Criticality & information transfer in neural networks

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What is Criticality?

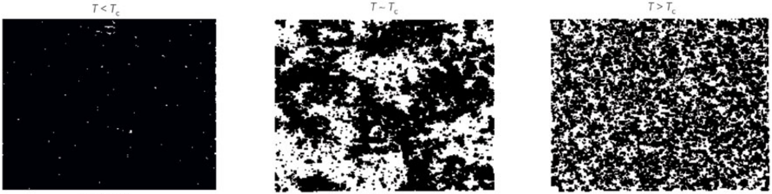
- Anything with a power law.

What is Criticality?

- But really...

What is Criticality?

- Near a critical phase transition, as we vary a parameter of the system the correlation lengths diverge in space and the dynamics slow. The spartial correlations take the form of power law distributions.
- The canonical example



Subcritical Critical Supercritical

The case for Criticality in Neuroscience

From Chialvo, Emergent Complex Dynamics, Nature Physics 6 2010:

- Brain has necessary hardware to show criticalty
- Healthy brains lack a preferred time scale
- fMRI degree distributions are power-law
- Neuronal avalances have power law size distributions
- All known models that display complex emergent behavior display criticality

These arguments all make the case for a resemblance but lack any reason why an organism would want a critical nervous system.

Hypothesis

The dynamics of systems near a critical phase transition enhance information propagation through the system and naturally enact certain encoding strategies.

To this end we (are) examining simple systems that display a critical transition:

Does the Ising model naturally perform compressive sensing but only near the phase transition?

Is dynamical synchronization to a measured signal optimized near criticality?

Neural network: dynamical model

A model of N interconnected "neurons" that obey the dynamics:

$$\frac{dV_i}{dt} = -V_i + \sum_j J_{ij} S_j$$

where S_i is the "activity" of a neuron, defined as

$$S_i \equiv tanh(g\tilde{J}V_i).$$

Actually it might as well be any sigmoidal function of V_i. The sigmoidal behavior limits the coupling to avoid runaway unstable solutions (also biophysically motivated, like a saturation of the interneuronal coupling).

Random neural network: dynamical model

The randomness lies in J_{ij}. Each entry is a Gaussian-distributed random variable with the statistics

$$\langle J_{ij} \rangle = 0$$
$$\langle J_{ij} J_{kl} \rangle = \frac{\tilde{J}^2}{N} \delta_{ik} \delta_{jl}.$$

In general, the dynamics of this kind of system are non-relaxational (i.e. the system does not converge to a global minimum of an energy function) since the couplings are asymmetric.

In the large-N limit, the dynamics can be reduced to the form

$$\frac{dV_i}{dt} = -V_i + \eta_i$$

and there exist a broad range of chaotic solutions.

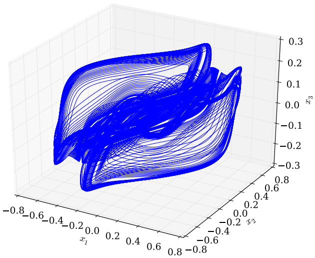
Random neural network: dynamical model

With these definitions, the quantity $g\tilde{J}$ acts like a control parameter for the dynamics of the quantity

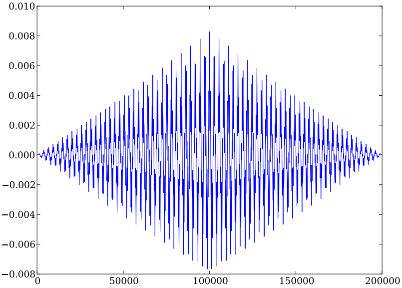
$$\Delta(t) \equiv \langle V_i(t_0 + t) V_i(t_0) \rangle$$

Roughly, when $g\tilde{J}$ crosses above 1, the dynamics undergo a qualitative change in behavior that looks much like the phase transition in the Ising model; Δ acts as a qualifier for the behavior.

$g\tilde{J} = 1.1$

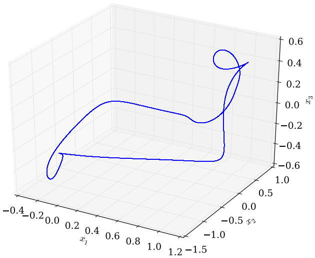


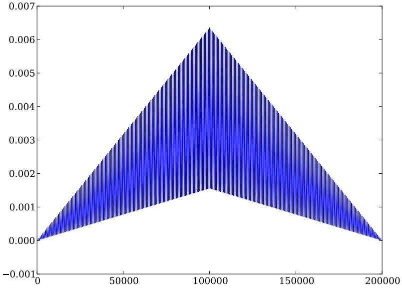
0.3
0.2
0.1
0.0
−0.1
−0.2
−0.3
x_3
−0.8 −0.6 −0.4 −0.2 0.0 0.2 0.4 0.6 0.8
x_1
0.8 0.6 0.4 0.2 0.0 −0.2 −0.4 −0.6 −0.8
x_2



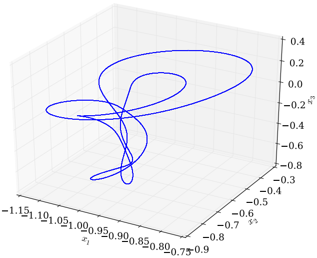
0.010
0.008
0.006
0.004
0.002
0.000
−0.002
−0.004
−0.006
−0.008
0 50000 100000 150000 200000

$g\tilde{J} = 1.2$

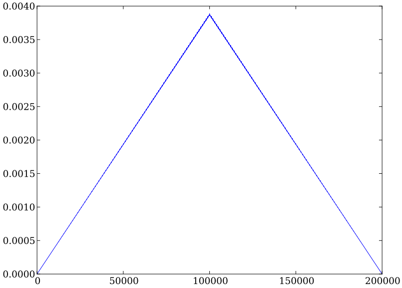

0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
x_3
1.0
0.5
0.0
−0.5
−1.0
−1.5
x_2
−0.4
−0.2
0.0
0.2
0.4
0.6
0.8
1.0
1.2
x_1


0.007
0.006
0.005
0.004
0.003
0.002
0.001
0.000
−0.001
0
50000
100000
150000
200000

$g\tilde{J} = 1.26$

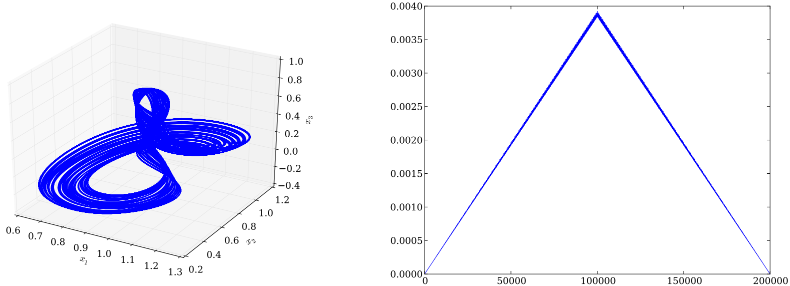


0.4
0.2
0.0
−0.2
−0.4
−0.6
−0.8
x_3
−0.3
−0.4
−0.5
−0.6
−0.7
−0.8
−0.9
x_2
−1.15
−1.10
−1.05
−1.00
−0.95
−0.90
−0.85
−0.80
−0.75
x_1

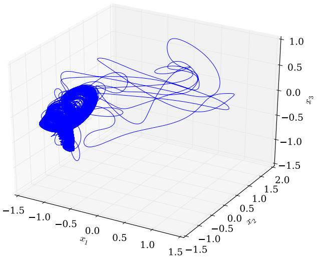


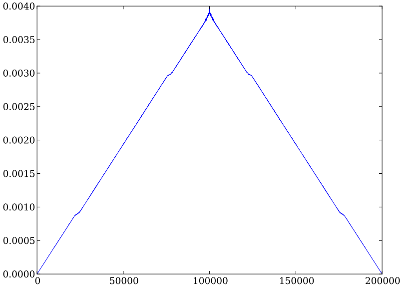
0.0040
0.0035
0.0030
0.0025
0.0020
0.0015
0.0010
0.0005
0.0000
0
50000
100000
150000
200000

$g\tilde{J} = 1.266$

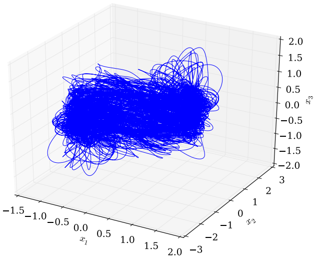


$g\tilde{J} = 1.2668$

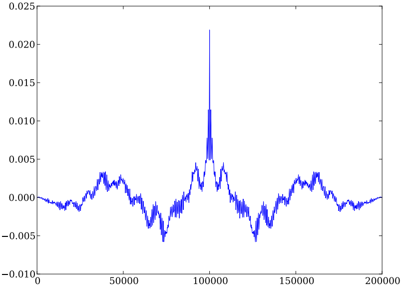




$g\tilde{J} = 1.27$

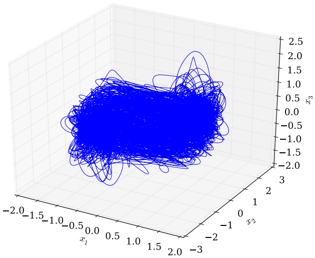


2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
x_3
3
2
1
0
−1
−2
−3
x_2
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0
x_1

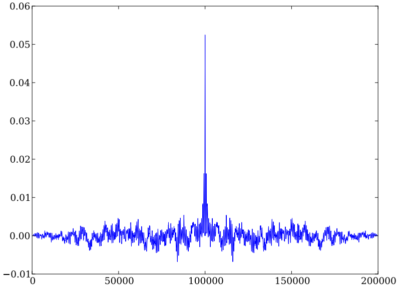


0.025
0.020
0.015
0.010
0.005
0.000
−0.005
−0.010
0
50000
100000
150000
200000

$g\tilde{J} = 1.3$



2.5
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
x_3
−2.0 −1.5 −1.0 −0.5 0.0 0.5 1.0 1.5 2.0
x_1
3
2
1
0
−1
−2
−3
x_2



0.06
0.05
0.04
0.03
0.02
0.01
0.00
−0.01
0
50000
100000
150000
200000