## Problem 13.

S = The product of any irrational number and any nonzero rational number is irrational

- a. Write a negation for S.
- b. Prove S by contradiction.

 $\exists d \notin \mathbb{Q} \text{ and } f \in \mathbb{Q} \ni d \cdot f \in \mathbb{Q}$ 

*Proof.* (by contradiction) Suppose not.

Then, there exists an irrational number, call it d and a nonzero rational number, call it f such that the product of d and f is a rational.

By definition of rational,

f = h/j and  $d \cdot f = k/l$  for some integers h, j, k, and l with  $h = j \neq 0$  and  $l \neq 0$  since f is a nonzero rational number.

$$d \cdot \frac{h}{j} = \frac{k}{l}$$
 by substitution 
$$d = \frac{kj}{lh}$$
 by algebra

 $lh \neq 0$  by zero-product property so d is a ratio of integers with a nonzero denominator. So  $d \in \mathbb{Q}$ , hence the supposition is false and the given statement is true

 $\Rightarrow \Leftarrow$ 

## Problem 22.

**Theoerm:** For every real number r, if  $r^2$  is irrational then r is irrational.

Write what you would suppose and what you would need to show to prove this statement by:

a. contradiction

Suppose not.

That is suppose  $\exists r \in \mathbb{R} \ni r^2 \notin \mathbb{Q}$  and  $r \in \mathbb{Q}$ .

We need to show that  $r \in \mathbb{Q}$ .

b. contraposition

Suppose  $r \in \mathbb{Q}$ .

We need to show that  $r^2 \in \mathbb{Q}$ .

## Problem 24.

Prove by contraposition and contradiction

**Theoerm:** The reciprocal of any irrational number is irrational.

*Proof.* (by contradiction) Suppose not.

Then  $\exists k \notin \mathbb{Q} \ni \frac{1}{k} \in \mathbb{Q}$ 

By definition of a rational,  $1/k = a/b \ni a, b \in \mathbb{Z}$  with  $b \neq 0$ 

$$\frac{1}{k}(b) = \frac{a}{b}(b)$$
 by algebra 
$$\frac{b}{k} = a$$
 
$$k = \frac{b}{a}$$

Since b and k are both non zeroes, this implies that a is non zero a and b are both integers and k is a ratio of integers with a nonzero denominator. Hence k is a rational.

: the supposition is false and the given statement is true

 $\Rightarrow \leftarrow$ 

*Proof.* (by contraposition) Let  $r \in \mathbb{R} \ni \frac{1}{r} \in \mathbb{Q}$ 

By definition of rational,  $r = \frac{a}{b}$  for some integer a and b with  $b \neq 0$ 

$$\frac{1}{r} = \frac{1}{a/b}$$
 
$$r = \frac{b}{a}$$
 by algebra

a and b are both integers and r is a ratio of integers with a nonzero denominator, Hence r is a rational.