

Problem 23.

True or false?

If k is any even integer and m is any odd integer, then $(k + 2)^2 - (m - 1)^2$ is even. Explain.

False.

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| 1. The sum, product, and difference of any two even integers are even. | 5. The sum of any odd integer and any even integer is odd. |
| 2. The sum and difference of any two odd integers are even. | 6. The difference of any odd integer minus any even integer is odd. |
| 3. The product of any two odd integers is odd. | 7. The difference of any even integer minus any odd integer is odd. |
| 4. The product of any even integer and any odd integer is even. | |

Proof. Let k be an even integer and m be an odd integer.

- $k + 2$ is even since k and 2 are even and the sum of even integers is even by 1.)
- $(k + 2)^2$ is even since $k + 2$ is even and the product of even integers is even by 1.)
- $m - 1$ is even since m and 1 are odd and the sum and difference of any two odd integers are even by 2.)
- $(m - 1)^2$ is odd since $m - 1$ is odd and the product of odd integers is odd by 3.)
- $(k + 2)^2 - (m - 1)^2$ is odd since $(k + 2)^2$ is even and $(m - 1)^2$ is odd and the difference of any even integer minus any odd integer is odd by 7.)

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