1. Multiply the following matrices using Strassen's Matrix Multiplication Algorithm.

$$\begin{array}{cc} A_{11} & A_{12} \end{array}\qquad\qquad \begin{array}{cc} B_{11} & B_{12} \end{array}$$

$$A = \left(\begin{array}{cc|cc} 2 & 4 & 6 & 3 \\ 1 & 2 & 2 & 1 \\ \hline 3 & 1 & 1 & 3 \\ 1 & 1 & 1 & 1 \end{array}\right)_{4\times 4} \qquad B = \left(\begin{array}{cc|cc} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 2 \\ \hline 2 & 1 & 1 & 2 \\ 3 & 1 & 1 & 3 \end{array}\right)_{4\times 4}$$

$$\begin{array}{cc} A_{21} & A_{22} \end{array}\qquad\qquad \begin{array}{cc} B_{21} & B_{22} \end{array}$$

$A_{ij}, B_{ij} \Rightarrow 2\times 2$

$$S_1 = B_{12} - B_{22} = \begin{pmatrix} 0 & -1 \\ 0 & -1 \end{pmatrix} \qquad S_6 = B_{11} + B_{22} = \begin{pmatrix} 2 & 3 \\ 2 & 4 \end{pmatrix}$$

$$S_2 = A_{11} + A_{12} = \begin{pmatrix} 8 & 7 \\ 3 & 3 \end{pmatrix} \qquad S_7 = A_{12} - A_{22} = \begin{pmatrix} 5 & 0 \\ 1 & 0 \end{pmatrix}$$

$$S_3 = A_{21} + A_{22} = \begin{pmatrix} 4 & 4 \\ 2 & 2 \end{pmatrix} \qquad S_8 = B_{21} + B_{22} = \begin{pmatrix} 3 & 3 \\ 4 & 4 \end{pmatrix}$$

$$S_4 = B_{21} - B_{11} = \begin{pmatrix} 1 & 0 \\ 2 & 0 \end{pmatrix} \qquad S_9 = A_{11} - A_{21} = \begin{pmatrix} -1 & 3 \\ 0 & 1 \end{pmatrix}$$

$$S_5 = A_{11} + A_{22} = \begin{pmatrix} 3 & 7 \\ 2 & 3 \end{pmatrix} \qquad S_{10} = B_{11} + B_{12} = \begin{pmatrix} 2 & 2 \\ 2 & 3 \end{pmatrix}$$

$$P_1 = A_{11} \cdot S_1 = \begin{pmatrix} 2 & 4 \\ 1 & 2 \end{pmatrix} \cdot \begin{pmatrix} 0 & -1 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 0 & -6 \\ 0 & -3 \end{pmatrix}$$

$$P_2 = S_2 \cdot B_{22} = \begin{pmatrix} 8 & 7 \\ 3 & 3 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix} = \begin{pmatrix} 15 & 37 \\ 6 & 15 \end{pmatrix}$$

$$P_3 = S_3 \cdot B_{11} = \begin{pmatrix} 4 & 4 \\ 2 & 2 \end{pmatrix} \cdot \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 8 & 8 \\ 4 & 4 \end{pmatrix}$$

$$P_4 = A_{22} \cdot S_4 = \begin{pmatrix} 1 & 3 \\ 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 2 & 0 \end{pmatrix} = \begin{pmatrix} 7 & 0 \\ 3 & 0 \end{pmatrix}$$

$$P_5 = S_5 \cdot S_6 = \begin{pmatrix} 3 & 7 \\ 2 & 3 \end{pmatrix} \cdot \begin{pmatrix} 2 & 3 \\ 2 & 4 \end{pmatrix} = \begin{pmatrix} 20 & 37 \\ 10 & 18 \end{pmatrix}$$

$$P_6 = S_7 \cdot S_8 = \begin{pmatrix} 5 & 0 \\ 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 3 & 3 \\ 4 & 4 \end{pmatrix} = \begin{pmatrix} 15 & 15 \\ 3 & 3 \end{pmatrix}$$

$$P_7 = S_9 \cdot S_{10} = \begin{pmatrix} -1 & 3 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 2 & 2 \\ 2 & 3 \end{pmatrix} = \begin{pmatrix} 4 & 7 \\ 2 & 3 \end{pmatrix}$$

$$C_{11} = P_5 + P_4 - P_2 + P_6 = \begin{pmatrix} 27 & 15 \\ 10 & 6 \end{pmatrix}$$

$$C_{12} = P_1 + P_2 = \begin{pmatrix} 15 & 31 \\ 6 & 12 \end{pmatrix}$$

$$C_{21} = P_3 + P_4 = \begin{pmatrix} 15 & 8 \\ 7 & 4 \end{pmatrix}$$

$$C_{22} = P_5 + P_1 - P_3 - P_7 = \begin{pmatrix} 8 & 16 \\ 4 & 8 \end{pmatrix}$$

C_{11} C_{12}

$$C = \left(\begin{array}{cc|cc} 27 & 15 & 15 & 31 \\ 10 & 6 & 6 & 12 \\ \hline 15 & 8 & 8 & 16 \\ 7 & 4 & 4 & 8 \end{array}\right)$$

C_{21} C_{22}

(Ans.)