

## Physics and Thermodynamics

### Basic Thermodynamics

Ideal gas Eq.  $p = \rho r T$ ,  $r = \frac{R}{M}$

Heat Capacity  $Q = mc\Delta T$

First Princ.  $\Delta U = Q - W$

Idem., diff.  $c_p dT = \frac{dp}{\rho}$ ,  $c_v dT = -pdV$

Latent Heat  $Q = mL$

L Dependence with  $T$   $L = L_0 + (c_{pw} - c)T(^{\circ}\text{C})$

Poisson Eqs.  $\gamma = \frac{c_p}{c_v} \simeq 1.4$ ,  $\kappa = \frac{r}{c_p} \simeq 0.286$ ,

$$pV^{\gamma} = \text{constant}, \quad Tp^{-\kappa} = \text{constant}$$

### Constant values and units

Universal Gas Constant  $R = 8.314472 \text{ J mol}^{-1} \text{ K}^{-1}$

Dry Air Gas Constant  $r_d = 287 \text{ J kg}^{-1} \text{ K}^{-1}$

Water Gas Constant  $r_w = 461 \text{ J kg}^{-1} \text{ K}^{-1}$

Dry air heat capacity  $c_{pd} = 1006 \text{ J mol}^{-1} \text{ K}^{-1}$

Water vapor heat capac.  $c_{pw} = 1846 \text{ J mol}^{-1} \text{ K}^{-1}$

Water latent heat of vap.  $L_v = 2.257 \times 10^6 \text{ J kg}^{-1}$

Water latent heat of fuss.  $L_f = 3.34 \times 10^5 \text{ J kg}^{-1}$

### Radiation Heat Transfer

Wien's Law  $\lambda_M T = 2.898 \times 10^{-3} \text{ m K}$

Stefan-Boltzmann Law  $M_e = \varepsilon \sigma T^4$

### Constant values and units

Boltzmann Constant  $\sigma = 5.6704 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$

## Water Vapor and Humidity

Moist air  $r$  constant  $\bar{r} = qr_w + (1 - q)r_d$ .

Relative Humidity  $h = 100 \frac{e}{E} \simeq 100 \frac{a}{A} \simeq 100 \frac{m}{M}$

Mixing Ratio  $\frac{e}{P} = \frac{m}{\epsilon + m}$

Specific Humidity  $q = \frac{a}{\rho}$  and  $q = \frac{m}{m+1}$

Absolute Humidity  $e = ar_w T$

Useful Relations  $\frac{de}{e} = \frac{dm}{m} + \frac{dP}{P}$  and  $\frac{dh}{h} = \frac{dP}{P} - \frac{dE}{E}$ .

### Clausius-Clapeyron Equation

$$\frac{dE}{dT} = \frac{LE}{r_w T^2} \quad (\text{differential form})$$

$$\ln \frac{E}{E_0} = \frac{L}{r_w} \left( \frac{1}{T_0} - \frac{1}{T} \right)$$

$$\ln \frac{h}{h_0} = \frac{L}{r_w} \left( \frac{1}{T} - \frac{1}{T_0} \right) \quad (\text{only isobaric})$$

### Magnus formula

$$E(T) = A \times \exp \frac{B T[^{\circ}\text{C}]}{C + T[^{\circ}\text{C}]} \quad [\text{hPa}].$$

Water:  $A = 6.1094$ ,  $B = 17.625$ ,  $C = 243.04$

Ice:  $A = 6.1121$ ,  $B = 22.587$ ,  $C = 273.86$

### Virtual Temperature

$$\bar{r}T = rT_v \Rightarrow T_v = T \left( 1 + \frac{3}{5}q \right).$$

### Equivalent Temperature

$$T_e = T + \frac{mL}{c_p} \simeq T + 2a \text{ (g m}^{-3}\text{)}.$$

### Wet-bulb Temperature

$$(c_{pd} + mc_{pw})(T - T_w) = L[M(T_w) - m]$$

$$T_e \simeq T_w + \frac{M(T_w)L}{c_{pd}} \simeq T_h + 2A(T_h)$$

### Constant values and units

Molecular mass ratio  $\epsilon = \frac{M_w}{M_d} = \frac{r_d}{r_w} = 0.622$

Water Gas Constant  $r_w = 461 \text{ J kg}^{-1} \text{ K}^{-1}$

## Atmospheric Processes

Potential Temperature  $\theta = \left( \frac{1000}{P} \right)^{r/c_p} T$

Adiab. Elevation(linear)  $T(z) = T_0 \left( 1 - \frac{\Gamma z}{T_0} \right)$

Adiab. Elevation(exact)  $T(z) = T_0 \left( 1 - \alpha \frac{z}{T_0} \right)^{\Gamma/\alpha}$

Tropospheric Lapse Rate  $T'(z) = T'_0 - \alpha z$

Equilibrium Height  $z_e$  such that  $T'(z_e) = T(z_e)$ .

Stability Index  $\eta = g \frac{\Gamma - \alpha}{T^{\gamma}}$   
 $dh/dT$  in an adiabatic ascent

$$\frac{dh}{dT} = \frac{h}{T} \left( \frac{\bar{c}_p}{r_d} - \frac{L}{r_w T} \right).$$

Exact  $h(T)$  in an adiabatic ascent

$$\ln \frac{h}{h_0} = \frac{\bar{c}_p}{\bar{r}} \ln \frac{T}{T_0} + \frac{\epsilon L}{r_d} \left( \frac{1}{T} - \frac{1}{T_0} \right).$$

Pseudoadiab. Ascent Lapse Rate  $-LdM \simeq c_p dT - VdP$

$$\Gamma_{pseud} = \Gamma \frac{P + \epsilon \frac{LE}{RT}}{P + \epsilon \frac{L}{c_p} \frac{dE}{dT}}$$

Approximate  $h(T)$  in an adiabatic ascent

$$\frac{h}{h_0} = \left( \frac{T}{T_0} \right)^{\frac{\bar{c}_p}{\bar{r}}} - \frac{\epsilon L}{r_d T_0}.$$

Ferrel Formula  $z_s = 122(T_0 - \tau_0) \text{ (m)}$

Väisälä Formula  $z_s = 188(T(^{\circ}\text{C}) + 105) \frac{\log_{10} \frac{100}{h_0}}{\log_{10} \frac{100}{h_0} + 5.1}$

### Constant values and units

Adiabatic Lapse Rate  $\Gamma = \frac{g}{c_{pd}} = 9.8 \text{ K km}^{-1}$

## Polytropic Processes

Polyt. Heat Capacity  $\begin{cases} c_p \rightarrow c_p - c \\ c_v \rightarrow c_v - c \end{cases}$

Polyt. Heat Capacity  $\delta q = cdT$

First Principle Polyt.  $(\bar{c}_p - c)dT + \frac{T}{T} g dz = 0$

## Winds

Gradient Pressure Accel.  $\vec{a}_p = -\frac{1}{\rho} \vec{\nabla} P \rightarrow -\frac{1}{\rho} \frac{dP}{dx}$

Coriolis Accel.  $a_c = 2v\Omega \sin \phi$  where  $\phi$  = latitude  
and  $\Omega = 2\pi/T_{rot}$  angular velocity.

Geostrophic Vel.  $v_g = \frac{1}{\rho f} \frac{dP}{dx}$  where  $f = 2\Omega \sin \phi$ .

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