March 10, 2015

Problem 1: (Computational Task) Radioisotopes are often used in industrial facilities to assess the mechanical integrity (e.g., existence of micro-fractures) of equipment and pipeline network system. A prescribed quantity of isotopes is diluted in an inert solvent $\left(\rho = 877.15 \text{ kg.m}^{-3}\right)$ and is continuously injected at constant mass flow rate of 200 kg.s^{-1} into a pipe of 30 inches of diameter and 60 meters of length. In order to better analyse the data, a system engineer needs to calculate the expected distribution of radioisotopes throughout the pipe during the first 30 seconds of the test-injection. She decides to represent the concentration of radioisotopes, $\mathcal{C}(x,t)$ $\left(\text{in } \mu\text{g.m}^{-3}\right)$, as a 1D problem using a forward discretisation scheme in space and time (FDM) resulting in,

$$\mathcal{C}_i^{j+1} = \mathcal{C}_i^j - \frac{u\Delta t}{\Delta x}\left(\mathcal{C}_{i+1}^j - \mathcal{C}_i^j\right)$$

where $i \in \{1, 2, ..., N_x\}$ and $j \in \{1, ..., N_t\}$ are spatial- and time-increment indexes, respectively. She assumed:

- The domain is divided into $N_x = 10$ nodes;
- Time step size (Δt) of 0.5 seconds;
- Initial condition $\left(\text{in } \mu\text{g.m}^{-3}\right)$ is given by,

$$\mathcal{C}(x, t=0) = \begin{cases} 3.0 & \text{for } x \leqslant 3.0 \text{ m} \\ 10.0 & \text{for } 3.0 < x \leqslant 25.0 \text{ m} \\ \dfrac{10.0}{1.0 + e^{0.1(x-60.0)}} & \text{elsewhere.} \end{cases}$$

- *Ghost-node*: $x_{N_x+1} = x_{N_x}$
- Dirichlet boundary condition: $\mathcal{C}(x=0,t)$=0.15 μg.m^{-3}

(a) Write a code (in Matlab, Python, C or Fortran) that calculates $\mathcal{C}_i^j$ for $j \in \{1, ..., 30\}$. Plot $x \times \mathcal{C}_i^j$ for times (a) t = 0s, (b) t = 5s, (c) t = 10s and (d) t = 15s. **[40 Marks]**

(b) At t = 11s, the observed concentration (obtained from non-invasive sensor measurements) was:

$$\mathcal{C}^{\text{obs}}(x, t=15s) = (0.15 \;\; 10.07 \;\; 9.93 \;\; 10.31 \;\; 9.89 \;\; 9.75 \;\; 9.73 \;\; 9.35 \;\; 6.01 \;\; 10.02)^T$$

The sensors were placed in the same coordinates as the original simulated mesh grid. How would the engineer quantitatively assess the accuracy of her simulated data with respect to the measurements? **[10 Marks]**

Problem 2: Generate an algorithm (as pseudo-code or block-diagram) for the Gauss-Jordan method **[10 Marks]**.

Problem 3: Given,

$$\underline{\underline{A}} = \begin{pmatrix} 5 & 0 & 3 & 1 \\ 4 & 19 & 10 & 3 \\ 1 & 4 & 8 & -1 \\ 0 & 3 & 1 & 7 \end{pmatrix} \quad \text{and} \quad \underline{b} = \begin{pmatrix} 8 \\ 25 \\ 18 \\ 72 \end{pmatrix}$$

(a) Calculate $\underline{\underline{A}}^{-1}$ using Gauss-Jordan method **(showing all steps) [5 Marks]**.

(b) For the system $\underline{\underline{A}}\ \underline{x} = \underline{b}$, calculate $\underline{x}$ via:

i. Gaussian elimination **(showing all steps) [5 Marks]**;

ii. (Computational Task) Gauss-Seidel and SOR with $\omega = 0.4$. Write a code (in Matlab, Python, C or Fortran) **[30 Marks]**.

Deliverables

1. Write a report containing the numerical solutions for the problems above along with the source code (Fortran, Python, C or Matlab) used in Problems 1a and 3(b)ii. Prepare the report as **PDF** file and submit it through Turnitin.
2. **Maximum 2 students / group**;
3. The filename of the report should be **EG5597_CA1_XXX.pdf** (**XXX** to be replaced by your surname(s)).
4. Submit your work by **Sunday, February 22nd 2015, 23:59** at the latest.
5. Penalties for late or non-submission are as follows:
   (a) Up to one week late, 2 CGS points deducted;
   (b) Up to two weeks late, 3 CGS point deducted;
   (c) More than two weeks late no marks awarded.
6. Remember to include in your electronic submission a completed plagiarism cover sheet.
7. Note that the submitted work is part of the continuous assessment which will contribute 1/6 to your EG5597 mark.