Problem 1 For the heating of a residence in a cold enviroment (-5°C), a heat pump was used. The engine was designed to maintain the house's interior temperature at 25°C. The compressor heat pump is driven by a heat engine working between 1000°C and 25°C. Assume that both cycles are reversibles, determine the ratio in which the heat pump and the heat engine share the heating load.

Problem 2 A refrigerating engine operates on the Bell-Coleman cycle (of 6 tonnes capacity) with upper limiting pressure of 5.2 bar. Pressure and temperature at the beginning of the compression stage are 1.0 bar and 16°C, respectively. The compressed air is cooled at constant pressure from a temperature of 41°C (flow entering the expansion cylinder). Assuming that both expansion and compression processes are adiabatic with $\gamma = 1.4$ and the latent heat of fusion of water ($\mathrm{L_f}$) is 336 kJ/kg determine:

(a) COP;

(b) Mass flow rate of air in circulation (kg/min);

(c) Piston displacement of compressor and expander, bore of compressor and expansion cylinders. The unit runs at 240 rpm. Assume that the stroke length is 200 mm;

(d) Power required to drive the unit.

Problem 3 The clearance volumetric efficiency (η_{cv}) in a compressor (Fig. 1) is defined as,

$$\eta_{\text{CV}} = \frac{V_1 - V_4}{V_1 - V_{4'}}$$

Assuming polytropic expansion, derive

$$\eta_{\text{cv}} = 1 + \mathcal{C}\left[1 - \left(\frac{P_d}{P_s}\right)^{1/\gamma}\right]$$

where P_d are P_s are the discharge and suction pressures, respectively and the clearance ratio, $\mathcal{C}$ is

$$\mathcal{C} = \frac{\text{Clearance Volume}}{\text{Swept Volume}}$$

Problem 4 Air enters the compressor of a cold air-standard Brayton cycle at 100 kPa, 300 K, with a mass flow rate of 6 kg.s^{-1}. The compressor pressure ratio is 10, and the turbine inlet temperature is 1400 K. The turbine and compressor each have isentropic efficiencies of 80%. Calculate:

(a) Thermal efficiency of the cycle;

(b) Net power (in kW).

Efficiencies of the turbine and compressor are expressed as (subscript 1 indicates the stream entering the compressor),

$$\eta_T = \frac{T_4 - T_3}{T_{4s} - T_3} \quad \text{and} \quad \eta_C = \frac{T_{2s} - T_1}{T_2 - T_1},$$

respectively, where the subscript s refers to isentropic processes.

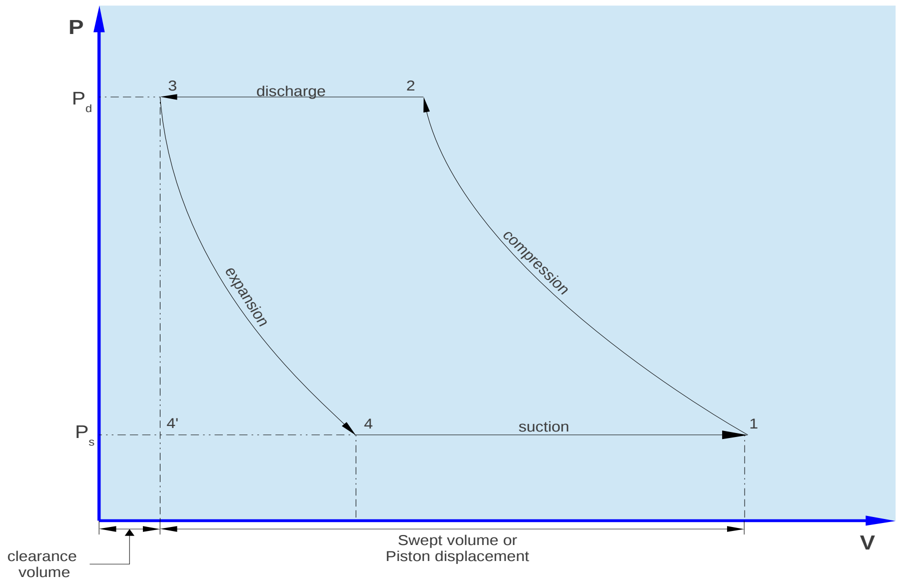


Figure 1: Strokes in the compressor (**Problem 3**).

Problem 5 An ice plant operates on the ideal vapour-compression cycle with superheated state using refrigerant fluid R134a. The refrigerant enters the compressor as saturated vapour at 0.15 MPa and leaves the condenser as saturated liquid at 0.7 MPa. Water enters the refrigerator cavity at 30°C and leaves as ice at -5°C. For an ice production rate of 10 kg per hour, determine the power input to the ice plant and the COP of the cycle. Also, sketch the PH and TS diagrams. Specific heats of ice and water are 2.1 and 4.18 kJ/(kg.K), respectively, and the latent heat of fusion of ice is 334 kJ/kg. Repeat the same procedure for ammonia and propane as refrigerat fluid.

Problem 6 A heat pump operates in a vapour-compression cycle using Refrigerant-22 (R-22) as working fluid. R-22 is compressed from saturated vapour at 2 bar to the condenser pressure of 12 bar. The isentropic efficiency of the compressor is of 80%. Saturated liquid enters the throttling valve at 12 bar. 80% of the heat rejected is transferred to the heated space which has a total heating requirement of 500 kJ/min. Determine:

(a) (A)-(F) in the Table below:

	Pressure (bar)	**Enthalpy (kJ/kg)**	**Entropy (kJ/kg.K)**	**State**
1	2.0	(A)	(B)	Saturated Vapour
2	12.0	(C)	–	(D)
3	12.0	(E)	–	–
4	–	(F)	–	–

(b) Mass flow rate of the R-22 in kg/min.

(c) Actual work in the compressor.

(d) Coefficient of performance.

Problem 7 R-22 is the refrigerant fluid in a geothermal heat pump system for a house (Fig. 2). The heat pump uses underground water from a well $\left(T_{\text{w}}^{\text{in}} = 13^{\text{o}}\text{C}; T_{\text{w}}^{\text{out}} = 7^{\text{o}}\text{C}\right)$ to produce a heating capacity of 4.2 tons. Determine:

(a) Volumetric flow rate of heated air to the house $\left(m^3/s\right)$;

(b) Isentropic efficiency (η_c) and power $\left(\dot{W}_c\right)$ of the compressor;

(c) Coefficient of Performance;

(d) Volumetric flow rate of water from the geothermal well (l/h);

(e) Sketch the TS diagram.

Given the heat capacity $\left(C_p^{\text{air}} = 1.005\dfrac{kJ}{kg.K}\right)$ and molar mass $\left(MW^{\text{air}} = 29\dfrac{kg}{kgmol}\right)$ of air and heat capacity of water $\left(C_p^{\text{water}} = 4.18\dfrac{kJ}{kg.K}\right)$.

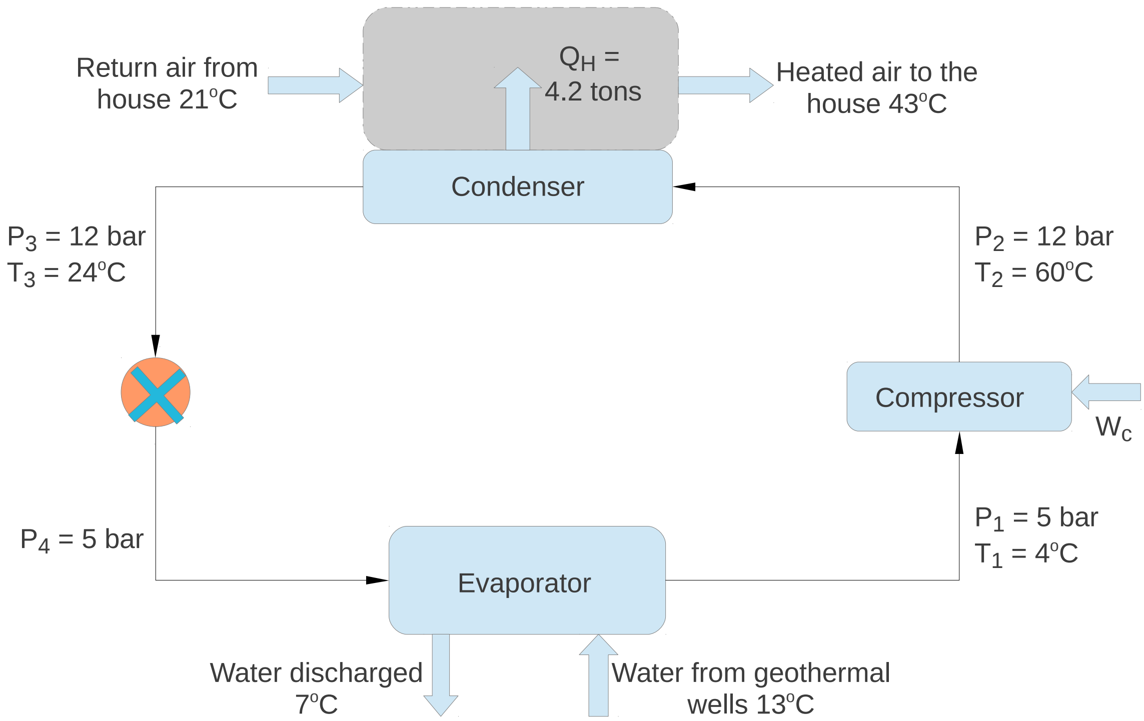
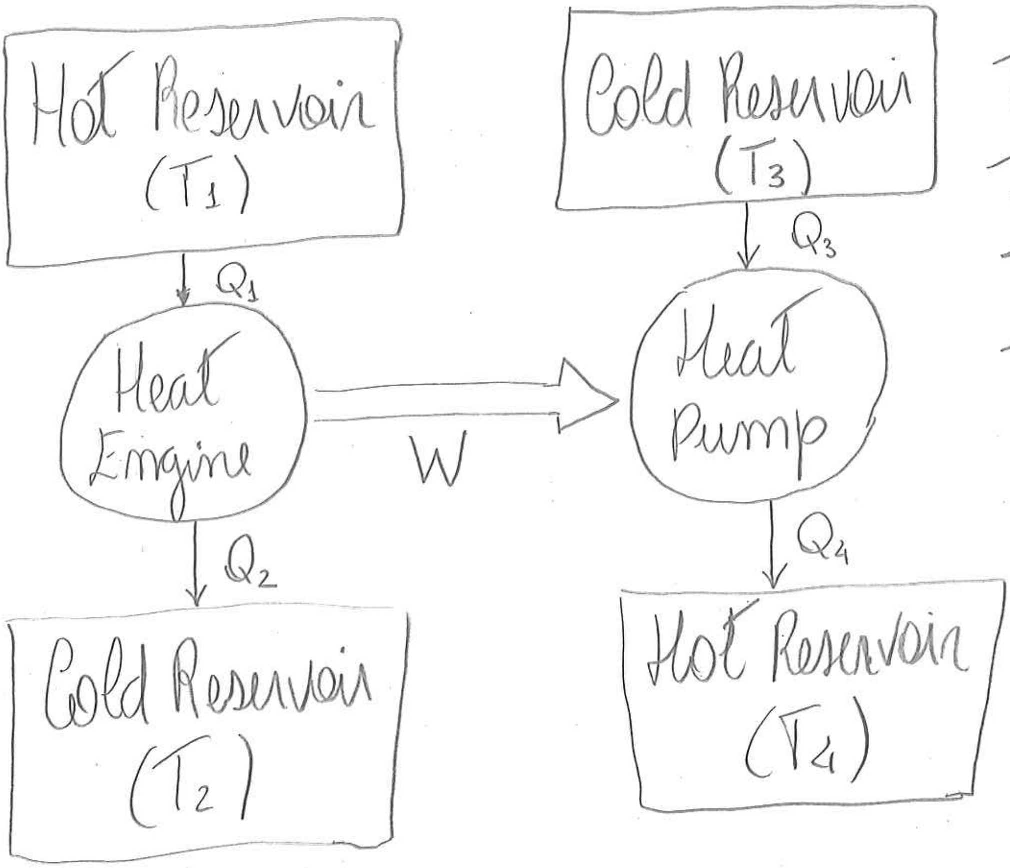


Figure 2: Heat pump cycle (**Problem 7**).

P1: Reversed Carnot Cycle (Heat Pump)



$T_1 = 1000\,°C = 1273.15\,K$

$T_2 = 25°C = 298.15\,K$

$T_3 = -5°C = 268.15\,K$

$T_4 = 25°C = 298.15\,K$

$Q_4/Q_1 = ?$

Both cycles are reversible, thus

$$\frac{Q_2}{Q_1} = \frac{T_2}{T_1} = 0.2342 \quad \text{and} \quad \frac{Q_4}{Q_3} = \frac{T_4}{T_3} = 1.1119$$

Energy balance around heat pump and heat engine:

$$W + Q_2 = Q_1 \;\therefore\; W = Q_1 - Q_2$$

$$Q_4 = Q_3 + W \;\therefore\; \underbrace{W = Q_4 - Q_3}$$

$$Q_1 - Q_2 = Q_4 - Q_3 \quad \times (1/Q_1)$$

$$1 - \underbrace{Q_2/Q_1}_{0.2342} = Q_4/Q_1 - Q_3/Q_1$$

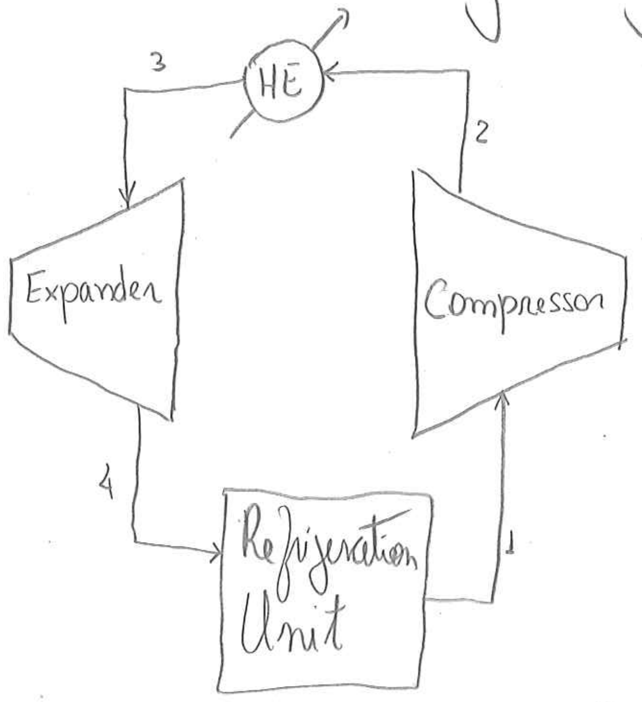
But

$$\frac{Q_3}{Q_1} = \underbrace{\frac{Q_3}{Q_4}}_{0.8994} \frac{Q_4}{Q_1} = 0.8994 \frac{Q_4}{Q_1}$$

$$1 - 0.2342 = Q_4/Q_1 - 0.8994\, Q_4/Q_1$$

$$\boxed{Q_4/Q_1 = 7.61}$$

P2: Reversed Brayton Cycle



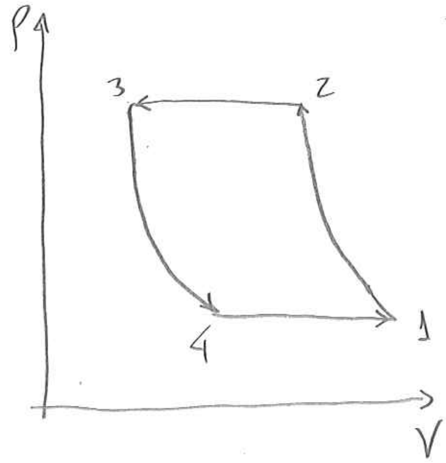
$T_1 = 16°C = 289.15\,K$

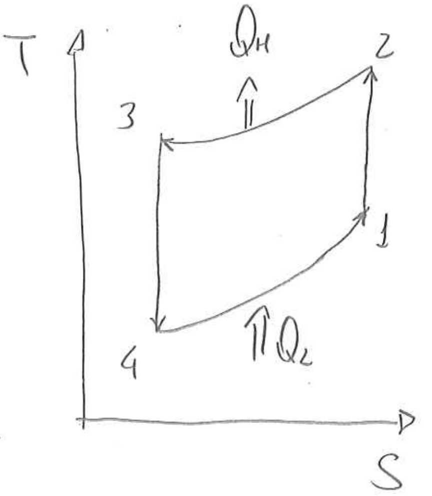
$T_3 = 41°C = 314.15\,K$

$P_2 = P_3 = 5.2$ bar

$P_1 = 1$ bar $= P_4$

$\gamma = 1.4$





- 1-2: Isentropic compression

$$T P^{\frac{1-\gamma}{\gamma}} = \text{constant}$$

$$T_1 P_1^{\frac{1-\gamma}{\gamma}} = T_2 P_2^{\frac{1-\gamma}{\gamma}}$$

$$T_2 = T_1 \left(P_1/P_2\right)^{\frac{1-\gamma}{\gamma}}$$

$$T_2 = 463.12\,K$$

- 3-4: Isentropic expansion

$$T_3 P_3^{\frac{1-\gamma}{\gamma}} = T_4 P_4^{\frac{1-\gamma}{\gamma}}$$

$$T_4 = T_3 \left(P_3/P_4\right)^{\frac{1-\gamma}{\gamma}}$$

$$T_4 = 196.14\,K$$

(a) COP: ?

$$COP = \frac{\text{Desired Effect}}{\text{Net Work}} = \frac{\text{Refrigerant Effect}}{\text{Heat Rejected - Heat Absorbed}}$$

$$COP = \frac{\dot{m} C_p (T_1 - T_4)}{\dot{m} C_p (T_2 - T_3) - \dot{m} C_p (T_1 - T_4)} = \frac{T_1 - T_4}{(T_2 - T_3) - (T_1 - T_4)}$$

$$COP = 1.66$$

(b) $\dot{m}$: ?

$$\underbrace{\text{Refrigerant Effect}}_{6 \text{ tonnes}} = \dot{m} C_p (T_1 - T_4)$$

$$6 \text{ ton} \times \frac{210 \text{ KJ/min}}{1 \text{ ton}} = \dot{m} \times 1.005 \frac{KJ}{Kg\,K} (289.15 - 196.14)$$

$$\dot{m} = 13.48 \text{ Kg/min}$$

(c) Piston displacement of the compressor is V_1 (and assuming ideal gas):

$$P_1 \dot{V}_1 = n R T_1 = \frac{\dot{m}}{MW} R T_1 \therefore \dot{V}_1 = \frac{\dot{m}}{MW} \frac{RT_1}{P_1}$$

$$\dot{V}_1 = 13.48 \frac{\cancel{Kg}}{min} \times \frac{\cancel{gmol}}{29\cancel{g}} \times 0.08314 \frac{\cancel{bar}\, m^3}{\cancel{Kgmol}.\cancel{K}} \times 289.15\cancel{K}$$

$$\times \frac{1}{1\cancel{bar}} \times \frac{1000\cancel{g}}{1\cancel{Kg}} \times \frac{1\cancel{Kgmol}}{1000\cancel{gmol}}$$

$$\dot{V}_1 = 11.17\ m^3/min$$

Swept volume per stroke: $\underbrace{}_{V_s}$ $\dfrac{11.17\ m^3/\cancel{min}}{240\ \dfrac{rotations}{\cancel{min}}}$

$$V_s^{(comp)} = 4.65 \times 10^{-2} m^3$$

$$V_s^{(comp)} = \frac{\pi D_c^2}{4} \times L = \frac{\pi D_c^2}{4} \times 0.2m$$ ← stroke length

$\underline{D_c = 0.54\ m}$ (diameter or bore of the compressing cylinder)

Piston displacement of the expander is

$$\dot{V}_4 = \frac{\dot{m}}{MW}\frac{RT_4}{P_4} \therefore \dot{V}_4 = 7.58\, m^3/min$$

$$\underbrace{\text{Swept volume per stroke}}_{V_s^{(exp)}} = \frac{7.58\, m^3/\cancel{min}}{240\, rot/\cancel{min}}$$

$$V_s^{(exp)} = 3.16 \times 10^{-2}\, m^3$$

$$V_s^{(exp)} = \frac{\pi D_e^2}{4} L \therefore D_e = 0.45\, m$$

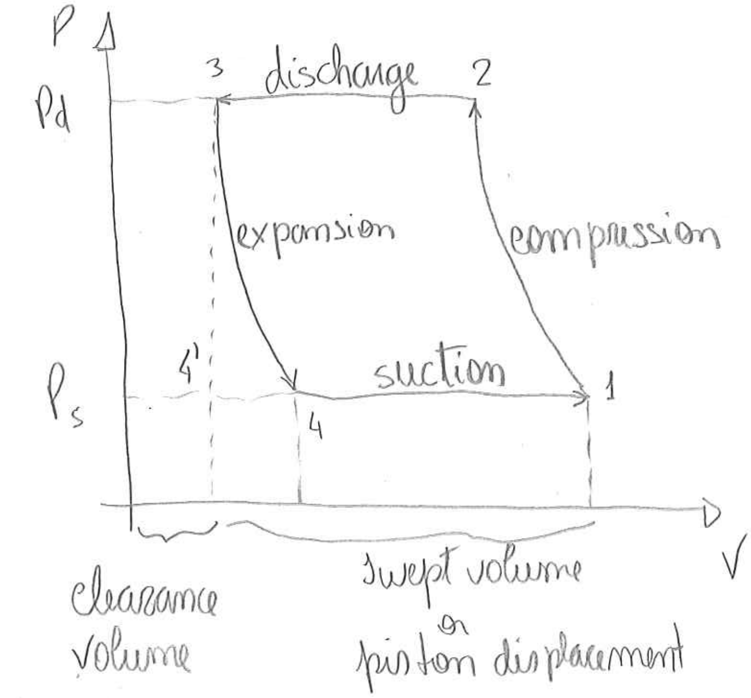
(d) Power: ?

$$COP = \frac{\text{Refrigerant Effect}}{\text{Net Work}}$$

$$1.66 = \frac{6\, ton \times 210\, KJ/min \times \frac{1\, min}{60 s}}{\dot{W}}$$

$$\dot{W} = 12.65\, \frac{KJ}{s} = 12.65\, KW$$

P3:



During the compression stage in the refrigeration cycle, the motion of the piston in the cylinder can be described by the 4 strokes:

- 4-1: suction;
- 1-2: compression;
- 2-3: discharge;
- 3-4: expansion.

Clearance volumetric efficiency is defined as:

$$\eta_{CV} = \frac{V_1 - V_4}{V_1 - V_{4'}} = \frac{V_1 - V_4}{V_1 - V_3} \quad (1)$$

In a polytropic expansion 3-4:

$$P V^{\gamma} = \text{constant}$$

$$P_3 V_3^{\gamma} = P_4 V_4^{\gamma}$$

$P_3 = P_d$ (discharge pressure)

$P_4 = P_s$ (suction pressure)

$$V_4 = V_3 \left(P_d / P_s \right)^{1/\gamma} \quad (2)$$

The clearance ratio is defined as:

$$C = \frac{\text{Clearance Volume}}{\text{Swept Volume}}$$

$$C = \frac{V_3}{V_1 - V_3} \quad (3)$$

Now rearranging Eqn. 1:

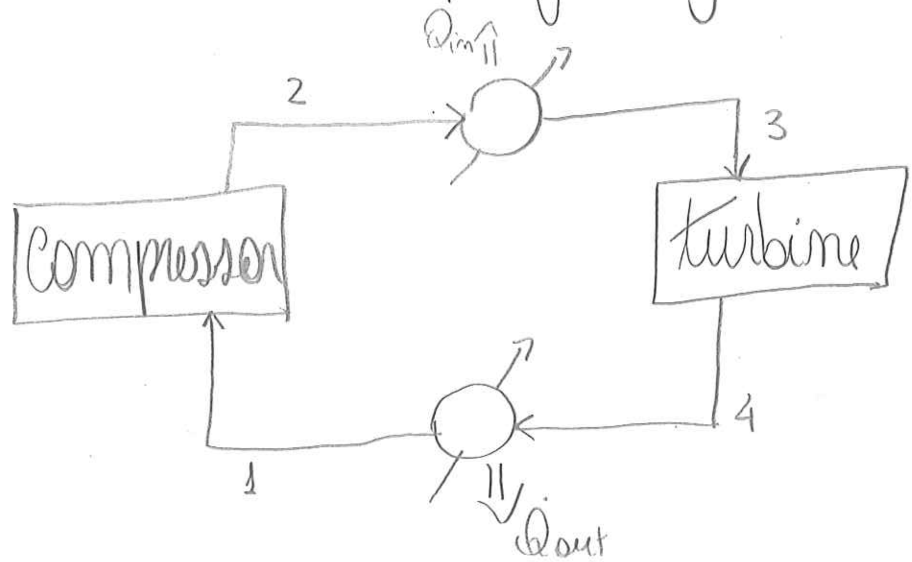
$$\eta_{cv} = \frac{V_1 - V_4}{V_1 - V_3} = \frac{(V_1 - V_{4'}) - (V_4 - V_{4'})}{V_1 - V_3} = \frac{(V_1 - V_3) - (V_4 - V_3)}{V_1 - V_3}$$

$$\eta_{cv} = 1 - \frac{V_4 - V_3}{V_1 - V_3} = 1 - \frac{V_3 (P_d / P_s)^{1/\gamma} - V_3}{V_1 - V_3}$$

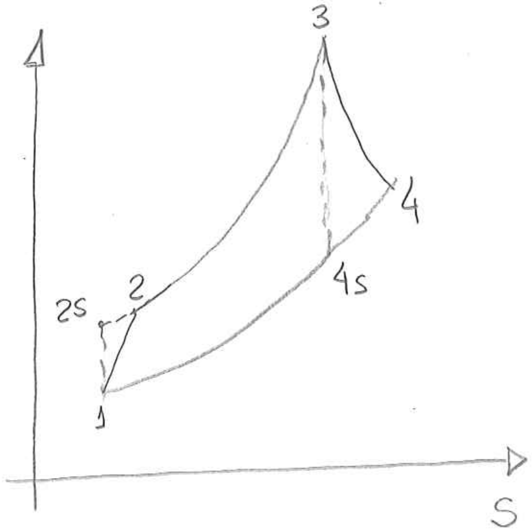
$$= 1 + \underbrace{\frac{V_3}{V_1 - V_3}}_{C} \left[1 - \left(\frac{P_d}{P_s} \right)^{1/\gamma} \right]$$

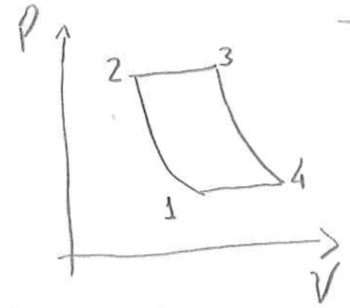
$$\boxed{\eta_{cv} = 1 + C \left[1 - \left(\frac{P_d}{P_s} \right)^{1/\gamma} \right]}$$

P4: Air-Standard Brayton Cycle



$$\left.\begin{array}{l} P_1 = 100\ \text{KPa} = 1\ \text{bar} \\ T_1 = 300\ \text{K} \\ \dot{m} = 6\ \text{Kg/s} \\ P_2 / P_1 = 10 \\ T_3 = 1400\ \text{K} \\ \eta_T = \eta_C = 80\% \end{array}\right\} \text{Given}$$





We need to determine T_2 & T_4. 1-2 and 3-4 are isentropic processes in ideal Brayton cycles. However, compression and expansion are not ideal in this problem, as indicated by the efficiencies. Thus, for the ideal (isentropic) compression (1-2s):

$$T P^{\frac{1-\gamma}{\gamma}} = \text{constant}$$

$$T_1 P_1^{\frac{1-\gamma}{\gamma}} = T_{2s} P_2^{\frac{1-\gamma}{\gamma}}$$

$$T_{2s} = T_1 \left(P_1/P_2 \right)^{\frac{1-\gamma}{\gamma}} = 579.20 \text{ K}$$

Using compressor efficiency,

$$\eta_c = \frac{T_{2s} - T_1}{T_2 - T_1} = \frac{579.20 - 300}{T_2 - 300} = 0.80$$

$$\boxed{T_2 = 649 \text{ K}}$$

Similarly for the expansion (3-4s):

$$T_{4s} = T_3 \left(P_3/P_4 \right)^{\frac{1-\gamma}{\gamma}}$$

$$\hookrightarrow P_3/P_4 = P_2/P_1$$

$$T_{4s} = 725.13 \text{ K}$$

$$\eta_T = \frac{T_4 - T_3}{T_{4s} - T_3} = \frac{T_4 - 1400}{725.13 - 1400} = 0.80$$

$$\boxed{T_4 = 860.10 \text{ K}}$$

(a) η : ?

$$\eta = \frac{\text{Net Work}}{\text{Heat Received}} = \frac{\dot{m}C_p[(T_3-T_2)-(T_4-T_1)]}{\dot{m}C_p(T_3-T_2)}$$

$$\eta = 1 - \frac{T_4-T_1}{T_3-T_2} = 0.2542 \therefore \boxed{25.42\%}$$

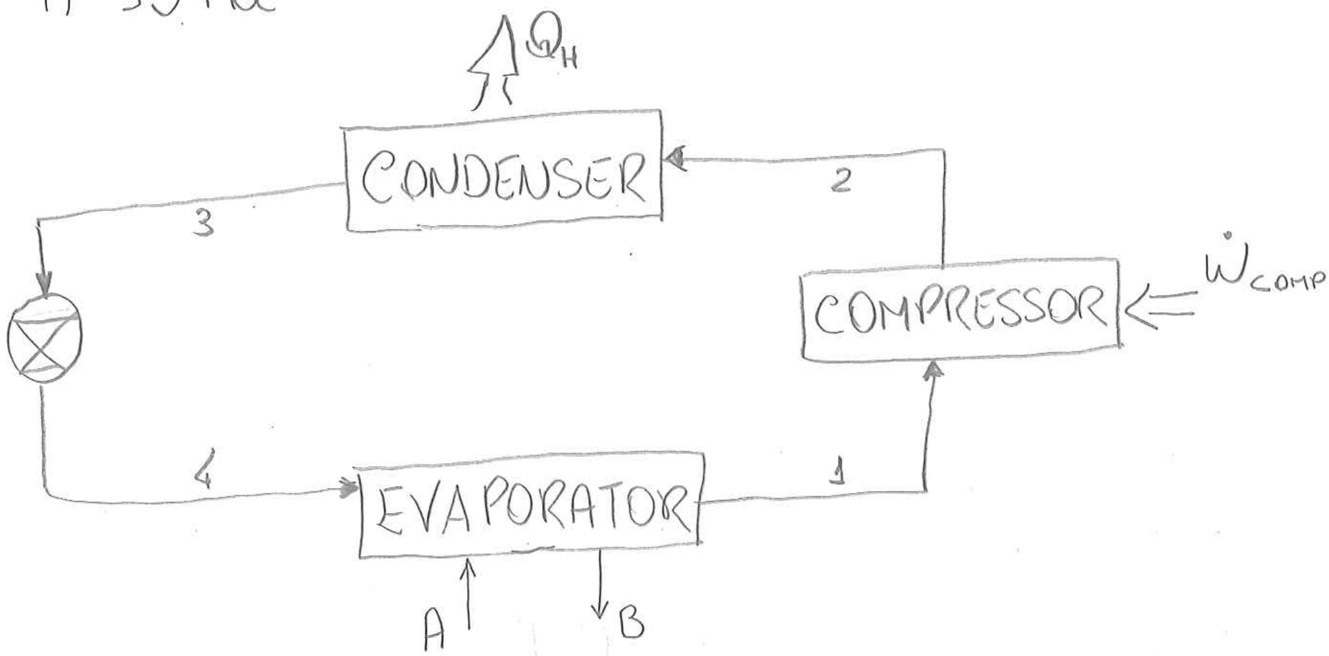
(b) Net Power ?

1.005 KJ/kg.K

$$\text{Net Power} = \dot{m}C_p[(T_3-T_2)-(T_4-T_1)]$$

$$\boxed{\text{Net Power} = 1151.13 \text{ KW}}$$

P5: Ideal Vapour-compression cycle
R-134a

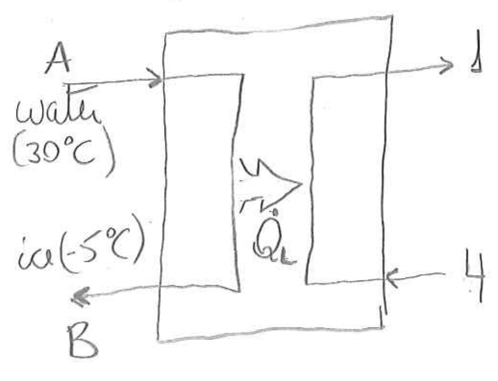


$P_1 = 0.15$ MPa (saturated vapour)
$P_3 = 0.70$ MPa (saturated liquid)
$T_A = 30°C$
$T_B = -5°C$ } water-ice : $\dot{m} = 10$ kg/h

Net Power, COP ?

We need to remove heat to convert 1 kg of water at 30°C into ice at -5°C:

Evaporator:



$$\dot{Q}_L = \dot{m}\, C_{p_w} (T_w - 0) + \dot{m}\, L_{f_{ice}} + \dot{m}\, C_{p_{ice}} (0 - T_{ice})$$

latent heat of ice

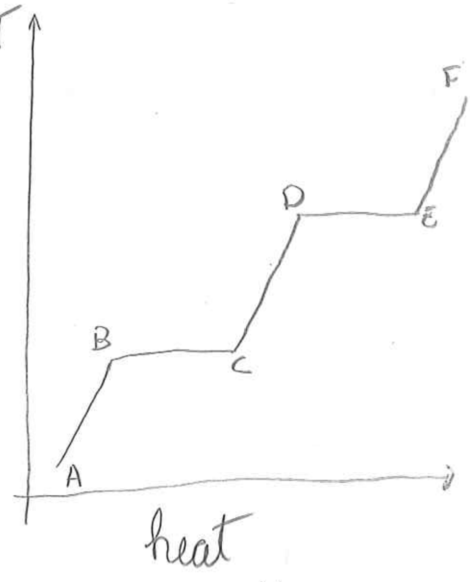
$$\dot{Q}_L = 10 \frac{Kg}{h} \times 4.18 \frac{KJ}{Kg.K} (30-0)K +$$

$$10 \frac{Kg}{h} \times 334 \frac{KJ}{Kg} + 10 \frac{Kg}{h} \times 2.1 \frac{KJ}{Kg.K} [0-(-5)] K$$

$$\dot{Q}_L = 4699 \frac{KJ}{h} = 1.31 \frac{KJ}{S}$$

$$= 1.31 \text{ KW}$$

(refrigerant effect)



A-B: SOLID

C-D: LIQUID

E-F: GAS

PHASE CHANGES:

- B-C: MELTING / FUSION
  C-B: SOLIDIFICATION
- D-E: VAPOURISATION
  E-D: CONDENSATION

Now calculating enthalpies for the R134a fluid stream:

- 1: $P_1 = 0.15$ MPa $= 1.5$ bar (saturated vapour)

(through linear interpolation

$T_1 = -17.21$ °C

$h_1 = 237.01$ KJ/Kg

$S_1 = 0.9309$ KJ/Kg.K

• 2: isentropic compression ($S_2 = S_1$)

$P_2 = P_3 = 0.70\ MPa = 7\ bar$

At 7bar, $S_g = 0.9080\ \frac{KJ}{Kg.K} < S_2$: thus the fluid is at superheated state, thus through linear interpolation:

$T_2 = 33.27\ °C$

$h_2 = 268.83\ KJ/Kg$

• 3: saturated liquid at $P_3 = 7\ bar$:

$h_3 = 86.78\ KJ/Kg$ $T_3 = 26.72\ °C$

$S_3 = 0.3242\ KJ/Kg.K$

• 4: isenthalpic expansion

$h_4 = h_3$

(a) Net Power ?

$$\dot{W}_{comp} = \dot{m}_R (h_2 - h_1)$$

$\dot{m}_R$? (mass flow rate of refrigerant fluid R134a)

From refrigerating effect

$$\dot{Q}_L = 1.31 \ \frac{KJ}{s} = \dot{m}_R (h_1 - h_4)$$

$$\boxed{\dot{m}_R = 8.72 \times 10^{-3} \ Kg/s}$$

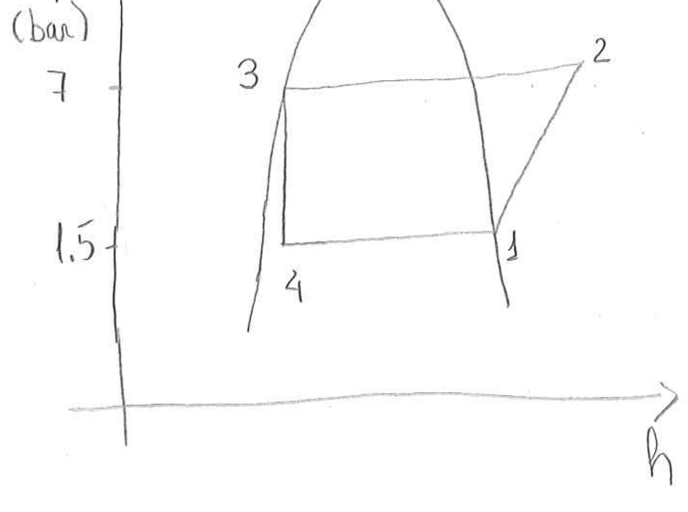
and the net power:

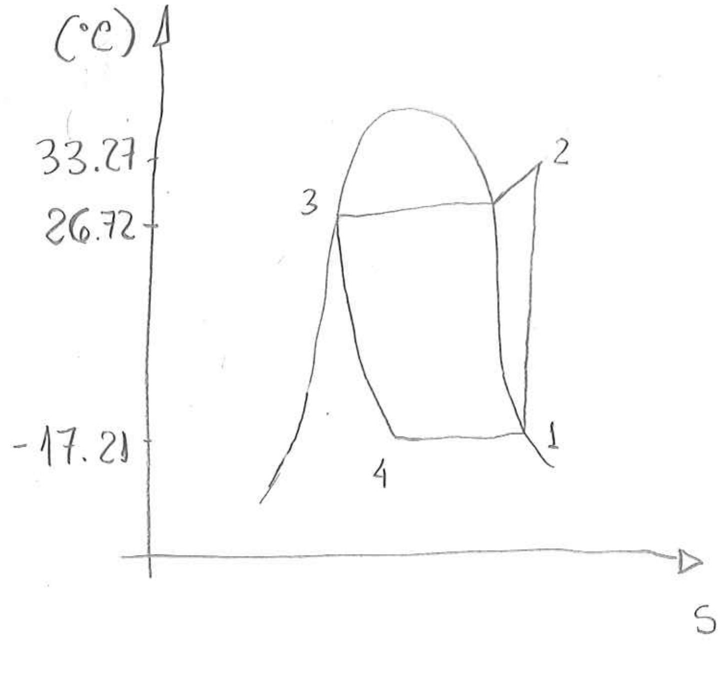
$$\dot{W}_{comp} = 0.2775 \ \frac{KJ}{s} \therefore \boxed{0.2775 \ KW}$$

(b) COP ?

$$COP = \frac{\text{Refrigerating effect}}{\text{Net Work}} = \frac{\dot{Q}_L}{\dot{W}_{comp}}$$

$$\boxed{COP = 4.72}$$





Using Ammonia (same procedure):

- $\underline{1}$: $P_1 = 1.5$ bar (saturated vapour):

  $T_1 = -25.22\,°C$

  $h_1 = 1410.61$ KJ/Kg

  $S_1 = 5.6973$ KJ/Kg.K

- $\underline{2}$: $P_2 = 7$ bar

  $S_2 = S_1 > S_g$ ($= 5.1576$ KJ/Kg.K) => superheated fluid

  linear interpolation

  $T_2 = 80.52\,°C$

  $h_2 = 1626.79$ KJ/Kg

- $\underline{3}$: $P_3 = 7$ bar (saturated liquid):

  $T_3 = 13.79\,°C$

  $h_3 = 244.69$ KJ/Kg

  $S_3 = 0.9394$ KJ/Kg.K

- $\underline{4}$: isenthalpic expansion

  $h_4 = h_3$

$\dot{m}_R = 1.12 \times 10^{-3}$ Kg/s

$\dot{W}_{comp} = 0.2421$ KW

COP $= 5.41$

Using Propane (same procedure):

- 1 : $P_1 = 1.5$ bar (saturated vapour):

  ↓ linear interpolation

  $T_1 = -33.91°C$ $\quad$ $s_1 = 1.802$ KJ/Kg.K

  $h_1 = 430.35$ KJ/Kg

- 2 : $P_2 = 7$ bar

  $s_2 = s_1 > s_g$ (= 1.733 KJ/Kg.K) ⇒ superheated fluid

  ↓ linear interpolation

  $T_2 = 24.06°C$

  $h_2 = 504.42$ KJ/Kg

- 3 : $P_3 = 7$ bar (saturated liquid):

  $T_3 = 13.41°C$

  $h_3 = 129.6$ KJ/Kg $\quad$ $s_3 = 0.495$ KJ/Kg.K

- 4 : isenthalpic expansion

  $h_4 = h_3$

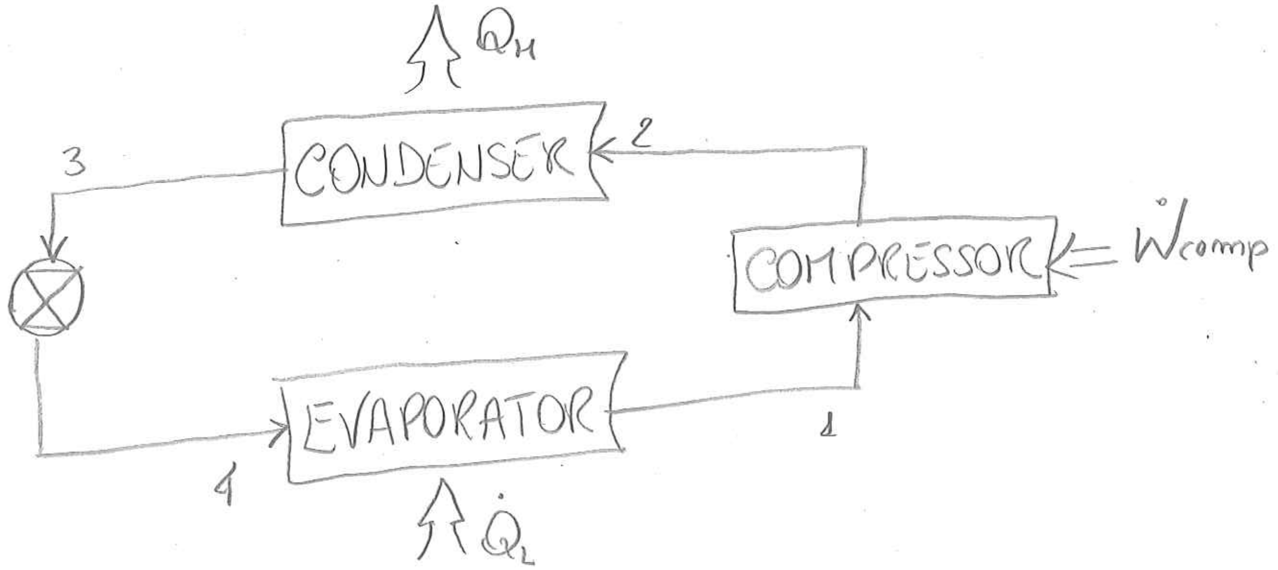
$\dot{m}_R = 4.36 \times 10^{-3}$ Kg/s

$\dot{W}_{comp} = 0.3229$ KW

COP = 4.06

	R-134a	Ammonia	Propane
$\dot{m}_R$ (Kg/s)	8.72×10^{-3}	1.12×10^{-3}	4.36×10^{-3}
$\dot{W}_{comp}$ (W)	277.5	242.1	322.9
COP	4.72	5.41	4.06

P6: Heat Pump – R22
(vapour compression cycle)



$$\begin{cases} P_1 = 2 \text{ bar (saturated vapour)} \\ P_2 = 12 \text{ bar} \\ \eta_{comp} = 0.80 \\ P_3 = 12 \text{ bar (saturated liquid)} \\ \dot{Q}_H' = 0.8\,\dot{Q}_H = -500 \text{ KJ/min} \end{cases}$$

Heat rejected in the heat pump cycle

$$\dot{Q}_H = -\frac{500}{0.8} = -10.42 \text{ KJ/s}$$

(a) Now calculating enthalpies for R22 fluid stream:

• 1: $P_1 = 2$ bar (sat. vapour): $\begin{cases} T_1 = -25.18\,°C \\ h_1 = 239.88 \text{ KJ/Kg} \quad (A) \\ s_1 = 0.9691 \text{ KJ/Kg.K} \quad (B) \end{cases}$

• 2: $P_2 = 12$ bar (isentropic compression):

$S_{2S} = S_1 > S_g$ ($= 0.8864$ KJ/Kg K) => superheated vapour (D)

linear interpolation

$h_{2S} = 285.28$ KJ/Kg

$$\eta_c = \frac{h_{2S} - h_1}{h_2 - h_1} = 0.8 \therefore h_2 = 296.63 \text{ KJ/Kg}$$

(C)

• 3: $P_3 = 12$ bar (saturated liquid):

$h_3 = 81.90$ KJ/Kg (E)

$S_3 = 0.3029$ KJ/Kg K

• 4: Isenthalpic expansion

$h_4 = h_3 = 81.90$ KJ/Kg (F)

$h_4 > h_f$ ($= 16.37$ KJ/Kg)

(b) $\dot{m}_R$?

$\dot{Q}_H = \dot{m}_R (h_3 - h_2) = -10.42$ KJ/s

$\dot{m}_R = 4.85 \times 10^{-2} \frac{Kg}{s} = 2.91$ Kg/min

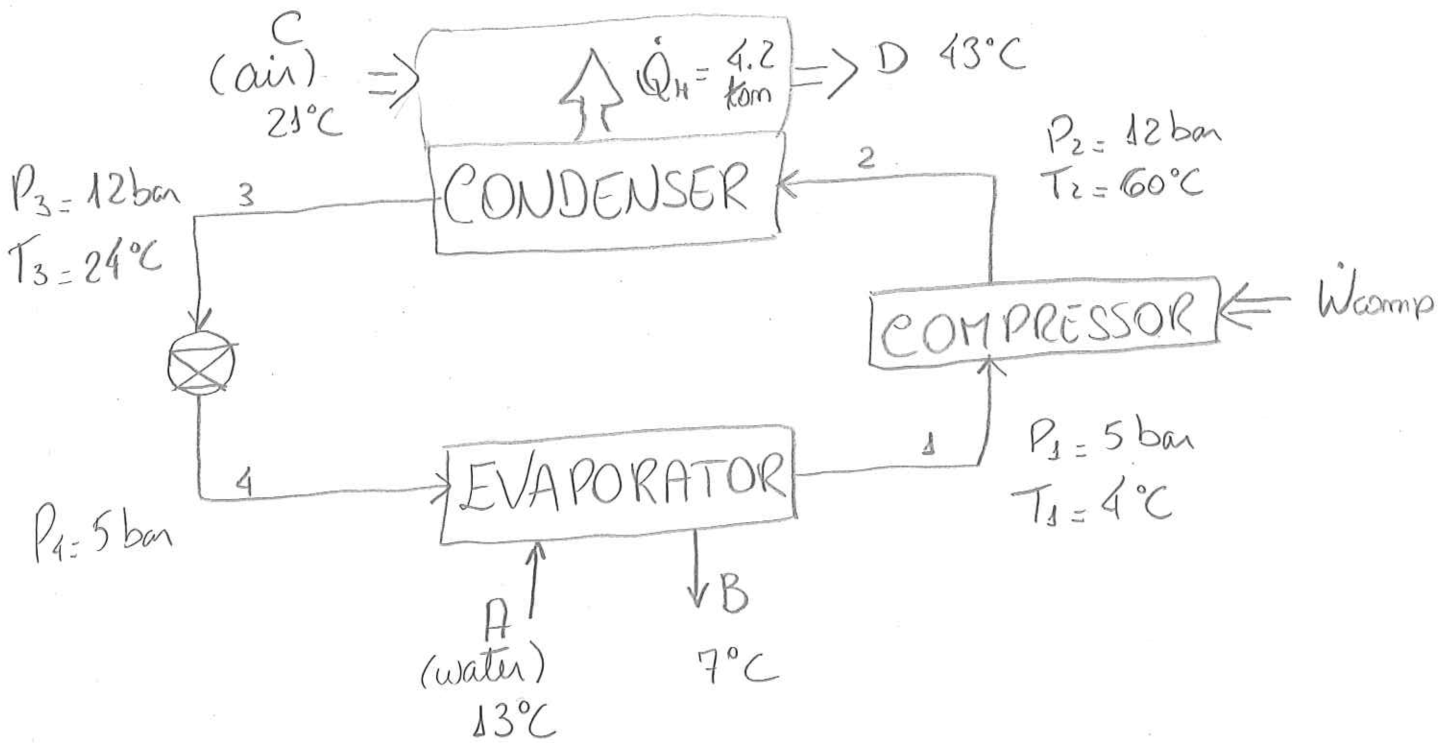
(c) $\dot{W}_{comp}$?

$$\dot{W}_{comp} = \dot{m}_R (h_2 - h_1) = 2.75 \text{ KW}$$

(d) COP ?

$$COP = \frac{|\dot{Q}_H|}{\dot{W}_{comp}} = 3.79$$

P7: Geothermal heat pump - R22



Calculating enthalpies:

• 1: $P_1 = 5$ bar, $T_1 = 4°C$ } $T_1 >> T_{sat} (= 0.12°C) \Rightarrow$ superheated vapour

↓ linear interpolation

$h_1 = 252.83$ KJ/Kg

$S_1 = 0.9372$ KJ/Kg.K

• 2: $P_2 = 12$ bar, $T_2 = 60°C$ } $T_2 >> T_{sat} (= 30.25°C) \Rightarrow$ superheat vapour

$h_2 = 284.43$ KJ/Kg

$S_2 = 0.9666$ KJ/Kg.K

• 3: $P_3 = 12$ bar, $T_3 = 24°C$ $\Big\{$ $T_3 << T_{sat}$ $(= 30.25°C)$ ⤳ subcooled liquid

$$h_3 = h_f(@\,24°C) = 74.04\ KJ/Kg$$

$$S_3 = S_f(@\,24°C) = 0.2772\ KJ/Kg.K$$

• 4: Isenthalpic expansion ($P_4 = 5$ bar)

$$h_4 = h_3 = 74.04\,KJ/Kg > h_f(@\,5\text{ bar})$$

$$x_4 = \frac{h_4 - h_f}{h_g - h_f} = 0.1406$$

$$x_4 = \frac{S_4 - S_f}{S_g - S_f} \therefore S_4 = 0.2831\ KJ/Kg\,K$$

(a) $\dot{V}_{air}^{(out)}$? (m^3/s)

$$\dot{Q}_H = \dot{m}_{air}\left(h_{out}^{air} - h_{in}^{air}\right) = \dot{m}_{air}\, Cp_{air}\left(T_{out}^{air} - T_{in}^{air}\right)$$

$$4.2\text{ ton} \times \frac{210\ KJ/min}{1\text{ ton}} \times \frac{1\,min}{60s} = \dot{m}_{air} \times 1.005\,\frac{KJ}{Kg.K} \times (43-21)K$$

$$\dot{m}_{air} = 0.6649\ Kg/s$$

ideal gas

$$P_{air}^{(out)}\,\dot{V}_{air}^{(out)} = m_{air} R\, T_{air}^{(out)} = \frac{\dot{m}_{air}}{MW_{air}} R\, T_{air}^{(out)}$$

$$\dot{V}_{air}^{(out)} = \frac{\dot{m}_{air}}{MW_{air}}\,\frac{R\,T_{air}^{(out)}}{P_{air}^{(out)}}$$

$$P_{air}^{(out)} = P_{atm} = 1.01325\text{ bar} \quad T_{air}^{(out)} = 43°C = 316.15K$$

$$\dot{V}_{air}^{(out)} = 0.6649\,\frac{\cancel{kg}}{s}\,\frac{\cancel{gmol}}{29\,\cancel{g}} \times 0.08314\,\frac{bar.m^3}{\cancel{Kgmol}.\cancel{K}} \times 316.15\cancel{K} \times \frac{1}{1.01325\,\cancel{bar}}$$

$$\times \frac{1000\,\cancel{g}}{1\,\cancel{Kg}} \times \frac{1\,\cancel{Kgmol}}{1000\,\cancel{gmol}}$$

$$\boxed{\dot{V}_{air}^{(out)} = 0.5948\,\frac{m^3}{s}}$$

(b) η_c ? $\dot{W}_{comp}$?

$$\eta_c = \frac{h_{2S} - h_1}{h_2 - h_1}$$

$P_2 = 12$ bar

$S_{2S} = S_1 = 0.9372$ KJ/Kg.K

↓ linear interpolation

$h_{2S} = 274.83$ KJ/Kg

$T_{2S} = 48.46\,°C$

$$\boxed{\eta_c = 0.6962 \therefore 69.62\%}$$

$$\dot{W}_{comp} = \dot{m}_R\,(h_2 - h_1)$$

$\dot{m}_R$?

The heating to house:

$$\dot{Q}_H = \dot{m}_R (h_3 - h_2)$$

$$-4.2 \text{ ton} \times \frac{210 \text{ kJ/min}}{1 \text{ ton}} \times \frac{1 \text{ min}}{60 \text{s}} = \dot{m}_R (74.04 - 284.43) \frac{\text{kJ}}{\text{kg}}$$

↑
heat leaving the system

$$\dot{m}_R = 0.0699 \text{ kg/s}$$

$$\boxed{\dot{W}_{comp} = 2.21 \text{ kW}}$$

(c) $$\boxed{COP = \frac{|\dot{Q}_H|}{\dot{W}_{comp}} = 6.65}$$

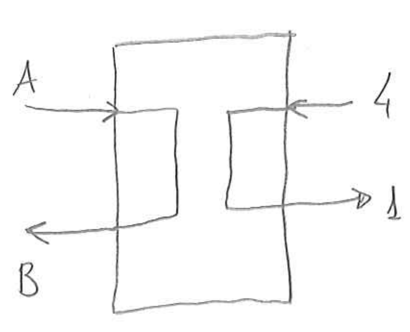
(d) $\dot{V}_w$ (l/h)

Energy balance across the evaporator:

$$Q_R + Q_w = 0 \therefore Q_R = -Q_w$$

$$\dot{m}_R (h_1 - h_4) = -\dot{m}_w (h_B - h_A)$$

$$\dot{m}_R (h_1 - h_4) = -\dot{m}_w C_{p_w} (T_B - T_A)$$



$$\dot{m}_w = 0.4983 \text{ kg/s}$$

At 13°C, the specific volume of the water (saturated liquid) is $1.0007 \times 10^{-3}\, m^3/kg$ (v_f). Thus the volumetric flow rate of water is

$$\dot{V}_w = \dot{m}_w v_f = 0.4983 \frac{kg}{s} \times 1.0007 \times 10^{-3} \frac{m^3}{kg}$$

$$\dot{V}_w = 0.0004986 \frac{m^3}{s} \times \frac{3600 s}{1 h} \times \frac{1 l}{10^{-3} m^3}$$

$$\boxed{\dot{V}_w = 1794.96\ l/h}$$