```
% Curran Robertson
% Lagrangian Equations of Motion of a Sliding Pendulum with Friction
 and
% Air Resistance
% April 16, 2023

clear all; clc; close all

% Symbols
syms m2 dfi fi x dx s ds t c mu ddfi dds ddx

% Constants
m1 = 1;
m2 = 1.5;
k = 25.6;
g = 9.81;
l = 0.8;

% Lagrangian ( L = T - V )
L = (1/2)*m1*dx^2 + (1/2)*m2*(dx+(l+s)*cos(fi)+ds*sin(fi))^2 +
 (1/2)*m2*((l+s)*sin(fi)+ds*cos(fi))^2 + (1/2)*m2*(l+s)^2*dfi^2 +
 m2*g*((l+s)*cos(fi)) - (1/2)*k*s^2;
L = expand(L);
L = simplify(L);
disp('L = ')
pretty(L)

% Partial Derivatives
    % DOF 1 : Fi
d1 = diff(L, dfi);
d2 = diff(L, fi);

    % DOF 2 : x
d4 = diff(L, dx);
d5 = diff(L, x);

    % DOF 3 : s
d7 = diff(L, ds);
d8 = diff(L, s);


% Time Derivatives
    % DOF 1 : Fi
syms dfi(t)
d3 = diff(subs(d1, dfi, dfi(t)), t);
    % DOF 2 : x
syms dx(t)
d6 = diff(subs(d4, dx, dx(t)), t);
    % DOF 3 : s
syms ds(t)
d9 = diff(subs(d7, ds, ds(t)), t);
```

```
% Eqns of Motion
    % DOF 1 : Fi
eqn1 = d3 - d2 == -c*dfi; % RHS = Dissipative force due to air
 resistance. Drag is proportional to velocity
    % DOF 2 : x
eqn2 = d6 - d5 == -mu*dx; % RHS = Dissipative force due to friction.
 Drag is proportional to velocity
    % DOF 3 : s
eqn3 = d9 - d8 == -c*ds; % RHS = Dissipative force due to air
 resistance. Drag is proportional to velocity

% Solutions
    % DOF 1 : Fi
ddfi = solve(subs(eqn1, diff(dfi(t), t), ddfi), ddfi);
disp('ddfi = ')
pretty(ddfi)
    % DOF 2 : x
ddx = solve(subs(eqn2, diff(dx(t), t), ddx), ddx);
disp('ddx = ')
pretty(ddx)
    % DOF 3 : s
dds = solve(subs(eqn3, diff(ds(t), t), dds), dds);
disp('dds = ')
pretty(dds)
```

```
L =
                                         2
 2
6 s   2943 cos(fi)   6 dx cos(fi)   6 dfi  s   2943 s cos(fi)   12 dfi
--- + ------------ + ------------ + -------- + -------------- +
 -------
 5         250            5            5            200            25

        2       2        2        2  2
    3 ds    5 dx    241 s    3 dfi  s    ds sin(2 fi) 6   3 dx s
 cos(fi)
  + ----- + ----- - ------ + --------- + -------------- +
 --------------
      4       4       20         4             5                 2

    3 ds dx sin(fi)   ds s sin(2 fi) 3   12
  + --------------- + ---------------- + --
           2                  2          25

ddfi =
  / 2943 sin(fi)              6 dx sin(fi)   2943 s sin(fi)
- | ------------ + c dfi(t) + ------------ + --------------
  \      250                        5              200

    ds cos(2 fi) 12   3 dx s sin(fi)
  - --------------- + -------------- - ds s cos(2 fi) 3
           5                2

                       /    2             \
```

```
     3 ds dx cos(fi) \ | 3 s    12 s   24 |
   - --------------- |/| ---- + ---- + -- |
            2        / \  2      5    25 /

ddx =
  2 mu dx(t)
- ----------
       5

dds =
                                                             2
981 cos(fi)   241 s                   2     2 c ds(t)   4 dfi
----------- - ----- + dx cos(fi) + dfi  s - --------- + ------
    100         15                              3          5

                    4
   + ds sin(2 fi) + -
                    5
```

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