Problem 38.
Use mathematical induction to verify the correctness of the formula obtained.
$t_k = t_{k-1} + 3k + 1$, for each integer $k \geq 1$
$t_0 = 0$

Proof. (by induction)
Let $P(n)$: $t_n = n + \dfrac{3n(n+1)}{2}$ for $n \geq 0$

$$P(n) \longrightarrow \qquad t_n = n + \frac{3n(n+1)}{2}$$

Basis step: $P(0) = 0 + \dfrac{3(0)(0+1)}{2}$. True since $t_0 = 0$

Inductive step: Let $k \in \mathbb{Z} \ni k \geq 0$. Assume $P(k)$, that is $t_k = k + \dfrac{3k(k+1)}{2}$

[**NTS:** $P(k+1)$: $t_{k+1} = (k+1) + \dfrac{3(k+1)(k+1+1)}{2}$]

$$\begin{aligned}
t_{k+1} &= t_k + 3(k+1) + 1 \\
&= k + \frac{3k(k+1)}{2} + 3(k+1) + 1 && \text{by substitution} \\
&= (k+1) + \frac{3k(k+1)}{2} + \frac{6(k+1)}{2} && \text{by algebra} \\
&= (k+1) + \frac{(k+1)}{2} \cdot 3(k+2) \\
&= (k+1) + \frac{3(k+1)(k+2)}{2}
\end{aligned}$$

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