Problem 11.
Use an element argument to prove each statement.
Assume that all sets are subsets of universal set U.

For all sets $A, B,$ and C,
$A \cap (B \setminus C) \subseteq (A \cap B) \setminus (A \cap C)$.

Proof. Let A, B and C be sets. Let $x \in A \cap (B \setminus C)$.
Then by definition of intersection, $x \in A$ and $x \in (B \setminus C)$.
$x \in (B \setminus C)$ so $x \in B$ and $x \notin C$ by definition of set difference.
Since $x \in A$ and $x \in B$, $x \in (A \cap B)$ by definition of intersection.
$x \notin (A \cap C)$ since $x \in A$ and $x \notin C$.
Since $x \in (A \cap B)$ and $x \notin (A \cap C)$ by definition of set difference.
Hence $x \in (A \cap B) \setminus (A \cap C)$
$\therefore A \cap (B \setminus C) \subseteq (A \cap B) \setminus (A \cap C)$.

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Problem 29.
Use the element method for proving a set equals the empty set to prove each statement.
Assume that all sets are subsets of a universal set U.

For all sets $A, B,$ and C,
$(A \setminus C) \cap (B \setminus C) \cap (A \setminus B) = \varnothing$

Proof. (by contradiction): Let A, B and C be sets.
Suppose not. That is, suppose $\exists x \in (A \setminus C) \cap (B \setminus C) \cap (A \setminus B)$
Then $x \in (A \setminus C)$ and $x \in (B \setminus C)$ and $x \in (A \setminus B)$ by definition of intersection.
So $x \in A$ and $x \in B$ and $x \notin C$ and $x \notin B$ by definition of set difference.
$\therefore x \in B$ and $x \notin B$, which is a contradiction.
So $(A \setminus C) \cap (B \setminus C) \cap (A \setminus B) = \varnothing$
$\Rightarrow\Leftarrow$

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