Meteorology & Climatology RefCard

Physics and Thermodynamics

Basic Thermodynamics

Ideal gas Eq. $p = \rho r T$, $r = \frac{R}{M}$
Heat Capacity $Q = mc\Delta T$
First Princ. $\Delta U = Q - W$
Idem., diff. $c_p dT = \frac{dp}{\rho}$, $c_v dT = -pdV$
Latent Heat $Q = mL$
L Dependence with T $L = L_0 + (c_{pw} - c)T(°\mathrm{C})$
Poisson Eqs. $\gamma = \frac{c_p}{c_v} \simeq 1.4$, $\kappa = \frac{r}{c_v} \simeq 0.286$,

$$pV^\gamma = \text{constant} \ , \ Tp^{\kappa-1} = \text{constant}$$

Constant values and units

Universal Gas Constant $R = 8.314\,472\,\mathrm{J\,mol^{-1}\,K^{-1}}$
Dry Air Gas Constant $r_d = 287\,\mathrm{J\,kg^{-1}\,K^{-1}}$
Water Gas Constant $r_w = 461\,\mathrm{J\,kg^{-1}\,K^{-1}}$
Dry air heat capacity $c_{pd} = 1006\,\mathrm{J\,mol^{-1}\,K^{-1}}$
Water vapor heat capac. $c_{pw} = 1846\,\mathrm{J\,mol^{-1}\,K^{-1}}$

Radiation Heat Transfer

Wien's Law $\lambda_M T = 2.898 \times 10^{-3}\,\mathrm{m\,K}$
Stefan-Boltzmann Law $M_e = \varepsilon \sigma T^4$

Constant values and units

Boltzmann Constant $\sigma = 5.6704 \times 10^{-8}\,\mathrm{W\,m^{-2}\,K^{-4}}$

Water Vapor and Humidity

Moist air r constant $\bar{r} = qr_w + (1-q)r_d$.
Relative Humidity $h = 100\frac{e}{E} \simeq 100\frac{a}{A} \simeq 100\frac{m}{M}$
Mixing Ratio $m = \frac{e}{P} = \frac{m}{\epsilon + m}$
Specific Humidity $q = \frac{m}{m+1}$
Absolute Humidity $e = ar_w T$
Useful Relations $\frac{de}{e} = \frac{dm}{m} + \frac{dP}{P}$ and $\frac{dh}{h} = \frac{dP}{P} - \frac{dE}{E}$.

Clausius-Clapeyron Equation

$$\frac{dE}{dT} = \frac{LE}{r_w T^2} \quad \text{(differential form)}$$
$$\ln\frac{E}{E_0} = \frac{L}{r_w}\left(\frac{1}{T_0} - \frac{1}{T}\right)$$
$$\ln\frac{h}{h_0} = \frac{L}{r_w}\left(\frac{1}{T} - \frac{1}{T_0}\right) \quad \text{(only isobaric)}$$

Magnus formula

$$E(T) = 6.1 \times 10^{\frac{7.45T(°\mathrm{C})}{234.07 + T(°\mathrm{C})}} \ \text{(hPa)} \ .$$

Virtual Temperature

$$\bar{r}T = rT_v \Rightarrow T_v = T\left(1 + \frac{3}{5}q\right) \ .$$

Equivalent Temperature

$$T_e = T + \frac{mL}{c_p} \simeq T + 2a\left(\mathrm{g\,m^{-3}}\right) \ .$$

Wet-bulb Temperature

$$(c_{pd} + mc_{pw})(T - T_w) = L[M(T_w) - m]$$
$$T_e \simeq T_w + \frac{M(T_w)L}{c_{pd}} \simeq T_h + 2A(T_h)$$

Constant values and units

Molecular mass ratio $\epsilon = \frac{M_w}{M_d} = \frac{r_d}{r_w} = 0.622$
Water Gas Constant $r_w = 461\,\mathrm{J\,kg^{-1}\,K^{-1}}$

Atmospheric Processes

Potential Temperature $\theta = \left(\frac{1000}{P}\right)^{r/c_p} T$
Adiab. Elevation(linear) $T(z) = T_0\left(1 - \frac{\Gamma z}{T_0'}\right)$
Adiab. Elevation(exact) $T(z) = T_0\left(1 - \alpha\frac{z}{T_0'}\right)^{\Gamma/\alpha}$
Tropospheric Lapse Rate $T'(z) = T_0' - \alpha z$
Equilibrium Height z_e such that $T'(z_e) = T(z_e)$.
Stability Index $\eta = g\frac{\Gamma - \alpha}{T'}$

dh/dT in an adiabatic ascent

$$\frac{dh}{dT} = \frac{h}{T}\left(\frac{\bar{c}_p}{\bar{r}_d} - \frac{L}{r_w T}\right) \ .$$

Exact $h(T)$ in an adiabatic ascent

$$\ln\frac{h}{h_0} = \frac{\bar{c}_p}{\bar{r}}\ln\frac{T}{T_0} + \frac{\epsilon L}{r_{\ d}}\left(\frac{1}{T} - \frac{1}{T_0}\right) \ .$$

Pseudoadiab. Ascent Lapse Rate $-LdM \simeq c_p dT - VdP$

$$\Gamma_{pseud} = \Gamma\frac{P + \epsilon\frac{LE}{RT}}{P + \epsilon\frac{L}{c_p}\frac{dE}{dT}}$$

Approximate h(T) in an adiabatic ascent

$$\frac{h}{h_0} = \left(\frac{T}{T_0}\right)^{\frac{\bar{c}_p}{\bar{r}} - \frac{\epsilon L}{r_d T_0}} \ .$$

Ferrel Formula $z_s = 122(T_0 - \tau_0)$ (m)
Väisälä Formula $z_s = 188\,(T(°\mathrm{C}) + 105)\,\frac{\log_{10}\frac{100}{h_0}}{\log_{10}\frac{100}{h_0} + 5.1}$

Constant values and units

Adiabatic Lapse Rate $\Gamma = \frac{g}{c_{pd}} = 9.8\,\mathrm{K\,km^{-1}}$

Polytropic Processes

Polyt. Heat Capacity $\begin{cases} c_p \to c_p - c \\ c_v \to c_v - c \end{cases}$
Polyt. Heat Capacity $\delta q = cdT$
First Principle Polyt. $(\bar{c}_p - c)dT + \frac{T}{T'}gdz = 0$

Winds

Gradient Pressure Accel. $\vec{a}_p = -\frac{1}{\rho}\vec{\nabla}P \to -\frac{1}{\rho}\frac{dP}{dx}$
Coriolis Accel. $a_c = 2v\Omega\sin\phi$ where ϕ = latitude and $\Omega = 2\pi/T_{rot}$ angular velocity.
Geostrophic Vel. $v_g = \frac{1}{\rho f}\frac{dP}{dx}$ where $f = 2\Omega\sin\phi$.