Criticality in neural networks paper title

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Abstract

It has been posited that biological neural networks, such as a brain, may naturally exist in critical states. We propose two mechanisms for signal transduction in two such networks as encoding strategies which are optimized by criticality. First, we examine compressive sensing in a 2-dimensional Ising model at or near its critical temperature. Secondly, we examine the dynamical synchronization capabilities of a random neural network model as it transitions into chaotic behavior. We propose that both techniques should be most successful at the critical state of either model.

1 Introduction

Criticality has enjoyed widespread success in the scientific community, being applied to problems as diverse as gene expression, starling flocks, and forest fires.[**?**] Originating in statistical physics, criticality is a set of properties of a system near a second order phase transition, in which the system possesses long range correlations in both space and time. These properties make it an attractive tool for describing the many complex systems that seem to display similar long range order

In the review "Emergent complex neural dynamics[**?**]" Chialvo makes a case for the brain exhibiting criticality. He pulls from several pieces of evidence including:

- The brain contains the *necessary* elements (a large network of nonlinear interacting elements) to display complex emergent behavior and thus criticality.
- EEG/MEG readings of healthy brains do not show a preferred time scale.
- All models that display emergent complex behavior also display criticality.
- Neuronal avalanches have a scale free size distribution matching a critical branching process.
- Degree distributions created from fMRI recordings are scale-free.

2 Dynamical synchronization technique

The model equations are coupled to a set of data through a linear term as

$$\frac{dx_i}{dt} = f_i(x) + \sum_j g_{ij} \left(y_j - x_j \right) \tag{1}$$

where $x(t)$ and $y(t)$ are the model and the data trajectories, respectively, and g_{ij} is a set of coupling constants. In practice, only a few of the couplings will be nonzero (corresponding to the components of the system which can be measured) and the matrix g_{ij} is diagonal, since in this scheme there is no obvious advantage to coupling different components of the system to each other.

In general, a model can be synchronized to a data set if enough measurements are made, and if the coupling is large enough. What constitutes "enough" seems to come on a case-by-case basis. Roughly, what is required is for the linear terms to regularize the (coupled) solution manifold by making the value of the largest Lyapunov exponent non-positive. However, the ratio of the i-th coupling term to the i-th term of the dynamics must remain small, so as to not wash out the physicality of the problem (in practice, this may also increase the speed of convergence to the actual solution). In other words, adjusting the coupling is a delicate balance for which there is no general procedure.

The success of the procedure, however, is not ultimately measured by the ability to synchronize, but by the ability of the model *make predictions* after the coupling is turned off, since in general we're only able to measure a few components of the system. In the case where we have access to all of the data states, however, we can additionally test the procedure by examining the normally unobservable states. This is called a *twin experiment* in the literature and is a useful technique for assessing the viability of the model to synchronize in a controlled setting.

3 Random neural network model

We now consider a neural network model, originally presented by [1], which is a dynamical system describing a network of N randomly connected elements each having a leaky capacitive term. The equations of motion describing the system are

$$\frac{dV_i}{dt} = -V_i + \sum_{j=1}^{N} J_{ij}\phi(V_j) \tag{2}$$

where $\phi(V_i)$ is a function which may be thought of as an "activity" proportional to the synaptic current between neurons i and j. One possible choice is a sigmoid function of V, i.e. $\phi_i = \tanh(\alpha V_i)$. This choice is both biologically motivated as acting to saturate synaptic activity as a function of membrane voltage, as well as mathematically to avoid highly unstable, runaway solutions to eq. (2). α acts as a control parameter on the turnaround rate of the synaptic activity around $V = 0$; in some sense it controls the "degree of nonlinearity" in the system.

The matrix J_{ij} describes the connectivity in the network; in this particular model, J_{ij} is chosen to be a Gaussian random matrix with elements distributed according to the statistics

$$\langle J_{ij} \rangle = 0, \quad \langle J_{ij} J_{kl} \rangle = \frac{\tilde{J}^2}{N} \, \delta_{ik}\delta_{jl} \left(1 - \delta_{ij}\right) \left(1 - \delta_{kl}\right) \tag{3}$$

which means synaptic connections are, in general, asymmetric and totally decorrelated. This also means that, on average, half of the connections are inherently inhibitory and half are excitatory. We use "inherently" because the sign of the synaptic current into neuron i switches depending on the sign of V_j.

A detailed mathematical treatment of this model in the limit of large N is given in [1]. Under the replacement $\alpha \to g\tilde{J}$ in the expression for $\phi(V_i)$, the model undergoes a "phase transition" when the control parameter $g\tilde{J}$ reaches a critical value of 1. This is manifest in the structure of the attracting solution manifold in the $\{V_i\}$ state space, which acquires a positive Lyapunov exponent; in other words a family of chaotic solutions to (2) appears.

The similarity of this transition to the phase transition in the 2D Ising model near T_{crit} is most apparent in the behavior of the quantity

$$\Delta(t) \equiv \langle V_i(t_0)V_i(t_0 + t) \rangle \tag{4}$$

3.1 Numerical analysis

We examined the model for $N = 256$ neurons with a single instantiation of J_{ij}. Ideally, we would like to scale up the simulation size to a larger N, perhaps near 1000, and to gather statistics about an *ensemble* of models parameterized by different instantiations of J_{ij}. However, limited by time, we now present said results as at least a preliminary examination of the model.

First, we performed a comparison of numerical results to the analytic results of [1]. For a single randomly drawn instantiation of J_{ij}, (2) was integrated forward in time using the `LSODA` solver in

`scipy.integrate`. The initial conditions were drawn randomly from a ball of radius 1 near the origin $V_i = 0$. We found that above $g\tilde{J} = 1$, limit cycle solutions appeared in a Hopf bifurcation from an attracting fixed point at the origin (which became unstable in the bifurcation).

References

[1] H Sompolinsky, A Crisanti, and H. Sommers. Chaos in random neural networks. *Physical Review Letters*, 61(3):259–262, July 1988.