Prove the each statements. Use the theorem on polynomial orders and results from the theorems and exercises in Section 5.2

Problem 33.
$1^3 + 2^3 + 3^3 + \cdots + n^3$ is $\theta(n^4)$

Proof.

$$\begin{aligned} 1^3 + 2^3 + 3^3 + \cdots + n^3 &= \left[\frac{n(n+1)}{2}\right]^2 \\ &= \left(\frac{n^2+n}{4}\right)\left(\frac{n^2+n}{4}\right) \\ &= \frac{n^4+2n^3+n^2}{16} \end{aligned}$$

$\frac{n^4+2n^3+n^2}{16}$ is $\theta(n^4)$ by the theorem on polynomial orders. ∎

Problem 35.
$5 + 10 + 15 + 20 + 25 + \cdots + 5n$ is $\theta(n^2)$

Proof.

$$\begin{aligned} 5(1+2+3+\cdots+n) &= 5(\frac{n(n+1)}{2}) \\ &= \frac{5}{2}(n^2+n) \\ &= \frac{5}{2}n^2 + \frac{5}{2}n \end{aligned}$$

$\frac{5}{2}n^2 + \frac{5}{2}n$ is $\theta(n^2)$ by the theorem on polynomial orders. ∎

Problem 37.
$\sum_{k=1}^{n}(k+3)$ is $\theta(n^2)$

Proof.

$$\begin{aligned}\sum_{k=1}^{n} k + 3\sum_{k=1}^{n} &= \frac{n(n+1)}{2} + 3(n)\\ &= \frac{n^2+n}{2} + 3n\\ &= \frac{n^2+7n}{2}\end{aligned}$$

$n^2 + 7n2$ is $\theta(n^2)$ by the theorem on polynomial orders.

■

Problem 39.
$\sum_{k=3}^{n}(k^2-2k)$ is $\theta(n^3)$

Proof.

$$\begin{aligned}\sum_{k=3}^{n} k^2 &= \frac{n(n+1)(2n+1)}{6} - 1^2 - 2^2\\ \sum_{k=3}^{n} 2k &= 2\left(\frac{n(n+1)}{2} - 1 - 2\right)\\ \sum_{k=3}^{n} k^2 - \sum_{k=3}^{n} 2k &= \left(\frac{n(n+1)(2n+1)}{6} - 5\right) - 2\left(\frac{n(n+1)}{2} - 3\right)\\ &= \frac{n(2n^2+3n+1)}{6} - 5 - n^2 + n - 6\\ &= \frac{2n^3+3n^2+n-30}{6} - \frac{6n^2+6n-36}{6}\\ &= \frac{2n^3-3n^2-5n+6}{6}\end{aligned}$$

$\frac{2n^3-3n^2-5n+6}{6}$ is $\theta(n^3)$ by the theorem on polynomial orders.

■