

November 12, 2022



► $f(x) = x^3 - 4x$, average rate of change over the interval $[-2, 3]$

$$\frac{\Delta f}{\Delta x} = \frac{\Delta y}{\Delta x}$$

x	$y = f(x)$
-2	$y = f(-2) = (-2)^3 - 4(-2) = -8 + 8 = 0$
3	$y = f(3) = (3)^3 - 4(3) = 27 - 12 = 15$

+5() +15

$$= \frac{15}{5} = \boxed{3}$$

► $h(x) = \frac{1}{8}x^3 - x^2$

x	$y = h(x)$
-2	$\frac{1}{8}(-2)^3 - (-2)^2 = \frac{1}{8}(-8) - (4) = -1 - 4 = -5$
2	$\frac{1}{8}(2)^3 - (2)^2 = \frac{1}{8}(8) - 4 = 1 - 4 = -3$

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 - (-5)}{2 - (-2)} = \frac{-3 + 5}{2 + 2} = \frac{2}{4} = \boxed{\frac{1}{2}}$$

$$\triangleright g(x) = -\frac{x^2}{4} + 7$$

$$x=0, g(0) = -\frac{(0)^2}{4} + 7 \quad x=4, g(4) = -\frac{4^2}{4} + 7$$

$$= -\frac{0}{4} + 7$$

$$= 7$$

$$(0, 7)$$

$$= -\frac{16}{4} + 7$$

$$= -4 + 7$$

$$(4, 3)$$

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 7}{4 - 0} = \frac{-4}{4} = \boxed{-1}$$

$$\triangleright h(x) = x^2 - 1$$

$$x=-3, h(-3) = (-3)^2 - 1 \quad x=-1, h(-1) = (-1)^2 - 1$$

$$= 9 - 1$$

$$= 8$$

$$(-3, 8)$$

$$= 1 - 1$$

$$= 0$$

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 8}{-1 - (-3)} \quad (-1, 0)$$

$$= \frac{-8}{-1 + 3} = \frac{-8}{2} = \boxed{-4}$$

$$\triangleright h(t) = (t+3)^2 + 5$$

$$\begin{aligned} t = -5, h(-5) &= (-5+3)^2 + 5 \\ &= (-2)^2 + 5 \\ &= 4 + 5 \\ &= 9 \end{aligned}$$

$$(-5, 9)$$

$$\begin{aligned} t = -1, h(-1) &= (-1+3)^2 + 5 \\ &= (2)^2 + 5 \\ &= 4 + 5 \\ &= 9 \end{aligned}$$

$$(-1, 9)$$

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{9 - 9}{-1 + 5} = \frac{0}{4} = \boxed{0}$$

$$\triangleright h(t) = (t+3)^2 + 5$$

$$(-2, 6) \quad (0, 14)$$

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{14 - 6}{0 - (-2)} = \frac{8}{0 + 2} = \frac{8}{2} = 4$$

$$f(x) = x^2 + 10$$

$$(-2, 14) \quad (-1, 11)$$

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{11 - 14}{-1 - (-2)} = \frac{-3}{-1 + 2} = \frac{-3}{1} = \boxed{-3}$$

$$\triangleright h(x) = x^2 - 1$$

$$\triangleright (-t^3 + 5t^2 - 6t) + (8t^2 - 8t)$$

$$-t^3 + 5t^2 + 8t^2 - 6t - 8t$$

$$\boxed{-t^3 + 13t^2 - 14t}$$

$$\triangleright -3c^4 + 0c^3 - 5c^2 - 7c$$

$$4c^4 + 8c^3 + 2c^2 + 0c$$

$$\boxed{c^4 + 8c^3 - 3c^2 - 7c}$$

$$\triangleright (-5a^3 - 2a^2) + (6a^3 + 9a^2 + 8a)$$

$$\boxed{a^3 + 7a^2 + 8a}$$

$$\triangleright (-2k^3 - 7k^2 + 5k) + (6k^2 + 3k)$$

$$\boxed{-2k^3 - k^2 + 8k}$$

$$\triangleright (5p^3 - p) - (7p^2 + 4p - 3)$$

$$5p^3 - p - 7p^2 - 4p + 3$$

$$\boxed{5p^3 - 7p^2 - 5p + 3}$$

$$\triangleright (f^3 - 5f + 25) - (4f^2 - 12f + 9)$$

$$f^3 - 5f + 25 - 4f^2 + 12f - 9$$

$$\boxed{f^3 - 4f^2 + 7f + 16}$$

$$\triangleright (-8y^2 - 9y) - (-8y^3 + 9y^2 - 5y)$$

$$-8y^2 - 9y + 8y^3 - 9y^2 + 5y$$

$$\boxed{8y^3 - 17y^2 - 4y}$$

$$\triangleright (6a^3 + 7a^2) - (5a^3 + 9a^2 + a)$$

$$6a^3 + 7a^2 - 5a^3 - 9a^2 - a$$

$$\boxed{a^3 - 2a^2 - a}$$

$$\triangleright (6x^2 + 3x - 9) - (-2x^2 + 4x - 1)$$

$$6x^2 + 3x - 9 + 2x^2 - 4x + 1$$

$$\boxed{8x^2 - x - 8}$$

$$\triangleright -8r^2 + 11r - 6$$

$$-7r^2 - 9r + 14$$

$$\boxed{-15r^2 + 2r + 8}$$

$$\triangleright (p^2 - 5p + 4) - (4p^2 - 9p + 11)$$

$$p^2 - 5p + 4 - 4p^2 + 9p - 11$$

$$\boxed{-3p^2 + 4p - 7}$$

$$\triangleright b^2 + 8b - 9 - (11b^2 - 4b + 7)$$

$$b^2 + 8b - 9 - 11b^2 + 4b - 7$$

$$\boxed{-10b^2 + 12b - 16}$$

$$\triangleright (-3a^2 + 2a - 5) - (-2a^2 + a + 6)$$

$$-3a^2 + 2a - 5 + 2a^2 - a - 6$$

$$\boxed{-a^2 + a - 11}$$

$$\triangleright (6y^2 + 2y + 5) - (5y^2 - 6y - 11)$$

$$6y^2 + 2y + 5 - 5y^2 + 6y + 11$$

$$\boxed{y^2 + 8y + 16}$$

$$\triangleright (7x^2 - 3x + 10) - (-4x^2 + 6x - 4)$$

$$7x^2 - 3x + 10 + 4x^2 - 6x + 4$$

$$\boxed{11x^2 - 9x + 14}$$

$$\triangleright (-3y^2 - 5y - 2) + (-7y^2 + 5y + 2)$$

$$\boxed{-10y^2}$$

$$\Rightarrow (10a^2 + 3a + 25) - (9a^2 - 6a + 5)$$

$$10a^2 + 3a + 25 - 9a^2 + 6a - 5$$

$$\boxed{a^2 + 9a + 20}$$

$$\Rightarrow 6c^2 - 2c - 1$$

$$-4c^2 + 7c + 5$$

$$\boxed{2c^2 + 5c + 4}$$

$$\Rightarrow g(t) = -(t-1)^2 + 5$$

$$g(-4) = -(-4-1)^2 + 5 \quad g(5) = -(5-1)^2 + 5$$

$$= -(-5)^2 + 5$$

$$= -(4)^2 + 5$$

$$= -25 + 5$$

$$= -16 + 5$$

$$= -20$$

$$= -11$$

$$(-4, -20)$$

$$(5, -11)$$

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{-11 - (-20)}{5 - (-4)} = \frac{-11 + 20}{5 + 4} = \frac{9}{9} = \boxed{1}$$

$$\Rightarrow (-5z^5)(6z^6) = \boxed{-30z^{11}}$$

$$\Rightarrow 2x^4(x^2 + 8x + 15)$$

$$\boxed{2x^6 + 16x^5 + 30x^4}$$

$$\Rightarrow \frac{4w^3(5w^2 - 3w - 4)}{20w^5 - 12w^4 - 16w^3}$$

$$\Rightarrow \frac{4x^3(x^3 + 3x^2 + 2x)}{4x^6 + 12x^5 + 8x^4}$$

$$\Rightarrow \frac{3c^4(c^2 - 4c + 3)}{3c^6 - 12c^5 + 9c^4}$$

$$\Rightarrow \frac{3h(-h^2 + 2h - 1)}{-3h^3 + 6h^2 - 3h}$$

$$\Rightarrow \frac{-5k^2(k^2 - 4k)}{-5k^4 + 20k^3}$$

$$\Rightarrow \frac{8r^2(r^2 - 2)}{8r^4 - 16r^2}$$

$$\Rightarrow \frac{3x(x^2 - 5x + 6)}{3x^3 - 15x^2 + 18x}$$

$$\Rightarrow \frac{-n^2(n^2 + 5n + 6)}{-n^4 - 5n^3 - 6n^2}$$

$$\Rightarrow \frac{-t(-4t^3 + 8t^2)}{4t^4 - 8t^3}$$

$$\Rightarrow \frac{7x^2(3x^2 + 8x + 7)}{21x^4 + 56x^3 + 49x^2}$$

$$\Rightarrow \frac{m(-m^3 + m^2 + 3m)}{-m^4 + m^3 + 3m^2}$$

$$\Rightarrow 3b^2(4b^4 + 2b - 6)$$

$$12b^6 + 6b^3 - 18b^2$$

$$\Rightarrow -p^2(p^2 - p - 1)$$

$$-p^4 + p^3 + p^2$$

$$\Rightarrow 4w^3(5w^2 - 3w - 4)$$

$$20w^5 - 12w^4 - 16w^3$$

$$\Rightarrow (w^2 + 2)(w^2 + 3w + 9)$$

$$w^4 + 3w^3 + 9w^2 + 2w^2 + 6w + 18$$

$$w^4 + 3w^3 + 11w^2 + 6w + 18$$

$$\Rightarrow (k^2 + 3)(k^2 + 7)$$

$$k^4 + 7k^2 + 3k^2 + 21$$

$$k^4 + 10k^2 + 21$$

$$\Rightarrow (b^3 + b^2)(b^2 + 7b + 4)$$

$$b^5 + 7b^4 + 4b^3 + b^4 + 7b^3 + 4b^2$$

$$b^5 + 8b^4 + 11b^3 + 4b^2$$

$$\Rightarrow (2t + 1)(t^2 + 7t + 6)$$

$$2t^3 + 14t^2 + 12t + t^2 + 7t + 6$$

$$2t^3 + 15t^2 + 19t + 6$$

$$\Rightarrow (1 - 4m)(m^2 - 3m + 8)$$

$$m^2 - 3m + 8 - 4m^3 + 12m^2 - 32m$$

$$-4m^3 + 13m^2 - 35m + 8$$

$$\Rightarrow (q^2 - 2q + 5)(q^2 - 5)$$

$$q^4 - \cancel{5q^2} - 2q^3 + 10q + \cancel{5q^2} - 25$$

$$\boxed{q^4 - 2q^3 + 10q - 25}$$

$$\Rightarrow (d^2 + 3)(d^2 + 2d + 1)$$

$$d^4 + 2d^3 + d^2 + 3d^2 + 6d + 3$$

$$\boxed{d^4 + 2d^3 + 4d^2 + 6d + 3}$$

$$\Rightarrow (3y^2 + 2y^5)(3y^2 - 2y^5) = a^2 - b^2 = (3y^2)^2 - (2y^5)^2 = \boxed{9y^4 - 4y^{10}}$$

$$\Rightarrow (5 - 2x^4)(5 + 2x^4)$$

$$25 + 10x^4 - \cancel{10x^4} - 4x^8$$

$$\boxed{-4x^8 + 25}$$

$$\Rightarrow (9c^2 + c^6)(9c^2 - c^6)$$

$$81c^4 - \cancel{9c^8} + \cancel{9c^8} - c^{12}$$

$$\boxed{-c^{12} + 81c^4}$$

$$\Rightarrow (8 - 3a^2)(2a^2 + 6)$$

$$16a^2 + 48 - 6a^4 - 18a^2$$

$$\boxed{-6a^4 - 2a^2 + 48}$$

$$\Rightarrow (c^2 - 6)(2c^2 + 3c - 1)$$

$$2c^4 + 3c^3 - c^2 - 12c^2 - 18c + 6$$

$$\boxed{2c^4 + 3c^3 - 13c^2 - 18c + 6}$$

$$\begin{aligned} &\triangleright (8-n^7)(8+n^7) \\ &64 + \cancel{8n^7} - \cancel{8n^7} - n^{14} \\ &\boxed{-n^{14} + 64} \end{aligned}$$

$$\begin{aligned} &\triangleright (4m^5 - 5m^6)(4m^5 + 5m^6) \\ &(4m^5)^2 - (-5m^6)^2 \\ &16m^{10} - (25m^{12}) \\ &\boxed{-25m^{12} + 16m^{10}} \end{aligned}$$

$$\begin{aligned} &\triangleright (3-8w^2)^2 \\ &(3-8w^2)(3-8w^2) \\ &9 - 24w^2 - 24w^2 + 64w^4 \\ &\boxed{64w^4 - 48w^2 + 9} \end{aligned}$$

$$\begin{aligned} &\triangleright (6y^2 + 5y^4)^2 \\ &(6y^2 + 5y^4)(6y^2 + 5y^4) \\ &36y^4 + 30y^6 + 30y^6 + 25y^8 \\ &\boxed{25y^8 + 60y^6 + 36y^4} \end{aligned}$$

$$\begin{aligned} &\triangleright (4+7a^2)^2 \\ &(4+7a^2)(4+7a^2) \\ &16 + 28a^2 + 28a^2 + 49a^4 \\ &49a^4 + 56a^2 + 16 \end{aligned}$$

$$\begin{aligned} &\triangleright (2-7m^6)^2 \\ &(2-7m^6)(2-7m^6) \\ &4 - 14m^6 - 14m^6 + 49m^{12} \\ &\boxed{49m^{12} - 28m^6 + 4} \end{aligned}$$

$$\begin{aligned} &\triangleright (2x^3 + 4x^5)^2 \\ &(2x^3 + 4x^5)(2x^3 + 4x^5) \\ &4x^6 + 8x^8 + 8x^8 + 16x^{10} \\ &\boxed{16x^{10} + 16x^8 + 4x^6} \end{aligned}$$

$$\Rightarrow (4t^3 - 5)^2$$

$$(4t^3 - 5)(4t^3 - 5)$$

$$16t^6 - 20t^3 - 20t^3 + 25$$

$$\boxed{16t^6 - 40t^3 + 25}$$

$$\Rightarrow (8w^4 + w^3)^2$$

$$(8w^4 + w^3)(8w^4 + w^3)$$

$$64w^8 + 8w^7 + 8w^7 + w^6$$

$$\boxed{64w^8 + 16w^7 + w^6}$$

$$\Rightarrow (5 + 6b^3)^2$$

$$(5 + 6b^3)(5 + 6b^3)$$

$$25 + 30b^3 + 30b^3 + 36b^6$$

$$\boxed{36b^6 + 60b^3 + 25}$$

$$\Rightarrow (7b^5 - b^2)^2$$

$$(7b^5 - b^2)(7b^5 - b^2)$$

$$49b^{10} - 7b^7 - 7b^7 + b^4$$

$$\boxed{49b^{10} - 14b^7 + b^4}$$

$$\Rightarrow (9y^5 + 2)^2$$

$$(9y^5 + 2)(9y^5 + 2)$$

$$81y^{10} + 18y^5 + 18y^5 + 4$$

$$\boxed{81y^{10} + 36y^5 + 4}$$

$$\Rightarrow (3a^2 - 1)(-3a^2 + 5)$$

$$-9a^4 + 15a^2 + 3a^2 - 5$$

$$\boxed{-9a^4 + 18a^2 - 5}$$

$$\Rightarrow (4b^2 + 3)(4b^2 - 3)$$

$$16b^4 - \cancel{12b^2} + \cancel{12b^2} - 9$$

$$\boxed{16b^4 - 9}$$

$$\triangleright (2z-1)(z^2-2z+1)$$

$$2z^3 - 4z^2 + 2z - z^2 + 2z - 1$$

$$\boxed{2z^3 - 5z^2 + 4z - 1}$$

$$\triangleright (2x^4+3x^3)(2x^4-3x^3)$$

$$4x^8 - 6x^7 + 6x^7 - 9x^6$$

$$\boxed{4x^8 - 9x^6}$$

$$\triangleright (5a^3-2)(5a^3+2)$$

$$25a^6 + 10a^3 - 10a^3 - 4$$

$$\boxed{25a^6 - 4}$$

$$\triangleright 4b^2(b^3-8)$$

$$\boxed{4b^5 - 32b^2}$$

$$\triangleright 4x^3(x^3+3x^2+2x)$$

$$\boxed{4x^6 + 12x^5 + 8x^4}$$

$$\triangleright (-p^2+4p-3)(p^2+2)$$

$$-p^4 - 2p^2 + 4p^3 + 8p - 3p^2 - 6$$

$$\boxed{-p^4 + 4p^3 - 5p^2 + 8p - 6}$$

$$\triangleright (7x^2-3x+10) - (-4x^2+6x-4)$$

$$7x^2 - 3x + 10 + 4x^2 - 6x + 4$$

$$\boxed{11x^2 - 9x + 14}$$

$$\triangleright f(x) = x^2 + 10$$

$$f(-2) = (-2)^2 + 10 \quad f(-1) = (-1)^2 + 10$$

$$= 4 + 10 \quad = 1 + 10$$

$$= 14 \quad = 11$$

$$(-2, 14) \quad (-1, 11)$$

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{11 - 14}{-1 + 2} = \frac{-3}{1} = \boxed{-3}$$

$$\Rightarrow (4t^3 - 5)^2$$

$$(4t^3 - 5)(4t^3 - 5)$$

$$16t^6 - 20t^3 - 20t^3 + 25$$

$$\boxed{16t^6 - 40t^3 + 25}$$

$$\Rightarrow (b^2 + 8b - 9) - (11b^2 - 4b + 7)$$

$$b^2 + 8b - 9 - 11b^2 + 4b - 7$$

$$\boxed{-10b^2 + 12b - 16}$$