Last time we took the schrodinger equation and transformed it to the radial equation in center of mass frame.

$$\left(= \frac{\hbar^2}{m}\frac{1}{r^2}\partial_r r^2 \partial_r + \frac{1}{4}m\omega^2 r^2 + \frac{ke^2}{r}\right) + \left(-\frac{\hbar^2}{4m}\frac{1}{R^2}\partial_R R^2 \partial_R + m\omega^2 R^2 + \frac{\hbar l(l+1)}{R^2}\right)\psi(r)\theta(R) = (E_r + E_R)\psi(r)\theta(R) \tag{1}$$

we are only interested in the

$$\left(= \frac{\hbar^2}{m}\frac{1}{r^2}\partial_r r^2 \partial_r + \frac{1}{4}m\omega^2 r^2 + \frac{ke^2}{r}\right)\psi(r) = E_r\psi(r) \tag{2}$$

we can make the equation dimentionless which gives

$$\left(-\partial_\rho^2 + \rho^2 + \frac{\beta}{\rho}\right)u(\rho) = \lambda u(\rho) \left(-\partial_\rho^2 + V(\rho)\right)u(\rho) = \lambda u(\rho) \tag{3}$$

Discretize

$$u(\rho) \to u(\rho_i) = u_i \rho \to \rho_i = \rho_0 + ih h = \frac{b-a}{n+1} a = 0 \tag{4}$$

the linear algebra equation is

$$\hat{A} = \begin{pmatrix} \frac{2}{h^2} + V_1 & -\frac{1}{h^2} & & & \\ -\frac{1}{h^2} & \frac{2}{h^2} + V_2 & -\frac{1}{h^2} & & \\ & & \ddots & & \\ & & & & -\frac{1}{h^2} \\ & & -\frac{1}{h^2} & \frac{2}{h^2} + V_{N-1} & \end{pmatrix} \tag{5}$$

$$\hat{A}x = \lambda x \tag{6}$$

$$S^T A x = \lambda S^T x \tag{7}$$

$$S^T A S(S^T x) = \lambda(S^T x) \tag{8}$$

Eignevalues unchanged under orthogonal transforms
Eigenvectros change, but the length is preserved
Dot product between vectors is conserved