Q.1 Question 1

A steam power plant operates with coupled regenerative and reheat Rankine cycle with 2 connected turbines as shown in Fig. 1. Primary steam is supplied by the boiler at 120 bar and 565°C. Conditions for water/steam flows are described in Table 1.

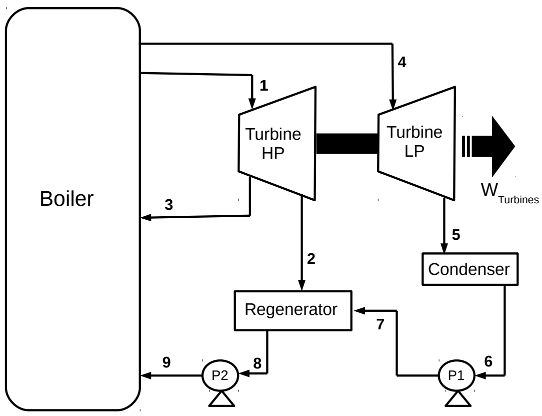

Figure 1: *Reheat and regenerative Rankine cycle with 2 turbines.*

Table 1: *Thermodynamic table of the reheat and regenerative Rankine cycle.*

Stage	**P (bar)**	**T (^{o}C)**	**State**	**H (kJ.kg^{-1})**	**S (kJ.(kg.K)$^{-1}$)**	**Steam Quality**
1	120	565	**(a)**	**(b)**	**(c)**	–
2	3	–	wet vapour	**(d)**	–	**(e)**
3	3	250	–	–	–	–
4	3	475	–	–	–	–
5	0.06	–	wet vapour	**(f)**	–	**(g)**
6	–	–	sat.liquid	**(h)**	–	–
7	3	–	–	**(i)**	–	–
8	3	–	–	–	–	–
9	120	–	–	**(j)**	–	–

(a) In Table 1, determine *(a)-(j)*. [10 marks]

Solution:

In order to fill the Table we need to calculate the thermodynamic properties for each stage of the cycle:

Stage 1: *The fluid leaving the boiler towards the first turbine is at 120 bar and 565°C. This is well above the saturation temperature ($T_{sat} = 324.60^oC$) and we can thus confirm that the fluid is superheated steam . At such pressure, the superheated steam tables (SHST) results in (through linear interpolation):* **[1/10]**

[1/10] $H_1 = 3518.77\dfrac{kJ}{kg}$ *and*

[1/10] $S_1 = 6.6983\dfrac{kJ}{kg.K}$.

Stage 2: *Isentropic expansion in HP Turbine at* $P_2 = 3$ *bar* $\Leftrightarrow$ $S_2 = S_1 =$
[1/10] $6.6983\dfrac{kJ}{kg.K}$. *The fluid is at wet vapour state. The quality of the steam is*

$$x_2 = \frac{S_2 - S_f}{S_g - S_f} = \frac{6.6983 - 1.6716}{6.9909 - 1.6716} = 0.9450$$

[1/10] *Now calculating the enthalpy,*

$$x_2 = \frac{H_2 - H_f}{H_g - H_f} = 0.9450 \Leftrightarrow H_2 = 2605.72\frac{kJ}{kg}$$

Stage 3: *The fluid at* $P_3 = P_2 = 3.0$ *bar and* $T_3 = 250^oC$ $(>> T_{sat} = 133.5^oC)$ *is superheated steam with* $H_3 = 2967.6\dfrac{kJ}{kg}$ *and* $S_3 = 7.517\dfrac{kJ}{kg.K}$.

Stage 4: *The steam leaves the boiler towards the LP turbine at* $P_4 = P_3 = 3.0$ *bar and* $T_4 = 475^oC$ *(also as superheated steam) with (via linear interpolation)* $H_4 = 3433.33\dfrac{kJ}{kg}$ *and* $S_4 = 8.252\dfrac{kJ}{kg}$.

Stage 5: *Isentropic expansion with* $P_5 = 0.060$ *bar* $($*with* $S_5 = S_4)$. *The quality of*
[1/10] *the steam is*

$$x_5 = \frac{S_5 - S_f}{S_g - S_f} = \frac{8.252 - 0.521}{8.330 - 0.521} = 0.99$$

[1/10] *and the enthalpy,*

$$x_5 = 0.99 = \frac{H_5 - H_f}{H_g - H_f} = \frac{H_5 - 151.5}{2567.4 - 151.5} \Leftrightarrow H_5 = 2543.24\frac{kJ}{kg}$$

Stage 6: *The fluid leaves the condenser at* $P_6 = P_5 = 0.06$ *bar is saturated liquid*
[1/10] *with* $H_6 = H_f\,(0.06\ bar) = 151.5\dfrac{kJ}{kg}$

Stage 7: *Saturated and incompressible liquid leaving the pump towards the regener-*
[1/10] *ator at* $P_7 = P_2 = 3.0$ *bar,*

$$\begin{aligned} H_7 &\approx H_6 + V_6\,(P_7 - P_6) \\ &\approx 151.5\frac{kJ}{kg} + 0.001006\frac{m^3}{kg}\,(3 - 0.06)\ bar \times \frac{10^5\dfrac{N}{m^2}}{1\ bar} \times \frac{1\ J}{1\ N.m} \times \frac{10^{-3}\ kJ}{1\ J} \\ &\approx 151.795\ \frac{kJ}{kg} \end{aligned}$$

Stage 8: *Saturated liquid water leaving the regenerator at* $P_8 = 3.0$ *bar with* $H_8 = H_f\,(3.0\ bar) = 561.4\dfrac{kJ}{kg}$ *and* $V_8 = 0.001068\dfrac{m^3}{kg}$.

[1/10] **Stage 9:** *Saturated and incompressible liquid at* $P_9 = 120$ *bar,*

$$\begin{aligned} H_9 &\approx H_8 + V_8\,(P_9 - P_8) \\ &\approx 561.4\frac{kJ}{kg} + 0.001068\frac{m^3}{kg}\,(120 - 3)\ bar \times \frac{10^5\dfrac{N}{m^2}}{1\ bar} \times \frac{1\ J}{1\ N.m} \times \frac{10^{-3}\ kJ}{1\ J} \\ &\approx 573.97\ \frac{kJ}{kg} \end{aligned}$$

Thus the Table becomes:

Stage	P (bar)	T (^{o}C)	State	H (kJ.kg^{-1})	S (kJ.(kg.K)$^{-1}$)	Steam Quality
1	*120*	*565*	**Superheated vapour**	**3518.77**	**6.6983**	–
2	*3*	–	*wet vapour*	**2605.72**	–	**0.9450**
3	*3*	*250*	–	–	–	–
4	*3*	*475*	–	–	–	–
5	*0.06*	–	*wet vapour*	**2543.24**	–	**0.99**
6	–	–	*sat.liquid*	**151.5**	–	–
7	*3*	–	–	**151.8**	–	–
8	*3*	–	–	–	–	–
9	*120*	–	–	**573.97**	–	–

(b) Calculate the fraction (as %) of steam supplied to the low-pressure (LP) turbine. [2 marks]

Solution:
Energy balance in the regenerator, assuming total mass of water of ($m_T =$) *1 kg, and that a fraction,* $\mathcal{F}$*, is bled-off from the HP turbine to the regenerator, and the remaining water-steam,* $1 - \mathcal{F}$ *is conducted back to the boiler*

$$m_T H_8 = \mathcal{F} H_2 + (1 - \mathcal{F}) H_7 \Rightarrow \mathcal{F} = 0.1669kg$$

Thus, $m_2 = \mathcal{F} = 0.1669$ *kg and* $m_3 = m_4 = m_5 = m_6 = m_7 = (1 - \mathcal{F}) = 0.833$ *kg*
[2/2] $\Longrightarrow$ *83.3% of the steam was supplied to the LP turbine.*

(c) Determine the heat supplied by the boiler. [2 marks]
Solution:
[2/2] *The heat supplied by the boiler* (Q_{Boiler}) *can be calculated through the energy balance,*

$$Q_{Boiler} = [m_T H_1 + (1 - \mathcal{F}) H_4] - [(1 - \mathcal{F}) H_3 + m_T H_9] \Rightarrow Q_{Boiler} = 3332.75 \frac{kJ}{kg}$$

(d) Determine the thermal efficiency of the cycle, [6 marks]

$$\eta = \frac{W_{Total}}{Q_{Boiler}} = \frac{\sum W_{\text{Turbines}} - \sum W_{\text{Pumps}}}{Q_{Boiler}}$$

Solution:
Now, in order to calculate the thermal efficiency of the cycle,

$$\eta = \frac{W_{Total}}{Q_{Boiler}} = \frac{\sum W_{Turbines} - \sum W_{Pumps}}{Q_{Boiler}}$$

We need to calculate the work associated with the turbines and pumps.

[1/6] **HP Turbine:** $W_{T,HP} = m_T H_1 - [\mathcal{F} H_2 + (1 - \mathcal{F}) H_3] = 611.86 \frac{kJ}{kg}$

[1/6] **LP Turbine:** $W_{T,LP} = (1 - \mathcal{F})(H_4 - H_5) = 741.44 \frac{kJ}{kg}$

[**1/6**] **Pump 1:** $W_{P,1} = (1 - \mathcal{F})\,(H_7 - H_6) = 0.246\dfrac{kJ}{kg}$

[**1/6**] **Pump 2:** $W_{P,2} = m_T\,(H_9 - H_8) = 12.57\dfrac{kJ}{kg}$

[**2/6**] *Thus the thermal efficiency of the cycle is,*

$$\eta = \frac{\sum W_{Turbines} - \sum W_{Pumps}}{Q_{Boiler}} = \frac{1339.76}{3332.27} = 0.4022$$

To solve this problem, you should assume that the saturated liquid streams are incompressible, and therefore $dH = VdP$ (where H, V and P are enthalpy, volume and pressure, respectively). Quality of the steam is expressed as

$$x_j = \frac{\Psi_j - \Psi_f}{\Psi_g - \Psi_f} \quad \text{with } \Psi = \{H, S\}$$

Q.2 **Question 2**

Refrigerant R-134a is used in a geothermal heat pump system (Fig. 2) to a storage in an industrial facility at 40°C. The heat pump uses underground water from a well to produce a heating capacity of 6 tons. Determine:

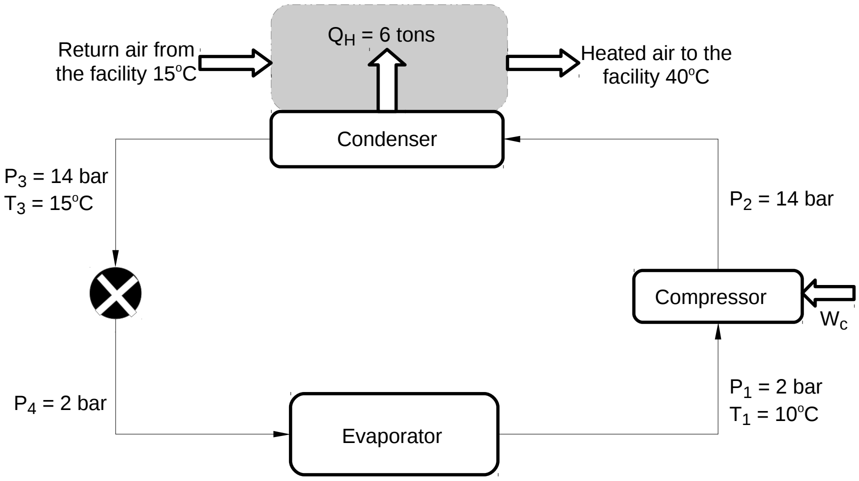

Figure 2: *Heat pump cycle.*

1. Enthalpies and Entropies: H_i, S_i with $i = \{1, 2, 3, 4\}$; [8 marks]
   Solution:
   Calculating all enthalpies and entropies of the cycle:

   Stage 1: *The refrigerant fluid leaves the evaporator at $P_1 = 2$ bar and $T_1 = 10^oC >> T_{sat} = -10.09^oC$, thus the fluid is at superheated vapour state and*
[2/8] *with $H_1 = 258.89\dfrac{kJ}{kg}$ and $S_1 = 0.9898\dfrac{kJ}{kg.K}$.*

   Stage 2: *Isentropic compression, $S_2 = S_1$, and assuming ideal compressor at $P_2 =$*
[2/8] *14 bar. Via linear interpolation, $H_2 = 303.66\dfrac{kJ}{kg}$ and $T_2 = 77.08^oC$.*

   Stage 3: *The fluid leaves the condenser at $P_3 = 14$ bar and $T_3 = 15^oC$ ($<< T_{sat} = 52.4^oC$)*
[2/8] *is a sub-cooled liquid with $H_3 = 125.26\dfrac{kJ}{kg}$ and $S_3 = 0.4453\dfrac{kJ}{kg.K}$.*

[2/8] **Stage 4:** *Isenthalpic process, $H_4 = H_3$, at $P_4 = 2$ bar. Calculating the quality of the vapour,*

$$x_4 = \frac{H_4 - H_f}{H_g - H_f} = \frac{125.26 - 36.84}{241.30 - 36.84} = 0.4325$$

   and the entropy,

$$x_4 = \frac{S_4 - S_f}{S_g - S_f} = \frac{S_4 - 0.1481}{0.9253 - 0.1481} \Rightarrow S_4 = 0.4842\frac{kJ}{kg.K}$$

2. Volumetric flow rate of heated air to the room (m^3/s); [3 marks]
   Solution:
   In order to calculate the volumetric flow rate of heated air, we first need to determine
[1/3] *the mass flow rate,*

$$Q_H = \dot{m}_{air}\left(H^{air}_{out} - H^{air}_{in}\right) = \dot{m}_{air} C^{air}_p \left(T^{air}_{out} - T^{air}_{in}\right)$$

$$6\ ton \times \frac{210\frac{kJ}{min}}{1\ ton} \times \frac{1\ min}{60s} = \dot{m}_{air} \times 1.004\frac{kJ}{kg.K}\left(40-15\right)^o C$$

$$\dot{m}_{air} = 0.8367\frac{kg}{s}$$

[2/3] *Now for the volumetric flow rate* ($with\ T = 40^oC\ \ and\ \ P = 1.01325\ bar$)

$$\begin{aligned}\dot{V}^{out}_{air} &= \dot{m}_{air} V^{out}_{air} = \dot{m}_{air}\frac{RT^{out}_{air}}{P_{air}} \\ &= 0.8367\frac{kg}{s} \times 287\frac{J}{kg.K} \times \frac{(40+273.15)\ \ K}{1.0125\ \ bar} \times \frac{1\ \ N.m}{1\ \ J} \times \frac{1\ \ bar}{10^5\ \ N/m^2} \\ &= 0.7426\frac{m^3}{s}\end{aligned}$$

3. Mass flow rate of the R-134a refrigerant fluid; [3 marks]
   Solution:

[3/3] *The mass flow rate of the refrigerant fluid R-134a can be calculated as,*

$$Q_H = \dot{m}_R\left(H_2 - H_3\right) \Rightarrow \dot{m}_R = 0.1177\frac{kg}{s}$$

4. Compressor power (W_C) in kW; [3 marks]
   Solution:

[3/3]

$$W_C = \dot{m}_R\left(H_2 - H_1\right) \Rightarrow W_C = 5.27\ kW$$

5. Coefficient of performance $\left(\mathrm{COP} = \frac{Q_H}{W_C}\right)$; [3 marks]
   Solution:

[3/3]

$$COP = \frac{Q_H}{W_C} = 3.98$$

Given the heat capacity, $C^{\mathrm{air}}_p = 1.004\ kJ.(kg.K)^{-1}$, and molecular weight, $MW^{\mathrm{air}} = 28.97\ kg.kgmol^{-1}$ of air. Assume that air behaves as an ideal gas. Quality of the vapour is expressed as

$$x_j = \frac{\Psi_j - \Psi_f}{\Psi_g - \Psi_f} \quad \text{with } \Psi = \{H, S\}$$

Q.3 Question 3

A steady flow energy device formed of a turbine with one inlet (labelled 1), and two outlets (labelled 2 and 3), does work on an ideal gas at a rate of 120 kW. The specific gas constant $R = 287\,\mathrm{J\,kg^{-1}\,K^{-1}}$ and the specific heat capacity at constant pressure $c_p = 1003\,\mathrm{J\,kg^{-1}\,K^{-1}}$, while the known conditions at the inlet and each outlet are given in table 2.

Table 2: *Inlet and outlet conditions for the steady flow device*

Inlet/Outlet	Area A (m^2)	Volume flux q (m^3 s^{-1})	Temperature T (°C)	Pressure p (Pa)	Height z (m)
1	0.1	2.0	20	p_1	0.0
2	0.1	1.0	50	200000	10.0
3	0.05	1.0	90	100000	4.0

(a) The steady flow energy equation for a steady flow device with one inlet and one outlet is

$$\frac{\dot{Q} - \dot{W}_s}{\dot{m}} = \left(c_p T_{\text{outlet}} + \frac{u_{\text{outlet}}^2}{2} + g z_{\text{outlet}}\right) - \left(c_p T_{\text{inlet}} + \frac{u_{\text{inlet}}^2}{2} + g z_{\text{inlet}}\right),$$

where u is the gas velocity, $\dot{Q}$ is the rate of heat addition and $\dot{W}_s$ is the rate at which shaft work is done on the gas. Explain how this equation should be changed to model the device described above. [4 marks]

Solution:

The mass flux entering the device $\dot{m}_1$ is now divided between two outlets. Therefore the modified mass conservation equation would be

$$\dot{m}_1 = \dot{m}_2 + \dot{m}_3.$$

[1/4]

With two outlets, the flux of energy must also be split between the two outlets, which gives

$$\dot{Q} - \dot{W}_s = \dot{m}_2\left(c_p T_2 + \frac{u_2^2}{2} + g z_2\right) + \dot{m}_3\left(c_p T_3 + \frac{u_3^2}{2} + g z_3\right) - \dot{m}_1\left(c_p T_1 + \frac{u_1^2}{2} + g z_1\right).$$

[3/4]

(b) Calculate the gas velocity at the inlet and each outlet; [3 marks]

Solution:

The gas velocities are given by

$$u_1 = \frac{q_1}{A_1} = \frac{2.0\,m^3\,s^{-1}}{0.1\,m^2} = 20\,m\,s^{-1},$$
$$u_2 = \frac{q_2}{A_2} = \frac{1.0\,m^3\,s^{-1}}{0.1\,m^2} = 10\,m\,s^{-1},$$
$$u_3 = \frac{q_3}{A_3} = \frac{1.0\,m^3\,s^{-1}}{0.05\,m^2} = 20\,m\,s^{-1}.$$

[3/3]

(c) Determine the pressure at the inlet (p_1); [5 marks]

Solution:

The densities at the outlets can be obtained via the ideal gas equation

$$\rho_2 = \frac{p_2}{RT_2} = \frac{200000\,Pa}{287\,J\,kg^{-1}\,K^{-1}\,(50+273.15)\;K} = 2.1564726\,kg\,m^{-3},$$
$$\rho_3 = \frac{p_3}{RT_3} = \frac{100000\,Pa}{287\,J\,kg^{-1}\,K^{-1}\,(90+273.15)\;K} = 0.9594714\,kg\,m^{-3}.$$

[2/5]

Mass conservation gives $\dot{m}_1 = \dot{m}_2 + \dot{m}_3$ *where the mass flux* $\dot{m} = \rho q$. *Therefore*

$$\rho_1 = \frac{\rho_2 q_2 + \rho_3 q_3}{q_1} = \frac{\left(2.1564726\,kg\,m^{-3} \times 1\,m^3\,s^{-1}\right) + \left(0.9594714\,kg\,m^{-3} \times 1\,m^3\,s^{-1}\right)}{2\,m^3\,s^{-1}}$$
$$=1.557972\,kg\,m^{-3}$$

[2/5]

The pressure at the inlet is now

$$p_1 = \rho_1 R T_1 = 1.557972\,kg\,m^{-3} \times 287\,J\,kg^{-1}\,K^{-1} \times (20+273.15)\;K = 131078.49\,Pa.$$

[1/5]

(d) What is the relative percentage error if the gravitational potential energy terms are neglected when calculating the rate of heat transfer $\dot{Q}$? [8 marks]

Solution:

The mass fluxes at each inlet and exit are

$$\dot{m}_1 = \rho_1 q_1 = 1.557972\,kg\,m^{-3} \times 2\,kg/s = 3.1159440\,kg/s,$$
$$\dot{m}_2 = \rho_2 q_2 = 2.1564726\,kg\,m^{-3} \times 1\,kg/s = 2.1564726\,kg/s,$$
$$\dot{m}_3 = \rho_3 q_3 = 0.9594714\,kg\,m^{-3} \times 1\,kg/s = 0.9594714\,kg/s.$$

[1/8]

The energy fluxes without gravitational potential energy are

$$c_p T_1 + \frac{u_1^2}{2} = 294229.45\,m^2/s^2,$$
$$c_p T_2 + \frac{u_2^2}{2} = 324169.45\,m^2/s^2,$$
$$c_p T_3 + \frac{u_3^2}{2} = 364439.45\,m^2/s^2.$$

[1/8]

The energy fluxes with gravitational potential energy are

$$c_p T_1 + \frac{u_1^2}{2} + g z_1 = 294229.45\,m^2/s^2,$$
$$c_p T_2 + \frac{u_2^2}{2} + g z_2 = 324267.55\,m^2/s^2,$$
$$c_p T_3 + \frac{u_3^2}{2} + g z_3 = 364478.69\,m^2/s^2.$$

[1/8]

The rate of heat addition without gravitational potential energy is

$$\begin{aligned}\dot{Q}_{with} =& \dot{W}_s + \dot{m}_2\left(c_pT_2 + \frac{u_2^2}{2}\right) + \dot{m}_3\left(c_pT_3 + \frac{u_3^2}{2}\right) - \dot{m}_1\left(c_pT_1 + \frac{u_1^2}{2}\right)\\ =& -120000 + 699062.53 + 349669.25 - 916802.49\\ =& 11929.28\ W.\end{aligned}$$

[2/8] *Note turbine does work on gas so $\dot{W}_s$ is negative.*

The rate of heat addition with gravitational potential energy is

$$\begin{aligned}\dot{Q}_{without} =& \dot{W}_s + \dot{m}_2\left(c_pT_2 + \frac{u_2^2}{2} + gz_2\right) + \dot{m}_3\left(c_pT_3 + \frac{u_3^2}{2} + gz_3\right) - \dot{m}_1\left(c_pT_1 + \frac{u_1^2}{2} + gz_1\right)\\ =& -120000 + 699274.08 + 349706.9 - 916802.49\\ =& 12178.48\ W.\end{aligned}$$

[2/8] *Note turbine does work on gas so $\dot{W}_s$ is negative.*

The relative percentage difference between the rate of heat addition with and without the gravitational potential energy terms is

$$100\%\left|\frac{\dot{Q}_{with} - \dot{Q}_{without}}{\dot{Q}_{with}}\right| = 100\%\left|\frac{12178.48 - 11929.28}{12178.48}\right| = 2.0462\%.$$

[1/8]

Q.4 Question 4

(a) Gas flows along a pipe of slowly varying cross section in the direction of increasing x. By considering the rate of change of the mass of gas within a small section of pipe and the gas mass fluxes into and out of this section of pipe, show that

$$\frac{\partial}{\partial t}(\rho A) + \frac{\partial}{\partial x}(\rho u A) = 0.$$

Here the gas velocity u, density ρ and pipe cross section A are all functions of x and time t. [5 marks]

Solution:

[1/5] *The total mass contained in a section of pipe of length Δx is $\rho A \Delta x$.*

The rate of change of mass in this section of pipe equals the mass flux entering the pipe $\rho u A$ minus the mass flux leaving the other end of the pipe section

$$\rho u A + \Delta x \frac{\partial}{\partial x}(\rho u A).$$

[2/5] *[Obtained via a linearized Taylor expansion.]*

Hence mass conservation implies

$$\frac{\partial}{\partial t}(\rho A) = \rho u A - \rho u A - \Delta x \frac{\partial}{\partial x}(\rho u A).$$

[2/5] *and hence dividing by Δx and rearranging gives the result.*

(b) Explain what is meant by a steady flow and show that for steady flow in a pipe of uniform cross section

$$\frac{1}{\rho}\frac{\mathrm{d}\rho}{\mathrm{d}x} + \frac{1}{u}\frac{\mathrm{d}u}{\mathrm{d}x} = 0.$$

[4 marks]

Solution:

The flow is steady if the density, velocity and cross section depend only on x and not time t. In this case the result from (a) gives

$$\frac{\mathrm{d}}{\mathrm{d}x}(\rho u A) = 0.$$

[2/4]

If the pipe has uniform cross section, then A is constant. Therefore via the chain rule

$$\rho \frac{\mathrm{d}u}{\mathrm{d}x} + u \frac{\mathrm{d}\rho}{\mathrm{d}x} = 0.$$

[2/4] *Finally dividing by ρu gives the required result.*

(c) For steady compressible flow in a uniform pipe, using the conservation of energy and the laws of thermodynamics; changes in the entropy s, is related to changes in the pressure pressure p, density ρ and velocity u, through

$$T\,\mathrm{d}s + \frac{\mathrm{d}p}{\rho} + u\,\mathrm{d}u = 0.$$

Hence show that the change in pressure and the change in entropy as fluid flows are related through

$$\left(1 - \frac{u^2}{c^2}\right)\frac{\mathrm{d}p}{\mathrm{d}x} = -\rho T\left(1 + \frac{u^2\beta}{c_p}\right)\frac{\mathrm{d}s}{\mathrm{d}x}.$$

In the derivation of this result, you may additionally assume that a change in gas density

$$\mathrm{d}\rho = \frac{\mathrm{d}p}{c^2} - \frac{\rho\beta T}{c_p}\mathrm{d}s,$$

where c is the speed of sound, β is the thermal expansion coefficient and c_p is the specific heat capacity as constant pressure. [8 marks]

Solution:

Variations over x imply

$$T\frac{\mathrm{d}s}{\mathrm{d}x} + \frac{1}{\rho}\frac{\mathrm{d}p}{\mathrm{d}x} + u\frac{\mathrm{d}u}{\mathrm{d}x} = 0, \quad \textit{and} \quad \frac{\mathrm{d}\rho}{\mathrm{d}x} = \frac{1}{c^2}\frac{\mathrm{d}p}{\mathrm{d}x} - \frac{\rho\beta T}{c_p}\frac{\mathrm{d}s}{\mathrm{d}x}.$$

Using the second equation, we eliminate the $\frac{\mathrm{d}\rho}{\mathrm{d}x}$ from the mass conservation equation to give

$$\rho\frac{\mathrm{d}u}{\mathrm{d}x} + \frac{u}{c^2}\frac{\mathrm{d}p}{\mathrm{d}x} - \frac{u\rho\beta T}{c_p}\frac{\mathrm{d}s}{\mathrm{d}x} = 0.$$

[2/8]

Next we eliminate $\frac{\mathrm{d}u}{\mathrm{d}x}$ to give

$$\frac{T}{u}\frac{\mathrm{d}s}{\mathrm{d}x} + \frac{1}{\rho u}\frac{\mathrm{d}p}{\mathrm{d}x} = -\frac{\mathrm{d}u}{\mathrm{d}x} = \frac{u}{\rho c^2}\frac{\mathrm{d}p}{\mathrm{d}x} - \frac{u\beta T}{c_p}\frac{\mathrm{d}s}{\mathrm{d}x}.$$

[2/8]

Collecting all the terms involving $\frac{\mathrm{d}p}{\mathrm{d}x}$ on one side and all the terms involving $\frac{\mathrm{d}s}{\mathrm{d}x}$ on the other side

$$\frac{1}{\rho u}\frac{\mathrm{d}p}{\mathrm{d}x} - \frac{u}{\rho c^2}\frac{\mathrm{d}p}{\mathrm{d}x} = -\frac{u\beta T}{c_p}\frac{\mathrm{d}s}{\mathrm{d}x} - \frac{T}{u}\frac{\mathrm{d}s}{\mathrm{d}x}.$$

$$\left(\frac{1}{\rho u} - \frac{u}{\rho c^2}\right)\frac{\mathrm{d}p}{\mathrm{d}x} = -T\left(\frac{u\beta}{c_p} + \frac{1}{u}\right)\frac{\mathrm{d}s}{\mathrm{d}x}.$$

[3/8]

If we multiply by ρu, then

$$\left(1 - \frac{u^2}{c^2}\right)\frac{\mathrm{d}p}{\mathrm{d}x} = -\rho T\left(1 + \frac{u^2\beta}{c_p}\right)\frac{\mathrm{d}s}{\mathrm{d}x}.$$

[1/8]

(d) Define a Mach number, and (with reference to the result derived in part (c)), explain how the pressure changes as a compressible fluid flows subsonically along a pipe of uniform cross section. [3 marks]

Solution:

The Mach number Ma *is the ratio of the actual speed* u *to the speed of sound* c*, i.e.*

$$\mathrm{Ma} = \frac{u}{c}.$$

[1/3]

For subsonic flow $\mathrm{Ma} < 1$*, so the coefficient of* $\frac{\partial p}{\partial x}$ *is positive. The density, temperature and velocity are all positive, so the* $-\rho T\left(1 + \frac{u^2\beta}{c_p}\right)$ *is negative, while the entropy must*

[1/3] *increase with flow along the pipe (i.e.* $\frac{\mathrm{d}s}{\mathrm{d}x} > 0$*).*

[1/3] *Consequently the pressure must fall as fluid flows along the pipe.*

Q.5 Question 5

(a) Define the specific humidity ω. Assuming both dry air and water vapour behave like ideal gases with specific gas constants $R_a = 0.2871$ kJ/(kg.K) and $R_v = 0.4615$ kJ/(kg.K), respectively, show that

$$\omega = \frac{0.622 p_v}{p - p_v},$$

where p is the absolute pressure and p_v is the partial pressure of water vapour. [5 marks]

Solution:

The specific humidity ω is the ratio of the mass of water vapour m_v to the mass of
[1/5] *dry air m_a in a volume V , i.e.*

$$\omega = \frac{m_v}{m_a}.$$

[1/5]

In terms of densities

$$\omega = \frac{\rho_v V}{\rho_a V} = \frac{\rho_v}{\rho_a}.$$

Treat both water vapour and dry air as ideal gases, so that

$$\omega = \frac{p_v}{R_v T}\frac{R_a T}{p_a} = \frac{0.622 p_v}{p_a}.$$

[2/5]

Finally the partial pressure of water vapour and dry air $p_v + p_a = p$, so

$$\omega = \frac{0.622 p_v}{p - p_v}.$$

[1/5]

(b) Define the relative humidity φ, and hence show that

$$\omega = \frac{0.622 \varphi p_g}{p - \varphi p_g},$$

where the saturation pressure of water is denoted p_g. [3 marks]

Solution:

The relative humidity φ is the ratio of the mass of water vapour m_v to the mass of water vapour at saturation m_g, i.e.

$$\varphi = \frac{m_v}{m_g}.$$

[1/3]

Again using the ideal gas equation

$$\varphi = \frac{m_v}{m_g} = \frac{\rho_v V}{\rho_g V} = \frac{p_v}{R_v T}\frac{R_v T}{p_g} = \frac{p_v}{p_g}.$$

[1/3]

[1/3] *Therefore writing $p_v = \varphi p_g$ in the equation from part (a), gives the required result.*

(c) Air enters an air-conditioning system at 1 atm, 35°C and 60% relative humidity, at a rate of $12\,\mathrm{m^3/min}$. Saturated air leaves the air-conditioning system at a temperature of 16°C. The moisture in the air that condenses during the process is removed at 16°C, while the specific enthalpy of liquid water at 16°C is 67.22 kJ/kg.

i) Determine the rate of moisture removal from the air; [7 marks]

Solution:

The inlet and outlet are at 1 atm (= 101.325 kPa) so we can determine the conditions using the psychrometric chart, which gives

$$h_1 = 90\,kJ/kg \text{ dry air}, \qquad h_2 = 43\,kJ/kg \text{ dry air},$$
$$\omega_1 = 0.022\,kg \text{ water}/kg \text{ dry air}, \qquad \omega_2 = 0.011\,kg \text{ water}/kg \text{ dry air},$$
$$V_1 = 0.905\,m^3/\,kg \text{ dry air}.$$

[3/7]

In the cooling section:

$$\begin{aligned} &\textit{Conservation of dry air:} && \dot{m}_{a_1} = \dot{m}_{a_2} = \dot{m}_a, \\ &\textit{Conservation of water vapour:} && \dot{m}_{a_1}\omega_1 = \dot{m}_{a_2}\omega_2 + \dot{m}_w, \\ & && \Rightarrow \dot{m}_w = \dot{m}_a\left(\omega_1 - \omega_2\right). \end{aligned}$$

[2/7]

The mass flux of dry air is given by

$$\dot{m}_a = \frac{\dot{V}_1}{V_1} = \frac{12\,m^3/min}{0.905\,m^3/\,kg \text{ dry air}} = 13.26\,kg/min.$$

[1/7]

Hence the conservation of water vapour gives

$$\begin{aligned} \dot{m}_w =& \dot{m}_a\left(\omega_1 - \omega_2\right) \\ =& 13.26\,kg/min \times (0.022 - 0.011) \\ =& 0.15\,kg/min. \end{aligned}$$

[1/7]

ii) Determine the rate of heat removal from the air. [5 marks]

Solution:

In the cooling section:

$$\textit{Conservation of energy:} \qquad \dot{Q} = \dot{m}_a\left(h_2 - h_1\right) + \dot{m}_w h_w.$$

[1/5]

The heat supplied to the system $\dot{Q}$ is given by the energy conservation equation

$$\begin{aligned} \dot{Q} =& \dot{m}_a\left(h_2 - h_1\right) + \dot{m}_w h_w \\ =& 13.26\,kg/min\,(43 - 90)\ kJ/kg + (0.15\,kg/min \times 67.22\,kJ/kg) \\ =& -623.204\,kJ/min + 9.804\,kJ/min \\ =& -613.4\,kJ/min. \end{aligned}$$

[3/5]

[1/5] *The rate of heat addition $\dot{Q}$ is negative indicating heat removal from the cooling section. Therefore the rate of heat removal is 613.4 kJ/min.*

END OF PAPER