

Problem 28.

Theorem: For all integers a, b , and c , if $a \mid bc$ then $a \mid b$ or $a \mid c$.

Proof. Let $a, b, c \in \mathbb{Z} \ni a \mid bc$
 Since $a \mid bc$, $\exists d \in \mathbb{Z} \ni ad = bc$

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Problem 29.

Theorem: For all integers a and b , if $a \mid b$ then $a^2 \mid b^2$.

Proof. Let $a, b \in \mathbb{Z} \ni a$ divides b
 By definition of divisibility,

$$\exists k \in \mathbb{Z} \ni b = ak$$

$$\begin{aligned} b^2 &= (ak)^2 && \text{by substitution} \\ &= (ak)(ak) && \text{by algebra} \\ &= a(kak) && \text{by associative property} \end{aligned}$$

Let $t = (kak)$, $t \in \mathbb{Z}$ since $a, k \in \mathbb{Z}$

The sum and products of integers is an integer.

Therefore, $a^2 \mid b^2$ by definition of divisibility since $b^2 = at$ where $t \in \mathbb{Z}$ by substitution

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