

Problem 40.

Prove that for every integer $n \geq 2$, $P(n+1, 3) = n^3 - n$

Proof. Let $n \in \mathbb{Z} \ni n \geq 2$

$$\begin{aligned} P(n+1, 3) &= \frac{(n+1)!}{((n+1)-3)!} \\ &= \frac{(n+1)!}{(n-2)!} \\ &= \frac{(n+1)n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}} && \text{by algebra} \\ &= n(n+1)(n-1) \\ &= n^3 - n \end{aligned}$$

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