Nonlinear optimization

- optimization problem in standard form
- solving optimization problems
- convex optimization problems
- Topics and goals

Optimization problem in standard form

$$\begin{array}{rl} \min & f_0(x) \\ \text{s.t.} & f_i(x) \leq 0, \quad i = 1, \ldots, m \\ & h_i(x) = 0, \quad i = 1, \ldots, p \end{array}$$

- $x \in \mathbb{R}^n$ is the optimization variable
- $f_0 : \mathbb{R}^n \to \mathbb{R}$ is the objective or cost function
- $f_i : \mathbb{R}^n \to \mathbb{R}$, $i = 1, \ldots, m$, are the inequality constraint functions
- $h_i : \mathbb{R}^n \to \mathbb{R}$ are the equality constraint functions

optimal solution: x has smallest value of f_0 among all vectors that satisfy the constraints

Solving optimization problems

general optimization problem

- very difficult to solve
- methods involve some compromise, e.g., very long computation time, or not always finding the solution

exceptions: certain problem classes can be solved efficiently and reliably

- linear programming problems
- convex optimization problems

Convex optimization problem

$$\begin{aligned} \min \quad & f_0(x) \\ \text{s.t.} \quad & f_i(x) \le 0, \quad i = 1, \dots, m \\ & h_i(x) = 0, \quad i = 1, \dots, p \end{aligned}$$

- objective and inequality constraint functions are convex:

$$f_i\big(\alpha x + (1-\alpha)y\big) \le \alpha f_i(x) + (1-\alpha) f_i(y), \quad \text{for } \alpha \in [0,1]$$

- equality constraint functions are linear
- includes linear programs as special cases

solving convex optimization problems

- no analytical solution
- reliable and efficient algorithms
- computation time (roughly) proportional to $\max\{n^3, n^2m, F\}$, where F is cost of evaluating f_is and their first and second derivatives
- almost a technology

using convex optimization

- often difficult to recognize
- many tricks for transforming problems into convex form
- surprisingly many problems can be solved via convex optimization

Topics and goals

topics

- convex sets, functions, optimization problems
- duality
- algorithms: gradient methods, qusi-Newton methods, Newton methods, etc.

goals

- recognize/formulate problems as convex optimization problems
- develop code for problems of moderate size
- characterize optimal solution, give limits of performance, etc.