Q.1 Question 1

A binary geothermal power station is operated with brine extracted at 90°C and reinjected at 30°C. Propane (n-C_3) is used as working fluid in the Rankine cycle to produce power (W_T) in a turbine (isentropic expansion) with efficiency (η_T) of 90%. After condensated, n-C_3 is driven to a heat exchanger (with thermal efficiency of 68%) and the cycle continues. The mass flow rate of n-C_3 ($\dot{m}_{C3}$) is 250 kg.s^{-1} and the heat capacity (C_p) of brine is 3565.5 J.(kg.K)$^{-1}$. Conditions for n-C_3 and brine flows are described in Table 1.

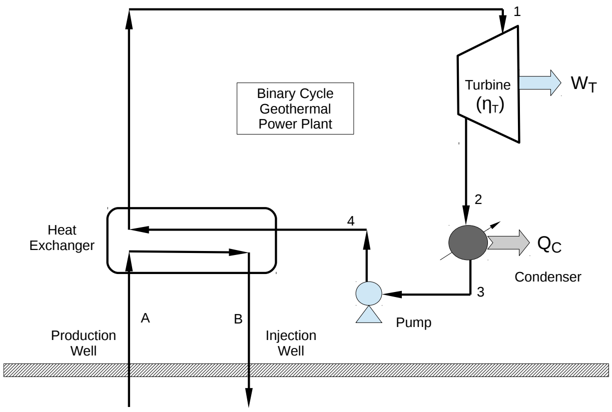


Table 1: *Thermodynamic table of the geothermal binary cycle.*

Stage	P (bar)	T (^{o}C)	State	H (kJ.kg^{-1})	S (kJ.(kg.K)$^{-1}$)
1	16	50	**(a)**	**(b)**	**(c)**
2	6	–	wet vapour	**(d)**	–
3	6		sat. liquid	**(e)**	–
4	16		**(f)**	**(g)**	–
A	–	90	–	–	–
B	–	30	–	–	–

(a) In Table 1, determine *(a)-(g)*. [7 marks]

Solution:

In order to fill the Table we need to calculate the thermodynamic properties for each stage of the cycle:

Stage 1: *At P_1 = 16 bar, $T_1 = 50^o C > T_{sat}(P_1) = 46.89^o C$. Therefore the fluid is*
[1/7] *at* **superheated state** . *From the superheated table for n-C_3 at P_1 and T_1, we can obtain:*
[1/7] $\mathbf{H_1 = 522.5\ kJ.kg^{-1}}$ *and*
[1/7] $\mathbf{S_1 = 1.733\ kJ.(kg.K)^{-1}}$.

Stage 2: *At P_2 = 6 bar, the fluid is wet vapour after the isentropic expansion. We should first calculate the quality of the vapour in an ideal expansion (using values of entropy/enthapy obtained from the saturated n-C_3 table at P_2.*

$$x_{2s} = \frac{S_{2s} - S_f}{S_g - S_f} = \frac{1.733 - 0.446}{1.737 - 0.446} = 0.9969$$

now to calculate the ideal enthalpy,

$$x_{2s} = 0.9969 = \frac{H_{2s} - H_f}{H_g - H_f} = \frac{H_{2s} - 115.3}{478.3 - 115.3} \iff H_{2s} = 477.17\frac{kJ}{kg}$$

As the efficiency of the turbine is of 90%,

$$\eta_{Turbine} = 0.90 = \frac{H_2 - H_1}{H_{2s} - H_1} = \frac{H_2 - 522.5}{477.17 - 522.5} \iff \mathbf{H_2 = 481.70\frac{kJ}{kg}}$$

[1/7]

Stage 3: *At $P_3 = P_2 = 6$ bar, the fluid leaving the condenser towards the pump is saturated liquid, and the enthalpy and specific volume are the same of the liquid phase obtained fromthe saturated table:*

[1/7] $\mathbf{H_3} = H_f\,(P = 6\ bar) = \mathbf{115.3\ kJ.kg^{-1}}$

$V_3 = V_f\,(P = 6\ bar) = 1.931{\times}10^{-3}\ m^3.kg^{-1}$

[1/7] **Stage 4:** *The fluid leaving the pump is* **sub-cooled liquid** . *As there is no heat loss in the pump, we can assume $dH \approx VdP$, therefore*

$$\mathbf{H_4} = H_3 + V_3\,(P_4 - P_3) = 115.3\frac{kJ}{kg} + 1.931 \times 10^{-3}\frac{m^3}{kg}\,(16 - 6)\,bar = \mathbf{117.23\frac{kJ}{kg}}$$

[1/7]

Thus the Table becomes:

Stage	**P (bar)**	**T (°C)**	**State**	**H (kJ.kg^{-1})**	**S (kJ.(kg.K)$^{-1}$)**
1	*16*	*50*	**superheated vapour**	**522.5**	**1.733**
2	*6*	–	*wet vapour*	**481.70**	–
3	*6*		*sat. liquid*	**115.3**	–
4	*16*		**sub-cooled liquid**	**117.23**	–
A	–	*90*	–	–	–
B	–	*30*	–	–	–

(b) Calculate the power produced by the turbine (W_T) in MW. [1 marks]
Solution:

$$\mathbf{W_T} = \dot{m}_{C3}\,(H_1 - H_2) = 250\frac{kg}{s} \times (522.5 - 481.70)\,\frac{kJ}{kg} = 10200\frac{kJ}{s} = \mathbf{10.2MW}$$

[1/1]

(c) Assuming that the heat exchanger has an efficiency of 68%, calculate the mass flow rate of brine in $kg.s^{-1}$. [3 marks]
Solution:
The heat extracted by the n-C_3 $\left(\dot{Q}_{C3}\right)$ fluid in the heat exchanger can be easily calculated by

$$\mathbf{\dot{Q}_{C3}} = \dot{m}_{C3}\,(H_1 - H_4) = \mathbf{101317.5\frac{kJ}{s}}$$

[1/3] *Assuming that the heat extracted from the geothermal fluid (brine), $\dot{Q}_{gf}$ is transferred* *to the n-C_3 stream with efficiency of 68%,*

$$\eta_{HE} = 0.68 = \frac{\dot{Q}_{C3}}{\dot{Q}_{gf}} \iff \mathbf{\dot{Q}_{gf} = 148996.32 \frac{kJ}{s}}$$

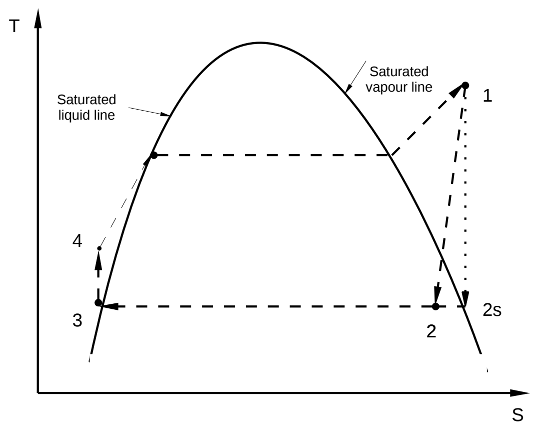
[1/3] *With the heat generated by the geothermal fluid and the inlet/outlet fluid temperatures, we can now calculate the brine mass flow rate for the associated heat transferred,*

$$\dot{Q}_{gf} = 148996.32 \frac{kJ}{s} = \dot{m}_{gf} C_p \left(T_A - T_B \right) \iff \mathbf{\dot{m}_{gf} = 696.57 \frac{kg}{s}}$$

[1/3]

(d) Sketch the temperature × entropy (TS) diagram for the process indicating the liquid and vapour saturated lines and each stage of the n-C_3 Rankine cycle. [4 marks]

Solution:



[5/5]

(e) Dry-steam, flash-steam and binary-cycle power plants are considerend the three main conversion technologies in geothermal systems. Describe the flash-steam process. [4 marks]

Solution:

[0.5/4] *(a) It is the most common geothermal power plant system and usually operates at temperatures above 150° C;*

[1/4] *(b) It utilises water below the boiling point at reservoir conditiions that suffer an isenthalpic flash at lower surface pressures;*

[1/4] *(c) During the flash, steam is driven to a turbine to undertae an isentropic expansion producing work (i.e., power);*

[1/4] *(d) After moving the turbine, the steam is condensed and reinjected;*

[0.5/4] *(e) The liquid fluid produced during the flash is chemically treated and re-injected into the well.*

(f) Temperature gradient (∇T) between upper and deep layers of rocks (i.e., near the surface and at large depths) can lead to geothermal circulation. Define thermal buoyancy and its links to thermal convection. [6 marks]

Solution:

Let's consider a geothermal reservoir with dimension $\underline{X}\,(= x, y, z)$ *with* **imposed**
[1/6] **temperature gradient** $\left(\nabla T = \frac{\partial T}{\partial z}\right)$ *and saturated with fluid with density*

$$\rho = \rho\left(T, p, \underline{X}, salinity,\ etc\right).$$

[1/6] *Under static conditions, pressure can be expressed as* $p = \rho g z$. *Algebraic expressions, known as* **equations of state (EOS), are designed to correlate density, tem-**
[1/6] **perature, pressure and any other thermodynamic potential** . *The pressure can be obtained by integrating the above equation through the depth,*

$$p(z) = \int_0^z \rho(z) g dz$$

Thermal buoyancy is a physical phenomena in which cold and denser fluid at low depth $(z \to 0)$ **displaces warm and lighter fluid at larger depth push-**
[3/6] **ing the warmer fluid upwards.**

To solve this problem, you should assume that the saturated liquid streams are incompressible, and therefore $dH = VdP$ (where H, V and P are enthalpy, volume and pressure, respectively). Quality of the vapour is expressed as

$$x_j = \frac{\Psi_j - \Psi_f}{\Psi_g - \Psi_f} \quad \text{with } \Psi = \{H, S\}$$

where S is the entropy. Efficiency of the turbine (η_{Turbine}) and the heat exchanger (η_{HE}) are given by,

$$\eta_{\text{Turbine}} = \frac{H_2 - H_1}{H_{2s} - H_1} \quad \text{and} \quad \eta_{\text{HE}} = \frac{\dot{Q}_{C3}}{\dot{Q}_{gf}}$$

where H_{2s} is the enthalpy of stream 2 assuming ideal turbine performance (i.e., reversible expansion). $\dot{Q}_{C3}$ and $\dot{Q}_{gf}$ are the heat associated with the n-C_3 and brine streams, respectively, at the heat exchanger.