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Assignment 2

Exercise 1

(a) Find a Θ evaluation for the function $(4n+1)8^{\log(n^2)}$.

$\Theta(n^7)$

(b) Give an example of two functions $t_1(n)$ and $t_2(n)$ that satisfy the relations: $t_1(n) = \Theta(n^2)$, $t_2(n) = \Theta(n^2)$ and $t_1(n) - t_2(n) = o(n^2)$.

$t_1(n) = 3n^2 + 3n$

$t_2(n) = 3n^2 - 2$

$3n^2 - 3n^2 + 3n - 2$

$3n - 2$

(c) Give an example of a function $t_1(n)$ such that $t_1(n) = \Theta(t_1(2n))$.

$t_1(n) = n^2 + n - 10$

$t_1(2n) = 2n^2 + 2n - 10$

$t_1(2n) = \Theta(t_1(2n))$

(d) Give an example of a function $t_2(n)$ such that $t_2(n) = o(t_2(2n))$.

$t_2(n) = n^n$

$t_2(2n) = 2n^{2n}$

$2n^{2n} > n^n$

Exercise 2

Indicate whether A is O, o, Ω, ω, or Θ of B. Answer should be "yes" or "no" in each box. Assume that $k \geq 1$, $\varepsilon > 1$, and $c > 1$ are constants.

A	B	O	o	Ω	ω	Θ
$\log^k(n)$	n^ε	yes	yes	no	no	no
n^k	c^n	no	no	yes	yes	no
2^n	$2^{n/2}$	no	no	yes	yes	no
$n^{\log c}$	$c^{\log n}$	yes	no	yes	no	yes
$\log n!$	$\log n^n$	yes	no	yes	no	yes

Exercise 3

For each of the following program fragments, give a $\Theta(\cdot)$ estimation of the running time as a function of n.

(a)
```
sum = 0;
for (int i = 0; i< n * n; i++) {
    for(int j =0;  j < n/2; j++)
        sum++;
}
```

(b)
```
sum = 0;
for (int i = 0; i< n; i++) {
    sum++;}

for(int j = 0;  j < n/2; j++){
    sum++;}
```

(c)
```
sum = 0;
for (int i = 0; i< n * n; i++) {
    for(int j = 0;  j < n * n; j++)
        sum++
}
```

(d)
```
sum = 0;
for (int i = 1; i< n; i = 2*i)
        sum++
```

(e)
```
sum = 0;
for (int i = 0; i< n; i++) {
    for(int j = 1;  j < n * n; j = 2*j)
        sum++
}
```

(a) $\Theta(n^2)$

(b) $\Theta(n)$

(c) $\Theta(n^2)$

(d) $\Theta(\log n)$

(e) $\Theta(n \log n)$

Exercise 4

(a) Compute the sum $S_1 = 500 + 501 + 502 + 503 + \ldots + 999$ (the sum of all integers from 500 to 999).

$$N = 500$$

$$S_1 = \frac{N(500 + 999)}{2}$$

$$= 374,750$$

(b) Compute the sum $S_2 = 1 + 3 + 5 + \ldots + 999$ (the sum of all odd integers from 1 to 999).

$$S_2 = \frac{500(1 + 999)}{2}$$

$$= 250,000$$

(c) A group of 30 persons need to form a committee of 3 person. How many such committees are possible?

$$\binom{30}{3} = \frac{30!}{(30-3)! \cdot 3!}$$

$$= 4,060$$

4,060 committees are possible.

(d) Let C_n be the number of committees of 4 persons selected from a group of n persons. Is the estimation $C_n = o(n^3)$ correct? Justify your answer.

$$\binom{n}{4} = \frac{n!}{(n-4)! \cdot 4!}$$

$$= \frac{n(n-1)(n-2)(n-3)\cancel{(n-4)(n-5)}\ldots}{\cancel{(n-4)(n-5)}\ldots \cdot 4!}$$

$$= \frac{n(n-1)(n-2)(n-3)}{4!}$$

$$= \Theta(n^4)$$

The estimation of $C_n = o(n^3)$ is incorrect since $n^4 > n^3$.

Exercise 5

Find a $\Theta(\cdot)$ evaluation for the sum

$$S = 1\sqrt{1} + 2\sqrt{2} + \ldots + n\sqrt{n}.$$

Find a function f such that $S = \Theta(f(n))$

The summation is monotonically increasing, so:

$$\int_0^n f(x)\,dx \leq S_n \leq \int_1^{n+1} f(x)\,dx$$

Lower bound

$$\int_0^n x\sqrt{x}\,dx = \int_0^n x^{3/2}\,dx$$

$$= \frac{x^{(3/2)+1}}{3/2+1}$$

$$= \frac{2}{5}x^{5/2}\Big|_0^n$$

$$= \frac{2}{5}n^{5/2} - 0$$

$$= \Theta(n^{5/2})$$

Upper bound

$$\int_1^{n+1} x\sqrt{x}\,dx = \frac{2}{5}x^{5/2}\Big|_1^{n+1}$$

$$= \frac{2}{5}(n+1)^{5/2} - \frac{2}{5}(1)^{5/2}$$

$$= O(n^{5/2})$$

$$\therefore \Theta(n^{5/2})$$