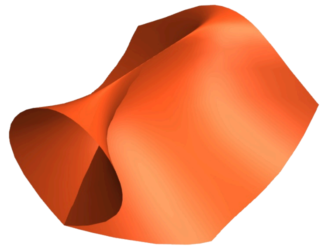
A τ/μ-TYPE INEQUALITY FOR FRONTAL MAP GERMS

A map germ $f:(\mathbb{C}^n,S)\to(\mathbb{C}^{n+1},0)$ is frontal if there is a $\nu:(\mathbb{C}^n,S)\to T^*\mathbb{C}^{n+1}$ along f such that $\nu(df\circ\xi)=0$ for all $\xi:(\mathbb{C}^n,S)\to T\mathbb{C}^n$ and $0\notin\nu(S)$.



$f(x,y)=(x,y^2,x^3y^3+y^5)$

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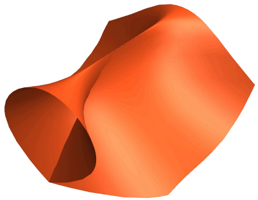

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A frontal f is stable if for any deformation f_u with (u,f_u) frontal we can find diffeomorphisms Ψ_u,ϕ_u such that $f_u=\Psi_u\circ f\circ\phi_u^{-1}$. We also have a Tjurina-type invariant called the $\mathscr{F}_e$-codimension.

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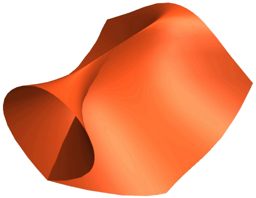

$f(x,y) = (x, y^2, x^3y^3 + y^5)$

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If $M_1, \dots, M_{n-1}$ are the minors of df and $\langle M_1, \dots, M_n \rangle = \langle \beta \rangle$, then f is a wave front if $\tilde{f}(x) = \left(f(x), \frac{M_1}{\beta}(x), \dots, \frac{M_n}{\beta}(x) \right)$ is an immersion.

A τ/μ-TYPE INEQUALITY FOR FRONTAL MAP GERMS

Let f_u be a stable deformation of f. If f has corank 1 or it is a wave front(*), then $\mathrm{Im}\, f_u \simeq \bigvee_{j=1}^{m} S^n$ for $u \neq 0$. We call $\mu_{\mathcal{F}}(f) = m$ the frontal Milnor number of f.

(*) With $(2n - rk(f), n+1)$ nice dimensions!

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Theorem:
If f has finite $\mathcal{F}$-codimension, then $\mu_{\mathcal{F}}(f) = e\left(\mathfrak{m}_r, \dfrac{J(G)}{J_y(G)}\right)$ $(u \in \mathbb{C}^r)$. Moreover if f is a wave front,

$$\mu_{\mathcal{F}}(f) = \dim \frac{J(G)}{J_y(G)} \otimes \mathfrak{m}_r \implies \mu_{\mathcal{F}}(f) \geq \operatorname{codim}_{\mathcal{F}_e}(f),$$

(Equality if f is quasihomogeneous)

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Formulas for $\mu_{\mathscr{F}}(f)$:

- ⋆ f wave front: If $\operatorname{Im} f$ is the discriminant of $H:(\mathbb{C}^N,0)\to(\mathbb{C}^{n+1},0)$,

$$\mu_{\mathscr{F}}(f)=\mu_\Delta(H)$$

Discriminant Milnor number

- ⋆ f plane curve: If κ is the number of cusps,

$$\mu_{\mathscr{F}}(f)-\mu_{I}(f)=\kappa$$

- ⋆ f corank 1 surface: If $D_+(f)$ is the source transverse double point curve,

$$\mu_{\mathscr{F}}(f)=\frac{1}{2}\Big(\mu(D_+(f),0)+S+\overline{W}-1\Big)-K-2T$$

¡MUCHAS GRACIAS!

¿ALGUNA PREGUNTA?