

Algorithm 1: Connecting Pairs of Persons

Pseudocode:

Problem: Determine the minimum number of swaps required to ensure that all couples are seated together in a row.

Input: An array/vector of integers representing individuals in seats, where consecutive integers represent couples.

Output: The minimum number of swaps needed to arrange the couples side by side.

Constraints: The number of individuals is always even, with unique, consecutive IDs for each person in a $2n$ -sized array/vector.

Function FindPartnerPosition(row, partner)

for each seat in row

 if the person in the seat is the partner

 return the seat number

Function ConnectCouples(row)

swaps = 0

for i from 0 to length of row - 1, step by 2

 person1 = row[i]

 partner = find the partner of person1

 if row[i + 1] is not the partner

 partnerPosition = FindPartnerPosition(row, partner)

 swap the person in seat i + 1 with the partner

 increase swap count by 1

return swaps

Proving Efficiency Using Step Counts

```
Function FindPartnerPosition(row, partner)
for each seat in row                                // n
    if the person in the seat is the partner        // 1 comparison per iteration
        return the seat number                    // 1

Function ConnectCouples(row)
swaps = 0                                            // 1
for i from 0 to length of row - 1, step by 2      // n/2
    person1 = row[i]                                // 1
    partner = find the partner of person1          // 1

    if row[i + 1] is not the partner                // 1
        swap(row[i + 1], row[partnerPosition])    // n
        swap the person in seat i + 1 with the partner // 1
        increase swap count by 1                  // 1
return swaps                                        // 1
```

FindPartnerPosition Function

$O(n + 1 + 1) = \mathbf{O(n)}$ for worst case time complexity.

Function ConnectCouples(row)

First we'll handle the loop. The number of iterations will be $n/2$. Taking the rest of the steps inside the loop: $O(1 + 1 + 1 + n + 1 + 1) = O(n + 5)$.

Since the number of iterations will be iterations is $n/2$, we'll multiply that with $(n + 5)$.

$$T(n) = n/2 * (n + 5) = (n^2)/2 + 5n/2.$$

We'll add the 1 for initializing swaps and the 1 for returning swaps.

$$T(n) = (n^2)/2 + 5n/2 + 2$$

This will make this function $\mathbf{O(n^2)}$ for worst case time complexity.

Overall:

$$T(n) = O(n) + O(n^2)$$

$\mathbf{O(n^2)}$ would be the overall worst case time complexity.