Answers 7

Q1. It depends!!

If interested in *forces on other structures* then water is more viscous, because surface stresses are given by $\tau = \mu(\partial u/\partial y)$ and water has a larger *dynamic* viscosity μ:

$\mu_{water} = 1.0 \times 10^{-3}$ Pa s, $\mu_{air} = 1.8 \times 10^{-5}$ Pa s

The transfer of momentum within the fluid itself, however, depends on the *force per unit mass* and hence on the *kinematic* viscosity $\nu = \mu/\rho$:

$\nu_{water} = 1.0\times10^{-6}$ m^2 s^{-1}, $\nu_{air} = 1.5\times10^{-5}$ m^2 s^{-1}

In this sense, water is actually less viscous than air.

Q2.

$$\mathrm{Re} = \frac{\rho U D}{\mu} = \frac{UD}{\nu}$$

$$\Rightarrow \qquad U = \frac{\nu}{D}\mathrm{Re}$$

Setting Re = 2300, D = 0.05 m and using the values of ν from above,

water: $U_{crit} = 0.046 \text{ m s}^{-1}$

air: $U_{crit} = 0.69 \text{ m s}^{-1}$

Q3.
At Re = 500 the flow will be laminar. The velocity profile is parabolic.
At Re = 50 000 the flow is fully turbulent and there will be a much flatter profile.

Look at the inflow profiles for Coursework 2 to see what these profiles look like in the related situation of channel flow.

The total shear stress varies linearly in both cases and is zero on the axis.

Q4.
A boundary layer adjacent to a solid boundary arises because of the need to satisfy the no-slip (zero-relative-velocity) condition at the surface.

An "adverse" pressure gradient – pressure increasing in the direction of flow – occurs when the flow as a whole is decelerating; for example, because a channel is expanding. This pressure gradient tries to retard the flow and has the most significant effect near the surface where the flow is already moving slowly. If the retarding effect of pressure is bigger than the viscous force from the main stream trying to drag fluid forward then reversed flow will occur. Where forward-moving and backward-moving streams meet, the flow must flow away from the surface to ensure mass conservation – the phenomenon of flow separation.

Turbulent eddies are much more effective than viscous forces at transferring forward momentum from the free stream toward the surface, so helping to prevent the backflow that leads to flow separation. Deliberately tripping the flow (e.g. with a wire or other surface roughness) makes it turbulent rather than laminar and delays or prevents separation.

Q5.

Mean: $$\bar{u} = \frac{\sum u}{N}$$

Variances: $$\overline{u'^2} \equiv \overline{(u-\bar{u})^2} \quad = \overline{u^2} - \bar{u}^2 \quad = \frac{\sum u^2}{N} - \bar{u}^2$$

Covariances: $$\overline{u'v'} \equiv \overline{(u-\bar{u})(v-\bar{v})} \quad = \overline{uv} - \bar{u}\ \bar{v} \quad = \frac{\sum uv}{N} - \bar{u}\bar{v}$$

Similarly for the other components. It is probably simplest to do the lot with Microsoft Excel.

$$\bar{u} = 4.0, \quad \bar{v} = 0.0, \quad \overline{u'^2} = 0.305, \quad \overline{v'^2} = 0.088, \quad \overline{u'v'} = -0.089$$

Q6.

(a)

$$P_{22} = -2(\overline{vu}\frac{\partial V}{\partial x} + \overline{vv}\frac{\partial V}{\partial y} + \overline{vw}\frac{\partial V}{\partial z})$$

$$P_{33} = -2(\overline{wu}\frac{\partial W}{\partial x} + \overline{wv}\frac{\partial W}{\partial y} + \overline{ww}\frac{\partial W}{\partial z})$$

$$P_{23} = -(\overline{vu}\frac{\partial W}{\partial x} + \overline{vv}\frac{\partial W}{\partial y} + \overline{vw}\frac{\partial W}{\partial z}) - (\overline{wu}\frac{\partial V}{\partial x} + \overline{wv}\frac{\partial V}{\partial y} + \overline{ww}\frac{\partial V}{\partial z})$$

$$P_{31} = -(\overline{wu}\frac{\partial U}{\partial x} + \overline{wv}\frac{\partial U}{\partial y} + \overline{ww}\frac{\partial U}{\partial z}) - (\overline{uu}\frac{\partial W}{\partial x} + \overline{uv}\frac{\partial W}{\partial y} + \overline{uw}\frac{\partial W}{\partial z})$$

(b) In simple shear flow these simplify considerably to:

$$P_{11} = -2\overline{uv}\frac{\partial U}{\partial y}, \qquad P_{22} = P_{33} = 0$$

$$P_{12} = -\overline{vv}\frac{\partial U}{\partial y}, \qquad P_{23} = P_{31} = 0$$

(In the P_{31} component it is assumed that a simple shear flow is 2-dimensional, and hence $\overline{wv} = 0$, because, on average, w is as likely to be positive as negative.)

Looking at the normal-stress production (P_{11}, P_{22}, P_{33}), energy is preferentially transferred from the mean flow into the streamwise velocity fluctuation u. It is subsequently redistributed among the other components by pressure fluctuations.

Since

$$k = \tfrac{1}{2}(\overline{u^2} + \overline{v^2} + \overline{w^2})$$

we have

$$P^{(k)} = \tfrac{1}{2}(P_{11} + P_{22} + P_{33})$$

Hence, in simple shear,

$$P^{(k)} = -\overline{uv}\frac{\partial U}{\partial y}$$