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Assignment 4

Exercise 1

For each of the following functions, give a $\Theta(t(n))$ estimation with the simplest possible $t(n)$.

1. $13n^2 - 2n + 56$
2. $2.5 \log(n) + 2$
3. $n(12 + \log n)$
4. $1 + 2 + 3 + \ldots + 2n$
5. $1 + 2 + 3 + \ldots + n^2$
6. $\log(n^3) + 10$
7. $\log(n^3) + n \log n$
8. $n \log(n^3) + n \log n$
9. $2^{2 \log n} + 5n + 1$

Exercise 2

1. Evaluate the following postfix arithmetic expression: $1034 - 5 * /$
2. Convert the following infix arithmetic expression to postfix notation: $(((2+3)*5)-15)$

Exercise 3

Consider the following algorithms A and B for the problem of computing $2^n \pmod{317}$

```
Algorithm A.

mod_exp_A(n) {
   if (n == 0) return 1;
   else {
          t = mod_exp_A(n/2);
          if (n is even) return t*t (mod 317);
          if (n is odd) return t*t*2 (mod 317);
}}
```

```
Algorithm B.

mod_exp_B(n) {
   if (n == 0) return 1;
   else {
          if (n is even)
            return mod_exp_B(n/2) * mod_exp_B(n/2) (mod 317);
          if (n is odd)
            return mod_exp_B(n/2) * mod_exp_B(n/2) *2 (mod 317);
}}
```

1. Write the reucrrence for the runtime $T_A(n)$ of algorithm A and solve the recurrence to find a $\Theta(\cdot)$ estimation of $T_A(n)$
2. Write the recurrence for the runtime $T_B(n)$ of algorithm B, and solve the recurrence to find a $\Theta(\cdot)$ estimation of $T_B(n)$.
3. Which algorithm is faster?

Exercise 4

Give a $\Theta(\cdot)$ evaluation for the runtime of the following code:

```
i= 1; x=0;
while(i <= n) {
       j=1;
       while (j <= i) { x=x+1; j= 2*j; }
       i= 2*i;
}
```

Assume that n is a power two. Then i from the outer loop takes successively the values: 1, 2, 2^2, 2^3, $\ldots, 2^{\log n}$