IIIT-Bangalore
CS 825: Graph Theory
Term II 2020-21: Projects

This document contains a list of projects to be done by the students enrolled for the CS 825 course on Graph Theory. Each project will be done by a team of three students, students can opt for their choice of project and team members. TAs will be co-ordinating the project and team choice, it will be allotted on a first-come first-serve basis, with no option to change it later.

The projects are categorized in to three groups: essays which are either surveys or on miscellaneous problems, presentations and implementation. Essays and implementation projects will be evaluated offline by the instructors and TAs, presentations will be scheduled and announced in LMS. Copying from books or internet is strictly not allowed, students can cite the references (websites, books and research papers) they used and write the details in their own words. If a project/code was found to have been copied, zero marks will be allotted for the same.

1 Survey on Graph Classes

For each graph class given below, give a survey of results known about the class. The survey should contain at least the following details:

- The definition of the graph class, along with equivalent definitions, if any.
- Any distinct characteristics of the class
- Computational status of the following problems on the graph class - Recognition of the graph class, Vertex Cover, Independent set, Coloring Problem, Dominating Set problem
- State of the art literature review of at least two problems given above in detail - a short description of the best known polynomial time/approximation/exponential algorithm or results on lower bounds.

1. Perfect graphs
2. Outerplanar graphs
3. Chordal graphs
4. Comparability graphs
5. Co-comparability graphs
6. Split graphs
7. Threshold graphs
8. Interval graphs
9. No-where dense graphs

10. Bounded Expansion graphs
11. Block graphs
12. Permutation graphs
13. Cographs
14. Claw free graphs
15. AT-free graphs
16. Line graphs of bipartite graphs
17. Distance-hereditary graphs
18. Weakly chordal graphs
19. Strongly chordal graphs

2 Miscellaneous Problems and Problems on Graph Algorithms

This section contains miscellaneous problems and algorithms related to graph theory. For each question, you are required to give a 30 minutes long seminar or submit an essay report (as given in the question), on the concerned theorem and/or algorithm with an illustrative example.

1. A *graphic sequence* is a list of non-negative numbers that is the degree sequence of some simple graph. A simple graph with degree sequence d *realizes* d. Give a detailed presentation of the algorithm by Havel and Hakimi that provides a condition to check if a given sequence of non-negative integers (in non-increasing order) is graphic or not.
2. Erdös and Gallai theorem provides a necessary and sufficient condition for a finite sequence of numbers to be a graphic sequence of a simple graph. Give a presentation on this theorem for simple graphs.
3. Give a presentation on matrix tree theorem that gives a polynomial time algorithm to count spanning trees.
4. Cayley's formula counts the number of trees with n vertices. Write a detailed essay on how to use the matrix tree theorem to prove Cayley's formula and also to compute $\tau(K_{r,s})$.
5. Give a presentation on the Hungarian algorithm, which takes as input a complete, weighted bipartite graph (weights on edges), and gives a maximum weight matching (also a perfect matching). Your presentation should also discuss the problem as the assignment problem and also provide details of the corresponding dual problem.

6. The blossom algorithm is an algorithm to construct maximum matchings on graphs. Read and present the algorithm developed by Edmonds in his paper titled "Paths, trees and flowers". The paper is available here: `https://www.cambridge.org/core/journals/canadian-journal-of-mathematics/article/paths-trees-and-flowers/08B492B72322C4130AE800C0610E0E21`.

7. Give a presentation on the Gale-Shapley algorithm for the stable matching problem. You have to include details regarding an application of this problem in auctions, in Economics.

8. Write an essay on any two applications of Menger's theorem.

9. Write an essay on the various notions of *centrality* of a graph, a notion that is used to identify the most important vertices in a graph with respect to a certain parameter. For example, the simplest notion of centrality is that of *degree centrality*, which is defined as the number of edges incident on a vertex.

10. Given a graph $G = (V, E)$, a *dominating set* $S \subseteq V$ is a set of vertices such that every vertex not in S has a neighbour in S. The *dominating number* is the minimum size of a dominating set of G. Write an essay on domination as an extremal problem, including some elementary results on dominating sets.

11. Give a short presentation on the Weak Perfect Graph Theorem along with a proof of the theorem.

12. Write an essay on algorithms for Planarity Testing. Explain at least one algorithm in detail.

13. Write an essay on solving the min-Coloring problem for Perfect Graphs. Also mention some algorithms that are applicable on sub-classes of perfect graphs. For example, there exists a linear time algorithm for bipartite graphs.

14. Give a short presentation on Planar Separator Theorem.

3 Implementation Projects

Implement the following algorithms using a programming language of your choice.

1. Hungarian Algorithm for Matching.

2. Blossom Algorithm for Matching.

3. Gale Shapley Algorithm.