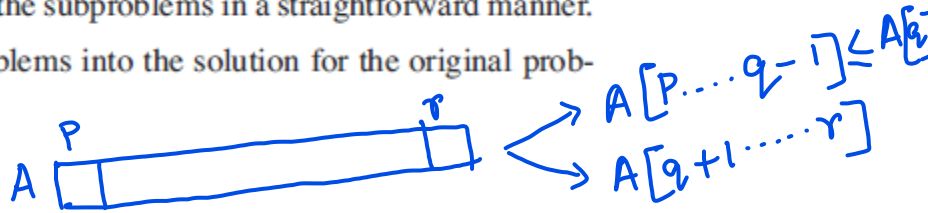


## Divide and Conquer:

1. Divide the problem into a number of subproblems that are smaller instances of the same problem.
2. Conquer the subproblems by solving them recursively. If the subproblem sizes are small enough, however, just solve the subproblems in a straightforward manner.
3. Combine the solutions to the subproblems into the solution for the original problem.

### Quicksort:



**Divide:** Partition (rearrange) the array  $A[p \dots r]$  into two (possibly empty) subarrays  $A[p \dots q-1]$  and  $A[q+1 \dots r]$  such that each element of  $A[p \dots q-1]$  is less than or equal to  $A[q]$ , which is, in turn, less than or equal to each element of  $A[q+1 \dots r]$ . Compute the index  $q$  as part of this partitioning procedure.

**Conquer:** Sort the two subarrays  $A[p \dots q-1]$  and  $A[q+1 \dots r]$  by recursive calls to quicksort.

**Combine:** Because the subarrays are already sorted, no work is needed to combine them: the entire array  $A[p \dots r]$  is now sorted.

QUICKSORT( $A, p, r$ )

```

1  if  $p < r$ 
2     $q = \text{PARTITION}(A, p, r)$ 
3    QUICKSORT( $A, p, q - 1$ )
4    QUICKSORT( $A, q + 1, r$ )

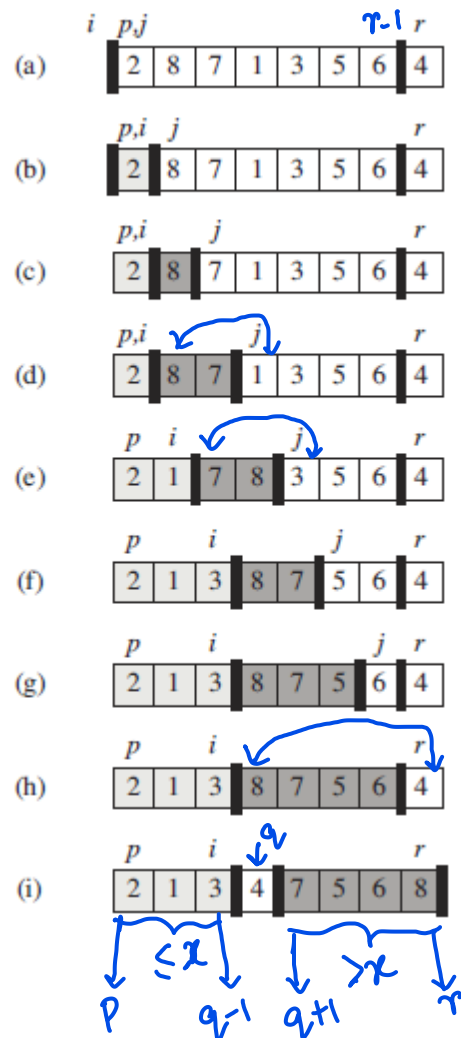
```

PARTITION( $A, p, r$ )

```

1   $x = A[r]$ 
2   $i = p - 1$ 
3  for  $j = p$  to  $r - 1$ 
4    if  $A[j] \leq x$ 
5       $i = i + 1$ 
6    exchange  $A[i]$  with  $A[j]$ 
7  exchange  $A[i + 1]$  with  $A[r]$ 
8  return  $i + 1$ 

```



$A[r] = 4 = x$   
 $2 \leq 4 \checkmark$   
 $8 \leq 4 \times$   
 $7 \leq 4 \times$   
 $1 \leq 4 \checkmark$   
 $3 \leq 4 \checkmark$   
 $5 \leq 4 \times$   
 $6 \leq 4 \times$

$x = A[r] \leadsto$  Pivot element  
 $A[p \dots r] \leadsto A[p \dots q-1] \quad A[q]$   
 $A[p \dots r] \leadsto A[q+1 \dots r]$

Time complexity:

Time req. for partition  $n-1$  elements, 0 element

Worst case:  $T(n) = T(n-1) + T(0) + \Theta(n)$   
 $= T(n-1) + \Theta(n)$

Const. time  $\Theta(n^2)$   
 $\Theta(n \log n)$

Best case:  $T(n) = 2T(n/2) + \Theta(n)$

Average case:  $T(n) = T(9n/10) + T(n/10) + cn$

9:1 proportion

