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% Curran Robertson
% Lagrangian Equations of Motion of a Cart, Pendulum, Pulley System
  with
% No Dissipative Forces.
% April 16, 2023

clear all; clc; close all

% Symbols
syms m1 m2 m3 r theta dtheta x dx g l ddtheta ddx

% Constants

% Lagrangian ( L = T - V )
L = (1/2)*m1*r^2*dtheta^2 + (1/2)*m1*(dx+r*cos(theta))^2 +
    (1/2)*m1*(r*cos(theta))^2 + (1/2)*m2*dx^2 + (1/2)*m3*dx^2 -
    m1*g*r*cos(theta) - m3*g*(1-x);
L = expand(L);
L = simplify(L);
disp('L = ')
pretty(L)

% Partial Derivatives
    % DOF 1 : theta
d1 = diff(L, dtheta) ;
d2 = diff(L, theta);
    % DOF 2 : x
d4 = diff(L, dx);
d5 = diff(L, x);

% Time Derivatives
    % DOF 1 : theta
syms dtheta(t)
d3 = diff(subs(d1, dtheta, dtheta(t)), t);
    % DOF 2 : x
syms dx(t) theta(t)
d4_update = dx(t)*m1 + dx(t)*m2 + dx(t)*m3 + m1*r*cos(theta(t));
d6 = diff(d4_update, t);

% Eqns of Motion
    % DOF 1 : theta
eqn1 = d3 - d2 == 0;
    % DOF 2 : x
eqn2 = d6 - d5 == 0;

% Solutions
    % DOF 1 : theta
ddtheta = solve(subs(eqn1, diff(dtheta(t), t), ddtheta), ddtheta);
disp('ddtheta = ')
pretty(ddtheta)
    % DOF 2 : x
ddx = solve(subs(eqn2, diff(dx(t), t), ddx), ddx);

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disp('ddx = ')
pretty(ddx)
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$$L = \frac{dx^2 m1}{2} + \frac{dx^2 m2}{2} + \frac{dx^2 m3}{2} + \frac{dtheta^2 m1 r^2}{2} - g l m3 + g m3 x$$

$$+ m1 r^2 \cos^2(\theta) + dx m1 r \cos(\theta) - g m1 r \cos(\theta)$$

$$ddtheta = \frac{dx \sin(\theta) - g \sin(\theta) + 2 r \cos(\theta) \sin(\theta)}{r}$$

$$ddx = \frac{m1 r \sin(\theta(t)) \frac{d}{dt} \theta(t) + g m3}{m1 + m2 + m3}$$

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