$\log_2 8 = 3 \quad \log_8 2 = \frac{1}{3}$

$\log_2 \frac{1}{8} = x = -3 \quad \log_8 \frac{1}{2} = -\frac{1}{3}$

$8^x = 2$

$x = \frac{1}{3}$

$8^{\frac{1}{3}} = 2$

$8^{-\frac{1}{3}} = \frac{1}{8^{\frac{1}{3}}} = \frac{1}{2}$

$\log_{81} \frac{1}{27}$

$81^x = \frac{1}{27}$

$81^{\frac{1}{4}} = \sqrt[4]{81} = 3$

$\left(81^{\frac{1}{4}}\right) = \frac{1}{27}$

$\left(81^{\frac{1}{4}}\right)^x = \frac{1}{3^3}$

$\left(81^{\frac{1}{4}}\right)^{-3} = 3^{-3} \Rightarrow \frac{1}{4} \cdot -\frac{3}{4} = -\frac{3}{4} = 81^{-\frac{3}{4}} = \frac{1}{27}$ $\boxed{-\frac{3}{4}}$

$\log_{16}\left(\frac{1}{2}\right) = \boxed{-\frac{1}{4}}$

$2^4 = 16$

$2^{-4} = \frac{1}{16}$

$16^{\frac{1}{4}} = 2$

$16^{-\frac{1}{4}} = \frac{1}{2}$

$\log_{125}(5) = \boxed{\frac{1}{3}}$

$5^3 = 125$

$125^{\frac{1}{3}} = \sqrt[3]{125} = 5$

$\log_{\frac{1}{5}}(5) = \boxed{-1}$

$\left(\frac{1}{5}\right)^x = 5$

$5^1 = 5$

$\left(\frac{1}{5}\right)^{-1} = \left(\frac{5}{1}\right)^1 = 5^1 = 5$

$\log_8\left(\frac{1}{32}\right) = \boxed{-\frac{5}{3}}$

$8^x = \frac{1}{32}$

$8^{\frac{1}{3}} = \sqrt[3]{8} = 2$

$\frac{1}{32} = \frac{1}{2^5} = 2^{-5}$

$\left(8^{\frac{1}{3}}\right)^x = 2^{-5}$

$\left(8^{\frac{1}{3}}\right)^{-5} = 2^{-5}$

$8^{-\frac{5}{3}} = 2^{-5}$

$\log_{25}\left(\frac{1}{5}\right) = \boxed{-\frac{1}{2}}$

$25^x = \frac{1}{5}$

$25^{\frac{1}{2}} = 5$

$25^{-\frac{1}{2}} = \frac{1}{5}$

$\log_{81}(3) = \boxed{\frac{1}{4}}$

$3^4 = 81$

$81^{\frac{1}{4}} = \sqrt[4]{81} = 3$

$\log_{\frac{1}{2}}(32) = \boxed{-5}$

$2^5 = 32$

$\left(\frac{1}{2}\right)^{-5} = \left(\frac{2}{1}\right)^5 = 2^5 = 32$

$\log_5\left(\frac{1}{125}\right) = -3$

$\boxed{5^{-3} = \frac{1}{125}}$

$100 = 10^2$

$\boxed{\log_{10}(100) = 2}$

X	$y = b^x$
0	1
1	4
2	16

x	$y = \log_b x$
1	$\log_b(1) = 0$
4	$\log_b(4) = 1$
16	$\log_b(16) = 2$

$\log_b(x)$ FLIPS X and Y

$b^{1.585} = 3$

$a = 1, b = 2$

$c = \frac{3}{2}, d = \frac{1}{2}$

$\log_b(a) = 0$

$\log_b(2) = 1.0$

$b^1 = 2, \boxed{b = 2}$

$2^0 = a$

$2^0 = 1$ $\boxed{a = 1}$

$\log_2(2c) = 1.585$

$2^{1.585} = 2c$

$\frac{3}{2} = \frac{2c}{2}$

$\boxed{c = \frac{3}{2}}$

$\log_2(10d) = 2.322$

$2^{2.322} = 10d$

$\frac{5}{10} = \frac{10d}{10}$

$\boxed{d = \frac{1}{2}}$

$0.25 = 2^{-2}$

$\log_2(0.25) = -2$

$\log_8(512) = 3$

$8^3 = 512$

x	$\log_6(x)$
1	0
3	1
9	2
27	3

x	$y = b^x$
0	1
1	3
2	9
3	27

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e \approx 2.7182818$$

$$\log_5(m) \div \log_{15}(m)$$

$$\frac{\log(m)}{\log(5)} \div \frac{\log(m)}{\log(15)}$$

$$\frac{\cancel{\log(m)}}{\log(5)} \cdot \frac{\log(15)}{\cancel{\log(m)}}$$

$$\frac{\log(15)}{\log(5)}$$

$$\boxed{\log_5(15)}$$

$$\log_4(16) \cdot \log_2(c)$$

$$\frac{\log(16)}{\cancel{\log(4)}} \cdot \frac{\cancel{\log(c)}}{\log(2)}$$

$$\frac{\log(16)}{\log(2)}$$

$$\log_2(16) = \boxed{4}$$

$$\frac{\log(3)}{\log_n(3)} = \frac{\log(3)}{1} \div \frac{\log_n(3)}{1}$$

$$\frac{\log(3)}{1} \div \frac{\log(3)}{\log(n)}$$

$$\frac{\cancel{\log(3)}}{1} \cdot \frac{\log(n)}{\cancel{\log(3)}} = \boxed{\log(n)}$$

$$\log_3(a) \cdot \log(3)$$

$$\frac{\log(a)}{\cancel{\log(3)}} \cdot \frac{\cancel{\log(3)}}{1} = \boxed{\log(a)}$$

Change of base formula:

$$\log_a(X) = y$$

$$\log_b(a^y) = \log_b(x)$$

$$\frac{y\log_b(a)}{\log_b(a)} = \frac{\log(b^x)}{\log(b^a)}$$

$$\log_a(x) = \frac{\log_b(x)}{\log_b(a)}$$

$$\log_3(25) = \frac{\log_{10}(25)}{\log_{10}(3)}$$

$$y = \log_a(x) = \frac{\log_b(x)}{\log_b(a)}$$

- $\log_2(3a)$

$\log_2(3) + \log_2(a)$

- $\log_5(2y) + \log_5(8)$

$\log_5(2y \cdot 8) = \log_5(16y)$

- $\log_b\left(\frac{4}{c}\right)$

$\log_b(4) - \log_b(c)$

- $\log(3z) - \log(8)$

$\log\left(\frac{3z}{8}\right)$

- $\log_7(x^5)$

$5\log_7(x)$

- $6\ln(y)$

$\ln(y^6)$

- $\log_3(20)$

$\frac{\log(20)}{\log(3)} \approx 2.727$

- $\log_7(400)$

$\frac{\log(400)}{\log(7)} \approx 3.079$

$\log_4(0.3)$

$\frac{\log(0.3)}{\log(4)} \approx \boxed{-0.868}$

$10^{2t-3} = 7$

$\log_{10}(7) = 2t - 3$

$+3 \quad +3$

$\log_{10}(7) + 3 = \frac{2t}{2}$

$\boxed{\frac{\log_{10}(7)+3}{2} = t}$

$\frac{2 \cdot 6^x}{2} = \frac{236}{2}$

$6^x = 118$

$\boxed{\log_6(118) = x}$

$\frac{6 \cdot e^y}{6} = \frac{300}{6}$

$e^y = 50$

$\log_e(50) = y$

$\boxed{3.912 \approx y}$

$\frac{5 \cdot 3^t}{5} = \frac{20}{5}$

$3^t = 4$

$\log_3(4) = t$

$\frac{\log(4)}{\log(3)}$

$\boxed{\approx 1.262}$

$$\frac{3 \cdot 10^{4t}}{3} = \frac{522}{3}$$

$$10^{4t} = 174$$

$$\frac{\log_{10}(174)}{4} = \frac{4t}{4}$$

$$\boxed{t = \frac{\log_{10}(174)}{4}}$$

$$\frac{4 \cdot 5^{2x}}{4} = \frac{300}{4}$$

$$5^{2x} = 75$$

$$\log_5(75) = 2x$$

$$x = \frac{\log_5(75)}{2} \approx \boxed{1.341}$$

$$\frac{-2 \cdot 3^{0.2z}}{-2} = \frac{-400}{-2}$$

$$3^{0.2z} = 200$$

$$\frac{\log_3(200)}{0.2} = \frac{0.2z}{0.2}$$

$$\frac{\log_3(200)}{0.2} \approx \boxed{24.114}$$

$(2^x - 3)(2^x - 4) = 0$

$2^x - 3 = 0$ $\quad$ $2^x - 4 = 0$

$+4$

$2^x = 3$ $\quad$ $2^x = 4$

$\log_2(3) = x$ $\quad$ $\log_2(4) = x$

$2 = x$

$\frac{-16 \cdot 10^{6x}}{-16} = \frac{-80}{-16}$

$10^{6x} = 5$

$\frac{\log_{10}(5)}{6} = \frac{6x}{6}$

$x = \frac{\log_{10}(5)}{6}$

$x \approx 0.116$

$\frac{11 \cdot 10^{5t}}{11} = \frac{20}{11}$

$10^{5t} = \frac{20}{11}$

$\frac{\log_{10}\left(\frac{20}{11}\right)}{5} = \frac{5t}{5}$

$t = \frac{\log_{10}\left(\frac{20}{11}\right)}{5} \approx 0.052$

$\frac{0.5 \cdot e^{4x}}{0.5} = \frac{13}{0.5}$

$e^{4x} = 26$

$\frac{\log_e(26)}{4} = \frac{4x}{4}$

$\frac{\log_e(26)}{4} \approx 0.815$

$$\frac{5 \cdot e^{-7x}}{5} = \frac{12}{5}$$

$$e^{-7x} = \frac{12}{5}$$

$$\frac{\log_e\left(\frac{12}{5}\right)}{-7} = \frac{-7x}{-7}$$

$$x = \frac{\log_e\left(\frac{12}{5}\right)}{-7}$$

$$x \approx \boxed{-0.125}$$

$$y = 5 \cdot 2^t$$

$$\frac{5 \cdot 2^t}{5} = \frac{1{,}111}{5}$$

$$2^t = \frac{1{,}111}{5}$$

$$\log_2\left(\frac{1{,}111}{5}\right) = t$$

$$\frac{\log_{10}\left(\frac{1{,}111}{5}\right)}{\log_{10}(2)} = t$$

$$\boxed{t \approx 7.796}$$

$$a^b = c$$

$$\log_a(c) = b$$

$$\frac{-100 \cdot 2^{0.5y}}{-100} = \frac{-5}{-100}$$

$$2^{0.5y} = \frac{1}{20}$$

$$\log_2\left(\frac{1}{20}\right) = 0.5y$$

$$\frac{\log_2\left(\frac{1}{20}\right)}{0.5}$$

$$\frac{500 \cdot 5^{\frac{y}{3}}}{500} = \frac{1}{500}$$

$$5^{y/3} = \frac{1}{500}$$

$$\log_5\left(\frac{1}{500}\right) = \frac{y}{3} \cdot 3$$

$$3\log_5\left(\frac{1}{500}\right) = y$$

$$y = -11.584$$

$$\frac{3 \cdot 2^{\frac{x}{5}}}{3} = \frac{150}{3}$$

$$2^{\frac{x}{5}} = 50$$

$$\log_2(50) = \frac{x}{5} \cdot 5$$

$$5\log_2(50) = x$$

$$4\log_2(37) = x$$

$$x \approx 20.838$$

$$\frac{6 \cdot 2^{\frac{x}{4}}}{6} = \frac{222}{6}$$

$$2^{\frac{x}{4}} = 37$$

$$\log_2(37) = \frac{x}{4} \cdot 4$$

$M(t) = 20 \cdot e^{-0.8t}$

$\frac{1}{20} = \frac{20 \cdot e^{-0.8t}}{20}$

$e^{-0.8t} = \frac{1}{20}$

$\frac{\log_e\left(\frac{1}{20}\right)}{-0.8t} = \frac{-0.8t}{-0.8t}$

$t = \frac{\log_e\left(\frac{1}{20}\right)}{-0.8}$

$\boxed{t \approx 3.74}$

$B(t) = 10 \cdot 2^{\frac{t}{12}}$

$B(120) = 10 \cdot 2^{\frac{120}{12}}$

$B(120) = 10 \cdot 2^{10}$

$= 10 \cdot 1024$

$= \boxed{10,240}$

$A = 2000 e^{0.1t}$

$\frac{2450}{2000} = \frac{2000 e^{0.1t}}{2000}$

$\frac{49}{40} = e^{0.1t}$

$\log_e\left(\frac{49}{40}\right) = \frac{0.1t}{0.1} \Rightarrow \boxed{t = \frac{\log_e\left(\frac{49}{40}\right)}{0.1}}$

$B(t) = 2500 \cdot 2^{0.01t}$

$B(25) = 2500 \cdot 2^{0.01(25)}$

$= 2500 \cdot 2^{0.25}$

$\approx \boxed{2973}$

$P(t) = 12{,}000 \cdot 2^{-\frac{t}{15}}$

$\frac{9000}{12000} = \frac{12000 \cdot 2^{-\frac{t}{15}}}{12000}$

$\frac{3}{4} = 2^{-\frac{t}{15}}$

$\log_2\left(\frac{3}{4}\right) = -\frac{t}{15}$

$-15 \log_2\left(\frac{3}{4}\right) = t$

$t = -15 \cdot \frac{\log\left(\frac{3}{4}\right)}{\log(2)} \approx \boxed{6.23}$

$V = 22500 \cdot 10^{-\frac{t}{12}}$

$\frac{10000}{22500} = \frac{22500 \cdot 10^{-\frac{t}{12}}}{22500}$

$\frac{4}{9} = 10^{-\frac{t}{12}}$

$\log_{10}\left(\frac{4}{9}\right) = -\frac{t}{12}$

$-12 \log_{10}\left(\frac{4}{9}\right) = t$

▶ $\log_a(18) \cdot \log_3(a)$

$$\frac{\log(18)}{\log(a)} \cdot \frac{\log(a)}{\log(3)}$$

$$\frac{\log(18)}{\log(3)}$$

$$\boxed{\log_3(18)}$$

▶ $B(t) = 2500 \cdot e^{0.025t}$

$$B(4) = 2500 \cdot e^{0.025(4)}$$

$$= 2500 \cdot e^{0.1}$$

$$= \boxed{2762.93}$$

▶ $\frac{5 \cdot 2^{-3t}}{5} = \frac{45}{5}$

$$2^{-3t} = 9$$

$$\frac{\log_2(9)}{-3} = \frac{-3t}{-3}$$

$$\boxed{t = -\frac{\log_2(9)}{3}}$$

$$\frac{0.5 \cdot 10^{8t}}{0.5} = \frac{73}{0.5}$$

$$10^{8t} = 146$$

$$\frac{\log_{10}(146)}{8} = \frac{8t}{8}$$

$$\boxed{t = \frac{\log_{10}(146)}{8}}$$

$$\boxed{t = 0.271}$$

$$5\log_2(48)$$

$$\log_2(48^5)$$

$$\frac{\log(48^5)}{\log(2)} \approx \boxed{27.925}$$

$$\frac{20 \cdot 7^{3y}}{20} = \frac{5}{20}$$

$$7^{3y} = \frac{1}{4}$$

$$\frac{\log_7\left(\frac{1}{4}\right)}{3} = \frac{3y}{3}$$

$$\frac{\left(\frac{\log\left(\frac{1}{4}\right)}{\log(7)}\right)}{3} = y \rightarrow y \approx \boxed{-0.237}$$

$3\log(2)$

$\log(2^3)$

$\boxed{\log(8)}$

$M(t) = 20 \cdot e^{-0.8t}$

$\frac{1}{20} = \frac{20 \cdot e^{-0.8t}}{20}$

$e^{-0.8t} = \frac{1}{20}$

$\frac{\ln\left(\frac{1}{20}\right)}{-0.8} = \frac{-0.8t}{-0.8}$

$\frac{\ln\left(\frac{1}{20}\right)}{-0.8} = t$

$\boxed{t \simeq 3.74}$

$$\frac{9 \cdot e^{2z}}{9} = \frac{54}{9}$$

$$e^{2z} = 6$$

$$\frac{\ln(6)}{2} = \frac{2z}{2}$$

$$z = \frac{\ln(6)}{2}$$

$$z \approx 0.896$$

$$\log_{16}(4) = \frac{1}{2}$$

$$4^2 = 16$$

$$16^{\frac{1}{2}} = \sqrt{16} = 4$$

$$\log_7(a) \cdot \log_b(7)$$

$$\frac{\log(a)}{\log(7)} \cdot \frac{\log(7)}{\log(b)}$$

$$\frac{\log(a)}{\log(b)} = \log_b(a)$$

- $6 \log_7(681)$

$$6 \cdot \frac{\log(681)}{\log(7)}$$

$$6 \cdot 3.35244785$$

$$\boxed{20.115}$$

- $0.01 = 10^{-2}$

$$\log_{10}(0.01) = -2$$

- $\frac{5 \cdot 7^{2y}}{5} = \frac{175}{5}$

$$7^{2y} = 35$$

$$\frac{\log_7(35)}{2} = \frac{2y}{2}$$

$$y = \frac{\log_7(35)}{2}$$