Problem 583

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1 Problem

There is a square field $ABCD$ near my house. My dog Fido is tethered to a pole so that the distance from the pole to A is 17 feet, to B is 24 feet, and to C is 25 feet. Find the area of the square field, and determine whether the pole is inside or outside the square.

2 Solution

Let s be the length of each side of the square $ABCD$. Place the square in $\mathbb{R}^2$ as follows: $A(0, s)$, $B(0, 0)$, $C(s, 0)$, and $D(s, s)$. With the pole at the point (x, y) such that $AP = 17$ft, $BP = 24$ ft, and $CP = 25$ ft. we can construct a circle of radius 17 ft centered at A, a circle of radius 24 ft centered at B, and a circle of radius 25 ft centered at C, such that all three circles intersect at point P. The equations of these three circles are:

$$x^2 + (y - s)^2 = 17^2 \tag{1}$$

$$x^2 + y^2 = 24^2 \tag{2}$$

$$(x - s)^2 + y^2 = 25^2 \tag{3}$$

By substituting (2) into (1) and (3), we find

$$x = \frac{s^2 - 49}{2s} \text{ and } y = \frac{s^2 + 287}{2s}$$

Substituting both into equation (2) gives us $s^4 - 914s^2 + 42385 = 0$. By quadratic formula $s^2 = 49$ or $s^2 = 865$. If $s^2 = 49$, we have $P = (0, 24)$, the pole is outside the square. If $s^2 = 865$, we have $P = (\frac{408}{\sqrt{865}}, \frac{576}{\sqrt{865}}) \approx (13.87, 19.58)$. The pole is inside the square. Therefore the pole can either be inside or outside the square.