Solution of the Problems – Exam (August 2012/13)

Question 1: Internal combustion operating on dual cycle, given

$T_1 = 30^{\mathrm{o}}\mathrm{C}$, $P_1 = 1$ bar

Cylinder bore: 0.25 m , Stroke length: 0.40 m

$r_c = 9$, $r_e = 5$

$C_p = 1.0\dfrac{kJ}{kg.K}$, $C_v = 0.71\dfrac{kJ}{kg.K}$

(a) Calculating T_i and P_i, with i $\in \{1,2,3,4,5\}$ at all strokes:

1–2: Isentropic Compression

$$P_1V_1^n = P_2V_2^n \Longrightarrow P_2 = P_1\left(\frac{V_1}{V_2}\right)^n = 1 \times 9^{1.25} = 15.59 \text{ bar}$$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{n-1} = 9^{1.25-1} = 1.732 \Longrightarrow T_2 = 525.07 \text{ K}$$

3–4: Addition of Heat at Constant Pressure Calculating T_3 and T_4:

$$\frac{T_4}{T_3} = \frac{V_4}{V_3} = \rho = \frac{r_c}{r_e} = \frac{9}{5} = 1.8 \Longrightarrow T_4 = 1.8T_3$$

The problem states that heat liberated at constant pressure is twice the heat liberated at constant colume, i.e.,

$$C_p\left(T_4 - T_3\right) = 2C_v\left(T_3 - T_2\right).$$

Substituting T_2 and T_4,

$$1.0\left(1.8T_3 - T_3\right) = 2 \times 0.71\left(T_3 - 525.07\right) \Longrightarrow T_3 = 1202.58 \text{ K}$$

$$\frac{P_3}{T_3} = \frac{P_2}{T_2} \Longrightarrow P_3 = \frac{15.59}{525.07}1202.58 - 35.70 \text{ bar}$$

$$P_4 = P_3 \quad \text{and} \quad T_4 = 1.8T_3 = 2164.65 \text{ K}$$

4–5: Isentropic Expansion

$$P_4V_4^n = P_5V_5^n \Longrightarrow P_5 = P_4\left(\frac{V_4}{V_5}\right)^n = P_4\frac{1}{r_e^n} = 35.70\frac{1}{5^{1.25}} = 4.775 \text{ bar}$$

$$\frac{T_5}{T_4} = \left(\frac{V_4}{V_5}\right)^{n-1} \Longrightarrow T_5 = T_4\frac{1}{r_e^{n-1}} = 1447.59 \text{ K}$$

Stroke	P (bar)	T (K)
1	1.0	303.15
2	15.59	525.07
3	35.70	1202.58
4	35.70	2164.65
5	4.78	1447.59

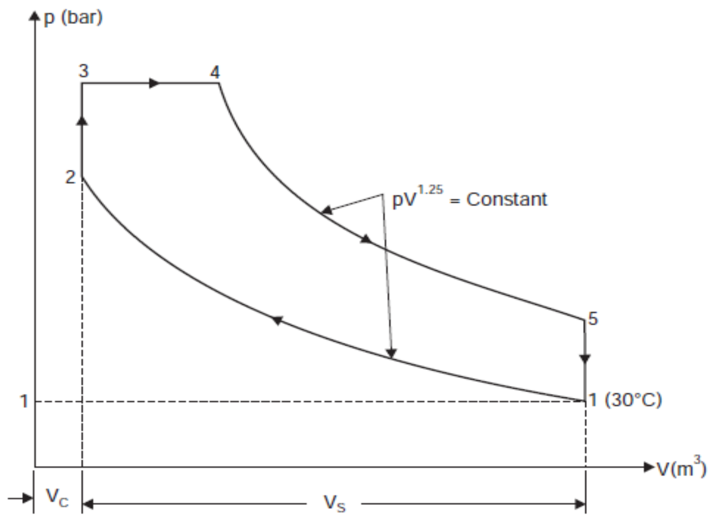


Figure 1: Pv diagram for Question 1.

(b) Skecth of the Pv diagram in Fig. b.

(c) Calculating MEP:

$$
\begin{aligned}
MEP &= \frac{1}{V_s}\left[P_3\left(V_4 - V_3\right) + \frac{P_4V_4 - P_5V_5}{n-1} - \frac{P_2V_2 - P_1V_1}{n-1}\right] \\
&= \frac{1}{r_c - 1}\left[P_3\left(\rho - 1\right) + \frac{P_4\rho - P_5 r_c}{n-1} - \frac{P_2 - P_1 r_c}{n-1}\right] \\
&= \frac{1}{9-1}\left[35.70\left(1.8-1\right) + \frac{35.70 \times 1.8 - 4.78 \times 9}{1.25 - 1} - \frac{15.59 - 1 \times 9}{1.25 - 1}\right] \\
&= 10.895 \text{ bar}
\end{aligned}
$$

(d) The work done per cycle is defined as

$$W = MEP \times V_s$$

where

$$V_s = \frac{\pi}{4}D^2L = \frac{3.1415}{4}\left(0.25\right)^2 \times 0.4 = 0.0196 \text{ m}^3$$

Thus

$$W = 10.895 \text{ bar } \times 0.0196 \text{ m}^3 = 0.213542 \text{ bar.m}^3 = 21.354 \text{ kJ}$$

The heat supplied to the cycle is given by

$$Q_{\text{cycle}} = mQ_s = m\left[C_p\left(T_3 - T_2\right) + C_p\left(T_4 - T_3\right)\right]$$

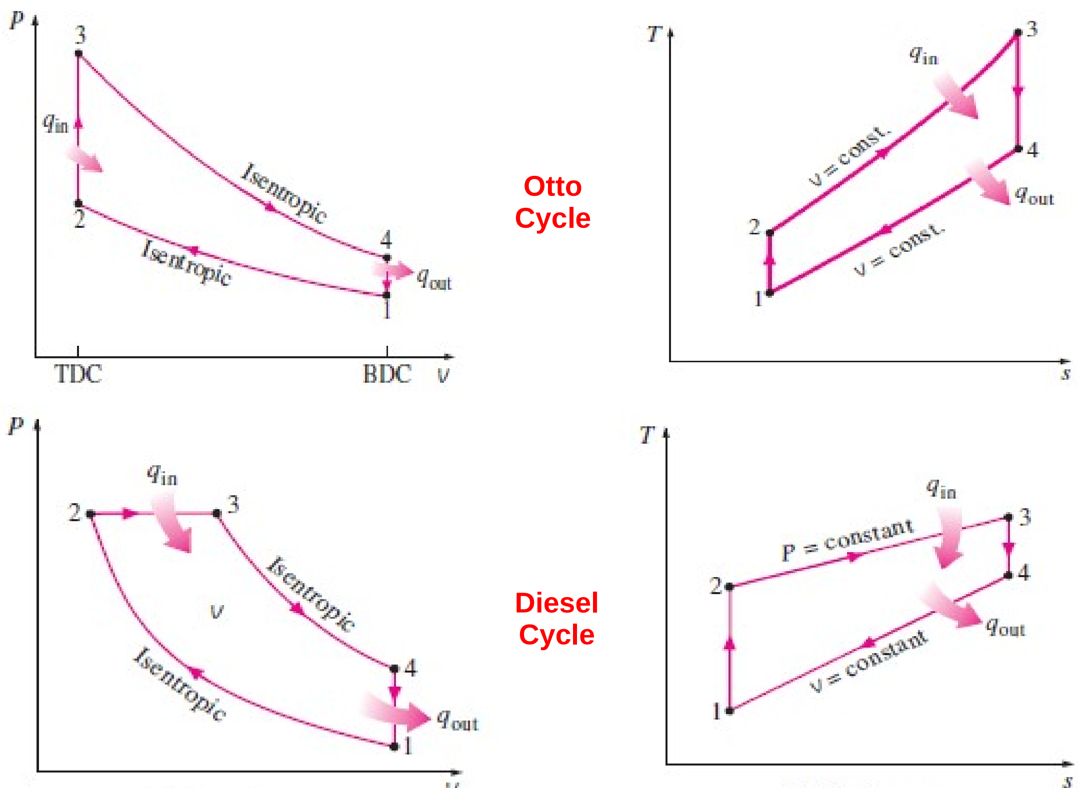
we need to calculate the mass (*m*) of air in the cylinder:

$$V_1 = V_s + V_c = \frac{r_c}{r_c - 1} V_s = \frac{9}{9-1} \times 0.0196 = 0.02209 \text{ m}^3$$

$$m = \frac{P_1 V_1 MW}{RT_1} = \frac{1 \text{ bar} \times 0.02209 \text{ m}^3 \times 29 \dfrac{\text{g}}{\text{gmol}}}{8.3144621 \times 10^{-5} \dfrac{\text{m}^3\text{.bar}}{\text{K.gmol}} \times 303.15 \text{ K}} = 25.4141 \text{ g}$$

$$\begin{aligned} \text{Heat supplied} &= mQ_s \\ &= 0.02541 \left[0.71 \left(1202.58 - 525,07\right) + 1.0 \left(2164.65 - 1202.58\right)\right] \\ &= 36.67 \text{ kJ} \end{aligned}$$

(e)



(a) P- v diagram

(b) T-s diagram

Question 2:

(a)

$$\begin{aligned}
\text{Amount of petroleum material}\quad & \\
\text{thermally equivalent to the biodiesel} &= \left(75000 \times 103 \times \frac{37}{43}\right) \text{ kg releasing on burning} \\
&= \left[\left(75000 \times 103 \times \frac{37}{43}\right) \times \frac{44}{14}\right] \times 10^{-3} \text{ tonnes of } CO_2 \\
&= 0.2 \text{ million tonnes of carbon dioxide.}
\end{aligned}$$

(b) Trans-esterification with methanol.

(c) It can be transported long distances by tanker, reducing the need for pipelines.

(d)

$$\begin{aligned}
\text{Rate of electricity production} &= \\
&= 1.2 \times 109\text{kg} \times 55 \times 106\text{J.kg}^{-1} \times \frac{0.35}{3600 \times 365 \times 24}\text{W} \\
&= 730\text{MW}
\end{aligned}$$

Question 3:

(a) Symbols:

$\dot{Q}$ **-** the rate of heat added to the fluid;
$\dot{W}_s$ **-** the rate of shaft work done by the fluid;
$\dot{m}$ **-** the mass flux through the device;
c_p **-** the specific heat capacity at constant pressure;
T **-** the fluid temperature;
u **-** the fluid velocity;
χ_2 **-** the property χ evaluated at the device outlet;
χ_1 **-** the property χ evaluated at the device inlet;

[3 Marks]

Assumptions:

- Steady fluid flow through the turbine;
- The fluid is an ideal gas;
- The velocity, pressure, internal energy and potential energy over each inlet or outlet can be replaced by their respective averages;
- No work due to viscosity, chemical, radiological etc;

[Half mark for each up to 2 Marks]

(b) For a steady flow device with two inlets (labelled 1 and 2) and one outlet (labelled 3), the steady flow energy equation can be written as

$$\dot{Q} - \dot{W}_s = \dot{m}_3\left(c_pT_3 + \frac{u_3^2}{2} + gz_3\right) - \dot{m}_1\left(c_pT_1 + \frac{u_1^2}{2} + gz_1\right) - \dot{m}_2\left(c_pT_2 + \frac{u_2^2}{2} + gz_2\right).$$

[2 Marks]

The corresponding steady mass conservation equation is

$$\dot{m}_1 + \dot{m}_2 = \dot{m}_3.$$

[1 Marks]

(c) (i) the volume flux is related to the velocity through

$$\text{Fluid velocity, } u = \frac{\text{Volume flux}}{\text{Cross-sectional area, } A}.$$

[1 Mark of 2]

At inlet 1,

$$u_1 = \frac{1.8\,\text{m}^3/\text{s}}{0.03\,\text{m}^2} = 60\,\text{m/s},$$
$$u_2 = \frac{0.5\,\text{m}^3/\text{s}}{0.1\,\text{m}^2} = 5\,\text{m/s},$$
$$u_3 = \frac{20\,\text{m}^3/\text{s}}{0.5\,\text{m}^2} = 40\,\text{m/s},$$

[1 Mark of 2]

(ii) The fluid density at inlet 1 and outlet 3 can be calculated directly

$$\rho_1 = \frac{p_1}{RT_1} = \frac{200000\,\text{Pa}}{300\,\text{J/(kg K)}\,(80 + 273.15)\ K} = 1.8878\,\text{kg/m}^3,$$
$$\rho_3 = \frac{p_3}{RT_3} = \frac{110000\,\text{Pa}}{300\,\text{J/(kg K)}\,(30 + 273.15)\ K} = 1.2095\,\text{kg/m}^3.$$

[1 Mark of 6]

The mass flux at inlet 1 and outlet 3 are given by

$$\dot{m}_1 = \rho_1 u_1 A_1 = 1.8878\,\text{kg/m}^3 \times 60\,\text{m/s} \times 0.03\,\text{m}^2 = 22.6533\,\text{kg/s},$$
$$\dot{m}_3 = \rho_3 u_3 A_3 = 1.2095\,\text{kg/m}^3 \times 40\,\text{m/s} \times 0.5\,\text{m}^2 = 24.1904\,\text{kg/s}.$$

[1 Mark of 6]

Now the mass flux through the inlet satisfies

$$\dot{m}_1 + \dot{m}_2 = \dot{m}_3,$$

[1 Mark of 6]

i.e.

$$\dot{m}_2 = \dot{m}_3 - \dot{m}_1 = 24.1904\,\text{kg/s} - 22.6533\,\text{kg/s} = 1.5372\,\text{kg/s}.$$

[1 Mark of 6]

Consequently the density

$$\rho_2 = \frac{\dot{m}_2}{u_2 A_2} = \frac{1.5372\,\text{kg/s}}{5\,\text{m/s} \times 0.1\,\text{m}^2} = 3.0744\,\text{kg/m}^3,$$

[1 Mark of 6]

and the pressure

$$p_2 = \rho_2 R T_2 = 3.0744\,\text{kg/m}^3 \times 300\,\text{J/(kg K)} \times (70 + 273.15)\ K = 316490\,\text{Pa} = 316\,\text{kPa}.$$

[1 Mark of 6]

(iii) Now we can use the SFEE in the form

$$\dot{Q} - \dot{W}_s = \dot{m}_3\left(c_p T_3 + \frac{u_3^2}{2} + g z_3\right) - \dot{m}_1\left(c_p T_1 + \frac{u_1^2}{2} + g z_1\right) - \dot{m}_2\left(c_p T_2 + \frac{u_2^2}{2} + g z_2\right).$$

to calculate the rate at which heat is added to the gas.

$$\begin{aligned}
\dot{Q} =& -80000 + \dot{m}_3\left((800 \times 303.15) + \frac{40^2}{2} + (9.81 \times 0.5)\right) \\
& - \dot{m}_1\left((800 \times 353.15) + \frac{60^2}{2} + (9.81 \times 0.2)\right) \\
& - \dot{m}_2\left((800 \times 343.15) + \frac{5^2}{2} + (9.81 \times 1.2)\right) \\
=& -80000 + 5886138 - 6440820 - 422024 \\
=& -1056707.0\,\text{W} = -1057\,\text{kW}.
\end{aligned}$$

[2 Marks]

The rate at which heat added to the gas is negative indicating the gas in the device heats the surroundings. **[1 Mark]**

Question 4: The schematic and Ts diagrams are given bellow (Fig.).

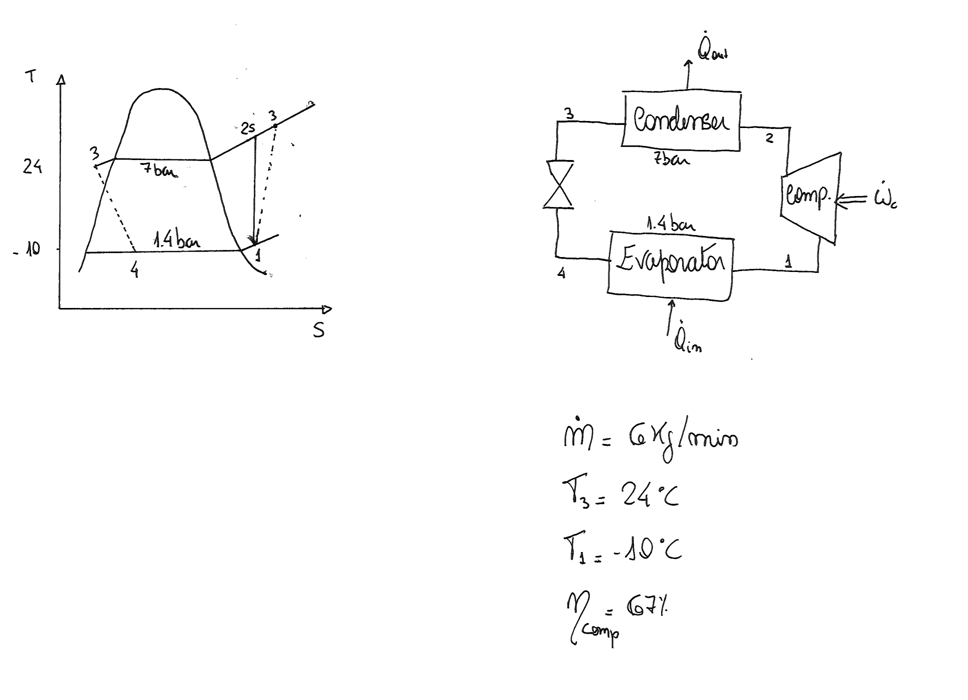


Figure 2: Schematic and Ts diagram for Question 4.

(a) *State 1:* At $T_1 = 10$ºC and $P_1 = 1.4$ bar, the thermodynamic table for *R-134a* gives $H_1 = 243.40$ kJ/kg and $S_1 = 0.9606$ kJ/(kg..K).

(b) *State 2:* For isentropic compression to $P_2 = 7$ bar, $S_{2s} = S_1$. From linear interpolation, we can find $H_{2s} = 278.06$ kJ/kg. Now, using the (given) compressor efficiency,

$$\eta = \frac{H_{2s} - H_1}{H_2 - H_1} = 0.67 \Longrightarrow H_2 = 245.13 \text{ kJ/kg}, \quad S_2 = 1.0135 \text{ kJ/(kg.K)}$$

(c) *State 3:* For $P_3 = 7$ bar and $T_3 = 24$ºC, the thermodynamic table gives $H_3 = H_f\,(T = 297.15 \text{ K}) = 82.90$ kJ/kg and $S_3 = 0.3113$ kJ/(kg.K)

(d) *State 4:* Throttling process $\Longrightarrow$ $H_4 = H_3 = 82.90$ kJ/kg and $S_4 = 0.33011$ kJ/(kg.K)

The coefficient of performance, β is given by

$$\beta = \frac{H_1 - H_4}{H_2 - H_1} = 3.10$$

And the refrigerant capacity is

$$\dot{Q}_{in} = \dot{m}\,(H_1 - H_4) = \left(6\frac{\text{kg}}{\text{min}}\right)(243.40 - 82.90)\,\frac{\text{kJ}}{\text{kg}}\left[\frac{1\text{ ton}}{211\text{ kJ/min}}\right] = 4.564\text{ tons}$$

Question 5: The specific humidity ω is the ratio of the mass of water vapour m_v, to the mass of dry air m_a and satisfies the equation

$$\omega = \frac{m_v}{m_a}.$$

As both water vapour and dry air behave like ideal gases, in some arbitrary volume V,

$$\omega = \frac{m_v}{m_a} = \frac{\rho_v}{\rho_a} = \frac{p_v}{R_v T}\frac{R_a T}{p_a} = \frac{R_a p_v}{R_v p_a}.$$

The partial pressures of dry air and water vapour satisfy $p_a = p - p_v$. Hence

$$\omega = \frac{R_a p_v}{R_v \left(p - p_v\right)}.$$

The saturation pressure of water $p_{v,\text{sat}}$ is the maximum partial pressure of water vapour a gas can contain at a give temperature before water starts condensing out of the gas. The relative humidity φ is the ratio of the partial pressure of water vapour to the saturation pressure of water

$$\varphi = \frac{p_v}{p_{v,\text{sat}}}.$$

Hence eliminating p_v from the previous expression gives

$$\omega = \frac{R_a \varphi p_{v,\text{sat}}}{R_v \left(p - \varphi p_{v,\text{sat}}\right)}.$$

[5 Marks]

If the inlet from inside is labelled 1, the inlet from outside is labelled 2 and the outlet with the mixture is labelled 3, then

$$\begin{aligned} \dot{m}_{a_1} + \dot{m}_{a_2} =& \dot{m}_{a_3}, && \text{Mass conservation of dry air,} \\ \omega_1 \dot{m}_{a_1} + \omega_2 \dot{m}_{a_2} =& \omega_3 \dot{m}_{a_3}, && \text{Mass conservation of water vapour,} \\ h_1 \dot{m}_{a_1} + h_2 \dot{m}_{a_2} =& h_3 \dot{m}_{a_3}, && \text{Energy conservation,} \end{aligned}$$

where $\dot{m}_a$ is a mass flux of air, h is the enthalpy and ω is the specific humidity. **[3 Marks of 7]**

Eliminating $\dot{m}_3$ from the water vapour conservation equation gives

$$\omega_1 \dot{m}_{a_1} + \omega_2 \dot{m}_{a_2} = \omega_3 \left(\dot{m}_{a_1} + \dot{m}_{a_2}\right) = \omega_3 \dot{m}_{a_1} + \omega_3 \dot{m}_{a_2}.$$

Collecting together terms involving $\dot{m}_1$ and terms involving $\dot{m}_2$ gives

$$\begin{aligned} \omega_1 \dot{m}_{a_1} - \omega_3 \dot{m}_{a_1} =& \omega_3 \dot{m}_{a_2} - \omega_2 \dot{m}_{a_2} \\ \Rightarrow \left(\omega_1 - \omega_3\right) \dot{m}_{a_1} =& \left(\omega_3 - \omega_2\right) \dot{m}_{a_2}, \end{aligned}$$

and hence rearranging gives

$$\frac{\dot{m}_{a_2}}{\dot{m}_{a_1}} = \frac{\omega_1 - \omega_3}{\omega_3 - \omega_2} = \frac{\omega_3 - \omega_1}{\omega_2 - \omega_3}.$$

[2 Marks of 7]

Similarly eliminating $\dot{m}_3$ from the energy conservation equation gives

$$h_1\dot{m}_{a_1} + h_2\dot{m}_{a_2} = h_3\left(\dot{m}_{a_1} + \dot{m}_{a_2}\right) = h_3\dot{m}_{a_1} + h_3\dot{m}_{a_2}.$$

Collecting together terms involving $\dot{m}_1$ and terms involving $\dot{m}_2$ gives

$$\begin{aligned} h_1\dot{m}_{a_1} - h_3\dot{m}_{a_1} &= h_3\dot{m}_{a_2} - h_2\dot{m}_{a_2} \\ \Rightarrow (h_1 - h_3)\,\dot{m}_{a_1} &= (h_3 - h_2)\,\dot{m}_{a_2}, \end{aligned}$$

and hence rearranging gives

$$\frac{\dot{m}_{a_2}}{\dot{m}_{a_1}} = \frac{h_1 - h_3}{h_3 - h_2} = \frac{h_3 - h_1}{h_2 - h_3} = \frac{\omega_3 - \omega_1}{\omega_2 - \omega_3},$$

as required. **[2 Marks of 7]**

(a) The partial pressure of dry air at 1 is given by

$$p_{a_1} = p_1 - p_{v,\text{sat}_1} = 100000\,\text{Pa} - 1818.747\,\text{Pa} = 98181.253\,\text{Pa}.$$

The specific volume of dry air

$$V_1 = \frac{R_a T_1}{p_{a_1}} = \frac{287.1\,\text{kJ/(kg K)}\,(16 + 273.15)}{98181.253\,\text{Pa}} = \ \text{m}^3\text{/kg}$$

[1 Mark of 3]

The mass flux of dry air through inlet 1 is given by

$$\dot{m}_{a_1} = \frac{\text{volume flux}}{V_1} = \frac{1\,\text{m}^3\text{/s}}{0.8455\,\text{m}^3\text{/kg}} = 1.1827\,\text{kg/s}.$$

[1 Mark of 3]

The mass flux of dry air through inlet 2 is given by

$$\dot{m}_{a_2} = \dot{m}_{a_3} - \dot{m}_{a_1} = 1.8\,\text{kg/s} - 1.1827\,\text{kg/s} = 0.6173\,\text{kg/s}$$

[1 Mark of 3]

(b) The gas entering the system through inlet 1 is saturated ($\varphi_1 = 1$), and therefore the specific humidity

$$\begin{aligned} \omega_1 &= \frac{R_a\varphi_1 p_{v,\text{sat}_1}}{R_v\left(p_1 - \varphi_1 p_{v,\text{sat}_1}\right)} = \frac{287.1\,\text{kJ/(kg K)} \times 1 \times 1818.747\,\text{Pa}}{461.5\,\text{kJ/(kg K)}\,(100000\,\text{Pa} - 1 \times 1818.747\,\text{Pa})} \\ &= 0.0115\,\text{kg H}_2\text{O/ kg dry air}. \end{aligned}$$

[1 Mark]

(c) Using

$$\frac{\dot{m}_{a_2}}{\dot{m}_{a_1}} = \frac{\omega_3 - \omega_1}{\omega_2 - \omega_3}.$$

Rearranging

$$(\omega_2 - \omega_3)\,\frac{\dot{m}_{a_2}}{\dot{m}_{a_1}} = (\omega_3 - \omega_1)\,.$$

Collecting terms involving ω_3 gives

$$\omega_2 \frac{\dot{m}_{a_2}}{\dot{m}_{a_1}} + \omega_1 = \omega_3 + \omega_3 \frac{\dot{m}_{a_2}}{\dot{m}_{a_1}} = \omega_3 \left(1 + \frac{\dot{m}_{a_2}}{\dot{m}_{a_1}}\right).$$

Therefore

$$\omega_3 = \frac{\omega_1 + \omega_2 \frac{\dot{m}_{a_2}}{\dot{m}_{a_1}}}{1 + \frac{\dot{m}_{a_2}}{\dot{m}_{a_1}}}$$

[2 Marks of 4]

The mass flux ratio

$$\frac{\dot{m}_{a_2}}{\dot{m}_{a_1}} = \frac{0.6173}{1.1827} = 0.5219$$

[1 Mark of 4]

Therefore

$$\omega_3 = \frac{0.0115 + (0.5219 \times 0.0182)}{1 + 0.5219} = 0.0138\,\text{kg H}_2\text{O/ kg dry air.}$$

[1 Mark of 4]