Problem 6a & 6b

$$\begin{aligned}\text{Let } A &= \{x \in \mathbb{Z} \mid x = 5a + 2 \text{ for some integer } a\},\\ B &= \{y \in \mathbb{Z} \mid y = 10b - 3 \text{ for some integer } b\},\\ C &= \{z \in \mathbb{Z} \mid z = 10c + 7 \text{ for some integer } c\}.\end{aligned}$$

Prove or disprove each of the following statements.
a. $A \subseteq B$ *False*

Disproof. $12 \in A$ since $12 = 5(2) + 2$ but $12 \notin B$
since $12 = 10b - 3 \iff 15 = 10b$ for some integer b but $10 \nmid 15$. ∎

b. $B \subseteq A$ *True*

Proof. Let $x \in B$ then $x \in \mathbb{Z} \ni x = 10b - 3$ for some integer b.

$$\begin{aligned} x &= 10b - 3 \\ &= 5(2b-1) + 2 && \text{by algebra}\end{aligned}$$

Let $a = 2b - 1$. $a \in \mathbb{Z}$ by the closure of integers by addition and multiplication.
So $x = 5a + 2$ where $a \in \mathbb{Z}$.
$\therefore x \in A$. ∎

Problem 32b

Suppose $X = \{a, b\}$ and $Y = \{x, y\}$. Find $\mathcal{P}(X \times Y)$

$X \times Y = \{(a,x), (a,y), (b,x), (b,y)\}$

$$\begin{aligned}\mathcal{P}(X \times Y) = \{&\emptyset,\ \{(a,\ x)\},\ \{(a,\ y)\},\ \{(b,\ x)\},\ \{(b,\ y)\},\\ &\{(a,\ y),\ (a,\ x)\},\ \{(b,\ x),\ (a,\ x)\},\\ &\{(b,\ y)\},\ (a,\ x),\ \{(b,\ x),\ (a,\ y)\},\\ &\{(b,\ y),\ (a,\ y)\},\ \{(b,\ y),\ (b,\ x)\},\\ &\{(b,\ x),\ (a,\ y),\ (a,\ x)\},\\ &\{(b,\ y),\ (a,\ y),\ (a,\ x)\},\\ &\{(b,\ y),\ (b,\ x),\ (a,\ x)\},\\ &\{(b,\ y),\ (b,\ x),\ (a,\ y)\},\\ &\{(a,\ x),\ (a,\ y),\ (b,\ x),\ (b,\ y)\}\end{aligned}$$