THIRD EDITION

Feedback Control of Dynamic Systems

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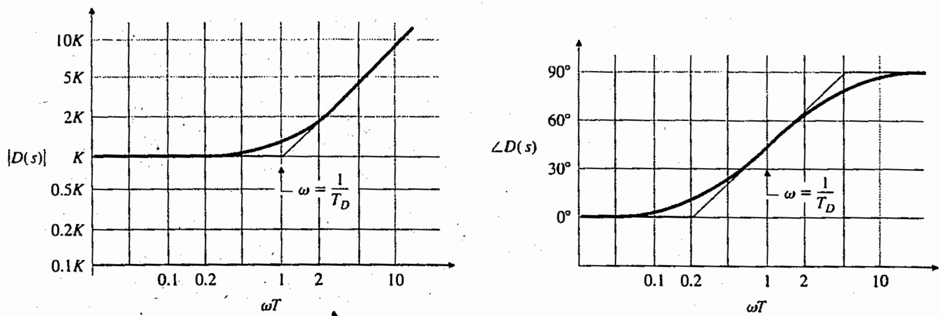
FIGURE 6.50
Frequency response of PD control



6.7.2 Lead Compensation

In order to alleviate the high-frequency amplification of the PD compensation, a first-order pole is added in the denominator at frequencies higher than the breakpoint of the PD compensator. Thus the phase increase (or lead) still occurs, but the amplification at high frequencies is limited. The resulting **lead compensation** has a transfer function of

Lead compensation

$$D(s) = K\,\frac{Ts + 1}{\alpha Ts + 1}, \tag{6.35}$$

where $\alpha < 1$. Figure 6.51 shows the frequency response of this lead compensation. Note that a significant amount of phase lead is still provided, but with much less amplification at high frequencies. A lead compensator is generally used whenever a substantial improvement in damping of the system is required.

The phase contributed by the lead compensation in Eq. (6.35) is given by

$$\phi = \tan^{-1}(T\omega) - \tan^{-1}(\alpha T\omega).$$

It can be shown (see Problem 6.42) that the frequency where the phase is maximum is given by

$$\omega_{max} = \frac{1}{T\sqrt{\alpha}}. \tag{6.36}$$

The maximum phase contribution, that is, the peak of the $\angle D(s)$ curve in Fig.

6.51, corresponds to

$$\sin\phi_{\max} = \frac{1-\alpha}{1+\alpha} \tag{6.37}$$

or

$$\alpha = \frac{1-\sin\phi_{\max}}{1+\sin\phi_{\max}}.$$

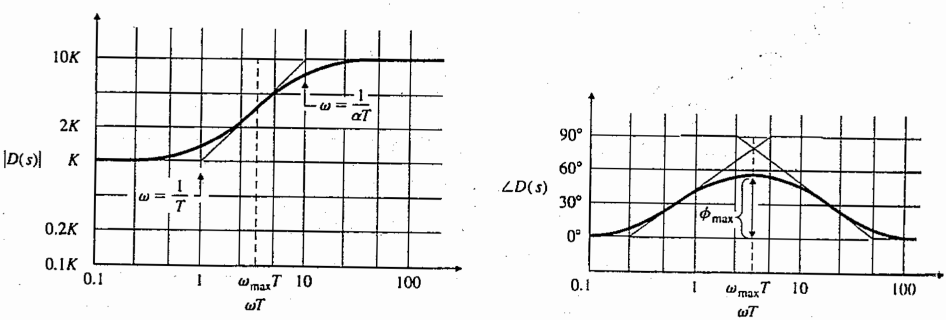
Another way to look at this is the following: The maximum frequency occurs midway between the two break-point frequencies (sometimes called corner frequencies) on a logarithmic scale.

$$\begin{aligned}\log\omega_{\max} &= \log\frac{1/\sqrt{T}}{\sqrt{\alpha T}} \\ &= \log\frac{1}{\sqrt{T}} + \log\frac{1}{\sqrt{\alpha T}} \\ &= \frac{1}{2}\left[\log\left(\frac{1}{T}\right) + \log\left(\frac{1}{\alpha T}\right)\right],\end{aligned} \tag{6.38}$$

as shown in Fig. 6.51. Alternatively, we may state these results in terms of the pole-zero locations. Rewriting $D(s)$ as

$$D(s) = K\frac{(s+z)}{(s+p)}, \tag{6.39}$$

FIGURE 6.51
Lead-compensation frequency response with $1/\alpha = 10$



we get

$$\omega_{max} = \sqrt{|z||p|} \tag{6.40}$$

and

$$\log \omega_{max} = \tfrac{1}{2}(\log|z| + \log|p|). \tag{6.41}$$

These results agree with the previous ones if we let $z = -1/T$ and $p = -1/\alpha T$ in Eqs. (6.36) and (6.38).

For example, a lead compensator with a zero at $s = -2$ ($T = 0.5$) and a pole at $s = -10$ ($\alpha T = 0.1$), thus $\alpha = \frac{1}{5}$ would yield the maximum phase lead at

$$\omega_{max} = \sqrt{2 \cdot 10} = 4.47 \text{ rad/sec.}$$

Lead ratio $= \dfrac{1}{\alpha}$

The amount of phase lead at the midpoint depends only on α in Eq. (6.37) and is plotted in Fig. 6.52. For $\alpha = \frac{1}{5}$, Fig. 6.52 shows that $\phi_{max} = 40°$. Note from the figure that we could increase the phase lead up to 90° using higher values of the **lead ratio** $1/\alpha$; however, Fig. 6.51 shows that increasing values of $1/\alpha$ also produces higher amplifications at higher frequencies. Thus our task is to select a value of $1/\alpha$ that is a good compromise between an acceptable phase margin and an acceptable noise sensitivity at high frequencies. Usually a single lead compensation can contribute a maximum of 60° to the phase. If a greater phase lead is needed, then a double lead compensation would be required, where

$$D(s) = K\left(\frac{Ts + 1}{\alpha Ts + 1}\right)^2.$$

Even if a system had negligible amounts of noise present and the pure derivative compensation of Eq. (6.34) were acceptable, a continuous compensation would look more like Eq. (6.35) than Eq. (6.34) because of the impossibility of building a pure differentiator. No physical system—mechanical

FIGURE 6.52
Maximum phase increase for lead compensation