The Finite-Volume Method for Computational Fluid Dynamics

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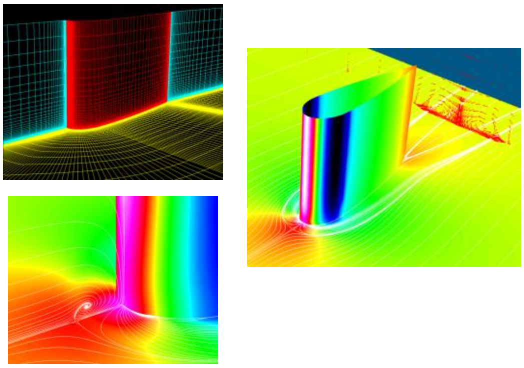
1. Introduction

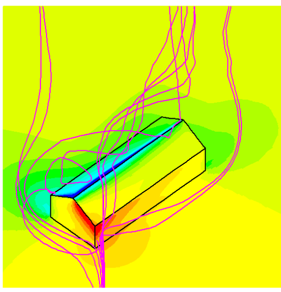
What is … Computational Fluid Dynamics?

The use of computers and numerical methods to solve problems involving fluid flow

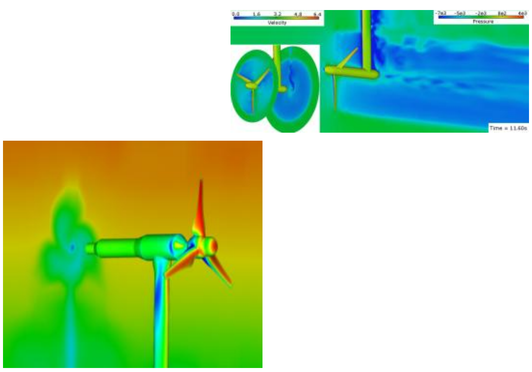
Aerodynamics



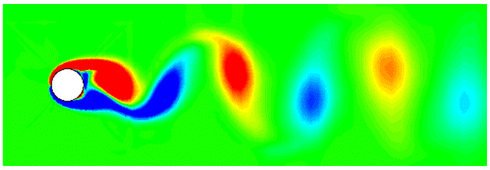
Wind Loading



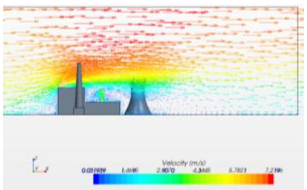
Turbine Technology

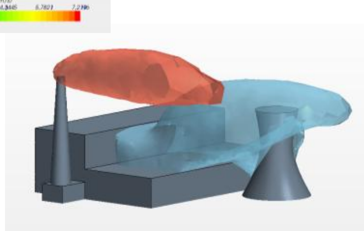


Vortex Shedding

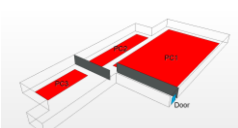


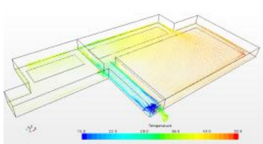
Dispersion of Pollution

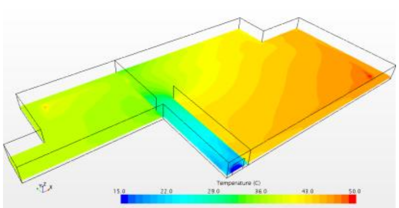




Ventilation





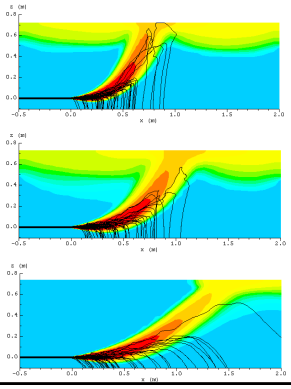


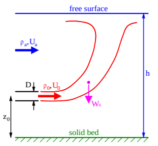
Particle-Laden Plumes (Sea Outfalls)

(ex302)
ρ_a=1040 kg/m^3

(ex322)
ρ_a=1020 kg/m^3

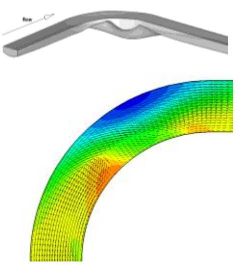
(ex336)
ρ_a=1020 kg/m^3
U_a=0.026 m/s



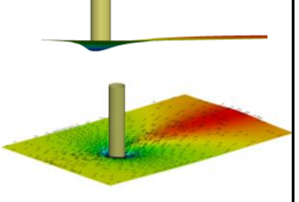


Sediment Scour

River bend

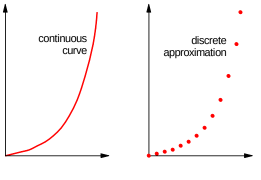


Bridge pier

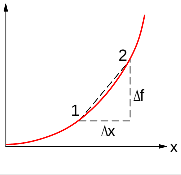


Discretisation

Variables:



Equations:



$$\frac{df}{dx} \approx \frac{\Delta f}{\Delta x} = \frac{f_2 - f_1}{x_2 - x_1}$$

Basic Principles of CFD

1. **Discretise space:**
   replace field variables (ρ, u, v, w, p, ...) by values at a finite number of **nodes**
2. **Discretise equations:**
   continuum equations $\rightarrow$ **algebraic** equations
3. **Solve:**
   large system of **simultaneous equations**

Stages of a CFD Analysis

1. **Pre-processing:**
   - formulate problem (equations and boundary conditions)
   - construct computational mesh
2. **Solving:**
   - discretise
   - solve
3. **Post-processing:**
   - analyse
   - visualise (graphs and plots)

Fluid-Flow Equations

- **Mass:** change of mass = 0
- **Momentum:** change of momentum = force × time
- **Energy:** change of energy = work + heat
- (**Other constituents**)

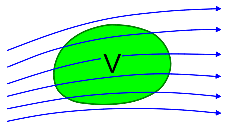
In fluid mechanics, these are normally expressed in **rate** form

Form of Equations

- **Integral** (control-volume)
- **Differential**

Integral (Control-Volume) Approach

Consider the budget of any physical property in a **control volume**



$$\begin{pmatrix}\text{RATE OF CHANGE}\\ \textit{inside } V\end{pmatrix} + \begin{pmatrix}\text{NET FLUX}\\ \textit{through boundary of } V\end{pmatrix} = \begin{pmatrix}\text{SOURCE}\\ \textit{inside } V\end{pmatrix}$$

$$\begin{pmatrix}\text{RATE OF CHANGE}\\ \textit{inside } V\end{pmatrix} + \begin{pmatrix}\text{ADVECTION} + \text{DIFFUSION}\\ \textit{through boundary of } V\end{pmatrix} = \begin{pmatrix}\text{SOURCE}\\ \textit{inside } V\end{pmatrix}$$

→ **Finite-volume** method for CFD

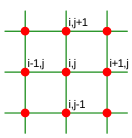
Differential Equations For Fluid Flow

- Derived by considering the rate of change at a **point**
- Discretisation gives the **finite-difference** method for CFD
- Several types:
  - fixed-point ("Eulerian"): **conservative**
  - moving with the flow ("Lagrangian"): **non-conservative**
  - **derived variables**; e.g. potential flow

Main Methods for CFD

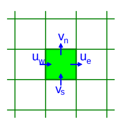
- **Finite-difference:**
  - discretise **differential** equations

$$0 = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \approx \frac{u_{i+1,j} - u_{i-1,j}}{2\Delta x} + \frac{v_{i,j+1} - v_{i,j-1}}{2\Delta y}$$



- **Finite-volume:**
  - discretise **control-volume** equations

$$0 = \textit{net mass outflow} = (\rho uA)_e - (\rho uA)_w + (\rho vA)_n - (\rho vA)_s$$

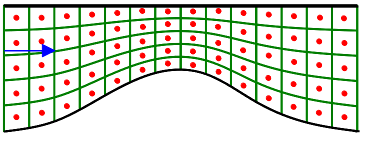


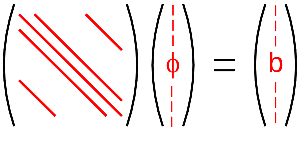
- **Finite-element:**
  - represent solution as a weighted sum of **basis functions**

$$u(\mathbf{x}) = \sum u_\alpha S_\alpha(\mathbf{x})$$

Advantages of the Finite-Volume Method in CFD

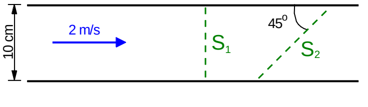
- Rigorously enforces **conservation**
- **Flexible** in terms of:
  - **geometry**
  - **fluid phenomena**
- Directly relatable to **physical quantities**





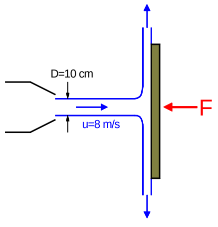
Example Q1

Water (density 1000 kg m^{-3}) flows at 2 m s^{-1} through a circular pipe of diameter 10 cm. What is the mass flux C across the surfaces S_1 and S_2?



Example Q2

A water jet strikes normal to a fixed plate as shown. Compute the force F required to hold the plate fixed.



Example Q3

An explosion releases 2 kg of a toxic gas into a room of dimensions 30 m $\times$ 8 m $\times$ 5 m. Assuming the room air to be well-mixed and to be vented at a speed of 0.5 m s^{-1} through an aperture of area 6 m^2, calculate:

(a) the initial concentration of gas in ppm by mass;
(b) the time taken to reach a safe concentration of 1 ppm.

(For air, density = 1.20 kg m^{-3}.)