2 Homework: Simulating Hodgkin-Huxley Neurons

In this homework we will simulate the Hodgkin-Huxley (HH) model of action potential generation in the squid giant axon. The equations describing the HH dynamics with a resting potential of -70 mV are replicated here:

$$C_m \frac{dV}{dt} = -I_{Na} - I_K - I_L + I_{ext} \tag{1}$$
$$I_{Na} = g_{Na}\, m^3 h\, (V - E_{Na}) \tag{2}$$
$$I_K = g_K\, n^4\, (V - E_K) \tag{3}$$
$$I_L = g_L\, (V - E_L) \tag{4}$$

with parameters:

$$\begin{array}{rclrcl} C_m & = & 1\,\mu F/cm^2 & & & \\ E_{Na} & = & 45\,mV; & g_{Na} & = & 120\,mS/cm^2 \\ E_K & = & -82\,mV; & g_K & = & 36\,mS/cm^2 \\ E_L & = & -59.387\,mV; & g_L & = & 0.3\,mS/cm^2 \end{array} \tag{5}$$

where the dynamics of gating variables are:

$$\frac{dm}{dt} = \alpha_m(V)\,(1-m) - \beta_m(V)\,m \tag{6}$$
$$\frac{dh}{dt} = \alpha_h(V)\,(1-h) - \beta_h(V)\,h \tag{7}$$
$$\frac{dn}{dt} = \alpha_n(V)\,(1-n) - \beta_n(V)\,n \tag{8}$$

and is determined by the rate functions (all rates in units $1/ms$ and voltages in mV):

$$\alpha_m(V) = 0.1\,(V+45)/(1-\exp(-(V+45)/10)) \tag{9}$$
$$\beta_m(V) = 4\,\exp(-(V+70)/18) \tag{10}$$
$$\alpha_h(V) = 0.07\,\exp(-(V+70)/20) \tag{11}$$
$$\beta_h(V) = 1/(1+\exp(-(V+40)/10)) \tag{12}$$
$$\alpha_n(V) = 0.01\,(V+60)/(1-\exp(-(V+60)/10)) \tag{13}$$
$$\beta_n(V) = 0.125\,\exp(-(V+70)/80) \tag{14}$$

1. Plot α_m, β_m, α_h, β_h, α_n, and β_n as functions of membrane voltage V, for V from -90 to +70 mV.

2. Start with just the leak current I_L, leaving out I_{Na} and I_K. (You do not need the gating variables m, h, and n since they are only involved with I_{Na} and I_K). The differential equations become:

$$C_m \frac{dV}{dt} = -I_L + I_{ext}$$

3. Now add I_{Na}, I_K, and their gating variables m, h, and n to observe spiking. Plot the membrane voltage V and the gating variables n, m, and h as a function of time t. Use the same injected current I_{ext} setup as in part 2.2.

   Increase the amount of injected current further. What happens as you increase the current? What happens when you go above 160 $\mu A/cm^2$?

   Note: Make sure to find an appropiate set of initial conditions by running to model with no injected current until it reaches a steady-state.

4. What is the approximate threshold potential (the minimum **voltage** which causes the neuron to spike)? How would you modify the HH parameters and equations to increase the threshold potential (and thus increase the amount of injected current I_{ext} required for spiking)?