

Luis Gascon, Ethan Webb, Femi Dosumu

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Assignment 2

Exercise 1

- (a) Find a Θ evaluation for the function $(4n + 1)8^{\log(n^2)}$.

$$\Theta(n^7)$$

- (b) Give an example of two functions $t_1(n)$ and $t_2(n)$ that satisfy the relations:
 $t_1(n) = \Theta(n^2)$, $t_2(n) = \Theta(n^2)$ and $t_1(n) - t_2(n) = o(n^2)$.

$$t_1(n) = 3n^2 + 3n$$

$$t_2(n) = 3n^2 - 2$$

$$3n^2 - 3n^2 + 3n - 2$$

$$3n - 2$$

- (c) Give an example of a function $t_1(n)$ such that $t_1(n) = \Theta(t_1(2n))$.

$$t_1(n) = n^2 + n - 10$$

$$t_1(2n) = 2n^2 + 2n - 10$$

$$t_1(2n) = \Theta(t_1(2n))$$

- (d) Give an example of a function $t_2(n)$ such that $t_2(n) = o(t_2(2n))$.

$$t_2(n) = n^n$$

$$t_2(2n) = 2n^{2n}$$

$$2n^{2n} > n^n$$

Exercise 2

Indicate whether A is O , o , Ω , ω , or Θ of B . Answer should be “yes” or “no” in each box. Assume that $k \geq 1$, $\varepsilon > 1$, and $c > 1$ are constants.

A	B	O	o	Ω	ω	Θ
$\log^k(n)$	n^ε	yes	yes	no	no	no
n^k	c^n	no	no	yes	yes	no
2^n	$2^{n/2}$	no	no	yes	yes	no
$n^{\log c}$	$c^{\log n}$	yes	no	yes	no	yes
$\log n!$	$\log n^n$	yes	no	yes	no	yes

Exercise 3

For each of the following program fragments, give a $\Theta(\cdot)$ estimation of the running time as a function of n .

- (a)

```
sum = 0;
for (int i = 0; i < n * n; i++) {
    for(int j = 0; j < n/2; j++)
        sum++;
}
```
- (b)

```
sum = 0;
for (int i = 0; i < n; i++) {
    sum++;}

for(int j = 0; j < n/2; j++){
    sum++;}
```
- (c)

```
sum = 0;
for (int i = 0; i < n * n; i++) {
    for(int j = 0; j < n * n; j++)
        sum++
}
```
- (d)

```
sum = 0;
for (int i = 1; i < n; i = 2*i)
    sum++
```
- (e)

```
sum = 0;
for (int i = 0; i < n; i++) {
    for(int j = 1; j < n * n; j = 2*j)
        sum++
}
```

- (a) $\Theta(n^2)$
(b) $\Theta(n)$
(c) $\Theta(n^2)$
(d) $\Theta(\log n)$
(e) $\Theta(n \log n)$

Exercise 4

- (a) Compute the sum $S_1 = 500 + 501 + 502 + 503 + \dots + 999$ (the sum of all integers from 500 to 999).

$$\begin{aligned} N &= 500 \\ S_1 &= \frac{N(500 + 999)}{2} \\ &= 374,750 \end{aligned}$$

- (b) Compute the sum $S_2 = 1 + 3 + 5 + \dots + 999$ (the sum of all odd integers from 1 to 999).

$$\begin{aligned} S_2 &= \frac{500(1 + 999)}{2} \\ &= 250,000 \end{aligned}$$

- (c) A group of 30 persons need to form a committee of 3 person. How many such committees are possible?

$$\begin{aligned} \binom{30}{3} &= \frac{30!}{(30-3)! \cdot 3!} \\ &= 4,060 \end{aligned}$$

4,060 committees are possible.

- (d) Let C_n be the number of committees of 4 persons selected from a group of n persons. Is the estimation $C_n = o(n^3)$ correct? Justify your answer.

$$\begin{aligned} \binom{n}{4} &= \frac{n!}{(n-4)! \cdot 4!} \\ &= \frac{n(n-1)(n-2)(n-3)\cancel{(n-4)}\cancel{(n-5)} \dots}{\cancel{(n-4)}\cancel{(n-5)} \dots \cdot 4!} \\ &= \frac{n(n-1)(n-2)(n-3)}{4!} \\ &= \Theta(n^4) \end{aligned}$$

The estimation of $C_n = o(n^3)$ is incorrect since $n^4 > n^3$.

Exercise 5

Find a $\Theta(\cdot)$ evaluation for the sum

$$S = 1\sqrt{1} + 2\sqrt{2} + \dots + n\sqrt{n}.$$

Find a function f such that $S = \Theta(f(n))$

The summation is monotonically increasing, so:

$$\int_0^n f(x) dx \leq S_n \leq \int_1^{n+1} f(x) dx$$

Lower bound

$$\begin{aligned} \int_0^n x\sqrt{x} dx &= \int_0^n x^{3/2} dx \\ &= \frac{x^{(3/2)+1}}{3/2+1} \\ &= \frac{2}{5} x^{5/2} \Big|_0^n \\ &= \frac{2}{5} n^{5/2} - 0 \\ &= \Theta(n^{5/2}) \end{aligned}$$

Upper bound

$$\begin{aligned} \int_1^{n+1} x\sqrt{x} dx &= \frac{2}{5} x^{5/2} \Big|_1^{n+1} \\ &= \frac{2}{5} (n+1)^{5/2} - \frac{2}{5} (1)^{5/2} \\ &= O(n^{5/2}) \end{aligned}$$

$$\therefore \Theta(n^{5/2})$$