

November 12, 2022



$$\Rightarrow \frac{2x^4 - 3x}{x} = \frac{x(2x^3 - 3)}{2x^4 - 3x} \Rightarrow \boxed{2x^3 - 3}$$

$$\Rightarrow \frac{x^3 - 5x^2}{x} \Rightarrow x(x^2 - 5x) \Rightarrow \boxed{x^2 - 5x}$$

$$\Rightarrow \frac{3x^5 - x}{x} = \frac{3x^5}{x} - \frac{x}{x} = \boxed{3x^4 - 1}$$

$$\Rightarrow \frac{2x^5 + 5x^3}{x} = \frac{2x^5}{x} + \frac{5x^3}{x} = \boxed{2x^4 + 5x^2}$$

$$\Rightarrow \frac{18x^4 - 3x^2 + 6x - 4}{6x}$$

$$\frac{18x^4}{6x} - \frac{3x^2}{6x} + \frac{6x}{6x} - \frac{4}{6x}$$

$$\boxed{3x^3 - \frac{1}{2}x + 1x^0 - \frac{2}{3x}}$$

$$\triangleright \frac{4x^3 - x^2 + 3}{x}$$

$$\frac{4x^3}{x} - \frac{x^2}{x} + \frac{3}{x}$$

$$\boxed{4x^2 - x + \frac{3}{x}}$$

$$\triangleright \frac{4x^3 - 3x + 1}{x}$$

$$\frac{4x^3}{x} - \frac{3x}{x} + \frac{1}{x}$$

$$\boxed{4x^2 - 3 + \frac{1}{x}}$$

$$\triangleright \frac{6x^5 - 2x^4 - 1}{x}$$

$$\frac{6x^5}{x} - \frac{2x^4}{x} - \frac{1}{x}$$

$$\boxed{6x^4 - 2x^3 - \frac{1}{x}}$$

$$\triangleright \frac{5x^2 + x + 7}{x}$$

$$\frac{5x^2}{x} + \frac{x}{x} + \frac{7}{x}$$

$$\boxed{5x + 1 + \frac{7}{x}}$$



$$\begin{array}{r} x+2 \\ 2 \overline{) 2x+4} \\ \underline{-2x} \end{array}$$

$$\boxed{x+2}$$

$$\begin{array}{r} 4 \\ \underline{-4} \\ 0 \end{array}$$



$$\begin{array}{r} x+1 \\ x+4 \overline{) x^2+5x+4} \\ \underline{-x^2-4x} \end{array}$$

$$\begin{array}{r} x+4 \\ \underline{-x-4} \end{array}$$

0

$$\boxed{x+1}$$



$$\begin{array}{r} x+2 \\ x+1 \overline{) x^2+3x+6} \\ \underline{-(x^2+x)} \end{array}$$

$$\begin{array}{r} 2x+6 \\ \underline{-2x-2} \end{array}$$

4

$$\boxed{x+2, r. 4}$$

or

$$\boxed{x+2 + \frac{4}{x+1}}$$

$$\begin{array}{r}
 3x-2 \\
 x^2+1 \overline{) 3x^3-2x^2+7x-4} \\
 \underline{-3x^3+0x^2-3x} \\
 -2x^2+4x-4 \\
 \underline{+2x^2+0x+2} \\
 4x-2
 \end{array}$$

$$\boxed{3x-2 + \frac{4x-2}{x^2+1}}$$

$$\begin{array}{r}
 x+5 \\
 x+2 \overline{) x^2+7x+10} \\
 \underline{-x^2-2x} \\
 5x+10 \\
 \underline{-5x-10} \\
 0
 \end{array}$$

$$\boxed{x+5}$$

$$\begin{array}{r}
 x^2-49 \\
 x+7 \overline{) x^2-49} \\
 \underline{-(x+7)(x-7)} \\
 0
 \end{array}
 = \frac{(x+7)(x-7)}{x+7}$$

$$\boxed{x-7}$$

$$\begin{array}{r}
 x+2 \\
 x-5 \overline{) x^2-3x-10} \\
 \underline{-x^2+5x} \\
 2x-10 \\
 \underline{-2x+10} \\
 0
 \end{array}$$

$$\boxed{x+2}$$

$$\begin{array}{r}
 x^2+10x+25 \\
 x+5 \overline{) x^2+10x+25} \\
 \underline{-(x+5)(x+5)} \\
 0
 \end{array}
 = \frac{(x+5)(x+5)}{x+5}$$

$$\boxed{x+5}$$

$$\Rightarrow \frac{x^2 + 7x + 10}{x+2} = \frac{(x+5)(\cancel{x+2})}{\cancel{x+2}} = \boxed{x+5}$$

$$\Rightarrow x+1 \overline{) \begin{array}{r} x-3 \\ x^2 - 2x - 8 \\ -x^2 - x \\ \hline -3x - 8 \\ +3x + 3 \\ \hline -5 \end{array}}$$

$$\boxed{x-3 - \frac{5}{x+1}}$$

$$\Rightarrow x+3 \overline{) \begin{array}{r} x+2 \\ x^2 + 5x + 5 \\ -x^2 - 3x \\ \hline 2x + 5 \\ -2x - 6 \\ \hline -1 \end{array}}$$

$$\boxed{x+2 - \frac{1}{x+3}}$$

$$\Rightarrow x-2 \overline{) \begin{array}{r} x+2 \\ x^2 + 0x - 5 \\ -x^2 + 2x \\ \hline 2x - 5 \\ -2x + 4 \\ \hline -1 \end{array}}$$

$$\boxed{x+2 - \frac{1}{x-2}}$$

$$\Rightarrow x+2 \overline{) \begin{array}{r} x+5 \\ x^2 + 7x + 12 \\ -x^2 - 2x \\ \hline 5x + 12 \\ -5x - 10 \\ \hline 2 \end{array}}$$

$$\boxed{x+5 + \frac{2}{x+2}}$$

$$\begin{array}{r} \text{▶} \quad \frac{x^2 - 7}{x + 3} = x + 3 \overline{) \begin{array}{r} x^2 + 0x - 7 \\ - x^2 - 3x \quad \downarrow \\ \hline -3x - 7 \\ + 3x + 9 \\ \hline 2 \end{array}} \end{array}$$

$$\boxed{x - 3 + \frac{2}{x + 3}}$$

$$\begin{array}{r} \text{▶} \quad x + 5 \overline{) \begin{array}{r} x^2 + 0x - 28 \\ - x^2 - 5x \quad \downarrow \\ \hline -5x - 28 \\ + 5x + 25 \\ \hline -3 \end{array}} \end{array}$$

$$\boxed{x - 5 - \frac{3}{x + 5}}$$

$$\begin{array}{r} \text{▶} \quad x + 1 \overline{) \begin{array}{r} x^2 - 2x - 8 \\ - x^2 - x \quad \downarrow \\ \hline -3x - 8 \\ + 3x + 3 \\ \hline -5 \end{array}} \end{array}$$

$$\boxed{x - 3 - \frac{5}{x + 1}}$$

$$\begin{array}{r} \text{▶} \quad x - 2 \overline{) \begin{array}{r} x^2 + 0x - 5 \\ - x^2 + 2x \quad \downarrow \\ \hline 2x - 5 \\ - 2x + 4 \\ \hline -1 \end{array}} \end{array}$$

$$\boxed{x + 2 - \frac{1}{x - 2}}$$

$$\begin{array}{r}
 \overline{) X+5} \\
 X+2 \overline{) X^2 + 7X + 12} \\
 \underline{-X^2 - 2X} \downarrow \\
 5X + 12 \\
 \underline{-5X - 10} \\
 2
 \end{array}$$

$$X+5 + \frac{2}{X+2}$$

$$\begin{array}{r}
 \overline{) X+1} \\
 X-1 \overline{) X^2 + 0X + 2} \\
 \underline{-X^2 + X} \downarrow \\
 X + 2 \\
 \underline{-X + 1} \\
 3
 \end{array}$$

$$X+1 + \frac{3}{X-1}$$

$$\begin{array}{r}
 \overline{) X-3} \\
 X+1 \overline{) X^2 - 2X - 8} \\
 \underline{-X^2 - X} \downarrow \\
 -3X - 8 \\
 \underline{+3X + 3} \\
 -5
 \end{array}$$

$$X-3 - \frac{5}{X+1}$$

$$\begin{array}{r}
 \overline{) X-5} \\
 X+5 \overline{) X^2 + 0X - 28} \\
 \underline{-X^2 - 5X} \downarrow \\
 -5X - 28 \\
 \underline{+5X + 25} \\
 -3
 \end{array}$$

$$X-5 - \frac{3}{X+5}$$

$$X-1 + \frac{7}{X-2}$$

$$X+3 + \frac{13}{X-3}$$

$$3x^2 - 2x + 1 + \frac{5}{x+2}$$

$$\begin{array}{r}
 \begin{array}{r} 3x^2 - 3x + 4 \end{array} \\
 x+1 \overline{) 3x^3 + 0x^2 + x - 11} \\
 \underline{- 3x^3 - 3x^2} \downarrow \\
 -3x^2 + x \downarrow \\
 \underline{+ 3x^2 + 3x} \downarrow \\
 4x - 11 \\
 \underline{- 4x - 4} \\
 -15
 \end{array}$$

$$3x^2 - 3x + 4 - \frac{15}{x+1}$$

$$\begin{array}{r}
 \begin{array}{r} 3x^2 + 7x \end{array} \\
 x+1 \overline{) 3x^3 + 10x^2 + 7x + 0} \\
 \underline{- 3x^3 - 3x^2} \\
 7x^2 + 7x \downarrow \\
 \underline{- 7x^2 - 7x} \\
 0
 \end{array}$$

$$3x^2 + 7x$$

$$\begin{array}{r}
 \begin{array}{r} 2x^2 - x + 3 \end{array} \\
 x-6 \overline{) 2x^3 - 13x^2 + 9x - 16} \\
 \underline{- 2x^3 + 12x^2} \downarrow \\
 -x^2 + 9x \downarrow \\
 \underline{+ x^2 - 6x} \\
 3x - 16 \\
 \underline{- 3x + 18} \\
 2
 \end{array}$$

$$2x^2 - x + 3 + \frac{2}{x-6}$$

$$\begin{array}{r}
 \begin{array}{r} 4x^2 + 2x + 1 \end{array} \\
 x-4 \overline{) 4x^3 - 14x^2 - 7x - 4} \\
 \underline{- 4x^3 + 16x^2} \downarrow \\
 2x^2 - 7x \downarrow \\
 \underline{- 2x^2 + 8x} \downarrow \\
 x - 4 \\
 \underline{- x + 4} \\
 0
 \end{array}$$



$$\begin{array}{r} 4x^2 + 11x - 3 \\ x + 2 \overline{) 4x^3 + 19x^2 + 19x - 6} \\ \underline{- 4x^3 - 8x^2} \downarrow \\ 11x^2 + 19x \\ \underline{- 11x^2 - 22x} \downarrow \\ -3x - 6 \\ \underline{+ 3x + 6} \\ 0 \end{array}$$

$$p(x) = (x+2)(4x^2 + 11x - 3)$$

$$4x^2 + 11x - 3$$

$$a \cdot b = 4 \cdot -3 = -12$$

$$a + b = 11$$

$$12 \cdot -1 = -12$$

$$12 - 1 = 11$$

$$4x^2 + 12x - x - 3$$

$$(4x^2 + 12x) + (-x - 3)$$

$$4x(x+3) - 1(x+3)$$

$$(4x-1)(x+3)$$

$$p(x) = (4x-1)(x+3)(x+2)$$

$$\triangleright p(x) = x^3 + 9x^2 - 108$$

$$\begin{array}{r}
 x^2 + 3x - 18 \\
 x + 6 \overline{) x^3 + 9x^2 + 0x - 108} \\
 \underline{- x^3 - 6x^2} \\
 3x^2 \\
 \underline{- 3x^2 - 18x} \\
 -18x - 108 \\
 \underline{+ 18x + 108} \\
 0
 \end{array}$$

$$p(x) = (x^2 + 3x - 18)(x + 6)$$

$$p(x) = (x + 6)(x - 3)(x + 6)$$

$$p(x) = (2x + 3)(x + 2)(x + 5)$$

$$\triangleright p(x) = 2x^3 + 17x^2 + 41x + 30$$

$$\begin{array}{r}
 2x^2 + 7x + 6 \\
 x + 5 \overline{) 2x^3 + 17x^2 + 41x + 30} \\
 \underline{- 2x^3 - 10x^2} \\
 7x^2 \\
 \underline{- 7x^2 - 35x} \\
 6x + 30 \\
 \underline{- 6x - 30} \\
 0
 \end{array}$$

$$p(x) = (2x^2 + 7x + 6)(x + 5)$$

$$a \cdot b = 2 \cdot 6 = 12$$

$$a + b = 7$$

$$4 \cdot 3 = 12$$

$$4 + 3 = 7$$

$$(2x^2 + 4x) + (3x + 6)$$

$$2x(x + 2) + 3(x + 2)$$

$$(2x + 3)(x + 2)$$

$$\begin{array}{r}
 x^2 - x - 6 \\
 x + 1 \overline{) x^3 + 0x^2 - 7x - 6} \\
 \underline{-x^3 - x^2} \\
 -x^2 - 7x \\
 \underline{+x^2 + x} \\
 -6x - 6 \\
 \underline{+6x + 6} \\
 0
 \end{array}$$

$$p(x) = (x^2 - x - 6)(x + 1)$$

$$p(x) = (x - 3)(x + 2)(x + 1)$$

$$p(x) = (5x + 1)(x - 2)(x - 7)$$

$$\begin{array}{r}
 5x^2 - 9x - 2 \\
 x - 7 \overline{) 5x^3 - 44x^2 + 61x + 14} \\
 \underline{-5x^3 + 35x^2} \\
 -9x^2 + 61x \\
 \underline{+9x^2 - 63x} \\
 -2x + 14 \\
 \underline{+2x - 14} \\
 0
 \end{array}$$

$$p(x) = (5x^2 - 9x - 2)(x - 7)$$

$$5x^2 - 9x - 2$$

$$a \cdot b = 5 \cdot -2 = -10$$

$$a + b = -9$$

$$-10 \cdot 1 = -10$$

$$-10 + 1 = -9$$

$$(5x^2 - 10x) + (x - 2)$$

$$5x(x - 2) + 1(x - 2)$$

$$\triangleright p(x) = x^3 - 21x - 20$$

$$\begin{array}{r}
 x^2 + 5x + 4 \\
 x-5 \overline{) \cancel{x^3} + 0x^2 - 21x - 20} \\
 \underline{-\cancel{x^3} + 5x^2} \quad \downarrow \\
 5x^2 - 21x \\
 \underline{-5x^2 + 25x} \quad \downarrow \\
 4x - 20 \\
 \underline{-4x + 20} \\
 0
 \end{array}$$

$$p(x) = (x^2 + 5x + 4)(x - 5)$$

$$p(x) = (x+4)(x+1)(x-5)$$

Polynomial Remainder Theorem

$f(x) \Rightarrow$ polynomial divide by $x - a \Rightarrow$ remainder $f(a)$

$\Rightarrow f(x) = 3x^2 - 4x + 7$ divided by $x - 1$

$$\begin{array}{r} 3x - 1 \\ x - 1 \overline{) 3x^2 - 4x + 7} \\ \underline{-3x^2 + 3x} \\ -x + 7 \\ \underline{+x - 1} \\ 6 \end{array}$$

$$x - a = 3x - 1$$

$$a = 1$$

$$f(x) = 3x^2 - 4x + 7$$

$$f(1) = 3(1)^2 - 4(1) + 7$$

$$= 3 - 4 + 7$$

$$= -1 + 7$$

$$= 6$$

$$\boxed{6 = 6}$$



$$\Rightarrow p(x) = -3x^3 - 4x^2 + 10x - 7 ; x-2$$

$$a = 2$$

$$\frac{p(x)}{(x-a)} \Rightarrow r = p(a)$$

$$a = 2$$

$$r = p(2) = -3(2)^3 - 4(2)^2 + 10(2) - 7$$

$$= -3(8) - 16 + 20 - 7$$

$$= -24 - 16 + 20 - 7$$

$$= -40 + 20 - 7$$

$$= -20 - 7$$

$$= \boxed{-27}$$

$$\Rightarrow p(x) = x^3 + 2x^2 + cx + 10$$

$$x-5$$

$$p(5) = (5)^3 + 2(5)^2 + c(5) + 10$$

$$0 = 125 + 50 + 5c + 10$$

$$0 = 185 + 5c$$

$$-185 - 185$$

$$\frac{-185}{5} = \frac{5c}{5}$$

$$\boxed{c = -37}$$

$$\Rightarrow p(x) = cx^3 - 15x - 68$$

$$p(4) = c(4)^3 - 15(4) - 68$$

$$0 = 64c - 60 - 68$$

$$0 = 64c - 128$$

$$128 = 64c \xrightarrow{+128} \boxed{c = 2}$$

$$\triangleright p(x) = -x^3 + cx^2 - 4x + 3$$

$$p(3) = -(3)^3 + c(3)^2 - 4(3) + 3$$

$$0 = -27 + 9c - 12 + 3$$

$$0 = -36 + 9c$$

$$\frac{36}{9} = \frac{9c}{9}$$

$$\boxed{c = 4}$$

$$\triangleright p(x) = x^3 - 4x^2 + cx + 33$$

$$p(-3) = (-3)^3 - 4(-3)^2 + c(-3) + 33$$

$$0 = -27 - 4(9) - 3c + 33$$

$$0 = -27 - 36 - 3c + 33$$

$$0 = -30 - 3c$$

$$30 = -3c \quad \boxed{c = -10}$$



$$\begin{array}{r}
 5x^2 - 14x + 8 \\
 x+1 \overline{) 5x^3 - 9x^2 - 6x + 8} \\
 \underline{- 5x^3 - 5x^2} \quad \downarrow \\
 -14x^2 - 6x \\
 \underline{+ 14x^2 + 14x} \quad \downarrow \\
 8x + 8 \\
 \underline{- 8x - 8} \\
 0
 \end{array}$$

$$p(x) = (5x^2 - 14x + 8)(x+1)$$

$$a \cdot b = 5 \cdot 8 = 40$$

$$a + b = -14$$

$$-10 \cdot -4 = 40$$

$$-10 - 4 = -14$$

$$(5x^2 - 10x)(-4x + 8)$$

$$5x(x-2) - 4(x-2)$$

$$(5x-4)(x-2)$$



$$\begin{array}{r}
 x + 7 \\
 x-1 \overline{) x^2 + 6x - 4} \\
 \underline{- x^2 + x} \quad \downarrow \\
 7x - 4 \\
 \underline{- 7x + 7} \\
 3
 \end{array}$$

$$x + 7 + \frac{3}{x-1}$$



$$p(x) = (5x-4)(x-2)(x+1)$$

$$\begin{array}{r}
 x^2 + 4x + 4 \\
 x-1 \overline{) x^3 + 3x^2 + 0x - 4} \\
 \underline{-x^3 + x^2} \quad \downarrow \\
 4x^2 + 0x \\
 \underline{-4x + 4} \quad \downarrow \\
 4x - 4 \\
 \underline{-4x + 4} \\
 0
 \end{array}$$

$$p(x) = (x^2 + 4x + 4)(x-1)$$

$$p(x) = (x+2)(x+2)(x-1)$$

$$p(x) = 2x^3 - x^2 - 25x - 12$$

$$\begin{array}{r}
 2x^2 - 7x - 4 \\
 x+3 \overline{) 2x^3 - x^2 - 25x - 12} \\
 \underline{-2x^3 - 6x^2} \quad \downarrow \\
 -7x^2 - 25x \\
 \underline{+7x^2 + 21x} \quad \downarrow \\
 -4x - 12 \\
 \underline{+4x + 12} \\
 0
 \end{array}$$

$$p(x) = (2x^2 - 7x - 4)(x+3)$$

$$\begin{array}{r}
 x - 4 \\
 x+4 \overline{) x^2 + 0x + 1} \\
 \underline{-x^2 - 4x} \quad \downarrow \\
 -4x + 1 \\
 \underline{+4x + 16} \\
 17
 \end{array}$$

$$x - 4 + \frac{17}{x+4}$$

$$p(x) = (2x^2 - 7x - 4)(x + 3)$$

$$a \cdot b = 2 \cdot -4 = -8$$

$$a + b = -7$$

$$-8 \cdot 1 = -8$$

$$-8 + 1 = -7$$

$$(2x^2 - 8x) + (x - 4)$$

$$2x(x - 4) + 1(x - 4)$$

$$(2x + 1)(x - 4)$$

$$p(x) = (2x + 1)(x - 4)(x + 3)$$

$$\frac{x^2 - 9x + 14}{x - 5}$$

$$x - 5$$

$$x - 4$$

$$x - 5 \overline{) x^2 - 9x + 14}$$

$$\underline{-x^2 + 5x} \quad \downarrow$$

$$-4x + 14$$

$$\underline{+4x - 20}$$

$$-6$$

$$x - 4 - \frac{6}{x - 5}$$