

$$\begin{array}{lll}
 k = 1.381 \times 10^{-23} \text{ J} \cdot \text{K}^{-1} & q = 1.602 \times 10^{-19} \text{ C} & R = 8.314 \text{ J} \cdot \text{K}^{-1} \cdot \text{mol}^{-1} \\
 F = 9.648 \times 10^4 \text{ C} \cdot \text{mol}^{-1} & C_m = 1.0 \text{ } \mu\text{F}/\text{cm}^2 & E_L = -59.387 \text{ mV} \\
 \bar{g}_{Na} = 120 \text{ mS}/\text{cm}^2 & \bar{g}_K = 36 \text{ mS}/\text{cm}^2 & g_L = 0.3 \text{ mS}/\text{cm}^2
 \end{array}$$

Three squids were placed in three separate saltwater tanks. In one tank, the person in charge accidentally added too much salt; in another, they added too little. Consider the following concentrations of sodium and potassium ions in each tank:

	Tank #1	Tank #2	Tank #3
[Na ⁺]	50 mM	71 mM	100 mM
[K ⁺]	4 mM	6 mM	8 mM

1. [5 points] Assuming all squids maintain the same intracellular concentrations for both sodium (12 mM) and potassium (155 mM), calculate the Nernst reversal potentials for these ion species in each of the three tanks ($E_{Na,1-3}$, $E_{K,1-3}$). Assume room temperature of 20 °C.

Under these assumptions one can derive the current density carried by a single ionic species S traveling along a single axis perpendicular to the membrane as a function of membrane voltage. This is referred to as the *GHK current density equation*. The *Nernst reversal potential* can subsequently be derived. With this, we obtain:

$$J_S = z_S^2 P_S V_m \frac{[S]_i - [S]_o e^{-z_S V_m / V_t}}{1 - e^{-z_S V_m / V_t}} \quad E_S = \frac{V_t}{z_S} \ln \frac{[S]_o}{[S]_i}$$

where J_S is the current, E_S is the reversal potential, V_m is the voltage across the membrane, $V_t = \frac{kT}{q} = \frac{RT}{F}$ is the thermal voltage, z_S is the valence of the ion S , P_S is the permeability of S , and $[S]_i$ and $[S]_o$ are the intracellular and extracellular concentrations of the ion.

For the K^+ channel type with $z_K = +1$ the current equation and reversal potential reduce to:

$$J_K = P_K V_m \frac{[K]_i - [K]_o e^{-V_m / V_t}}{1 - e^{-V_m / V_t}} \quad E_K = V_t \ln \frac{[K]_o}{[K]_i}$$

Since the GHK equation is complex it is usually approximated as a linear equation as follows:

$$J'_K = g_K (V_m - E_K) \quad g_K = P_K \frac{[K]_o [K]_i}{[K]_o - [K]_i} \ln \frac{[K]_o}{[K]_i}$$

where g_K is the K^+ conductance. The conductance is directly related to the permeability as shown above.

Similar expressions hold for Na^+ and Cl^- , with $z_{Na} = +1$ and $z_{Cl} = -1$.

Values to Use

$V_t = 26.7 \text{ mV}$ (thermal voltage at body temperature)

$C_m = 1 \text{ } \mu\text{F}/\text{cm}^2$ (standard membrane capacitance)

S	$g_S \text{ (mS}/\text{cm}^2)$	$[S]_o \text{ (}\mu\text{M)}^\dagger$	$[S]_i \text{ (}\mu\text{M)}^\dagger$
Na^+	12.0	145	10
K^+	4.0	5	140
Cl^-	0.1	110	4

Problems

1. *Linear and nonlinear current approximations.*

Plot J_K and J'_K as a function of V_m ranging between -200 mV and $+200 \text{ mV}$. Compare the two currents. Repeat for Na^+ and Cl^- . How good is the approximation, particularly in the range around the reversal potentials? How are the ions different?

For the case when $V_m \gg V_t$, what is a good approximation of J_K ? And when $V_m \ll -V_t$? Compare both approximations with the GHK equation results by plotting.