6 GHz Low-noise Amplifier Design

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***Abstract*—A Low-noise Amplifier is designed at 6 Ghz. The design utilizes Microstrip matching networks at the input and the output on a Duroid RO3006 substrate circuit board. The LNA design achieves a noise figure of 0.81dB, a gain of 13.1dB, and an input and output VSWR of 1.5.**

***Index Terms*—Low-noise amplifiers, Radiofrequency amplifiers, Microstrip, Impedance matching**

I. Introduction

LOW noise amplifiers are essential elements for any radio frequency receiver. The need for a large dynamic range and their position early in the receiver chain dictates stringent noise requirements. However the amplifier also needs a high gain to effectively amplify the small signals received and to reduce the noise contribution of subsequent stages in the RF chain. Proper matching to the other stages requires an acceptable VSWR at the input and output.

The LNA is designed to operate at 6 GHz. This frequency lies in the IEEE C-band. This band is suitable for satellite communications, some wireless communication applications, and radiolocation applications [1].

The LNA is designed using a pHEMT GaAs Fet from Sirenza Microdevices [2]. The design uses microstrip open shunt matching networks fabricated on a Duroid RO3006 substrate circuit board. The DC biasing network utilizes surface mount inductors and capacitors.

II. Design

A. Stability Considerations

The design begins by considering the stability of the transistor. The Rollett stability factor, K, and delta were calculated for the transistor. K was found to be 1.05 and delta was found to be 0.25, making it unconditionally stable with K > 1 and $|\Delta| < 1$.

B. Initial Analysis

In order to get an overview of the achievable performance of the LNA with regard to noise figure and gain, an initial analysis of each of these parameters was performed. In this initial analysis the performance limit of each parameter was calculated and then trade-offs between each of the parameters was considered graphically on the Smith Chart.

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First the noise figure was considered. The minimum noise figure of 0.7 dB at 6GHz and an optimal reflection coefficient of $\Gamma_S = 0.28\angle 179^o$ was obtained from the datasheet.

The achievable gain of the LNA at 6GHz was then considered. The maximum stable gain and maximum transducer gain were calculated. The maximum gains are given below.

$$MSG = 15.69\ dB$$
$$G_{T,max} = G_{P,max} = G_{A,max} = 14.27\ dB$$

The constant gain circles were then plotted on the smith chart. The available power gain circles were plotted in the Γ_S plane as shown in Fig. 1 and Fig. 3 in order to better understand the tradeoffs between gain and noise. The operating power gain circles were plotted in the Γ_L plane as shown in Fig. 2 and Fig. 4 in order to get a better understanding of the trade-offs between gain and VSWR.

C. Minimum Noise Figure Design

Our goal of achieving a minimum noise figure seemed like a natural starting point for the design. The optimal source reflection coefficient, Γ_{OPT}, was selected for Γ_S. Then a conjugate match was selected at the output to set Γ_L. The performance of this configuration was calculated and is shown below.

$$\Gamma_S = \Gamma_{OPT} = 0.280\angle 179^o$$
$$\Gamma_L = \Gamma_{ML,OPT} = 0.364\angle 118^o$$
$$NF = NF_{min} = 0.7dB$$
$$G_P = 13.55dB \quad G_A = 12.42dB \quad G_T = 12.42dB$$
$$VSWR_{IN} = 2.84 \qquad VSWR_{OUT} = 1.00$$

Next constant output VSWR circles were plotted for a $VSWR_{OUT}$ of 1.6, 1.8, and 2.0 as shown in Fig.2. Along each of these constant $VSWR_{OUT}$ circles the $VSWR_{IN}$ was calculated for 8 choices of Γ_L evenly spaced along each circle. The minimum $VSWR_{IN}$ of the 8 points along each circle was calculated and indicated on the Smith chart by a green-colored marker. These calculations were carried out using MATLAB. Detailed results of the calculations are given in Appendix A. Finally a Γ_L that had a balanced $VSWR_{IN}$ and $VSWR_{OUT}$ was selected. The performance of this configuration was calculated and is shown below.

$$\Gamma_S = \Gamma_{OPT} = 0.280\angle 179^o$$
$$\Gamma_L = \Gamma_{L,OPT} = 0.155\angle -179^o$$
$$NF = NF_{min} = 0.7dB$$
$$G_P = 13.55dB \quad G_A = 12.42dB \quad G_T = 12.42dB$$
$$VSWR_{IN} = 1.95 \qquad VSWR_{OUT} = 2.00$$

D. *Final Design*

Although choosing Γ_{OPT} for Γ_S produces a minimum noise figure it can be clearly seen from the constant noise figure and available power gain circles in Fig. 3 that a whole 1dB of gain can be added at the expense of only 0.1 dB higher noise figure. However further increasing the noise figure provides a significantly smaller increase in gain. Hence Γ_S was selected to have a noise figure of 0.8db and a gain that was 1 dB below the maximum available power gain.

Once Γ_S was selected, constant $VSWR_{OUT}$ circles at a $VSWR_{OUT}$ of 1.8, 1.5, and 1.3 were plotted in the Γ_L plane shown in Fig. 4 and $VSWR_{IN}$ was calculated at 8 points around each circle. The minimum $VSWR_{IN}$ on each constant $VSWR_{OUT}$ circle was found and highlighted in green on the Γ_L

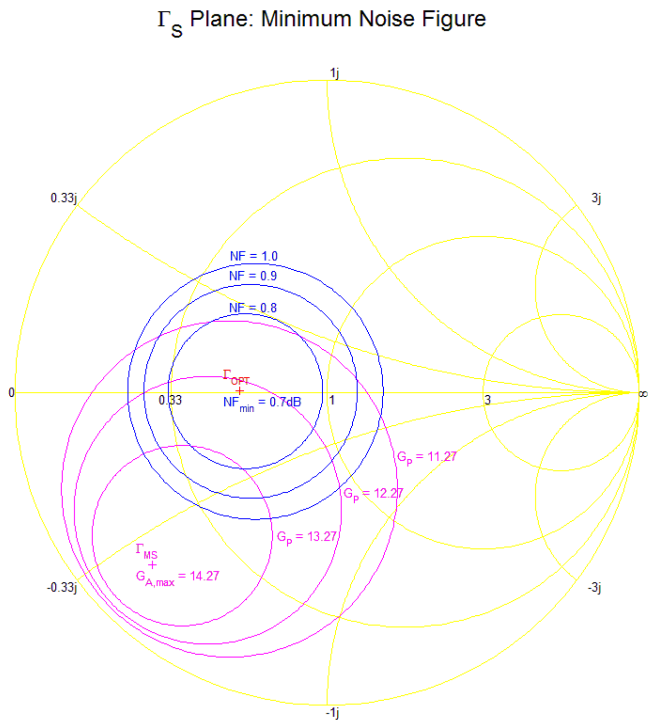


Fig. 1. Γ_S plane for the minimum noise figure design with $\Gamma_S = \Gamma_{OPT}$

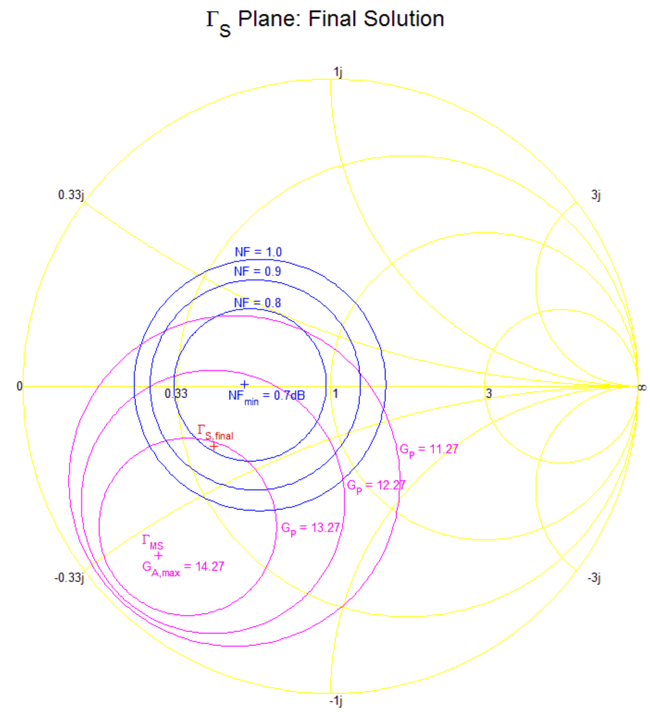


Fig. 3. Γ_S plane for the final design

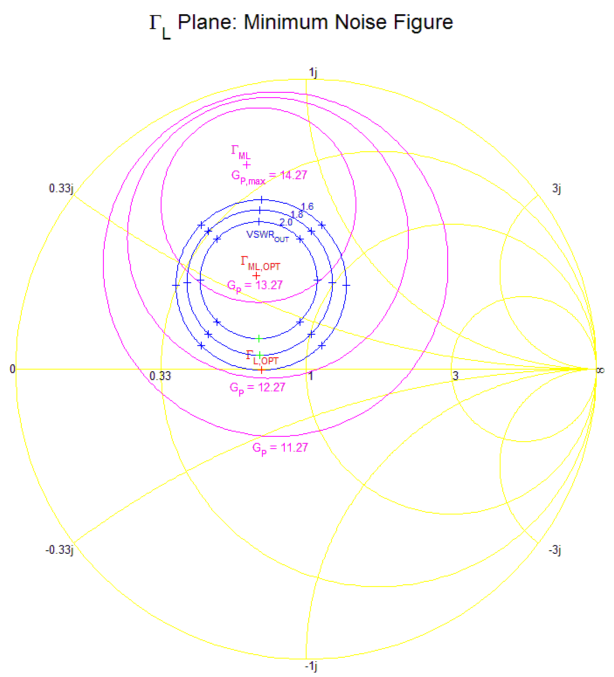


Fig. 2. Γ_L plane for the minimum noise figure design with $\Gamma_S = \Gamma_{OPT} = 0.28\angle 179^{o}$ and (a) $\Gamma_L = \Gamma_{ML,OPT} = 0.364\angle 118^{o}$ and (b) $\Gamma_L = \Gamma_{L,OPT} = 0.155\angle -179^{o}$

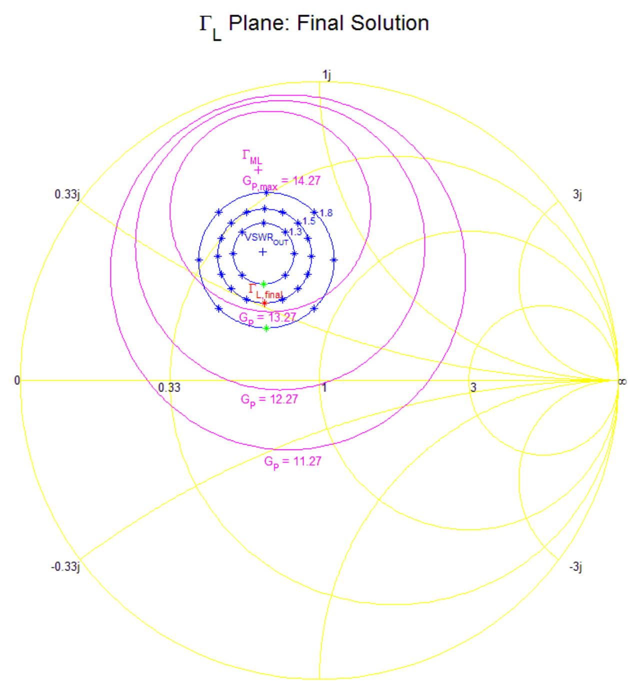


Fig. 4. Γ_L plane for the final design

plane. Once Γ_L had been selected the number of points for which $VSWR_{IN}$ was calculated was doubled to see if a smaller $VSWR_{IN}$ could be achieved, but the original selected Γ_L remained the minimum. Γ_L was selected for a good VSWR that balanced a $VSWR_{OUT}$ of 1.5 and $VSWR_{IN}$ of 1.58.

The performance of the final design was calculated as:

$$\Gamma_S = 0.427\angle{-153^o}$$
$$\Gamma_L = 0.318\angle 125^o$$
$$NF = NF_{min} = 0.786dB$$
$$G_P = 13.36dB \quad G_A = 13.31dB \quad G_T = 13.14dB$$
$$VSWR_{IN} = 1.58 \qquad VSWR_{OUT} = 1.50$$

E. Microstrip lengths

The electrical lengths of the required open shunt and series microstrip lines were first calculated by hand as shown in attached Smith charts 3 and 4 in appendix B. The ADS Smith chart matching network utility was also used to calculate the electrical lengths and matched the hand calculations well. The electrical lengths can be seen in Fig. 5.

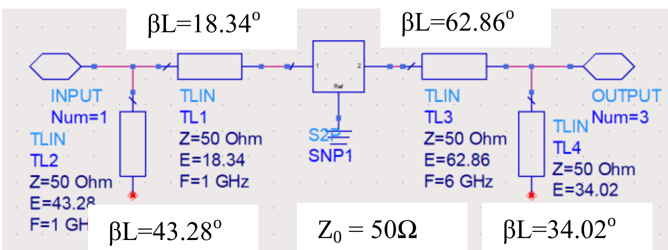


Fig. 5. Ideal T-line test bench

F. DC Biasing Network

S-parameter data for two unique biasing conditions were supplied by the manufacturer in the datasheet of the provided GaAs pHEMT fet [2]. The quiescent point of V_{DS} = 3.0v and I_{DS} = 20mA was selected for its lower minimum noise figure.

The DC biasing network must supply a constant DC gate voltage, drain voltage and drain current to the transistor while keeping the RF signals from corrupting the supply. A simple DC biasing scheme utilizing two RF chokes to bias the drain and gate is shown in the final schematic in fig. 16 was selected from [3]. The gate voltage is selected to set the drain current to 20 mA while the drain voltage is set directly to 3.0v. The values of the RF choke inductors need to be large enough to ensure good isolation of the DC bias source from the RF input and output. The DC decoupling capacitors on the supply lines also need to be large enough to filter out noise and any remaining RF on the supplies. The design currently requires a bipolar supply, but other DC biasing networks could be used if only a single supply was available.

III. Simulation and Optimization

Agilent Advanced Design System (ADS) was used to simulate the LNA using both ideal and microstrip transmission line models.

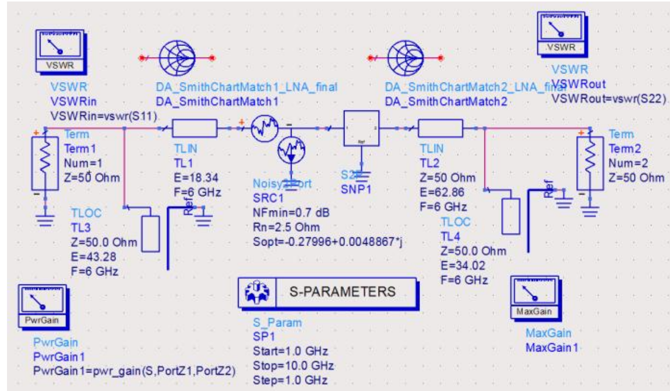


Fig. 6. Ideal T-line test bench

NF_6GHz	PwrGain	VSWRin_6GHz	VSWRout_6GHz
0.789	13.147	1.562	1.506

Fig. 7. Ideal T-line simulation performance

A. Ideal Transmission line matching

Simulations utilizing the ideal transmission line model were performed using the test bench shown in fig. 6 to confirm the hand calculated performance of the design. The hand calculated and simulated measurements matched up well as can be seen from the simulation results in fig.7.

B. Microstrip Matching Network Simulation.

Next microstrip transmission lines were used to get a more accurate simulation of the performance of the amplifier design. The circuit board substrate used a Duroid 3006 substrate. The electrical and mechanical properties of the substrate were found in the substrate's datasheet [4]. Then these properties were input into the simulator and LineCalc was utilized to calculate the widths and lengths of the microstrip lines. The microstrip lengths and widths are shown in the layout shown in fig. 17.

A test bench was created in ADS as shown in Fig. 8. This test bench was used to test the gain, noise figure and the input and output VSWRs. The performance of this design is

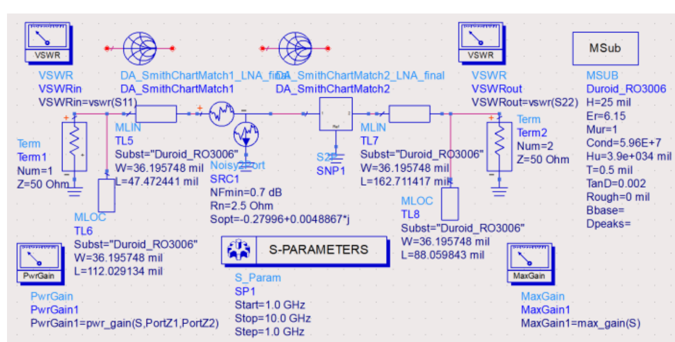


Fig. 8. Micro-strip T-line test bench

NF_6GHz	PwrGain	VSWRin_6GHz	VSWRout_6GHz
0.806	13.100	1.556	1.495

Fig. 9. Micro-strip T-line simulated performance

summarized in Fig. 9. The noise and gain were degraded slightly with the microstrip T-lines, however the $VSWR_{IN}$ and VSWROUT were improved slightly.

C. *Optimization of the design*

The optimization of the design was done using test bench shown in Fig. 10 and the optimization cockpit shown in Fig. 11 to adjust the length of the micro-strip lines. It was used to make only very subtle trade-offs in the design. The optimization was able to slightly increase the power gain at the expense of a slight (0.005 dB) increase in noise figure. It was also able to better balance the input and output VSWR by increasing the $VSWR_{IN}$ slightly and decreasing the $VSWR_{OUT}$ slightly.

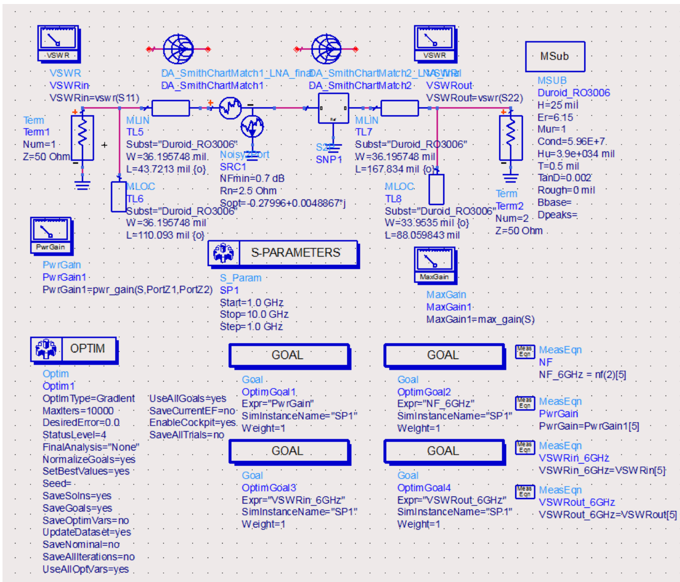


Fig. 10. Optimization test bench

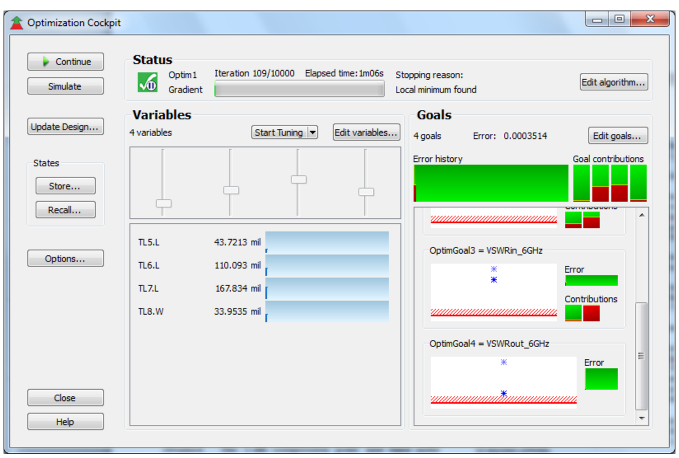
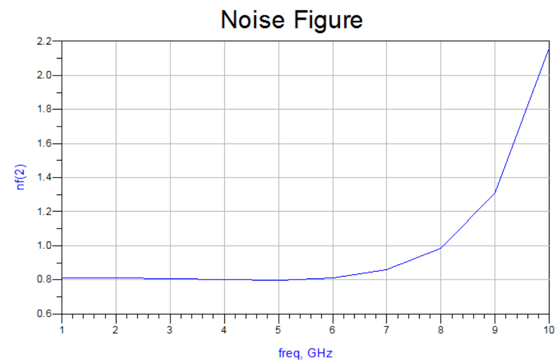


Fig. 11. Micro-strip T-line simulated performance

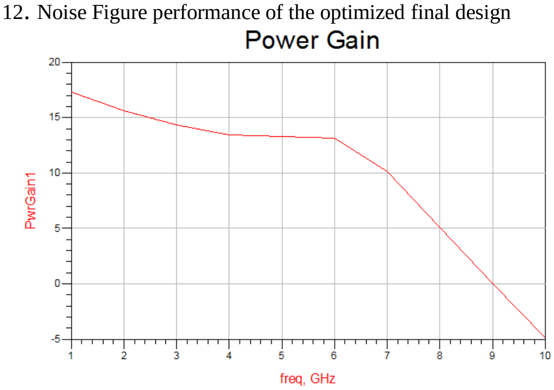
D. *Final Design Performance*

Following the optimization of the design the final simulated performance was measured using the optimization test bench shown in Fig. 10. The noise figure performance is shown in fig. 12, the gain performance in fig. 13 and the VSWR performance in fig. 14. A summary of the performance at 6 GHz is given in fig. 15. The final schematic incorporating the



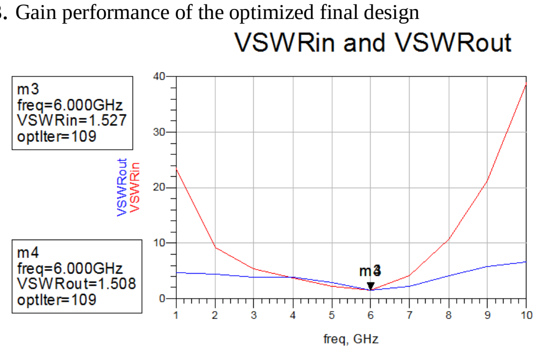
freq	nf(2) optIter=109
1.000 GHz	0.811
2.000 GHz	0.809
3.000 GHz	0.804
4.000 GHz	0.799
5.000 GHz	0.798
6.000 GHz	0.811
7.000 GHz	0.858
8.000 GHz	0.985
9.000 GHz	1.307

Fig. 12. Noise Figure performance of the optimized final design



freq	PwrGain1 optIter=109
1.000 GHz	17.251
2.000 GHz	15.618
3.000 GHz	14.347
4.000 GHz	13.419
5.000 GHz	13.282
6.000 GHz	13.124
7.000 GHz	10.166
8.000 GHz	5.133
9.000 GHz	0.020

Fig. 13. Gain performance of the optimized final design



freq	VSWRin optIter=109	VSWRout optIter=109
1.000 GHz	23.579	4.699
2.000 GHz	9.214	4.369
3.000 GHz	5.399	3.876
4.000 GHz	3.739	3.833
5.000 GHz	2.235	2.935
6.000 GHz	1.527	1.508
7.000 GHz	4.077	2.153
8.000 GHz	10.660	4.097
9.000 GHz	21.242	5.779

Fig. 14. VSWR performance of the optimized final design

transistor, DC bias network and matching networks is shown in fig. 16.

Final Design Performance	
Specification	Simulated Value
Noise Figure	0.811 dB
Power Gain	13.12 dB
Input VSWR	1.527
Output VSWR	1.508

Fig. 15. Final Performance of the optimized final design

E. *Circuit Board Layout*

The layout of the circuit board was designed using the calculated values from the optimized final design. The input, output, and ground pads will need to be added and their effects added into the simulation, along with the corners that connect the series and open circuit shunt transmission lines. The pad capacitance for the transistor should also be considered in the simulation, along with the bond wire inductance.

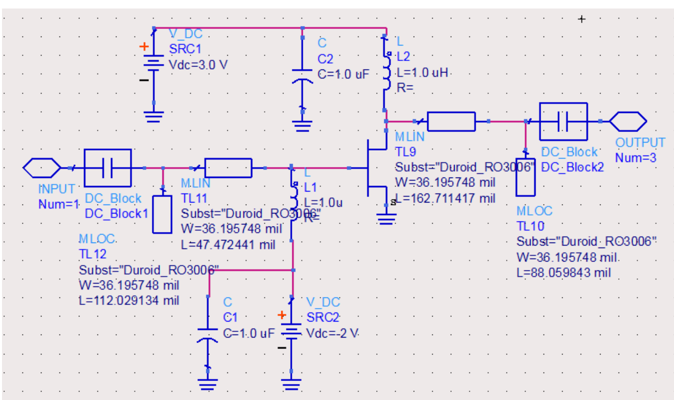


Fig. 16. Final schematic of the optimized final design

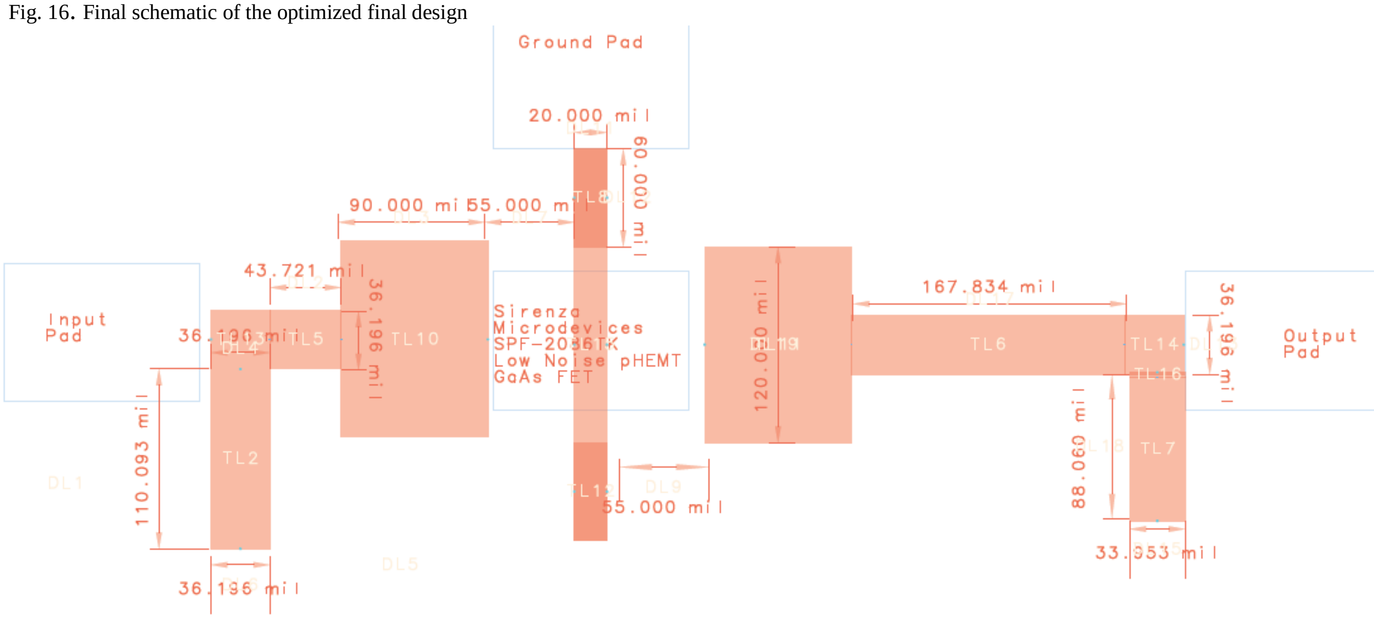


Fig. 17. Design Layout

IV. Conclusion

A low-noise amplifier has been designed at 6.0 GHz. A noise figure of 0.811dB has been achieved with a gain of X. The $VSWR_{IN}$ of X was balanced with the $VSWR_{OUT}$ of Y. The DC biasing network has been implemented with RF chokes isolating the DC voltage supplies from RF corruption.

Further design work could include considerations of the linearity and bandwidth of the amplifier. In order to simulate the linearity a device model for the GaAs fet would need to be obtained. The 1-dB compression point and third order intercept values could then be simulated in ADS. The design simulation accuracy could also be improved by including a model of the input, output and ground pads along with the associated bond wires.

Appendix

Please see attached Appendices
- Appendix A: Matlab Results (output)
- Appendix B: Smith Charts
- Appendix C: Matlab Script utilizing [5]

Acknowledgment

Curtis Mayberry thanks Prof. John Papapolymerou for a great semester and looks forward to ECE6360.

References

[1] U.S. Department of Commerce National Telecommunications and Information Administration, *United States Frequency Allocations* Available:http://www.ntia.doc.gov/files/ntia/publications/spectrum_wall_chart_aug2011.pdf

[2] Sirenza Microdevices, S*PF-2086TK Low Noise pHEMT GaAs FET Datasheet*

[3] G. Gonzalez, “Microwave Transistor Amplifiers,” in *Plastics*, 2nd ed. , Upper Saddle River, NJ: Prentice Hall, 1997, pp. 281.

[4] Rogers Corporation, *RO3000 Series Circuit Materials Datasheet*

[5] Warren du Plessis, *The Smith Chart Circles Toolbox*, Available: http://www.mathworks.com/matlabcentral/fileexchange/20960-the-smith-chart-circles-toolbox/content/smith_circles/smith_circles.m

Appendix A: Matlab Results

Hand Calculations

```
------
Stability: Unconditionally Stable
k = 1.0545, |del| = 0.24529
MSG = 15.6937dB
rML = 0.73451 < 106.1876
Gp,max = Gt,max = Ga,max = 14.2659 dB
Plotted Gp values (dB):
   13.2659
   12.2659
   11.2659

Plotted Ga values (dB):
   13.2659
   12.2659
   11.2659

Plotted NF values (dB):
    0.8000
    0.9000
    1.0000

--
Minimum Noise figure (rS = rOPT & rL = rML_opt)
rS = 0.28 < 179
rL = 0.36413 < 118.0992
VSWRin = 2.835
VSWRout = 1
NF = 0.7dB
Gp = 13.5493dB
Ga = 12.4201dB
Gt = 12.4201dB
--
Try VSWRout = 2
VSWRin = 2.1106   rL = 0.37146 < 167.2023    theta = -0.75 pi rad
VSWRin = 1.9525   rL = 0.15477 < -178.6412   theta = -0.5 pi rad
VSWRin = 2.1941   rL = 0.097755 < 57.3223    theta = -0.25 pi rad
VSWRin = 2.7799   rL = 0.32129 < 64.4181     theta = 0 pi rad
VSWRin = 3.6996   rL = 0.50009 < 83.9418     theta = 0.25 pi rad
VSWRin = 4.4675   rL = 0.60342 < 104.8576    theta = 0.5 pi rad
VSWRin = 3.92     rL = 0.61524 < 126.0699    theta = 0.75 pi rad
VSWRin = 2.779    rL = 0.53371 < 147.1141    theta = 1 pi rad
min VSWRin = 1.9525
--
Choose theta = -pi/2
rS = rOPT = 0.28 < 179
rS = 0.28 < 179
rL = 0.15477 < -178.6412
VSWRin = 1.9525
```

VSWRout = 2
NF = 0.7dB
Gp = 12.3859dB
Ga = 12.4201dB
Gt = 11.9086dB
--
Try VSWRout = 1.8
VSWRin = 2.2037   rL = 0.35751 < 160.2077   theta = -0.75 pi rad
VSWRin = 2.0516   rL = 0.16621 < 163.3319   theta = -0.5 pi rad
VSWRin = 2.2655   rL = 0.12238 < 81.5737   theta = -0.25 pi rad
VSWRin = 2.7911   rL = 0.31188 < 72.9752   theta = 0 pi rad
VSWRin = 3.5711   rL = 0.47572 < 87.8396   theta = 0.25 pi rad
VSWRin = 4.1572   rL = 0.57139 < 106.1806   theta = 0.5 pi rad
VSWRin = 3.7272   rL = 0.58236 < 125.2841   theta = 0.75 pi rad
VSWRin = 2.7964   rL = 0.5068 < 143.9542   theta = 1 pi rad
min VSWRin = 2.0516
--
Try VSWRout = 1.6
VSWRin = 2.3154   rL = 0.34711 < 151.8579   theta = -0.75 pi rad
VSWRin = 2.1754   rL = 0.19416 < 147.3769   theta = -0.5 pi rad
VSWRin = 2.3556   rL = 0.16506 < 97.3015   theta = -0.25 pi rad
VSWRin = 2.8032   rL = 0.30862 < 82.9145   theta = 0 pi rad
VSWRin = 3.424   rL = 0.44931 < 92.676   theta = 0.25 pi rad
VSWRin = 3.8388   rL = 0.53354 < 107.848   theta = 0.5 pi rad
VSWRin = 3.5227   rL = 0.54325 < 124.2925   theta = 0.75 pi rad
VSWRin = 2.8124   rL = 0.47656 < 140.0097   theta = 1 pi rad
min VSWRin = 2.1754

Give up a little NF for a larger increase in gain
Lose 0.1dB of NF, add 1dB of gain
Choose rS = 0.42713 < -152.8189
Plotted Gp values (dB):
13.2659   12.2659   11.2659

Plotted Ga values (dB):
13.2659   12.2659   11.2659

Plotted NF values (dB):
0.8000   0.9000   1.0000

--
Try VSWRout = 1.8
VSWRin = 1.5453   rL = 0.4152 < 144.338   theta = -0.75 pi rad
VSWRin = 1.4267   rL = 0.24941 < 135.2243   theta = -0.5 pi rad
VSWRin = 1.6305   rL = 0.24264 < 93.9614   theta = -0.25 pi rad
VSWRin = 2.0458   rL = 0.4054 < 82.968   theta = 0 pi rad
VSWRin = 2.6146   rL = 0.56289 < 91.7065   theta = 0.25 pi rad

VSWRin = 3.0288   rL = 0.65347 < 105.7201   theta = 0.5 pi rad
VSWRin = 2.6997   rL = 0.65602 < 120.9454   theta = 0.75 pi rad
VSWRin = 2.0076   rL = 0.56999 < 135.0981   theta = 1 pi rad
min VSWRin = 1.4267

Solution:
rS = 0.42713 < -152.8189
rL = 0.24941 < 135.2243
VSWRin = 1.4267
VSWRout = 1.8
NF = 0.78634dB
Gp = 13.08dB
Ga = 13.3135dB
Gt = 12.9436dB
--
Try VSWRout = 1.5
VSWRin = 1.8197   rL = 0.48486 < 132.6655   theta = -0.875 pi rad
VSWRin = 1.6758   rL = 0.42437 < 133.949   theta = -0.75 pi rad
VSWRin = 1.5939   rL = 0.36467 < 131.8954   theta = -0.625 pi rad
VSWRin = 1.5776   rL = 0.31773 < 125.2462   theta = -0.5 pi rad
VSWRin = 1.623   rL = 0.2981 < 114.4113   theta = -0.375 pi rad
VSWRin = 1.721   rL = 0.31394 < 103.2956   theta = -0.25 pi rad
VSWRin = 1.8616   rL = 0.35856 < 96.1031   theta = -0.125 pi rad
VSWRin = 2.0359   rL = 0.41751 < 93.5917   theta = 0 pi rad
VSWRin = 2.2323   rL = 0.47837 < 94.5708   theta = 0.125 pi rad
VSWRin = 2.4295   rL = 0.53276 < 97.7886   theta = 0.25 pi rad
VSWRin = 2.5903   rL = 0.57527 < 102.3662   theta = 0.375 pi rad
VSWRin = 2.6663   rL = 0.60247 < 107.7188   theta = 0.5 pi rad
VSWRin = 2.6219   rL = 0.61242 < 113.4302   theta = 0.625 pi rad
VSWRin = 2.4648   rL = 0.60446 < 119.1599   theta = 0.75 pi rad
VSWRin = 2.2438   rL = 0.5791 < 124.5709   theta = 0.875 pi rad
VSWRin = 2.0158   rL = 0.53816 < 129.2595   theta = 1 pi rad
min VSWRin = 1.5776

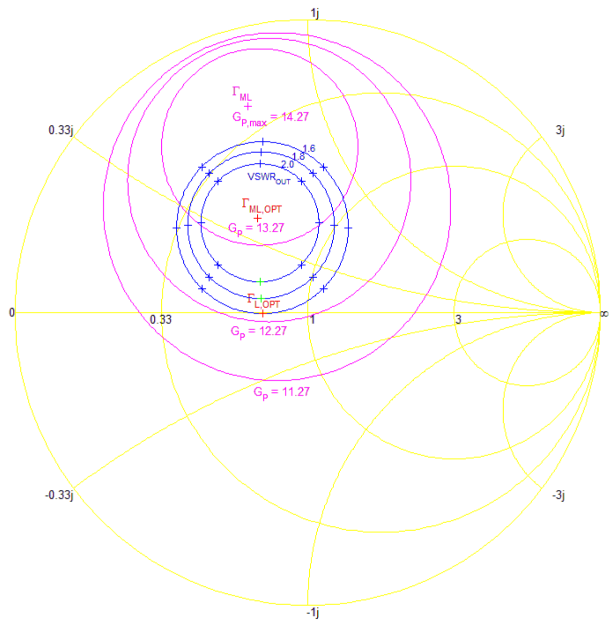
Solution:
rS = 0.42713 < -152.8189
rL = 0.31773 < 125.2462
VSWRin = 1.5776
VSWRout = 1.5
NF = 0.78634dB
Gp = 13.3599dB
Ga = 13.3135dB
Gt = 13.1362dB
--
Try VSWRout = 1.3
VSWRin = 1.7908   rL = 0.43726 < 126.3086   theta = -0.75 pi rad
VSWRin = 1.7169   rL = 0.37268 < 120.0804   theta = -0.5 pi rad
VSWRin = 1.8126   rL = 0.37055 < 108.0259   theta = -0.25 pi rad

VSWRin = 2.0312 rL = 0.43288 < 101.296 theta = 0 pi rad
VSWRin = 2.2841 rL = 0.50968 < 103.0015 theta = 0.25 pi rad
VSWRin = 2.4155 rL = 0.55865 < 109.534 theta = 0.5 pi rad
VSWRin = 2.2959 rL = 0.56006 < 117.536 theta = 0.75 pi rad
VSWRin = 2.0205 rL = 0.51342 < 124.2287 theta = 1 pi rad
min VSWRin = 1.7169

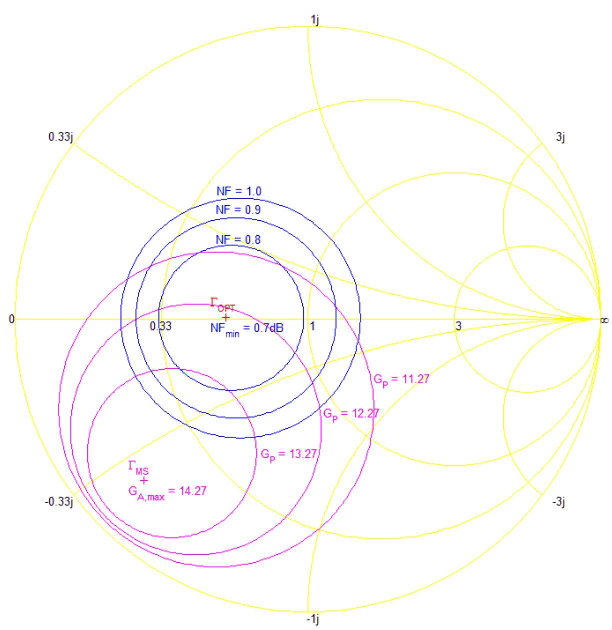
Solution:
rS = 0.42713 < -152.8189
rL = 0.37268 < 120.0804
VSWRin = 1.7169
VSWRout = 1.3
NF = 0.78634dB
Gp = 13.5524dB
Ga = 13.3135dB
Gt = 13.239dB

FINAL SOLUTION:
rS = 0.42713 < -152.8189
rL = 0.31773 < 125.2462
VSWRin = 1.5776
VSWRout = 1.5
NF = 0.78634dB
Gp = 13.3599dB
Ga = 13.3135dB
Gt = 13.1362dB

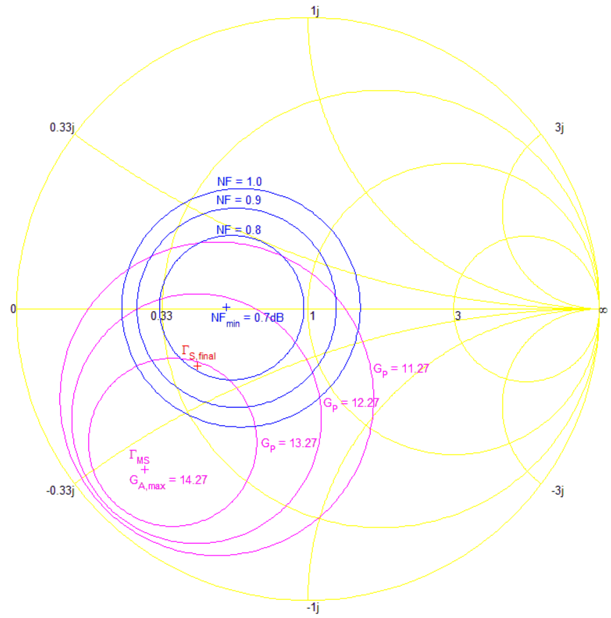
Γ_L Plane: Minimum Noise Figure



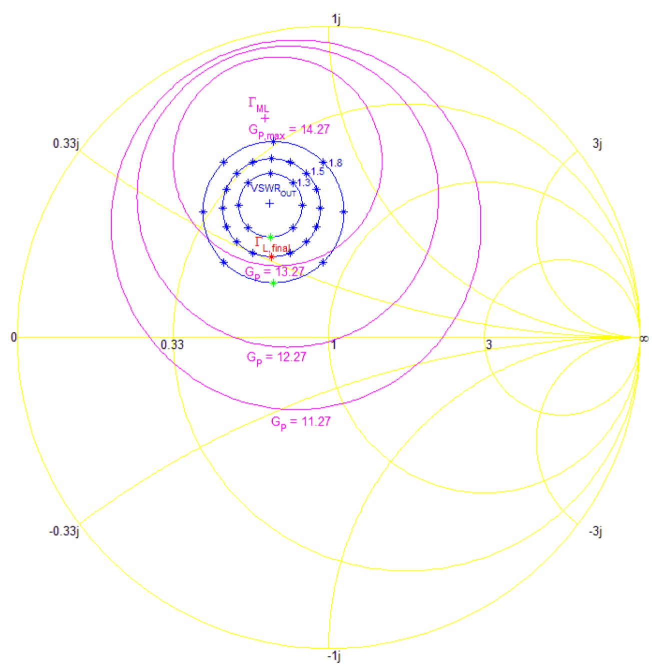
Γ_S Plane: Minimum Noise Figure



Γ_S Plane: Final Solution



Γ_L Plane: Final Solution



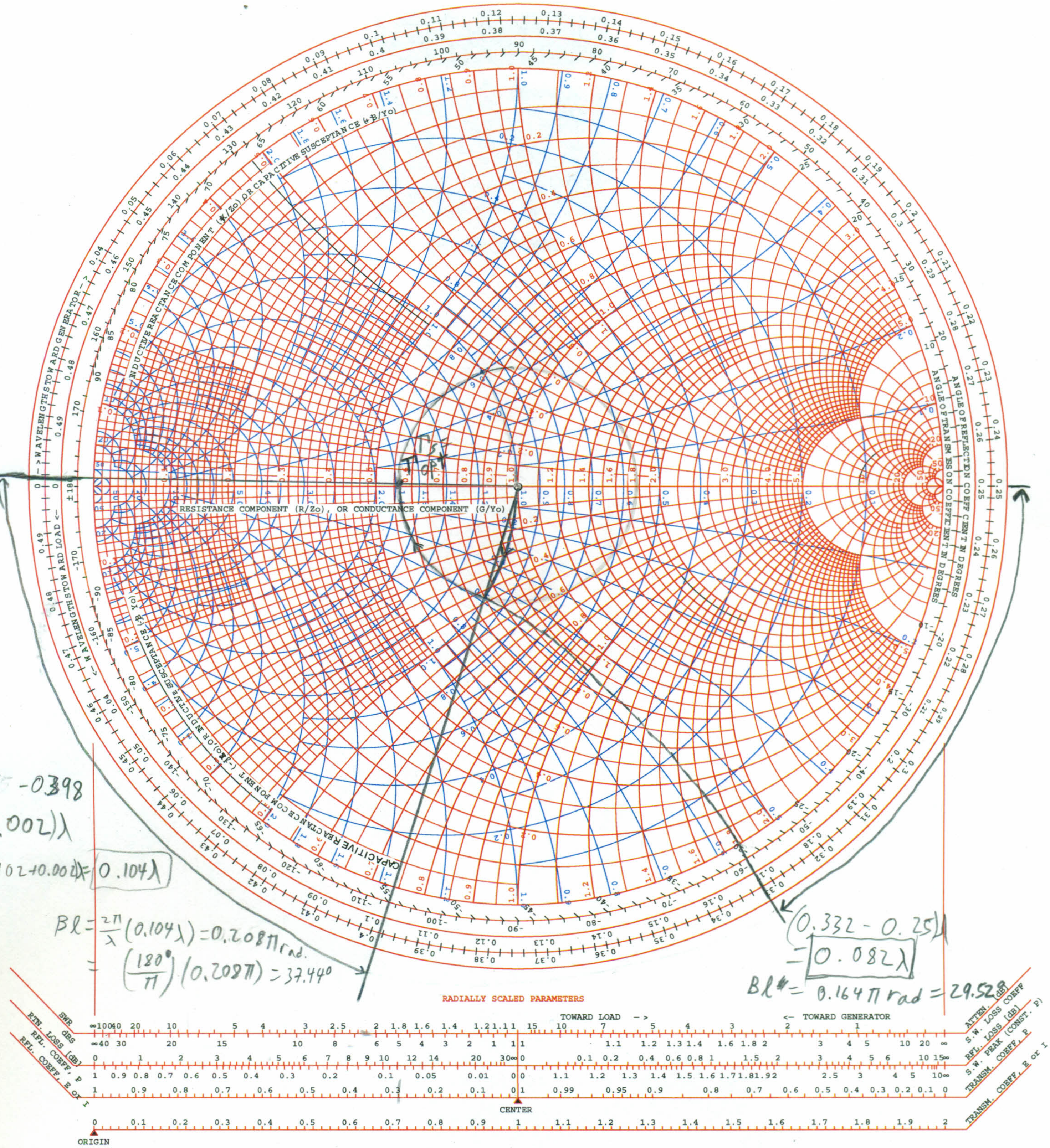
Appendix B: Smith Charts

4 Smith Charts Attached

Γ_s Plane

NAME Curtis Mayberry	TITLE Optimum Noise Figure Solution $\Gamma_s = \Gamma_{opt} = 0.28\angle 179^\circ$ $\Gamma_L = 0.155\angle -179^\circ$	DWG. NO. 1
SMITH CHART ENGS 120	COLOR BY J. COLVIN, UNIVERSITY OF FLORIDA, 1997	DATE

NORMALIZED IMPEDANCE AND ADMITTANCE COORDINATES



0.002λ

$(0.5 - 0.398 + 0.002)\lambda$

$(0.102 + 0.002) = \boxed{0.104\lambda}$

$\beta l = \frac{2\pi}{\lambda}(0.104\lambda) = 0.208\pi \text{ rad.}$

$= \left(\frac{180^\circ}{\pi}\right)(0.208\pi) = 37.44^\circ$

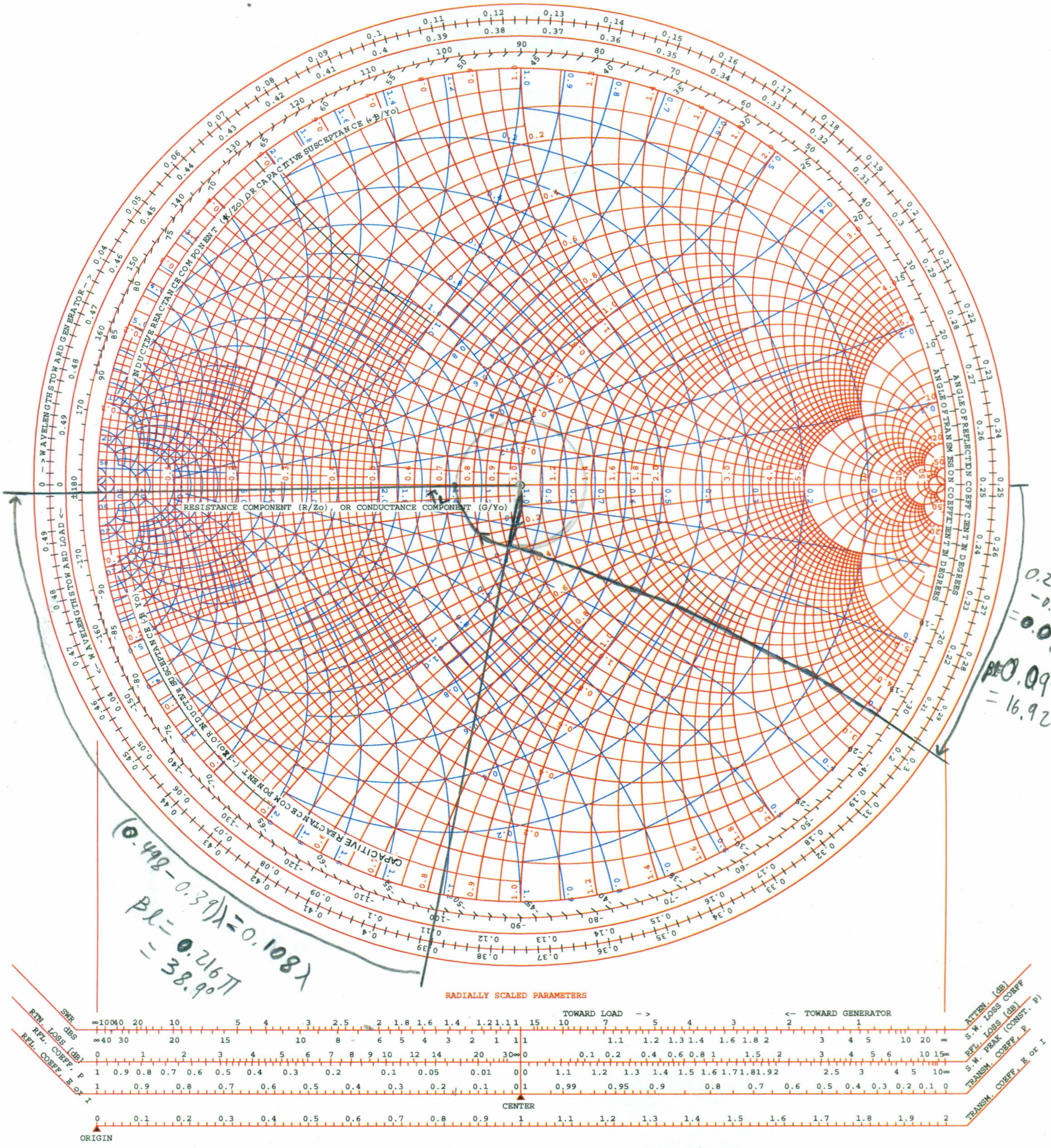
$(0.332 - 0.25)\lambda = \boxed{0.082\lambda}$

$\beta l^* = 0.164\pi \text{ rad} = 29.52^\circ$

Γ_L Plane

NAME Curtis Mayberry	TITLE Optimum Noise Figure Solution $\Gamma_S = \Gamma_{opt} = 0.28 \angle 179°$ $\Gamma_L = 0.155 \angle -179°$	DWG. NO. 2
SMITH CHART ENGS 120	COLOR BY J. COLVIN, UNIVERSITY OF FLORIDA, 1997	DATE

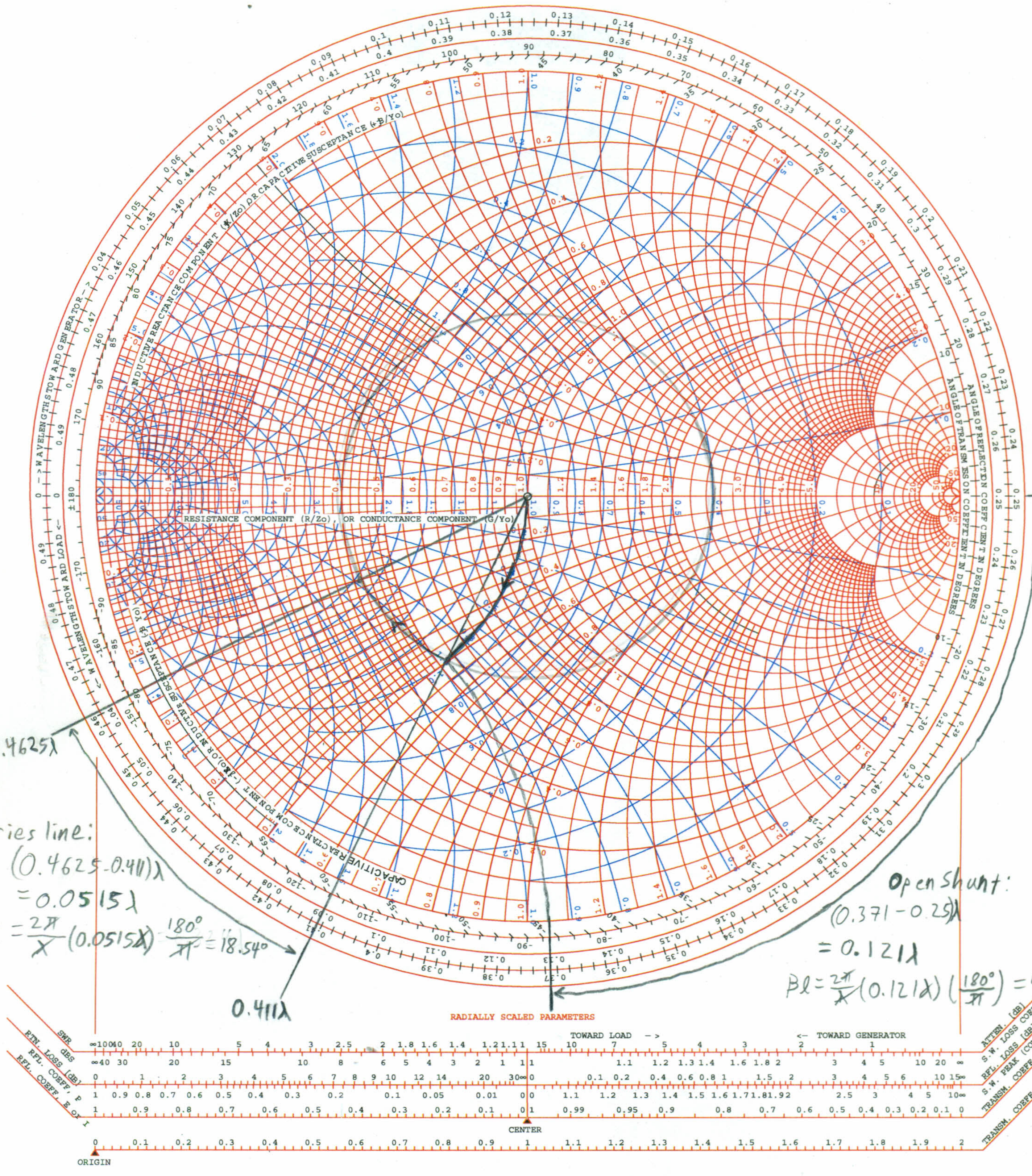
NORMALIZED IMPEDANCE AND ADMITTANCE COORDINATES



Γ_s Plane

NAME	TITLE	DWG. NO.
Curtis Mayberry	Final Hand Calculation Solution: $\Gamma_s = 0.427 \angle -153^\circ$ $\Gamma_L = 0.318 \angle -125^\circ$	3
SMITH CHART ENGS 120	COLOR BY J. COLVIN, UNIVERSITY OF FLORIDA, 1997	DATE

NORMALIZED IMPEDANCE AND ADMITTANCE COORDINATES



0.4625λ

0.411λ

Series line:

$(0.4625 - 0.411)\lambda$

$= 0.0515\lambda$

$\beta l = \frac{2\pi}{\lambda}(0.0515\lambda)\frac{180^\circ}{\pi} = 18.54^\circ$

Open Shunt:

$(0.371 - 0.25)\lambda$

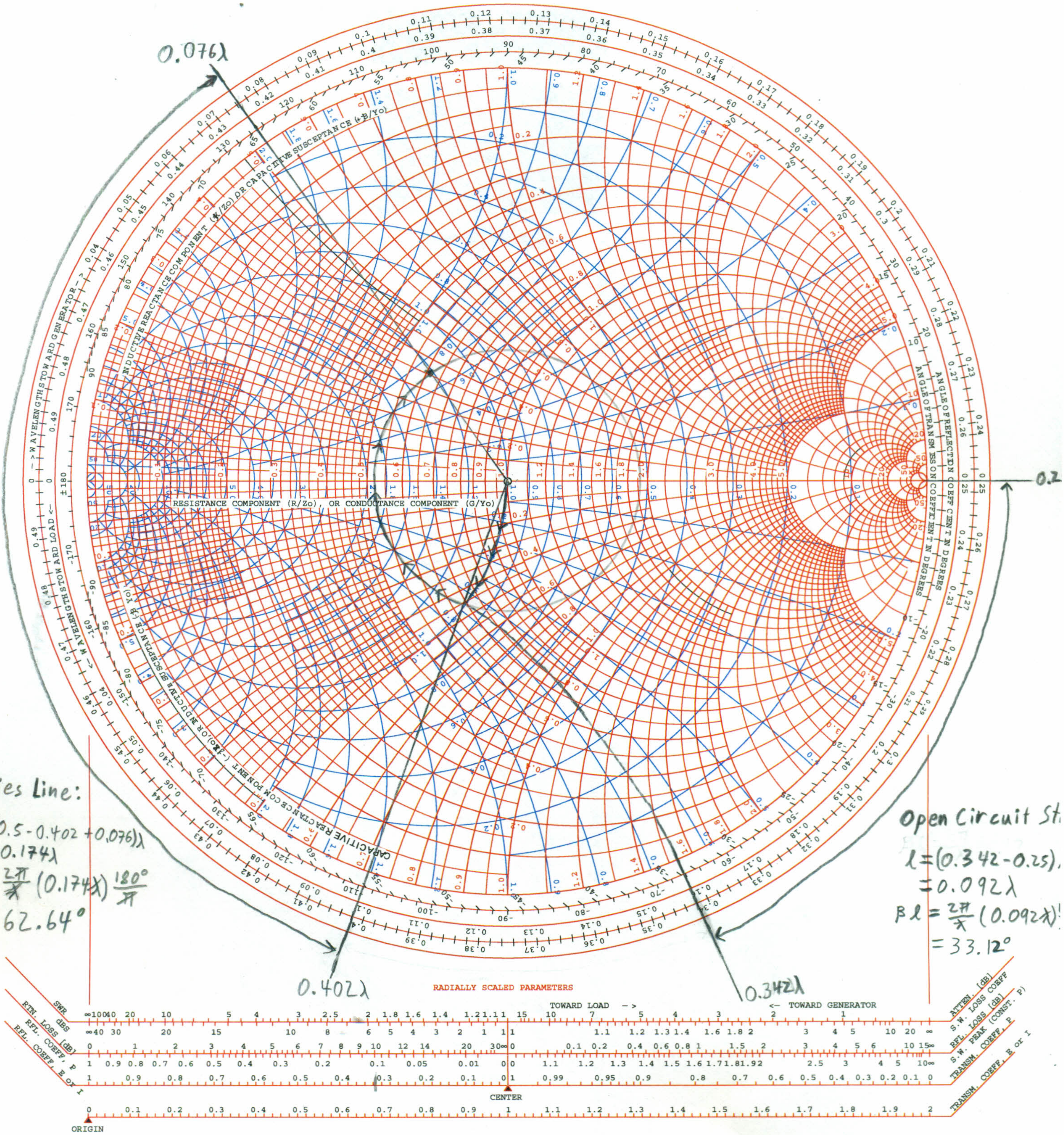
$= 0.121\lambda$

$\beta l = \frac{2\pi}{\lambda}(0.121\lambda)\left(\frac{180^\circ}{\pi}\right) = 43.56^\circ$

Γ_L Plane

NAME Curtis Mayberry	TITLE Final Hand Calculation Solution $\Gamma_S = 0.427 \angle -153^\circ$ $\Gamma_L = 0.318 \angle -125^\circ$	DWG. NO. 4
SMITH CHART ENGS 120	COLOR BY J. COLVIN, UNIVERSITY OF FLORIDA, 1997	DATE

NORMALIZED IMPEDANCE AND ADMITTANCE COORDINATES



Series Line:

$l = (0.5 - 0.402 + 0.076)\lambda$
$= 0.174\lambda$
$\beta l = \frac{2\pi}{\lambda}(0.174\lambda)\frac{180^\circ}{\pi}$
$= 62.64^\circ$

Open Circuit St

$l = (0.342 - 0.25)\lambda$
$= 0.092\lambda$
$\beta l = \frac{2\pi}{\lambda}(0.092\lambda)$
$= 33.12^\circ$

Appendix C: Matlab Script

Utilizes the *Smith Chart Circles Toolbox* (see references)

```matlab
% Microstrip LNA Project Hand Calculations
% ECE4415
% Author: Curtis Mayberry
%
%   Reference:
%   G. Gonzalez, "Microwave Transistor Amplifiers - Analysis and Design,"
%   2nd ed., Prentice Hall, 1997.

function Smith_project
clear all
close all
disp(' ')
disp(' ')
disp(' ')
disp(' ')
disp(' ')
disp('------')
s = [ (0.44*exp( j*137.1/180*pi)) (0.1 *exp( j* 9.5/180*pi)) ; ...
      (3.71*exp( j*16.7 /180*pi)) (0.29*exp(-j* 102.9/180*pi)) ];
fmin = 0.7; % dB
rOPT = (0.28*exp( j*179/180*pi));
rn   = 0.05;

% Check stability
[stable, k, del] = rollett_stability(s);
% del = calc_del(s);
% k =calc_k(s);
if(stable)
    disp(['Stability: Unconditionally Stable'])
else
    disp(['Stability: Not Unconditionally Stable']);
end
disp(['k = ' num2str(k) ', |del| = ' num2str(abs(del))])

% Setup planes
rS_plane_fig = figure(1);
% set(1,'Position',[1290 9 624 932]); % Communications Studio screen position
set(1,'Position',[-1911 9 944 988]); % Home screen position
rS_plane = gca;
smith;
title('\Gamma_S Plane: Minimum Noise Figure ','FontSize', 18)
rL_plane_fig = figure(2);
% set(2,'Position',[1929 9 624 932]); % Communications Studio screen position
set(2,'Position',[-951 9 944 988]); % Home screen position
rL_plane = gca;
smith;
title('\Gamma_L Plane: Minimum Noise Figure ','FontSize', 18)

% Stability Circles
% [out_centre, out_radius] = stability_circles(s, 'i',rS_plane)
% [in_centre, in_radius] = stability_circles(s, 'o',rL_plane)

% Maximum Stable Gain
```

```
MSG = 10*log10(abs(s(2,1))/abs(s(1,2)));
disp(['MSG = ' num2str(MSG) 'dB']);

%  Find Max Gp
Gp_max = (abs(s(2,1))/abs(s(1,2)))*(k-sqrt(k^2-1)); % Gonzalez eq
3.7.8/3.6.10
gp_max = Gp_max / (abs((s(2,1))))^2;
Gp_max_dB = 10*log10(Gp_max);
C2 = s(2,2)-del*conj(s(1,1));
rML = gp_max*conj(C2)/...
      (1+gp_max*((abs(s(2,2)))^2-(abs(del))^2));
  rMS = (calc_rin(s,rML))';
disp(['rML = ' num2str(abs(rML)) ' < ' num2str(angle(rML)*(180/pi))]);
disp(['Gp,max = Gt,max = Ga,max = ' num2str(Gp_max_dB) ' dB']);

% Gp Circles
%  Plot Constant Gp circles in rL plane
Gp_values = gp_circles(s,10*log10(maximum_gain(s)) - [1 2 3 ],rL_plane);
disp('Plotted Gp values (dB): ');
disp(Gp_values');

Gp_Label_Loc = [0.5+1i*0.33...
                0.58-1i*0.09...
                0.6-1i*0.38];
plot(rL_plane,rML,'m+')
text(real(rML)-.05,imag(rML)+.045,
'\Gamma_M_L','Parent',rL_plane,'Color','magenta','Interpreter','tex');
text(real(rML)-.05,imag(rML)-.045, ['G_P_,_m_a_x = '
num2str(Gp_max_dB,4)],'Parent',rL_plane,'Color','magenta','Interpreter','tex'
);
text(real(z_to_r(Gp_Label_Loc(1))),imag(z_to_r(Gp_Label_Loc(1))), ['G_P = '
num2str(Gp_values(1),4)],'Parent',rL_plane,'Color','magenta','Interpreter','t
ex');
text(real(z_to_r(Gp_Label_Loc(2))),imag(z_to_r(Gp_Label_Loc(2))), ['G_P = '
num2str(Gp_values(2),4)],'Parent',rL_plane,'Color','magenta','Interpreter','t
ex');
text(real(z_to_r(Gp_Label_Loc(3))),imag(z_to_r(Gp_Label_Loc(3))), ['G_P = '
num2str(Gp_values(3),4)],'Parent',rL_plane,'Color','magenta','Interpreter','t
ex');

% Ga Circles
%  Plot Constant Ga circles in rS plane
Ga_values = ga_circles(s,10*log10(maximum_gain(s)) - [1 2 3 ],rS_plane);
disp('Plotted Ga values (dB): ');
disp(Ga_values');

Ga_Label_Loc = [0.48-1i*0.6...
                0.88-1i*0.66...
                1.4-1i*0.66];
plot(rS_plane,rMS,'m+')
text(real(rMS)-.05,imag(rMS)+.045,
'\Gamma_M_S','Parent',rS_plane,'Color','magenta','Interpreter','tex');
text(real(rMS)-.05,imag(rMS)-.045, ['G_A_,_m_a_x = '
num2str(Gp_max_dB,4)],'Parent',rS_plane,'Color','magenta','Interpreter','tex'
);
```

```
text(real(z_to_r(Ga_Label_Loc(1))),imag(z_to_r(Ga_Label_Loc(1))), ['G_P = '
num2str(Ga_values(1),4)],'Parent',rS_plane,'Color','magenta','Interpreter','t
ex');
text(real(z_to_r(Ga_Label_Loc(2))),imag(z_to_r(Ga_Label_Loc(2))), ['G_P = '
num2str(Ga_values(2),4)],'Parent',rS_plane,'Color','magenta','Interpreter','t
ex');
text(real(z_to_r(Ga_Label_Loc(3))),imag(z_to_r(Ga_Label_Loc(3))), ['G_P = '
num2str(Ga_values(3),4)],'Parent',rS_plane,'Color','magenta','Interpreter','t
ex');

% Plot Constant NF circles in rS plane
noise_circles(fmin, rOPT, rn, (0.8:0.1:1) + 1e-5, rS_plane)
disp('Plotted NF values (dB): ');
disp((0.8:0.1:1)');

NF_Label_Loc = [0.46+1i*0.3...
                0.41+1i*0.4...
                0.373+1i*0.457];
text(real(rOPT)-.05,imag(rOPT)-.045, 'NF_m_i_n =
0.7dB','Parent',rS_plane,'Color','blue','Interpreter','tex');
text(real(z_to_r(NF_Label_Loc(1))),imag(z_to_r(NF_Label_Loc(1))), 'NF =
0.8','Parent',rS_plane,'Color','blue','Interpreter','tex');
text(real(z_to_r(NF_Label_Loc(2))),imag(z_to_r(NF_Label_Loc(2))), 'NF =
0.9','Parent',rS_plane,'Color','blue','Interpreter','tex');
text(real(z_to_r(NF_Label_Loc(3))),imag(z_to_r(NF_Label_Loc(3))), 'NF =
1.0','Parent',rS_plane,'Color','blue','Interpreter','tex');




%%%%%%%%%%%%%%%%%
% See what happens with rS = rOPT, rL = rout' = rMLopt

rML_opt = conj(s(2,2)+(s(1,2)*s(2,1)*rOPT)/(1-s(1,1)*rOPT));
VSWRin_opt = calc_VSWRin(s,rOPT,rML_opt);
VSWRout_opt = calc_VSWRout(s,rOPT,rML_opt);
Gp_opt = 10*log10(calc_Gp(s,rML_opt));
Ga_opt = 10*log10(calc_Ga(s,rOPT));
Gt_opt = 10*log10(calc_Gp(s,rML_opt));
plot(rL_plane,rML_opt,'r+')
text(real(rML_opt)-.05,imag(rML_opt)+.045,
'\Gamma_M_L_,_O_P_T','Parent',rL_plane,'Color','red','Interpreter','tex');


disp('--')
disp('Minimum Noise figure (rS = rOPT & rL = rML_opt)')
print_solution(s,rOPT,rML_opt,fmin,rn,rOPT)
% disp(['rOPT = ' num2str(abs(rOPT)) ' < ' num2str((180/pi)*angle(rOPT))]);
% disp(['rML_opt = ' num2str(abs(rML_opt)) ' < '
num2str((180/pi)*angle(rML_opt))]);
% disp(['VSWRin = ' num2str(VSWRin_opt)]);
% disp(['VSWRout = ' num2str(VSWRout_opt)]);
% disp(['Gp_opt = ' num2str(Gp_opt)]);

%%%%%%%%%%%%%%%%%
```

```
% See what happens with rS = rOPT,
%  Now choose rL for a VSWR not equal to 1.
%  Then draw constant VSWRout = 2 circle and see if a rL on this circle
%   gives a reasonable VSWRin.

% VSWRout Circle
% Try VSWRout = 2
disp('--')
disp('Try VSWRout = 2')
% VSWRout Circle
VSWRout = 2;
[VSWRin_min, rL_VSWRin_min, theta_VSWRin_min] =
plot_VSWRoutCircle(rL_plane,s,rOPT,VSWRout,8);
text(real(z_to_r(0.51+1i*0.7)),imag(z_to_r(0.51+1i*0.7)),
'2.0','Parent',rL_plane,'Color','blue','FontSize',7,'Interpreter','tex');
text(real(z_to_r(0.45+1i*0.55)),imag(z_to_r(0.45+1i*0.55)),
'VSWR_O_U_T','Parent',rL_plane,'Color','blue','FontSize',7,'Interpreter','tex
');
disp(['min VSWRin = ' num2str(VSWRin_min)]);
% Choose min. VSWRin
rL = rL_VSWRin_min;

plot(rS_plane, rOPT, 'r+');
text(real(rOPT)-.05,imag(rOPT)+.045,
'\Gamma_O_P_T','Parent',rS_plane,'Color','red','Interpreter','tex');
plot(rL_plane, rL, 'r+');
text(real(rL)-.05,imag(rL)+.045,
'\Gamma_L_,_O_P_T','Parent',rL_plane,'Color','red','Interpreter','tex');


disp('--')
disp('Choose theta = -pi/2')
disp(['rS = rOPT = ' num2str(abs(rOPT)) ' < '
num2str((180/pi)*angle(rOPT))]);
print_solution(s,rOPT,rL,fmin,rn,rOPT)

%%%%%%%%%%%%%%%%%
% Try a smaller value of VSWRout = 1.8
disp('--')
disp('Try VSWRout = 1.8')
% VSWRout Circle
VSWRout = 1.8;
[VSWRin_min, rL_VSWRout_min, theta_VSWRout_min] =
plot_VSWRoutCircle(rL_plane,s,rOPT,VSWRout,8);
text(real(z_to_r(0.51+1i*0.765)),imag(z_to_r(0.51+1i*0.765)),
'1.8','Parent',rL_plane,'Color','blue','FontSize',7,'Interpreter','tex');
disp(['min VSWRin = ' num2str(VSWRin_min)]);

%%%%%%%%%%%%%%%%%
% Try a smaller value of VSWRout = 1.6
disp('--')
disp('Try VSWRout = 1.6')
% VSWRout Circle
VSWRout = 1.6;
[VSWRin_min, rL_VSWRout_min, theta_VSWRout_min] =
plot_VSWRoutCircle(rL_plane,s,rOPT,VSWRout,8);
```

```
text(real(z_to_r(0.51+1i*0.83)),imag(z_to_r(0.51+1i*0.83)),
'1.6','Parent',rL_plane,'Color','blue','FontSize',7,'Interpreter','tex');
disp(['min VSWRin = ' num2str(VSWRin_min)]);

%%%%%%%%%%%%%%%%%%%%%%
% Final Solution
% Give up a little NF for a larger increase in gain
% Lose 0.1dB of NF, add 1dB of gain
% Trade off can be seen on rS plane as you move from rSopt
%  to rS
rS = rOPT - 0.1 - j*0.2;
% rS = rOPT - 0.088 - j*0.21; % Not really much of an improvement...

disp('  ')
disp('-----')
disp('Give up a little NF for a larger increase in gain')
disp('Lose 0.1dB of NF, add 1dB of gain')
disp(['Choose rS = ' num2str(abs(rS)) ' < ' num2str((180/pi)*angle(rS))])


% Setup a new set of plots
% Setup planes
rS_plane_fig = figure(3);
% set(3,'Position',[1290 9 624 932]); % Communications Studio screen position
set(3,'Position',[-1911 9 944 988]); % Home screen position
rS_plane = gca;
smith;
title('\Gamma_S Plane: Final Solution ','FontSize', 18)
rL_plane_fig = figure(4);
% set(4,'Position',[1929 9 624 932]); % Communications Studio screen position
set(4,'Position',[-951 9 944 988]); % Home screen position
rL_plane = gca;
smith;
title('\Gamma_L Plane: Final Solution ','FontSize', 18)

% Gp Circles
%  Plot Constant Gp circles in rL plane
Gp_values = gp_circles(s,10*log10(maximum_gain(s)) - [1 2 3 ],rL_plane);
disp('Plotted Gp values (dB): ');
disp(Gp_values);
Gp_Label_Loc = [0.54+1i*0.245...
                0.58-1i*0.09...
                0.6-1i*0.38];
plot(rL_plane,rML,'m+')
text(real(rML)-.05,imag(rML)+.045,
'\Gamma_M_L','Parent',rL_plane,'Color','magenta','Interpreter','tex');
text(real(rML)-.05,imag(rML)-.045, ['G_P_,_m_a_x = '
num2str(Gp_max_dB,4)],'Parent',rL_plane,'Color','magenta','Interpreter','tex'
);
text(real(z_to_r(Gp_Label_Loc(1))),imag(z_to_r(Gp_Label_Loc(1))), ['G_P = '
num2str(Gp_values(1),4)],'Parent',rL_plane,'Color','magenta','Interpreter','t
ex');
text(real(z_to_r(Gp_Label_Loc(2))),imag(z_to_r(Gp_Label_Loc(2))), ['G_P = '
num2str(Gp_values(2),4)],'Parent',rL_plane,'Color','magenta','Interpreter','t
ex');
```

```
text(real(z_to_r(Gp_Label_Loc(3))),imag(z_to_r(Gp_Label_Loc(3))), ['G_P = '
num2str(Gp_values(3),4)],'Parent',rL_plane,'Color','magenta','Interpreter','t
ex');

% Ga Circles
%  Plot Constant Ga circles in rS plane
Ga_values = ga_circles(s,10*log10(maximum_gain(s)) - [1 2 3 ],rS_plane);
disp('Plotted Ga values (dB): ');
disp(Ga_values);
Ga_Label_Loc = [0.48-1i*0.6...
                0.88-1i*0.66...
                1.4-1i*0.66];
plot(rS_plane,rMS,'m+')
text(real(rMS)-.05,imag(rMS)+.045,
'\Gamma_M_S','Parent',rS_plane,'Color','magenta','Interpreter','tex');
text(real(rMS)-.05,imag(rMS)-.045, ['G_A_,_m_a_x = '
num2str(Gp_max_dB,4)],'Parent',rS_plane,'Color','magenta','Interpreter','tex'
);
text(real(z_to_r(Ga_Label_Loc(1))),imag(z_to_r(Ga_Label_Loc(1))), ['G_P = '
num2str(Ga_values(1),4)],'Parent',rS_plane,'Color','magenta','Interpreter','t
ex');
text(real(z_to_r(Ga_Label_Loc(2))),imag(z_to_r(Ga_Label_Loc(2))), ['G_P = '
num2str(Ga_values(2),4)],'Parent',rS_plane,'Color','magenta','Interpreter','t
ex');
text(real(z_to_r(Ga_Label_Loc(3))),imag(z_to_r(Ga_Label_Loc(3))), ['G_P = '
num2str(Ga_values(3),4)],'Parent',rS_plane,'Color','magenta','Interpreter','t
ex');

% Plot Constant NF circles in rS plane
noise_circles(fmin, rOPT, rn, (0.8:0.1:1) + 1e-5, rS_plane)
disp('Plotted NF values (dB): ');
disp((0.8:0.1:1));
NF_Label_Loc = [0.46+1i*0.3...
                0.41+1i*0.4...
                0.373+1i*0.457];
plot(rS_plane,rOPT,'b+')
text(real(rOPT)-.05,imag(rOPT)-.045, 'NF_m_i_n =
0.7dB','Parent',rS_plane,'Color','blue','Interpreter','tex');
text(real(z_to_r(NF_Label_Loc(1))),imag(z_to_r(NF_Label_Loc(1))), 'NF =
0.8','Parent',rS_plane,'Color','blue','Interpreter','tex');
text(real(z_to_r(NF_Label_Loc(2))),imag(z_to_r(NF_Label_Loc(2))), 'NF =
0.9','Parent',rS_plane,'Color','blue','Interpreter','tex');
text(real(z_to_r(NF_Label_Loc(3))),imag(z_to_r(NF_Label_Loc(3))), 'NF =
1.0','Parent',rS_plane,'Color','blue','Interpreter','tex');



% VSWRout Circle
text(real(z_to_r(0.4+1i*0.53)),imag(z_to_r(0.4+1i*0.53)),
'VSWR_O_U_T','Parent',rL_plane,'Color','blue','FontSize',7,'Interpreter','tex
');
disp('--')
disp('Try VSWRout = 1.8')
VSWRout = 1.8;
[VSWRin_min, rL_VSWRout_min, theta_VSWRout_min] =
plot_VSWRoutCircle(rL_plane,s,rS,VSWRout,8);
```

```
text(real(z_to_r(0.51+1i*0.86)),imag(z_to_r(0.51+1i*0.83)),
'1.8','Parent',rL_plane,'Color','blue','FontSize',7,'Interpreter','tex');
rML_rS = conj(s(2,2)+(s(1,2)*s(2,1)*rS)/(1-s(1,1)*rS));
plot(rL_plane,rML_rS, 'b+');
disp(['min VSWRin = ' num2str(VSWRin_min)]);
rL = rL_VSWRout_min;
plot(rS_plane,rS,'r+')

disp(' ')
disp('Solution:')
print_solution(s,rS,rL,fmin,rn,rOPT)

% VSWRout Circle
disp('--')
disp('Try VSWRout = 1.5')
VSWRout = 1.5;
[VSWRin_min, rL_VSWRout_min, theta_VSWRout_min] =
plot_VSWRoutCircle(rL_plane,s,rS,VSWRout,16);
text(real(z_to_r(0.51+1i*0.76)),imag(z_to_r(0.51+1i*0.76)),
'1.5','Parent',rL_plane,'Color','blue','FontSize',7,'Interpreter','tex');

disp(['min VSWRin = ' num2str(VSWRin_min)]);
rL = rL_VSWRout_min;
plot(rS_plane,rS,'r+')

rL_final_solution = rL;

disp(' ')
disp('Solution:')
print_solution(s,rS,rL,fmin,rn,rOPT)

% VSWRout Circle
disp('--')
disp('Try VSWRout = 1.3')
VSWRout = 1.3;
[VSWRin_min, rL_VSWRout_min, theta_VSWRout_min] =
plot_VSWRoutCircle(rL_plane,s,rS,VSWRout,8);
text(real(z_to_r(0.51+1i*0.68)),imag(z_to_r(0.51+1i*0.68)),
'1.3','Parent',rL_plane,'Color','blue','FontSize',7,'Interpreter','tex');
disp(['min VSWRin = ' num2str(VSWRin_min)]);
rL = rL_VSWRout_min;
plot(rS_plane,rS,'r+')

disp(' ')
disp('Solution:')
print_solution(s,rS,rL,fmin,rn,rOPT)

plot(rL_plane,rL_final_solution,'r*')

disp(' ')
disp('-----')
disp('FINAL SOLUTION:')
print_solution(s,rS,rL_final_solution,fmin,rn,rOPT)
text(real(rS)-.05,imag(rS)+.047,
'\Gamma_S_,_f_i_n_a_l','Parent',rS_plane,'Color','red','Interpreter','tex');
```

```
text(real(rL_final_solution)-.05,imag(rL_final_solution)+.042,
'\Gamma_L_,_f_i_n_a_l','Parent',rL_plane,'Color','red','Interpreter','tex');
```

```
%%%%%%%%%%%%%
% Functions %
%%%%%%%%%%%%%
function Gp_gammaL_to_gammaS(s,del,Cp_rl,Rp_rl,Gp)
Cp_rs = (((1-s(2,2)*Cp_rl)*(s(1,1)-del*Cp_rl)')-
((Rp_rl)^2*(del')*s(2,2)))/...
        ((abs(1-(s(2,2)*Cp_rl)))^2-Rp_rl^2*abs(s(2,2))^2);

Rp_rs = (Rp_rl*abs(s(1,2)*s(2,1)))/...
        (abs(abs(1-s(2,2)*Cp_rl)^2-Rp_rl^2*abs(s(2,2))^2));

disp('--');
disp(['Gp = ' Gp 'dB']);
disp(['Cp_rs = ' num2str(abs(Cp_rs)) ' < ' num2str((180/pi)*angle(Cp_rs))]);
disp(['Rp_rs = ' num2str(abs(Rp_rs))]);

% Convert points in rL plane to pts in rS plane
function rS = rLplane_to_rSplane(rL,s)
rS = (s(1,1)+(s(1,2)*s(2,1)*rL)/(1-s(2,2)*rL))';

% Gonzalez eq. 3.8.6
function rb_mag = calc_rb_mag(rL,rout)
rb_mag = abs((rout-(rL'))/(1-rout*rL));

function del = calc_del(s)
del = s(1,1)*s(2,2) - S(1,2)*S(2,1);

function k = calc_k(s)
del = s(1,1)*s(2,2) - S(1,2)*S(2,1);
k = (1-(abs(s(1,1)))^2-(abs(s(2,2)))^2+(abs(del))^2)/...
    (2*abs(s(1,2)*s(2,1)));

function rin = calc_rin(s,rL)
rin = s(1,1)+(s(1,2)*s(2,1)*rL)/...
             (1-s(2,2)*rL);

% Gonzalez eq. 2.6.5
function rout = calc_rout(s,rS)
rout = s(2,2)+(s(1,2)*s(2,1)*rS)/...
              (1-s(1,1)*rS);
function VSWRin = calc_VSWRin(s,rS,rL)
rin = calc_rin(s,rL);
ra_mag = abs((rin-rS')/(1-rin*rS));
VSWRin = (1+ra_mag)/(1-ra_mag);
```

```
function VSWRout = calc_VSWRout(s,rS,rL)
rout = calc_rout(s,rS);
rb_mag = abs((rout-rL')/(1-rout*rL));
VSWRout = (1+rb_mag)/(1-rb_mag);

function Gp = calc_Gp(s,rL)
rin = calc_rin(s,rL);
Gp = (1/(1-(abs(rin)).^2)*(abs(s(2,1))).^2*(1-(abs(rL)).^2)/((abs(1-
s(2,2)*rL)).^2));

% Calculate ra/rb
function rab = calc_rab(VSWR)
rab = (VSWR-1)/(VSWR+1);

function Ga = calc_Ga(s,rS)
rout = calc_rout(s,rS);
Ga = ((1-abs(rS)^2)/(abs(1-s(1,1)*rS))^2)*(abs(s(2,1)))^2*(1/(1-
(abs(rout))^2));

% Calculates the Noise Figure in dB
function F = calc_NF(Fmin, rn, rOPT, rS)
F = 10.^(Fmin/10) + (4*rn*(abs(rS-rOPT))^2)/...
           ((1-(abs(rS))^2)*(abs(1+rOPT))^2); % Gonzalez 4.3.4
 F = 10 * log10(F);
% yS = (1-rS)/(1+rS); % Gonzalez eq 4.3.2
% yOPT = (1-rOPT)/(1+rOPT);
% F = 10.^(Fmin/10) + (rn/(real(yS)))*(abs(yS-yOPT))^2; % Gonzalez eq 4.3.4

function Gt = calc_Gt(s,rS,rL)
rin = calc_rin(s,rL);
Gt = ((1-abs(rS)^2)/abs(1-rin*rS)^2) * (abs(s(2,1))^2) * ((1-
abs(rL)^2)/(abs(1-s(2,2)*rL)^2));

% function Gp_circles_on_rS_plane(s)

% Calculates the reflection coefficient of a normalized impedance z
function r = z_to_r(z)
r = (z-1)/(z+1);

% plots the VSWRoutCircle and selects the minimum VSWRin on that circle.
function [VSWRin_min, rL_VSWRin_min, theta_VSWRin_min] =
plot_VSWRoutCircle(rL_plane,s,rS,VSWRout,N)
rb = calc_rab(VSWRout);
rout = calc_rout(s,rS);
Cvo = (rout'*(1-abs(rb)^2))/...
      (1-abs(rb*rout)^2);
Rvo = (abs(rb)*(1-abs(rout)^2))/...
      (1-abs(rb*rout)^2);
smith_circles(Cvo,Rvo,'b-',256,rL_plane)
% Test 8 points on the constant VSWRout circle
for idx = 1:N
    theta(idx) = -pi+(2*pi/N)*idx; % -pi+pi/4:pi/4:pi
    rL(idx) = Cvo+Rvo*exp(j*theta(idx));
    VSWRin(idx) = calc_VSWRin(s,rS,rL(idx));
```

```
    disp_theta{idx} = ['theta = ' num2str(theta(idx)/pi) ' pi rad'];
    disp_rL{idx} = ['rL = ' num2str(abs(rL(idx))) ' < '
num2str((180/pi)*angle(rL(idx)))];
    disp_VSWRin{idx} = ['VSWRin = ' num2str(VSWRin(idx))];
    disp_spacing{idx} = [32 32 32];
    plot(rL_plane, rL, 'b*');
%     disp('-')
%     disp(['theta = ' num2str(theta/pi) ' pi rad'])
%     disp(['rL = ' num2str(abs(rL)) ' < ' num2str((180/pi)*angle(rL))]);
%     disp(['VSWRin = ' num2str(VSWRin)])
end
% Display Table of pts on VSWRout circle
disp_theta = char(disp_theta);
disp_rL = char(disp_rL);
disp_VSWRin = char(disp_VSWRin);
disp_spacing = char(cellfun(@char,disp_spacing','UniformOutput',false));
disp([disp_VSWRin disp_spacing disp_rL disp_spacing disp_theta]);

% Calculate min VSWRin
[VSWRin_min min_idx] = min(VSWRin);
theta_VSWRin_min = theta(min_idx);
rL_VSWRin_min = rL(min_idx);
plot(rL_plane, rL_VSWRin_min, 'g*');

% Print Solution
% Prints important information about a given rS, rL solution
function print_solution(s,rS,rL,Fmin,rn,rOPT)
F = calc_NF(Fmin, rn, rOPT, rS);
VSWRin = calc_VSWRin(s,rS,rL);
VSWRout = calc_VSWRout(s,rS,rL);
Gp = calc_Gp(s,rL);
Ga = calc_Ga(s,rS);
Gt = calc_Gt(s,rS,rL);

disp(['rS = ' num2str(abs(rS)) ' < ' num2str((180/pi)*angle(rS))]);
disp(['rL = ' num2str(abs(rL)) ' < ' num2str((180/pi)*angle(rL))]);
disp(['VSWRin = ' num2str(VSWRin)])
disp(['VSWRout = ' num2str(VSWRout)])
disp(['NF = ' num2str(F) 'dB'])
disp(['Gp = ' num2str(10*log10(Gp)) 'dB'])
disp(['Ga = ' num2str(10*log10(Ga)) 'dB'])
disp(['Gt = ' num2str(10*log10(Gt)) 'dB'])
```