum_i y_i P_{c,i}$$

# VLE

## Simple Models: Raoult's Law

- Dew point and bubble point calculations:
  - BUBL P: calculate  $y_i$  and  $P$  for given  $x_i$  and  $T$
  - DEW P: calculate  $x_i$  and  $P$  for given  $y_i$  and  $T$
  - BUBL T: calculate  $y_i$  and  $T$  for given  $x_i$  and  $P$
  - DEW T: calculate  $x_i$  and  $T$  for given  $y_i$  and  $P$
- Worked example:

Prepare a graph showing  $P$  vs  $x_1$  and  $P$  vs  $y_1$  for  $T = 75$  degC using Raoult's law for the binary system acetonitrile(1)/nitromethane(2)



# VLE

## Simple Models: Modified Raoult's Law

- To account for deviations from ideal solution behaviour in liquid phase:

$$y_i P = x_i \gamma_i P_i^{sat} \quad (i = 1, 2, \dots, N)$$

$x_i$  liquid phase mole fraction

$y_i$  gas phase mole fraction

$P_i^{sat}$  vapour pressure

$N$  number of chem. Species

$\gamma_i$  activity coefficient

- Activity coefficients are typically a function of temperature and chemical composition, e.g.

$$\ln \gamma_i = A(T) \cdot x_j^2 \quad (i, j = 1, 2, \dots, N)$$

# VLE

## Simple Models: Henry's Law

- Raoult's law requires vapour pressure data
  - Not applicable if temperature is above the critical temperature of one species
  - No gas dissolution in liquid
- Henry's law:

$$y_i P = x_i \mathcal{H}_i \quad (i = 1, 2, \dots, N)$$

$\mathcal{H}_i$  Henry's constant

Gas	$\mathcal{H}/\text{bar}$	Gas	$\mathcal{H}/\text{bar}$
Acetylene	1,350	Helium	126,600
Air	72,950	Hydrogen	71,600
Carbon dioxide	1,670	Hydrogen sulfide	550
Carbon monoxide	54,600	Methane	41,850
Ethane	30,600	Nitrogen	87,650
Ethylene	11,550	Oxygen	44,380

Henry's constants for gases dissolved in water at 25°C

# VLE

## K-Value Correlations

- Equilibrium ratio  $K_i$

$$K_i = \frac{y_i}{x_i}$$

- With Raoult's law

$$K_i = \frac{P_i^{sat}}{P}$$

- Worked example:  
Determine dew point and bubble point pressures for a mixture of 10% methane, 20% ethane, and 70% propane at 50 degF.

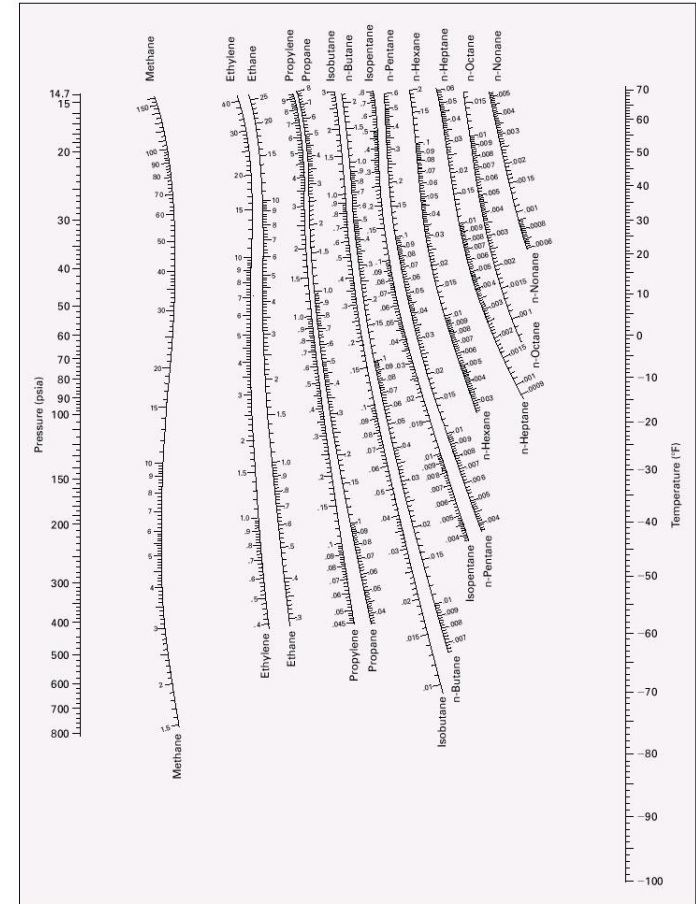


Figure 10.13:  $K$ -values for systems of light hydrocarbons. Low-temperature range. (Reproduced by permission from C. L. DePriester, *Chem. Eng. Progr. Symp. Ser. No. 7*, vol. 49, p. 41, 1953.)

# VLE

## Flash Calculations

- Flash: liquid at  $P \geq P_{\text{bubblepoint}}$   $\rightarrow$   $P$  reduction  
 $\rightarrow$  liquid flashes/partially evaporates to produce a 2-phase system in equilibrium
- Introduction of useful quantities:
  - Moles of liquid:  $\mathcal{L}$  with mole fractions  $\{x_i\}$
  - Moles of vapour:  $\mathcal{V}$  with mole fractions  $\{y_i\}$
  - Overall mole fraction:  $\{z_i\}$