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Assignment 7

Exercise 1. Show similarly to Fig 8.3 on page 198 in the textbook, how RadixSort sorts the following arrays:

1. 34, 9134, 20134, 29134, 4, 134
2. 4, 34, 134, 9134, 20134, 29134
3. 29134, 20134, 9134, 134, 34, 4

Exercise 2. Present an $O(n)$ algorithm that sorts n positive integer numbers $a_1, a_2, \ldots, a_n$ which are known to be bounded by $n^2 - 1$ (so $0 \leq a_i \leq n^2 - 1$, for every $i = 1, \ldots, n$. Use the idea of textbook).

Note that in order to obtain $O(n)$ you have to do Radix Sort by writing the numbers in a suitable base. Recall that the runtime of Radix Sort is $O(d(n + k))$, where d is the number of digits, and k is the base, so that the number of digits in the base is also k. The idea is to represent each number in a base k chosen so that each number in $\{0, 1, \ldots, n^2 - 1\}$ requires only 2 "digits," so $d = 2$. Explain what is the base that you choose and how the digits of each number are calculated, in other words how you convert from base 10 to the base. Note that you cannot use the base 10 representation, because $n^2 - 1$ (which is the largest possible value) requires $\log_{10}(n^2 - 1)$ digits in base 10, which is obviously not constant and therefore you would not obtain an $O(n)$-time algorithm. By the same argument we see that no base k that is constant works, therefore k has to depend on n. In your explanations you need to indicate the formula that gives k as a function of n, and show that $d = 2$ "digits" are enough to represent all the numbers in the range $\{0, 1, \ldots, n^2 - 1\}$.

Illustrate your algorithm by showing on paper similar to Fig. 8.3, page 198 in the textbook (make sure you indicate clearly the columns) how the algorithm sorts the following sequence of 12 positive integers:

45, 98, 3, 82, 132, 71, 72, 143, 91, 28, 7, 45.

In this example $n = 12$, because there are 12 positive numbers in the sequence bounded by $143 = 12^2 - 1$.