

Vol4_23.3

April 15, 2024

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[ ]: import numpy as np
      from matplotlib import pyplot as plt
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1 23.6

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[ ]: def my_LQG(A,B,H,Q,M,V,W,R,x0,T):
    S = [None]*(T+1)
    P = [None]*(T+1)
    #setup the LQR gain here
    P[T] = M
    for k in range(T, -1, -1):
        var = R+(B.T@P[k])@B
        P[k-1] = (A.T@P[k])@A + Q - (A.T@P[k]@B)@np.linalg.pinv(var)@(B.
↪T@P[k]@A)

    u = [None]*T
    K = [None]*(T+1)
    L = [None]*T
    S = [None]*(T+1)
    S[0] = np.eye(2)
    #setup the Kalman gain
    for k in range(1,T):
        S[k] = A@(S[k-1]-S[k-1]@H.T@np.linalg.pinv(H@S[k-1]@H.T+ V)@H@S[k-1])@A.
↪T+W
    #now compute both gain matrices at each step
    #note that np.squeeze is only used because this term is - actually a scalar.
    ↪..
    K[0] = -B.T@P[1]@A/(np.squeeze(B.T@P[1]@B)+R)
    for k in range(1,T):
        K[k] = -B.T@P[k+1]@A/(np.squeeze(B.T@P[k+1]@B)+R)
        L[k] = S[k-1]@H.T@np.linalg.pinv(H@S[k-1]@H.T+V)
    full_state = [None]*(T+1)
    full_state[0] = np.array([x0]).T
    observed_state = [None]*(T+1)
    #finally we are ready to compute the solution.
    u[0] = K[0]@full_state[0]
    for k in range(1,T):
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        full_state[k] = A@full_state[k-1]+B@u[k-1] + np.random.
        ↪multivariate_normal(np.zeros(len(W)), cov=W)
        observed_state[k] = H@full_state[k] + np.random.multivariate_normal(np.
        ↪zeros(len(V)), cov=V)
        u[k] = -K[k]@observed_state[k]
        return u, observed_state, full_state

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2 23.7

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[ ]: A = np.array([[1,1],[0,1]])
      B = np.array([[0],[1]])
      H = np.diag([1,0])
      Q, M = np.eye(2), np.eye(2)
      V, W = 10*np.eye(2), 20*np.eye(2)
      x0 = np.array([1,0])
      R = 1

      u, obs, state = my_LQG(A, B, H, Q, M, V, W, R, x0, 100)
      print(u)
      print(obs)
      print(state)

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