UNIVERSITY OF ABERDEEN SESSION 2016–17

EM40JN

Degree Examination in EM40JN Heat and Momentum Transfer

13th December 2016 2 pm – 5 pm

PLEASE NOTE THE FOLLOWING

(i) You **must not** have in your possession any material other than that expressly permitted in the rules appropriate to this examination. Where this is permitted, such material **must not** be amended, annotated or modified in any way.

(ii) You **must not** have in your possession any material that could be determined as giving you an advantage in the examination.

(iii) You **must not** attempt to communicate with any candidate during the exam, either orally or by passing written material, or by showing material to another candidate, nor must you attempt to view another candidate's work.

(iv) You **must not** take to your examination desk any electronic devices such as mobile phones or other smart devices. The only exception to this rule is an approved calculator.

Failure to comply with the above will be regarded as cheating and may lead to disciplinary action as indicated in the Academic Quality Handbook Section 7 and particularly Appendix 7.1

Notes:

(i) Candidates ARE permitted to use an approved calculator.

(ii) Candidates ARE permitted to use the Engineering Mathematics Handbook.

(iii) Data sheets are attached to the paper.

Candidates should attempt *all* questions.

Question 1

a) Describe the physical interpretation of the Grashof number for natural convection. Describe each of its terms and write down an equation for the temperature at which temperature-dependent terms in Gr should be evaluated. [5 marks]

b) An electric heater of 0.032 m diameter and 0.85 m in length is used to heat a room. Calculate the electrical input (i.e. the sum of heat transferred by convection and radiation) to the heater when the bulk of the air in the room is at 24°C, the walls are at 12°C, and the surface of the heater is at 532°C. For convective heat transfer from the heater, assume the heater is a horizontal cylinder and the Nusselt number is given by

$$\text{Nu} = 0.38(\text{Gr})^{0.25}$$

where all properties are evaluated at the film temperature. You may assume air is an ideal gas, giving $\beta = T^{-1}$. Take the emissivity of the heater surface as $\epsilon = 0.62$ and assume that the surroundings are black. All other properties should be calculated using the steam tables provided. [10 marks]

c) Using index notation, prove the following vector calculus identity:

$$\nabla^2 f\, g = f\, \nabla^2 g + 2(\nabla f) \cdot (\nabla g) + g\, \nabla^2 f$$

[5 marks]

Note: You must treat f and g as functions of x, y, z.

Question 2

A wire-coating die consists of a cylindrical wire of radius, $\kappa\,R$, moving horizontally at a constant velocity, v_{wire}, along the axis of a cylindrical die of radius, R. You may assume the pressure is constant within the die (it is not pressure driven flow) but the flow is driven by the motion of the wire (it is "axial annular Couette flow"). Neglect end effects and assume an isothermal system.

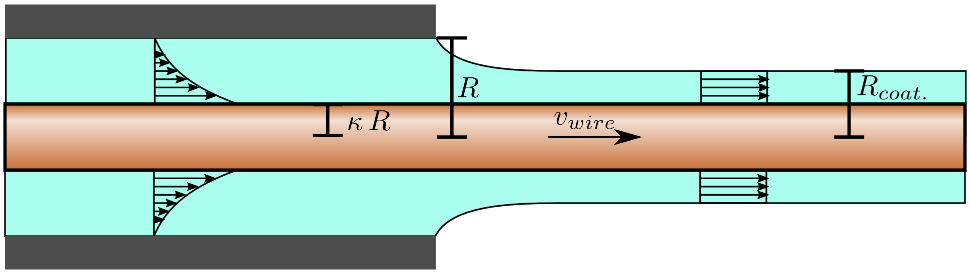


Figure 1: *Diagram of a wire coating die.*

a) State the two relevant boundary conditions for the flow within the die and how they arise. [2 marks]

b) The stress profile for an annular system is of the following form

$$\frac{1}{r}\frac{\partial}{\partial r} r\,\tau_{rz} = -\frac{\partial p}{\partial z} + \rho\,g_z.$$

Derive the following expression for the flow profile

$$v_z = \frac{v_{wire}}{\ln\kappa}\ln\left(\frac{r}{R}\right).$$

[9 marks]

c) Derive the following expression for the volumetric flow-rate of liquid through the die

$$\dot{V}_z = -\pi\,R^2\,v_{wire}\left(\kappa^2 + \frac{1-\kappa^2}{2\ln\kappa}\right).$$

[5 marks]

Note: You will need the integration identity

$$\int x\,\ln(x)\,\mathrm{d}x = \frac{x^2}{2}\left(\ln(x) - \frac{1}{2}\right).$$

d) Derive an expression for the outer radius of the coating, $R_{coat.}$, far away from the die exit. [4 marks]

Question 3

To explore the effect of using a temperature-dependent thermal conductivity, consider heat flowing through an annular (pipe) wall of inside radius R_0 and an outside radius R_1. It is assumed that thermal conductivity varies linearly with temperature from $k_0(T = T_0)$ to $k_1(T = T_1)$ where T_0 and T_1 are the inner and outer wall temperatures respectively.

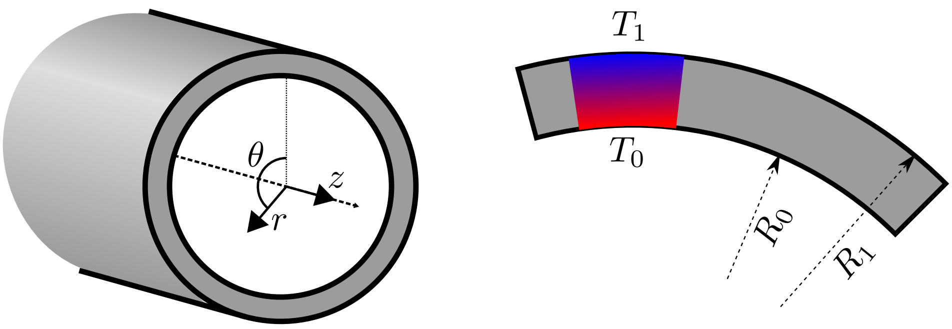


Figure 2: *Conduction through an annular(pipe) wall.*

a) Derive the following energy balance equation

$$\frac{\partial}{\partial r} r\, q_r = 0,$$

and state ALL assumptions required. [7 marks]

b) Derive the following expression for the temperature profile

$$Q_r = \frac{2\pi\, L}{\ln\left(\frac{R_0}{R_1}\right)} \frac{k_1 + k_0}{2} (T_1 - T_0),$$

where L is the length of the pipe/annulus. [10 marks]
Note: You will need the following identity:

$$T_1^2 - T_0^2 = (T_1 + T_0)(T_1 - T_0).$$

c) Compare this expression to the standard expression for conduction in pipe walls (with constant thermal conductivity), what can you observe? [3 marks]

Question 4

Consider laminar flow within a pipe. The only prior knowledge you should assume is that the pressure drop must be a function of pipe diameter D, viscosity μ, density ρ, and average velocity $\langle v_z \rangle$, i.e.,

$$\Delta p/l = f\left(D, \rho, \mu, \langle v_z \rangle\right).$$

a) Perform dimensional analysis on the pressure drop per unit length, $\Delta p/l$, and determine the relevant dimensionless groups. [12 marks]

b) Compare this to the exact solution, known as the Hagen-Poiseuille equation, as given below.

$$\dot{V}_z = \pi \left(\frac{-\Delta p}{l} + \rho\, g_z \right) \frac{R^4}{8\,\mu}.$$

Determine the form of the unknown function, f. [5 marks]

c) Comment on why dimensional analysis is so important. Also comment on why redundant dimensionless groups arise (as an example, consider the relationship between friction factor C_f and the Reynolds number). [3 marks]

Question 5

(a) A new material is to be developed for bearing balls in a new rolling-element bearing. For annealing (heat treatment) each bearing ball, a sphere of radius $r_o = 5$ mm, is heated in a furnace until it reaches to the equilibrium temperature of the furnace at 400°C. Then, it is suddenly removed from the furnace and subjected to a two-step cooling process.

Stage 1: Cooling in an air flow of 20°C for a period of time t_{air} until the center temperature reaches 335°C. For this situation, the convective heat transfer coefficient of air is assumed constant and equal to $h = 10$ W/(m^2.K). After the sphere has reached this specific temperature, the second step is initiated.

Stage 2: Cooling in a well-stirred water bath at 20°C, with a convective heat transfer coefficient of water $h = 6000$ W/(m^2.K).

The thermophysical properties of the material are $\rho = 3000$ kg/m^3, $\kappa = 20$ W/(m.K), $C_p = 1000$ J/(kg.K). Determine:

(a) The time t_{air} required for *Stage 1* of the annealing process to be completed; [5 marks]

(b) The time t_{water} required for *Stage 2* of the annealing process during which the center of the sphere cools from 335°C (the condition at the completion of *Stage 1*) to 50°C. [6 marks]

(b) A double-pipe (shell-and-tube) heat exchanger is constructed of a stainless steel ($\kappa = 15.1$ W/(m.°C) inner tube of inner diameter D$_i$ = 1.5 cm and outer diameter D$_o$ = 1.9 cm and an outer shell of inner diameter 3.2 cm. The convective heat transfer coefficient is h$_i$ = 800 W/(m^2.°C) on the inner surface of the tube and h$_o$ = 1200 W/(m^2.°C) on the outer surface. For a fouling factor R$_{f,i}$ = 0.0004 m^2.°C/W on the tube side and R$_{f,o}$ = 0.0001 m^2.°C/W on the shell side, determine:

(a) The thermal resistance of the heat exchanger per unit length (in °C/W) and; [3 marks]

(b) The overall heat transfer coefficients, U$_i$ and U$_o$ (in W/(m^2.°C)) based on the inner and outer surface areas of the tube, respectively. [6 marks]

END OF PAPER

DATASHEET

General balance equations:

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot \rho\, \boldsymbol{v} \qquad \text{(Mass/Continuity)} \quad (1)$$

$$\frac{\partial C_A}{\partial t} = -\nabla \cdot \boldsymbol{N}_A + \sigma_A \qquad \text{(Species)} \quad (2)$$

$$\rho \frac{\partial \boldsymbol{v}}{\partial t} = -\rho\, \boldsymbol{v} \cdot \nabla \boldsymbol{v} - \nabla \cdot \boldsymbol{\tau} - \nabla\, p + \rho\, \boldsymbol{g} \qquad \text{(Momentum)} \quad (3)$$

$$\rho\, C_p \frac{\partial T}{\partial t} = -\rho\, C_p\, \boldsymbol{v} \cdot \nabla\, T - \nabla \cdot \boldsymbol{q} - \boldsymbol{\tau} : \nabla\, \boldsymbol{v} - p\, \nabla \cdot \boldsymbol{v} + \sigma_{energy} \qquad \text{(Heat/Energy)} \quad (4)$$

In Cartesian coordinate systems, ∇ can be treated as a vector of derivatives. In curvelinear coordinate systems, the directions $\hat{\boldsymbol{r}}$, $\hat{\boldsymbol{\theta}}$, and $\hat{\boldsymbol{\phi}}$ depend on the position. For convenience in these systems, look-up tables are provided for common terms involving ∇.

Cartesian coordinates (with index notation examples)
where s is a scalar, $\boldsymbol{v}$ is a vector, and $\boldsymbol{\tau}$ is a tensor.

$$\nabla s = \nabla_i s = \left[\frac{\partial\, s}{\partial x}, \frac{\partial\, s}{\partial y}, \frac{\partial\, s}{\partial z}\right]$$

$$\nabla^2 s = \nabla_i \nabla_i s = \frac{\partial^2\, s}{\partial x^2} + \frac{\partial^2\, s}{\partial y^2} + \frac{\partial^2\, s}{\partial z^2}$$

$$\nabla \cdot \boldsymbol{v} = \nabla_i v_i = \frac{\partial\, v_x}{\partial x} + \frac{\partial\, v_y}{\partial y} + \frac{\partial\, v_z}{\partial z}$$

$$\nabla \cdot \boldsymbol{\tau} = \nabla_i\, \tau_{ij}$$

$$[\nabla \cdot \boldsymbol{\tau}]_x = \frac{\partial\, \tau_{xx}}{\partial x} + \frac{\partial\, \tau_{yx}}{\partial y} + \frac{\partial\, \tau_{zx}}{\partial z}$$

$$[\nabla \cdot \boldsymbol{\tau}]_y = \frac{\partial\, \tau_{xy}}{\partial x} + \frac{\partial\, \tau_{yy}}{\partial y} + \frac{\partial\, \tau_{zy}}{\partial z}$$

$$[\nabla \cdot \boldsymbol{\tau}]_z = \frac{\partial\, \tau_{xz}}{\partial x} + \frac{\partial\, \tau_{yz}}{\partial y} + \frac{\partial\, \tau_{zz}}{\partial z}$$

$$\boldsymbol{v} \cdot \nabla \boldsymbol{v} = v_i\, \nabla_i\, v_j$$

$$[\boldsymbol{v} \cdot \nabla \boldsymbol{v}]_x = v_x \frac{\partial\, v_x}{\partial x} + v_y \frac{\partial\, v_x}{\partial y} + v_z \frac{\partial\, v_x}{\partial z}$$

$$[\boldsymbol{v} \cdot \nabla \boldsymbol{v}]_y = v_x \frac{\partial\, v_y}{\partial x} + v_y \frac{\partial\, v_y}{\partial y} + v_z \frac{\partial\, v_y}{\partial z}$$

$$[\boldsymbol{v} \cdot \nabla \boldsymbol{v}]_z = v_x \frac{\partial\, v_z}{\partial x} + v_y \frac{\partial\, v_z}{\partial y} + v_z \frac{\partial\, v_z}{\partial z}$$

Cylindrical coordinates

where s is a scalar, $\boldsymbol{v}$ is a vector, and $\boldsymbol{\tau}$ is a tensor. All expressions involving $\boldsymbol{\tau}$ are for symmetrical $\boldsymbol{\tau}$ only.

$$\nabla s = \left[\frac{\partial s}{\partial r}, \ \frac{1}{r}\frac{\partial s}{\partial \theta}, \ \frac{\partial s}{\partial z}\right]$$

$$\nabla^2 s = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial s}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 s}{\partial \theta^2} + \frac{\partial^2 s}{\partial z^2}$$

$$\nabla \cdot \boldsymbol{v} = \frac{1}{r}\frac{\partial}{\partial r}(r\,v_r) + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z}$$

$$[\nabla \cdot \boldsymbol{\tau}]_r = \frac{1}{r}\frac{\partial}{\partial r}(r\,\tau_{rr}) + \frac{1}{r}\frac{\partial \tau_{r\theta}}{\partial \theta} - \frac{1}{r}\tau_{\theta\theta} + \frac{\partial \tau_{rz}}{\partial z}$$

$$[\nabla \cdot \boldsymbol{\tau}]_\theta = \frac{1}{r}\frac{\partial \tau_{\theta\theta}}{\partial \theta} + \frac{\partial \tau_{r\theta}}{\partial r} + \frac{2}{r}\tau_{r\theta} + \frac{\partial \tau_{\theta z}}{\partial z}$$

$$[\nabla \cdot \boldsymbol{\tau}]_z = \frac{1}{r}\frac{\partial}{\partial r}(r\,\tau_{rz}) + \frac{1}{r}\frac{\partial \tau_{\theta z}}{\partial \theta} + \frac{\partial \tau_{zz}}{\partial z}$$

$$[\boldsymbol{v} \cdot \nabla \boldsymbol{v}]_r = v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z\frac{\partial v_r}{\partial z}$$

$$[\boldsymbol{v} \cdot \nabla \boldsymbol{v}]_\theta = v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r\,v_\theta}{r} + v_z\frac{\partial v_\theta}{\partial z}$$

$$[\boldsymbol{v} \cdot \nabla \boldsymbol{v}]_z = v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}$$

Spherical coordinates

where s is a scalar, $\boldsymbol{v}$ is a vector, and $\boldsymbol{\tau}$ is a tensor. All expressions involving $\boldsymbol{\tau}$ are for symmetrical $\boldsymbol{\tau}$ only.

$$\nabla s = \left[\frac{\partial s}{\partial r}, \ \frac{1}{r}\frac{\partial s}{\partial \theta}, \ \frac{1}{r\,\sin\theta}\frac{\partial s}{\partial \phi}\right]$$

$$\nabla^2 s = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial s}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial s}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 s}{\partial \phi^2}$$

$$\nabla \cdot \boldsymbol{v} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\,v_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(v_\theta\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi}$$

$$[\nabla \cdot \boldsymbol{\tau}]_r = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\,\tau_{rr}\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\tau_{r\theta}\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial \tau_{r\phi}}{\partial \phi} - \frac{\tau_{\theta\theta} + \tau_{\phi\phi}}{r}$$

$$[\nabla \cdot \boldsymbol{\tau}]_\theta = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\,\tau_{r\theta}\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\tau_{\theta\theta}\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial \tau_{\theta\phi}}{\partial \phi} + \frac{\tau_{r\theta}}{r} - \frac{\cot\theta}{r}\tau_{\phi\phi}$$

$$[\nabla \cdot \boldsymbol{\tau}]_\phi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\,\tau_{r\phi}\right) + \frac{1}{r}\frac{\partial \tau_{\theta\phi}}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial \tau_{\phi\phi}}{\partial \phi} + \frac{\tau_{r\theta}}{r} + \frac{2\cot\theta}{r}\tau_{\theta\phi}$$

$$[\boldsymbol{v} \cdot \nabla \boldsymbol{v}]_r = v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_r}{\partial \phi} - \frac{v_\theta^2 + v_\phi^2}{r}$$

$$[\boldsymbol{v} \cdot \nabla \boldsymbol{v}]_\theta = v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_\theta}{\partial \phi} + \frac{v_r\,v_\theta - v_\phi^2\cot\theta}{r}$$

$$[\boldsymbol{v} \cdot \nabla \boldsymbol{v}]_\phi = v_r\frac{\partial v_\phi}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\phi}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi} + \frac{v_r\,v_\phi + v_\theta\,v_\phi\cot\theta}{r}$$

Rectangular		Cylindrical		Spherical	
q_x	$-k\frac{\partial T}{\partial x}$	q_r	$-k\frac{\partial T}{\partial r}$	q_r	$-k\frac{\partial T}{\partial r}$
q_y	$-k\frac{\partial T}{\partial y}$	q_θ	$-k\frac{1}{r}\frac{\partial T}{\partial \theta}$	q_θ	$-k\frac{1}{r}\frac{\partial T}{\partial \theta}$
q_z	$-k\frac{\partial T}{\partial z}$	q_z	$-k\frac{\partial T}{\partial z}$	q_ϕ	$-k\frac{1}{r\,\sin\theta}\frac{\partial T}{\partial \phi}$
τ_{xx}	$-2\,\mu\frac{\partial v_x}{\partial x}+\mu^B\,\nabla\cdot\boldsymbol{v}$	τ_{rr}	$-2\,\mu\frac{\partial v_r}{\partial r}+\mu^B\,\nabla\cdot\boldsymbol{v}$	τ_{rr}	$-2\,\mu\frac{\partial v_r}{\partial r}+\mu^B\,\nabla\cdot\boldsymbol{v}$
τ_{yy}	$-2\,\mu\frac{\partial v_y}{\partial y}+\mu^B\,\nabla\cdot\boldsymbol{v}$	$\tau_{\theta\theta}$	$-2\,\mu\left(\frac{1}{r}\frac{\partial v_\theta}{\partial \theta}+\frac{v_r}{r}\right)+\mu^B\,\nabla\cdot\boldsymbol{v}$	$\tau_{\theta\theta}$	$-2\,\mu\left(\frac{1}{r}\frac{\partial v_\theta}{\partial \theta}+\frac{v_r}{r}\right)+\mu^B\,\nabla\cdot\boldsymbol{v}$
τ_{zz}	$-2\,\mu\frac{\partial v_z}{\partial z}+\mu^B\,\nabla\cdot\boldsymbol{v}$	τ_{zz}	$-2\,\mu\frac{\partial v_z}{\partial z}+\mu^B\,\nabla\cdot\boldsymbol{v}$	$\tau_{\phi\phi}$	$-2\,\mu\left(\frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi}+\frac{v_r+v_\theta\cot\theta}{r}\right)+\mu^B\,\nabla\cdot\boldsymbol{v}$
τ_{xy}	$-\mu\left(\frac{\partial v_x}{\partial y}+\frac{\partial v_y}{\partial x}\right)$	$\tau_{r\theta}$	$-\mu\left(r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right)+\frac{1}{r}\frac{\partial v_r}{\partial \theta}\right)$	$\tau_{r\theta}$	$-\mu\left(r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right)+\frac{1}{r}\frac{\partial v_r}{\partial \theta}\right)$
τ_{yz}	$-\mu\left(\frac{\partial v_y}{\partial z}+\frac{\partial v_z}{\partial y}\right)$	$\tau_{\theta z}$	$-\mu\left(\frac{1}{r}\frac{\partial v_z}{\partial \theta}+\frac{\partial v_\theta}{\partial z}\right)$	$\tau_{\theta\phi}$	$-\mu\left(\frac{\sin\theta}{r}\frac{\partial}{\partial \theta}\left(\frac{v_\phi}{\sin\theta}\right)+\frac{1}{r\sin\theta}\frac{\partial v_\theta}{\partial \phi}\right)$
τ_{xz}	$-\mu\left(\frac{\partial v_x}{\partial z}+\frac{\partial v_z}{\partial x}\right)$	τ_{zr}	$-\mu\left(\frac{\partial v_r}{\partial z}+\frac{\partial v_z}{\partial r}\right)$	$\tau_{\phi r}$	$-\mu\left(\frac{1}{r\sin\theta}\frac{\partial v_r}{\partial \phi}+r\frac{\partial}{\partial r}\left(\frac{v_\phi}{r}\right)\right)$

Table 1: *Fourier's law for the heat flux and Newton's law for the stress in several coordinate systems. Please remember that the stress is symmetric, so* $\tau_{ij}=\tau_{ji}$.

Viscous models:
Power-Law Fluid:

$$|\tau_{xy}| = k\left|\frac{\partial v_x}{\partial y}\right|^n \tag{5}$$

Bingham-Plastic Fluid:

$$\frac{\partial v_x}{\partial y} = \begin{cases} -\mu^{-1}\left(\tau_{xy}-\tau_0\right)) & \text{if } \tau_{xy} > \tau_0 \\ 0 & \text{if } \tau_{xy} \leq \tau_0 \end{cases}$$

Dimensionless Numbers

$$\text{Re} = \frac{\rho\,\langle v\rangle\,D}{\mu} \qquad \text{Re}_H = \frac{\rho\,\langle v\rangle\,D_H}{\mu} \qquad \text{Re}_{MR} = -\frac{16\,L\,\rho\,\langle v\rangle^2}{R\,\Delta p} \tag{6}$$

The hydraulic diameter is defined as $D_H = 4\,A/P_w$.

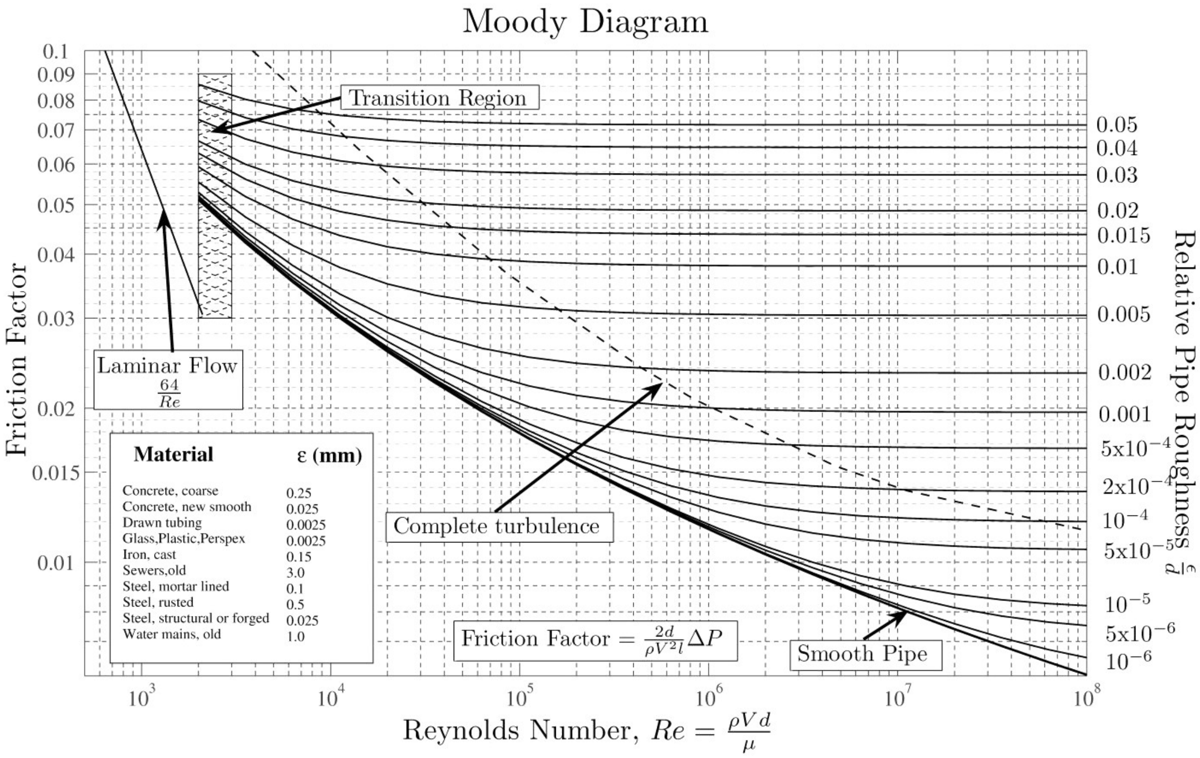
Single phase pressure drop calculations in pipes:
Darcy-Weisbach equation:

$$\frac{\Delta p}{L} = -\frac{C_f\,\rho\,\langle v\rangle^2}{R} \tag{7}$$

where $C_f = 16/Re$ for laminar Newtonian flow. For turbulent flow of Newtonian fluids in smooth pipes, we have the Blasius correlation:

$$C_f = 0.079\,\mathrm{Re}^{-1/4} \qquad \text{for } 2.5\times10^3 < \mathrm{Re} < 10^5 \text{ and smooth pipes.}$$

Otherwise, you may refer to the Moody diagram.



Laminar Power-Law fluid:

$$\dot{V} = \frac{n\,\pi\,R^3}{3\,n+1}\left(\frac{R}{2\,k}\right)^{\frac{1}{n}}\left(-\frac{\Delta p}{L}\right)^{\frac{1}{n}}$$

Two-Phase Flow:
Lockhart-Martinelli parameter:

$$X^2 = \frac{\Delta p_{liq.-only}}{\Delta p_{gas-only}}$$

Pressure drop calculation:

$$\Delta p_{two-phase} = \Phi^2_{liq.}\,\Delta p_{liq.-only} = \Phi^2_{gas}\,\Delta p_{gas-only}$$

Chisholm's relation:

$$\Phi^2_{gas} = 1 + c\,X + X^2$$

$$\Phi^2_{liq.} = 1 + \frac{c}{X} + \frac{1}{X^2} \qquad c = \begin{cases} 20 & \text{turbulent liquid \& turbulent gas} \\ 12 & \text{laminar liquid \& turbulent gas} \\ 10 & \text{turbulent liquid \& laminar gas} \\ 5 & \text{laminar liquid \& laminar gas} \end{cases}$$

Farooqi and Richardson expression for liquid hold-up in co-current flows of Newtonian fluids and air in horizontal pipes:

$$h = \begin{cases} 0.186 + 0.0191\,X & 1 < X < 5 \\ 0.143\,X^{0.42} & 5 < X < 50 \\ 1/\left(0.97 + 19/X\right) & 50 < X < 500 \end{cases}$$

Heat Transfer:
Stefan-Boltzmann constant $\sigma = 5.6703 \times 10^{-8}$ W/m^2 K^4

Heat Transfer Dimensionless numbers:

$$\text{Nu} = \frac{h\,L}{k} \qquad \text{Pr} = \frac{\mu\,C_p}{k} \qquad \text{Gr} = \frac{g\,\beta\,(T_w - T_\infty)\;L^3}{\nu^2}$$

Resistances

$$Q = U_T\,A_T\,\Delta T = R_T^{-1}\,\Delta T \qquad Q_{rad.} = \sigma\,\varepsilon\,A\left(T_\infty^4 - T_w^4\right) = h_{rad.}\,A\,(T_\infty - T_w)$$

	Conduction Shell Resistances			Radiation
	Rect.	Cyl.	Sph.	
R	$\dfrac{X}{k\,A}$	$\dfrac{\ln\left(R_{outer}/R_{inner}\right)}{2\,\pi\,L\,k}$	$\dfrac{R_{inner}^{-1} - R_{outer}^{-1}}{4\,\pi\,k}$	$\left[A\,\varepsilon\,\sigma\,(T_\infty^2 + T_w^2)\,(T_\infty + T_w)\right]^{-1}$

Natural Convection

Ra $=$ Gr Pr	C	m
$< 10^4$	1.36	1/5
10^4–10^9	0.59	1/4
$> 10^9$	0.13	1/3

Table 2: *Natural convection coefficients for isothermal vertical plates in the empirical relation Nu $\approx C\,(\text{Gr}\,\text{Pr})^m$.*

For isothermal vertical cylinders, the above expressions for isothermal vertical plates may be used but must be scaled by a factor, F:

$$F = \begin{cases} 1 & \text{for } (D/H) < 35\,\text{Gr}_H^{-1/4} \\ 1.3\left[H\,D^{-1}\,\text{Gr}_D^{-1}\right]^{1/4} + 1 & \text{for } (D/H) \geq 35\,\text{Gr}_H^{-1/4} \end{cases}$$

where D is the diameter and H is the height of the cylinder. The subscript on Gr indicates which length is to be used as the critical length to calculate the Grashof number.

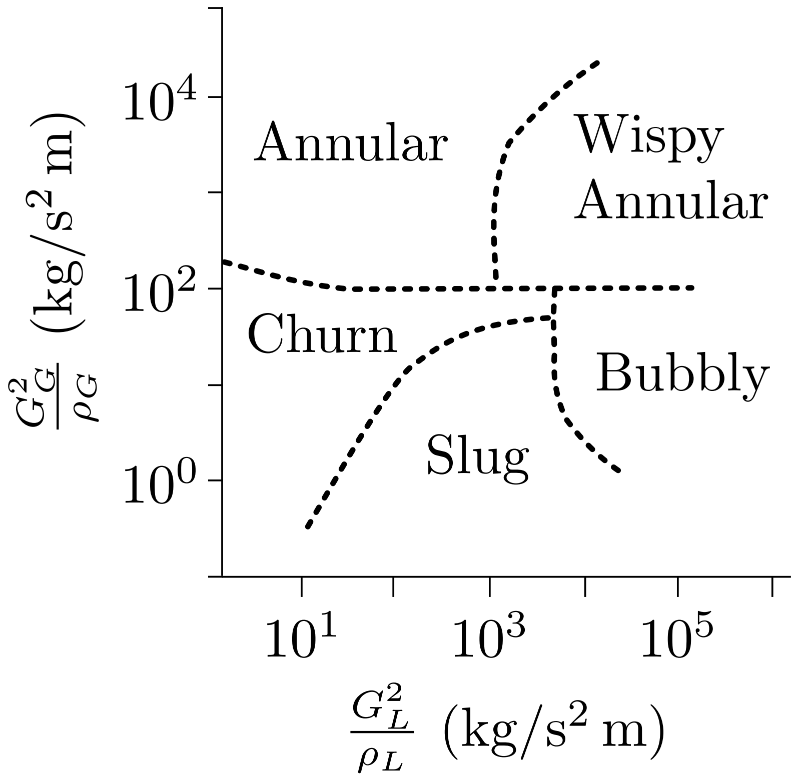


Figure 3: *Hewitt-Taylor flow pattern map for multiphase flows in vertical pipes.*

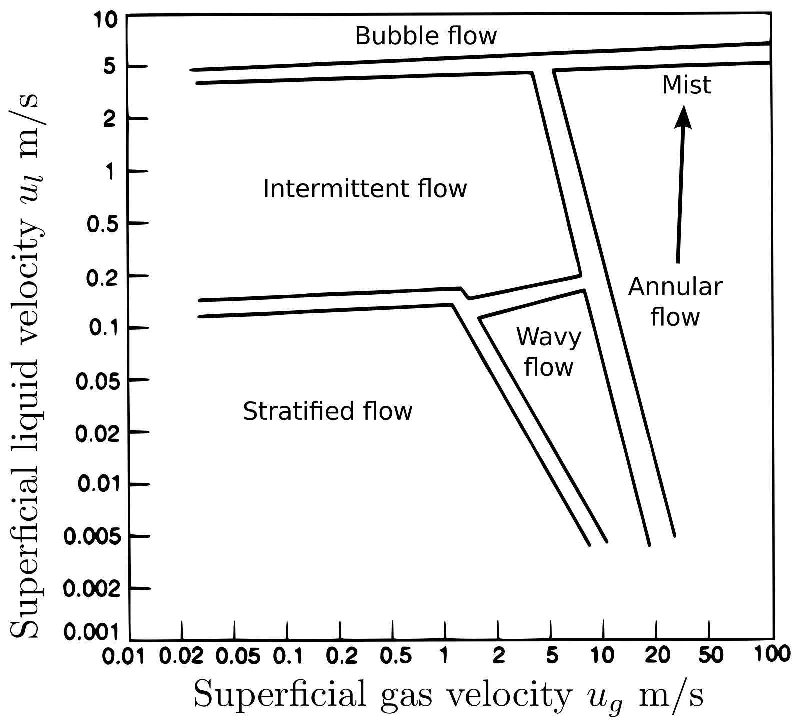


Figure 4: *Chhabra and Richardson flow pattern map for horizontal pipes.*

Churchill and Chu expression for natural convection from a horizontal pipe:

$$\mathsf{Nu}^{1/2} = 0.6 + 0.387 \left\{ \frac{\mathsf{Gr}\,\mathsf{Pr}}{\left[1 + (0.559/\mathsf{Pr})^{9/16}\right]^{16/9}} \right\}^{1/6} \qquad \text{for } 10^{-5} < \mathsf{Gr}\,\mathsf{Pr} < 10^{12}$$

Forced Convection:
Laminar flows:

$$\mathsf{Nu} \approx 0.332\,\mathsf{Re}^{1/2}\,\mathsf{Pr}^{1/3}$$

Well-Developed turbulent flows in smooth pipes:

$$\mathsf{Nu} \approx \frac{(C_f/2)\mathsf{Re}\,\mathsf{Pr}}{1.07 + 12.7(C_f/2)^{1/2}\left(\mathsf{Pr}^{2/3} - 1\right)} \left(\frac{\mu_b}{\mu_w}\right)^{0.14}$$

Boiling:
Forster-Zuber pool-boiling coefficient:

$$h_{nb} = 0.00122 \frac{k_L^{0.79}\,C_{p,L}^{0.45}\,\rho_L^{0.49}}{\gamma^{0.5}\,\mu_L^{0.29}\,h_{fg}^{0.24}\,\rho_G^{0.24}} (T_w - T_{sat})^{0.24} (p_w - p_{sat})^{0.75}$$

Mostinski correlations:

$$h_{nb} = 0.104\,p_c^{0.69}\,q^{0.7} \left[1.8\left(\frac{p}{p_c}\right)^{0.17} + 4\left(\frac{p}{p_c}\right)^{1.2} + 10\left(\frac{p}{p_c}\right)^{10}\right]$$

$$q_c = 3.67 \times 10^4\,p_c \left(\frac{p}{p_c}\right)^{0.35} \left[1 - \frac{p}{p_c}\right]^{0.9}$$

(**Note**: for the Mostinski correlations, the pressures are in units of bar)
Condensing:
Horizontal pipes

$$h = 0.72\left(\frac{k^3\,\rho^2\,g_x\,E_{latent}}{D\,\mu\,(T_w - T_\infty)}\right)^{1/4}$$

Lumped capacitance method:

$$\mathsf{Bi} = \frac{h\,L_c}{\kappa}$$

$$L_c = V/A \qquad \text{for } \mathsf{Bi} < 0.1$$

1-D Transient Heat Conduction:

$$Fo = \frac{\alpha \Delta t}{(\Delta x)^2}$$

$$\theta_{\text{wall}} = \frac{T(x,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \cos\left(\frac{\lambda_1 x}{L}\right), \qquad \theta_{\text{cyl}} = \frac{T(r,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \mathbf{J_0}\left(\frac{\lambda_1 r}{r_0}\right)$$

$$\theta_{\text{sph}} = \frac{T(r,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \frac{\sin\left(\frac{\lambda_1 r}{r_0}\right)}{\frac{\lambda_1 r}{r_0}}$$

$$\left(\frac{\mathcal{Q}}{\mathcal{Q}_{\text{max}}}\right)_{\text{wall}} = 1 - \theta_{0,\text{wall}} \frac{\sin \lambda_1}{\lambda_1}, \quad \left(\frac{\mathcal{Q}}{\mathcal{Q}_{\text{max}}}\right)_{\text{cyl}} = 1 - 2\theta_{0,\text{cyl}} \frac{\mathbf{J_1}}{\lambda_1}$$

$$\left(\frac{\mathcal{Q}}{\mathcal{Q}_{\text{max}}}\right)_{\text{sph}} = 1 - 3\theta_{0,\text{sph}} \frac{\sin \lambda_1 - \lambda_1 \cos \lambda_1}{\lambda_1^3}$$

TABLE 4–2

Coefficients used in the one-term approximate solution of transient one-dimensional heat conduction in plane walls, cylinders, and spheres (Bi = hL/k for a plane wall of thickness $2L$, and Bi = hr_o/k for a cylinder or sphere of radius r_o)

	Plane Wall		*Cylinder*		*Sphere*	
Bi	λ_1	A_1	λ_1	A_1	λ_1	A_1
0.01	0.0998	1.0017	0.1412	1.0025	0.1730	1.0030
0.02	0.1410	1.0033	0.1995	1.0050	0.2445	1.0060
0.04	0.1987	1.0066	0.2814	1.0099	0.3450	1.0120
0.06	0.2425	1.0098	0.3438	1.0148	0.4217	1.0179
0.08	0.2791	1.0130	0.3960	1.0197	0.4860	1.0239
0.1	0.3111	1.0161	0.4417	1.0246	0.5423	1.0298
0.2	0.4328	1.0311	0.6170	1.0483	0.7593	1.0592
0.3	0.5218	1.0450	0.7465	1.0712	0.9208	1.0880
0.4	0.5932	1.0580	0.8516	1.0931	1.0528	1.1164
0.5	0.6533	1.0701	0.9408	1.1143	1.1656	1.1441
0.6	0.7051	1.0814	1.0184	1.1345	1.2644	1.1713
0.7	0.7506	1.0918	1.0873	1.1539	1.3525	1.1978
0.8	0.7910	1.1016	1.1490	1.1724	1.4320	1.2236
0.9	0.8274	1.1107	1.2048	1.1902	1.5044	1.2488
1.0	0.8603	1.1191	1.2558	1.2071	1.5708	1.2732
2.0	1.0769	1.1785	1.5995	1.3384	2.0288	1.4793
3.0	1.1925	1.2102	1.7887	1.4191	2.2889	1.6227
4.0	1.2646	1.2287	1.9081	1.4698	2.4556	1.7202
5.0	1.3138	1.2403	1.9898	1.5029	2.5704	1.7870
6.0	1.3496	1.2479	2.0490	1.5253	2.6537	1.8338
7.0	1.3766	1.2532	2.0937	1.5411	2.7165	1.8673
8.0	1.3978	1.2570	2.1286	1.5526	2.7654	1.8920
9.0	1.4149	1.2598	2.1566	1.5611	2.8044	1.9106
10.0	1.4289	1.2620	2.1795	1.5677	2.8363	1.9249
20.0	1.4961	1.2699	2.2880	1.5919	2.9857	1.9781
30.0	1.5202	1.2717	2.3261	1.5973	3.0372	1.9898
40.0	1.5325	1.2723	2.3455	1.5993	3.0632	1.9942
50.0	1.5400	1.2727	2.3572	1.6002	3.0788	1.9962
100.0	1.5552	1.2731	2.3809	1.6015	3.1102	1.9990
∞	1.5708	1.2732	2.4048	1.6021	3.1416	2.0000

TABLE 4–3

The zeroth- and first-order Bessel functions of the first kind

η	$J_0(\eta)$	$J_1(\eta)$
0.0	1.0000	0.0000
0.1	0.9975	0.0499
0.2	0.9900	0.0995
0.3	0.9776	0.1483
0.4	0.9604	0.1960
0.5	0.9385	0.2423
0.6	0.9120	0.2867
0.7	0.8812	0.3290
0.8	0.8463	0.3688
0.9	0.8075	0.4059
1.0	0.7652	0.4400
1.1	0.7196	0.4709
1.2	0.6711	0.4983
1.3	0.6201	0.5220
1.4	0.5669	0.5419
1.5	0.5118	0.5579
1.6	0.4554	0.5699
1.7	0.3980	0.5778
1.8	0.3400	0.5815
1.9	0.2818	0.5812
2.0	0.2239	0.5767
2.1	0.1666	0.5683
2.2	0.1104	0.5560
2.3	0.0555	0.5399
2.4	0.0025	0.5202
2.6	−0.0968	−0.4708
2.8	−0.1850	−0.4097
3.0	−0.2601	−0.3391
3.2	−0.3202	−0.2613

Figure 5: *Coefficients for the 1D transient equations.*

Overall Heat Transfer Coefficient:

$$\dot{\mathcal{Q}} = \frac{\Delta T}{\mathcal{R}} = UA\Delta T = U_i A_i \Delta T = U_o A_o \Delta T$$

$$\mathcal{R} = R_i + R_{\text{wall}} + R_o = \frac{1}{h_i A_i} + \frac{\ln D_o/D_i}{2\pi\kappa L} + \frac{1}{h_o A_o}$$

Fouling Factor:*

$$\mathcal{R} = \frac{1}{h_i A_i} + \frac{R_{\text{f,i}}}{A_i} + R_{\text{wall}} + \frac{R_{\text{f,o}}}{A_o} + \frac{1}{h_o A_o}$$

LMTD Method:

$$\dot{\mathcal{Q}} = UA_s\Delta T_{\text{lm}} \text{ with } \Delta T_{\text{lm}} = \frac{(T_{\text{hot,out}} - T_{\text{cold,out}}) - (T_{\text{hot,in}} - T_{\text{cold,in}})}{\ln\left(\frac{T_{\text{hot,out}} - T_{\text{cold,out}}}{T_{\text{hot,in}} - T_{\text{cold,in}}}\right)}.$$

Diffusion Dimensionless Numbers

$$\text{Sc} = \frac{\mu}{\rho\, D_{AB}} \qquad \text{Le} = \frac{k}{\rho\, C_p\, D_{AB}}$$

Diffusion

General expression for the flux:

$$\boldsymbol{N}_A = \boldsymbol{J}_A + x_A \sum_B \boldsymbol{N}_B$$

Fick's law:

$$\boldsymbol{J}_A = -D_{AB}\,\nabla C_A$$

Stefan's law:

$$N_{s,r} = -D\frac{c}{1-x}\frac{\partial x}{\partial r}$$

Misc

$$P\,V = n\,R\,T \qquad R \approx 8.314598 \text{ J K}^{-1} \text{ mol}^{-1}$$

Table 10-3 | Heat-exchanger effectiveness relations.

$N = \mathrm{NTU} = \dfrac{UA}{C_{min}}$ $\quad C = \dfrac{C_{min}}{C_{max}}$

Flow geometry	Relation
Double pipe:	
Parallel flow	$\epsilon = \dfrac{1-\exp[-N(1+C)]}{1+C}$
Counterflow	$\epsilon = \dfrac{1-\exp[-N(1-C)]}{1-C\exp[-N(1-C)]}$
Counterflow, $C = 1$	$\epsilon = \dfrac{N}{N+1}$
Cross flow:	
Both fluids unmixed	$\epsilon = 1-\exp\left[\dfrac{\exp(-NCn)-1}{Cn}\right]$ where $n = N^{-0.22}$
Both fluids mixed	$\epsilon = \left[\dfrac{1}{1-\exp(-N)} + \dfrac{C}{1-\exp(-NC)} - \dfrac{1}{N}\right]^{-1}$
C_{max} mixed, C_{min} unmixed	$\epsilon = (1/C)\{1-\exp[-C(1-e^{-N})]\}$
C_{max} unmixed, C_{min} mixed	$\epsilon = 1-\exp\{-(1/C)[1-\exp(-NC)]\}$
Shell and tube:	
One shell pass, 2, 4, 6, tube passes	$\epsilon = 2\left\{1+C+(1+C^2)^{1/2} \times \dfrac{1+\exp[-N(1+C^2)^{1/2}]}{1-\exp[-N(1+C^2)^{1/2}]}\right\}^{-1}$
Multiple shell passes, $2n$, $4n$, $6n$ tube passes ($\epsilon_p =$ effectiveness of each shell pass, $n =$ number of shell passes)	$\epsilon = \dfrac{[(1-\epsilon_p C)/(1-\epsilon_p)]^n - 1}{[(1-\epsilon_p C)/(1-\epsilon_p)]^n - C}$
Special case for $C = 1$	$\epsilon = \dfrac{n\epsilon_p}{1+(n-1)\epsilon_p}$
All exchangers with $C = 0$	$\epsilon = 1 - e^{-N}$

Table 10-4 | NTU relations for heat exchangers.

$C = C_{min}/C_{max}$ $\quad \epsilon =$ effectiveness $\quad N = \mathrm{NTU} = UA/C_{min}$

Flow geometry	Relation
Double pipe:	
Parallel flow	$N = \dfrac{-\ln[1-(1+C)\epsilon]}{1+C}$
Counterflow	$N = \dfrac{1}{C-1}\ln\left(\dfrac{\epsilon-1}{C\epsilon-1}\right)$
Counterflow, $C = 1$	$N = \dfrac{\epsilon}{1-\epsilon}$
Cross flow:	
C_{max} mixed, C_{min} unmixed	$N = -\ln\left[1+\dfrac{1}{C}\ln(1-C\epsilon)\right]$
C_{max} unmixed, C_{min} mixed	$N = \dfrac{-1}{C}\ln[1+C\ln(1-\epsilon)]$
Shell and tube:	
One shell pass, 2, 4, 6, tube passes	$N = -(1+C^2)^{-1/2} \times \ln\left[\dfrac{2/\epsilon-1-C-(1+C^2)^{1/2}}{2/\epsilon-1-C+(1+C^2)^{1/2}}\right]$
All exchangers, $C = 0$	$N = -\ln(1-\epsilon)$

Figure 6: *NTU relations*