
Homework 5: Seafloor subsidence due to cooling

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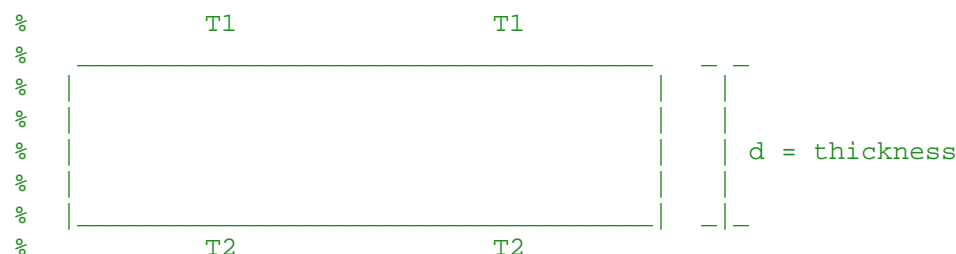
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Part 1: Conductive heat flow

Part 1 - Step 1: A model

Imagine an infinitely long and wide solid plate. The plate has thickness d . The temperature at the top of the plate is T_1 and the temperature at the bottom of the plate is T_2 . Draw a diagram of this plate and label these parameters.



Part 1 - Step 2: Heat flow

Assuming that $T_2 > T_1$, the rate of change of heat flow per unit area up through the plate is proportional to $(T_2 - T_1)/d$.

In fact the rate of heat flow per unit area (Q) down through the plate is $Q = -k(T_2 - T_1)/d$, where k is called the thermal conductivity. Q is the rate of heat flow per unit area and has units Wm^{-2} ; k has units $Wm^{-1}C^{-1}$. Why does the heat flow down in this equation?

```
% Heatflow is negative because we are losing heat as we are moving
away
% from the source.
% If heatflow were positive, heat would be gained as it moves away
from
% the source.
```

Part 1 - Step 3: Thermal conductivities

Give the values of thermal conductivity for the following items. Make sure to cite where you found this value and make sure the units are in the SI units given in step 2.

```
% Silver: 406.0 (W/mK)
% Magnesium:156 (W/mK)
% Glass:0.8 (W/mK)
% Rock:2-7 (W/mK)
% Wood:0.12-0.04 (W/mK)
% Sources used to determine thermal conductivity:
% http://hyperphysics.phy-astr.gsu.edu/hbase/tables/thrcn.html and
% http://www.engineeringtoolbox.com/thermal-conductivity-d_429.html
```

Part 1 - Step 4: The heat transport equation

```
% Let's assume that the temperature of the upper surface(i.e. at  $z$ ) is
 $T$ 
% and at the temperature at the lower surface (i.e.  $z+dz$ ) is  $T+dT$ .
Substitute these values
% into equation 1 (Think about how to replace  $d$ ,  $T_1$ , and  $T_2$ ).
% In the limit that  $dz$  goes to zero, some part of the right-hand side
of
% your new equation becomes a derivative.
% Write this derivative.
%  $Q = -k([T+dT] - T)/[Z-dZ] - Z$ 
% Write the heat flow (equation 1) using this derivative
%  $Q = -k(dt/dz)$ 
% Write this same equation using the gradient operator
%  $Q = -kdT$ 
```

Part 1 - Step 5: The conservation equation

```
% Use the heat transport equation and compute the derivative  $dQ/dz$  and
% insert this into the conservation equation.
%  $(cpP)(dT/dt) = A - k(dT^2/dZ^2)$ 
% Assume that internal heat generation is zero, write the updated the
% conservation equation from the previous step.
% What is the value of  $k$  in this case?
% The value of  $k$  is in  $m^2/s$ ,  $k = (k/cpP)$ 
```

Part 2: Oceanic lithosphere cooling

Part 2 - Step 1: Setup the model domain and compute

Part 2 - Step 1: Nested for loop with erf function

```
tic
z = 1:100;
t = 1:100;
k = 31.556;
To = 640;
Tl = [];

for ii = 1:length(z);
    for jj = 1:length(t);
        Tl(ii,jj) = To.*erf((ii)./(2*sqrt(k.*(jj))));
    end
end

figure(1);clf
imagesc(Tl);
xlabel('Time [Ma]');
ylabel('Depth [km]');
c = colorbar;
c.Label.String = 'Temperature [C]';
colormap(jet)
toc
```

c =

ColorBar (Temperature [C]) with properties:

Location: 'eastoutside'

Limits: [6.4277 640]

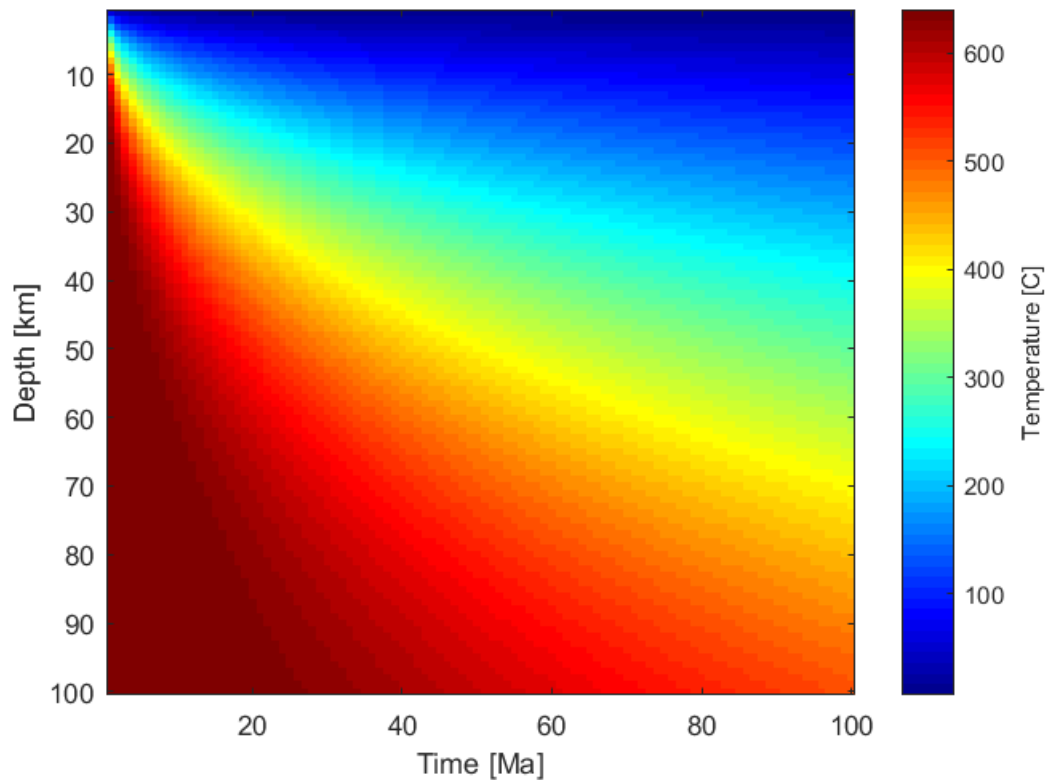
FontSize: 9

Position: [0.8226 0.1105 0.0476 0.8152]

Units: 'normalized'

Use GET to show all properties

Elapsed time is 0.152349 seconds.



Part 2 - Step 1: Nested for loop with erf function and Meshgrid

```
tic
[X,Y] = meshgrid(z,t);
T1 = [];

for ii = 1:length(X);
    for jj = 1:length(Y);
        T1(ii,jj) = To.*erf((ii)./(2*sqrt(k.*(jj))));
    end
end
figure(2); clf
imagesc(T1);
xlabel('Time [Ma]');
ylabel('Depth [km]');
c = colorbar;
c.Label.String = 'Temperature [C]';
colormap(jet)
toc

% 1. Use two for loops to compute the function T(z,t).
% 2. Use the meshgrid() command to setup matrices for z and t and then
% compute the function T(z,t).
% Extra credit: Time how long it takes to compute T(z,t) using the two
```

```
% methods. If you don't notice a difference, try decreasing the sample  
% interval in z and t. Discuss the differences in computation time.  
% The first figure took 1.34 seconds to run on TREX. The second figure  
% takes 0.45 seconds to run.
```

```
c =
```

```
ColorBar (Temperature [C]) with properties:
```

```
Location: 'eastoutside'
```

```
Limits: [6.4277 640]
```

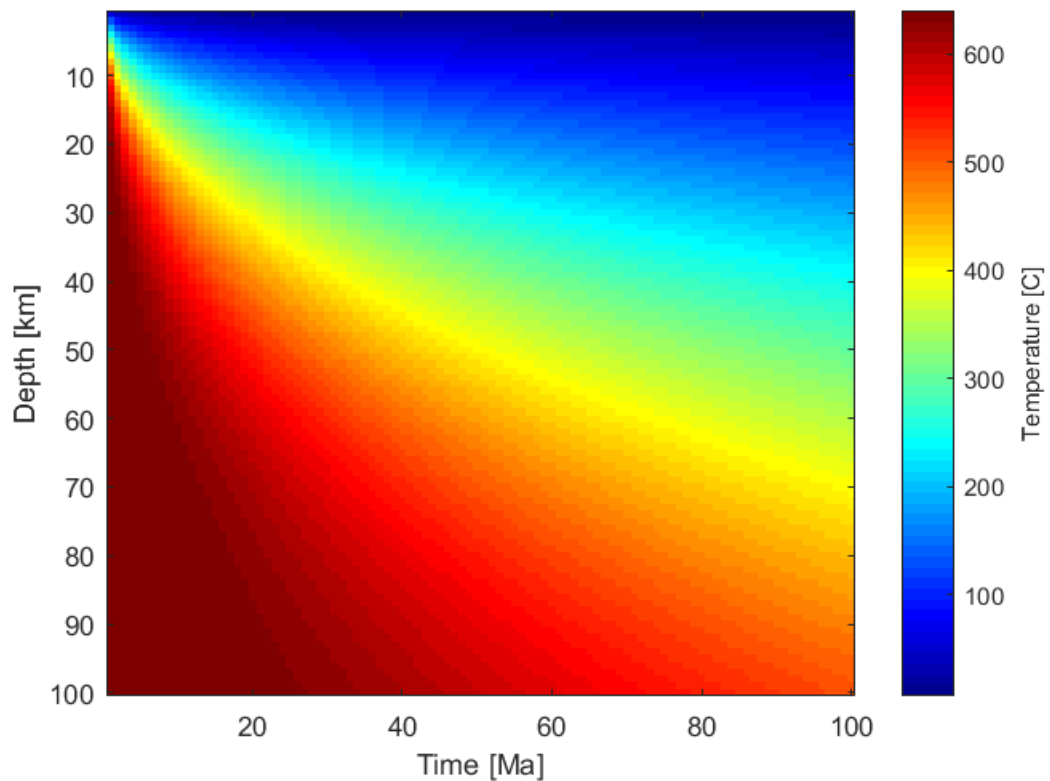
```
FontSize: 9
```

```
Position: [0.8226 0.1105 0.0476 0.8152]
```

```
Units: 'normalized'
```

```
Use GET to show all properties
```

```
Elapsed time is 0.140675 seconds.
```



Part 2 - Step 2: Analyze the model output

```
% Answer the following questions in as much detail as possible.
```

```
% Does your model make sense given the boundary conditions used to  
derive
```

```
% the solution?
% Yes
% What controls the rate at which the temperature decays?
% Depth and Time, but mostly time.
% How could we convert this model from age to distance from ridge
  axis?
% It could follow a similar gradient, only temperature would decrease
  with
% distance from axis.
% What would be a more appropriate boundary condition at  $T(z=0)$  given
% what we know about the oceans?
% Is 640C an appropriate value for the temperature at a mid-ocean
  ridge?
% Why or why not?
% Yes, because the ridge is where the crust is cracked, and magma is
% pushing upwards. It is active tectonism. Without a high temperature
  to
% facilitate the excretion of mass, there would be no spreading.
```

Part 3: Plate velocity and the depth of oceans

Part 3 - Step 1: Load and plot sea-floor depth data

Load the file spreadingData.mat in MATLAB. This will load the structure Bath.

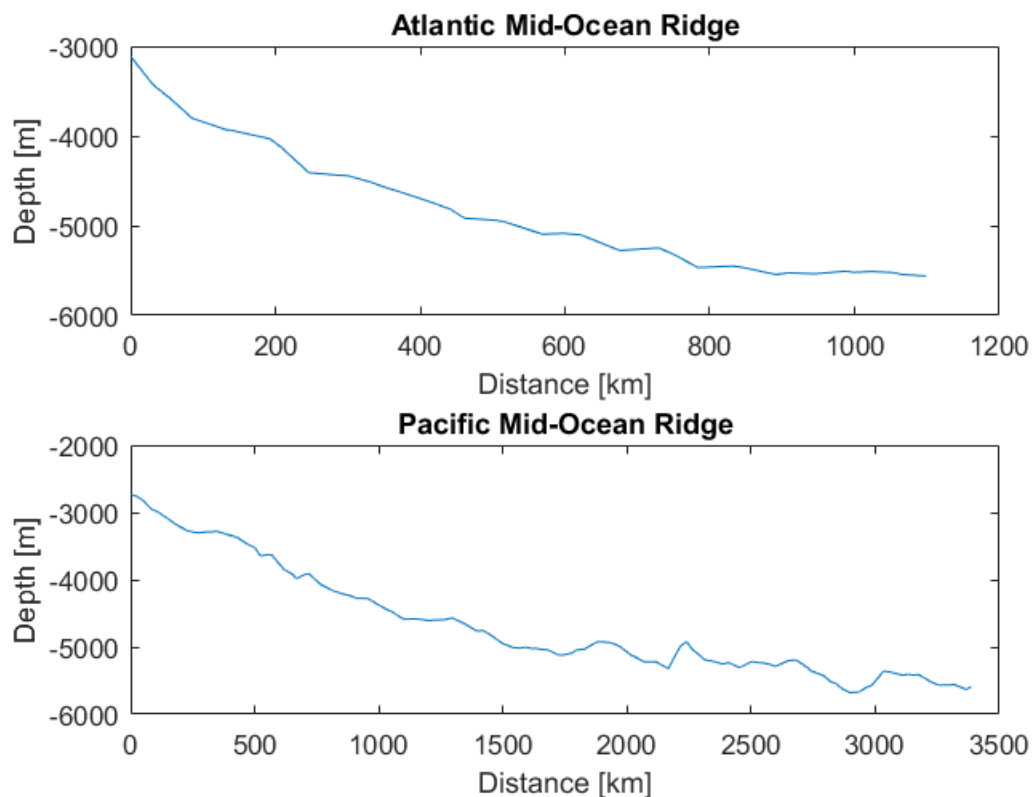
```
load('spreadingData.mat');
% These data are for the Pacific and Atlantic Oceans. The fields that
  end
% in z are the ocean depths [m] measured along a profile normal to a
% mid-ocean ridge. The fields that end in x are the distance [km] from
  the
% ridge where the data were collected. Plot depth vs. distance from
  the
% ridge; plot each ocean in its own subplot. make sure to label axes.

figure(3);clf
subplot(2,1,1);
load('spreadingData.mat');
plot(extractfield(Bath,'atlanticx'),(extractfield(Bath,'atlanticz')));
xlabel('Distance [km]')
ylabel('Depth [m]')
title('Atlantic Mid-Ocean Ridge')

subplot(2,1,2);
plot(extractfield(Bath,'pacificx'),(extractfield(Bath,'pacificz')));
xlabel('Distance [km]')
ylabel('Depth [m]')
title('Pacific Mid-Ocean Ridge')

% What does 2.65 represent in the equation (3)?
```

```
% It is an offset of the ridge axis depth.  
  
% What are the plate velocities [km/Ma] that match your data the  
% closest?  
% List both oceans.  
% I will finish this after work. crc  
  
% Convert these plate velocities to [cm/yr] and compare with an  
% estimate  
% from the literature from these ocean basins. Do your estimates agree  
% with  
% published Pacific and Atlantic Ocean spreading rates?  
% Gotta add these in
```



Part 3 - Step 2: A half-space model

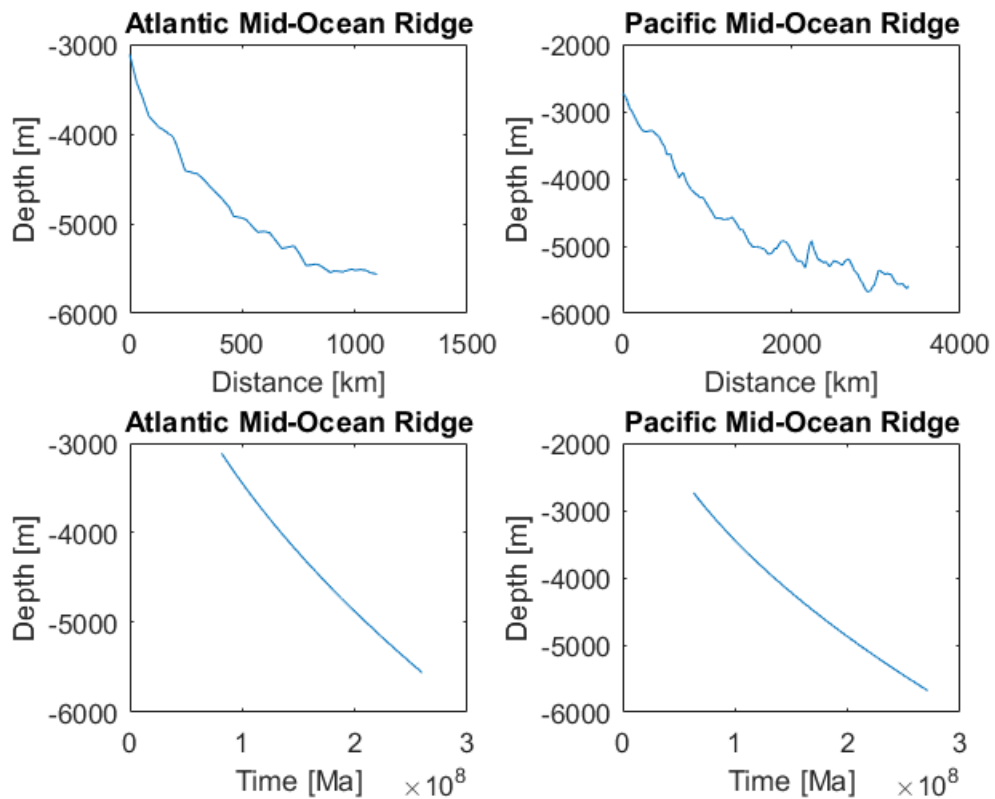
```
atlanticT = ((extractfield(Bath,'atlanticz')-2.65)/0.345).^2;  
pacificT = ((extractfield(Bath,'pacificz')-2.65)/0.345).^2;  
  
atlanticV = (extractfield(Bath,'atlanticx')./atlanticT);  
pacificV = (extractfield(Bath,'pacificx')./pacificT);  
  
figure(4);clf  
subplot(2,2,1);  
load('spreadingData.mat');  
plot(extractfield(Bath,'atlanticx'),(extractfield(Bath,'atlanticz')));
```

```
xlabel('Distance [km]')
ylabel('Depth [m]')
title('Atlantic Mid-Ocean Ridge')

subplot(2,2,2);
plot(extractfield(Bath,'pacificx'),(extractfield(Bath,'pacificz')));
xlabel('Distance [km]')
ylabel('Depth [m]')
title('Pacific Mid-Ocean Ridge')

subplot(2,2,3);
load('spreadingData.mat');
plot(atlanticT,(extractfield(Bath,'atlanticz')));
xlabel('Time [Ma]')
ylabel('Depth [m]')
title('Atlantic Mid-Ocean Ridge')

subplot(2,2,4);
plot(pacificT,(extractfield(Bath,'pacificz')));
xlabel('Time [Ma]')
ylabel('Depth [m]')
title('Pacific Mid-Ocean Ridge')
```

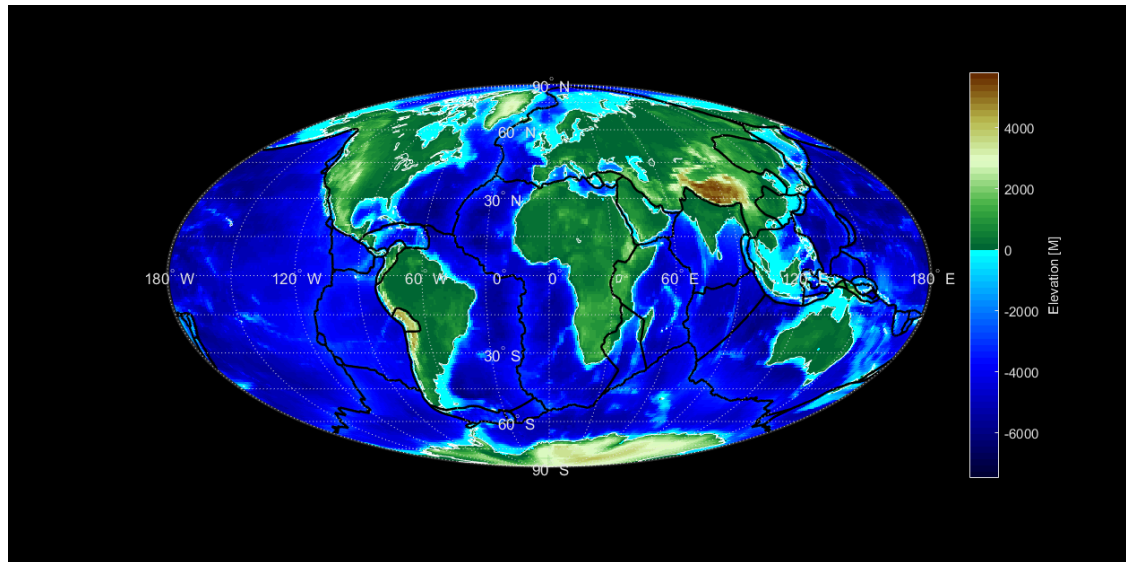


Part 4: Global oceanic plate ages

Part 4 - Step 1: Load topo data and plot seafloor depths

```
load('topo'); % Load global topography
load('coastlines'); % load coastlines
[platelat, platelon] = importPlates('All_boundaries.txt');

h = figure(5);
h.InvertHardcopy = 'off'; % ensures color of saved fig. matches
    display
h.Color = 'k'; % (1 pt.) % changes fig. color to black
h.Position = [100 100 1000 500]; % specifies location/size of fig's
    drawable area
h.PaperPositionMode = 'auto'; % preserves fig's aspect ratio when
    printing
% setup the map axes
ax = axesm('Mollweid', 'Frame', 'on', 'Grid', 'on'); % sets map
    projection, inserts globe outline, inserts grid
setm(ax, 'MLabelLocation', 60); % sets lon. labels to every 60 degrees
setm(ax, 'PLabelLocation', 30); % sets lat. labels to every 30 degrees
mlabel('MLabelParallel', 0); % sets lon. label location to the equator
plabel('PLabelMeridian', -25); % sets lat. label location to prime
    meridian
axis('off'); % prevents axes display
setm(ax, 'FontColor', [0.9 0.9 0.9]); % brightens text
setm(ax, 'GColor', [0.9 0.9 0.9]); % brightens grid
LAT = topolatlim(1):topolatlim(2);
LON = topolonlim(1):topolonlim(2);
[lon, lat] = meshgrid(LON, LAT); % compute the lat/lon of every grid
    point in topo
pcolorm(lat, lon, topo); % plot the matrix of elevations on the map
hold on;
demcmap(topo); % give it a better colormap
c = colorbar; %
c = colorbar('color', [0.9 0.9 0.9]); % changes text color on color
    bar
c.Label.String = 'Elevation [M]';
plotm(coastlat, coastlon, 'w', 'LineWidth', 0.5); %plots coast lines
plotm(platelat, platelon, 'k', 'LineWidth', 1.5); % plots plate
    boundaries
```



Part 4 - Step 2: Kill the topography and get the units right

```
[iy, ix] = find(topo > 0);

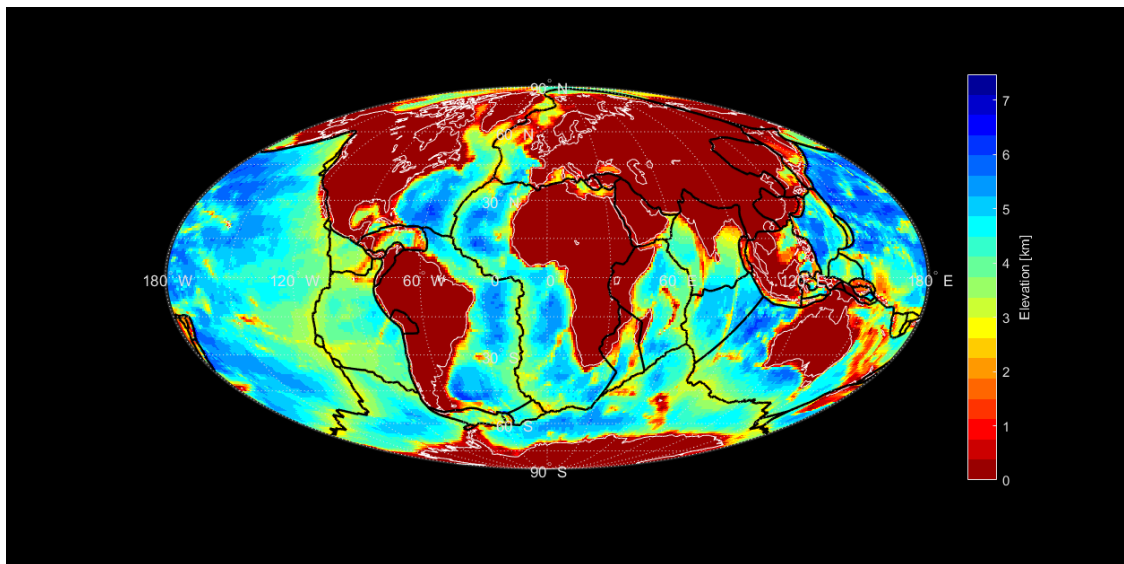
ocean = topo; % creates another matrix to hold values from the forloop

% for loop finds values of elevations that are negative
for ii = 1 : size( ix, 1 )
    ocean( iy(ii), ix(ii) ) = 0;
end

ocean = ocean.*-1e-3; % creates ocean depths, converts units, makes
    them positive

h = figure(6);
h.InvertHardcopy = 'off'; % ensures color of saved fig. matches
    display
h.Color = 'k'; % (1 pt.) % changes fig. color to black
h.Position = [100 100 1000 500]; % specifies location/size of fig's
    drawable area
h.PaperPositionMode = 'auto'; % preserves fig's aspect ratio when
    printing
% setup the map axes
ax = axesm('Mollweid','Frame', 'on', 'Grid', 'on'); % sets map
    projection, inserts globe outline, inserts grid
setm(ax,'MLabelLocation',60); % sets lon. labels to every 60 degrees
setm(ax,'PLabelLocation',30); % sets lat. labels to every 30 degrees
mlabel('MLabelParallel',0); % sets lon. label location to the equator
plabel('PLabelMeridian',-25); % sets lat. label location to prime
    meridian
axis('off'); % prevents axes display
setm(ax,'FontColor',[0.9 0.9 0.9]); % brightens text
```

```
setm(ax,'GColor',[0.9 0.9 0.9]); % brightens grid
LAT = topolatlim(1):topolatlim(2);
LON = topolonlim(1):topolonlim(2);
[lon, lat] = meshgrid(LON,LAT); % compute the lat/lon of every grid
point in topo
pcolorm(lat,lon,ocean); % plot the matrix of elevations on the map
hold on;
demcmap(ocean); % give it a better colormap
c = colorbar; %
c = colorbar('color', [0.9 0.9 0.9]); % changes text color on color
bar
c.Label.String = 'Elevation [km]';
plotm(coastlat, coastlon, 'w', 'LineWidth', 0.5); %plots coast lines
plotm(plateat,plateon, 'k', 'LineWidth', 1.5); % plots plate
boundaries
colormap(flipud(jet(20))); %Flipud inverts the array of colors on the
colormap and adds 20 increments of color scale
```

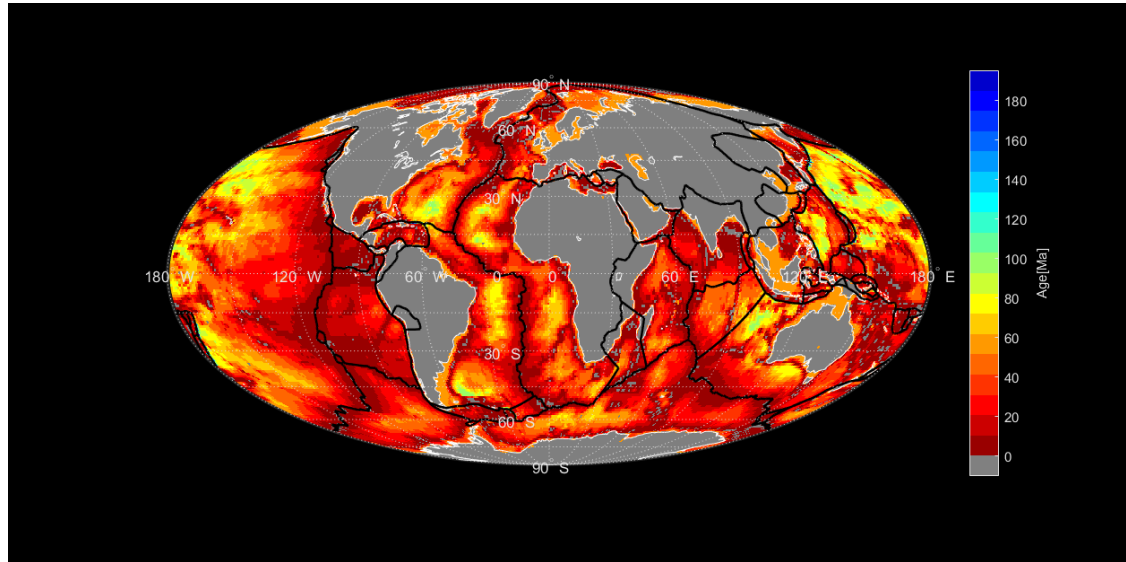


Part 4 - Step 3 Compute sea-floor age

```
d = ocean; % resaves matrix to hold values of depth for later
% for loop finds values of elevations that are negative
t = zeros(size(d));
for ii = 1:size(d)
    for jj = 1:length(d)
        t(ii,jj) = (((d(ii,jj) - 2.65)/0.345))^2;
    end
    t(t==59.000210039907586)=-10;
end

h = figure(7);
h.InvertHardcopy = 'off'; % ensures color of saved fig. matches
display
h.Color = 'k'; % (1 pt.) % changes fig. color to black
```

```
h.Position = [100 100 1000 500]; % specifies location/size of fig's
    drawable area
h.PaperPositionMode = 'auto'; % preserves fig's aspect ratio when
    printing
% setup the map axes
ax = axesm('Mollweid','Frame', 'on', 'Grid', 'on'); % sets map
    projection, inserts globe outline, inserts grid
setm(ax,'MLabelLocation',60); % sets lon. labels to every 60 degrees
setm(ax,'PLabelLocation',30); % sets lat. labels to every 30 degrees
mlabel('MLabelParallel',0); % sets lon. label location to the equator
plabel('PLabelMeridian',-25); % sets lat. label location to prime
    meridian
axis('off'); % prevents axes display
setm(ax,'FontColor',[0.9 0.9 0.9]); % brightens text
setm(ax,'GColor',[0.9 0.9 0.9]); % brightens grid
LAT = topolatlim(1):topolatlim(2);
LON =  topolonlim(1):topolonlim(2);
[lon, lat] = meshgrid(LON,LAT); % compute the lat/lon of every grid
    point in topo
pcolorm(lat,lon,t); % plot the matrix of elevations on the map
hold on;
c = colorbar; %
c = colorbar('color', [0.9 0.9 0.9]); % changes text color on color
    bar
c.Label.String = 'Age[Ma]';
plotm(coastlat, coastlon, 'w', 'LineWidth', 0.5); %plots coast lines
plotm(platelat, platelon, 'k', 'LineWidth', 1.5); % plots plate
    boundaries
cmap=(flipud(jet(20))); % Initializes color map as a variable (cmap)
    so later strings can be input to change plotting conditions.
% Flipud inverts the array of colors on the colormap and adds 20
    increments of color scale
cmap = [0.5 0.5 0.5 ; cmap]; % Allows us to create a grey color for
    plotting negative ages.
cmap(end,:) = []; % Recreates an array to include grey within the jet
    color scheme.
colormap(cmap); % Compiles array of cmap RGB values and plots the new
    color scheme.
```



Part 4 - Step 5: Discuss your results

% Does your map of ocean ages make sense given the plate boundaries?
% Yes, only near spreading centers and ocean basins.

% What is the oldest age in your map?
% ~180 Ma

% Where does this oldest age occur and does this make sense geologically?
% Off the north-east coast of Japan, the accepted age is near 160 Ma for
% that area. It makes sense geologically because it is near a
% subduction
% zone where the oldest crust should be.

% Where do the youngest ages occur? Does this conform to your
% knowledge of
% oceanic lithosphere generation?
% Conforming to the model, the youngest ages occur along Mid-ocean
% ridges.

% Are there any assumptions that have gone into this model that might
% not
% be accurate?
% Spreading rate is not equal on both sides of the ridge axis. Certain
% geographic areas were younging when getting close to continental
% shelves;
% however, they should have been aging. Our model predicts the
% Mediterranean
% sea as being ~20Ma, while accepted values are putting the
% Mediterranean sea as some of the oldest oceanic crust.

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