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Übungsblatt 5

(Besprechung am 14.11.2024)

1. General halting problem

Prove that the general halting problem K is undecidable.

2. Decidability

Let $f: \mathbb{N} \to \mathbb{N}$ be an arbitrary function. Prove the decidability of the following set.

 $A = \{m \in \mathbb{N} \mid \text{there exists an } n \in \mathbb{N} \text{ with } f(n) \text{ defined and } f(n) \geq m\}$

3. Decidability

Let M_0, M_1, \ldots the Gödel numbering of all RAMs discussed in the lecture. Which of the following sets is decidable and which is not. Prove your statements.

- (a) $A = \{n \in \mathbb{N} \mid \text{there are 2 distinct primes } p, q \ge 2 \text{ and } i, j \ge 1 \text{ with } n = p^i q^j \}.$
- (b) $C = \{i \in \mathbb{N} \mid M_i \text{ does not halt on even inputs } n \in \mathbb{N} \}.$
- (c) $D = \{i \in \mathbb{N} \mid M_i \text{ computes the function } c_{K_0}\}.$
- (d) $E = \{i \in \mathbb{N} \mid M_i \text{ halts on at least 2024 inputs of } \mathbb{N} \}.$

4. Differences of square numbers

Prove that the set $A = \{d \in \mathbb{N} \mid \exists x, y \in \mathbb{N}, d = x^2 - y^2\}$ is in REC.

Hints

Exercise 1:

You can show the undecidability with the help of the special halting problem K_0 and Property 2.75.

Exercise 2:

The function f is not necessarily total nor necessarily computable. This task can be solved without using algorithms.

Exercise 3:

The undecidability of sets can always be shown in this task using Rice's theorem. If you apply Rice's theorem, check whether the requirements of the theorem are met.

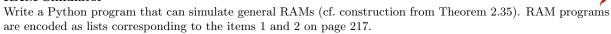
Exactly 2 of the sets are decidable.

Exercise 4:

Consider the distance of a square number $n_1 = x^2 \in \mathbb{N}$ to the next larger square number $n_2 = (x+1)^2$. Show that every square number x^2 for $x \ge 2$ is at least x+1 away from every other square number.

Extra tasks

1. RAM-Simulator



Example:

The Python program receives two lists as input. The first list contains the code of a RAM program, the second contains the arguments for its execution. Your Python program should return the function value computed by the RAM.

NOTE: When having written the program, your program can —via the constructions from the proofs of Theorem 2.33 and Corollary 2.39—be converted into a RAM, which proves that there are universal RAMs.