Design of a 1/2 Heat Exchanger

The Device is a 1 Shell Pass and a 2 Tube Pass Exchanger

The Design Equation for a Heat Exchanger

$$Q_H = UAF\frac{\left(\Delta T_2 - \Delta T_1\right)}{\ln\left(\frac{\Delta T_2}{\Delta T_1}\right)} = UAF\Delta T_{lm}$$

F is the correction to the ΔT for a non-ideal flow path. To determine this for this exchanger , noting that it is at the same time a co-current and a counter-current exchange, we have to solve some enrgy balances.

Overall Energy Balance

$$(wC_p\ \Delta T)_{hot} = (wC_p\Delta T)_{cold}$$

This leads to a ratio of thermal capacitances

$$R = \frac{\left(WC_p\right)_C}{\left(WC_p\right)_H} = \frac{C_C}{C_H} = \frac{T_{H1} - T_{H2}}{T_{C2} - T_{C1}}$$

or it can be written as

$$R = \frac{(WC_p)_{tube}}{(WC_p)_{shell}} = \frac{C_{tube}}{C_{shell}} = \frac{\Delta T_{shell}}{\Delta T_{tube}}$$

Shell Balance on a first section of tube (cold stream)

$$C_C\left(T_C'\Big|_z - T_C'\Big|_{z+\Delta z}\right) = U\left(T_C' - T_H\right)\frac{dA}{2}$$

Shell Balance on a second section of tube (cold stream)

$$C_C\left(T_z''\Big|_z - T_C''\Big|_{z+\Delta z}\right) = U\left(T_H - T_C''\right)\frac{dA}{2}$$

Overall Shell Balance on a second section of tube (hot stream)

$$C_H\left(T_H\Big|_z - T_H\Big|_{z+\Delta z}\right) = U\left(T_H - T_C''\right)\frac{dA}{2} + U\left(T_H - T_C'\right)\frac{dA}{2}$$

The corresponding differential equations.

$$\frac{C_C}{U}\frac{dT_C'}{dA} = \frac{T_H - T_C'}{2}$$

$$\frac{C_C}{U}\frac{dT_C''}{dA} = -\frac{T_H - T_C''}{2}$$

$$C_H\frac{dT_H}{dA} = -U\frac{\left(T_H - T_C''\right)}{2} - U\frac{\left(T_H - T_C'\right)}{2}$$

If we normalize the distance term to

$$dn \equiv \frac{UdA}{C_C}$$

In this representation the equations are easier to formulate:

$$\frac{dT_C'}{dn} = \frac{T_H - T_C'}{2}$$

$$\frac{dT_C''}{dn} = -\frac{T_H - T_C''}{2}$$

$$\frac{1}{R}\frac{dT_H}{dn} = \frac{\left(T_C'' - T_H\right)}{2} + \frac{\left(T_C' - T_H\right)}{2}$$

Energy balance from z to L

$$C_H\left(T_H - T_{Hz}\right) = C_C\left(T_C'' - T_C'\right)$$

This becomes

$$\frac{1}{R}\left(T_H - T_{Hz}\right) = \left(T_C'' - T_C'\right)$$

Eliminate all the T_C variables, from the equations and the overall energy balance, we obtain

$$\frac{dT_C''}{dn} = \frac{dT_C'}{dn} + \frac{1}{R}\frac{dT_H}{dn} = -\frac{T_H}{2} + \frac{T_C' + \frac{1}{R}\left(T_H - T_{Hz}\right)}{2}$$

and

$$-\frac{\left(T_C' - T_H\right)}{2} + \frac{1}{R}\frac{dT_H}{dn} = -\frac{T_H}{2} + \frac{T_C' + \frac{1}{R}\left(T_H - T_{Hz}\right)}{2}$$

The final form of the temperature equation is

$$\frac{1}{R}\frac{d^2T_H}{dn^2} + \frac{dT_H}{dn} - \frac{1}{4R}\left(T_H - T_{Hz}\right) = 0$$

Boundary Conditions

$T_H = T_{H1}$ at $n = 0$

$T_H = T_{H2}$ at $n = n_L$ where $n_L = UA/C_C$

If we set a dimensionless T_H, we obtain

$$\theta = \frac{T_H - T_{H2}}{T_{H1} - T_{H2}}$$

and the equation

$$\frac{1}{R}\frac{d^2\theta}{dn^2} + \frac{d\theta}{dn} - \frac{1}{4R}\theta = 0$$

Boundary Conditions

$\theta = \theta_1$ at $n = 0$

$\theta = \theta_2$ at $n = n_L$

The solution requires algebraic gymnastics, but it produces

$$F = \frac{\dfrac{\sqrt{R^2+1}}{R-1}\ln\left(\dfrac{1-P}{1-PR}\right)}{\ln\left(\dfrac{\dfrac{2}{P}-1-R-\sqrt{R^2+1}}{\dfrac{2}{P}-1-R+\sqrt{R^2+1}}\right)}$$

where $P = \dfrac{T_{C2}-T_{C1}}{T_{H1}-T_{C1}}$