Problem 2.

Theorem: For every integer m, if m is even then $3m + 5$ is odd

Proof.
Let $m \in \mathbb{Z} \ni m$ is even
By definition of even, $m = 2k$ for some $k \in \mathbb{Z}$

$$\begin{aligned} 3m + 5 &= 3(2k) + 5 && \text{by substitution} \\ &= 6k + 5 && \text{by algebra} \\ &= 2(3k + 2) + 1 && \text{by factoring out 2} \end{aligned}$$

Let $t = 3k + 2$
$t \in \mathbb{Z}$ because $3, 2, k \in \mathbb{Z}$ and $\mathbb{Z}$ are closed under sums and products.
Therefore, $3m + 5 = 2t$ where $t \in \mathbb{Z}$, so $3m + 5$ is odd.

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Problem 9.

Theorem: If an integer greater than 4 is a perfect square, then the immediately preceding integer is not prime

Proof.
Let $n \in \mathbb{Z}$ with $n > 4 \ni n$ is a perfect square
By definition of a perfect square, $\exists\, n = k^2 \ni k \in \mathbb{Z}$

$$\begin{aligned} n - 1 &= k^2 - 1 && \text{by substitution} \\ &= (k-1)(k+1) && \text{by algebra} \end{aligned}$$

$$\begin{aligned} k^2 &> 4 && \text{by substitution} \\ k &> 2 && \text{by algebra} \\ k - 1 > 1 \;\&\; k + 1 &> 3 && \text{by algebra} \end{aligned}$$

$k^2 - 1$ is the product of two integers greater than 1.
Therefore, the immediately preceding integer is composite by definition of composite.

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Problem 23.

Theorem: The product of any even integer and any integer is even.
True

Proof.
Let $a \in \mathbb{Z} \ni a$ is even and $b \in \mathbb{Z}$.
By definition of even, $\exists\, n \in \mathbb{Z} \ni a = 2n$

Then

$$\begin{aligned} ab &= 2n \times b && \text{by substitution} \\ &= 2(n \times b) && \text{by factoring out 2} \end{aligned}$$

Let $t = n \times b$.
Then $t \in \mathbb{Z}$ because the products of integers are integers.
Hence, $ab = 2t$, thus ab is even by definition of even.

$\blacksquare$