Problem 583

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1 Problem

There is a square field ABCD near my house. My dog Fido is tethered to a pole so that the distance from the pole to A is 17 feet, to B is 24 feet, and to C is 25 feet. Find the area of the square field, and determine whether the pole is inside or outside the square.

2 Solution

Let s be the length of each side of the square ABCD. Place the square in \mathbb{R}^2 as follows: A(0,s), B(0,0), C(s,0), and D(s,s). With the pole at the point (x,y) such that AP=17ft, BP=24 ft, and CP=25 ft. we can construct a circle of radius 17 ft centered at A, a circle of radius 24 ft centered at B, and a circle of radius 25 ft centered at C, such that all three circles intersect at point P. The equations of these three circles are:

$$x^2 + (y - s)^2 = 17^2 \tag{1}$$

$$x^2 + y^2 = 24^2 \tag{2}$$

$$(x-s)^2 + y^2 = 25^2 (3)$$

By substituting (2) into (1) and (3), we find

$$x = \frac{s^2 - 49}{2s}$$
 and $y = \frac{s^2 + 287}{2s}$

Substituting both into equation (2) gives us $s^4-914s^2+42385=0$. By quadratic formula $s^2=49$ or $s^2=865$. If $s^2=49$, we have P=(0,24), the pole is outside the square. If $s^2=865$, we have $P=(\frac{408}{\sqrt{865}},\frac{576}{\sqrt{865}})\approx (13.87,19.58)$. The pole is inside the square. Therefore the pole can either be inside or outside the square.