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Theoretische Informatik
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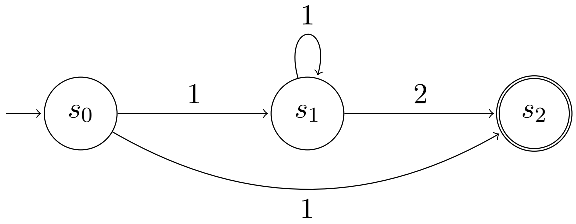
20.11.2024

Übungsblatt 7

(Besprechung am 28.11.2024)

1. **Constructions for NFAs**



$$
\begin{array}{rcl}
K & = & \text{the language over } \{1,2\} \text{ that is accepted by the above NFA} \\
L & = & \{w \in \{1,2\}^* \mid |w| \geq 2 \text{ and } w \text{ starts with 1 and ends with 2}\} \\
M & = & \{w \in \{1,2\}^* \mid |w| \geq 2 \text{ and the last but one symbol of } w \text{ is a 1}\}
\end{array}
$$

   (a) Specify NFAs for L and M in graphic representation and as tuples.

   (b) Construct a DFA for K by applying the power set construction for the above NFA. Draw the resulting DFA.

   (c) Construct —according to the procedure from Theorem 3.21— NFAs for $\overline{K}$, $L \cup M$ and $(L \cup M) \cdot \overline{K}$. Draw the resulting NFAs.

2. **NFA with only one accepting state**
   Why is the following statement true?

   For each set $L \in \text{FA}$ with $\varepsilon \notin L$ there is an NFA A with $L(A) = L$ that only has one accepting state.

Hints

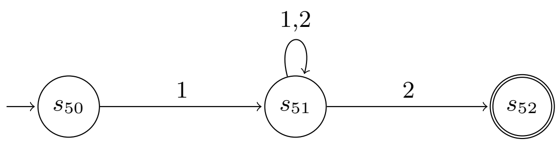
Exercise 1:

- When constructing the NFAs for L and M, use the possibility that for a given symbol and a given state NFAs can have several, but also no outgoing arrow from this state! Both languages can be accepted by automata which only have three states each.
- Proceed with the power set construction as in Example 3.18.

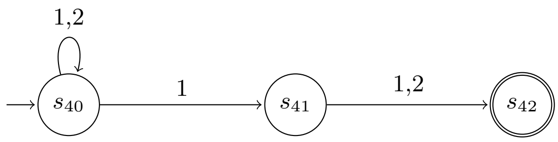
Exercise 2:
Use ideas similar to the constructions we used to prove the closure properties of FA

Solutions

Solution for exercise 1:
NFAs for L and M
$L = \{w \in \{1,2\}^* \mid |w| \geq 2 \text{ and } w \text{ starts with 1 and ends with 2}\}$



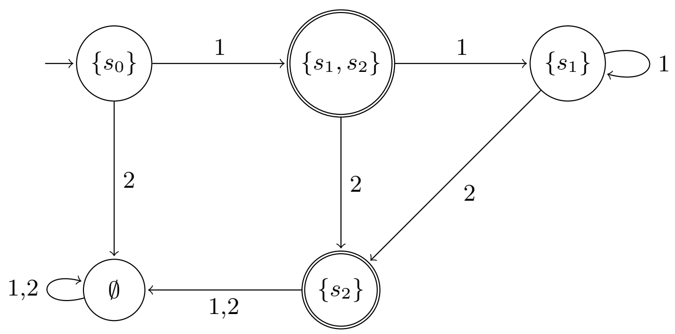
$M = \{w \in \{1,2\}^* \mid |w| \geq 2 \text{ and the last but one symbol of } w \text{ is a } 1\}$



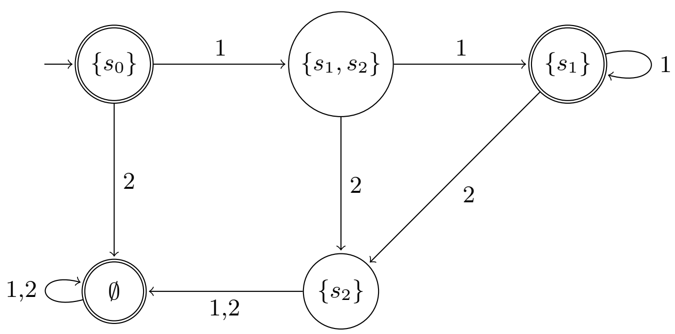
$$A_5 = (\{1,2\}, \{s_{50}, s_{51}, s_{52}\}, \delta_5, s_{50}, \{s_{52}\}) \text{ mit } \delta_5(s,a) = \begin{cases} \{s_{51}\} & \text{, if } s = s_{50} \text{ and } a = 1 \\ \emptyset & \text{, if } s = s_{50} \text{ and } a = 2 \\ \{s_{51}\} & \text{, if } s = s_{51} \text{ and } a = 1 \\ \{s_{51}, s_{52}\} & \text{, if } s = s_{51} \text{ and } a = 2 \\ \emptyset & \text{, if } s = s_{52} \end{cases}$$

$$A_4 = (\{1,2\}, \{s_{40}, s_{41}, s_{42}\}, \delta_4, s_{40}, \{s_{42}\}) \text{ mit } \delta_4(s,a) = \begin{cases} \{s_{40}, s_{41}\} & \text{, if } s = s_{40} \text{ and } a = 1 \\ \{s_{40}\} & \text{, if } s = s_{40} \text{ and } a = 2 \\ \{s_{42}\} & \text{, if } s = s_{41} \\ \emptyset & \text{, if } s = s_{42} \end{cases}$$

Power set construction yields the following DFA for K

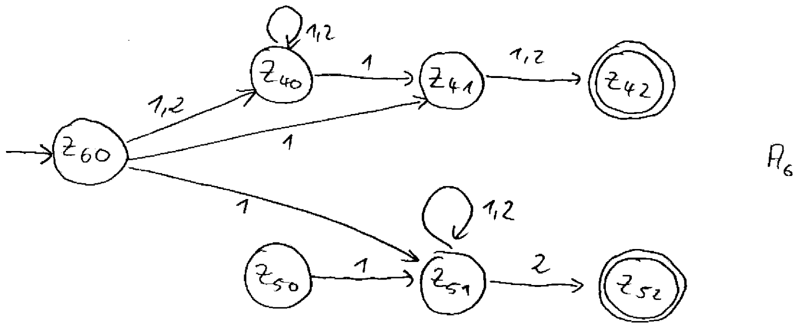


Construction for the complement yields the following DFA $\overline{K}$

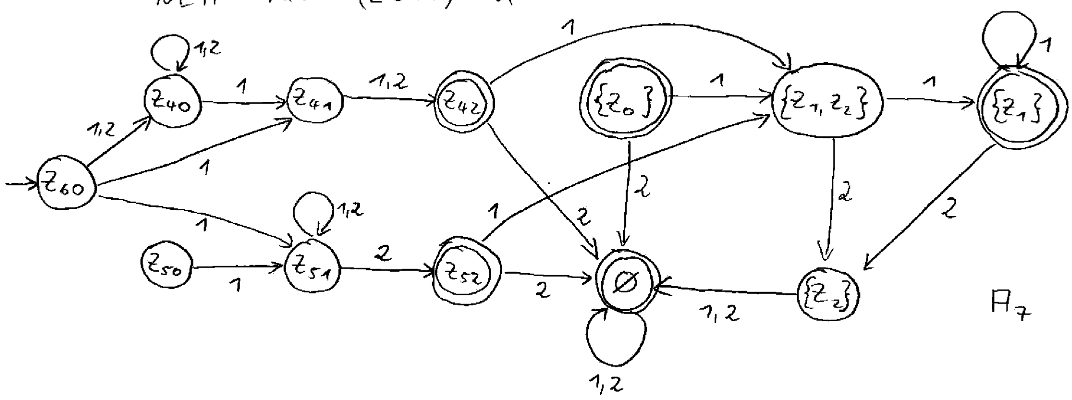


Constructions for union and intersection

NEA für $L \cup M$



NEA für $(L \cup M) \cdot \overline{K}$



Die Zustände z_{42} und z_{52} sind Endzustände, da $\varepsilon \in \overline{K}$

Solution for exercise 2:
Informal idea and description (sufficient for the exercise):

- We take a DFA for the language L.
- We make all states non-accepting.
- We add a new accepting state s^+.
- For each arrow that points to a former accepting state of the DFA, we add a copy of this arrow that points to s^+.

For every word w, after reading w, the NFA is among others in the state that the DFA is in after reading the same word. This is so because the NFA contains all transitions that the DFA has. By the newly added transitions, whenever the DFA is in an accepting state, the NFA is also in s^+.

Formal idea and proof: Let $L \in \mathrm{FA}$ and $\varepsilon \notin L$. Then there is a deterministic finite automaton $A = (\Sigma, S, \delta, s_0, F)$ with $L = L(A)$.
Let $A' = (\Sigma, S \cup \{s^+\}, \delta', s_0, \{s^+\})$ be an NFA with transition function

$$\delta'(s,a) = \begin{cases} \{\delta(s,a), s^+\} & \text{if } \delta(s,a) \in F, \\ \{\delta(s,a)\} & \text{if } \delta(s,a) \in S - F, \\ \emptyset & \text{if } s = s^+. \end{cases}$$

We now prove $L = L(A) = L(A')$. For that purpose, let $w \in \Sigma^* \setminus \{\varepsilon\}$ and $w = va$ for $v \in \Sigma^*, a \in \Sigma$ (w can be written like this as it is not the empty word). Then it holds:

$$\begin{aligned} w \in L(A') &\iff \overline{\delta'}(s_0, w) \cap \{s^+\} \neq \emptyset \\ &\iff \left(\bigcup_{s \in \overline{\delta'}(s_0, v)} \delta'(s,a) \right) \cap \{s^+\} \neq \emptyset \\ &\iff \exists s \in \overline{\delta'}(s_0, v) :\ \delta'(s,a) \cap \{s^+\} \neq \emptyset \\ &\iff \exists s \in \overline{\delta'}(s_0, v) :\ \delta(s,a) \cap F \neq \emptyset \\ &\iff \overline{\delta}(s_0, va) \cap F \neq \emptyset \\ &\iff w \in L(A) \end{aligned}$$