Heat, Mass and Momentum Conservation – Thermodynamics2 (EX3030/EM40JK)

Transient Heat Transfer

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Section Outline

Aims and Objectives

1. Lumped system analysis
2. Transient heat conduction

Suggested References

Literature relevant for this module:

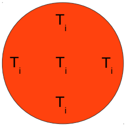
[1] J.P. Holman, ‘Heat Transfer’, 10^{th} Edition (McGraw-Hill): Chapters 4 and 10;

[2] F.P. Incropera, D.P. DeWitt, T.L.Bergman, A.S. Lavine, ‘introduction to Heat Transfer’ (John Willey & Sons): Chapters 5 and 11.

[3] L. Theodore, ‘Essential Engineering Calculations Series: Heat Transfer Applications for the Practicing Engineer’ (John Willey & Sons): Chapters 8 and 14.

Section Outline

Introduction



1. In the study of heat transfer, the simplest case analysis is when bodies are assumed *isothermal* and, therefore they can be treated as a lump system;
2. This means that the temperature of these bodies at any spatial coordinate can be expressed as function of time only, i.e., $T = T(t)$;

Thermal Energy Balance

1. Let's consider a body of mass m and arbitrary shape (with area A_s) and temperature T;
2. The body is assumed homogeneous and we can consider constant heat capacity at constant pressure, C_p, thermal conductivity, κ and;
3. The energy balance of an isothermal body during time interval dt, subject to constant surrounding temperature T_∞, can be simplified as,
4. For $m = \rho V$ (where ρ and V are the density and volume of the body, respectively) and;
5. $dT = d\left(T_\infty - T\right)$;

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6. The energy balance,

$$hA_s\left(T_\infty - T\right)dt = mC_p dT \tag{1}$$

7. can be rearranged and integrated as (from time 0 to t),

8. From Eqn. 2:
   8.1 We can calculate either the temperature $T(t)$ of the body at a given time t or;
   8.2 The time t required for a body to reach temperature $T(t)$;
   8.3 Temperature $T(t)$ approaches the surrounding temperature T_∞ **exponentially**;
   8.4 A large value of b indicates a higher rate of temperature decay;
   8.5 Finally, b is proportional to A_s and inversely proportional to m and C_p of the body. This means that it takes longer to heat or cool a larger mass, in particular if it also has a large heat capacity.

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\end{aligned} \tag{2}$$

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1. The *Newton's law of cooling* can determine the rate of convection heat transfer,

$$\dot{Q}(t) = hA_s\left[T(t) - T_\infty\right] \qquad (\mathbf{W}) \tag{3}$$

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Criteria for Lumped System

1. The assumption of **lumped system** is not often the most appropriate. With the *characteristic length* $L_c = \dfrac{V}{A_s}$;
2. We can determine the dimensionless Biot Number (Bi),

$$ \tag{6} $$

3. **Lumped system** analysis assumes *uniform temperature distribution* throughout the body;
4. This assumption is correct **if** the convection coefficient is small and thermal conductivity is large, i.e.,;
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$$Bi \leq 0.1$$

Criteria for Lumped System – Example 1

1. Example: Two spheres are removed from a furnace and let to cool with air at 25°C under relatively low convection coefficient of 15 $\text{W.}\left(\text{m}^2.^\circ\text{C}\right)$. The spheres are made of copper, $\kappa_{\text{Cu}} = 401\ \text{W.}\left(\text{m.}^\circ\text{C}\right)^{-1}$, and coal, $\kappa_{\text{Coal}} = 0.2\ \text{W.}\left(\text{m.}^\circ\text{C}\right)^{-1}$. Can we apply the lumped method to both spheres?

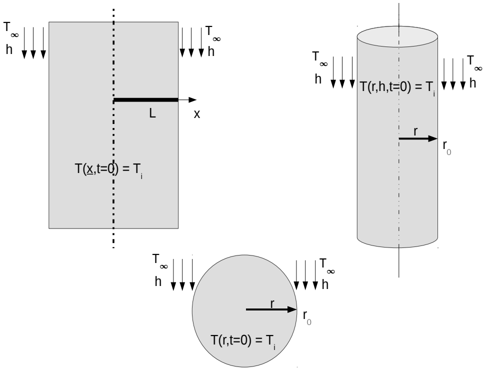
Criteria for Lumped System – Example 2

2. A steel ball of 5 cm in diameter and at uniform temperature of 450°C is suddenly placed in a controlled environment where temperature is kept at 100°C. The prescribed convection heat transfer coefficient is 10 W.$\left(\text{m}^2.^\circ\text{C}\right)^{-1}$. Calculate the time required for the ball to reach 150°C. Given $C_{p,\text{steel}} = 0.46$ kJ.kg^{-1}, $\kappa_{\text{steel}} = 35$ W. (m.°C) and $\rho_{\text{steel}} = 7.8 \times 10^3$kg.m^{-3}.

Section Outline

Introduction

1. In most transient heat transfer problems, the *Biot* dimensionless number is larger than 0.1 and the lumped-capacity method and Eqn. 2 can not be used.
2. In such cases, it is necessary to calculate the temperature distribution throught spatial and time domains, i.e., $T\left(\underline{x}, t\right)$, where $\underline{x}$ is the spatial-coordinate in 1-, 2- and 3-D;



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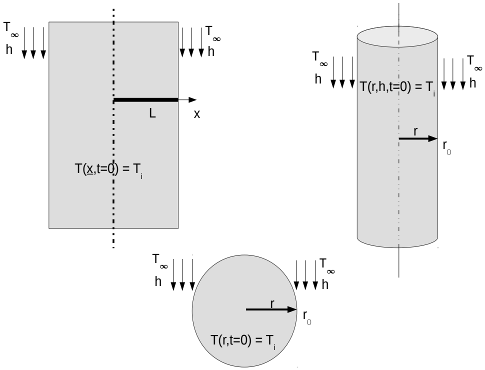


Plane Wall

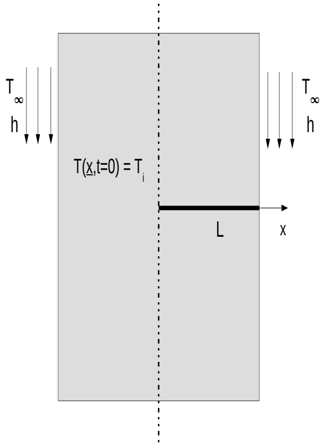
(a) Let's assume a plane wall of thickness $2L$ with initial uniform temperature distribution $T\left(\underline{x}, t=0\right) = T_i$;

(b) The wall is surrounded by a fluid with constant temperature T_∞ imposing a convective heat transfer coefficient h;

(c) Also, let's assume that the wall's height and width are infinite and the problem can be considered as 1-D;

(d) Finally, all thermo-physical properties are considered constant, and there is no heat generation;

(e) If the geometry is uniform, we can assume the axis of symmetry at $x = 0$;

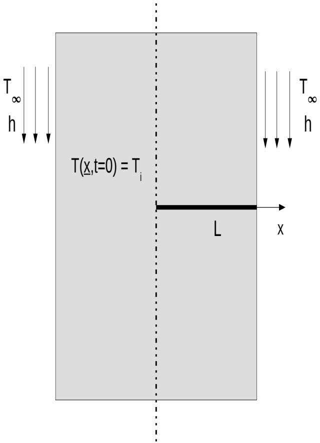


Plane Wall: 1-D Transient Heat Conduction at $0 \leq x \leq L$

Heat Equation

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{7}$$

where $\alpha = \kappa \left(\rho C_p\right)^{-1}$ is the heat diffusivity of the material.



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Boundary Conditions

$$\frac{\partial}{\partial x}T\left(x=0,t\right) = 0$$

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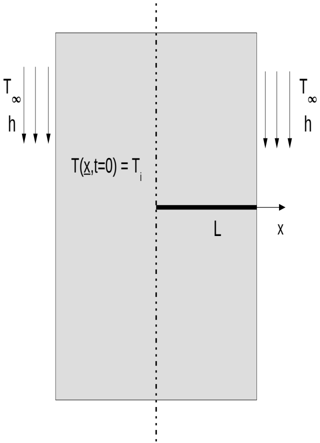

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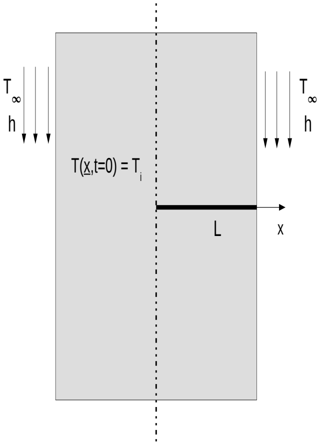
Initial Conditions

$$T\left(x, t = 0\right) = T_i$$



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(f) In order to solve Eqn. 1 with the associated boundary and initial conditions, we first need to *adimensionalise* this partial differential equation (PDE) with

$$\theta = \frac{T - T_\infty}{T_i - T_\infty}$$

(g) Leading to (in non-dimensional form):

(h) With *boundary conditions*:

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$$\frac{\partial^2 \theta}{\partial X^2} = \frac{\partial \theta}{\partial \tau} \tag{8}$$

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$$\frac{\partial \theta}{\partial X} = 0 \qquad \text{at } X = 0 \text{ and } \tau > 0$$

$$\frac{\partial \theta}{\partial X} = -Bi \cdot \theta \qquad \text{at } X = L \text{ and } \tau > 0$$

(i) And *initial conditions*:

$$\theta\left(X, \tau = 0\right) = 1$$

Plane Wall: 1-D Transient Heat Conduction at $0 \leq x \leq L$

(j) The solution of Eqn. 8 involves using Fourier series expansion,

$$\theta = \sum_{n=1}^{\infty} A_n \exp\left(-\lambda_n^2 \tau\right) \cos\left(\lambda_n X\right)$$

with

$$A_n = \frac{4 \sin \lambda_n}{2\lambda_n + \sin 2\lambda_n} \text{ and } \lambda_n \tan \lambda_n = Bi$$

(k) The solution for this infinite series converges rapidly with increasing time and for $\tau > 0.2$. We can use the first term and neglect the remaining terms;

(l) Solution for $\tau > 0.2$ can be found in the next slide for the 3 geometries considered here.

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1-D Transient Heat Conduction (with $\tau > 0.2$)

(a) Plane Wall:

$$\theta_{\text{wall}} = \frac{T(x,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \cos\left(\frac{\lambda_1 x}{L}\right) \tag{9}$$

(b) Cylinder:

$$\theta_{\text{cyl}} = \frac{T(r,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \mathbf{J_0}\left(\frac{\lambda_1 r}{r_0}\right) \tag{10}$$

(c) Sphere:

$$\theta_{\text{sph}} = \frac{T(r,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \frac{\sin\left(\frac{\lambda_1 r}{r_0}\right)}{\frac{\lambda_1 r}{r_0}} \tag{11}$$

(d) At the centre of the geometry: $\cos 0 = 1 = \mathbf{J_0}(0) = 1$ and the limit of $\frac{sinx}{x}$ is 1.

$$\theta_{0,\text{wall}} = \theta_{0,\text{cyl}} = \theta_{0,\text{sph}} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \tag{12}$$

A_1 and λ_1 are functions of the Bi dimensioneless number. Function J_0 is the zeroth-order Bessel function of the first kind (see Tables in Appendix 1).

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Energy Balance

(a) The maximum amount of heat that can be transferred from/to a body occurs when the temperature of the body changes from the initial temperature, T_i to ambient temperature, T_∞,

$$\mathcal{Q}_{\text{max}} = mC_p\left(T_\infty - T_i\right)$$

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Energy Balance

(d) We can perform the integration for all three studied geometries:

$$\text{Plane wall:} \quad \left(\frac{\mathcal{Q}}{\mathcal{Q}_{\text{max}}}\right)_{\text{wall}} = 1 - \theta_{0,\text{wall}}\frac{\sin\lambda_1}{\lambda_1} \tag{13}$$

$$\text{Cylinder:} \quad \left(\frac{\mathcal{Q}}{\mathcal{Q}_{\text{max}}}\right)_{\text{cyl}} = 1 - 2\theta_{0,\text{cyl}}\frac{\mathbf{J_1}}{\lambda_1} \tag{14}$$

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(g) Conditions for using the Heisler and Gröber charts:

(i) The body is initially at a uniform temperature;
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$$Fo = \frac{\alpha t}{L^2} \tag{16}$$

Example

A long 20-cm-diameter cylindrical shaft made of stainless steel 304 comes out of an oven at a uniform temperature of 600°C. The shaft is then allowed to cool slowly in an environment chamber at 200°C with an average heat transfer coefficient of (h) of 80 W$/\left(\text{m}^2.^\circ\text{C}\right)$. Determine the temperature at the center of the shaft 45 min after the start of the cooling process. Also, determine the heat transfer per unit length of the shaft during this time period. Given, for stainless steel 304 at room temperature:

$$\begin{array}{lcll}
\kappa & = & 14.9 & \text{W}/(\text{m}.^\circ\text{C}) \\
\rho & = & 7900 & \text{kg}/\text{m}^3 \\
C_p & = & 477 & \text{J}/(\text{kg}.^\circ\text{C}) \\
\alpha & = & 3.95\times10^{-6} & \text{m}^2/\text{s}
\end{array}$$

Introduction to Finite Difference Methods (FDM)

1. The analytical methods introduced in the previous section are easy to apply thorugh either Eqns. 9- 11 or the Heisler & Gröber charts;
2. These methods are however constraint to 1-D and simple geometries;
3. Numerical methods offer a great alternative to solve partial differential equations (PDE) by replacing differential equations by algebraic equations. In FDM, derivatives are replaced by differences;
4. In FDM, the derivatives are approximated using Taylor series expansions truncated at (relativeley) low-order;
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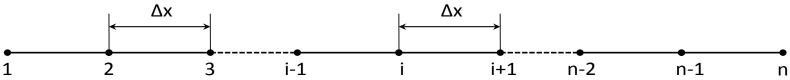
$$\cancelto{0}{\frac{\partial}{\partial t}(\rho\phi)} + \cancelto{0}{\frac{\partial}{\partial x}(\rho\mathbf{v}\phi)} = \nabla\cdot(\Gamma\nabla\phi) + \mathcal{S} \Longrightarrow \Gamma\frac{d^2\phi}{dx^2} + \mathcal{S} = 0 \qquad (17)$$

where ϕ is a scalar (e.g., temperature, concentration etc) and $\mathcal{S}$ is a source or sink term.

Introduction to Finite Difference Methods (FDM)

3. The diffusion term can be discretised using the Taylor series expansion:

4. If we subtract Eqn. 19 from 18, and sum them up:



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3. The diffusion term can be discretised using the Taylor series expansion:

truncation order

$$\phi_{i-1} = \phi_i - \Delta x \left(\frac{d\phi}{dx}\right)_i + \frac{(\Delta x)^2}{2!}\left(\frac{d^2\phi}{dx^2}\right)_i + \boxed{\mathcal{O}\left[(\Delta x)^3\right]} \tag{18}$$

$$\phi_{i+1} = \phi_i + \Delta x \left(\frac{d\phi}{dx}\right)_i + \frac{(\Delta x)^2}{2!}\left(\frac{d^2\phi}{dx^2}\right)_i + \mathcal{O}\left[(\Delta x)^3\right] \tag{19}$$

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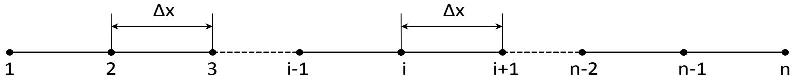

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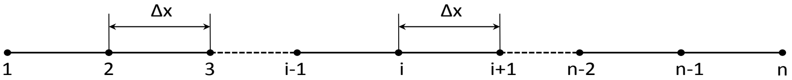
$$\left(\frac{d\phi}{dx}\right)_i = \frac{\phi_{i+1} - \phi_{i-1}}{2\Delta x} \quad \text{and} \quad \left(\frac{d^2\phi}{dx^2}\right)_i = \frac{\phi_{i-1} + \phi_{i+1} - 2\phi_i}{(\Delta x)^2}$$



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5. Thus, the diffusion term can be redefined as,

$$\Gamma\left(\frac{d^2\phi}{dx^2}\right)_2 = \Gamma\frac{\phi_{i-1}+\phi_{i+1}-2\phi_i}{(\Delta x)^2} \tag{20}$$

6. And the source term can be evaluated at point i, i.e., $\mathcal{S}_i = \mathcal{S}(\phi_i)$. Thus the discrete form of Eqn 17 is,

7. Here, we have a system of algebraic equations (for each point of the mesh) that can determine discrete values for ϕ;
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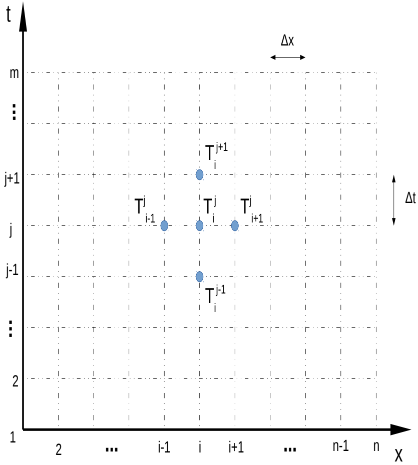
Solving 1D Transient HT Problems with FDM



1. The 1D transient heat conduction problem in a slab L with constant properties is represented by Eqn. 1,

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

subjected to prescribed boundary and initial conditions for $T(x, t)$.

2. In FDM, the spatial- and time-domains are divided into mesh grid. The points in both coordinates are separated by constant spacing, Δx and Δt;
3. Nodes x_1 and x_n are boundaries nodes whereas all other nodes are interior nodes;
4. Node t_1 represents the initial condition of the system.

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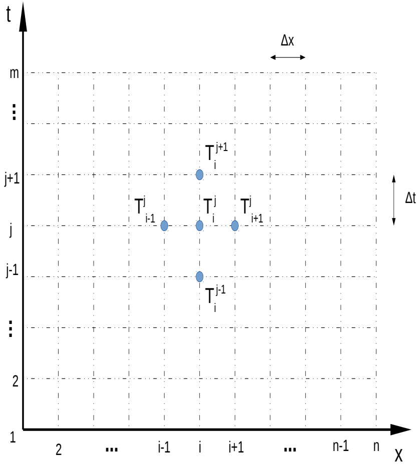

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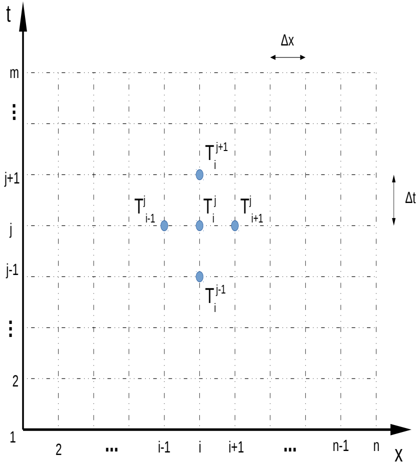

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Section Outline

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Coefficients and Bessel Functions

TABLE 4–2

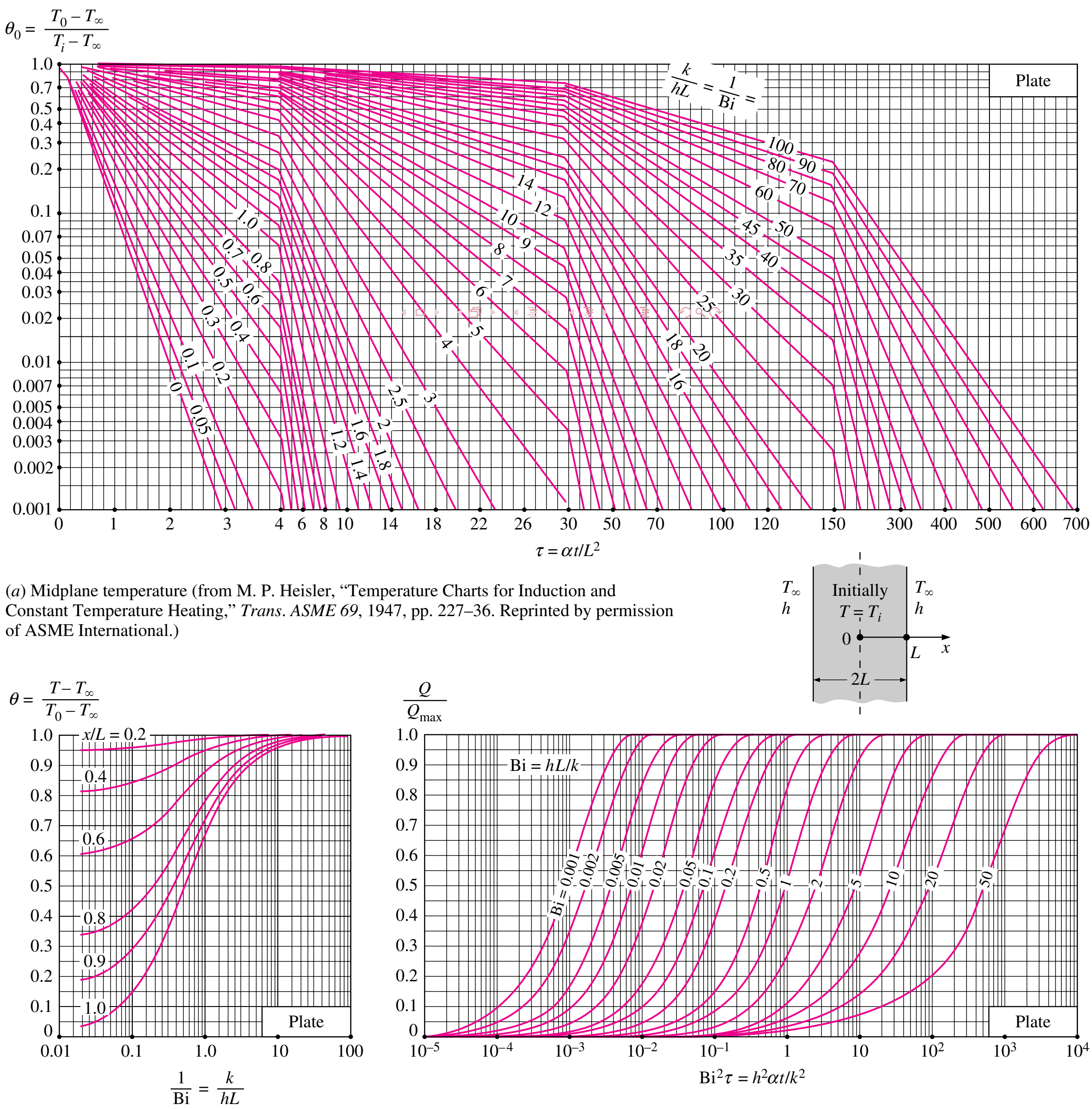
Coefficients used in the one-term approximate solution of transient one-dimensional heat conduction in plane walls, cylinders, and spheres (Bi = hL/k for a plane wall of thickness $2L$, and Bi = hr_o/k for a cylinder or sphere of radius r_o)

	Plane Wall		Cylinder		Sphere	
Bi	λ_1	A_1	λ_1	A_1	λ_1	A_1
0.01	0.0998	1.0017	0.1412	1.0025	0.1730	1.0030
0.02	0.1410	1.0033	0.1995	1.0050	0.2445	1.0060
0.04	0.1987	1.0066	0.2814	1.0099	0.3450	1.0120
0.06	0.2425	1.0098	0.3438	1.0148	0.4217	1.0179
0.08	0.2791	1.0130	0.3960	1.0197	0.4860	1.0239
0.1	0.3111	1.0161	0.4417	1.0246	0.5423	1.0298
0.2	0.4328	1.0311	0.6170	1.0483	0.7593	1.0592
0.3	0.5218	1.0450	0.7465	1.0712	0.9208	1.0880
0.4	0.5932	1.0580	0.8516	1.0931	1.0528	1.1164
0.5	0.6533	1.0701	0.9408	1.1143	1.1656	1.1441
0.6	0.7051	1.0814	1.0184	1.1345	1.2644	1.1713
0.7	0.7506	1.0918	1.0873	1.1539	1.3525	1.1978
0.8	0.7910	1.1016	1.1490	1.1724	1.4320	1.2236
0.9	0.8274	1.1107	1.2048	1.1902	1.5044	1.2488
1.0	0.8603	1.1191	1.2558	1.2071	1.5708	1.2732
2.0	1.0769	1.1785	1.5995	1.3384	2.0288	1.4793
3.0	1.1925	1.2102	1.7887	1.4191	2.2889	1.6227
4.0	1.2646	1.2287	1.9081	1.4698	2.4556	1.7202
5.0	1.3138	1.2403	1.9898	1.5029	2.5704	1.7870
6.0	1.3496	1.2479	2.0490	1.5253	2.6537	1.8338
7.0	1.3766	1.2532	2.0937	1.5411	2.7165	1.8673
8.0	1.3978	1.2570	2.1286	1.5526	2.7654	1.8920
9.0	1.4149	1.2598	2.1566	1.5611	2.8044	1.9106
10.0	1.4289	1.2620	2.1795	1.5677	2.8363	1.9249
20.0	1.4961	1.2699	2.2880	1.5919	2.9857	1.9781
30.0	1.5202	1.2717	2.3261	1.5973	3.0372	1.9898
40.0	1.5325	1.2723	2.3455	1.5993	3.0632	1.9942
50.0	1.5400	1.2727	2.3572	1.6002	3.0788	1.9962
100.0	1.5552	1.2731	2.3809	1.6015	3.1102	1.9990
∞	1.5708	1.2732	2.4048	1.6021	3.1416	2.0000

TABLE 4–3

The zeroth- and first-order Bessel functions of the first kind

η	$J_0(\eta)$	$J_1(\eta)$
0.0	1.0000	0.0000
0.1	0.9975	0.0499
0.2	0.9900	0.0995
0.3	0.9776	0.1483
0.4	0.9604	0.1960
0.5	0.9385	0.2423
0.6	0.9120	0.2867
0.7	0.8812	0.3290
0.8	0.8463	0.3688
0.9	0.8075	0.4059
1.0	0.7652	0.4400
1.1	0.7196	0.4709
1.2	0.6711	0.4983
1.3	0.6201	0.5220
1.4	0.5669	0.5419
1.5	0.5118	0.5579
1.6	0.4554	0.5699
1.7	0.3980	0.5778
1.8	0.3400	0.5815
1.9	0.2818	0.5812
2.0	0.2239	0.5767
2.1	0.1666	0.5683
2.2	0.1104	0.5560
2.3	0.0555	0.5399
2.4	0.0025	0.5202
2.6	−0.0968	−0.4708
2.8	−0.1850	−0.4097
3.0	−0.2601	−0.3391
3.2	−0.3202	−0.2613

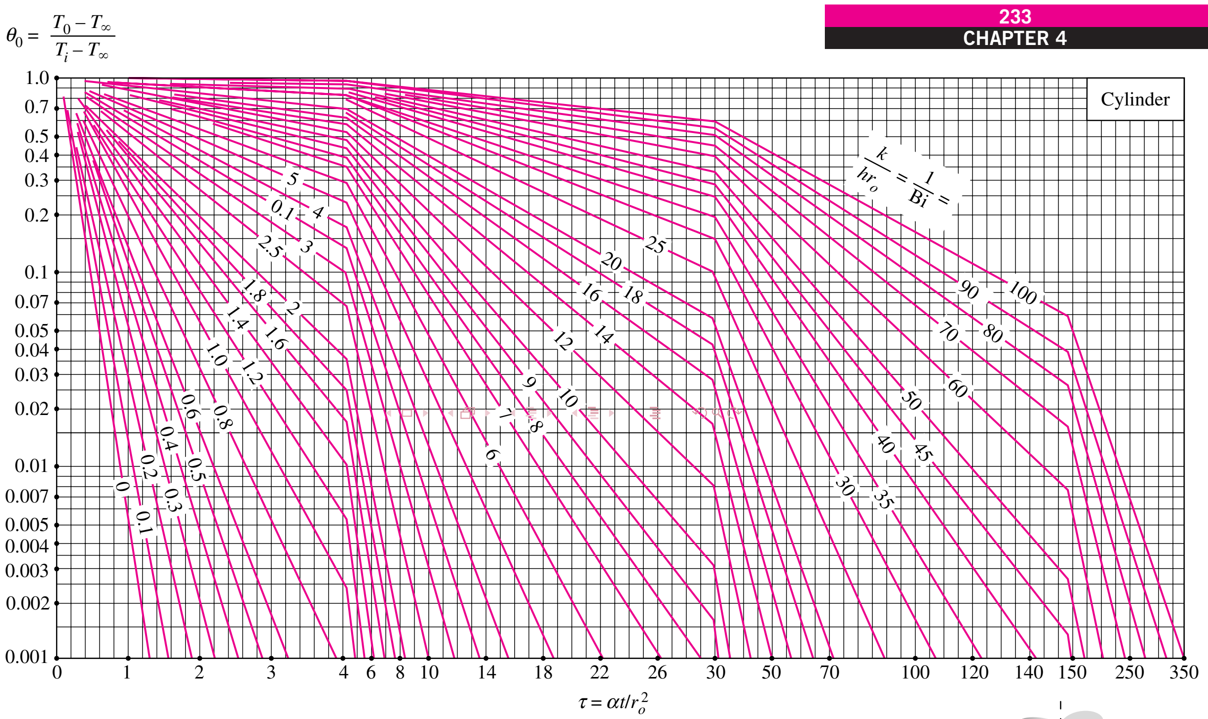


(b) Temperature distribution (from M. P. Heisler, "Temperature Charts for Induction and Constant Temperature Heating," *Trans. ASME 69*, 1947, pp. 227–36. Reprinted by permission of ASME International.)

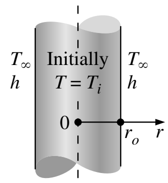
(c) Heat transfer (from H. Gröber et al.)

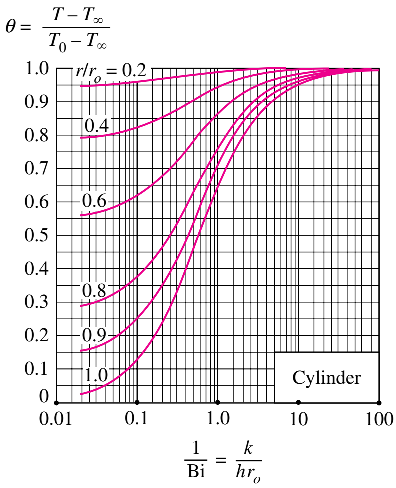
FIGURE 4–15

Transient temperature and heat transfer charts for a plane wall of thickness $2L$ initially at a uniform temperature T_i subjected to convection from both sides to an environment at temperature T_∞ with a convection coefficient of h.

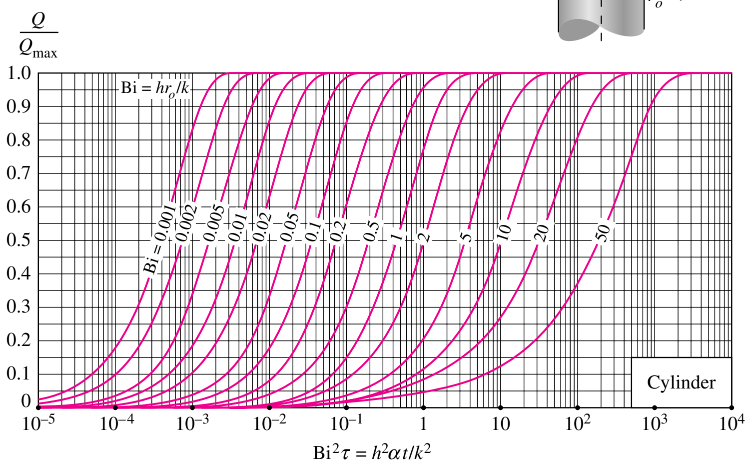


(*a*) Centerline temperature (from M. P. Heisler, "Temperature Charts for Induction and Constant Temperature Heating," *Trans. ASME 69*, 1947, pp. 227–36. Reprinted by permission of ASME International.)





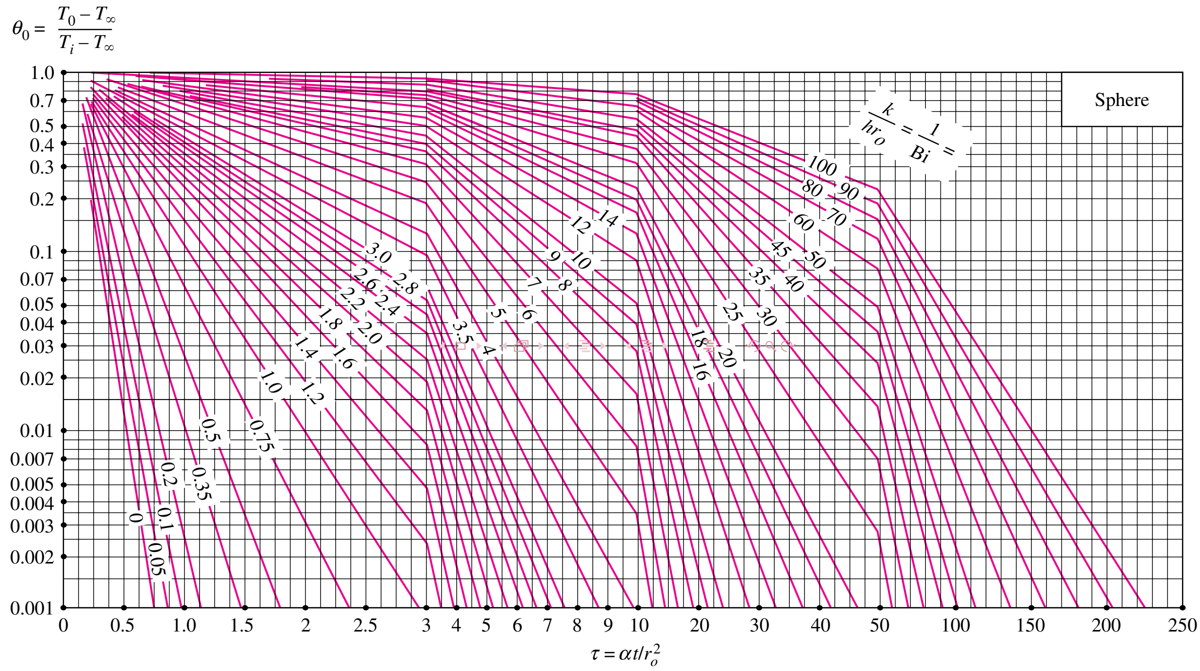
(*b*) Temperature distribution (from M. P. Heisler, "Temperature Charts for Induction and Constant Temperature Heating," *Trans. ASME 69*, 1947, pp. 227–36. Reprinted by permission of ASME International.)



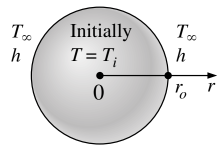
(*c*) Heat transfer (from H. Gröber et al.)

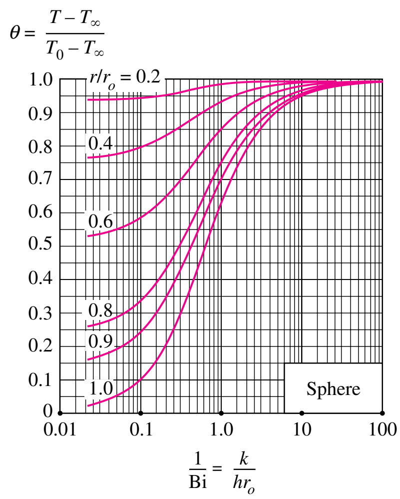
FIGURE 4–16

Transient temperature and heat transfer charts for a long cylinder of radius r_o initially at a uniform temperature T_i subjected to convection from all sides to an environment at temperature T_∞ with a convection coefficient of h.

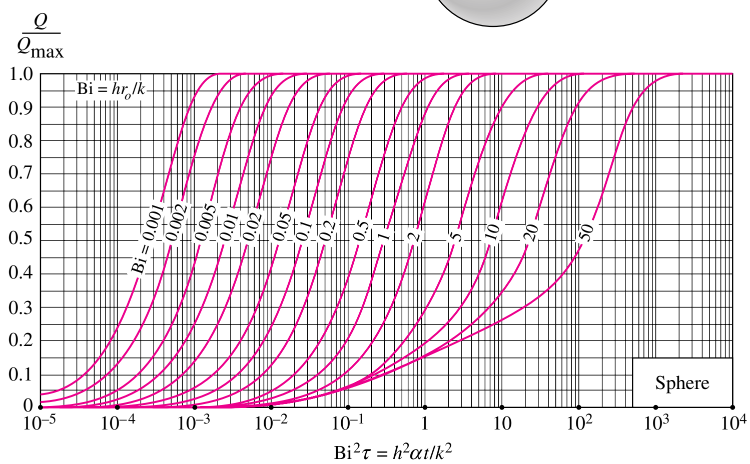


(*a*) Midpoint temperature (from M. P. Heisler, "Temperature Charts for Induction and Constant Temperature Heating," *Trans. ASME 69*, 1947, pp. 227–36. Reprinted by permission of ASME International.)





(*b*) Temperature distribution (from M. P. Heisler, "Temperature Charts for Induction and Constant Temperature Heating," *Trans. ASME 69*, 1947,



(*c*) Heat transfer (from H. Gröber et al.)

FIGURE 4–17

Transient temperature and heat transfer charts for a sphere of radius r_o initially at a uniform temperature T_i subjected to convection from all sides to an environment at temperature T_∞ with a convection coefficient of h.