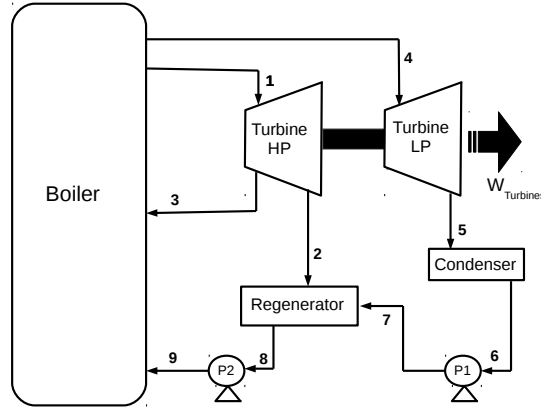


**Q.1 Question 1**

A steam power plant operates with coupled regenerative and reheat Rankine cycle with 2 connected turbines as shown in Fig. 1. Primary steam is supplied by the boiler at 120 bar and 565°C. Conditions for water/steam flows are described in Table 1.



**Figure 1:** Reheat and regenerative Rankine cycle with 2 turbines.

**Table 1:** Thermodynamic table of the reheat and regenerative Rankine cycle.

Stage	P (bar)	T (°C)	State	H (kJ.kg <sup>-1</sup> )	S (kJ.(kg.K) <sup>-1</sup> )	Steam Quality
1	120	565	(a)	(b)	(c)	—
2	3	—	wet vapour	(d)	—	(e)
3	3	250	—	—	—	—
4	3	475	—	—	—	—
5	0.06	—	wet vapour	(f)	—	(g)
6	—	—	sat.liquid	(h)	—	—
7	3	—	—	(i)	—	—
8	3	—	—	—	—	—
9	120	—	—	(j)	—	—

- (a) In Table 1, determine (a)-(j).

[10 marks]

**Solution:**

In order to fill the Table we need to calculate the thermodynamic properties for each stage of the cycle:

**Stage 1:** The fluid leaving the boiler towards the first turbine is at 120 bar and 565°C.

This is well above the saturation temperature ( $T_{sat} = 324.60^\circ\text{C}$ ) and we can thus confirm that the fluid is superheated steam. At such pressure, the superheated steam tables (SHST) results in (through linear interpolation):

[1/10]

$$H_1 = 3518.17 \frac{\text{kJ}}{\text{kg}} \text{ and}$$

[1/10]

$$S_1 = 6.6983 \frac{\text{kJ}}{\text{kg.K}}.$$

[1/10]

[1/10] **Stage 2:** *Isentropic expansion in HP Turbine at  $P_2 = 3$  bar  $\Leftrightarrow S_2 = S_1 = 6.6983 \frac{kJ}{kg.K}$ . The fluid is at wet vapour state. The quality of the steam is*

$$x_2 = \frac{S_2 - S_f}{S_g - S_f} = \frac{6.6983 - 1.6716}{6.9909 - 1.6716} = 0.9450$$

[1/10] *Now calculating the enthalpy,*

$$x_2 = \frac{H_2 - H_f}{H_g - H_f} = 0.9450 \Leftrightarrow H_2 = 2605.72 \frac{kJ}{kg}$$

**Stage 3:** *The fluid at  $P_3 = P_2 = 3.0$  bar and  $T_3 = 250^\circ C (>> T_{sat} = 133.5^\circ C)$  is superheated steam with  $H_3 = 2967.6 \frac{kJ}{kg}$  and  $S_3 = 7.517 \frac{kJ}{kg.K}$ .*

**Stage 4:** *The steam leaves the boiler towards the LP turbine at  $P_4 = P_3 = 3.0$  bar and  $T_4 = 475^\circ C$  (also as superheated steam) with (via linear interpolation)  $H_4 = 3433.33 \frac{kJ}{kg}$  and  $S_4 = 8.252 \frac{kJ}{kg}$ .*

[1/10] **Stage 5:** *Isentropic expansion with  $P_5 = 0.060$  bar (with  $S_5 = S_4$ ). The quality of the steam is*

$$x_5 = \frac{S_5 - S_f}{S_g - S_f} = \frac{8.252 - 0.521}{8.330 - 0.521} = 0.99$$

[1/10] *and the enthalpy,*

$$x_5 = 0.99 = \frac{H_5 - H_f}{H_g - H_f} = \frac{H_5 - 151.5}{2567.4 - 151.5} \Leftrightarrow H_5 = 2543.24 \frac{kJ}{kg}$$

[1/10] **Stage 6:** *The fluid leaves the condenser at  $P_6 = P_5 = 0.06$  bar is saturated liquid with  $H_6 = H_f(0.06 \text{ bar}) = 151.5 \frac{kJ}{kg}$*

[1/10] **Stage 7:** *Saturated and incompressible liquid leaving the pump towards the regenerator at  $P_7 = P_2 = 3.0$  bar,*

$$H_7 \approx H_6 + V_6 (P_7 - P_6)$$

$$\begin{aligned} &\approx 151.5 \frac{kJ}{kg} + 0.001006 \frac{m^3}{kg} (3 - 0.06) \text{ bar} \times \frac{10^5 \text{ kg}/(m.s^2)}{1 \text{ bar}} \times \frac{10^{-3} \frac{kJ}{kg}}{m^2/s^2} \times \frac{1}{0.61} \\ &\approx 151.795 \frac{kJ}{kg} \end{aligned}$$

**Stage 8:** *Saturated liquid water leaving the regenerator at  $P_8 = 3.0$  bar with  $H_8 = H_f(3.0 \text{ bar}) = 561.4 \frac{kJ}{kg}$  and  $V_8 = 0.001068 \frac{m^3}{kg}$ .*

[1/10] **Stage 9:** *Saturated and incompressible liquid at  $P_9 = 120$  bar,*

$$H_9 \approx H_8 + V_8 (P_9 - P_8)$$

$$\begin{aligned} &\approx 561.4 \frac{kJ}{kg} + 0.001068 \frac{m^3}{kg} (120 - 3) \text{ bar} \times \frac{10^5 \text{ kg}/(m.s^2)}{1 \text{ bar}} \times \frac{10^{-3} \frac{kJ}{kg}}{m^2/s^2} \times \frac{1}{0.61} \\ &\approx 573.90 \frac{kJ}{kg} \end{aligned}$$

Thus the Table becomes:

Stage	P (bar)	T (°C)	State	H (kJ.kg <sup>-1</sup> )	S (kJ.(kg.K) <sup>-1</sup> )	Steam Quality
1	120	565	Superheated vapour	3518.17	6.6983	–
2	3	–	wet vapour	2605.72	–	0.9450
3	3	250	–	–	–	–
4	3	475	–	–	–	–
5	0.06	–	wet vapour	2543.24	–	0.99
6	–	–	sat.liquid	151.5	–	–
7	3	–	–	151.8	–	–
8	3	–	–	–	–	–
9	120	–	–	573.90	–	–

- (b) Calculate the fraction (as %) of steam supplied to the low-pressure (LP) turbine.

[2 marks]

**Solution:**

Energy balance in the regenerator, assuming total mass of water of ( $m_T =$ ) 1 kg, and that a fraction,  $\mathcal{F}$ , is bled-off from the HP turbine to the regenerator, and the remaining water-steam,  $1 - \mathcal{F}$  is conducted back to the boiler.

$$m_T H_8 = \mathcal{F} H_2 + (1 - \mathcal{F}) H_7 \Rightarrow \mathcal{F} = 0.1669 \text{ kg}$$

[2/2]

Thus 83.3% (i.e.,  $1 - \mathcal{F}$ ) of the steam was supplied to the LP turbine.

- (c) Determine the heat supplied by the boiler.

[2 marks]

**Solution:**

[2/2]

The heat supplied by the boiler ( $Q_{\text{Boiler}}$ ) can be calculated through the energy balance,

$$Q_{\text{Boiler}} = [m_T H_1 + (1 - \mathcal{F}) H_4] - [(1 - \mathcal{F}) H_3 + m_T H_9] \Rightarrow Q_{\text{Boiler}} = 3332.27 \frac{\text{kJ}}{\text{kg}}$$

- (d) Determine the thermal efficiency of the cycle,

[6 marks]

$$\eta = \frac{W_{\text{Total}}}{Q_{\text{Boiler}}} = \frac{\sum W_{\text{Turbines}} - \sum W_{\text{Pumps}}}{Q_{\text{Boiler}}}$$

**Solution:**

Now, in order to calculate the thermal efficiency of the cycle,

$$\eta = \frac{W_{\text{Total}}}{Q_{\text{Boiler}}} = \frac{\sum W_{\text{Turbines}} - \sum W_{\text{Pumps}}}{Q_{\text{Boiler}}}$$

We need to calculate the work associated with the turbines and pumps.

[1/6]

**HP Turbine:**  $W_{T,HP} = m_T H_1 - [\mathcal{F} H_2 + (1 - \mathcal{F}) H_3] = 610.97 \frac{\text{kJ}}{\text{kg}}$

[1/6]

**LP Turbine:**  $W_{T,LP} = (1 - \mathcal{F}) (H_4 - H_5) = 741.53 \frac{\text{kJ}}{\text{kg}}$

[1/6] **Pump 1:**  $W_{P,1} = (1 - \mathcal{F})(H_7 - H_6) = 0.246 \frac{kJ}{kg}$

[1/6] **Pump 2:**  $W_{P,2} = m_T(H_9 - H_8) = 12.4 \frac{kJ}{kg}$

[2/6] *Thus the thermal efficiency of the cycle is,*

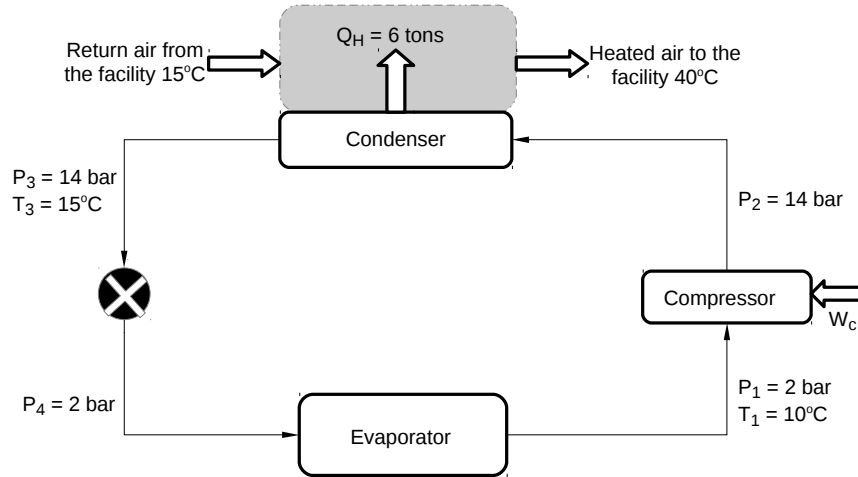
$$\eta = \frac{\sum W_{Turbines} - \sum W_{Pumps}}{Q_{Boiler}} = \frac{1339.76}{3332.27} = 0.4021$$

To solve this problem, you should assume that the saturated liquid streams are incompressible, and therefore  $dH = VdP$  (where  $H$ ,  $V$  and  $P$  are enthalpy, volume and pressure, respectively). Quality of the steam is expressed as

$$x_j = \frac{\Psi_j - \Psi_f}{\Psi_g - \Psi_f} \quad \text{with } \Psi = \{H, S\}$$

**Q.2 Question 2**

Refrigerant R-134a is used in a geothermal heat pump system (Fig. 2) to a storage in an industrial facility at 40°C. The heat pump uses underground water from a well to produce a heating capacity of 6 tons. Determine:



**Figure 2:** Heat pump cycle.

1. Enthalpies and Entropies:  $H_i$ ,  $S_i$  with  $i = \{1, 2, 3, 4\}$ ;

[8 marks]

**Solution:**

*Calculating all enthalpies and entropies of the cycle:*

**Stage 1:** The refrigerant fluid leaves the evaporator at  $P_1 = 2$  bar and  $T_1 = 10^\circ\text{C} \gg T_{\text{sat}} = -10.09^\circ\text{C}$ , thus the fluid is at superheated vapour state and with  $H_1 = 258.89 \frac{\text{kJ}}{\text{kg}}$  and  $S_1 = 0.9898 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$ .

[2/8]

**Stage 2:** Isentropic compression,  $S_2 = S_1$ , and assuming ideal compressor at  $P_2 = 14$  bar. Via linear interpolation,  $H_2 = 303.66 \frac{\text{kJ}}{\text{kg}}$  and  $T_2 = 77.08^\circ\text{C}$ .

[2/8]

**Stage 3:** The fluid leaves the condenser at  $P_3 = 14$  bar and  $T_3 = 15^\circ\text{C} (\ll T_{\text{sat}} = 52.4^\circ\text{C})$  is a sub-cooled liquid with  $H_3 = 125.26 \frac{\text{kJ}}{\text{kg}}$  and  $S_3 = 0.4453 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$ .

[2/8]

**Stage 4:** Isenthalpic process,  $H_4 = H_3$ , at  $P_4 = 2$  bar. Calculating the quality of the vapour,

[2/8]

$$x_4 = \frac{H_4 - H_f}{H_g - H_f} = \frac{125.26 - 36.84}{241.30 - 36.84} = 0.4325$$

and the entropy,

$$x_4 = \frac{S_4 - S_f}{S_g - S_f} = \frac{S_4 - 0.1481}{0.9253 - 0.1481} \Rightarrow S_4 = 0.4842 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

2. Volumetric flow rate of heated air to the room ( $\text{m}^3/\text{s}$ );

[3 marks]

**Solution:**

*In order to calculate the volumetric flow rate of heated air, we first need to determine the mass flow rate,*

[1/3]

$$Q_H = \dot{m}_{air} (H_{out}^{air} - H_{in}^{air}) = \dot{m}_{air} C_p^{air} (T_{out}^{air} - T_{in}^{air})$$

$$6 \text{ ton} \times \frac{210 \frac{kJ}{min}}{1 \text{ ton}} \times \frac{1 \text{ min}}{60s} = \dot{m}_{air} \times 1.004 \frac{kJ}{kg.K} (40 - 15)^{\circ} C$$

$$\dot{m}_{air} = 0.8367 \frac{kg}{s}$$

[2/3] Now for the volumetric flow rate (with  $T = 40^{\circ}C$  and  $P = 1.01325 \text{ bar}$ )

$$\dot{V}_{air}^{out} = \dot{m}_{air} V_{air}^{out} = \dot{m}_{air} \frac{RT_{air}^{out}}{P_{air}} \Rightarrow \dot{V}_{air}^{out} = 7.42 \times 10^{-4} \frac{m^3}{s}$$

3. Mass flow rate of the R-134a refrigerant fluid; [3 marks]

**Solution:**

[3/3] The mass flow rate of the refrigerant fluid R-134a can be calculated as,

$$Q_H = \dot{m}_R (H_2 - H_3) \Rightarrow \dot{m}_R = 0.1177 \frac{kg}{s}$$

4. Compressor power ( $W_C$ ) in kW; [3 marks]

**Solution:**

[3/3]

$$W_C = \dot{m}_R (H_2 - H_1) \Rightarrow W_C = 5.27 \text{ kW}$$

5. Coefficient of performance  $\left( \text{COP} = \frac{Q_H}{W_C} \right)$ ; [3 marks]

**Solution:**

[3/3]

$$\text{COP} = \frac{Q_H}{W_C} = 3.98$$

Given the heat capacity,  $C_p^{\text{air}} = 1.004 \text{ kJ} \cdot (\text{kg} \cdot \text{K})^{-1}$ , and molecular weight,  $MW^{\text{air}} = 28.97 \text{ kg} \cdot \text{kgmol}^{-1}$  of air. Assume that air behaves as an ideal gas. Quality of the vapour is expressed as

$$x_j = \frac{\Psi_j - \Psi_f}{\Psi_g - \Psi_f} \quad \text{with } \Psi = \{H, S\}$$

**Q.3 Question 3**

A steady flow energy device formed of a turbine with one inlet (labelled 1), and two outlets (labelled 2 and 3), does work on an ideal gas at a rate of 120 kW. The specific gas constant  $R = 287 \text{ J kg}^{-1} \text{ K}^{-1}$  and the specific heat capacity at constant pressure  $c_p = 1003 \text{ J kg}^{-1} \text{ K}^{-1}$ , while the known conditions at the inlet and each outlet are given in table 2.

**Table 2:** *Inlet and outlet conditions for the steady flow device*

Inlet/Outlet	Area $A \text{ (m}^2\text{)}$	Volume flux $q \text{ (m}^3 \text{ s}^{-1}\text{)}$	Temperature $T \text{ (}^\circ\text{C)}$	Pressure $p \text{ (Pa)}$	Height $z \text{ (m)}$
1	0.1	2.0	20	?	0.0
2	0.1	1.0	50	200000	10.0
3	0.05	1.0	90	100000	4.0

- (a) The steady flow energy equation for a steady flow device with one inlet and one outlet is

$$\frac{\dot{Q} - \dot{W}_s}{\dot{m}} = \left( c_p T_{\text{outlet}} + \frac{u_{\text{outlet}}^2}{2} + gz_{\text{outlet}} \right) - \left( c_p T_{\text{inlet}} + \frac{u_{\text{inlet}}^2}{2} + gz_{\text{inlet}} \right),$$

where  $u$  is the gas velocity,  $\dot{Q}$  is the rate of heat addition and  $\dot{W}_s$  is the rate at which shaft work is done on the gas. Explain how this equation should be changed to model the device described above. [4 marks]

**Solution:**

The mass flux entering the device  $\dot{m}_1$  is now divided between two outlets. Therefore the modified mass conservation equation would be

$$\dot{m}_1 = \dot{m}_2 + \dot{m}_3.$$

[1/4]

With two outlets, the flux of energy must also be split between the two outlets, which gives

$$\dot{Q} - \dot{W}_s = \dot{m}_2 \left( c_p T_2 + \frac{u_2^2}{2} + gz_2 \right) + \dot{m}_3 \left( c_p T_3 + \frac{u_3^2}{2} + gz_3 \right) - \dot{m}_1 \left( c_p T_1 + \frac{u_1^2}{2} + gz_1 \right).$$

[3/4]

- (b) Calculate the gas velocity at the inlet and each outlet;

[3 marks]

**Solution:**

The gas velocities are given by

$$\begin{aligned} u_1 &= \frac{q_1}{A_1} = \frac{2.0 \text{ m}^3 \text{ s}^{-1}}{0.1 \text{ m}^2} = 20 \text{ m s}^{-1}, \\ u_2 &= \frac{q_2}{A_2} = \frac{1.0 \text{ m}^3 \text{ s}^{-1}}{0.1 \text{ m}^2} = 10 \text{ m s}^{-1}, \\ u_3 &= \frac{q_3}{A_3} = \frac{1.0 \text{ m}^3 \text{ s}^{-1}}{0.05 \text{ m}^2} = 20 \text{ m s}^{-1}. \end{aligned}$$

[3/3]

- (c) Determine the pressure at the inlet;

[5 marks]

**Solution:***The densities at the outlets can be obtained via the ideal gas equation*

$$\rho_2 = \frac{p_2}{RT_2} = \frac{200000 \text{ Pa}}{287 \text{ J kg}^{-1} \text{ K}^{-1} (50 + 273.15) \text{ K}} = 2.1564726 \text{ kg m}^{-3},$$

$$\rho_3 = \frac{p_3}{RT_3} = \frac{100000 \text{ Pa}}{287 \text{ J kg}^{-1} \text{ K}^{-1} (90 + 273.15) \text{ K}} = 0.9594714 \text{ kg m}^{-3}.$$

[2/5]

*Mass conservation gives  $\dot{m}_1 = \dot{m}_2 + \dot{m}_3$  where the mass flux  $\dot{m} = \rho q$ . Therefore*

$$\rho_1 = \frac{\rho_2 q_2 + \rho_3 q_3}{q_1} = \frac{(2.1564726 \text{ kg m}^{-3} \times 1 \text{ m}^3 \text{ s}^{-1}) + (0.9594714 \text{ kg m}^{-3} \times 1 \text{ m}^3 \text{ s}^{-1})}{2 \text{ m}^3 \text{ s}^{-1}}$$

$$= 1.557972 \text{ kg m}^{-3}$$

[2/5]

*The pressure at the inlet is now*

$$p_1 = \rho_1 R T_1 = 1.557972 \text{ kg m}^{-3} \times 287 \text{ J kg}^{-1} \text{ K}^{-1} \times (20 + 273.15) \text{ K} = 131078.49 \text{ Pa}.$$

[1/5]

- (d) What is the relative percentage error if the gravitational potential energy terms are neglected when calculating the rate of heat transfer
- $\dot{Q}$
- ? [8 marks]

**Solution:***The mass fluxes at each inlet and exit are*

$$\dot{m}_1 = \rho_1 q_1 = 1.557972 \text{ kg m}^{-3} \times 2 \text{ kg/s} = 3.1159440 \text{ kg/s},$$

$$\dot{m}_2 = \rho_2 q_2 = 2.1564726 \text{ kg m}^{-3} \times 1 \text{ kg/s} = 2.1564726 \text{ kg/s},$$

$$\dot{m}_3 = \rho_3 q_3 = 0.9594714 \text{ kg m}^{-3} \times 1 \text{ kg/s} = 0.9594714 \text{ kg/s}.$$

[1/8]

*The energy fluxes without gravitational potential energy are*

$$c_p T_1 + \frac{u_1^2}{2} = 294229.45 \text{ m}^2/\text{s}^2,$$

$$c_p T_2 + \frac{u_2^2}{2} = 324169.45 \text{ m}^2/\text{s}^2,$$

$$c_p T_3 + \frac{u_3^2}{2} = 364439.45 \text{ m}^2/\text{s}^2.$$

[1/8]

*The energy fluxes with gravitational potential energy are*

$$c_p T_1 + \frac{u_1^2}{2} + g z_1 = 294229.45 \text{ m}^2/\text{s}^2,$$

$$c_p T_2 + \frac{u_2^2}{2} + g z_2 = 324267.55 \text{ m}^2/\text{s}^2,$$

$$c_p T_3 + \frac{u_3^2}{2} + g z_3 = 364478.69 \text{ m}^2/\text{s}^2.$$



[1/8]

The rate of heat addition without gravitational potential energy is

$$\begin{aligned}\dot{Q}_{with} &= \dot{W}_s + \dot{m}_2 \left( c_p T_2 + \frac{u_2^2}{2} \right) + \dot{m}_3 \left( c_p T_3 + \frac{u_3^2}{2} \right) - \dot{m}_1 \left( c_p T_1 + \frac{u_1^2}{2} \right) \\ &= -120000 + 916802.49 + 699062.53 + 349669.25 \\ &= 11929.28 \text{ W.}\end{aligned}$$

[2/8]

Note turbine does work on gas so  $\dot{W}_s$  is negative.

The rate of heat addition with gravitational potential energy is

$$\begin{aligned}\dot{Q}_{without} &= \dot{W}_s + \dot{m}_2 \left( c_p T_2 + \frac{u_2^2}{2} + g z_2 \right) + \dot{m}_3 \left( c_p T_3 + \frac{u_3^2}{2} + g z_3 \right) - \dot{m}_1 \left( c_p T_1 + \frac{u_1^2}{2} + g z_1 \right) \\ &= -120000 + 916802.49 + 699274.08 + 349706.9 \\ &= 12178.48 \text{ W.}\end{aligned}$$

[2/8]

Note turbine does work on gas so  $\dot{W}_s$  is negative.

The relative percentage difference between the rate of heat addition with and without the gravitational potential energy terms is

$$100\% \left| \frac{\dot{Q}_{with} - \dot{Q}_{without}}{\dot{Q}_{with}} \right| = 100\% \left| \frac{12178.48 - 11929.28}{11929.28} \right| = 2.0462\%.$$

[1/8]

**Q.4 Question 4**

- (a) Gas flows along a pipe of slowly varying cross section in the direction of increasing  $x$ . By considering the rate of change of the mass of gas within a small section of pipe and the gas mass fluxes into and out of this section of pipe, show that

$$\frac{\partial}{\partial t}(\rho A) + \frac{\partial}{\partial x}(\rho u A) = 0.$$

Here the gas velocity  $u$ , density  $\rho$  and pipe cross section  $A$  are all functions of  $x$  and time  $t$ . [5 marks]

**Solution:**

[1/5] *The total mass contained in a section of pipe of length  $\Delta x$  is  $\rho A \Delta x$ .*

*The rate of change of mass in this section of pipe equals the mass flux entering the pipe  $\rho u A$  minus the mass flux leaving the other end of the pipe section*

$$\rho u A + \Delta x \frac{\partial}{\partial x}(\rho u A).$$

[2/5] *[Obtained via a linearized Taylor expansion.]*

*Hence mass conservation implies*

$$\frac{\partial}{\partial t}(\rho A) = \rho u A - \rho u A - \Delta x \frac{\partial}{\partial x}(\rho u A).$$

[2/5] *and hence dividing by  $\Delta x$  and rearranging gives the result.*

- (b) Explain what is meant by a steady flow and show that for steady flow in a pipe of uniform cross section

$$\frac{1}{\rho} \frac{d\rho}{dx} + \frac{1}{u} \frac{du}{dx} = 0.$$

[4 marks]

**Solution:**

*The flow is steady if the density, velocity and cross section depend only on  $x$  and not time  $t$ . In this case the result from (a) gives*

$$\frac{d}{dx}(\rho u A) = 0.$$

[2/4]

*If the pipe has uniform cross section, then  $A$  is constant. Therefore via the chain rule*

$$\rho \frac{du}{dx} + u \frac{d\rho}{dx} = 0.$$

[2/4] *Finally dividing by  $\rho u$  gives the required result.*

- (c) For steady compressible flow in a uniform pipe, using the conservation of energy and the laws of thermodynamics; changes in the entropy  $s$ , is related to changes in the pressure  $p$ , density  $\rho$  and velocity  $u$ , through

$$T ds + \frac{dp}{\rho} + u du = 0.$$

Hence show that the change in pressure and the change in entropy as fluid flows are related through

$$\left(1 - \frac{u^2}{c^2}\right) \frac{dp}{dx} = -\rho T \left(1 + \frac{u^2 \beta}{c_p}\right) \frac{ds}{dx}.$$

In the derivation of this result, you may additionally assume that a change in gas density

$$d\rho = \frac{dp}{c^2} - \frac{\rho \beta T}{c_p} ds,$$

where  $c$  is the speed of sound,  $\beta$  is the thermal expansion coefficient and  $c_p$  is the specific heat capacity as constant pressure. [8 marks]

**Solution:**

*Variations over  $x$  imply*

$$T \frac{ds}{dx} + \frac{1}{\rho} \frac{dp}{dx} + u \frac{du}{dx} = 0, \quad \text{and} \quad \frac{d\rho}{dx} = \frac{1}{c^2} \frac{dp}{dx} - \frac{\rho \beta T}{c_p} \frac{ds}{dx}.$$

*Using the second equation, we eliminate the  $\frac{d\rho}{dx}$  from the mass conservation equation to give*

$$\rho \frac{du}{dx} + \frac{u}{c^2} \frac{dp}{dx} - \frac{u \rho \beta T}{c_p} \frac{ds}{dx} = 0.$$

[2/8]

*Next we eliminate  $\frac{du}{dx}$  to give*

$$\frac{T}{u} \frac{ds}{dx} + \frac{1}{\rho u} \frac{dp}{dx} = -\frac{du}{dx} = \frac{u}{\rho c^2} \frac{dp}{dx} - \frac{u \beta T}{c_p} \frac{ds}{dx}.$$

[2/8]

*Collecting all the terms involving  $\frac{dp}{dx}$  on one side and all the terms involving  $\frac{ds}{dx}$  on the other side*

$$\frac{1}{\rho u} \frac{dp}{dx} - \frac{u}{\rho c^2} \frac{dp}{dx} = -\frac{u \beta T}{c_p} \frac{ds}{dx} - \frac{T}{u} \frac{ds}{dx}.$$

$$\left(\frac{1}{\rho u} - \frac{u}{\rho c^2}\right) \frac{dp}{dx} = -T \left(\frac{u \beta}{c_p} + \frac{1}{u}\right) \frac{ds}{dx}.$$

[3/8]

*If we multiply by  $\rho u$ , then*

$$\left(1 - \frac{u^2}{c^2}\right) \frac{dp}{dx} = -\rho T \left(1 + \frac{u^2 \beta}{c_p}\right) \frac{ds}{dx}.$$

[1/8]

- (d) Define a Mach number, and (with reference to the result derived in part (c)), explain how the pressure changes as a compressible fluid flows subsonically along a pipe of uniform cross section. [3 marks]

**Solution:**

*The Mach number  $\text{Ma}$  is the ratio of the actual speed  $u$  to the speed of sound  $c$ , i.e.*

$$\text{Ma} = \frac{u}{c}.$$

[1/3]

*For subsonic flow  $\text{Ma} < 1$ , so the coefficient of  $\frac{\partial p}{\partial x}$  is positive. The density, temperature and velocity are all positive, so the  $-\rho T \left(1 + \frac{u^2 \beta}{c_p}\right)$  is negative, while the entropy must increase with flow along the pipe (i.e.  $\frac{ds}{dx} > 0$ ).*

[1/3]

*Consequently the pressure must fall as fluid flows along the pipe.*

[1/3]

**Q.5 Question 5**

- (a) Define the specific humidity  $\omega$ . Assuming both dry air and water vapour behave like ideal gases with specific gas constants  $R_a = 0.2871 \text{ kJ}/(\text{kg.K})$  and  $R_v = 0.4615 \text{ kJ}/(\text{kg.K})$ , respectively, show that

$$\omega = \frac{0.622p_v}{p - p_v},$$

where  $p$  is the absolute pressure and  $p_v$  is the partial pressure of water vapour. [5 marks]

**Solution:**

[1/5] *The specific humidity  $\omega$  is the ratio of the mass of water vapour  $m_v$  to the mass of dry air  $m_a$  in a volume  $V$ , i.e.*

$$\omega = \frac{m_v}{m_a}.$$

[1/5]

*In terms of densities*

$$\omega = \frac{\rho_v V}{\rho_a V} = \frac{\rho_v}{\rho_a}.$$

*Treat both water vapour and dry air as ideal gases, so that*

$$\omega = \frac{p_v}{R_v T} \frac{R_a T}{p_a} = \frac{0.622p_v}{p_a}.$$

[2/5]

*Finally the partial pressure of water vapour and dry air  $p_v + p_a = p$ , so*

$$\omega = \frac{0.622p_v}{p - p_v}.$$

[1/5]

- (b) Define the relative humidity  $\varphi$ , and hence show that

$$\omega = \frac{0.622\varphi p_g}{p - \varphi p_g},$$

where the saturation pressure of water is denoted  $p_g$ . [3 marks]

**Solution:**

*The relative humidity  $\varphi$  is the ratio of the mass of water vapour  $m_v$  to the mass of water vapour at saturation  $m_g$ , i.e.*

$$\varphi = \frac{m_v}{m_g}.$$

[1/3]

*Again using the ideal gas equation*

$$\varphi = \frac{m_v}{m_g} = \frac{\rho_v V}{\rho_g V} = \frac{p_v}{R_v T} \frac{R_g T}{p_g} = \frac{p_v}{p_g}.$$

[1/3]

[1/3] *Therefore writing  $p_v = \varphi p_g$  in the equation from part (a), gives the required result.*

- (c) Air enters an air-conditioning system at 1 atm, 35°C and 60% relative humidity, at a rate of 12 m<sup>3</sup>/min. Saturated air leaves the air-conditioning system at a temperature of 16°C. The moisture in the air that condenses during the process is removed at 16°C, while the specific enthalpy of liquid water at 16°C is 67.22 kJ/kg.

- i) Determine the rate of moisture removal from the air; [7 marks]

**Solution:**

*The inlet and outlet are at 1 atm (= 101.325 kPa) so we can determine the conditions using the psychrometric chart, which gives*

$$\begin{aligned} h_1 &= 80.0 \text{ kJ/kg dry air}, & h_2 &= 45.6 \text{ kJ/kg dry air}, \\ \omega_1 &= 0.022 \text{ kg water/kg dry air}, & \omega_2 &= 0.016 \text{ kg water/kg dry air}, \\ V_1 &= 0.866 \text{ m}^3/\text{kg dry air}. \end{aligned}$$

[3/7]

*In the cooling section:*

$$\begin{aligned} \text{Conservation of dry air:} & \quad \dot{m}_{a1} = \dot{m}_{a2} = \dot{m}_a, \\ \text{Conservation of water vapour:} & \quad \dot{m}_{a1}\omega_1 = \dot{m}_{a2}\omega_2 + \dot{m}_w, \\ & \Rightarrow \dot{m}_w = \dot{m}_a(\omega_1 - \omega_2). \end{aligned}$$

[2/7]

*The mass flux of dry air is given by*

$$\dot{m}_a = \frac{\dot{V}_1}{V_1} = \frac{12 \text{ m}^3/\text{min}}{0.866 \text{ m}^3/\text{kg dry air}} = 13.86 \text{ kg/min}.$$

[1/7]

*Hence the conservation of water vapour gives*

$$\begin{aligned} \dot{m}_w &= \dot{m}_a(\omega_1 - \omega_2) \\ &= 13.86 \text{ kg/min} \times (0.022 - 0.016) \\ &= 0.083 \text{ kg/min}. \end{aligned}$$

[1/7]

- ii) Determine the rate of heat removal from the air. [5 marks]

**Solution:**

*In the cooling section:*

$$\text{Conservation of energy:} \quad \dot{Q} = \dot{m}_a(h_2 - h_1) + \dot{m}_w h_w.$$

[1/5]

*The heat supplied to the system  $\dot{Q}$  is given by the energy conservation equation*

$$\begin{aligned} \dot{Q} &= \dot{m}_a(h_2 - h_1) + \dot{m}_w h_w \\ &= 13.86 \text{ kg/min} (45.6 - 80.0) \text{ kJ/kg} + (0.083 \text{ kg/min} \times 67.22 \text{ kJ/kg}) \\ &= -476.674 \text{ kJ/min} + 5.589 \text{ kJ/min} \\ &= -471.1 \text{ kJ/min}. \end{aligned}$$

[3/5]

*The rate of heat addition  $\dot{Q}$  is negative indicating heat removal from the cooling section. Therefore the rate of heat removal is 471.1 kJ/min.*

[1/5]

**END OF PAPER**