

**Problem 28.**

**Theorem:** For all integers  $a, b$ , and  $c$ , if  $a \mid bc$  then  $a \mid b$  or  $a \mid c$ .

*Proof.* Let  $a, b, c \in \mathbb{Z} \ni a \mid bc$   
 Since  $a \mid bc$ ,  $\exists d \in \mathbb{Z} \ni ad = bc$

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**Problem 29.**

**Theorem:** For all integers  $a$  and  $b$ , if  $a \mid b$  then  $a^2 \mid b^2$ .

*Proof.* Let  $a, b \in \mathbb{Z} \ni a$  divides  $b$   
 By definition of divisibility,

$$\exists k \in \mathbb{Z} \ni b = ak$$

$$\begin{aligned} b^2 &= (ak)^2 && \text{by substitution} \\ &= (ak)(ak) && \text{by algebra} \\ &= a(kak) && \text{by associative property} \end{aligned}$$

Let  $t = (kak)$ ,  $t \in \mathbb{Z}$  since  $a, k \in \mathbb{Z}$

The sum and products of integers is an integer.

Therefore,  $a^2 \mid b^2$  by definition of divisibility since  $b^2 = at$  where  $t \in \mathbb{Z}$  by substitution

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