Indeterminate numerals and their alternatives

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February 2, 2019

Abstract

This paper examines the use of English *some* with numerals, as in *twenty-some(thing)*, drawing parallels to *some*'s use as an epistemic indefinite. In its regular use as an epistemic indefinite, *some* signals uncertainty regarding the precise referent who satisfies a description. When modifying numerals, *some* has an identical flavor, in that it fails to commit the speaker to knowledge of the particular number that satisfies the descriptive claim in the sentence. I show that numerals modified by *some* in this way have both a lower-bounded interpretation (i.e., *twenty-some* is interpreted as *at least twenty-one*) and also an upper-bounded interpretation (*no more than twenty-nine*). Additionally, *some* cannot modify all numerals, which I argue is due to the syntax of numerals. The analysis is couched in a semantic framework where alternatives are represented semantically.

1 Introduction

Approximation in English can be expressed in various ways. For instance, the adverbials *almost* and *approximately* are some ways of expressing that a numerical expression should be construed approximately, that is, to express uncertainty regarding the precise number that expression should denote. Prepositions provide another way of expressing approximation.

(1) a. around ten people
b. between ten and twenty people
c. close to ten people

In this paper, I look at an another expression involving numerals and uncertainty in English. Part of what makes this construction theoretically interesting is its reliance on the epistemic indefinite *some*. This sets it apart syntactically from other instances of approximation, in that the element that is expressing approximation is not an adverbial or a preposition.

Examples of this approximation construction in English, which I will call "indeterminate numerals," are shown in (2). In these examples, *some* appears post-numerally, affixed to the preceding numeral. The interpretation in these examples is one where the indeterminate numeral expresses a range of possible numbers, but where the speaker doesn't know the precise number that satisfies the existential claim expressed by the sentence, as observed by Anderson (2015, 2016).

(2) a. Twenty-some people arrived.
b. His forty-some years of experience were devoted to human resources.
c. I could have it entirely full of small icons and fit a hundred some icons on one screen.

However, these numerals are restricted syntactically. *Some* is not a simple ad-numeral affix, but seems to be integrated within the syntactic structure of the numeral; only numerals which support additive composition, that is, can combined with other numerals with the meaning of addition, can support *some*.

(3) a. *five-some
  b. *ten-some
  c. *fifteen-some

Interestingly, these numerals are also curious in that they simultaneously express both an upper-bounded and lower-bounded meaning. *Twenty-some*, for instance, expresses that any number between 20 and 30 is a possibility. In some ways, this makes them superficially similar to modified numerals such as *at least ten* and *not more than twenty*, but also different from them in having this sort of two-sided meaning.

Discussion of the semantics of numerals has often gone hand-in-hand with that of canonical quantificational determiners like *every* and *most*, with the question of how and whether cardinal numerals differ from quantificational determiners in their type-theoretic properties. Possibilities include treating numerals as quantificational determiners (type $\langle et, \langle et, t\rangle\rangle$; e.g., Barwise and Cooper 1981, Hofweber 2005), as degree quantifiers (type $\langle dt, t\rangle$; see Kennedy 2015), as cardinality predicates (type $\langle e, t\rangle$; e.g., Landman 2003, Rothstein 2013) or predicate modifiers (type $\langle et, et\rangle$; e.g. Ionin and Matushansky 2006), and as degree-denoting terms (type d) with additional functional machinery mediating between the noun phrase and the numeral (e.g., Solt 2015).[1] Likewise, modified numerals like *at least sixty* and *no more than fifteen*, which indeterminate numerals bear some resemblance to, also have generated discussion as to their logical form, particularly about whether they are quantificational determiners (Barwise and Cooper 1981) or degree quantifiers (Nouwen 2010, Kennedy 2015). Indeed, some surveys of quantification devote space to discussion of both modified and unmodified numerals (for instance, Szabolcsi 2010: chapters 9 and 10).

The analysis I discuss in this paper expands on our understanding of these groups of expressions and how quantificational elements like indefinite determiners interact with degrees denoted in domains other than the adjective phrase. These indeterminate numerals show an interaction of degree and quantification due to how properties of the indefinite determiner *some* (particularly, it's ability to force the generation multiple Hamblin alternatives) interact with the numeral to produce quantification over sets of alternatives that vary by degree.

I structure this chapter in the following way. First, in section 2, I discuss additional background data on English indeterminate numerals as well as link them to the broader category of epistemic indefinites. Next, in section 3, I give a syntax for numerals in general that will be necessary to have for the analysis of indeterminate numerals. Section 4 lays out background on the alternative semantics used in the rest of the analysis in the paper. Section 5 develops an account of the semantics of ordinary numerals, while sections 6 and 7 develop the analysis of indeterminate numerals.

2 Background data

2.1 Expanding on the phenomenon

Modified numerals such as *at least 10* and *not more than 20* have bounded interpretations, either lower-bounded (like with *at least*) or upper-bounded (like with *not more than*). What sets indeterminate numerals apart from many other cases of modified numerals is that they are both lower-bounded and upper-bounded. For instance, the numerals in the examples in (2) are associated with the intervals as in (4). The salient fact about this interval is that its lower bound starts at the modified numeral, and has an upper-bound as determined by keeping the base of the modified numeral and increasing the multiplier by one unit. For

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[1] See Geurts 2006 for additional discussion of these issues.

instance, *twenty* is represented as 2×10, so by keeping the base 10 constant and increasing the multiplier from 2 to 3, we arrive at the upper-bound for *twenty-some*. Likewise, *hundred* is represented as 1×100, so the upper-bound of *hundred-some* is represented as 2×100.

(4) a. twenty-some $\rightsquigarrow (20, 30)$
b. forty-some $\rightsquigarrow (40, 50)$
c. hundred-some $\rightsquigarrow (100, 200)$

This makes indeterminate numerals different than approximators, such as *around* and *about*. Although they seem similar in that they involve a number that is close to what is being modified, *around* implicates a halo of numbers centered around the modified numeral (for instance, something like $[18 - 22]$ in (5)), while the interval for the indeterminate numeral is bounded on the lower end by the number denoted by the numeral.

(5) I saw around twenty dogs during my walk today.
(= I saw between 18 and 22 days during my walk today.)

It's tricky to show that there is a particular number that sets the lower bound, due to the epistemic requirement that the speaker do not know the precise number that satisfies the claim. But, if we pair an utterance with a fact about the world that the speaker learns later on, we can show that the utterance was either true or false. When we pair (6) with (7a), where the fact of the matter is that there was a number of dogs incompatible with *twenty-some*, namely 19 dogs, the sentence is judged false. However, if (6) is paired with (7b), where the fact is that there were actually 23 dogs the speaker saw, then the utterance is judged to be true. This shows that the utterance really is lower-bounded by the numeral that is being modified.

(6) I saw twenty-some dogs during my walk today.

(7) a. *Speaker later learns he saw only 19 dogs:*
(6) is judged to have been false.
b. *Speaker later learns he saw 23 dogs:*
(6) is judged to have been true.

Moreover, the lower-bound is at the modified numeral, but does not include the denotation of said numeral. The examples in (8) and (9) are quite marginal, providing evidence that the lower-bound for e.g. *twenty-some* does not include the number 20, but rather starts at 21.

(8) ??I saw twenty-some dogs today, namely exactly twenty.

(9) *(Situation: John ate exactly twenty cookies.)*
??John ate twenty-some cookies.

Returning to the question of how and where *some* is licensed, what we observe is that indeterminate numerals in English are only possible if the modified numeral is one that can combine additively with another numeral. When the numeral cannot combine additively with another numeral, as is the case with *one* through *nineteen*, an indeterminate numeral is impossible.

(10) a. *ten-some
b. *five-some

(11) a. *ten-five (expected: 15)
b. *five-one (expected: 6)

Moreover, *some* does not have to occur after the entire phrase corresponding to the numeral. If a smaller constituent can combine additively with another numeral, *some* can appear in that position, as in (12).

(12) More than half of the expenditure of eighty-some thousand dollars is for soft costs.

Not previously noticed, to the best of my knowledge, is that indeterminate numerals under attitude verbs can have their ignorance anchored to either the holder of the attitude or to the speaker. The example in (13) demonstrates this, where John either expressed uncertainty about the precise number of people at a party, or the speaker expresses uncertainty about the particular number of people John said were at the party.

(13) John said that twenty-some people were at the party.
   a. John said how many people were at the party, but I don't know precisely what he said.(speaker ignorance)
   b. John said something and expressed ignorance as to the precise number of people at the party. (subject ignorance)

Perhaps unsurprisingly, these same facts are also found with measure phrases. This shows that we are looking at a phenomenon that is quite generally related to measurement and degree, and not only to counting constructions within the DP.

(14) a. The Empire State Building is 440-some meters tall.
   b. He is 20-some years old.

An understanding of the position of *some* in the syntax of the numeral, the lower and upper-bound of the scale, and the ignorance in the construction form the basic desideratum of an account of English indeterminate numerals.

2.2 Indeterminate numerals as epistemic indefinites

The driving idea behind the analysis is that indeterminate numerals like *twenty-some* are a variety of epistemic indefinite. Epistemic indefinites are indefinites that convey ignorance on the part of the speaker as to the particular referent of some nominal expression. They are quite robustly attested cross-linguistically with examples in English (*some*), German (*irgendein*), Spanish (*algún*), Romanian (*vreun*), Hungarian (*vagy*), and Japanese (the *wh-ka* series of pronouns).[2]

Rather than express ignorance as to the identify of an individual, however, what the indeterminate numeral does is express ignorance as to the precise number that satisfies a description. In other words, while ordinary epistemic indefinites contribute uncertainty as to the witnessing individual for a linguistic description, indeterminate numerals contribute uncertainty with respect to the witnessing number. To motivate this view that indeterminate numerals really are epistemic indefinites, we have to first compare their properties with another well-known epistemic indefinite. The epistemic indefinites that I compare the indeterminacy-building element in indeterminate numerals to are *some* in its canonical determiner use, as well as Spanish *algún*.

Some implicates that the speaker doesn't know the precise identity of the person being referred to. The examples in (15) and (16) below (attributable to Strawson (1974)) demonstrate this contrast with *a* and *some*. While person B cannot ask the question about who was shot in the exchange in (15), due to person A having used *some*, this is allowed in (16), due to the indefinite *a* being compatible with knowledge on the part of the speaker.

(15) A: Some cabinet minister has been shot!
   B: #Who?

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[2] See Haspelmath 1997 and Alonso-Ovalle and Menéndez-Benito 2013 for overviews.

(16) A: A cabinet minister has been shot!
B: Who?

Comparing the behavior of the indeterminate numeral to *some*, we can see that it requires the same expression of ignorance. This is illustrated in (17), where someone cannot follow-up an utterance that uses an indeterminate numeral by asking for an exact quantity.

(17) A: Twenty-some students are taking my class this semester
B: #How many?

Alonso-Ovalle and Menéndez-Benito (2010) note that the ignorance inference with *algún* can be reinforced with other linguistic material. This sets it apart from presuppositional content and asserted content, which cannot be reinforced, due to being entailed. Thus, the fact that the ignorance inference can be reinforced suggests that the inference is not entailed, but is rather an implicature. (18) demonstrates this with *algún*, where the clause following *pero* 'but' reinforces the ignorance expressed in the first clause. (19) demonstrates an equivalent sentence in English, where the epistemic indefinite determiner *some* is used.

(18) María sale con algún estudiante del departamento de lingüística, pero no sé con
María goes out with ALGUN student of the department of linguistics, but not I know with
quién
whom.
'María is dating some student in the linguistics department, but I don't know who.'

(19) Mary is dating some student in the linguistics department, but I don't know who.

Likewise, the expression of ignorance in the indeterminate numeral can be reinforced, drawing an additional parallel between known epistemic indefinites like *some* and *algún* on one hand, and indeterminate numerals.

(20) Mary cooked twenty-some pies, but I don't know exactly how many.[3]

Alonso-Ovalle and Menéndez-Benito (2010) argue that Spanish *algún* is compatible with partial ignorance. A speaker using *algún* is not committed to total ignorance regarding the witness of an existential claim, merely that they cannot in principle narrow the domain of the indefinite to fewer than two choices. If *algún* required total ignorance, that all epistemic possibilities are available, examples such as the one in (21) would be malformed, due to restrictions being placed on the set of alternatives. As (21) show, however, *algún* doesn't require that all possibilities be open, only that there be at least two. This also seems to hold for *some*, in that a similar example in English is also perfectly licit in the same scenario.

(21) Scenario: María, Juan, and Pedro are playing hide-and-seek in their country house. Juan is hiding. María and Pedro haven't started looking for Juan yet. Pedro believes that Juan is not hiding in the garden or in the barn: he is sure that Juan is inside the house. Furthermore, Pedro is sure that Juan is not in the bathroom or in the kitchen. As far as he knows, Juan could be in any of the other rooms in the house. Pedro utters:

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[3] This example gets worse or even unacceptable if *exactly* is left off: **Mary cooked twenty-some pies, but I don't know how many.* My suspicion is that this is due to a clash between *twenty-some* committing the speaker to some measure of pies (just not an exact measure), and *I don't know how many* committing the speaker to total ignorance. Since the speaker does assert he knows some number, just not the precise number, he can't go on to further assert he doesn't know the number at all.

Juan tiene que estar en alguna habitación de la casa.
Juan has to be in ALGUNA room of the house
'Juan must be in a room of the house.' (based on Alonso-Ovalle and Menéndez-Benito 2010: (14) & (15))

(22) Juan must be in some room in the house.

Mendia (2018) makes a similar observation for indeterminate numerals; these numerals are also compatible with partial ignorance regarding the witnessing number, as shown in (23). A score in basketball is usually two points, but a triple is worth three points; adding additional information about the manner of scoring in this way serves to narrow down the set of possibilities for how much Michael Jordan actually scored.

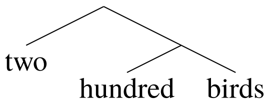
(23) That night Michael Jordan scored twenty-some points in triples. (Mendia 2018: (43))

To conclude this section, indeterminate numerals appear to pattern with other epistemic indefinites in that they also enforce an epistemic requirement on the speaker that the speaker not be able to make a precise claim as to the identity of the referent. With respect to numbers, this amounts to the speaker not being able to commit as to which particular number satisfies a description. This is similar to the behavior of *some* and *algún*. Moreover, like *algún* and *some*, the ignorance inference can be reinforced, making it pattern with implicatures rather than presuppositions and assertions. In the next sections, I'll develop an analysis of indeterminate numerals that uses insight from Alonso-Ovalle and Menéndez-Benito (2010)'s analysis of *algún*, and show how the ignorance inference can be generated as an implicature.

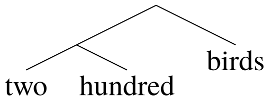
3 A syntax for numerals

3.1 Indeterminate numerals are in specifiers

The syntax of numerals has largely revolved around two competing approaches, what Danon (2012) calls the head-complement construction and the spec-head construction. Although the precise details regarding various proposals for these types of approaches vary, what primarily differentiates them is whether multiplicative numerals, such as *two hundred*, are constituents (to the exclusion of the NP they appear along with) or are represented hierarchically along the spine of the noun phrase. The possibilities are schematically represented in (24) and (25). In (24), the head-complement approach, the numerals are located along the spine of the tree, with the base numeral *hundred* taking the noun as its complement. This contrasts with the spec-head approach in (25), where the numeral itself is a constituent, to the exclusion of the noun.

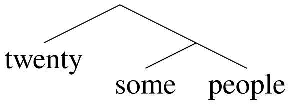
(24)


(head-complement)

(25)


(spec-head)

The facts regarding indeterminate numerals suggest that the appropriate structure in their case is a structure along the lines of (25), where the numeral (including *some*) form a constituent. The argument is as follows. Suppose that the indeterminate numeral structure is as in (26), where *some* has as its complement the NP.

(26)


(rejected analysis)

We observe that, in addition to *some*, English also allows for an equivalent structure containing *something*, as in (27).

(27) Similarly, Lauren and the other twenty-something people I observed had some structured group meals [...] (Google)

The ability of *something* to also appear in the indeterminate numeral construction is important, because it shows that *some* cannot be taking the NP as its complement. We are able to see that *something* cannot merge with an NP (possibly due to *some*'s complement already being filled by the noun *thing*). If the structure for indeterminate numerals were as in (26), with *something* in the same position as *some*, we would be forced to assume the existence of two separate instances of *something*: one that is incapable of appearing with a noun, and another that can have a noun as its complement, as in (29).

(28) some(*thing) people

(29) [twenty [something years]] old (rejected analysis)

Moreover, the existence of numeral internal *some* also points to *some* not taking the noun as a complement in examples such as *twenty-some people*.

(30) a. twenty-some thousand dollars
b. forty-some million Germans

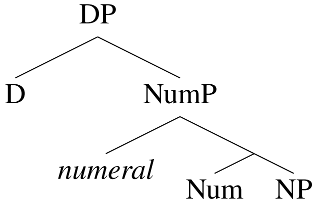
(31) [twenty [some [thousand dollars]]] (rejected analysis)

We might have analyzed these as *some* taking the numeral as a complement again, but as I point out in Anderson (2014), *some* plus a numeral gets a kind of approximative interpretation.[4] Thus, we would expect that the constituent *some thousand* get interpreted as an interval centered on 1000 (e.g., *some thousand* ≈ [990,1010]), and the entire indeterminate numeral have the interpretation "twenty counts of some thousand". But this is not what this means: *twenty-some thousand* is most naturally interpreted as a possible range of thousands, starting at twenty-one thousand and ending at twenty-nine thousand, as we would expect if *some* were a constituent with *twenty* and not *thousand*. Thus, the analysis in (31) must be rejected.

Therefore, I analyze *some* as forming a constituent with the numeral, and not with the noun following it.

In order to link the numerosity denoted by the numeral up with the noun, I assume that a covert element is used in order to overt provide a degree argument. Some theories suppose a covert typeshift or adjective MANY/MUCH, which does the job of providing a degree argument via a measure function over individuals, returning their cardinality. Taking an approach closer to Solt (2015) and others who assume functional material in the DP mediating between numerals and the lexical NP, I syntacticize the measure function and place it in a functional head sister to the NP, Num (semantics to follow in later sections). The resulting syntactic structure thus looks like the following in (32).

(32)



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[4] But see Stevens and Solt 2018, who argue that that construction is not truly an approximative on par with *around* and *about*.

3.2 ADD and complex numeral structure

Ionin and Matushansky (2006), although they argue that numerals have their position along the spine of the tree rather than being in a specifier, still need to address the fact that some numerals have additional complexity due to having an additional numeral they combine with additively, such as with *two hundred twenty* (= (2 × 100) + 20). Rather than analyze these as recursive head-complement structures, they propose that these numerals are actually coordinate structures, in English making use of a phonologically null *and*.

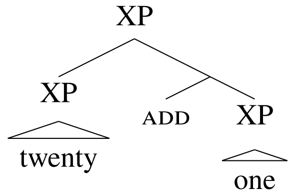
But, direct evidence can be found with languages that overtly realize this conjunction; for instance, as shown in (33), both Spanish and German overtly realize an element meaning *and* in at least some numerals with additive complements, and English even optionally allows for *and* in some environments, as shown in (34). (See Ionin and Matushansky 2006 for additional details.)

(33) a. fünfundzwanzig (German)
five.and.twenty
b. treinta y cinco (Spanish)
thirty and five

(34) one hundred (and) one

I take complex additive numerals, then, to have the structure in (35), which follows Ionin and Matushansky (2006) in the use of a covert coordination element. Departing slightly from Ionin and Matushansky, I call this ADD. This use of ADD builds on even earlier work by Hurford (1975), who develops an early account using phrase structure rules for how numerals are constructed in English and a selection of other languages. Hurford observes that syntactic positions are correlated with particular modes of composition (additive or multiplicative), and the use of a coordination-like element encoding the mode of composition essentially syntacticizes this earlier insight.[5]

(35)



3.3 NUMBER as the complement to *some*

As demonstrated previously, English indeterminate numerals are only possible with additive numeral constructions. I analyze the *some* component of the construction as being like a numeral, albeit an indefinite numeral. In keeping with the pragmatic parallels between *-some* in the indeterminate numeral and the more canonical determiner *some*, I analyze *some* in this construction as a determiner as well, taking an NP complement.

Moreoever, being in a complex numeral construction, *some* is combined with the numeral that it modifies via the ADD coordinator described in the previous section. The structure for indeterminate numerals is as in (36). I assume that the NP complement to *some* is a silent noun NUMBER. A covert nominal of this sort has been proposed to be at work in other phenomenon using numerals; Kayne (2005) proposes that *few* and *many* modify a silent noun NUMBER, while Zweig (2005) makes use of it in his syntax of numerals.

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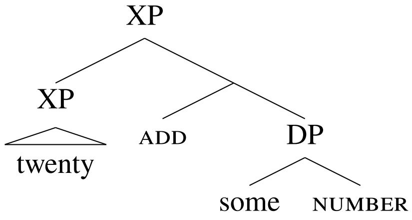
[5] Hurford (1975) predates many contemporary syntactic notions; additive and multiplicative composition are rules for semantic interpretation assigned to particular phrase structure rules, rather than read off of terminals in the tree as in our current tradition. Regardless, he clearly has the view that structure plays a role.

(36)



3.4 Blocking of illicit numerals

An issue that I will take as secondary in this paper is the issue of malformed numerals such as **twenty-eleven* (for *thirty-one*) and *forty-fifteen* (for *fifty-five*). The problem of how to constrain the compositionality of the numeral system has been vexing problem since at least Hurford (1975). One possibility that I speculate about in Anderson 2016 is the use of syntactic features corresponding to numerical bases (e.g., ones, tens, and hundreds). Additive numerals could use feature checking systems to ensure that the numerical base of their sister is smaller. Mendia (2018) independently develops a similar strategy, with the intention of generalizing to bases other than ten, but encodes this information in NUMBER rather than in features.

I'm skeptical at putting too much of the machinery regarding compositionality and numerals in the syntactic component itself, due to a lack of direct evidence for particular proposals (my own feature-checking proposal included). Moreover, conventions regarding numerical well-formedness can sometimes be flouted (discussed more below), which suggests to me that it is not (entirely) the syntactic component determining what the numerical form for a number should be. Instead, I propose that at least some of the work in ruling out particular numerals is handled by the pragmatic system.

For instance, Bogal-Allbritten (2010) proposes a neo-Gricean principle called **Avoid Synonymy** (see (37)), meant to explain the distribution of evaluativity with comparative aspect and absolute aspect marked verbs in Navajo.[6] Perhaps we might consider the use of such a pragmatic principle in generally ruling out marked examples of numerals.

(37) **Avoid Synonymy** (Bogal-Allbritten 2010)
Avoid a derivation producing an expression that has the same truth conditions as a competing derivation containing a less marked adjective.

More generally, we might consider the lack of forms such as *twenty-eleven* or *thirty-fourteen* as being ruled out by more general principles related to blocking, the phenomenon where marked forms are blocked by more unmarked forms. A canonical example is how the derivationally transparent but marked noun *stealer* is blocked by the lexicalized form *thief*; both have identical meanings, at least on a naive view, but the conventionalized form *thief* is preferred over the form *stealer*.

We might object that blocked forms do surface occasionally; *stealer* does have a meaning and occasionally surfaces, for instance. Blocked forms of numerals occasionally surface as well, although admittedly they are somewhat rarer. For instance, the numerical base for thousands in English can be re-expressed with a base for hundreds, provided the multiplicative numeral itself has an increase in its base (see (38)). Counting can be done incorrectly using numbers of too high a base (39a), and numerals constructed in this way can sometimes be used for humorous effect, such as when someone wants to make a comment on their age (39b).

(38) a. two thousand five hundred (=2500)
b. twenty-five hundred (=2500)

(39) a. ..., thirty-nine, thirty-ten, thirty-eleven, ...

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[6] See also Rett 2014 for additional discussion regarding evaluativity and markedness.

b. Well Rob, I just turned thirty-eleven (and I do give you credit for the phraseology on that), and I saw my first silver hairs about 3 years ago.[7]

Although how precisely blocking is to be formalized is still a matter of debate,[8] I do not think it is problematic to assume that blocking (whatever its nature) plays a role in the generation and subsequent filtering of possible numerals.

Additionally, we might also worry about the lack of numerals such as *somety-five*, where *some* appears in the multiplicative (rather than additive) position within the numeral. However, cross-linguistic data shows that some languages, such as Japanese, do allow for an indefinite-like element (e.g., *-ka* in Japanese) in both additive and multiplicative positions (see (40) and (41)).

(40) Juu -nan -nin -ka -ga kita. (Japanese)
ten -what -CL -KA -NOM came
'10 plus *x* people came.'

(41) Nan -juu -nin -ka -ga kita. (Japanese)
what -ten -CL -KA -NOM came
'*x* multiple 10 people came.'

That there exist languages that do allow for the equivalent of *somety-five* suggests to me that it is properties of the syntax of numerals in specific languages that determines whether or not indeterminate numerals that abstract over the multiplier can be constructed. More detailed study of the fine-grained syntax of numerals in English and Japanese (for instance) would need to be conducted to understand this variation.

4 Grammatical alternatives

One approach to the analysis of questions sentences has been to model them as being sets of propositions, following Hamblin (1973) and Karttunen (1977).[9] In this sort of approach, the meaning of a question is a set of propositions corresponding to answers to the question; a question such as *Who left?* would be represented as in (42), a set of propositions varying on the individual who did the leaving. What the meaning of question is, then, is a set of alternatives which raise an issue as to which alternative is the true alternative.

(42) $[\![\textit{Who left?}]\!] = \{\lambda w \exists x.\mathbf{leave}_w(x) \mid \mathbf{person}(x)\}$

I take the contribution of indeterminate numerals as expressing a set of alternatives. In expressing a set of alternatives, the speaker raises the issue of which of the alternatives holds true in the actual world, and implicates their ignorance, uncertainty, or indifference as to which alternative is the true alternative.

I explicitly represent the alternatives as part of the compositional semantic meaning of the sentence. The best known system that does this is that of Kratzer and Shimoyama 2002, who consider denotations to be sets of alternatives. Systems like this have been used to model not just the familiar cases of questions (Hamblin 1973), and focus (Rooth 1985), but also topichood (Büring 1997), indefinites (Alonso-Ovalle and Menéndez-Benito 2003), pronouns (Kratzer and Shimoyama 2002, Kratzer 2005), modified numerals (Coppock 2016), and scalar implicatures (Chierchia 2004).

In a Hamblinized system such as this, where alternatives are represented as part of the compositional semantics, it's necessary to have a mode of composition separate from ordinary Function Application (Heim and Kratzer 1998) that can put sets of functions together with their arguments—namely, what's necessary is to have a mode of composition where we can act like we're working with functions, but in reality be composing

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[7] http://belovedmonsterandme.blogspot.com/2007/02/grey-anatomy.html

[8] See Embick and Marantz (2008) for discussion.

[9] These two approaches are not quite equivalent; for Karttunen (1977), a question corresponds to the set of true answers.

sets of alternatives with each other. The intuition behind this new mode of composition, Pointwise Function Application, is to apply all the objects from one set of alternatives to all the objects from another set of alternatives pointwise, creating another set of alternatives. This is formalized in (43) below.

(43) **Pointwise Function Application** (based on Kratzer and Shimoyama 2002)
If α is a branching node with daughters β and γ, and $[\![\beta]\!]^{d,C} \subseteq D_{\sigma}$ and $[\![\gamma]\!]^{d,C} \subseteq D_{\langle\sigma,\tau\rangle}$, then $[\![\alpha]\!]^{d,C} = \{c(b) \mid b \in [\![\beta]\!]^{d,C} \wedge c \in [\![\gamma]\!]^{d,C}\}$

Singleton sets of alternatives compose in more or less the usual way; one member of the set of alternatives applies to the member of the other set. Where things get more interesting is when multiple alternatives are present. Function application applies pointwise, so that each alternative in the first set is applied to each alternative in the second. In this way, these alternatives "fan outwards" (to borrow phrasing from Coppock (2016: 472)), creating expanding sets of alternatives.

The set of alternatives generated by repeated application of the Pointwise Function Application rule is existentially closed via an existential closure operator in the tree. This operator is associated with the following rule:

(44) **Existential Closure** (adapted from Alonso-Ovalle 2006)
Where $[\![A]\!] \subseteq D_{\langle st,t\rangle}$, $[\![[\ \exists\ A\ ]]\!] = \{\lambda w.\exists p[p \in [\![A]\!] \wedge p(w)]\}$

This system will be put to use in the following sections.

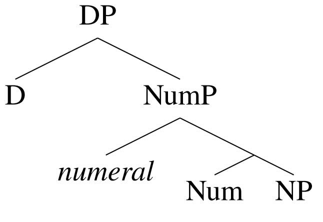
5 Semantics of ordinary numerals

First, I assume a degree semantics for cardinal numerals, following a similar move by Solt for quantity words such as *few* and *many*. Departing from Solt (2015), however, I treat simple numerals as directly denoting degrees, objects of type d. This makes a cardinal such as *twenty* have the denotation in (45). Note that this denotation has already been Hamblinized; in a non-Hamblinized system, *twenty* would simply denote the degree 20. Here, it denotes the set containing only the degree 20.

(45) $[\![\textit{twenty}]\!] = \{20\}$

Syntactically, numerals are inserted in the specifier of a NumP projection, as in (46), breaking with the syntax proposed by Ionin and Matushansky (2006) and more in line with proposals by Solt (2015) and others. NumP dominates the NP projection, but is still contained in DP. The role of Num head is to measure the cardinality of an individual (using a measure function for cardinality of individuals μ), and relate this to the denotation of the numeral in SpecNumP. How this is done is shown in (47).

(46)



(47) $[\![\text{Num}]\!] = \left\{\lambda f_{\langle e,st\rangle}\lambda d_{d}\lambda x\lambda w.\mu_{w}(x) = d \wedge f_{w}(x)\right\}$

Putting these pieces together, the derivation for *twenty people* would look as in (49).[10] Num takes the NP headed by the lexical noun as an argument, and their denotations compose via the Pointwise Function

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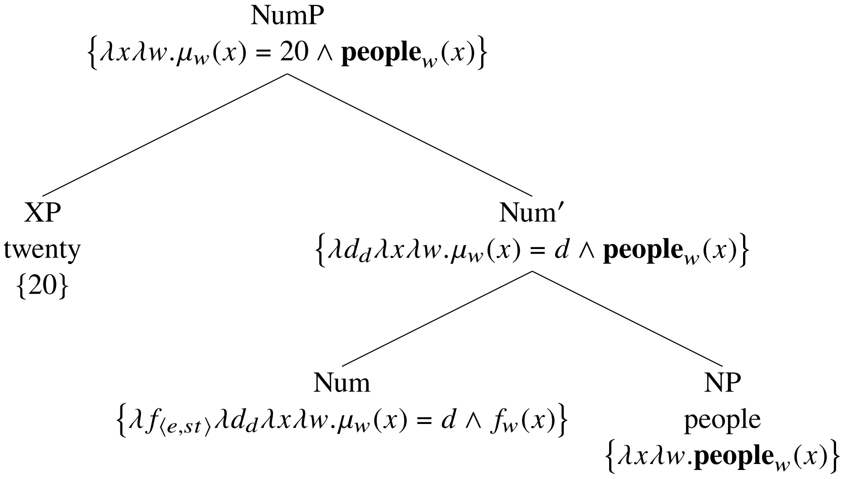
[10] It might be the case that *twenty* can be syntactically decomposed into *two* and *-ty*. This additional detail doesn't play a role in this paper, though see Mendia 2018 for discussion of multiplicative numerals with assumptions that are compatible with mine.

Application rule. This merges with the numeral, and the numeral saturates the degree argument of Num, resulting in the singleton set containing the intensional property of being a plurality of people who measure twenty.

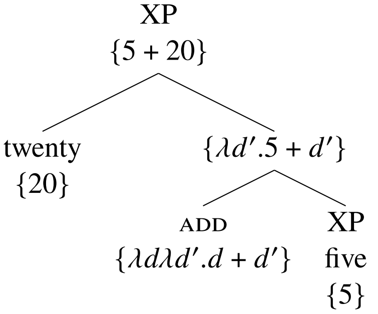
(48) $[\![\textit{people}]\!] = \{\lambda x \lambda w.\mathbf{people}_w(x)\}$

(49)



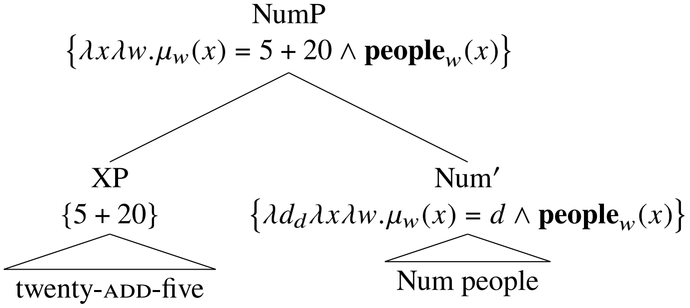
A complex, but non-indeterminate numeral can be given a similar analysis. First, the numeral is composed using ADD.

(50)



This numeral can then compose with its sister, an intermediate projection of NumP, via Pointwise Function Application.

(51)



In this way, no typeshift is necessary to get numerals to be an argument of Num. Numerals simply are names for degrees, and thus can directly serve as arguments to Num.

6 Analysis of indeterminate numerals

6.1 Previous analysis: Anderson (2015)

Anderson (2015) provides an analysis of English indeterminate numerals with *some*. In this analysis, the *some* element in the numeral merges with a phonologically null noun NUMBER. Similarly, the *some NUMBER* constituent combines with an additive numeral using a covert coordinate element ADD. However, an important difference is that, due to a typeclash between the semantics of Num and the indeterminate numeral, they must be lifted to the type of a generalized quantifier over degrees ($\langle dt, t\rangle$) and must raise out of the DP via quantifier raising. Schematically, this is shown in (52), where the indeterminate numeral raises to the left edge of TP and leaves behind a trace of type d as in the familiar Heim and Kratzer (1998) mode of analysis, suitably extended to type d. QR is necessary in order to fix the typeclash generated when an epistemic numeral is used; as Anderson argues, Partee's (1987) BE typeshift is not available with indeterminate numerals due to the numeral not being expressible as a singleton: these numerals must be represented as a set of degrees. In order to have the indeterminate numeral be type-compatible with its sister, the indeterminate numeral needs to be lifted to the type of a generalized quantifier over degrees, $\langle dt, t\rangle$, and then undergo quantifier raising.

(52) [*twenty-some*$_i$ λ_i [... [t_i [Num NP]]]]

However, this analysis has two problems. First, the DP itself is an island to movement, via familiar constraints on extraction out of definite DPs. Moreover, left-branch extraction of numerals is not generally permissible in English, and so it is suspicious that this construction would allow movement of the numeral, even if it is at LF; extraction of a numeral seems possible only when it pied-pipes the NP it counts over, as seen in (53).

(53) a. *[How many]$_i$ did John see [t_i dogs]? (John saw fifteen dogs.)
b. [How many dogs]$_i$ did John see t_i?

Additionally, the analysis also runs aground due to what Bhatt and Pancheva (2004) call the Heim-Kennedy Constraint.[11] The Heim-Kennedy constraint is based on the observation that DegPs do not take scope over QPs. Bhatt and Pancheva (2004) suggest that is should be considered as a constraint on degree abstraction, and not simply DegP, as schematized in (55). Given this formulation, the analysis in Anderson (2015) would violate the constraint, as quantifier raising of indeterminate numerals out of the DP involves degree abstraction.[12]

(54) **Heim-Kennedy Constraint** (as cited in Bhatt and Pancheva 2004: 15)
If the scope of a quantificational DP contains the trace of a DegP, it also contains that DegP itself.

(55) $*\lambda d \ldots \text{QP} \ldots d \ldots$ (Bhatt and Pancheva 2004: (25))

Taken together, these problems point in a different direction for the analysis of indeterminate numerals. An analysis where numerals are properties of degrees and must QR out of the DP cannot be correct, due to it violating several well-known constraints on movement in English. The analysis of indeterminate numerals I build in the following sections solves this problem.

6.2 The meaning of *some*

Based on the similarities with *algún*, I propose treating *some* in a similar way, in particular supposing that *some* triggers minimal domain widening via an anti-singleton constraint. This follows a similar idea by Weir (2012), who analyzes the determiner *some* as making use of a subset selection function f that is constrained

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[11] I thank Nicholas Fleisher (p.c.) for suggesting this line of thought.

[12] This also relies on an assumption that a QP takes scope over material within the QP.

to have a non-singleton codomain. In this way, *some* (like *algún*) can generate an implicature that the speaker cannot (or will not) narrow the domain to a single alternative, modeling the epistemic effect.

(56) $[\![some]\!] = \lambda f_{\langle et,et\rangle}\lambda P \lambda Q : \textbf{anti-singleton}(f).\exists x[f(P)(x) \wedge Q(x)]$ (Weir 2012: (14))

Where I will depart from this analysis is in treating *some* as a quantificational determiner. Rather, building on previous work in alternative semantics (Kratzer and Shimoyama 2002, Szabolcsi 2015), I consider *some* to actually signal the presence of two operations. The first is an existential operator at the clause level that provides existential closure over alternatives (flattening them into a single proposition). This operator has been mentioned already, as (44), repeated below. An analysis of *some* as a quantificational determiner, as in Anderson (2015) and Weir (2012), requires that *some* introduce existential quantification. In a fully Hamblinized semantic system like the one I develop here, indefinites introduce sets of alternatives (see also Alonso-Ovalle and Menéndez-Benito (2003), AnderBois (2011)), with the existential closure operator performing the role that existential quantification provided in the determiner *some*.

(44) **Existential Closure** (adapted from Alonso-Ovalle 2006)
Where $[\![A]\!] \subseteq D_{\langle st,t\rangle}$, $[\![[\ \exists\ A\ ]]\!] = \{\lambda w.\exists p[p \in [\![A]\!] \wedge p(w)]\}$

The second operation is marked by a morpheme ANTI-SINGLETON (abbreviated in trees as A-S). It has the role of ensuring that the alternatives generated by the constituent sister to ANTI-SINGLETON are a non-singleton set of alternatives. This mirrors in some respects the anti-singleton presupposition in Alonso-Ovalle and Menéndez-Benito's (2010) discussion of Spanish *algún*; where the anti-singleton presupposition in *algún* restricts the subset selection function in *algún* to having a non-singleton co-domain, ANTI-SINGLETON ensures that the set of alternatives generated at the point in the tree where ANTI-SINGLETON is merged are not a singleton. In English, ANTI-SINGLETON is spelled out as *some*.

ANTI-SINGLETON is somewhat usual: Because it needs access to the set of alternatives itself, rather than the individual alternatives within the set, it needs to exist in some sense "outside" of the normal composition rule in the semantic system I'm assuming, Pointwise Function Application. Instead, it combines with its argument via ordinary function application.[13] ANTI-SINGLETON takes a set of alternatives, presupposes that a contextually defined subset selection function yields a non-singleton subset of the set of alternatives, and then passes that subset up the tree for computation. This is given in (57).

(57) $[\![\text{ANTI-SINGLETON}]\!] = \lambda p_{\langle \sigma,t\rangle}\text{: } \textbf{anti-singleton}(f).f(p)$

What makes ANTI-SINGLETON important in this analysis is that, by using it, the speaker signals that they are forcing the semantic derivation to include at least two alternatives, and hence signaling that there are multiple epistemic possibilities at issue. In this way, by forcing multiple alternatives, the speaker can generate the implicature that they are ignorant towards which possibility is the true possibility in the world of evaluation.

7 An alternative semantics for indeterminate numerals

7.1 Upper-boundedness as implicature

Anderson (2015) speculates that the upper-boundedness of indeterminate numerals comes from competition with other, larger numerals, and that an implicature can be used to derive the upper-boundedness, although

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[13] As a reviewer points out, there is a tension here in that this analysis needs both the usual Function Application rule as well as Pointwise Function Application. Moreover, this representation format also muddies the distinction between sets of alternatives and characteristic sets. Moving to a compositional system such as that proposed by Charlow (2014, in press) might make resolve this situation, as a reviewer suggests. I'm sympathetic to such a move, but postpone the question of if and how my present analysis can be formulated with assumptions closer to Charlow's for another time.

how this works is not fully explained. Anderson (2016) suggests that the upper-bound can be given syntactically through the use of syntactic features encoding the next lower base, although the strategy is also not fully fleshed out. In this approach, *twenty* would check a base feature on its sister that indexes it with base 10^0. In a similar move, Mendia (2018) proposes to fix the denotation of NUMBER in such a way so as to give it precisely the correct base for its position within the syntactic structure.

Here, I want to return to the original intuition that it is the form of the numeral that is fixing the upper-boundedness, once again deriving it as an implicature. The benefit of this approach is that the upper-bounded constraint on the indeterminate numeral can be constructed compositionally using familiar tools from the analysis of scalar implicatures, rather than treated as a constraint on the construction as a whole. The essential idea will be to treat the upper-bounded interpretation as a kind of Q (quantity) implicature, an implicature generated by flouting Grice's Maxim of Quantity, the communicative principle that a speaker ought to be as informative as possible (Horn 1984, Grice 1957). In modifying one numeral rather than another, the speaker generates the inference, via the existence of a scale, that they could not commit themselves to the information content carried by members up the scale.

First, it is necessary to show that the lower-boundedness of indeterminate numerals is not an implicature, but is part of the asserted meaning of the sentence. This can be done by forcing a contradiction, as in (58).

(58) *There were thirty-some people at the party, and there weren't even thirty. (contradiction)

Next, (59) shows that the lower-bounded meaning component cannot be reinforced; these sentences sound redundant. This again suggests that it is part of the asserted meaning of the sentence. This is not surprising, I think, but showing this is necessary in order to make a distinction between the asserted and implicated components of indeterminate numerals.[14]

(59) a. ??There were thirty-some people at the party, definitely at least thirty.
b. ??I have a hundred-some stamps in my collection, definitely at least one hundred.

In comparison, the upper-bound of indeterminate numerals does seem to be reinforceable.

(60) a. There were thirty-some people at the party, definitely not more than forty.
b. I have a hundred-some stamps in my collection, definitely less than two hundred.

That the upper-bound is reinforceable, or at least more easily reinforceable than the lower-bound, suggests that it is not asserted as part of the conventional meaning of the indeterminate numeral; rather, the fact that it can be independently asserted and reinforced suggests that it behaves more like inferred meaning, e.g. an implicature.

How is this implicature calculated? Minimally, we need to consider what the competitors to a numeral like *twenty* are. What we notice is that for *twenty-some*, of course, the upper bound is set by 30; for *two hundred-some*, the upper bound is set by 300. The competitors for a numeral involved in an indeterminate numeral, at least for English indeterminate numerals formed from *some*, are formed by abstracting over the multiplier for the largest base in the numeral. The immediately relevant alternative is the next higher multiplier for the base. For *twenty* (2 × 10), the relevant alternative for the implicature is calculated by picking the next highest multiplier (3 × 10).

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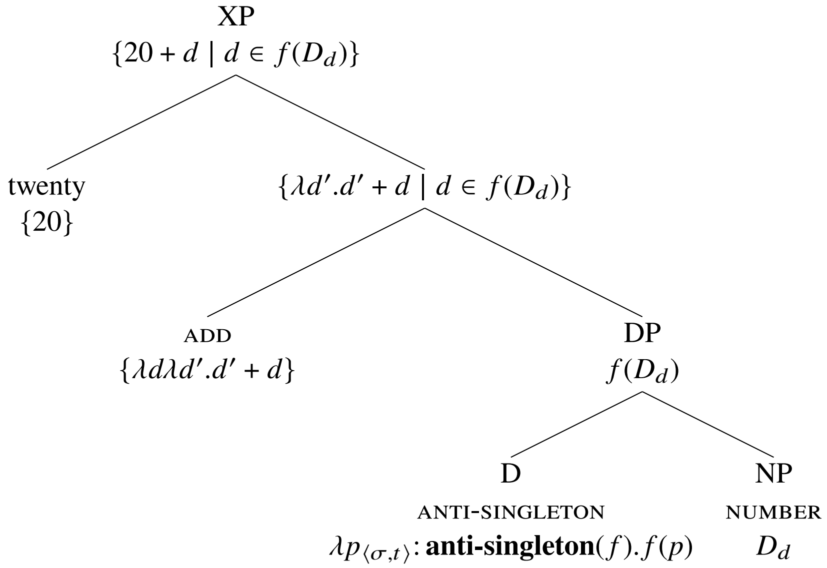
[14] There may be some ways of rescuing the sentences in (59), such as if the lower-bound is contextually relevant and the adverb *definitely* taken to be emphasizing meeting that bound. But, in out of the blue contexts, the preferred interpretation of (59) is one where *definitely* contributes information already asserted in the sentence, giving a sense of redundancy. I thank Ai Taniguchi (p.c.) for the observation that there is a context where these sentences are acceptable.

7.2 Computation of indeterminate numerals

The computation for an indeterminate numeral proceeds in largely the same fashion as for an ordinary numeral; both indeterminate numerals and ordinary numerals will be typed as degrees d, making them simply arguments to a Num head that mediates between the lexical NP and the numeral in SpecNumP. Where indeterminate numerals differ from ordinary numerals is in introducing into the derivation a non-singleton set of alternatives.

To begin, we'll consider the indeterminate numeral itself. First, ANTI-SINGLETON combines with NUMBER. I assume a weak semantics for NUMBER: it simply denotes the domain of degrees, D_d.[15] When ANTI-SINGLETON combines with NUMBER, it selects a subset of NUMBER. This set is guaranteed to have at least two members. For an indeterminate numeral such as *twenty-some*, the numeral will denote a set of degrees of the form $20 + d$, where d is a member of D_d, and this set will have at least two members. This derivation for *twenty-some* is given in (61).

(61)



Next, Num′ is applied pointwise to the denotation of the numeral; for each degree in the denotation of the numeral, Num′ applies to it, saturating Num's degree argument. This allows the alternatives from the numeral to continue to fan outwards throughout the course of the semantic derivation.

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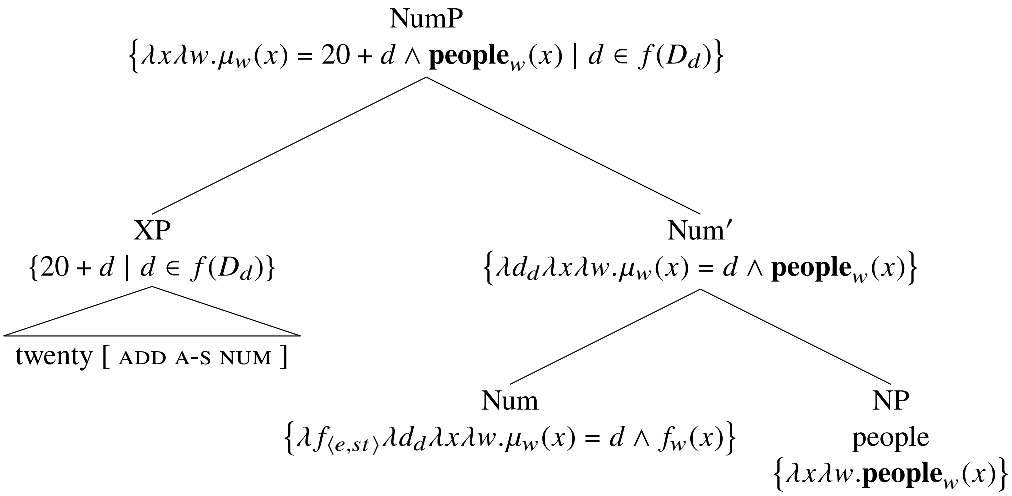
[15] It seems quite difficult for the indeterminate numeral to denote a fractional number, such as *twenty-some* denoting 25.5. If *some* NUMBER is just simply denoting the domain of degrees, it's somewhat unclear why this should be, given that some authors (e.g., Fox and Hackl (2007), for example) assume that the domain of degrees is a subset of the real numbers $\mathbb{R}$, and not of the integers. There's two options that come to mind here. One possibility would be to have NUMBER denote in the integers $\mathbb{Z}$ or in the natural numbers $\mathbb{N}$. A second possibility would be to have additional entailments stemming from a more general semantics of numerals that numerals necessarily count atomic individuals. An atomicity constraint of this type would then force *some* NUMBER to always denote an integer. I have very little else to say about these possibilities here, though, and leave the question for further research.

(62)



As argued for previously, the upper-bounded inference should be treated as an implicature. (As we can already see, it does not come directly from the semantics of the indeterminate numeral.) I assume that, at the level of NumP, an implicature is calculated based on competitors to the modified numeral. The relevant alternatives for the quantity implicature are given in (63b), where the next numeral "up" from the modified numeral, based on increasing the value of the multiplier in the numeral, sets an upper-bound for the measure function over individuals μ (see also section 7.1). The NumP and the implicature can be intersected to give the strengthened, upper-bounded interpretation in (64).

(63) a. NumP: $\{\lambda x\lambda w.\mu_w(x) = 20 + d \wedge \mathbf{people}_w(x) \mid d \in f(D_d)\}$
b. Implicature: $\{\lambda x\lambda w.\mu_w(x) < 30 \wedge \mathbf{people}_w(x)\}$

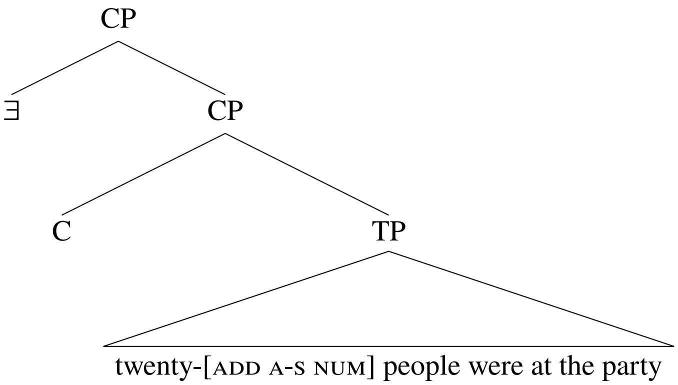
(64) Strengthened: $\{\lambda x\lambda w.\mu_w(x) = 20 + d \wedge \mu_w(x) < 30 \wedge \mathbf{people}_w(x) \mid d \in f(D_d)\}$

Setting the upper-bound in this way ensures that, no matter what the subset selected from D_d is, the addition of any member of that subset with the modified numeral will never be larger than the competitor. A schematization of these alternatives is given in (65).

(65)
$$\left\{\begin{array}{c} \lambda x\lambda w.\mu_w(x) = 20 + 1 \wedge \mu_w(x) < 30 \wedge \mathbf{people}_w(x), \\ \lambda x\lambda w.\mu_w(x) = 20 + 2 \wedge \mu_w(x) < 30 \wedge \mathbf{people}_w(x), \\ \ldots \\ \lambda x\lambda w.\mu_w(x) = 20 + 8 \wedge \mu_w(x) < 30 \wedge \mathbf{people}_w(x), \\ \lambda x\lambda w.\mu_w(x) = 20 + 9 \wedge \mu_w(x) < 30 \wedge \mathbf{people}_w(x) \end{array}\right\}$$

These alternatives percolate upward through the derivation, until arriving at the $\exists$ operator, which I will suppose is adjoined to CP. To recapitulate, the role of $\exists$ is to transform the set of alternatives it combines with into a singleton. To do this, it takes the alternatives which have been created in the course of the derivation, the set of epistemic possibilities, and asserts that one of them holds in the world of evaluation.

(66)



(67) $[\![\exists$ *twenty-[ADD A-S NUM] people were at the party*$]\!]$
$= \{\lambda w.\exists p[p \in [\![$*twenty-[ADD A-S NUM] people were at the party*$]\!] \wedge p(w)]\}$

In this way, indeterminate numerals generate a non-singleton set of alternatives, with the upper-bound given as a quantity implicature.

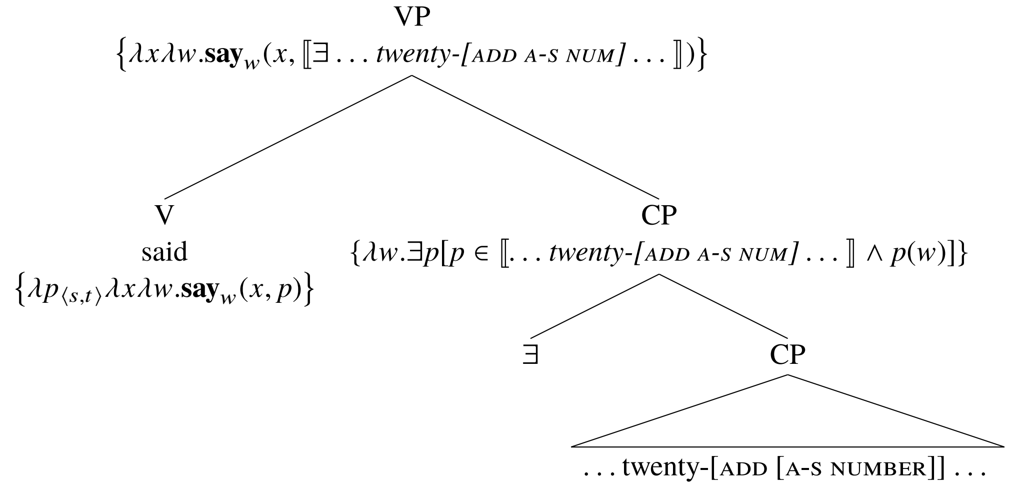
7.3 Verbs of saying and indeterminate numerals

The formalization I give here relies on an ∃ operator that can be inserted in the tree in order to close off the set of alternatives, e.g. to transform it into a set containing a single proposition. Adopting this system allows us to have an explanation for why the ignorance implicature can be anchored to different individuals in a sentence with a verb of saying, as in (68), where it is anchored to either the sentential subject (the Agent of the verb *say*) or the speaker.

(68) John said that there were twenty-some people at the party...
a. ... but he didn't know exactly how many. (subject)
b. ... but I don't know exactly how many (he said there were). (speaker)

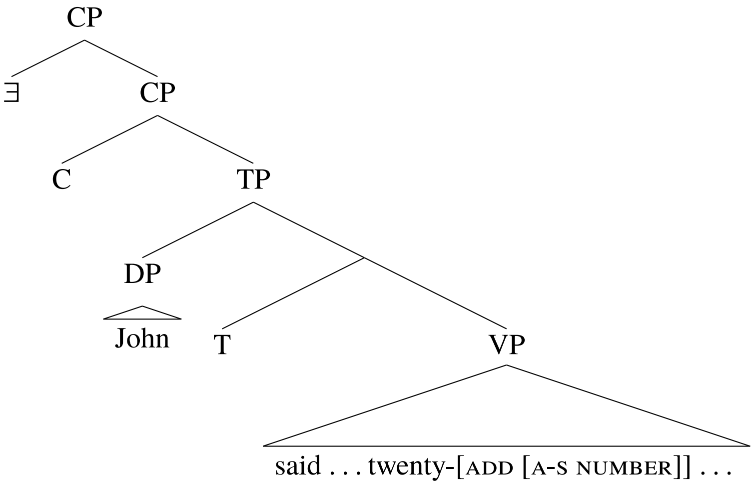
The anchoring of ignorance to different individuals can be essentially thought of as a relatively ordinary scope ambiguity; the relative scope of the ∃ operator with respect to the attitude verb determines when the set of propositional alternatives is flattened to a single proposition. If the operator scopes below the attitude verb, the alternatives are flattened into a single proposition, which is the proposition expressed by the one doing the saying.

(69)



On the other hand, if the ∃ operator scopes above the verb, as in (70), then the alternatives from the indeterminate numeral persist to the top of the clause. Due to the Pointwise Function Application rule, the verbal meaning is factored into the alternatives generated by the indeterminate numeral. The resulting set of alternatives is a set of alternatives that vary by which proposition was said. By allowing the alternatives to persist past the level of the verb, what is constructed is a set of alternatives that express possible propositions that were said, as in (71). Effectively, this creates ignorance about which particular proposition was uttered by someone, but only commits the speaker of the root clause to ignorance, not the person who said (the content of) the embedded clause. The logical form for this is provided in (71).

(70)



(71) $\{\lambda w.\mathbf{say}_w(\mathbf{john}, \lambda w \exists x \exists d.\mu_w(x) = 20 + d \wedge \mu_w(x) < 30 \wedge \ldots) \mid d \in f(D_d)\}$

8 Conclusion

This chapter investigated the use of *some* in forming approximate, uncertain meanings with numerals, what I call indeterminate numerals. These numerals have the structure and semantics of ordinary numerals (degree-denoting expressions), but are special in that they make use of an operator anti-singleton (spelled out as *-some*) that forces the generation of at least two alternatives. The generation of multiple alternatives models the uncertainty inherent to these numerals. These numerals gain an additional upper-bounded inference via a second quantity implicature, based on the value of the numeral that *-some* attaches to. This work shows how operators associated with quantificational elements such as indefinite determiners can interact with degrees when placed in certain syntactic configurations, and sheds light on the quantificational mechanisms used in computing ignorance over sets of degrees.

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