Notebook Screen Capture

Convolution Integral Simulation

For a continuous-time linear time invariant (LTI)system having impulse response $h(t)$ and input signal $x(t)$, the output, $y(t)$ can be written in terms of a convolution integral:

$$y(t) = \int_{-\infty}^{\infty} x(\lambda)h(t-\lambda)\,d\lambda = \int_{-\infty}^{\infty} h(\lambda)x(t-\lambda)\,d\lambda$$

Special Case

Consider $x(t) = u(t) - u(t-T)$ a rectangular pulse of duration Ts and $h(t) = ae^{-at}u(t)$ an exponential, where $a > 0$. **Note**: The impulse response is of the form of the well known RC lowpass filter if we let $a = 1/RC$.

Writing out and evaluating the convolution integral for the given $x(t)$ and $h(t)$ results in a piecewise solution involving three contiguous support intervals: (Case 1) $t < 0$, (Case 2) $0 \geq t < T$, and (Case 3) $t \geq T$. The integrand is zero for Case 1. Using the second form of the convolution integral, Case 2 evaluates to:

$$y(t) = \int_0^t ae^{-(t-\lambda)}\,d\lambda = -e^{-a\lambda}\Big|_0^t = 1 - e^{-at},\ 0 \leq t < T$$

For Case 3 we have

$$y(t) = \int_{t-T}^t ae^{-(t-\lambda)}\,d\lambda = -e^{-a\lambda}\Big|_{t-T}^t = e^{-a(t-T)}\left[1 - e^{-aT}\right],\ t \geq T$$

In summary:

$$y(t) = \begin{cases} 0, & t < 0 \\ 1 - e^{-at}, & 0 \leq t < T \\ e^{-a(t-T)}\left[1 - e^{-aT}\right], & t \geq T \end{cases}$$

Plot the piecewise solution:

```
# Let T = 1s and a = 5
figure(figsize=(6, 2))
T = 1; a = 5
tt = arange(-1, 3.001, .01)
yt = (1-exp(-a*tt))*(ssd.step(tt)-ssd.step(tt-T))\
      +exp(-a*(tt-T))*(1-exp(-a*T))*ssd.step(tt-T)
plot(tt, yt,'g')
```