

Currency Engineering

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2022

*The fundamental results presented in this paper are a re-exposition and extension of a series of unpublished papers by the physicist Henri D. Rathgeber (1908-1995). See endnotes for details.

Contents

1	Introduction	3
2	Introduction to Dynamic Systems and Control	5
2.1	Control System Design	5
2.2	System Specification	5
2.3	Brief History of Mechanic Feedback Mechanisms	5
2.4	Feedback	5
2.5	PID Controllers	6
2.6	How Feedback Systems Go Wrong	6
2.7	Errors and Noise	6
2.8	Internet	6
2.9	Discrete Model	6
3	Exchange Transactions	6
3.1	An Exchange Transaction Only Model	6
3.2	Price and Quantity	7
3.3	Aggregate Supply and Demand	9
3.4	Market Feedback	9
3.5	Aggregate Demand Feedback Model	10
3.6	Market Symmetry	10
3.7	Control of Price Level	11
4	Exchange Transactions and Errors	12
4.1	Information Theory	12
4.2	Aggregate Supply and Demand	12
5	Control	14
6	Time Transactions	15
6.1	Including Time Transactions in Our Model	15
6.2	Interest Control Failure	16
7	Solution to Interest Control Failure	18
8	Data	18

9	Contract Transactions	18
9.1	Mechanism	19
9.2	Negative Feedback	19
9.3	Positive Feedback	19
9.4	Proxy Accounts	19
9.5	Control of Price Level	21
9.6	Interaction with Time Transactions	21
9.7	Currency Design and Contract Transactions	22
10	External Transactions	22
10.1	External Transactions and Exchange Transactions	22
10.2	External Transactions and Time Transactions	23
10.3	External Transactions and Contract Transactions	23
10.4	Control of External Exchange	23
11	Other Transactions	23
12	Implementation	23
12.1	Goals	24
12.2	Liveness	24
12.3	Safety	24
13	Conclusion	25
13.1	Endnotes	26

1 Introduction

Economic participants drive markets towards equilibrium. This process, however, is constrained by the technical properties of the currency used. This paper describes these technical properties and shows how by correctly isolating and controlling previously coupled components effective control of currency can be achieved that is highly likely to lead to sustained, stable aggregate economic equilibrium with full-employment. Around 1741 David Hume[7] observed that real economic conditions were affected by increases in money supply, which he noted was contrary to the notion that increases in the price as unit of measurement should be independent of the real factors,

“If we consider any one kingdom by itself, it is evident, that the greater or less plenty of money is of no consequence; since the prices of commodities are always proportioned to the plenty of money, and a crown in Harry VII.’s time served the same purpose as a pound does at present ... It is indeed evident, that money is nothing but the representation of labour and commodities, and serves only as a method of rating or estimating them. Where coin is in greater plenty; as a greater quantity of it is required to represent the same quantity of goods; it can have no effect, either good or bad, taking a nation within itself; any more than it would make an alteration on a merchant’s books, if, instead of the Arabian method of notation, which requires few characters, he should make use of the Roman, which requires a great many ... **But notwithstanding this conclusion**, which must be allowed just, it is certain, that, since the discovery of the mines in America, industry has encreased in all the nations of Europe, except in the possessors of those mines; and this may justly be ascribed, amongst other reasons, to the encrease of gold and silver. Accordingly we find, that, in every kingdom, into which money begins to flow in greater abundance than formerly, every thing takes a new face: labour and industry gain life; the merchant becomes more enterprising, the manufacturer more diligent and skilful, and even the farmer follows his plough with greater alacrity and attention. This is not easily to be accounted for, if we consider only the influence which a greater abundance of coin has in the kingdom itself, by heightening the price of commodities, and obliging every one to pay a greater number of these little yellow or white pieces for every thing he purchases.”

Hume’s considerations suggest that any effective economic model requires an understanding of both how economic participants interact in the market but also an understanding of the role of money in those market transactions. A basic supply and demand model infers a macro-economic equilibrium where aggregate demand and aggregate supply are in equality, or equivalently, that aggregate excess supply and demand across markets should average out to close to zero. Contrary to this expectation, all countries experience a sustained state of unemployment and excess aggregate supply as shown in Figure 1.

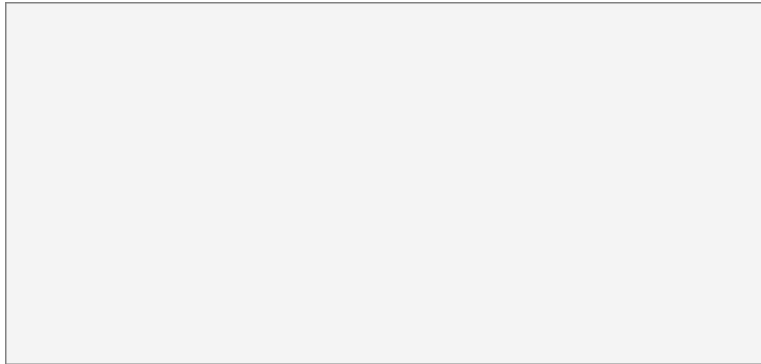


Figure 1: Inflation Rate vs. Unemployment Data

Given that any currency mechanism is absent from this model of supply and demand and that Hume’s observations suggest currencies have significant effects, it seems reasonable to find a way to include currency in our model. This leads to the question of what methods should be used to analyze the properties of currency. Because currency is at its essence a information structure the way to approach this problem is the same way as other similar systems, an example being the internet, as an engineering problem, and in particularly as a problem of the engineering and control of dynamic systems. Economic theory approaches economic problems similarly to the way medicine treats the human body, by applying relatively small changes, such as medicine, in response to various pathologies. Destructuring and rebuilding biological systems is impossible. Importantly however, the control system for economies, the currency, can indeed be destructured and rebuilt from first principles. Thus we take a different approach, viewing currency more like a robotics problem, something that can be built

from the bottom up. In general a currency is a set of accounts, eaching holding a number that can be transfered to other accounts, but we require a more concrete formulation of how economic participants *use* the currency, and so we categorize the way a currency is used into different transaction types. and these can be categorized into various types of transactions. We categorize the most common transactions into exchange transactions, time transactions, contract transactions and external transactions.

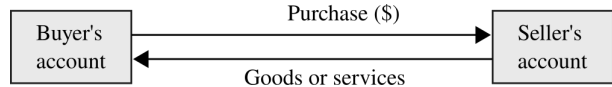


Figure 2: Exchange Transaction

An exchange transaction is a payment in return goods and services that occurs at one point in time.

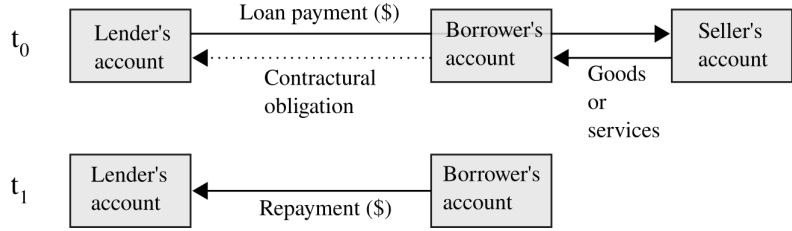


Figure 3: Time Transaction

A time transaction extends an exchange transaction and involves the lending of currency at t_0 . This money is then used by the borrower for an exchange transaction and a contractual obligation is established to the lender. At time t_1 the principal and interest is repaid to the lender.

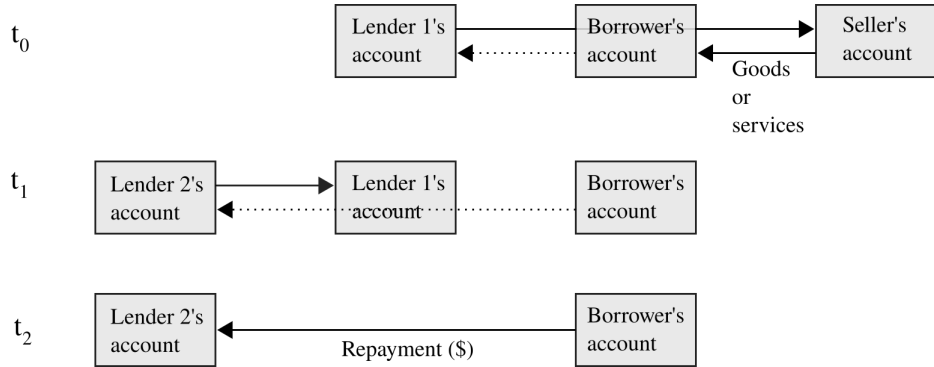


Figure 4: Contract Transactions

A contract transaction extends a time transaction and involves payment for a change in the status or ownership of a contractual obligation. We also consider external transactions between two currencies and other types of transactions.

Feedback regulators are self-adjusting mechanisms that work to achieve some desired conditions in the plant by taking measurements from the plant and feeding back the deviation between the measured values and the desired set point back into the plant.

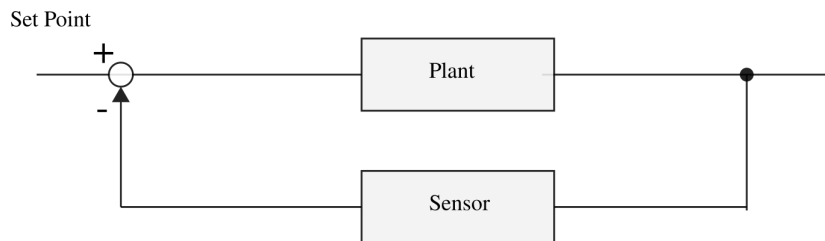


Figure 5: Feedback Schema

We represent the economy as a feedback regulator where the control variables adjusts money supply. Economic participants then interact by using the currency to make transactions.

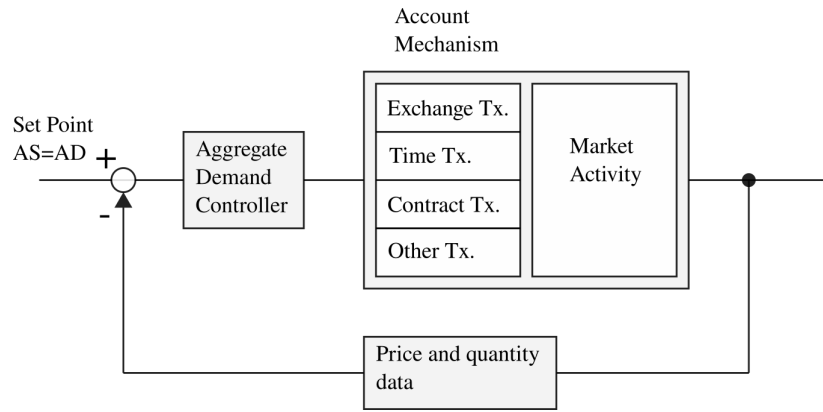


Figure 6: Economic Feedback Schema

Engineering method is applied to the the currency mechanism not the behaviour of people. The only assumption about the behaviour of economic participants is that they drive markets to equilibrium. Our general approach is to take a pared-down version of Figure 6 and step-by-step introduce each transaction type into the model, and then apply normal engineering analytical methods. We find that

1. Aggregate demand must be continuously increasing at a rate sufficient to compensate for errors. Without this requirement the currency constrains the space of possible transactions, preventing people from driving markets to macro-economic equilibrium.
2. All units written into contracts must be independent of the price level. Without this condition a positive-feedback instability can result in runaway behaviour under increases in the inflation rate.
3. Contract transactions must be prevented. Without this condition contract transactions can result in runaway behaviour in prices and difficulties in controlling an overly complex system. Preventing contract transactions allows for a currency design with precise control over aggregate demand.

A currency that implements these properties is highly likely to result in a economy with sustained stable aggregate equilibrium and full-employment.

2 Introduction to Dynamic Systems and Control

The purpose of this section is to present the field of dynamic systems and control to a reader who is not familiar with the topic. Our goal is to demonstrate the huge technological significance of control system design, and to make clear that the methods used in this paper are all very much standard practice. Other than the original insight of the application of control system design to the economic problem, the analysis is nothing new, albeit presented in a somewhat unusual way so as to provide a crash-course in the intuitions and basic analytical method while avoiding formal mathematical methods.

2.1 Control System Design

2.2 System Specification

2.3 Brief History of Mechanic Feedback Mechanisms

Prior to the Wright brother's first flight in December 1903, Orville and Wilbur Wright believed that the most fundamental problems they needed to solve were control problems. In September 1901 Wilbur Wright's first public presentation on the feasibility of heavier-than-air flight stated that "When this one feature [control] has been worked out the age of flying machines will have arrived, for all other difficulties are of minor importance." [19]

2.4 Feedback

feedforward

negative feedback

positive feedback

2.5 PID Controllers

PID controllers are very simple. PID controllers are remarkably robust and in universal use for a great variety of feedback control. So while they are not the only solution, they are a good starting model for thinking about feedback systems.

History

TODO: Huygens

TODO: Watt

TODO: Maxwell

(around 1912)

Elmer Sperry, who introduced the gyroscope invented by TODO to the USA, developed various automatic steering systems.

Elmer Sperry realized that the performance of the autopilot seemed uncanny, for he had been told that no one could possibly invent a device incorporating the intuition of an expert quartermaster. He had in fact analyzed the "intuition" problem when he first began work on the pilot in 1912 and had decided that the helmsman's intuition was, in essence, the ability to "ease off" and "meet" the helm.[21] Easing off involved lessening the rudder angle after the rudder had been put over and the ship had responded by swinging toward the desired heading; meeting was putting the rudder over to the other side (overthrow) to counter the tendency of the ship's angular momentum to carry it past the desired heading. If the helmsman did not perform these functions with skill, the ship continued to yaw. Sperry set out to devise a way of performing these functions mechanically and automatically.[22]

'Easing off' is 'Meeting' take the role of the 'D' component of a PID controller, where the controller responds to the *rate* at which the plant is moving towards the set point.

TODO: Integral?

2.6 How Feedback Systems Go Wrong

Wrong Model

Complexity

Delay

Not Smooth

Errors

Wrong Parameters

While we introduced PID controllers in some detail, we don't consider their application to our problem in detail. The general notion of feedback is critical to our model. To deepen our understanding of feedback controllers. The reason why contemporary economies are unstable and sub-optimal, running at high levels of unemployment is because we have the wrong model. Before the practical application of PID controllers to currency control systems, we must find a better model.

2.7 Errors and Noise

2.8 Internet

Introduction

TCP/IP

Congestion

Routing

2.9 Discrete Model

3 Exchange Transactions

3.1 An Exchange Transaction Only Model

We will start with a model simplified from Figure 6. that includes only exchange transactions.

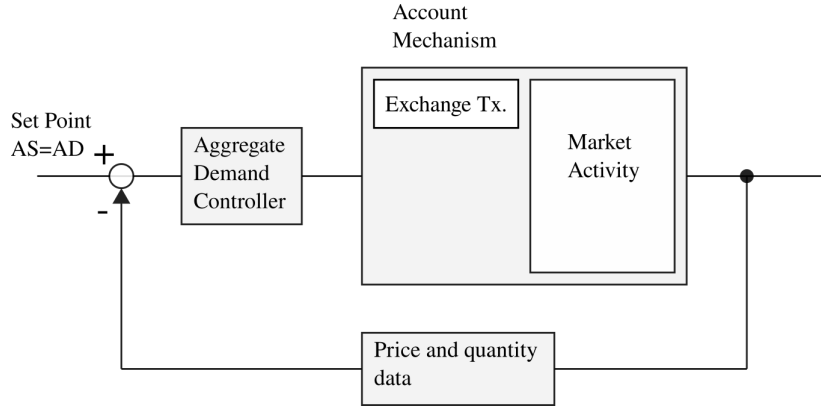


Figure 7: Exchange Only Feedback Schema

This diagram represents the global behavior of the system, which is the result of the aggregate effects of local behaviour. An obvious example, is that market behaviour at the global level is the result of a distributed system of many people interacting with each other in the context of markets. This is represented as the “Market Activity” box. Another example is that the “Exchange Tx.” box represents aggregate effects of all exchange transactions. We need a way to aggregate from individual transactions to sum variables that summarize their global, macro-level properties.

3.2 Price and Quantity

Starting with exchange transactions

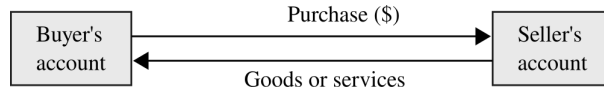


Figure 8: Exchange Transaction

we want to construct some aggregate variables. These aggregate variables could be sums, averages or some other kind of measures of centrality. We’ll categorize exchange transactions into different “goods categories”. We’ll use this term rather than say “goods and services” for brevity and because it more closely reflects our requirements. We’ll assume at this point that these goods categories are sufficiently well-defined and precise as not to introduce too much noise into our aggregate measures. For a given currency at a given period of time the transactions, we sum the quantities for each goods category.

$$(q_1, \dots, q_N)$$

and then let p_i be the average price of these transactions for a given good category. Let N be the total number of different goods categories. We’ll denote the units for these values as

$$([q_1], \dots, [q_N])$$

We’ll denote the average price for each q_i .

$$(p_1, \dots, p_N)$$

The units for the average prices are

$$\left(\left[\frac{\$}{p_1} \right], \dots, \left[\frac{\$}{p_N} \right] \right)$$

We cannot sum either these prices or quantities because they can have different units. But each payment $p_i q_i$ can be summed because its unit, using dimensional analysis, is

$$\left[\frac{\$}{q_i} \right] [q_i] = [\$]$$

So for a given time period, total payments for transactions for a currency is

$$F = p_i q_i + \dots + p_N q_N$$

We want to construct a aggregate measure of prices. As will become clear later in the paper we want a measure P that can be used to isolate purchasing power from the inflation rate. For example, if a person has a certain amount of currency in their account at time $t = 0$, and that they can buy a certain amount with this currency, we want to know how much money they must have in their account at time $t = 1$ so that they can buy the same amount. As written, the notion of “price level”, “purchasing power” and “amount” are vague and require mathematical treatment.

Suppose at a given time the transactions with quantites

$$\overline{Q} = (q_1, \dots, q_N) \text{ with corresponding prices } (p_1, \dots, p_N)$$

occurs. We'll introduce Alice. We'll feed Alice a 1 dollar and Alice will spend this 1 dollar by randomly selecting goods categories in such a way that in the limit they purchase quantities all in proportion to total transactions. In other words,

$$\varepsilon_{q_1, \dots, q_n} = c_1 \overline{Q}$$

where c_1 is a number between 0 and 1 and $\varepsilon_{q_1, \dots, q_n}$ is the expected value of the “abstract person”'s purchases. The c_1 represents the purchasing power of the Alice. It represents the proportion of all transactions over which Alice has the power to purchase, per dollar. While purchasing power is the amount of goods that will buy a given unit of currency, the price level is its reciprocal, the amount of currency that will buy a unit of quantity.

We need to make things more precise, and to check that our measure of price can be used as an index to isolate Alice's purchasing power from changes in the inflation rate. The motivate for this will become clear later in the paper.

First we'll do a static analysis, and try to construct meaningful measures of aggregate price and quantity for a time period that has

$$\overline{Q} = (q_1, \dots, q_N) \text{ with corresponding prices } (p_1, \dots, p_N)$$

Here, the vector \overline{Q} represents a collection of values, but we want to construct a single value that is a aggregate of these values. The problem is that each of these values may have different units. We can construct units that such as “one apple and two oranges”. The problem is that if we do this it is impossible to express “two apples and two oranges” in these units. But if we consider only a single period of time that has \overline{Q} transactions, then we can express this value in any unit of the form

$$([\gamma q_1], \dots, [\gamma q_N]) \tag{1}$$

for any number γ . So in fact there is not just one, but a set of feasible units in which to express \overline{Q} . We refer to any one of these set of units as a “basket of goods”. Given that we select one of these sets of units, we can then express \overline{Q} as some number Q , where the units given by equation (1) with a specific γ . The units of Q is this basket of goods. Now we can set the price level as the payment for this basket of goods. This value P is in units \$ per basket. And so

$$F = PQ$$

The units used depends on the \overline{Q} at a point in time. Of the proportion of q_i changes we would have to use a different unit, and so we couldn't do any dynamic analysis (comparisions across time). We can solve this problem by further constraining our units. As noted, for any \overline{Q} at any time we can construct a new unit. We further constrain this by setting the rule that given one basket of goods whose value of γ is arbitrarily chosen, we constrain all other baskets of goods, by setting their γ to a value such that all baskets of goods have the same average of quantities weighted by their share of trade.

$$\gamma' = \gamma \frac{\varepsilon([q_1], \dots, [q_N])}{\varepsilon([q_1'], \dots, [q_N'])} \tag{2}$$

Now, because γ depends only the average of the units in the baskets of goods, the distribution of q_i around this average becomes irrelevant. This means that *however* \overline{Q} changes, whatever its distribution, the relation $F = PQ$ is exact.

This section has involved various manipulations of units, most critically the ratio of dollar payments to a quantity of some good or service of a collection of different quantities of goods and services. Throughout the paper we will see that questions

of units are important. Section 4. will show that the price unit must change over time. In Section 6. we show that the price unit written into contracts must be constant over time. To isolate these different units and construct a control system that has sufficient degrees of freedom required for control, the use of indices is useful.

3.3 Aggregate Supply and Demand

We can now take a digital currency and measure aggregate properties F , P and Q from the set of transactions. We can also measure their relative change between two time periods.

$$\frac{\Delta F}{F} = \frac{\Delta P}{P} + \frac{\Delta Q}{Q}$$

We'll denote the rate of change of the price level as the inflation rate I . We'll denote the rate of rate of Q as the growth rate G , so

$$\frac{\Delta F}{F} = I + G \tag{3}$$

]

At the micro-level equilibration process for a market for a good or service is driven by excess demand and excess supply. The details of this process are the result of an aggregate of human decisions and is to a large-extent un-knowable, un-measurable and un-predictable. Nevertheless it is undeniable that such an equilibration process exists, and we can abstract this process it a reaction of prices and quantities to excess supply and excess demand at the level of a market for a good or service. Excess supply and excess demand can be measures in \$, and all the micro-level values for excess supply and demand can be summed to get a value for aggregate demand AD and aggregate supply AS .

While we can accurately measure transactions and calculate aggregate properties from these, the value of AD and AS are less concrete. We can't measure AD directly but we can see its effects in that if there is no AD or AS , then there is nothing to drive prices and quantities which will therefore remain constant. On the other hand, the unemployment rate is a good measure of excess AS . From AD and AS we can derive a set of agreements, which is the minimum of these two values (denominated in \$), and it is assumed that agreements maps directly to transactions.

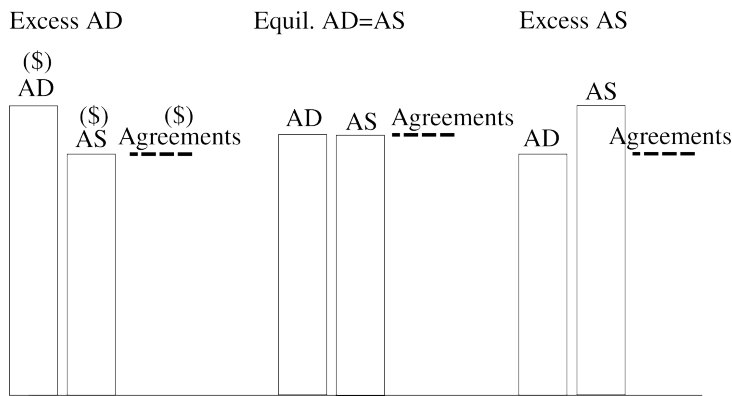


Figure 9: Agreements as a function of aggregate supply and demand

Agreements occur when both sellers and buyers want to transact and so agreements are the minimum of aggregate supply and aggregate demand as shown in Figure 9.

3.4 Market Feedback

We build here the simplest model that we can think with sufficient explanatory power. The use of supply and demand models of the decision making processes of economic participants in most useful in that it encapsulates the notion of markets being driven to equilibrium. We can capture the same notion by thinking of markets as feedback regulations. The value of this model of human behaviour is not that it is quantitatively useful (no-one knows how to predict the decision making process of economic participants with any accuracy) but that it encapsulates the notion of an equilibrating process. This process is drive by excess supply and excess demand.



Figure 10: Model of a market for a good or service

The aggregate equilibration process is driven by the summation of the excess supplys and excess demands of the individual goods markets it constitutes. We can also model the aggregate equilibration process as a feedback regulator.

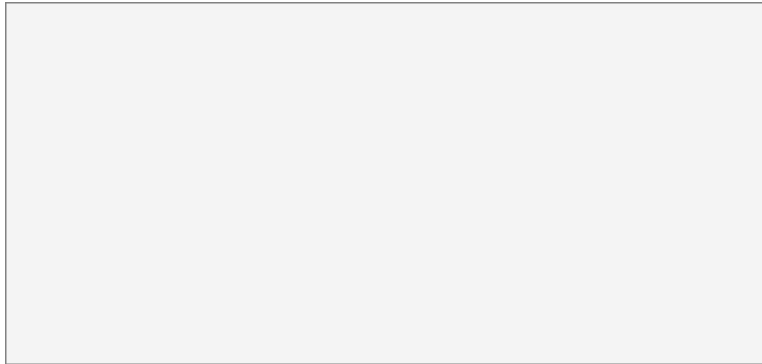


Figure 11: Model of excess aggregate demand/supply response

In the rest of the paper, only assumption we make about market behaviour is that this simple model we have presented here is sufficiently accurate for our purposes to explain the behaviour of markets. Accepting this assumption we focus our attention of the way the design of a currency impacts on this model.

3.5 Aggregate Demand Feedback Model

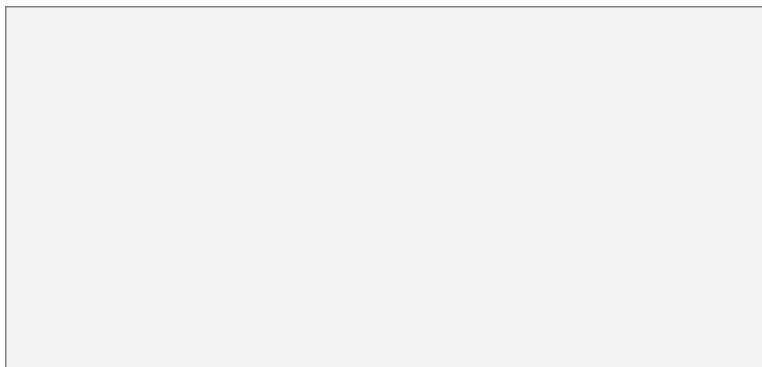


Figure 12: Feedback schema for the control of money supply

3.6 Market Symmetry

The model of excess supply and excess demand driving markets to equilibrium is the foundation of economic theory. The model we have presented is symmetrical in that there is no difference between markets being driven to equilibrium as a result of excess demand or markets being drive to equilibrium as a result

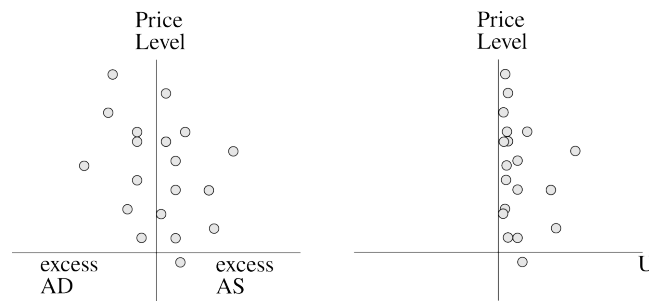


Figure 13: Symmetric Market Model

In fact if we take unemployment and inflation rate data for all countries we get results like as shown in Figure 14.

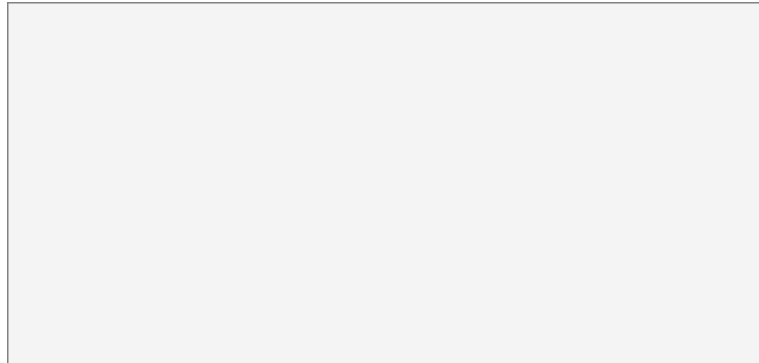


Figure 14: Unemployment and Inflation Rate

This simple model of supply and demand does not explain macro-economic data very well. The method invariably used by economists is to try to explain this anomalous behaviour as a property of decision making by economic participants as they interact in the context of markets. We will take our cue from David Hume's consideration that currency is important, and so apply methods suitable to understanding the mechanics of currencies.

3.7 Control of Price Level

In Section 3.3 we briefly introduced the idea that we could model macro-level market equilibration as a feedback regulator, that feeds back excess aggregate supply and excess aggregate demand back into the price and quantity adjustment process that people engage in when interacting with others in the context of markets.

Another important regulation process is the control of the inflation rate through control of aggregate demand.

Before examining exchange transactions, we will briefly consider the feedback control mechanism. Indexation is an important method for controlling distributed systems like a currency. An index is a single value that is utilized by multiple components. An example of indexation in digital currencies is Bitcoin's method for regulating the rate of production of blocks by Bitcoin miners. In this case the indexation is algorithmic rather than controlled by a central authority. TODO

Another example of indexation is

The simplest way to control the price level in a digital currency is to use indexation.

TODO

The feedback control loop does not work, however, in legacy currencies because the core currency doesn't account for all money. Most money in legacy currencies is banking money, termed M1, M2 and M3. Under these conditions, money authorities must use alternative methods to try and induce financial institutions to increase the amount of banking money mainly through changing the interest rate at which the money authority lends core currency to those financial institutions. At times this method has been effective at controlling the price level, at other times less so. The process lacks precision and has a long time-lag, making its use as the main mechanism for controlling economic conditions problematic. Its effectiveness is also determined by financial institution's ability and willingness to respond to decreases or increases in the monetary authority's core lending rate in a way the maps to increases or decreases in aggregate demand.

4 Exchange Transactions and Errors

4.1 Information Theory

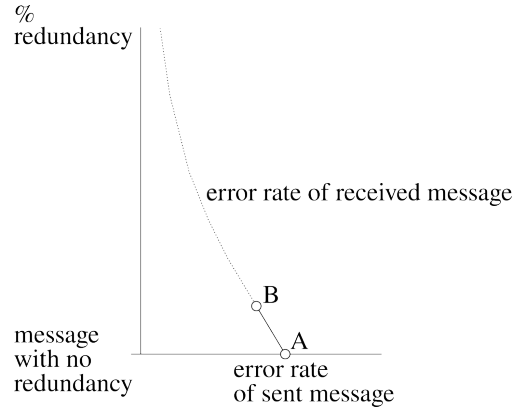


Figure 15: Intuition about Error Rates

Intuitively we might think about how adding redundancy to a message might change the error rate as in Figure 15. The original message with a certain error rate and no redundancy is represented by point A . If we add some redundancy to the error message we might be able to reduce the error rate of the received message to B . It seems that the problem is that for each piece of redundancy we add to the message, we introduce more errors, and so as we increase redundancy we would get decreasing returns on improved error rate. Claude Shannon proved mathematically that this intuition is incorrect, and that there exists a method of adding redundancy to our message such that if we add a H rate of redundancy we can remove all errors from the message. The correct bound on the possible error rate is show in Figure 4.2.

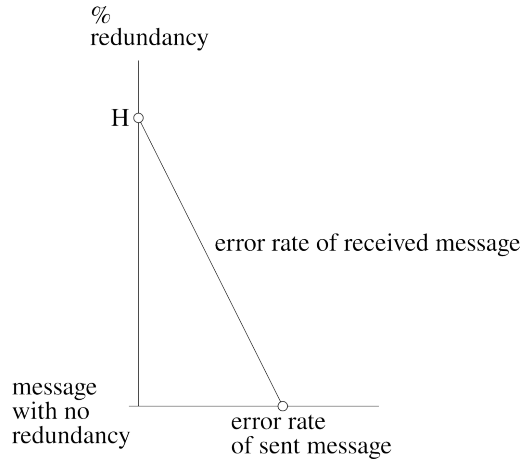


Figure 16: Shannon's Theorem of Noisy Channels

The value H indicates the extra length required of a message so as to compensate for all messages. It is determined by the probabilistic properties of the sent messages. We look into this in more details in ??.

4.2 Aggregate Supply and Demand

In Section 3.3 we presented a simple dynamic model of aggregate supply and demand. For any engineering system we must handle the effects of errors or noise on the system. In this section we will look at the effects of an error rate that reduces the per period aggregate agreements of an economy to the actual transactions that eventuate in that period.

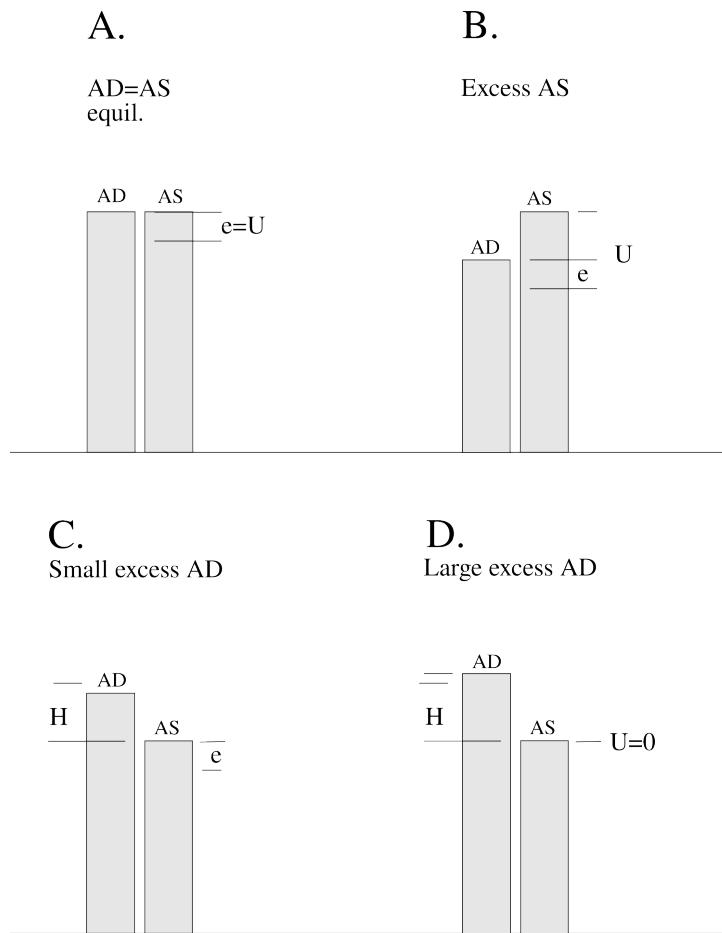


Figure 17: Unemployment as a Function of Aggregate Supply and Demand

Case A. $AD = AS$ equilibrium

In this case the level of agreements, measured in \$ is equal to aggregate supply and aggregate supply. There is no

Case B. Excess AS

Case C. Small excess AD

Case D. Large excess AD

We enumerated four cases above, but we can think about this more abstractly, as each \$ being a carrier for a message. We don't need to quantify the amount of information each unit of currency carries. We do need to know what the relative reduction in that quantity is as it is 'transmitted' across the economy. So currency can be thought of as a carrier of economic information that is distributed across the economy, and that its capability of reducing the error rate in this transmission process is dependent on providing redundancy through increases in aggregate amount of currency.

Whatever way we think about the problem, we can use Figure . to determine the increase in F required. The physical bound on market interactions is therefore shown in the Figure below. Case A, C and D are represented on the graph.

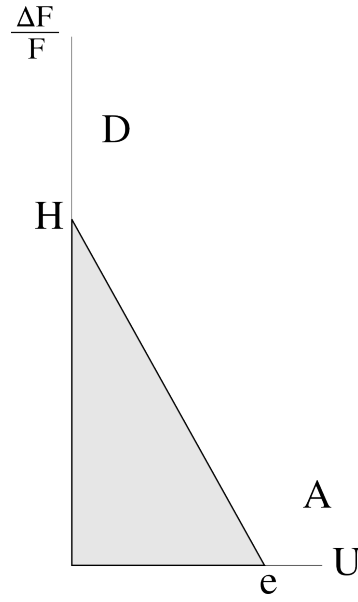


Figure 18: Grey Triangle is the Error Bound

The error bound is a physical constraint on the possibility space of economic interactions. The slope of the line is determined by the mathematical properties of the error rate, and in particular H .

From our simple model of aggregate equilibration in Section 3.4 and from equation ??

$$\frac{\Delta F}{F} = I + G \quad (4)$$

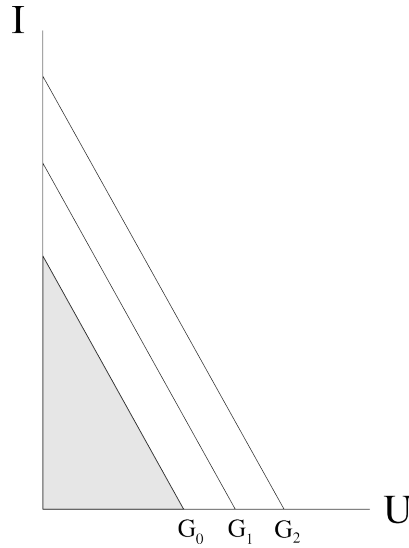


Figure 19: Bound on the Unemployment rate as a function of the Inflation Rate

we can show this bound in terms of the inflation rate I and the unemployment rate U for various different growth rates G , where $G_0 > G_1 > G_2$. Understanding this relation has been central to macroeconomic theory. For instance Fisher Fisher [3] in 1926, Phillips Phillips [11] in 1958 and Lucas [9] all present these relations in graphical form.

We have now been able to shed light on David Hume's problem. His question was why changing units of price should have any affect real quantities. We have now come, at least part-way to solving this problem. However, as we introduce more kinds of transactions into our system, we will find more challenges to the design of a currency that will result in sustained and stable market equilibration.

5 Control

Figure ?? is plotted with the inflation rate on the vertical axis and the unemployment rate on the horizontal axis in accordance to convention. However, the independent variable is the inflation rate, which we control through money supply as indicated

in 6. We can control the inflation rate, but in general the growth rate is external to our control. The unemployment bound is a function of I and G . Contrary to convention, the independent variable appears on the vertical axis while the dependent variable appears on the horizontal axis.

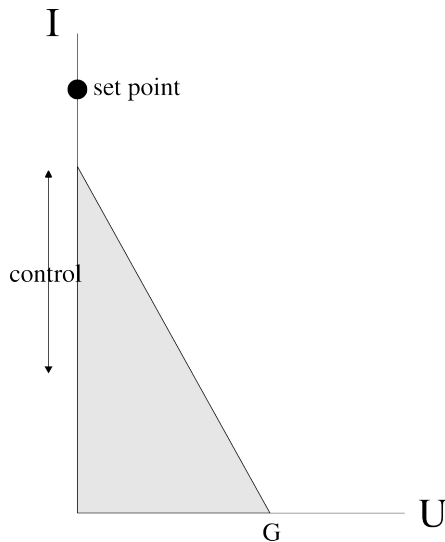


Figure 20: Control for Aggregate Market Equilibration

Decreasing the unemployment rate to zero requires increasing the inflation rate. At various times in the past, monetary authorities have attempted to reduce the unemployment rate by increasing money supply. At times this has been successful but at times it has resulted in increasing unemployment rates rather than decreasing unemployment rates. In the next section we will examine why this control solution has in many conditions been unsuccessful.

6 Time Transactions

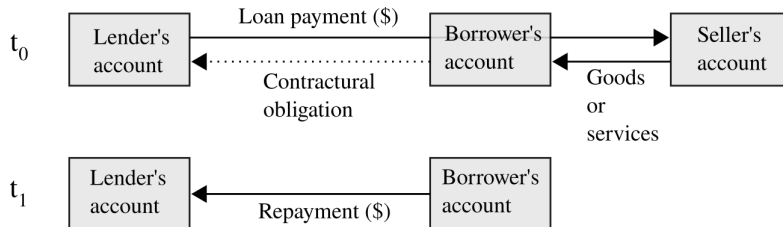


Figure 21: Time Transactions

6.1 Including Time Transactions in Our Model

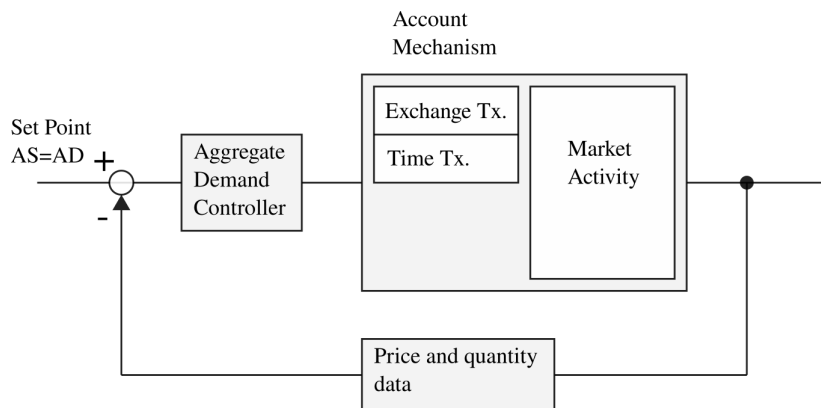


Figure 22: Exchange and Time Transaction Feedback Schema

In Section 4 we showed that there is a bound on the set of economic transactions that are possible, and that the error bound is determined by H (which we can consider as beyond our control), the inflation rate I and the growth rate G . We can control the inflation rate, but in general the growth rate is external to our control. We also showed that to achieve the goals of our currency requires a positive inflation rate. The unit in which we write prices is continuously changing. The unit used to denominate payments in contracts is generally this price unit. We show that this method of choosing interest rates is uncontrollable as discussed in Section 2.2 such that under conditions of a positive rate of change in the inflation rate, i.e. when

$$\frac{\Delta I}{I} > 0$$

then either the real costs of borrowing must increase or the output Q must decrease. Based on Figure 19 this results in unemployment/inflation tracking that appears as indicated in Figure ??.

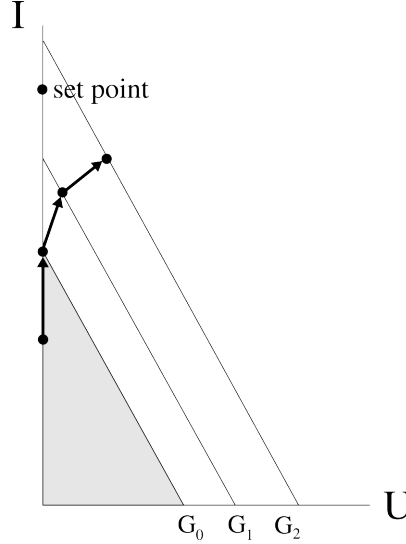


Figure 23: Interest-Inflation Coupling

Control failure occurs until the rate of change in the inflation rate drops to zero or becomes negative. The combination of the error bound and underspecification tracking leads to a fundamental control problem. To set conditions where aggregate equilibrium is possible, the inflation rate must be increased to region D as shown in Figure 19. As the inflation rate increases, uncontrollability tracking reduces the growth rate, increases the unemployment rate. The solution to this problem is to correct the units in which contracts are written. If a unit of account of constant value is used, real interest rates can be trivially specified because the unit directly measures real payments and so fully specifies our control problem. Given that a currency limits transactions to exchange transactions and time transactions, and that a unit of account of constant value is used, and given that there are no other unforeseen destabilizing effects, the currency can be controlled to maintain aggregate equilibrium. If a unit of account of constant value is not used, it is not possible for a currency to maintain aggregate equilibrium.

6.2 Interest Control Failure

We showed in Section 4. that the unit in which prices are denominated must be continuously changing.

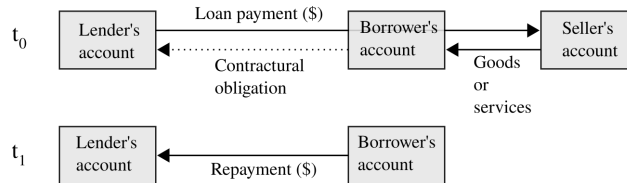


Figure 24: Time Transactions

This unit is used in time transactions to denominate the initial lending of currency, to denominate the repayment, and it is also common practice to use it to specify the repayment in a contract. Because this unit is continuously changing, the value

written into contracts must be written in such a way as to specify a real interest rate, i.e. an interest rate written in terms of purchasing power. Fisher [4] notes that in order to specify a real interest rate r then the value i needs to be written into contracts, where

$$i = (1 + r)(1 + I)$$

Whatever real interest rate r lenders and borrows agree upon, its is possible to write the value i into contracts to achieve this outcome. For a given principle k the repayment of the principal is $k(1 + I)$ and the repayment of interest on that principal is $k(1 + I)(1 + r)$, and so total the repayment of both principle and interest is

$$k(1 + r)(1 + I) = k(1 + I) + k(1 + I)(1 + r)$$

From the left-hand side of this equation we can see that the payment denominated in the unit of currency increases at the same rate as the inflation rate. Figure ?? illustrates that to maintain a market equilibration set point we need a feedback regulator that responds to deviations in the inflation rate from the set point. Not only does the inflation rate I changes, the rate of change of the inflation rate $\Delta I/I$ changes. This changes the requirement to control the real interest rate from a single-variable control problem, to a two variable specification problem. For every inflation rate I there are many rates of change of the inflation rate $\Delta I/I$ that affect the real interest rate of a contract. This means that to correctly specify a real interest rate, two number are required, not one. This is a case where our control system is uncontrollable. The real interest rate is a function of i , I and $\Delta I/I$.

$$r = (1 + i) \left(1 + I \left(1 + \frac{\Delta I}{I} \right) \right) = (1 + i) (1 + I + \Delta I) \quad (5)$$

Given that we only have one control, when two are required, we'll look at the effects of controlling the single control we do have.

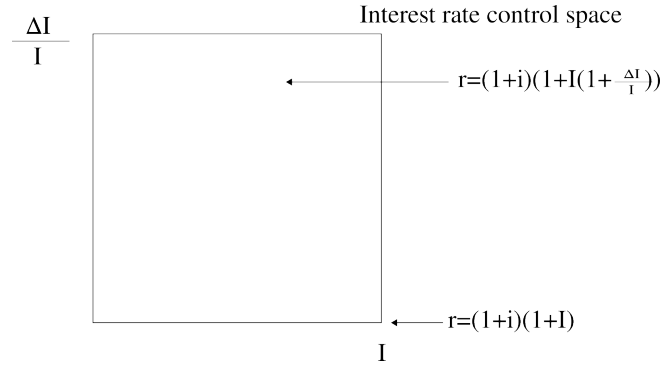


Figure 25: Interest Rate Control Space

We are limited to control options along the horizontal axis. Suppose we are given an r_0 that controls for a given I and where $\Delta I/I = 0$. And then suppose that the rate of change increases so that $\Delta I/I > 0$. The two possible options are to leave i as it is, or to increase i .

Case 1: No change to i

In this case the real interest rate facing lenders will decrease, causing some lender to reject a set of agreements that they would have previously accepted, resulting in decreases in Q and a decrease in the growth rate G .

Case 2: An increase in i

Under these conditions the real costs facing borrowers will increase, causing some lenders to reject a set of agreement that they would have previously accepted, resulting in decreases in Q and a decrease in the growth rate G .

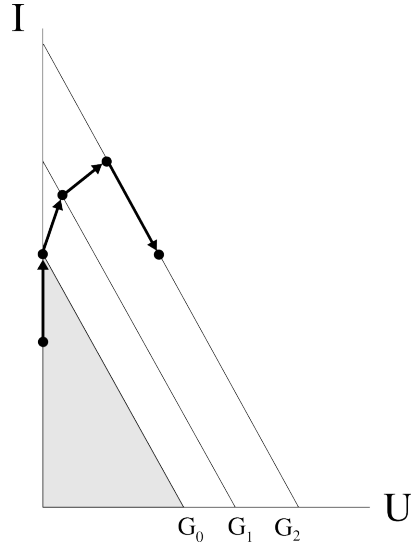


Figure 26: Interest-Inflation Coupling

Because of the shape of the error bound, we can see that if the inflation rate is increased sufficiently, the reduction in the unemployment rate due to increases in the inflation rate can compensate for *all* decreases in the growth rate due to interest control failure. It is possible to double-down on control-failure for a short period of time but maintenance of this state will result in hyper-inflation and high unemployment rates.

The combination of the error bound and interest control failure result in a set point of aggregate equilibrium being unstable, and any attempts to increase the inflation rate toward the set point will leave the system in a state worse, in the sense of a higher unemployment rate, than when it started.

7 Solution to Interest Control Failure

The solution to this problem is to use a unit that directly maps to real interest rates and is independent of the inflation rate, and so also independent of the rate of change of inflation. This unit is termed a unit of account of constant value. To map between this unit of account and prices, the unit of account is multiplied by the price level, and conversely to convert from a price to the unit of account we divide by the price level. The result is a unit that is independent of the exchange rate. Because the value written is independent of the inflation rate, equation (5) becomes

$$r = i$$

As a way of reducing uncertainty in house prices this unit, called the *unidad de formento*, has been used in Chile for several decades [15]. Without interest control failure, the inflation rate can be regulated to a set point in region D of Figure 18.

8 Data

9 Contract Transactions

9.1 Mechanism

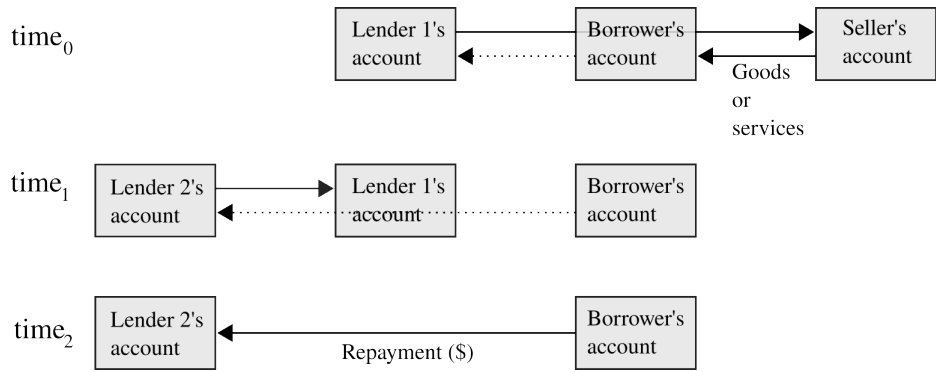


Figure 27: Contract Transactions

9.2 Negative Feedback

There are two money transfers in a contract transaction. The first is a transfer of money in exchange for a good. As with time transactions, interest rates are prices and are subject to excess supply/excess demand response.

9.3 Positive Feedback

A positive feedback can occur if people believe that they can resell something at a later time with a higher price. This may increase demand rather than reduce it as in the negative feedback case. This can occur with both exchange transactions and time transactions if the price of a good is increasing over time. We'll look at the proces in 11. Land and precious metals sometimes behave like this. We'll look at the proces in 11.

9.4 Proxy Accounts

Contract transactions can be used to construct proxy accounts such as bank accounts, that are external to the accounts of the digital currency.

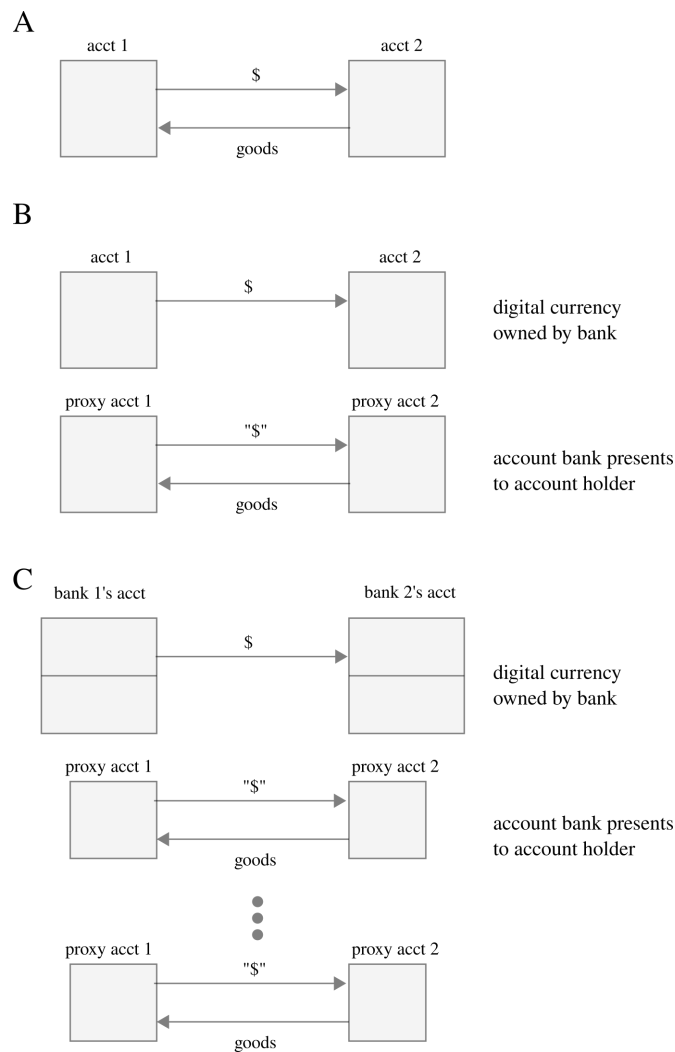


Figure 28: Proxy Accounts

Proxy accounts work by transacting contractual obligations, and adjusting those credits and debts without the need to complete all transactions in the original currency.

Figure ??, Part A, shows a standard between a standard digital currency. Part B shows the situation where account holders 1 and 2 have passed responsibility to the management of these accounts to a bank. The banks own accounts in the digital current, and present account holders 1 and 2 with a dollar value written on a piece of paper or on a computer, written in the diagram as "\\$". When a transaction is made by these two account holders, the bank mirrors this transaction in its digital currency accounts.

Part C shows the situation where bank 1 and bank 2 have many customers, and a single digital currency account that exactly mirrors the aggregate values in all the 'accounts' that the bank presents to the account holders.

It might be possible (and in reality is generally possible) for banks to agree with each other to redeem transactions rather than at the exact time of a transaction but over a certain period of time, such as a day. If the behaviour of the transactions is random meaning that the aggregate flow of transactions from bank 1 to bank 2 cancel out the aggregate flow of transactions from bank 2 to bank 1 over that time period, then banks can use a "fractional reserve system" and reduce that holdings of digital accounts, without any change in the circumstances of the bank account holders. Under a fractional reserve system the flow of transactions through banking accounts can be an order of magnitude greater than through the core digital currency accounts.

This process requires that banks utilize contract transactions between the banks, so that payments are not settled exactly at the time of transaction, are repayed by the end of the period. Time transactions cannot be used as time transactions require a flow of goods between the two parties. So making part C more explicit, we have figure ??.

9.5 Control of Price Level

As first discussed in Section 3.7, proxy accounts multiply the supply of money, and introduce problems for the control of the price level.

If there are not proxy accounts it is possible to directly control the total amount of currency through an index mechanism.

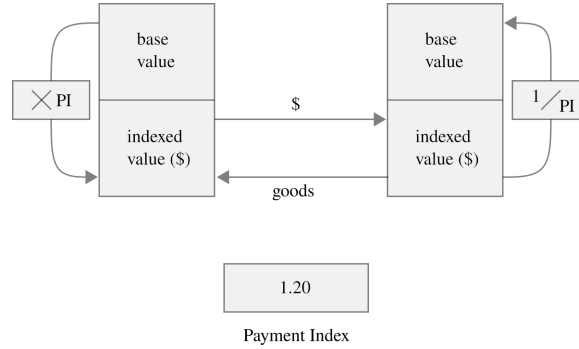


Figure 29: Demand Index Mechanism

The payment index is set by the monetary authority or by algorithm, and is globally accessible. Payment labelling and negotiation and agreements are done in dollars (\$). An account holds a base value, which remains constant when there are no transactions. The account holder however sees the dollar value of their account, which is constantly updated as the payment index changes. The dollar value of an account is the produce of the payment index and the base value. The account receiving a payment converts its value into a base value by dividing by the payment index and adding this value to the accounts current base value. Account holders are generally unaware of the base value, and observe a gradually changing dollar value, similar to the way a bank account appends interest payments. All the conversion between base value and dollar value is done automatically by the digital currency.

The monetary authority or algorithm increases or decreases the payment index in order to control aggregate excess supply or aggregate excess demand. In the presence of proxy accounts this control mechanism breaks down. Banking accounts become part of the set of all accounts that through market decisions made by account holders determine aggregate properties of the currency. Traditionally, monetary authorities try to control aggregate values by financial and banking regulations and through what are known as open-market operations, in which monetary authorities try to influence the degree to which banks have contractual obligations with each other by controlling the interest rate at which they can borrow the core digital account currency. This methods has at times been effective at controlling aggregate supply but at other times ineffective. In the absence of proxy accounts, control of the currency through a payment index is direct and likely to be timely, equitable and with high precision. Using this indexation, the tide rises all boats to exactly the same degree. Given that markets remain in equilibration, the proportional increase in purchasing power is exactly equal to the proportional increase in the price level for every account holder. There is no lag between an increase in the payment index and increase in dollar value in accounts.

9.6 Interaction with Time Transactions

Interest rates in traditional economies are highly correlated as a result of market activity. A rough working model is to think of an economy as having a single interest rate, qualified by “risk”. Both contract transactions and time transactions share this common interest rate. If the interest rate on contract transactions increases due to a positive feedback, at some point it will exceed the market interest rate for time transactions only, resulting in decreases in productive investment through time transactions while investment in a positive feedback bubble is occurring. The dynamics of these kinds of interactions are impossible to predict, both during a positive feedback event and subsequent to the positive feedback event when it is likely that a certain proportion of contracts from time and contract transactions default. In addition, contract transactions are unlimited in complexity of contractual arrangements. It is not possible to create control mechanisms to regulate instabilities that can occur as the result of the use of contract transactions.

9.7 Currency Design and Contract Transactions

To achieve a stable and sustained macro-level equilibrium currencies should be designed to prevent contract transactions for the following reasons:

1. Time transactions can be used for most, possibly all, productive investment.
2. Contract transactions destablity price level control.
3. Contract transactions can destabilize the interest rate which affects time transactions.
4. Positive feedback in contract transactions are the most common cause of financial bubbles and crashes.

10 External Transactions

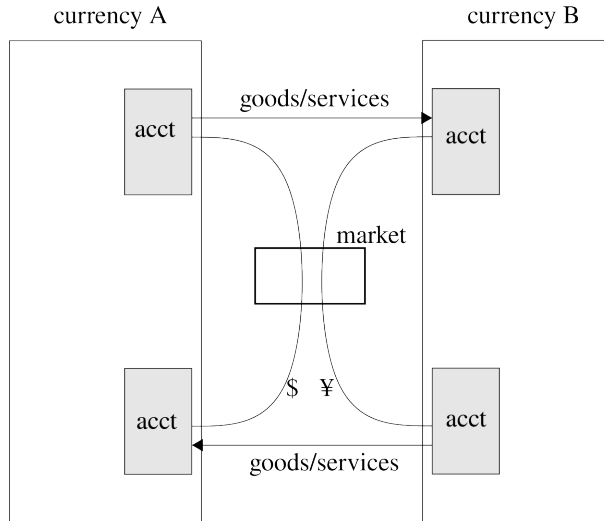


Figure 30: External Transactions

Figure 30 shows an external transaction. Without some extra form of coordination, implemented by some additional accounts, the transaction as it stands in the figure would be very difficult to coordinate. But it captures the important property of external transactions, that across the market boundaries the exchange rate is the rate of payments in currency *A* as a ratio of the rate of payments in currency *B*.

$$X = \frac{A}{B}$$

where X is the exchange rate and the unit is

$$\left[\frac{\$}{yen} \right]$$

Following our program, we'll look at how external transactions interact with other transaction types.

10.1 External Transactions and Exchange Transactions

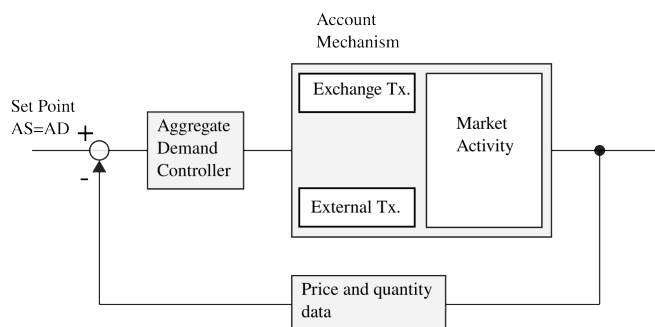


Figure 31: Interaction of External and Exchange Transactions

There is a negative feedback stabilizing process that brings the exchange rate into equilibrium with the relative price levels of the two currencies. This process was described by Cassel [2] in 1914. If

$$\frac{P_A}{P_B} > \frac{A}{B}$$

the goods and services in currency B are cheaper than goods and services in currency A , and so exports of goods and services from B to A increase, and exports of goods and services from A to B decrease. This process continues until an equilibrium where

$$\frac{P_A}{P_B} \doteq \frac{A}{B}$$

where \doteq represents the equilibrium state.

10.2 External Transactions and Time Transactions

Because time transactions also involve the exchange of goods and services, time transactions also contribute to the stabilization process described in the previous section. Repayments across the external transaction market are determined in earlier periods from short term to long term contracts, and therefore do not have the equilibrating “power” that exchange transactions do. Interest rate differentials across the two currencies are liable to cause unpredictable disturbances across the two currencies.

10.3 External Transactions and Contract Transactions

Contract transactions are only minimally connected to an initial exchange transactions, but can involve multiple and continuing payments for changes in contract status. This process is liable to cause strong disturbances unrelated to the equilibrating process, and therefore are highly likely to introduce disturbances to the exchange rate equilibrating process. Exchange rates across legacy currencies that have floating exchange rates can and often do experience rapid fluctuations in exchange rates.

10.4 Control of External Exchange

If only exchange transactions are allowed as external transactions the exchange rate should equilibrate to the relative prices levels of the two currencies, otherwise known as purchasing power parity. The introduction of contract transactions are highly likely to cause disturbances to both currencies, making it difficult to maintain macro-equilibration goals. Time transactions, represent a middle-ground between contract and exchange transactions.

11 Other Transactions

If the price a good or service can be resold at a higher price then another form of transaction is possible. This transaction can induce a positive feedback loop as an increase in prices over time cause other economic participants to enter the market, further increasing prices. Often part or all of the motivation for buying land or precious metals is to transact in this way. One method to prevent or limit this positive feedback instability is to use a separate currency for certain classes of goods category, for example to use a specific currency to real-estate transactions. By doing this it is possible to fix the exchange rate between the two currencies as an index that tracks the average increases in prices of real-estate. Then a conversion between the two currencies compensates for average changes in the price of this good.

12 Implementation

A useful abstraction or tool for working designing an implementation is the computer science abstraction of safety and liveness used for the design of concurrent and distributed systems [8]. Lamport’s core notion is that it is useful to categorize the implementation of systems into two different classes of requirements: liveness and safety, that require different forms of reasoning about. Liveness requirements are those properties that must exist in our design if we want to achieve our goals. Safety properties are those with which we wish to not have in our system if we wish to achieve our goals. But before we look at these we need to make our goals explicit.

12.1 Goals

Abstraction Layer over Accounts

To implement a currency design that does not constrain

we need to reconsider the interaction between the micro-level, the economic participants and owners of accounts, and the macro-level requirements.

The design strategy we take is to define the macro-level requirements and then to search for ways to control the currency so that the currency is likely to map out to those macro-level requirements.

We specify the macro-level requirements as

1. Sustained stability.
2. Macro-level equilibration.

Simple accounts that hold a number with some control of the aggregate value in all accounts is the digital equivalent of paper money. This design is insufficiently controlled to achieve our two macro-level requirements.

12.2 Liveness

12.3 Safety

TODO

To achieve these outcomes we apply a abstraction layer over these accounts. This abstraction layer does the following:

Exchange Transactions

1. Present to the user a value that is the product of the user's base account value and an single, global value that we call the demand index (D_x). This product is the value that users generally see and is the value in which price agreements are made. This value gradually changes as the demand index changes, appearing to users much as a bank account that has gradual increases as an interest rate is applied to it. As such, it presents no usability difficulties to the user.
3. All exchange transactions must be associated with a quantity and a goods or service category. An exchange transaction is only valid if both the seller and buyer confirm the same goods or service category. This serves three purposes.
 - a. It asserts that the transaction is an exchange transaction, and as such, has no associated contract with it beyond the current delivery of the goods or service. By doing this it confirms that there are no future legal commitments to any future repayment. This is required to limit the use of exchange transactions for the purpose of some other transaction category.
 - b. The data can be used, with necessary software mechanism to ensure the privacy of the data, to accurately calculate the price index.
 - c. It serves as a record for the seller and buyer to resolve any disputes, i.e. it acts as a receipt or record of agreement. Beyond the seller and buyer entering the same goods and service category, it is easy for seller and buyer to collude and provide incorrect information.
4. There are intermediate accounts. These are required because all exchange transactions must be associated

TODO

Time Transactions

2. Values that set future payments, in particular for repayments on time transactions, are denominated in a root value and the product of the price index (P_x). In this way, the purchasing power of that value remains absolutely constant, and as such there it present any possibility of inflation feedback. All repayments on time transactions must be defined at the time the money is borrowed and designated as a time transaction.

Contract Transactions

Contract transactions are prevented by the requirement that borrowers must make repayments to the same party that initially lent the money. There are possible ways that users could subvert these requirements which we discuss in a later section.

External Transactions

The provision of an exchange transaction mechanism will be deferred. The possibility of providing exchange transaction functionality with external currencies depends on the design of external currencies, in particular the accuracy of their price index, and their control over contract transactions. If we consider external transactions between two currencies of the kind we are presenting in this paper, then an external exchange rate fixed to the relative price index of the two currencies, and restricting transactions to exchange transactions by requiring the recording of goods or service category and quantity may be a suitable design. This kind of system would require a pair of special aggregate accounts specifically for external transactions

to pair up inflow and outflow. In this system, the pairing of inflow and outflow would halt if either account became empty. Another possibility would be to implement a floating exchange rate. The potential difficulty with this design is that it is relatively easy for users to use exchange accounts in lieu of time or contract transactions, and so it may be possible for users to engage in high-frequency capital interactions across the external boundary despite the disincentive of having no legal resort on repayment failure.

Usability

The controls we plan to implement will have minimal impact on any user who intends to use the currency for exchange or time transactions. There is an additional requirement for both seller and buyer to agree on a transaction and to record goods category. Indexed units of account have been used in Chile for a number of decades without significant usability problems, and the use of indexed units of account in a digital currency could potentially further simplify their use.

One important exception, however, are restrictions on the types of repayment schedules for time transactions. If repayment schedules are overly flexible, the repayments could possibly be used, given people's ingenuity in using financial mechanisms to enrich themselves, as a substitute for contract transactions. The extent to which this would happen in practice is unknown, and so starting with relatively strict controls and relaxing restrictions with experience is probably a reasonable approach.

Using One Transaction Category in Lieu of Another

There are various methods that users may potentially use to one category of transaction of another. The most serious risk is that users find ways to make contract transactions. The main barrier to preventing this is to ensure that repayments can only be made to the initial lender's account. The second barrier is to ensure that there is no legal protection to protect creditors from debtor's failure to make "repayments". These barriers should be sufficient on the condition that there are no significant economic incentives for engaging in such activity. In general such users will choose to use other currencies, rather than a currency with these barriers.

13 Conclusion

We summarize the theoretical components according to the level of confidence in the results.

The error effect and Fisher lines

This property is determined by physical properties and the inferences made from these rules are sufficiently precise and measurable to be subject to falsification.

Aggregate equilibrium

The property is an aggregate of people's behaviour that has been observed over long-periods or time with great consistency and is a plausible outcome of people's general incentives. Our model remains robust even if we relax this assumption to a large degree. Almost all fields of economic research are predicated implicitly or otherwise on the idea that this condition must be modified heavily to explain real-world conditions, and therefore require significant re-appraisal given that these effects are better explained as a property of currency design.

Inflation feedback and financial bubbles and crashes

These processes are best explained as positive feedbacks that, while their degree and timing cannot be reasonably determined quantitatively with precision, if unchecked will have clear consequences and these consequences have consistently been observed. These positive feedback processes are fundamentally uncontrollable with possible rapid changes that we cannot easily regulate. A successful control system will strongly limit the impact of positive feedback processes.

Market driven changes in aggregate quantities and prices

We cannot expect to be able to predict these changes, but the rate of change is not sufficiently fast that it cannot be controlled or adjusted for through feedback regulation.

We hope that this paper can lead to new directions in research. In all new engineering endeavours, theory and practice diverge. We can take advantage of the relative ease of building digital currencies to specific design specifications to make currency engineering into an experimental science. Given the theoretical foundations presented in this paper there is a reasonable high chance that we can design and build currencies that will not prevent markets from reaching equilibrium, that a stable equilibrium can be maintained indefinitely in response to changing conditions.

13.1 Endnotes

According to Rathgeber's notes, the error effect was discovered in 1974. Rathgeber's papers glossed over some matters that caused considerable confusion and doubt about the value of his work. I have tried to make explicit those sources of ambiguity. Rathgeber discusses the need to separate out the "functions of money". This was a source of confusion and lack of precision, which I tried to deal with using the notion of transactions as a way to delineate the different functions of money. Rathgeber did not make a clear distinction between a currency as a technical property as compared to the market behaviour of people, but the notion was implicit in his work. This was also a source a confusion, because people, quite reasonably, object to the application of engineering method directly to social interaction. Making explicit the difference between markets and currency hopefully clear that engineering method is applied to currencies, not social interaction. Rathgeber was working in the context of a single, national, centralized currency. I have extended the application of his ideas to digital currencies. At the time he wrote his private papers, he had access only to Australian inflation and unemployment data. I have extended this to data available up to the time of publication.

References

- [1] Stuart Bennet, *A Brief History of Automatic Control*, IEEE Control Systems Magazine, volume 16, issue 3, June 1996, pages 17-25, doi: 10.1109/37.506394.
- [2] Gustav Cassel, *Money and Foreign Exchange After 1914*, Macmillan, 1923, p.137.
- [3] Irving Fisher, *A Statistical Relation between Unemployment and Price Changes* International Labour Review, 1926, volume 13, pages 496-502, 1926.
- [4] Irving Fisher, *The Rate of Interest*, Macmillan Company, 1907.
- [5] R.W.Hamming, *Error Detecting and Error Correcting Codes*, The Bell System Technical Journal, volume 14, number 2, April 1950, pages 147-160.
- [6] T.P.Hughes, *Elmer Sperry: Inventor and Engineer* 1971.
- [7] David Hume, *Of Money, Essays, Moral, Political and Literary*, 1741.
- [8] Leslie Lamport, *Proving the Correctness of Multiprocess Programs* IEEE Transactions of Software Engineering, volume SE-3, number 2, March 1977.
- [9] Robert E. Lucas Jr., *Nobel Lecture: Money Neutrality*, The Journal of Political Economy, 1996, volume 104, pages 661-682.
- [10] David J.S. MacKay, *Information Theory, Inference, and Learning Algorithms*, Cambridge University Press, 2003.
- [11] A.W. Phillips, *The Relation between Unemployment and the Rate of Change of Money Wage Rates in the United Kingdom, 1861-1957*, Economica, 1958, volume 25, pages 283-299.
- [12] RFC 896, *Congestion Control in IP/TCP Internetworks*, John Nagle, January 1984.
- [13] Lawrence Roberts, *The Evolution of Packet Switching*, Proceedings of the IEEE, December 1978.
- [14] Claude E. Shannon, *A Mathematical Theory of Communication*, The Bell System Technical Journal, volume 27, pages 379-412, 1948.
- [15] Robert J Shiller, *Indexed Units of Account: Theory and Assessment of Historical Experience*, NBER Working Paper Series, volume 6356, 1998.
- [16] J. Clerk Maxwell, *On Governors*, March 5, 1968.
- [17] Nicolas Minorsky, *Directional Stability of Automatically Steered Bodies*, Naval Engineers Journal, volume 32, issue number 2, 1922.

- [18] Elmer A. Sperry, *Engineering Applications of the Gyroscope*, Journal of the Franklin Institute, volume 175, Number 5, May 1913.
- [19] Orville and Wilbur Wright, *The Wright Brothers Aeroplane*, Century Magazine, September 1908.