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Modeling forest fire by a paper-burning experiment, a realization of the interface growth mechanism

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We present an experiment on the propagation front of a flameless fire on a thin piece of paper. We find that fire fronts on a piece of paper follow quite well self-affine scaling statistics, with the roughening exponent χ around 0.70, well above the value 1/2 in the theoretical interface growth model using Gaussian noise. This discrepancy may be due to an anomalously singular behavior of the noise distribution, consistent with some recent studies.

Burning is one of the most common chemical processes in nature. Fire propagation on a two-dimensional substance is often encountered in daily life. From the outbreak of a forest fire to burning a piece of paper there are many common aspects, despite their huge scale difference. Various fire-spread models were studied in ecology [1] with the aim of understanding forest fires. The previous concern was about the final contours of burned regions by a fire started from one point in a random fuel-bed. Here we report a paper-burning experiment under rather idealized conditions. The paper used is only dozens of centimeters in size; however, due to its thinness, it may be regarded adequate to mimic much larger fuel-beds such as forests. Paper, like most other combustibles, contains all sorts of local inhomogeneities (noise for short), which influence fire propagation. Thus an initially straight fire front will develop into a rather rough shape, as a result of a combination of noise and the spreading dynamics. We are interested in the fluctuation statistics of fire fronts as they propagate.

Fire spread is just an example of the very general mechanism of interface growth, which is one of the current focuses of statistical physics [2]. Since the fire spread mechanism is local, it is plausible to view the fronts as being effectively modelled by a non-linear stochastic differential equation [3]. The theoretical prediction [2] based on many analytical calculations and computer simulations shows that fronts from interface growth should be self-affine fractals with a universal roughening exponent $\chi = 1/2$. Recent experiments [4,5], however, show significant deviation from this naive theoretical prediction.

We chose an optical lens cleaning paper (lens tissue) made by Whatman Paper Ltd., UK. In fact, almost any paper may serve the purpose, even this page of the journal. Our special choice is due to its extreme lightness, $9.1\,\mathrm{g/m^2}$ (or thickness $\sim 45\,\mu\mathrm{m}$), about one ninth of that of Xerox copy paper. This helps keeping the heat production by fire to a minimum, and this will in turn reduce further complication to fire propagation by strong air circulation. For the same reason, we want flameless, relatively slow burning. Paper usually does not sustain a flameless fire consistently. To ensure a uniform, continuous fuel-bed for fire propagation, we treat the paper with a solution of KNO_3, which is a common oxidization aid, used in many ordinary explosives. We adjust the KNO_3 concentration as a means of controlling fire propagation speed. There appears to be a lower threshold concentration below which fire would not be sustained continuously, and an upper threshold above which the flameless fire would burst into flame. The above concentration range is 0.87–$1.6\,\mathrm{g/m^2}$. The corresponding mean fire propagation speed lies within 5.5–8.2 mm/s. Our experiment is performed mostly for concentrations close to the lower threshold value, in line with our minimal heat consideration.

We carefully dry the treated paper while keeping it flat and smooth. The format is 46 cm wide and 110 cm long (high). We put the paper in an upright frame and apply a slight tension on its two lateral sides, making it immobile during burning. When fire approaches the metallic holders on the sides it stops upon touching. To avoid boundary effects we discard a region of a few centimeters on both sides in the analysis. The experiment takes place under normal room conditions with no forced ventilation. Fire is ignited from the bottom side by a straight electric heating wire. The initially straight fire front gradually develops into irregular shapes, on ever larger length scales.

We use a two-step digitization method: We take high resolution photographic pictures to record the fire front propagation. The pictures are then captured by a video camera into a (512×750) pixel buffer with multi-level grey scale. To obtain better resolution than that offered by ordinary video equipments, we digitize only sub-regions of a picture and they are then patched up into a whole in the computer. This indirect method allows in principle arbitrary resolution. We typically divide pictures into four patches and the effective pixel grid is thus about 512×3000. We identify the fire front as the burning region which is 0.5–3 mm wide in the vertical direction. We define the fire line as the sharp boundary between the burning part and the intact part. From the original two-dimensional data we single out (selecting desired grey scales) the one-dimensional fire line, with $h(x, t)$ denoting its position at point x (discrete after digitization) and time t, as shown in fig. 1.

The quantity of interest is the mean square deviation, or width $W(L, t)$,

$$W^2(L, t) = \left\langle \frac{1}{L} \sum_{x=1}^{L} [h(x, t) - \bar{h}(t)]^2 \right\rangle , \qquad \text{where } \bar{h}(t) = \frac{1}{L} \sum_{x=1}^{L} h(x, t) . \quad (1)$$

L ($0.05 \leqslant L \leqslant 18.3$ cm) is the length of a segment of a fire line, large compared to microscopic scales but still smaller than the paper's width (46 cm), to avoid boundary effects. $\langle \cdot \rangle$ denotes the average over all the segments, obtained from a single fire line by shifting the starting point ($x = 1$) each time.

Fig. 2 shows the data of $W(L, t)$ against L for a large and fixed t, on a double-log scale. One sees that in a region about two decades in the x-direction the data follow rather well a straight line, with the estimated slope $\sim 0.70 \pm 0.03$. It is remarkable that a fire line on a single sample already show reasonable scaling. Under almost identical conditions we average over 15 samples at the KNO_3 concentration ≈ 0.95 g/m^2. The sample-averaged χ value

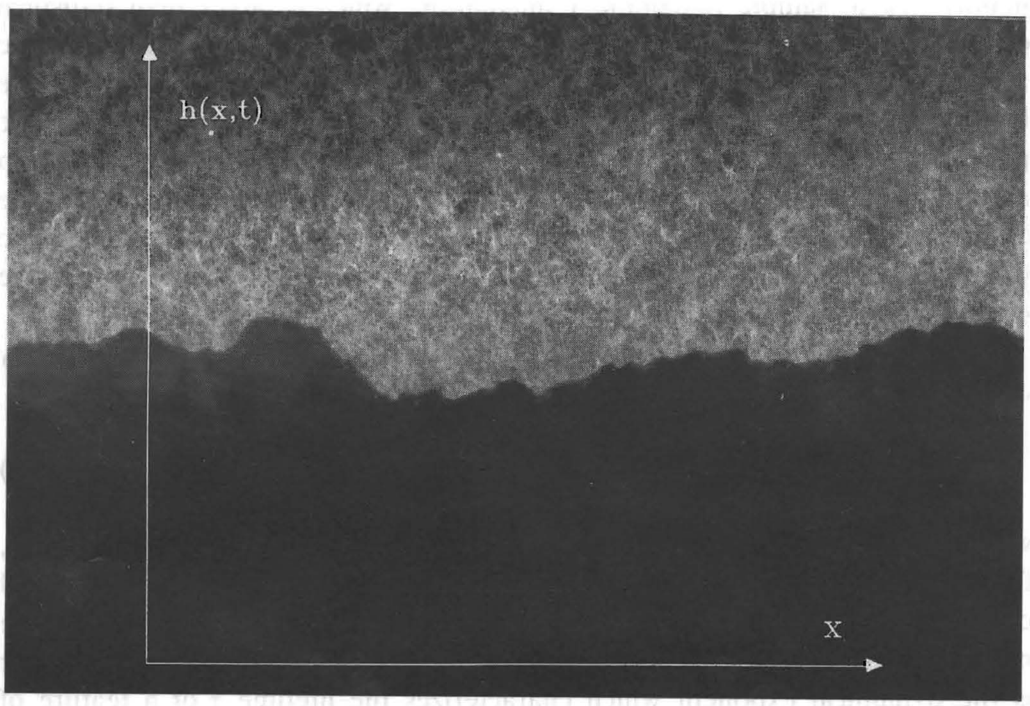

Fig. 1. This photo shows a segment of a piece of burning paper. The transverse size is about 8.5 cm, which is only a small part of the total (~46 cm). Fire is propagating upwards, the smoke indicates slow and laminar air circulation. The actual fire front is a few mm wide, but by using a strong background light the sharp fire line can be identified.

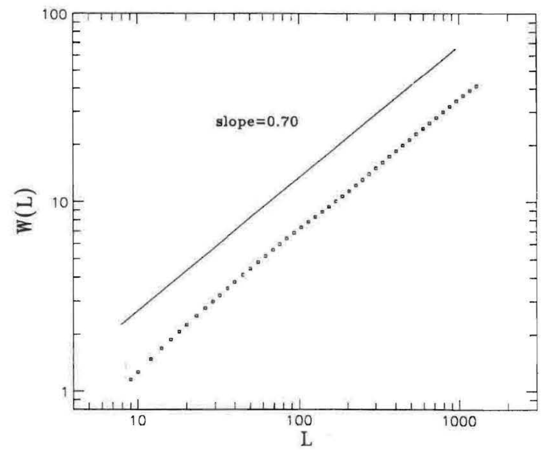

Fig. 2. The mean fire-line width $W(L)$ plotted versus the transverse length L. The data are from a single instantaneous fire line including the segment shown in fig. 1. The average is over different segments by shifting their starting point on the fire line. The data appear to follow a straight line of slope 0.70, over about two decades.

we estimate to be 0.71 ± 0.05. However, we find that the scaling exponent χ is not universal, rather it changes from sample to sample. (The last uncertainty is mainly due to sample to sample χ fluctuation, which is larger than statistical errors within a sample.) The highest point in fig. 2 represents 18.3 cm in the x-direction, the lowest about 0.13 cm. Below the lowest point, the data deviate from a straight line. Here the resolution approaches the scales of pores of the paper, where scaling is not expected. The resolution of the above two-step method proves more than sufficient for our large scale scaling purpose. For the case where the width $W \sim L^{\chi}$, for $\chi < 1$, the fire line is called self-affine in the literature [2]. Since $W/L \to 0$ as $L \to \infty$, on very large scales a fire front is more or less a straight line, viewed from afar.

From studies of computer simulation models of the fire-spread type, Family and Viscek [3] proposed the following scaling relation:

$$W(L, t) = L^{\chi} f(t/L^{z}) , \tag{2}$$

where $f(x)$ is the so-called scaling function who has a simple limit $f(x) \to \text{const}$, for $x \gg 1$. Thus for large t and finite L, where t is proportional to the vertical distance of the burned region (we let about $\frac{4}{5}$ of the paper burn out before measurement), we have $W \sim L^{\chi}$, where χ is called the roughening exponent. z is the dynamical exponent which characterizes the lifetime τ of a feature of length L on a fire line, $\tau \sim L^{z}$.

Presently there is considerable theoretical research on the general interface growth problem, fire front propagation being one of the main motives.

Following the current wisdom in interface growth [2], we consider the approximation that the fire front position $h(x, t)$ is modelled by the non-linear stochastic differential equation [3]

$$\dot{h}(x, t) = \nu \nabla^2 h + \lambda (\nabla h)^2 + \eta(x, t) , \tag{3}$$

where ν and λ are constants, and $\eta(x, t)$ is the noise which obeys the local condition $\langle \eta(x, t)\, \eta(x', t') \rangle = 2\delta(x - x')\, \delta(t - t')$. The left-hand side is the fire line advancement per unit time. The first term on the right-hand side is the diffusion term, which is believed to be always present in local propagation processes. The second, non-linear term is included to take into account the fact that the fire line advances in its local *normal* direction. The effective displacement is the projection in the vertical direction h and it is proportional to $\sqrt{1 + (\nabla h)^2} \sim 1 + \frac{1}{2}(\nabla h)^2 + \cdots$. Only the leading non-linear term $(\nabla h)^2$ is kept.

The last noise term represents instantaneous, local perturbation experienced by the advancing fire front. No matter how uniform a fuel-bed, for example, is made in the preparation of our paper, there are unavoidable local irregularities in the forms of random fibre network, porosity, chemical composition, etc. The noise contribution is actually fixed on the paper, not dependent on time. However, in our experiment, the fire front is moving everywhere, from its own point of view the irregularities are effectively a function of time. Fig. 3 is a microscope picture of magnification 20, where we see that the paper is actually a network of intertwined fibres. All sorts of random factors influence fire propagation, making it difficult to determine exact origins of noise.

Unlike in simulations and analytical studies where noise is taken as strictly local, we cannot rule out possible spatial correlations of the effective noise. Also the heat production does generate a hot air flow which will cause correlation between neighboring sites. Visual observation of the smoke indicates that the hot air has a slow flow in the vertical direction, and does not generate large transverse motion beyond a few centimeters. On the other hand, contributions from spatial correlation are rather well studied [6]: χ ranges from 1/2 to 2/3, the latter value corresponding to the extreme case of the infinite noise correlation limit $\langle \eta(x, t)\, \eta(x', t') \rangle = \delta(t - t')/|x - x'|^{\alpha}$, $\alpha \to 0$. We believe that the spatial correlation in our case is far from the infinite correlation case, and even in the extreme case spatial correlation cannot account for the large value of χ observed in the experiment.

For uncorrelated Gaussian or truncated noise, it has been shown analytically that the roughening exponent is $\chi = 1/2$. Many numerical simulations, not based on eq. (3) but directly on various growth rules, also yield $\chi \simeq 0.5$.

It was pointed out [7] that in real experiments χ may be non-universal, i.e.

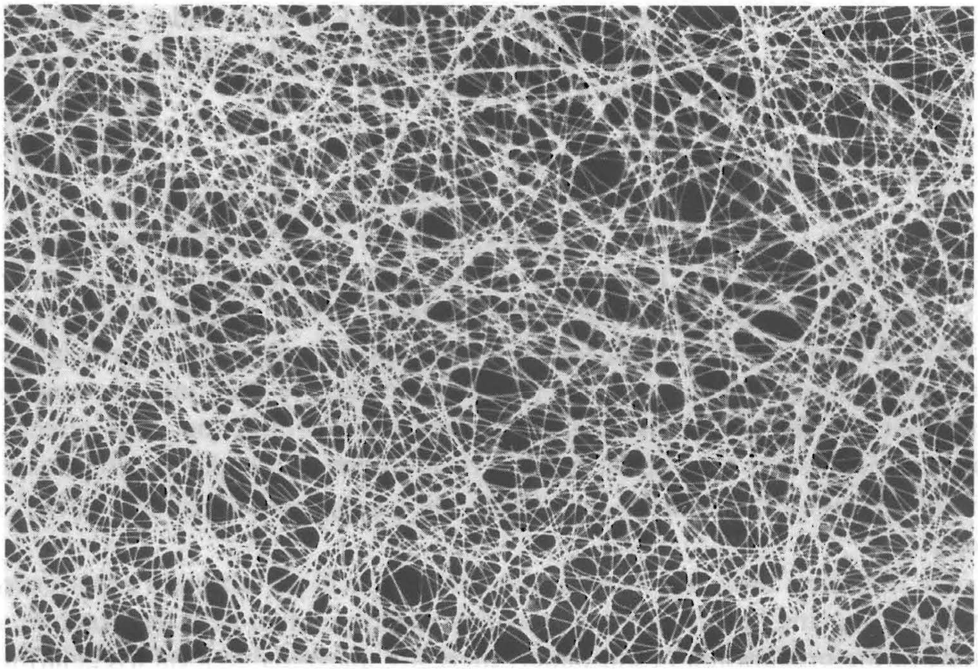

Fig. 3. The apparent uniform paper reveals much of its inhomogeneity at microscopic scales, in this photo at magnification 20. The intrinsic fibre network and uneven KNO_3 concentration on it all contribute to the noise affecting the fire propagation.

the scaling exponent can sensitively depend on, for instance, the higher moments of the noise distribution function, and it is always larger than the theoretical value. A hypothesis is advanced to suggest that the theoretical assumption of Gaussian or truncated noise may not materialize in reality. For uncorrelated noise with a power-law distribution one can show that χ can vary from 1/2 to 1, in numerical simulations [7]. The above result of $\chi > 1/2$ is consistent with this suggestion. Very recently other explanations have been proposed. One suggestion [8] is that pinning is important; this gives long crossover regions which are characterised by a larger pre-scaling exponent. In another work [9] the contact angle contributions in a fluid invasion problem are carefully examined. These alternative explanations can also account for the larger χ value, at least for the fluid displacement experiment [4].

The previous experiments on interface growth may be thought to share the same mechanism as fire propagation. Notably, one [4] is on the water–air immiscible displacement in a random medium; the other [5] is on bacterium-colony expansion on an agar nutrient plate. They report also non-universal, larger values of χ. In fact, power-law noise has been claimed to have been observed in one case [10]. However, these experiments suffer the drawback that bulk material on one or both sides of an interface still participate in the growth dynamics: like the retreating air and the advancing fluid in the fluid displacement experiment. A complete model should take these bulk materials into account as well. This may make an interface-only description like eq. (3) insufficient. In contrast, the present burning experiment is a strict *interface* phenomenon: burned paper becomes ash, thus has no further participation; intact paper does not change until reached by fire.

We have reported a simple paper burning experiment with a roughening exponent consistently larger than 1/2. The fire fronts show interesting self-affine scaling behavior. Besides its implications for real fire propagation like in a forest, our experiment can be regarded as a possible realization of eq. (3). Thus we can benefit much from recent theoretical progresses, and vice versa. We hope to gain insight into general fire-spread processes like forest fires. In fact, Sornette and the authors (in preparation) are analysing satellite pictures of big forest fires around the globe. Scales properly reduced, these pictures may resemble those of paper burning. Even though the arrangement of fibres in the paper and that of trees in a forest are very different, their ways of conducting fire are nevertheless largely similar, for we are interested in scales much larger than individual fibre or tree sizes, respectively.

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References

[1] D.G. Green, A.M. Gill and I.R. Noble, Ecolog. Modelling 20 (1983) 33.
[2] F. Family and T. Vicsek, Dynamics Of Fractal Surfaces (World Scientific, 1991, Singapore), and references therein.
J. Krug and H. Spohn, in: Solid Far From Equilibrium, C. Godreche, ed. (1990), to appear.
[3] M. Kardar, G. Parisi and Y.-C. Zhang, Phys. Rev. Lett. 56 (1986) 889.
[4] M.A. Rubio, C.A. Edwards, A. Dougherty and J.P. Gollub, Phys. Rev. Lett. 63 (1989) 1685.
V.K. Horváth, F. Family and T. Vicsek, J. Phys. A 24 (1991) L25.
[5] T. Vicsek, M. Cserzö and V.K. Horváth, Physica 167 (1990) 315.
[6] E. Medina, T. Hwa, M. Kardar and Y.-C. Zhang, Phys. Rev. A 39 (1989) 3035.
[7] Y.-C. Zhang, J. Phys. (Paris) 51 (1990) 2129; Physica A 170 (1990) 1.
[8] D. Kessler, H. Levine and Y. Tu, Phys. Rev. A 43 (1991) 4551.
G. Parisi, Europhys. Lett. 17 (1991) 673.
[9] N. Martys, M. Cieplak and M.O. Robbins, Phys. Rev. Lett. 66 (1991) 1058.
[10] V.K. Horváth, F. Family and T. Vicsek, Phys. Rev. Lett. 67 (1991) 3207.