For all problems in this tutorial, assume that air behaves as an ideal gas with constant heat capacities at room temperature $\left(MW = 29\ g.mol^{-1},\ C_p = 1.005\ kJ.(kg.K)^{-1}\ \text{and}\ C_v = 0.718\ kJ.(kg.K)^{-1}\right)$.

Problem 1 In a Carnot cycle, the maximum pressure and temperature are limited to 18 bar and 410°C. The ratio of isentropic compression and isothermal expansion are 6 and 1.5, respectively. Assuming the volume of the air at the beginning of isothermal expansion is 0.18 m^3, determine:

(a) Temperature and pressures at all stages of the cycle;

(b) Change in entropy $\left(\text{in kJ.K}^{-1}\right)$ during isothermal expansion. Entropy variation for ideal gases is given by

$$\frac{\Delta S}{R} = \int_{T_0}^{T} \frac{C_p^{ig}}{R}\frac{dT}{T} - \ln\frac{P}{P_0}.$$

(c) Mean thermal efficiency of the cycle;

(d) Mean effective pressure (MEP) of the cycle and;

(e) Theoretical power if there are 210 working cycles per minute.

Problem 2 An engine of 250 mm bore and 375 mm stroke works on ideal Otto cycle. The clearance volume is 0.00263 m^3. The initial pressure and temperature are 1 bar and 50ºC. If the maximum pressure is limited to 25 bar, determine: (a) air standard efficiency of the cycle and (b) MEP.

Problem 3 An engine with 200 mm cylinder diameter and 300 mm stroke works on ideal Diesel cycle. The initial pressure and temperature of air are 1 bar and 27ºC, repectively. The cut-off is 8% of the stroke. Calculate: (a) pressures and temperatures at all stages; (b) theoretical air-standard efficiency; (c) MEP; (d) power of the engine if the working cycles per minute are 380. Assume that the compression ratio (r) is 15 and the working fluid is air.

Problem 4 An ideal engine operates on the Carnot cycle using a perfects gas as the working fluid. The ratio of the greatest to the least volume is fixed as $x : 1$, the lower temperature of the cycle is also fixed, but the volume compression ratio r of the reversible adiabatic compression is variable. The ratio of specific heats is γ. Show that if the work done in the cycle is a maximum then,

$$(\gamma - 1)\ln\frac{x}{r} + \frac{1}{r^{\gamma-1}} - 1 = 0$$

Problem 5 An ideal Otto cycle has a volumetric compression ratio of 6, the lowest cycle pressure of 0.1 MPa and operates between temperature limits of 300.15 and 1842.15 K. Calculate the temperature and pressure after the isentropic expansion.

Problem 6 The volume ratios of compression and expansion for a diesel engine are 15.3 and 7.5, respectively. The pressure and temperature at the beginning of the compression are 1 bar and 27 °C. Assuming an ideal engine, determine the (a) MEP, (b) ratio of maximum pressure to MEP and (c) cycle efficiency. Assume that the volume at the end of the isentropic compression is 1 m^3.

Problem 7 In an air-standard Brayton cycle, the air enters the compressor at 0.1 MPa and 15°C. The pressure leaving the compressor is 1 MPa and the maximum temperature in the cycle is 1100°C. Determine:

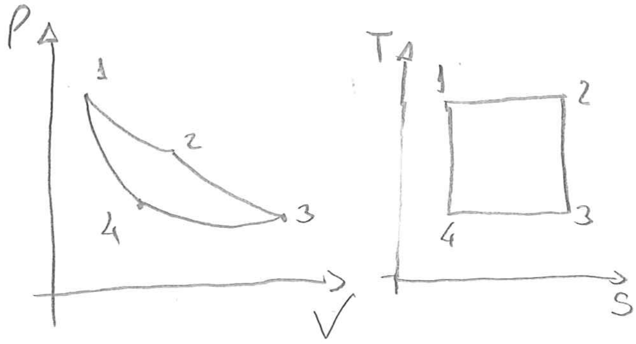
(a) Pressure and temperature at each point in the cycle;

(b) Work consumed by the compressor and produced by the turbine;

(c) Efficiency of the cycle;

(d) Assume that the efficiency of the compressor and the turbine are 80% and 85%, respectively, and the pressure drop between the compressor and the turbine is 15 kPa. Calculate the work in the compressor and turbine, and the efficiency of the cycle.

P1 Carnot cycle:



$P_{MAX} = P_1 = 18$ bar

$T_{MAX} = T_1 = T_2 = 683.15K$

ratio of isothermal expansion : $V_2/V_1 = 1.5$

ratio of isentropic compression : $V_4/V_1 = 6$

volume of the air in the beginning of the expansion : $V_1 = 0.18 m^3$

(a) Calculating T_i and P_i:

(i) 4-1: isentropic compression ($\gamma = 1.4$):

$$TV^{\gamma-1} = \text{constant}$$

$$T_1 V_1^{\gamma-1} = T_4 V_4^{\gamma-1}$$

$$T_1/T_4 = \underbrace{\left(V_4/V_1\right)}_{6}^{\gamma-1} \therefore T_4 = 333.62K = T_3$$

$$PV^{\gamma} = \text{constant}$$

$$P_1 V_1^{\gamma} = P_4 V_4^{\gamma}$$

$$P_1/P_4 = (V_4/V_1)^{\gamma} \therefore V_4 = 1.47 \text{ bar}$$

(ii) 1-2: isothermal expansion

$$P_1V_1 = P_2V_2 \therefore P_2 = \frac{P_1V_1}{V_2} = 12 \text{ bar}$$

1/1.5

(iii) 2-3: isentropic expansion

$$PV^{\gamma} = \text{constant}$$

$$P_2V_2^{\gamma} = P_3V_3^{\gamma}$$

$$P_3 = P_2\underbrace{\left(\frac{V_2}{V_3}\right)}_{V_1/V_4}^{\gamma}$$

$$P_3 = 0.98 \text{ bar}$$

Using:

$$TV^{\gamma-1} = \text{constant}$$

$$T_2V_2^{\gamma-1} = T_3V_3^{\gamma-1}$$

$$T_2/T_3 = (V_3/V_2)^{\gamma-1}$$

$$T_1/T_4 = (V_4/V_1)^{\gamma-1}$$

But $T_2/T_3 = T_1/T_4$:

$$\boxed{V_3/V_2 = V_4/V_1}$$

	1	2	3	4
T	683.15	683.15	333.62	333.62
P	18	12	0.98	1.47

$[T] = K$

$[P] = \text{bar}$

(b) Change in entropy [kJ/K] during the isothermal expansion 1-2:

$$\frac{\Delta S}{R} = \cancel{\int_{T_0}^{T} \frac{C_p^{ig}}{R}\frac{dT}{T}} - \ln P/P_0 = -\ln P_2/P_1$$

0 (isothermal)

$$[\text{kJ/mol.K}] \leftarrow \Delta S = -R\ln P_2/P_1$$

$$\Delta S = -\frac{P_1V_1}{mT_1}\ln(P_2/P_1)$$

$$m\Delta S = 1.9230\times10^{-3}\ \frac{\text{bar m}^3}{K}$$

$$\boxed{m\Delta S = 0.192 \text{ kJ/K}}$$

(c) $\eta_{carnot} = \dfrac{\text{Heat Supplied} - \text{Heat Rejected}}{\text{Heat Supplied}}$

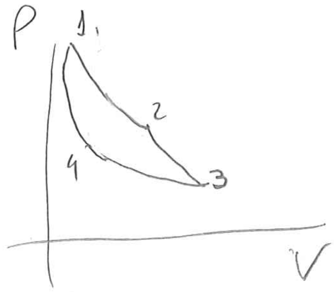
$= \dfrac{Q_S - Q_R}{Q_S}$

$Q_S = T_1 (S_2 - S_1) = 131.16 \text{ KJ}$

$Q_R = T_4 (S_3 - S_4) = T_4 (S_2 - S_1) = 64.06 \text{ KJ}$

$\eta_{carnot} = 0.5116 \therefore 51.16\%$

(d) $MEP = \dfrac{\text{Work done}}{\text{Stroke volume}} = \dfrac{Q_S - Q_R}{V_S}$



$V_S = V_3 - V_1 \rightarrow 0.18 m^3$

$\dfrac{V_3}{V_1} = \dfrac{V_3}{V_2} \underbrace{\dfrac{V_2}{V_1}}_{1.5} = \underbrace{\dfrac{V_4}{V_1}}_{6} \dfrac{V_2}{V_1} = 9$

$V_S = V_3 - V_1 = 9V_1 - V_1 = 8V_1 = 1.44 m^3$

$MEP = 46597.22 \text{ J/m}^3 = 0.46597 \text{ bar}$

(e) $\text{Power} = (Q_S - Q_R) \times N_{cycles} = (131.16 - 64.06) \text{ KJ} \times \dfrac{210}{60s}$

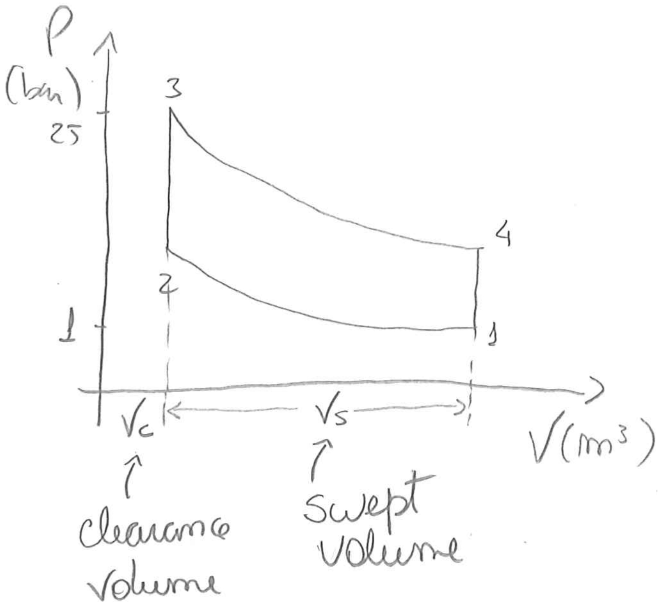
$\text{Power} = 234.85 \text{ KW}$

P2: Ideal Otto cycle:

$$\left\{\begin{array}{l} \text{Bore: } D = 2.5\times10^{-1}\,m \\ \text{Stroke length: } l = 3.75\times10^{-1}\,m \\ \text{Clearance Volume: } V_c = 2.63\times10^{-3}\,m^3 \\ \text{Initial pressure: } P_1 = 1\text{ bar} \\ \text{Initial temperature: } T_1 = 323.15\,K \\ \text{Maximum pressure: } P_3 = 25\text{ bar} \end{array}\right\} \begin{array}{l} (a)\ \eta_{otto} \\ (b)\ MEP \end{array} \Big\}\ ?$$

(a)

$$\eta_{otto} = 1 - \frac{1}{r^{\gamma-1}}$$



compression ratio (r)

$$r = V_1/V_2 = \frac{V_s + V_c}{V_c}$$

$$V_s = \frac{\pi}{4} D^2 l = 1.84\times10^{-2}\,m^3$$

$$r = 8 \therefore \eta_{otto} = 0.5647 \therefore \underline{56.47\%}$$

(b) $$MEP = \frac{P_1 r\left[(r^{\gamma-1}-1)(r_p-1)\right]}{(\gamma-1)(r-1)}$$

$$r_p = P_3/P_2 = P_4/P_1 \therefore P_2 : ?$$

1-2 : isentropic :

$$PV^{\gamma} = \text{constant}$$

$$P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$$

$$P_2 = P_1 (V_1 / V_2)^{\gamma} = 1\,\text{bar} \times \Big(\overbrace{\frac{V_s + V_c}{V_c}}^{r}\Big)^{\gamma}$$

$$P_2 = 18.38\ \text{bar}$$

$$\hookrightarrow r_p = P_3 / P_2 = 1.36$$

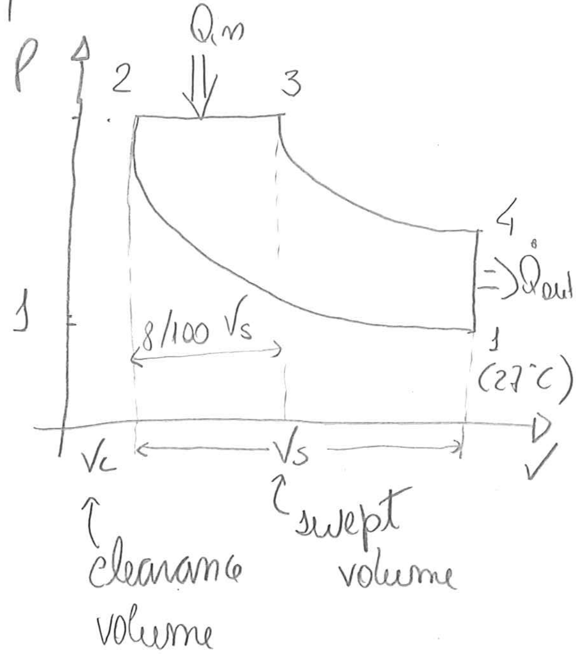
$$\underline{MEP = 1.334\ \text{bar}}$$

P3: Ideal Diesel

Cylinder diameter: $D = 2\times10^{-1}$ m
Stroke length: $L = 3\times10^{-1}$ m
Initial pressure: $P_1 = 1$ bar
Initial temperature: $T_1 = 300.15$ K
$r = V_1/V_2 = 15$

Cut-off: $8/100\ V_s$

P_i, T_i: ? MEP: ?
η_{Diesel}: ? Power ?



(a) P_i, T_i: ?

↳ we also need V_i to calculate P_i & T_i:

$$V_1 = V_s + V_c = V_s + \frac{V_s}{r-1}$$

$$V_s = \frac{\pi D^2}{4} L = 9.42\times10^{-3}\ m^3$$

$$V_1 = 1.01\times10^{-2}\ m^3 = V_4 \therefore V_2 = V_1/r$$

$$V_2 = 6.73\times10^{-4}\ m^3$$

$r = V_1/V_2$

$$V_2 = \frac{V_1}{r} = \frac{V_s + V_c}{r} =$$

$$V_2 = \frac{V_s + V_2}{r} = \frac{V_s}{r} + \frac{V_2}{r}$$

$$V_2\left(1 - \frac{1}{r}\right) = \frac{V_s}{r}$$

$$V_2\left(\frac{r-1}{\cancel{r}}\right) = \frac{V_s}{\cancel{r}}$$

$$\boxed{V_2 = \frac{V_s}{r-1} = V_c}$$

(i) 1-2: isentropic compression

PV^γ = constant

$$P_1 V_1^\gamma = P_2 V_2^\gamma \therefore P_2 = P_1 \overbrace{(V_1/V_2)}^{15}{}^\gamma$$

$$P_2 = 44.31\text{ bar} = P_3$$

$$TV^{\gamma-1} = \text{constant}$$

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

$$T_2 = T_1 (V_1/V_2)^{\gamma-1} = 886.70\,K$$

$$\boxed{\begin{array}{l} TV^{\gamma-1} = \text{const.} \\ TP^{\frac{1-\gamma}{\gamma}} = \text{const.} \\ PV^{\gamma} = \text{const.} \end{array}}$$

(ii) 2-3: Expansion at constant pressure

$$\frac{V_2}{T_2} = \frac{V_3}{T_3} \therefore T_3 = \frac{V_3 T_2}{V_2} \quad (V_3 : ?)$$

Cut-off: $0.08\,V_s = V_3 - V_2$

$V_3 = 1.43 \times 10^{-3}\,m^3$

Cut-off ratio:

$\rho = V_3/V_2 = 2.12$

$T_3 = 1884.07\,K$

(iii) 3-4: Isentropic expansion

$$PV^{\gamma} = \text{constant}$$

$$P_3 V_3^{\gamma} = P_4 V_4^{\gamma} \therefore P_4 = P_3 (V_3/V_4)^{\gamma}$$

$$P_4 = P_3 \times \Big(\underbrace{\frac{V_3}{V_2}}_{\rho} \times \underbrace{\frac{V_2}{V_4}}_{1/r}\Big)^{\gamma} = 44.31 (2.12 \times 1/15)^{1.4}$$

$V_4 \to V_1$

$P_4 = 2.86$ bar

strokes	1	2	3	4
P (bar)	1	44.31	44.31	2.86
T (K)	300.15	886.70	1884.07	861.38
V (m^3)	1.01×10^{-2}	6.73×10^{-4}	1.43×10^{-3}	1.01×10^{-2}

$$TV^{\gamma-1} = \text{constant}$$

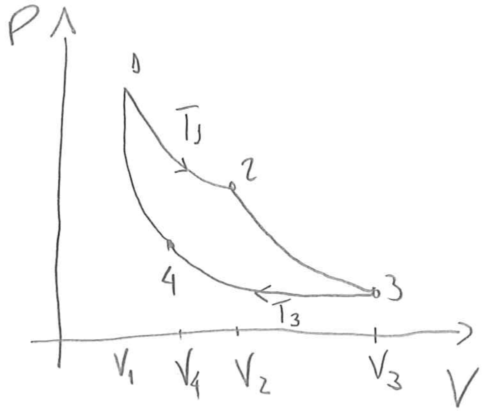
$$T_3 V_3^{\gamma-1} = T_4 V_4^{\gamma-1}$$

$$T_4 = T_3 (V_3/V_4)^{\gamma-1}$$

$$T_4 = 861.38\,K$$

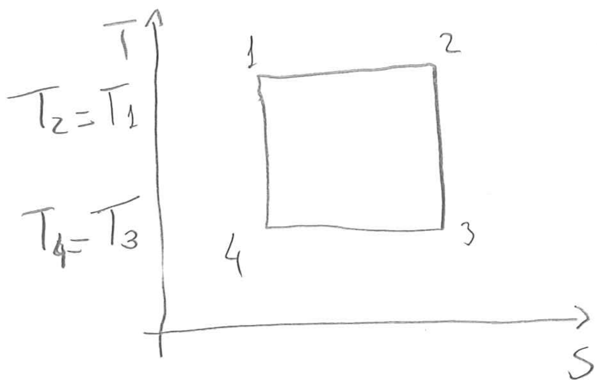
P4: Ideal Carnot

$$\begin{cases} V_3/V_1 = x \\ r = V_4/V_1 \text{ (not fixed)} \end{cases}$$



In the isotherms, since:

compression ratio = expansion ratio



$$\frac{V_3}{V_4} = \frac{V_2}{V_1}$$

We can redefine V_3/V_4 with the known variables

$$\frac{V_3}{V_4} = \frac{V_3}{V_1} \cdot \frac{V_1}{V_4} = x \cdot \frac{1}{r} = \frac{x}{r}$$

The work done per unit mass of gas is

W = heat supplied - heat rejected

$$W = RT_1 \ln(x/r) - RT_4 \ln(x/r)$$

$$W = R(T_1 - T_4)\ln(x/r)$$

We know the isentropic change:

$$TV^{\gamma-1} = \text{const}$$

$$T_1 V_1^{\gamma-1} = T_4 V_4^{\gamma-1}$$

$$\frac{T_1}{T_4} = \left(\frac{V_4}{V_1}\right)^{\gamma-1} = r^{\gamma-1}$$

$$T_1 = T_4 r^{\gamma-1}$$

Thus

$$W = R\left(T_4 r^{\gamma-1} - T_4\right)\ln(x/r)$$

$$\underline{W = RT_4\left(r^{\gamma-1} - 1\right)\ln(x/r)}$$

The maximum work is obtained by differentiating W wrt r:

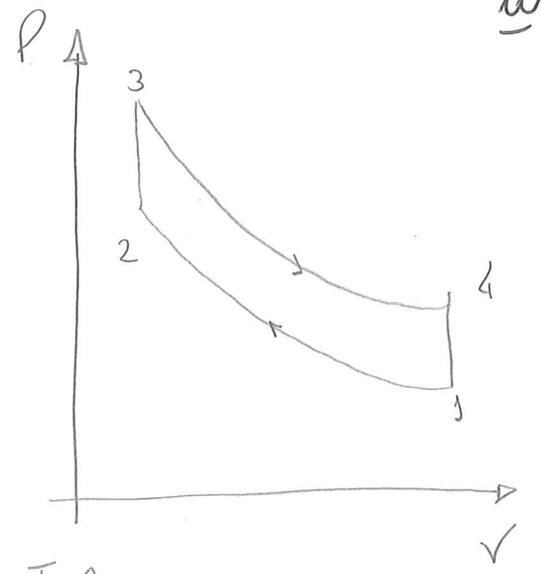
$$\frac{dW}{dr} = RT_4\left[(\gamma-1)r^{\gamma-2}\ln(x/r) + \left(r^{\gamma-1}-1\right)\frac{r}{x}\left(-\frac{x}{r^2}\right)\right] = 0$$

$$(\gamma-1)\ln(x/r) = \frac{\left(r^{\gamma-1}-1\right)}{r}\,\frac{1}{r^{\gamma-2}} = \frac{r^{\gamma-1}-1}{r^{\gamma-1}}$$

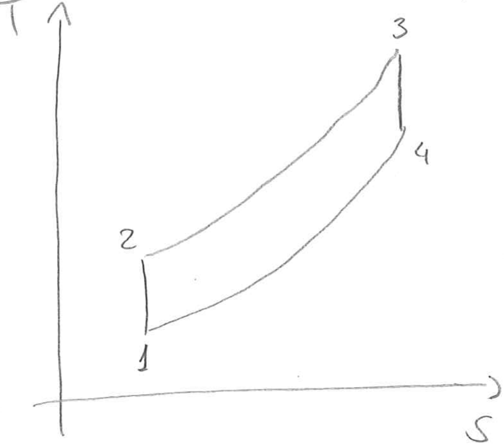
$$(\gamma-1)\ln(x/r) = 1 - \frac{1}{r^{\gamma-1}}$$

$$\boxed{(\gamma-1)\ln(x/r) + \frac{1}{r^{\gamma-1}} - 1 = 0}$$

P5: Ideal Otto cycle



$$\begin{cases} \dfrac{V_1}{V_2} = \dfrac{V_4}{V_3} = r = 6 \\ P_1 = 0.1\ \text{MPa} = 1\ \text{bar} \\ T_1 = 300.15\ \text{K} \\ T_3 = 1842.15\ \text{K} \end{cases}$$



$P_4, T_4 : ?$

(i) 1-2 : isentropic compression

$PV^{\gamma} = \text{constant}$

$P_1 V_1^{\gamma} = P_2 V_2^{\gamma} \therefore P_2 = P_1 (V_1/V_2)^{\gamma} = 12.29\ \text{bar}$

$TV^{\gamma-1} = \text{constant}$

$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1} \therefore T_2 = T_1 (V_1/V_2)^{\gamma-1} = 614.61\ \text{K}$

(ii) 2-3 : heat addition @ const. volume

$P/T = \text{const.}$

$P_2/T_2 = P_3/T_3 \therefore P_3 = P_2 T_3/T_2 = 36.84\ \text{bar}$

(iii) 3-4 : isentropic expansion

$TV^{\gamma-1} = \text{const.}$

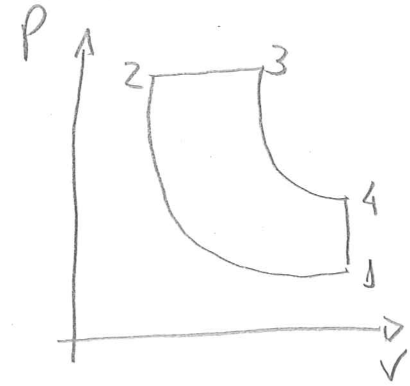
$T_3 V_3^{\gamma-1} = T_4 V_4^{\gamma-1} \therefore T_4 = T_3 (V_3/V_4)^{\gamma-1}$

$\underline{T_4 = 899.63\ \text{K}}$

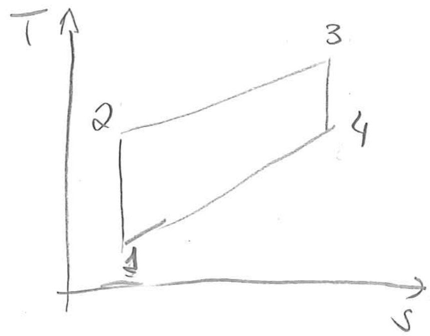
$PV^\gamma = const.$

$P_3 V_3^\gamma = P_4 V_4^\gamma \therefore P_4 = P_3 (V_3/V_4)^\gamma = 3.00\ bar$

P6: Diesel Engine:



$$\begin{cases} V_1/V_2 = 15.3 = r \\ V_4/V_3 = 7.5 \\ P_1 = 1\,\text{bar} \\ T_1 = 27°C = 300.15\,K \\ V_2 = 1\,m^3 \end{cases}$$



Determiming P_i and T_i:

(i) 1-2: adiabatic compression

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1} \therefore T_2 = 893.75\,K$$

$$P_1 V_1^{\gamma} = P_2 V_2^{\gamma} \therefore P_2 = 45.56\,\text{bar}$$

(ii) 2-3: heat addition @ const pressure

$$\frac{V_2}{T_2} = \frac{V_3}{T_3} \therefore T_3 = T_2 \frac{V_3}{V_2} = T_2 \frac{V_3}{\underset{\overset{\shortparallel}{V_4}}{V_1}} \times \frac{V_1}{V_2} = 1823.25\,K$$

(iii) 3-4: adiabatic expansion

$$T_3 V_3^{\gamma-1} = T_4 V_4^{\gamma-1} \therefore T_4 = 814.37\,K$$

$$P_3 V_3^{\gamma} = P_4 V_4^{\gamma} \therefore P_4 = 2.71\,\text{bar}$$

(a) MEP ?

$$MEP = \frac{W_{net}}{V_{MAX} - V_{MIN}}$$

$$MEP = \frac{m\left[C_p(T_3 - T_2) - C_v(T_4 - T_1)\right]}{V_1 - V_2}$$

calculating $\underline{m}$ from (ideal gas)

$$P_2 V_2 = m R T_2 = \frac{m}{MW} R T_2$$

$$m = \frac{P_2 V_2}{R T_2} \times MW$$

$$m = \frac{45.56\,\cancel{bar} \times 1\,\cancel{m^3}}{0.08314\,\frac{\cancel{bar\,m^3}}{\cancel{Kgmol \times K}} \times 893.75\,\cancel{K} \times \frac{1\,\cancel{Kgmol}}{10^3\,\cancel{gmol}}} \times 29\,g/\cancel{gmol}$$

$$m = 17780.98\ g = 17.78\ Kg$$

$$MEP = \frac{17.78\,\cancel{Kg}\left[1.005\,\frac{KJ}{\cancel{Kg\,K}}(1823.25 - 893.75)\cancel{K} - 0.718\,\frac{KJ}{\cancel{Kg\,K}}(814.37 - 300.15)\cancel{K}\right]}{\underbrace{15.3\,V_2 - V_2}_{14.3\,V_2 \to 1\,m^3}}$$

$$MEP = 702.42\ KJ/m^3 = 7.02\ bar$$

(b) P_2/MEP: ?

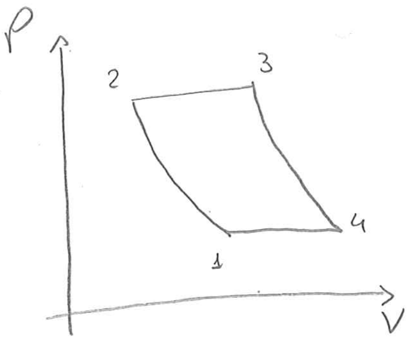
$$P_2/MEP = 6.49$$

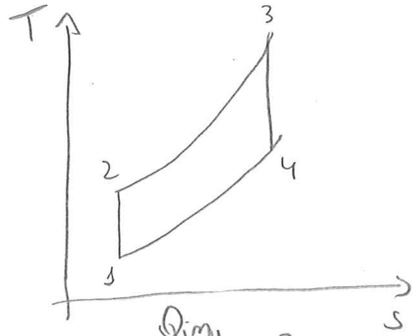
(c) η_{Diesel} : ?

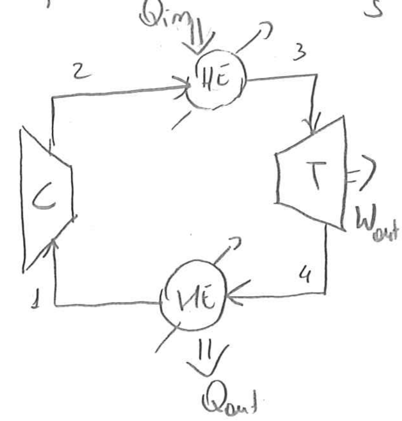
$$\eta_{Diesel} = \frac{W_{net}}{\text{Heat Supplied}} = \frac{m\left[C_p(T_3-T_2)-C_v(T_4-T_1)\right]}{mC_p(T_3-T_2)}$$

$\eta_{Diesel} = 0.6048 \therefore 60.48\%$

P7: Brayton cycle







$$\begin{cases} T_1 = 15°C = 288.15\,K \\ P_1 = 0.1\,MPa = 1\,bar \\ P_2 = 1\,MPa = 10\,bar \\ T_3 = 1100°C = 1373.15\,K \end{cases}$$

(a) P_i, T_i : ?

(i) 1-2 : isentropic compression

$$TP^{\frac{1-\gamma}{\gamma}} = \text{constant}$$

$$T_1 P_1^{\frac{1-\gamma}{\gamma}} = T_2 P_2^{\frac{1-\gamma}{\gamma}} \therefore T_2 = T_1 (P_1/P_2)^{\frac{1-\gamma}{\gamma}}$$

$$T_2 = 556.33\,K$$

(ii) 3-4 : isentropic expansion

$$T_3 P_3^{\frac{1-\gamma}{\gamma}} = T_4 P_4^{\frac{1-\gamma}{\gamma}} \therefore T_4 = T_3 \left(\frac{\overset{P_2}{\overset{\parallel}{P_3}}}{\underset{P_1}{\underset{\parallel}{P_4}}}\right)^{\frac{1-\gamma}{\gamma}} = 711.22\,K$$

	1	2	3	4
T (K)	288.15	556.33	1373.15	711.22
P (bar)	1	10	10	1

(b) W_C, W_T : ?

$$W_C = h_2 - h_1 = C_p(T_2 - T_1) = 1.005 \frac{kJ}{kg\,K} (556.33 - 288.15)\,K$$

$W_C = 269.52\ KJ/Kg$

$W_T = h_4 - h_3 = C_p(T_4 - T_3) = 1.005 \frac{KJ}{Kg.K} (711.22 - 1373.15)K$

$W_T = -665.24\ KJ/Kg$

(c) $\eta_{Brayton}$: ?

$$\eta_{Brayton} = \frac{|W_{net}|}{Q_{in}} = \frac{|W_T + W_C|}{Q_{in}} = \frac{|-665.24 + 269.52|}{820.90}$$

$= 0.4821 \therefore 48.21\%$

$Q_{in} = h_3 - h_2 = C_p(T_3 - T_2)$

$Q_{in} = 820.90\ KJ/Kg$

(d) $\left\{ \begin{array}{l} \eta_C = 0.80 \\ \eta_T = 0.85 \\ \Delta P_{2\text{-}3} = 15\ KPa \end{array} \right\}$ As the cycle is no longer ideal, temperature after the turbine calculated in (a) is T_{2s} (ideal) and

$$\eta_C = \frac{h_{2s} - h_1}{h_2 - h_1} = \frac{T_{2s} - T_1}{T_2 - T_1} = \frac{556.33 - 288.15}{T_2 - 288.15} = 0.80$$

$T_2 = 623.38\ K$ (actual temperature)

And across the turbine:

$$\eta_T = \frac{h_3 - h_4}{h_3 - h_{4s}} = \frac{T_3 - T_4}{T_3 - T_{4s}}$$

However: $\Delta P = P_2 - P_3$

$P_3 = P_2 - \Delta P = 9.85 \text{ bar}$

Thus: $T_3 P_3^{\frac{1-\gamma}{\gamma}} = T_{4s} P_4^{\frac{1-\gamma}{\gamma}} \therefore T_{4s} = T_3 (P_3/P_4)^{\frac{1-\gamma}{\gamma}}$

$T_{4s} = 714.30 \text{ K}$ (ideal temp)

$$\eta_T = \frac{1373.15 - T_4}{1373.15 - 714.30} = 0.85$$

$\underline{T_4 = 813.13 \text{ K}}$ (actual temperature)

$$\begin{cases} W_c = C_p(T_2 - T_1) = 336.91 \text{ KJ/Kg} \\ W_T = C_p(T_4 - T_3) = -562.82 \text{ KJ/Kg} \end{cases}$$

$$\eta^{actual}_{Brayton} = \frac{|W_{net}|}{Q_{in}} = \frac{|336.91 - 562.82|}{1.005(1373.15 - 623.38)}$$

$$\eta^{actual}_{Brayton} = 0.30 \therefore 30\%$$