Answers 5

Classroom Example 2

(a) Velocities – with correction:

$$u_B = 11 + 2(p'_A - p'_B)$$
$$u_D = 14 + 2(p'_C - p'_D)$$
$$v_C = 8 + 3(p'_A - p'_C)$$
$$v_D = 5 + 3(p'_B - p'_D)$$

Apply mass conservation for each cell in turn. Since densities and areas are the same on each face they may be omitted.

Cell A:

$$u_B - 5 + v_C - 15 = 0$$
$$\Rightarrow \quad 11 + 2(p'_A - p'_B) - 5 + 8 + 3(p'_A - p'_C) - 15 = 0$$
$$\Rightarrow \quad 5p'_A - 2p'_B - 3p'_C = 1$$

Cell B:

$$5 - u_B + v_D - 10 = 0$$
$$\Rightarrow \quad 5 - 11 - 2(p'_A - p'_B) + 5 + 3(p'_B - p'_D) - 10 = 0$$
$$\Rightarrow \quad -2p'_A + 5p'_B - 3p'_D = 11$$

Cell C:

$$u_D - 10 + 5 - v_C = 0$$
$$\Rightarrow \quad 14 + 2(p'_C - p'_D) - 10 + 5 - 8 - 3(p'_A - p'_C) = 0$$
$$\Rightarrow \quad -3p'_A + 5p'_C - 2p'_D = -1$$

Cell D:

$$20 - u_D + 10 - v_D = 0$$
$$\Rightarrow \quad 20 - 14 - 2(p'_C - p'_D) + 10 - 5 - 3(p'_B - p'_D) = 0$$
$$\Rightarrow \quad -3p'_B - 2p'_C + 5p'_D = -11$$

Assembling these into a single matrix equation:

$$\begin{pmatrix} 5 & -2 & -3 & 0 \\ -2 & 5 & 0 & -3 \\ -3 & 0 & 5 & -2 \\ 0 & -3 & -2 & 5 \end{pmatrix} \begin{pmatrix} p'_A \\ p'_B \\ p'_C \\ p'_D \end{pmatrix} = \begin{pmatrix} 1 \\ 11 \\ -1 \\ -11 \end{pmatrix}$$

Gaussian elimination as follows

$$R2 \to 5R2 + 2R1 \qquad R3 \to 5R3 + 3R1 \qquad \begin{pmatrix} 5 & -2 & -3 & 0 \\ 0 & 21 & -6 & -15 \\ 0 & -6 & 16 & -10 \\ 0 & -3 & -2 & 5 \end{pmatrix} \begin{pmatrix} p'_A \\ p'_B \\ p'_C \\ p'_D \end{pmatrix} = \begin{pmatrix} 1 \\ 57 \\ -2 \\ -11 \end{pmatrix}$$

$$R2 \to R2/3 \qquad R3 \to R3/2 \qquad \begin{pmatrix} 5 & -2 & -3 & 0 \\ 0 & 7 & -2 & -5 \\ 0 & -3 & 8 & -5 \\ 0 & -3 & -2 & 5 \end{pmatrix} \begin{pmatrix} p'_A \\ p'_B \\ p'_C \\ p'_D \end{pmatrix} = \begin{pmatrix} 1 \\ 19 \\ -1 \\ -11 \end{pmatrix}$$

$$R3 \to 7R3 + 3R2 \qquad R4 \to 7R4 + 3R2 \qquad \begin{pmatrix} 5 & -2 & -3 & 0 \\ 0 & 7 & -2 & -5 \\ 0 & 0 & 50 & -50 \\ 0 & 0 & -20 & 20 \end{pmatrix} \begin{pmatrix} p'_A \\ p'_B \\ p'_C \\ p'_D \end{pmatrix} = \begin{pmatrix} 1 \\ 19 \\ 50 \\ -20 \end{pmatrix}$$

$$R3 \to R3/50 \qquad R4 \to R4/20 \qquad \begin{pmatrix} 5 & -2 & -3 & 0 \\ 0 & 7 & -2 & -5 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & -1 & 1 \end{pmatrix} \begin{pmatrix} p'_A \\ p'_B \\ p'_C \\ p'_D \end{pmatrix} = \begin{pmatrix} 1 \\ 19 \\ 1 \\ -1 \end{pmatrix}$$

$$R4 \to R4 + R3 \qquad \begin{pmatrix} 5 & -2 & -3 & 0 \\ 0 & 7 & -2 & -5 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} p'_A \\ p'_B \\ p'_C \\ p'_D \end{pmatrix} = \begin{pmatrix} 1 \\ 19 \\ 1 \\ 0 \end{pmatrix}$$

p'_D can be chosen arbitrarily; we could set an arbitrary convenient value, but it is perfectly legitimate to leave it as undetermined.

Back-substituting:

$$p'_C - p'_D = 1$$

$$\Rightarrow \qquad p'_C = 1 + p'_D$$

$$7p'_B - 2p'_C - 5p'_D = 19$$

$$\Rightarrow \qquad p'_B = \frac{19 + 2p'_C + 5p'_D}{7} \qquad = p'_D + 3$$

$$5p'_A - 2p'_B - 3p'_C = 1$$

$$\Rightarrow \qquad p'_A = \frac{1 + 2p'_B + 3p'_C}{5} \qquad = p'_D + 2$$

Finally, use these to correct the velocity:

$$
\begin{aligned}
u_B &= 11 + 2(p'_A - p'_B) &&= 9 \\
u_D &= 14 + 2(p'_C - p'_D) &&= 16 \\
v_C &= 8 + 3(p'_A - p'_C) &&= 11 \\
v_D &= 5 + 3(p'_B - p'_D) &&= 14
\end{aligned}
$$

with p'_D simply cancelling in each case.

(b) Although (non-unique) velocity corrections alone could have been computed to make the system mass-consistent, the only way to ensure that it also remains a solution of the momentum equation is to relate velocity and pressure corrections in the form implied by the momentum equation.

Q1.
(a) $(p_w - p_e)A$

(b) Since $V = A\Delta x$, the net force per unit volume is

$$\frac{(p_w - p_e)A}{A\Delta x} = \frac{p_w - p_e}{\Delta x}$$

(c) The average pressure gradient is

$$\frac{p_e - p_w}{\Delta x}$$

The net pressure force per unit volume is *minus* the pressure gradient. Shrinking the control volume to a point, the pressure force in the x-direction, per unit volume, is $-\dfrac{\partial p}{\partial x}$.

Q2.

The net pressure force component in x or y direction on a face is given by:

pressure × *projected* area *into* the cell.

Hence,

$$F_x = \sum_{faces} p(-\Delta y)$$

$$F_y = \sum_{faces} p\Delta x$$

where (Δx, Δy) is the cell-edge vector as the cell boundary is traversed anticlockwise. (You shouldn't worry too much about the signs – it is always obvious from the geometry whether an individual pressure-force contribution is positive or negative.)

Traversing the given cell anticlockwise, starting from the lowest face (edge in 2d):

$$F_x = 5\times 2 + 7\times(-4) + 9\times 2 \quad = 0$$

$$F_y = 5\times 4 + 7\times(-2) - 9\times 2 \quad = -12$$

Q3.

(a) The discretised *momentum* equation for a single finite-volume cell can be written in the form (e.g. for the *x*-component of momentum):

$$a_P u_P - \sum_F a_F u_F = A(p_w - p_e) + other \;\; forces$$

Thus, there is a relationship between the velocity at any location and the *centred pressure difference* across it:

$$u_p = d_p(p_w - p_e) + \cdots$$

or, in terms of centred differences:

$$u = -d\Delta p + \cdots$$

If this relationship is translated to the velocities on cell faces then the *mass-conservation* equation yields (focusing on the *x*-directed faces):

$$\begin{aligned} 0 &= (\rho uA)_e - (\rho uA)_w + \cdots \\ &= (\rho dA)_e (p_P - p_E) - (\rho dA)_w (p_W - p_P) + \cdots \end{aligned}$$

hence producing a discrete equation between pressure values at nodes.

(b) If velocity and pressure are co-located then, because interpolation must be applied to determine pressures on cell faces, the momentum equation gives (in index notation):

$$\begin{aligned} u_i &= d_i \left[\tfrac{1}{2}(p_{i-1} + p_i) - \tfrac{1}{2}(p_i + p_{i+1})\right] + \cdots \\ &= \tfrac{1}{2} d_i \left[p_{i-1} - p_{i+1}\right] + \cdots \end{aligned}$$

Thus, the net pressure force depends only on the difference in pressure between *alternate*, not *successive* nodes.

If linear interpolation is used to get the velocities on cell faces in the continuity equation then

$$\begin{aligned} 0 &= (\rho uA)_{i+1/2} - (\rho uA)_{1-1/2} + \cdots \\ &= (\rho A)\left[\tfrac{1}{2}(u_i + u_{i+1}) - \tfrac{1}{2}(u_{i-1} + u_i)\right] + \cdots \\ &= \tfrac{1}{2}(\rho A)\left[u_{i+1} - u_{i-1}\right] + \cdots \\ &= \tfrac{1}{4}(\rho A)\left[d_{i+1}(p_i - p_{i+2}) - d_{i-1}(p_{i-2} - p_i)\right] + \cdots \end{aligned}$$

Hence, in a co-located arrangement with linear interpolation for advective fluxes both momentum and continuity equations lead to relations between pressures at alternate rather than successive nodes, leading to decoupling of the odd and even nodal values.

The interpolation procedure of Rhie and Chow separately interpolates pressure and non-pressure ("*pseudo-velocity*") parts to the cell face. i.e. if the momentum equation predicts a velocity-pressure linkage of the form

$$u = \hat{u} - d\Delta p$$

then the value on a cell face is determined by

$$u_{face} = \hat{u}_{face} - d_{face}\Delta p$$

where Δ denotes a centred difference. The pseudo-velocity has first to be worked out at *nodes* by inverting the relationship:

$$\hat{u} = u + d\Delta p$$

with the centred pressure difference this time being worked out across the cell.

(c)

	•		•		•		•
u	5		4		3		2
p	0.6		0.7		1.1		1.6
p_{face}		0.65		0.9		1.35	
$\hat{u}_{node}\ (= u + 3\Delta p)$			4.75		4.35		
$\hat{u}_{face}$				4.55			
$u_{face} (= \hat{u}_{face} - 3\Delta p)$				3.35			

The velocity on cell face *f* is 3.35.

(d) SIMPLE method:

(i) Solve the momentum equations with the current pressure p^*.

(ii) Rewrite the mass-conservation equation as a pressure-correction equation using the velocity-correction formulae:

$u \to u^* - d\Delta p'$ (applied at *scalar-cell faces*)

(iii) Solve the resulting pressure-correction equation and correct both velocity and pressure:

$u \to u^* - d\Delta p'$ (applied at *velocity nodes*)

$p \to p^* + p'$

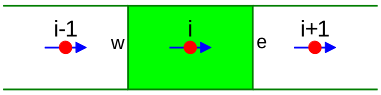
Repeat (i)–(iii) until both momentum and continuity equations are satisfied simultaneously.

Q4.

(a) If there is co-located storage of velocity and pressure with linear interpolation for cell-face values then both momentum and mass equations only relate pressures at alternate nodes. This leads to large oscillations in the pressure field because pressure values at nodes with odd numbers are unconnected with those at even numbers – i.e. *odd-even decoupling*.

In the <u>momentum</u> equation the net pressure force involves

$$p_w - p_e = \tfrac{1}{2}(p_{i-1} + p_i) - \tfrac{1}{2}(p_i + p_{i+1})$$
$$= \tfrac{1}{2}(p_{i-1} - p_{i+1})$$

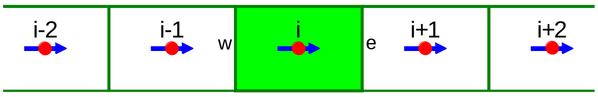


Hence, the discretised momentum equation has the form:

$$u_i = \tfrac{1}{2} d_i (p_{i-1} - p_{i+1}) + \cdots$$

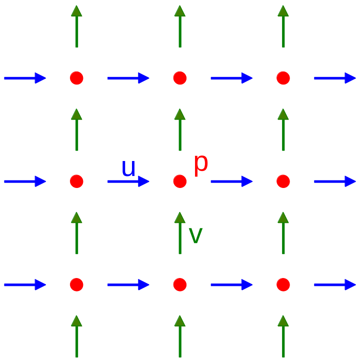
In the <u>continuity</u> equation the net mass flux depends on

$$u_w - u_e = \tfrac{1}{2}(u_{i-1} + u_i) - \tfrac{1}{2}(u_i + u_{i+1})$$
$$= \tfrac{1}{2}(u_{i-1} - u_{i+1})$$
$$= \tfrac{1}{4}[d_{i-1}(p_{i-2} - p_i) - d_{i+1}(p_i - p_{i+2})] + \cdots$$



Thus, both mass and momentum equations only produce links between pressures at alternate nodes, leading to odd-even decoupling.

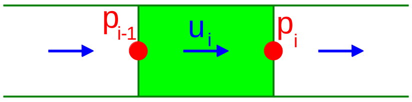
(b) In the *staggered-grid* arrangement, velocity components are stored half-way between the pressure nodes that drive them.



On a **Cartesian** mesh …

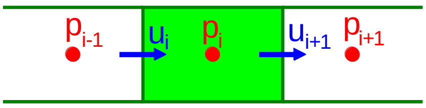
In the <u>momentum</u> equation pressure is stored at precisely the points required to compute the pressure force.

$$u_i = d_i (p_{i-1} - p_i) + \cdots$$



In the <u>continuity</u> equation velocity is stored at precisely the points required to compute mass fluxes. The net mass flux involves:

$$u_{i+1} - u_i = d_{i+1}(p_i - p_{i+1}) - d_i(p_{i-1} - p_i) + \cdots$$
$$= -d_i p_{i-1} + (d_i + d_{i+1}) p_i - d_{i+1} p_{i+1}$$



In both cases no interpolation is required for cell-face values and there is a strong linkage between **successive**, rather than **alternate** pressure nodes, avoiding odd-even decoupling.

<u>Advantages</u>

- No interpolation required; variables are stored where they are needed.
- No problem of odd-even pressure decoupling

<u>Disadvantages</u>

- Added geometrical complexity from multiple sets of nodes and control volumes.
- If the mesh is not Cartesian then the velocity nodes may cease to lie between the pressure nodes that drive them.

(c) In the Rhie-Chow algorithm the pressure and non-pressure parts of the velocity

$$u = \hat{u} - d\Delta p$$

are separately interpolated to the cell face.

(i) First work out the non-pressure part ("*pseudovelocity*") at nodes:

$\hat{u} = u + d\Delta p$ with centred difference Δp from interpolated face values

(ii) Then linearly interpolate $\hat{u}$ and d to the cell face:

$u_{face} = \hat{u}_{face} - d_{face}\Delta p$ with centred difference Δp taken from adjacent nodes

(d) (i) If p_W = 1.2 then we have the following.

		•		•		•		•		
u		2		2		3		5		
p		1.2		1.2		0.8		0.6		
p_{face}			1.2		1.0		0.7			Interpolation of p
$\hat{u}$				1.6		2.4				$\hat{u} = u + 2(p_e - p_w)$
$\hat{u}_{face}$					2.0					Interpolation of $\hat{u}$
u_{face}					2.8					$u_{face} = \hat{u}_{face} - 2(p_E - p_W)$

Answer: u_{face} = 2.8

(ii) If p_W = 1.0 then the pressure gradient, and hence Δp, is uniform. The velocity on the cell face can then be obtained by linear interpolation without repeating the whole calculation; i.e.

u_{face} = 2.5

(e)

The net pressure force component in x or y direction on a particular face (edge, in 2-d) is given by:

pressure × *projected* area *into* the cell.

Hence,

$$F_x = \sum_{faces} p(-\Delta y)$$

$$F_y = \sum_{faces} p\Delta x$$

where the (Δx, Δy) are the cell-edge vectors as the boundary is traversed anticlockwise.

Thus,

$$F_x = 3\times(-3) \quad +9\times 1 \quad +5\times 2 \quad +4\times 0 \quad = \quad 10$$

$$F_y = 3\times 1 \quad +9\times(-4) \quad +5\times 1 \quad +4\times 2 \quad = \quad -20$$

east *north* *west* *south*

Q5.
Corrected velocities:

$$u_1 = 4$$
$$u_2 = 3 - 4(p'_2 - p'_1)$$
$$u_3 = 5 - 4(p'_3 - p'_2)$$
$$u_4 = 6 - 4(p'_4 - p'_3)$$

Mass conservation for cells centred on the internal pressure nodes yields pressure-correction equations as follows.

<u>Cell 1</u>

$$u_2 - u_1 = 0$$
$$\Rightarrow \quad 3 - 4(p'_2 - p'_1) - 4 = 0$$
$$\Rightarrow \quad 4p'_1 - 4p'_2 = 1$$

<u>Cell 2</u>

$$u_3 - u_2 = 0$$
$$\Rightarrow \quad 5 - 4(p'_3 - p'_2) - 3 + 4(p'_2 - p'_1) = 0$$
$$\Rightarrow \quad -4p'_1 + 8p'_2 - 4p'_3 = -2$$

<u>Cell 3</u>

$$u_4 - u_3 = 0$$
$$\Rightarrow \quad 6 - 4(p'_4 - p'_3) - 5 + 4(p'_3 - p'_2) = 0$$
$$\Rightarrow \quad -4p'_2 + 8p'_3 - 4p'_4 = -1$$

Solving à la tri-diagonal matrix algorithm …
From the first,

$$4p'_1 = 4p'_2 + 1$$

Then, from the second,

$$-(4p'_2 + 1) + 8p'_2 - 4p'_3 = -2 \qquad \text{or} \qquad 4p'_2 = 4p'_3 - 1$$

Then, from the third,

$$-(4p'_3 - 1) + 8p'_3 - 4p'_4 = -1 \qquad \text{or} \qquad 4p'_3 = 4p'_4 - 2$$

Hence,

$$p'_3 = p'_4 - \tfrac{1}{2}$$
$$p'_2 = p'_4 - \tfrac{3}{4}$$
$$p'_1 = p'_4 - \tfrac{1}{2}$$

(p'_4 can be chosen as anything convenient, since it is only pressure *differences* that matter).

Substituting these into the velocity-correction formulae gives

$$u_1 = u_2 = u_3 = u_4 = 4$$

as anticipated.

Q6.

(a) Pressure-correction methods are *iterative* methods for the simultaneous solution of mass and momentum equations, which work by making small corrections to the pressure field to "nudge" the velocity field toward mass conservation.

They consist of alternate iterations of:

- the momentum equation with the current pressure;
- a pressure-correction equation, derived by substituting the velocity-pressure relationship implied by the momentum equation into the continuity equation.

(b)

- SIMPLE is iterative, PISO is non-iterative;
- PISO is always time-dependent; SIMPLE may be steady or time-dependent.

(c) Total flow rates:

inflow: $6A + 6A = 12A$

outflow: $12A + 4A = 16A$

where A is the face area of a cell.

For global mass conservation we require "total flow in = total flow out" and hence the outflow velocities must be multiplied by a scaling factor

$$\frac{12}{16} = 0.75$$

The outflow velocities then become

$u_E = 9, \quad u_F = 3$

(d)

Applying mass conservation to scalar cells A – D in turn we require (on dividing by the common cell face area A):

$$u_C - 6 + 0 - v_A = 0$$
$$u_D - 6 + v_A - 0 = 0$$
$$9 - u_C + 0 - v_C = 0$$
$$3 - u_D + v_C - 0 = 0$$

Substituting for the velocities in terms of the current velocities and pressure corrections:

$$2 + 3(p'_A - p'_C) - 6 + 0 - 1 - 2(p'_B - p'_A) = 0$$
$$1 + 3(p'_B - p'_D) - 6 + 1 + 2(p'_B - p'_A) - 0 = 0$$
$$9 - 2 - 3(p'_A - p'_C) + 0 - (-2) - 2(p'_D - p'_C) = 0$$
$$3 - 1 - 3(p'_B - p'_D) + (-2) + 2(p'_D - p'_C) - 0 = 0$$

Hence,

$$\begin{pmatrix} 5 & -2 & -3 & 0 \\ -2 & 5 & 0 & -3 \\ -3 & 0 & 5 & -2 \\ 0 & -3 & -2 & 5 \end{pmatrix} \begin{pmatrix} p'_A \\ p'_B \\ p'_C \\ p'_D \end{pmatrix} = \begin{pmatrix} 5 \\ 4 \\ -9 \\ 0 \end{pmatrix}$$

Solve by Gaussian elimination.

$R2 \to 5R2 + 2R1$
$R3 \to 5R3 + 3R1$

$$\begin{pmatrix} 5 & -2 & -3 & 0 \\ 0 & 21 & -6 & -15 \\ 0 & -6 & 16 & -10 \\ 0 & -3 & -2 & 5 \end{pmatrix} \begin{pmatrix} p'_A \\ p'_B \\ p'_C \\ p'_D \end{pmatrix} = \begin{pmatrix} 5 \\ 30 \\ -30 \\ 0 \end{pmatrix}$$

$R3 \to 7R3 + 2R2$
$R4 \to 7R4 + R2$

$$\begin{pmatrix} 5 & -2 & -3 & 0 \\ 0 & 21 & -6 & -15 \\ 0 & 0 & 100 & -100 \\ 0 & 0 & -20 & 20 \end{pmatrix} \begin{pmatrix} p'_A \\ p'_B \\ p'_C \\ p'_D \end{pmatrix} = \begin{pmatrix} 5 \\ 30 \\ -150 \\ 30 \end{pmatrix}$$

$R4 \to 5R4 + R3$

$$\begin{pmatrix} 5 & -2 & -3 & 0 \\ 0 & 21 & -6 & -15 \\ 0 & 0 & 100 & -100 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} p'_A \\ p'_B \\ p'_C \\ p'_D \end{pmatrix} = \begin{pmatrix} 5 \\ 30 \\ -150 \\ 0 \end{pmatrix}$$

p'_D can be chosen arbitrarily; (pressure is fixed only up to a constant).

Back-substituting:

$$100p'_C - 100p'_D = -150 \quad \Rightarrow \quad p'_C = p'_D - 1.5$$
$$21p'_B - 6p'_C - 15p'_D = 30 \quad \Rightarrow \quad p'_B = p'_D + 1$$
$$5p'_A - 2p'_B - 3p'_C = 5 \quad \Rightarrow \quad p'_A = p'_D + 0.5$$

Finally, update velocity:

$$u_C = 2 + 3(p'_A - p'_C) \quad = 8$$
$$u_D = 1 + 3(p'_B - p'_D) \quad = 4$$
$$v_A = 1 + 2(p'_B - p'_A) \quad = 2$$
$$v_C = -2 + 2(p'_D - p'_C) \quad = 1$$

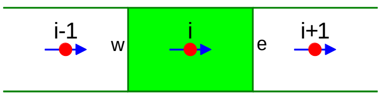
Q7.
(a)
(i) Pressure-correction methods are *iterative* methods for the simultaneous solution of mass and momentum equations, which work by making small corrections to the pressure field to "nudge" the velocity field toward mass conservation.

They consist of alternate iterations of:
- the momentum equation with the current pressure;
- a pressure-correction equation, derived by substituting the velocity-pressure relationship implied by the momentum equation into the continuity equation.

(ii) If there is co-located storage of velocity and pressure with linear interpolation for cell-face values then both momentum and mass equations only relate pressures at alternate nodes. This leads to large oscillations in the pressure field because pressure values at nodes with odd numbers are unconnected with those at even numbers – i.e. *odd-even decoupling*.

In the <u>momentum</u> equation the net pressure force involves

$$p_w - p_e = \tfrac{1}{2}(p_{i-1} + p_i) - \tfrac{1}{2}(p_i + p_{i+1}) = \tfrac{1}{2}(p_{i-1} - p_{i+1})$$

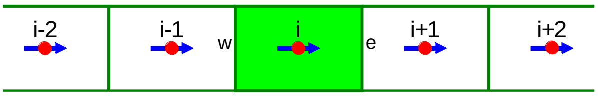


Hence, the discretised momentum equation has the form:

$$u_i = \tfrac{1}{2} d_i (p_{i-1} - p_{i+1}) + \cdots$$

In the <u>continuity</u> equation the net mass flux depends on

$$\begin{aligned} u_w - u_e &= \tfrac{1}{2}(u_{i-1} + u_i) - \tfrac{1}{2}(u_i + u_{i+1}) \\ &= \tfrac{1}{2}(u_{i-1} - u_{i+1}) \\ &= \tfrac{1}{4}[d_{i-1}(p_{i-2} - p_i) - d_{i+1}(p_i - p_{i+2})] + \cdots \end{aligned}$$



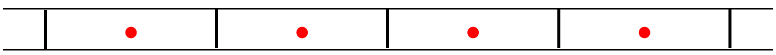
Thus, both mass and momentum equations only produce links between pressures *at alternate nodes*, leading to odd-even decoupling and large oscillations in the pressure field.

(b) In the Rhie-Chow algorithm we write

$$u = \hat{u} - d\Delta p$$

and interpolate pressure and non-pressure parts separately to the cell face. $\hat{u}$ is worked out at nodes by the inverse relation $= \hat{u} + d\Delta p$

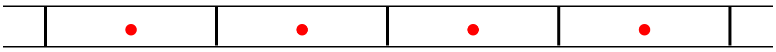
(i)



u	4		2		2		3		
p	0.3		0.5		0.4		0.3		
p_{face}		0.4		0.45		0.35			(by interpolation)
$\hat{u}$			2.2		1.6				$\hat{u} = u + 4(p_e - p_w)$
$\hat{u}_{face}$				1.9					(by interpolation)
u_{face}				<u>2.3</u>					$u_{face} = \hat{u}_{face} - 4(p_E - p_W)$

Answer: $u_f = 2.3$; (more than would be obtained by linear interpolation because the scheme is trying to smooth out the local pressure maximum to the left of the face).

(ii)



u	4		2		2		3	
p	0.3		0.3		0.4		0.3	
p_{face}		0.3		0.35		0.35		(by interpolation)
$\hat{u}$			2.2		2			$\hat{u} = u + 4(p_e - p_w)$
$\hat{u}_{face}$				2.1				(by interpolation)
u_{face}				$\underline{1.7}$				$u_{face} = \hat{u}_{face} - 4(p_E - p_W)$

Answer: u_f = 1.7; (less than would be obtained by linear interpolation because the scheme is trying to smooth out the local pressure maximum to the right of the face).

(c)

(i) The net pressure force component in x or y direction on a face is given by:

pressure × *projected* area *into* the cell

Hence, accounting for direction if the cell is traversed anticlockwise:

$$F_x = \sum_{faces} p(-\Delta y)$$

$$F_y = \sum_{faces} p\Delta x$$

Thus, starting from the lowest face:

$$F_x = 3\times 0 + 5\times(-5) + 2\times 5 \quad = -15$$

$$F_y = 3\times 4 + 5\times(-1) + 2\times(-3) \quad = 1$$

Answer: $\mathbf{F}$ = (–15, 1).

(ii) Since "force = mass × acceleration" we need the mass of the cell.

Since it has unit depth and is a triangle in plan, the cell has volume

$$\tfrac{1}{2}\times base \times height \quad = \tfrac{1}{2}\times 4\times 5 \quad = 10$$

Since ρ = 1.0, the mass is 10 units also. Hence,

$$\mathbf{a} = \frac{1}{m}\mathbf{F} \quad = \frac{1}{10}\times(-15, 1) \quad = (-1.5, 0.1)$$

Answer: $\mathbf{a} = (-1.5, 0.1)$.