

Problem 28.

For all integers a, b , and c , if $a \mid bc$ then $a \mid b$ or $a \mid c$.

False

Counter-example. Let $a = 4, b = 6, c = 10$

$$a \mid bc = 4 \mid 60$$

$$4 \nmid 6 \text{ \& } 4 \nmid 10$$

\therefore by counter-example, the statement is false. ■

Problem 29.

Theorem: For all integers a and b , if $a \mid b$ then $a^2 \mid b^2$.

Proof. Let $a, b \in \mathbb{Z} \ni a$ divides b

By definition of divisibility, $\exists k \in \mathbb{Z} \ni b = ak$

$$b^2 = a^2k^2 \qquad \text{by substitution}$$

Let $t = (k^2), t \in \mathbb{Z}$ by the closure of integers by multiplication

Therefore, $a^2 \mid b^2$ by definition of divisibility since $b^2 = a^2t$ where $t \in \mathbb{Z}$ by substitution ■