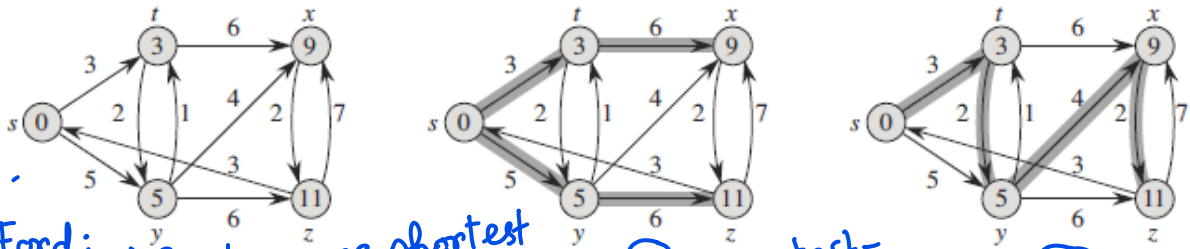
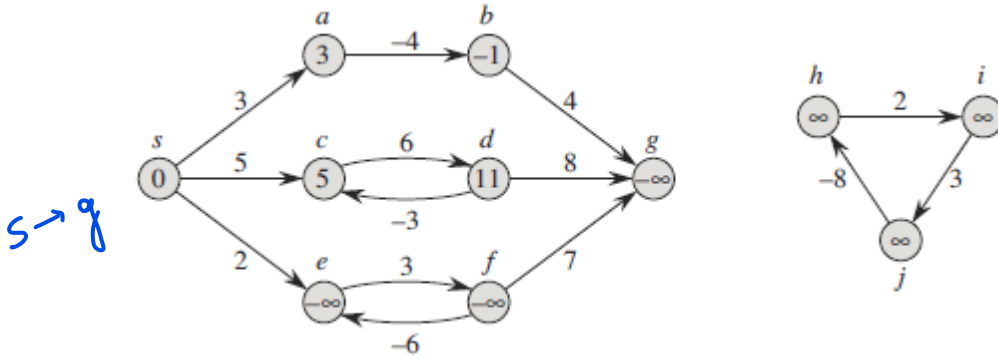


$$w(p) = \sum_{i=1}^k w(v_{i-1}, v_i)$$

$$\delta(u, v) = \begin{cases} \min\{w(p) : u \xrightarrow{p} v\} & \text{if there is a path from } u \text{ to } v \\ \infty & \text{otherwise.} \end{cases}$$



Bellman-Ford: → Single source shortest path problem

✓ INITIALIZE-SINGLE-SOURCE(G, s)

- 1 for each vertex $v \in G.V$
- 2 $v.d = \infty$
- 3 $v.\pi = \text{NIL}$
- 4 $s.d = 0$

① Shortest-path tree
Edge weights may be negative.

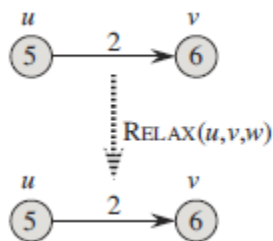
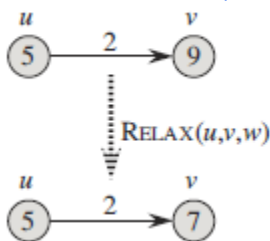
② Shortest-path tree

✓ RELAX(u, v, w)

- 1 if $v.d > u.d + w(u, v)$
- 2 $v.d = u.d + w(u, v)$
- 3 $v.\pi = u$

$$9 > 5 + 2$$

$$6 \nless 5 + 2$$



BELLMAN-FORD(G, w, s)

1 INITIALIZE-SINGLE-SOURCE(G, s) $\rightarrow \checkmark$

2 for $i = 1$ to $|G.V| - 1$

3 for each edge $(u, v) \in G.E \rightarrow$ all edges of $G.E$

4 RELAX(u, v, w)

5 for each edge $(u, v) \in G.E$

6 if $v.d > u.d + w(u, v)$

7 return FALSE

8 return TRUE

Time complexity

$O(V \cdot E)$

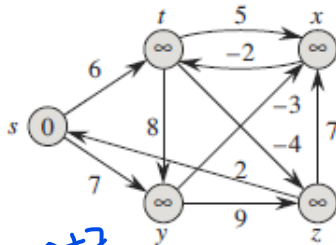
$V \cdot E$

E

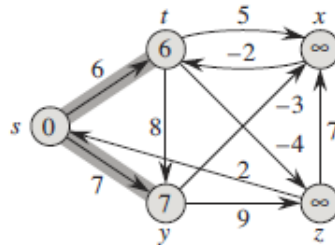
to find out any -ve weighted cycle
Having -ve weighted cycle

$$\infty > 0 + 6 = 6$$

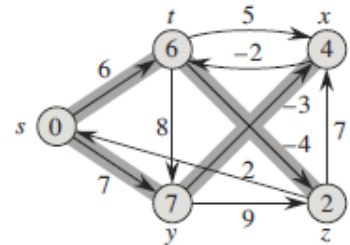
$$\infty > 0 + 7 = 7$$



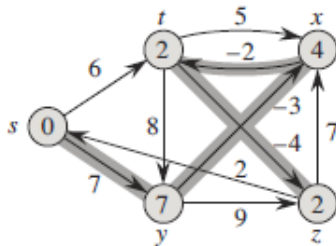
(a)



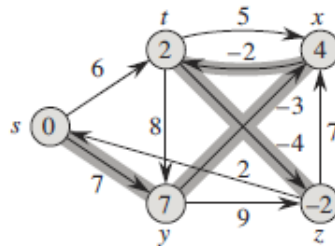
(b)



(c)



(d)



(e)

Cost Path

$s \rightarrow y$ 7 ($s \rightarrow y$)

$s \rightarrow t$ 2 ($s \rightarrow y \rightarrow x \rightarrow t$)

$s \rightarrow x$ 4 ($s \rightarrow y \rightarrow x$)

$s \rightarrow z$ -2 ($s \rightarrow y \rightarrow x \rightarrow t \rightarrow z$)

Shortest-Path tree

