# Solution of the Problems – Exam (May 2013/14)

**Question 1:** Reheat and Regenerative Rankine cycle with two turbines:

(a) In order to fill the Table we need to calculate the thermodynamic properties for each stage of the cycle

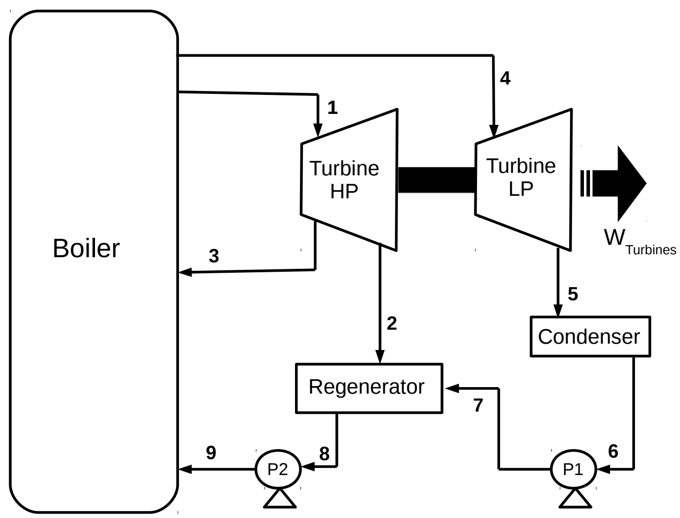


Figure 1: Reheat and regenerative Rankine cycle with 2 turbines.

**Stage 1:** The fluid leaving the boiler towards the first turbine is at 120 bar and 565$^o$C. This is well above the saturation temperature ($T_{\text{sat}} = 324.60^{\text{o}}\text{C}$) and we can thus confirm that the fluid is superheated steam (1 Mark). At such pressure, the superheated steam tables (SHST) results in (through linear interpolation) $H_1 = 3518.17\dfrac{kJ}{kg}$ (1 Mark) and $S_1 = 6.6983\dfrac{kJ}{kg.K}$ (1 Mark).

**Stage 2:** Isentropic expansion in *HP Turbine* at $P_2 = 3$ bar $\Leftrightarrow \quad S_2 = S_1 = 6.6983\dfrac{kJ}{kg.K}$. The fluid is at wet vapour state. The quality of the steam is

$$x_2 = \frac{S_2 - S_f}{S_g - S_f} = \frac{6.6983 - 1.6716}{6.9909 - 1.6716} = 0.9450 \quad (1 Mark)$$

Now calculating the enthalpy,

$$x_2 = \frac{H_2 - H_f}{H_g - H_f} = 0.9450 \Leftrightarrow H_2 = 2605.72\frac{kJ}{kg} \quad (1 Mark)$$

**Stage 3:** The fluid at $P_3 = P_2 = 3.0$ bar and $T_3 = 250^oC\,(>> T_{\text{sat}} = 133.5^oC)$ is superheated steam with $H_3 = 2967.6\dfrac{kJ}{kg}$ and $S_3 = 7.517\dfrac{kJ}{kg.K}$.

**Stage 4:** The steam leaves the boiler towards the LP turbine at $P_4 = P_3 = 3.0$ bar and $T_4 = 475^oC$ (also as superheated steam) with (via linear interpolation) $H_4 = 3433.33\dfrac{kJ}{kg}$ and $S_4 = 8.252\dfrac{kJ}{kg}$.

**Stage 5:** Isentropic expansion with $P_5 = 0.060$ bar (with $S_5 = S_4$). The quality of the steam is

$$x_5 = \frac{S_5 - S_f}{S_g - S_f} = \frac{8.252 - 0.521}{8.330 - 0.521} = 0.99\ (1Mark)$$

and the enthalpy,

$$x_5 = 0.99 = \frac{H_5 - H_f}{H_g - H_f} = \frac{H_5 - 151.5}{2567.4 - 151.5} \Leftrightarrow H_5 = 2543.24\frac{kJ}{kg}\quad (1Mark)$$

**Stage 6:** The fluid leaves the condenser at $P_6 = P_5 = 0.06$ bar is saturated liquid with $H_6$=$H_f$ (0.06 bar) $= 151.5\dfrac{kJ}{kg}$ (1 Mark).

**Stage 7:** Saturated and incompressible liquid leaving the pump towards the regenerator at $P_7 = P_2 = 3.0$ bar,

$$\begin{aligned} H_7 &\approx H_6 + V_6(P_7 - P_6) \\ &\approx 151.5\frac{kJ}{kg} + 0.001006\frac{m^3}{kg}(3 - 0.06)\text{ bar} \times \frac{10^5 kg/(m.s^2)}{1\text{ bar}} \times \frac{10^{-3}\frac{kJ}{kg}}{m^2/s^2} \times \frac{1}{0.61} \\ &\approx 151.795\ \frac{kJ}{kg}\quad (1\ Mark) \end{aligned}$$

**Stage 8:** Saturated liquid water leaving the regenerator at $P_8 = 3.0$ bar with $H_8 = H_f(3.0\text{ bar}) = 561.4\dfrac{kJ}{kg}$ and $V_8 = 0.001068\dfrac{m^3}{kg}$.

**Stage 9:** Saturated and incompressible liquid at $P_9 = 120$ bar,

$$\begin{aligned} H_9 &\approx H_8 + V_8(P_9 - P_8) \\ &\approx 561.4\frac{kJ}{kg} + 0.001068\frac{m^3}{kg}(120 - 3)\text{ bar} \times \frac{10^5 kg/(m.s^2)}{1\text{ bar}} \times \frac{10^{-3}\frac{kJ}{kg}}{m^2/s^2} \times \frac{1}{0.61} \\ &\approx 573.90\ \frac{kJ}{kg}\quad (1\ Mark) \end{aligned}$$

Thus the Table becomes:

| Stage | P (bar) | T ($^oC$) | State | H (kJ.kg$^{-1}$) | S (kJ.(kg.K)$^{-1}$) | Steam Quality |
|---|---|---|---|---|---|---|
| **1** | 120 | 565 | **Superheated vapour** | **3518.17** | **6.6983** | – |
| **2** | 3 | – | wet vapour | **2605.72** | – | **0.9450** |
| **3** | 3 | 250 | – | – | – | – |
| **4** | 3 | 475 | – | – | – | – |
| **5** | 0.06 | – | wet vapour | **2543.24** | – | **0.99** |
| **6** | – | – | sat.liquid | **151.5** | – | – |
| **7** | 3 | – | – | **151.8** | – | – |
| **8** | 3 | – | – | – | – | – |
| **9** | 120 | – | – | **573.90** | – | – |

(b) Energy balance in the regenerator, assuming total mass of water of ($m_{\text{T}} =$) 1 kg, and that a fraction, $\mathcal{F}$, is bled-off from the HP turbine to the regenerator, and the remaining water-steam, $1 - \mathcal{F}$ is conducted back to the boiler.

$$m_{\text{T}} H_8 = \mathcal{F} H_2 + (1 - \mathcal{F})\, H_7 \Rightarrow \mathcal{F} = 0.1669 kg$$

Thus 83.3% $(1 - \mathcal{F})$ of the steam was supplied to the LP turbine (2 Marks).

(c) The heat supplied by the boiler ($Q_{\text{Boiler}}$) can be calculated through the energy balance,

$$Q_{\text{Boiler}} = [m_{\text{T}} H_1 + (1 - \mathcal{F})\, H_4] - [(1 - \mathcal{F})\, H_3 + m_{\text{T}} H_9] \Rightarrow Q_{\text{Boiler}} = 3332.27 \frac{kJ}{kg}$$

(2 Marks).

(d) Now, in order to calculate the thermal efficiency of the cycle,

$$\eta = \frac{W_{Total}}{Q_{Boiler}} = \frac{\sum W_{\text{Turbines}} - \sum W_{\text{Pumps}}}{Q_{\text{Boiler}}}$$

We need to calculate the work associated with the turbines and pumps.

**HP Turbine:** $W_{\text{T,HP}} = m_{\text{T}} H_1 - [\mathcal{F} H_2 + (1 - \mathcal{F})\, H_3] = 610.97 \frac{kJ}{kg}$ (1 Mark)

**LP Turbine:** $W_{\text{T,LP}} = (1 - \mathcal{F})\, (H_4 - H_5) = 741.53 \frac{kJ}{kg}$ (1 Mark)

**Pump 1:** $W_{\text{P,1}} = (1 - \mathcal{F})\, (H_7 - H_6) = 0.246 \frac{kJ}{kg}$ (1 Mark)

**Pump 2:** $W_{\text{P,2}} = m_{\text{T}}\, (H_9 - H_8) = 12.4 \frac{kJ}{kg}$ (1 Mark)

Thus the thermal efficiency of the cycle is,

$$\eta = \frac{\sum W_{\text{Turbines}} - \sum W_{\text{Pumps}}}{Q_{\text{Boiler}}} = \frac{1339.76}{3332.27} = 0.4021 \quad (2 Marks)$$

**Question 2:**

**Question 3:**

**Question 4:** Figure 2 shows the refrigeration cycle operating as a heat pump with R-134a.

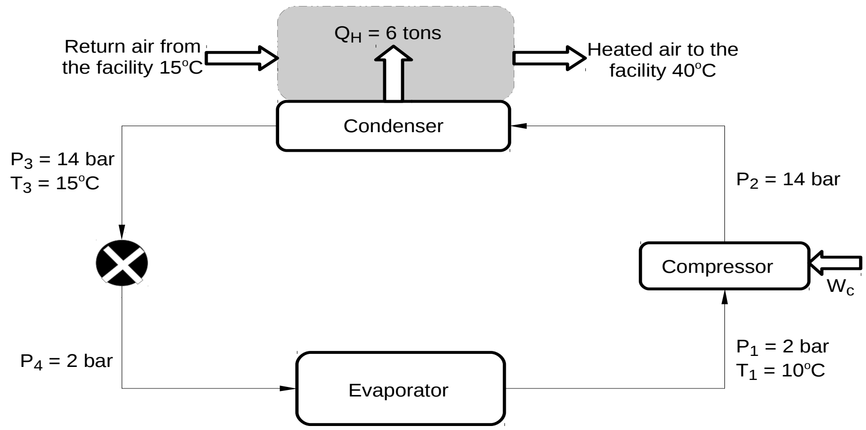


Figure 2: Heat pump cycle (Question 4).

(a) Calculating all enthalpies and entropies of the cycle:

**Stage 1:** The refrigerant fluid leaves the evaporator at $P_1 = 2$ bar and $T_1 = 10^oC >> T_{\text{sat}} = -10.09^oC$, thus the fluid is at *superheated vapour* state and with $H_1 = 258.89\frac{kJ}{kg}$ (1 Mark) and $S_1 = 0.9898\frac{kJ}{kg.K}$ (1 Mark).

**Stage 2:** Isentropic compression, $S_2 = S_1$ (1 Mark), and assuming ideal compressor at $P_2 = 14$ bar. Via linear interpolation, $H_2 = 303,66\frac{kJ}{kg}$ (1 Mark) and $T_2 = 77.08^oC$.

**Stage 3:** The fluid leaves the condenser at $P_3 = 14$ bar and $T_3 = 15^oC$ ($<< T_{\text{sat}} = 52.4^oC$) is a *sub-cooled liquid* with $H_3 = 125.26\frac{kJ}{kg}$ (1 Mark) and $S_3 = 0.4453\frac{kJ}{kg.K}$ (1 Mark).

**Stage 4:** Isenthalpic process, $H_4 = H_3$ (1 Mark), at $P_4 = 2$ bar. Calculating the quality of the vapour,

$$x_4 = \frac{H_4 - H_f}{H_g - H_f} = \frac{125.26 - 36.84}{241.30 - 36.84} = 0.4325$$

and the entropy,

$$x_4 = \frac{S_4 - S_f}{S_g - S_f} = \frac{S_4 - 0.1481}{0.9253 - 0.1481} \Rightarrow S_4 = 0.4842\frac{kJ}{kg.K} \quad \text{(1 Mark)}$$

(b) In order to calculate the volumetric flow rate of heated air, we first need to determine the mass flow rate,

$$
\begin{aligned}
&Q_H = \dot{m}_{\text{air}} \left(H_{\text{out}}^{\text{air}} - H_{\text{in}}^{\text{air}}\right) = \dot{m}_{\text{air}} C_p^{\text{air}} \left(T_{\text{out}}^{\text{air}} - T_{\text{in}}^{\text{air}}\right) \\
&6 \text{ ton} \times \frac{210 \dfrac{kJ}{\text{min}}}{1 \text{ ton}} \times \frac{1 \text{ min}}{60s} = \dot{m}_{\text{air}} \times 1.004 \frac{kJ}{kg.K} (40 - 15)^{\text{o}} C \\
&\dot{m}_{\text{air}} = 0.8367 \frac{kg}{s}
\end{aligned}
$$

Now for the volumetric flow rate (with $T = 40^{\text{o}}C$ and $P = 1.01325$ bar)

$$
\dot{V}_{\text{air}}^{\text{out}} = \dot{m}_{\text{air}} V_{\text{air}}^{\text{out}} = \dot{m}_{\text{air}} \frac{RT_{\text{air}}^{\text{out}}}{P_{\text{air}}} \Rightarrow \dot{V}_{\text{air}}^{\text{out}} = 7.42 \times 10^{-4} \frac{m^3}{s} \text{ (3 Marks)}
$$

(c) The mass flow rate of the refrigerant fluid R-134a can be calculated as,

$$
Q_H = \dot{m}_R (H_2 - H_3) \Rightarrow \dot{m}_R = 0.1177 \frac{kg}{s} \text{ (3 Marks)}
$$

(d) The compressor power,

$$
W_C = \dot{m}_R (H_2 - H_1) \Rightarrow W_C = 5.27 \ kW \quad \text{(3 Marks)}
$$

(e) And the COP,

$$
\text{COP} = \frac{Q_H}{W_C} = 3.98 \quad \text{(3 Marks)}
$$

**Question 5:** 1. This is a dual combustion cycle and the $TS$ (2 Marks) and $PV$ (2 Marks) diagrams are:

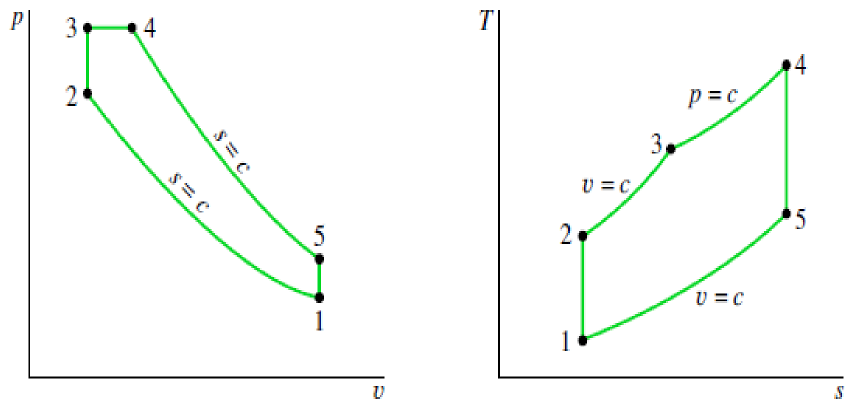


Figure 3: Dual combustion cycle – $PV$ and $TS$ (rhs) diagrams (Question 5).

2. Thermal analysis, for $m$ kg of air:
   (a) Total heat supplied (2–3 + 3–4): $m\left[C_v\left(T_3 - T_2\right) + C_p\left(T_4 - T_3\right)\right]$ (2 Marks)
   (b) Total heat rejected (5–1): $mC_v\left(T_5 - T_1\right)$ (2 Marks)
   (c) Net Work: $W_{\text{net}} = m\left[C_v\left(T_3 - T_2\right) + C_p\left(T_4 - T_3\right) - C_v\left(T_5 - T_1\right)\right]$ (2 Marks)